This paper develops a model of dynamic pricing with menu cost for a monopolistic retail store. By examining the prices of two brands of curry paste, the model shows that frequent price changes appear to be the optimal price policy. The key reason behind this strategy is that customers differ in their willingness to pay, depending on whether they purchase the product for immediate consumption or to add to their inventory at home.

The empirical results strongly support the model’s predictions that: (1) stores tend to lower prices when (a) the share of customers still holding inventory is low, and when (b) the expected number of customers is high; and (2) demand is negatively dependent on the duration of the lower price and positively dependent on the duration of the higher, regular price. Unlike in models that posit a negative dependence of current demand on past prices, the findings support the theory that inventory accumulation (when the price is low) and decumulation (when the price is high) drive short-run fluctuations in demand.

**Introduction**

Strong and frequent price changes suggest that retailers are attempting to dynamically price discriminate to increase their profits. Figure 1 plots the daily price of House Vermont Curry, a national brand of curry paste sold at one of the stores of a Japanese supermarket chain. The figure, which traces daily prices from 1 July 1991 until 31 August 1996, shows extremely frequent price changes: prices last on average only 5.3 days, with single-day prices comprising more than 40 per cent of the price durations recorded. Another visible characteristic is the presence of two focal prices – 253 yen and 198 yen – which comprise 77 per cent of the observations, even though there are 64 other prices recorded in the data (Figure 2 gives the frequency distribution).

Not surprisingly, frequent price changes induce customers to wait for the price to be marked down to buy the product: among the seven stores that belong to that particular supermarket chain, the price was below 198 yen for 1,862 days, or 15 per cent of the 12,032 days observed, during which time the stores sold 130,394 units, or 45.6 per cent, of the total sales of 286,812.
Figure 1 Daily price of house brand at store 1 (price in yen)

Figure 2 Frequency distribution of daily price (per cent)
What is the underlying logic behind this pricing pattern? Although it is clear that the customers do recognise the pattern and adjust their purchasing accordingly, it is not at all obvious why these stores employ this strategy.

In this paper we develop a model of store sales by extending Varian’s pioneering work (Varian 1980). We then empirically estimate the optimal pricing model, with an emphasis on inventory accumulation by consumers. We join the small but rapidly growing number of researchers who have used scanner data to investigate the underlying mechanism of price adjustments of individual brands or at individual stores. We surveyed the prices of curry paste sold at 18 different supermarkets stores in Japan over the five-year period. Intertemporal pricing patterns, such as in Figure 1, are quite common, although the stores in this study exhibited an exceptionally high frequency of price changes. The most obvious and natural starting point of the analysis is to take the pattern as a reflection of fluctuations in demand. For example, Slade (1998) combines a model of menu costs with a model of customer capital (goodwill), which is the driving factor behind price changes in the model.

Following Varian (1980) and Sobel (1984), a variety of random-pricing models have been developed showing equilibria supported by mixed strategies in pricing games. These models show that current demand is positively dependent on past prices. This is because it is necessary to maintain a high price for an extended period to build up a large amount of latent demand from bargain hunters. Pesendorfer (1998) exploits this idea and develops a simple model of store sales to explain the pricing of ketchup sold at a large supermarket. Each price cycle ends with a sale, which is short lived and generates a large amount of purchases. Another common implication of these models is that the length of the price cycle depends on how quickly bargain hunters are drawn to the store in relation to the cost of delaying the sale (lost business). This suggests that exogenous changes in demand predict the occurrence of a sale. Warren and Barsky (1995) find evidence that a large predicted increase in the number of shoppers significantly raises the probability that the store will hold a sale.

Price cycles are endogenous in these models, even in the absence of exogenous shocks. Large fluctuations in demand and in the number of price markdowns must therefore be the outcome of intertemporal price discrimination.

This paper builds upon the intuitively appealing idea that retail stores use a high-low pricing strategy to dynamically price discriminate. The driving force of our model is inventory accumulation by customers. Customers differ in the maximum price they are willing to pay, depending on whether they purchase the product for immediate consumption or to add to their
inventory. This is the source of price discrimination that retail stores exploit. Consumers try to concentrate their purchasing to when the price is marked down. Hence trading is characterised by long periods of relatively stable and low purchasing, punctuated by increased buying at a lower price for a short period. The fact that our data shows a positive and significant correlation between current demand and the past price provides support for this theory. The evidence in this paper also suggests that when the store charges a high price, the number of customers waiting for the sale price steadily increases, whereas the stock of these customers is rapidly depleted when a lower price is set.5

We develop a model that incorporates all these features of dynamic price discrimination.6 A key assumption is that the flow of customers is exogenous not only to the stores but also to the stock of customers. Because the stores in our sample sell a large variety of grocery items, it is unlikely that customers time their shopping solely to take advantage of the price of any particular product. This implies that their pent-up demand for inventory cannot be satisfied during a single sale. This crucial property gives rise to shorter but still non-negligible price durations even at prices below the regular price.

A second important feature of the model is the source of heterogeneity of customers. We assume that each customer is able to store the product at home. The heterogeneity arises because the demand for immediate consumption and for inventory differs. The amount of inventory that can be held is restricted by limits on storage space in Japanese households and because food has a limited shelf life.7 These restrictions have a very important effect on demand, possibly even more important than the amount of time the price is discounted.8

First we explain why retail stores might employ a dynamic pricing policy. The analysis starts with a simulation of a static price policy, which shows that a retail store chooses between two alternatives: a static high price, at which customers purchase only for immediate consumption; and a static low price, at which customers are marginally induced also to buy the product to store. This analysis provides results that serve as the benchmark for the subsequent analysis of dynamic price discrimination. We look at the optimal price policy both with and without menu cost (the cost of changing prices). Low variations in the volume of customers lead the store to time the sale to heavy shopping periods. In considering the impact of a periodic increase in the number of customers, we indeed find that stores time the low-price period to heavy shopping days. The results contradict those of other models that predict that current demand will be negatively related to the past price. We then estimate non-linear
regressions and obtain the two key parameters of the model. The result is used to construct
the key unobservable variable, the ratio of customers without inventory. The estimation of an
ordered probit model shows this variable to be significant in explaining a price change.
Finally, we estimate a cross-section of regressions using a few important endogenous
variables on three key parameters of the model.

A model of dynamic price discrimination

We consider a local retail store that is a monopolist and caters to a stable population of regular
customers. We presume there is no rivalry or price competition from other retailers,\(^9\) and that
the volume of customers is constant and normalized to unity. The store sells a large variety
of grocery items and has a regular base of customers who purchase a variety of goods. The
model focuses on one particular brand of a good, holding all other aspects of the store’s
activities as given. We assume that each customer visits the store with probability \(s\) for any
given day. In other words, we presume that other potential customers decide the cost of
shopping is too high on that particular day. Thus \(s\) is also the number of shoppers each day.
These assumptions on consumption and shopping patterns are convenient shortcuts to
characterise an environment in which a firm sets the price of single product.

Customers are far-sighted and may purchase the good today for future consumption if
the price is right. We assume a constant cost \(\varepsilon\) of holding one unit of inventory per unit of
time. For simplicity, we assume that the cost of storing more than one unit is prohibitively
large so that each customer has at most one unit of the good stored.\(^{10}\)

The customers’ preference for the good is random: on each day, with probability \(c\), the
consumption of one unit of the good generates a utility (in money terms) of \(u\). Otherwise the
customer gets no utility from consumption. At the beginning of each day, each customer
observes his or her own preference and shopping cost, and then makes his or her shopping
decision.\(^{11}\)

The store faces a constant marginal cost, \(\omega\), per unit. Denote by \(\sigma\) the (menu) cost of
a price change. Another simplifying assumption is a zero time discount, which can be justified
in this framework because the relevant time horizon for storage and price adjustment is likely
to be short relative to the length of time the price is discounted.\(^{12}\)
**Static pricing**

We start the analysis with a simple benchmark case of a store that is exogenously constrained to offer a time-invariant price. Since the store cannot distinguish between different types of customers, the only choice is a constant price for the good.\(^\text{13}\)

The pricing problem is further simplified by noting that in effect only two types of potential customers exist: those who are buying to add to their inventory at home and those who are purchasing for immediate consumption. If the store decides not to cater to the first type of customer, the optimal price is obviously the maximum that it can charge without losing the second type of customer. Hence we have:

\[
\pi_H = u
\]

In this case, the demand for the good is given by:

\[
D_H = sc
\]

because only those who visit the store and plan to consume the good immediately will buy the product. Therefore the profit per day under this policy is given by:

\[
\Pi_H = (u - \omega)sc
\]

On the other hand, suppose that the store lowers the price enough to induce the first type of customer to buy. Denote by \(\pi\) the share of customers without one unit of inventory. On each day, \(s\) customers visit the shop, among which \(sc\pi\) of them purchase two one unit for immediate consumption and another to store, \(sc(1 - \pi)\) purchase one unit for consumption, \(s(1 - c)\pi\) also purchase one unit for inventory, and \(s(1 - c)(1 - \pi)\) do not purchase any. Summing these, we get:

\[
D_L = s(c + \pi)
\]

Denote by \(p_L\) the optimal price in this case (yet to be determined). We have:
By comparing equations (1) and (2), we obtain:

\[
\Pi_H < \Pi_L \iff \frac{p_L - \omega}{\omega} < \frac{c}{u - \omega} > c + \pi
\]  \hspace{1cm} (3)

If \( p = p_L \) forever, the far-sighted customer without inventory will purchase the good to store at home whenever visiting the shop; whereas the inventory will be consumed with probability \( c \), which will not be replenished if the consumer does not shop for the day, which occurs with probability \( (1 - s) \). Thus we get:

\[
\Delta \pi = \pi_t - \pi_{t-1} = c(1 - s)(1 - \pi_{t-1}) - s\pi_{t-1}
\]

Hence the steady state level of \( \pi \) is given by:

\[
\Pi^* = \frac{c(1 - s)}{c(1 - s) + s}
\]  \hspace{1cm} (4)

Therefore condition (3) is rewritten as:

\[
\Pi_H < \Pi_L \iff \frac{p_L - \omega}{\omega} < \frac{c(1 - s) + s}{1 + c(1 - s)}
\]  \hspace{1cm} (3')

Let us now consider the determination of \( p_L \). Denote by \( V_i \) \((i = 0, 1)\) the expected present value of a net utility stream for a customer with \( i (= 0, 1) \) units of inventory of the good at home. We have:

\[
\begin{align*}
    rV_0 &= sc(\bar{u} + \bar{v}) + s(1 - c)\bar{v} \\
    rV_1 &= -\varepsilon + sc\bar{u} + c(1 - s)(\bar{u} - \bar{v}) \\
    \bar{u} &\equiv u - p_L \\
    \bar{v} &\equiv W_L - p_L, \quad W_L \equiv V_1 - V_0
\end{align*}
\]
Solving for \( \bar{v} \), we get:

\[
\bar{v} = \frac{c(1-s)\mu - \epsilon - rp_L}{r + s + c(1-s)}
\]

Setting the time discount rate at \( r = 0 \), we get:

\[
\bar{v} = \frac{c(1-s)\mu - \epsilon}{s + c(1-s)}
\]

The customer without inventory is willing to purchase to store the good if, and only if, doing so yields a net gain in expected utility. Hence the maximum that the store can charge is given by:

\[
c(1-s)\mu - \epsilon = 0
\]

or,

\[
p_L = W_L \equiv u - \frac{\epsilon}{c(1-s)}
\]

Therefore, the previous condition (3') can be written as:

\[
\Pi_H \lesssim \Pi_L \lesssim R \lesssim R_s, \quad R_s = \frac{(1-s)}{1+c(1-s)}, \quad R = \frac{\epsilon}{(u-\omega)c(1-s)}
\]

Note that \( R \) is the ratio of the cost of holding inventory \( \left[ \frac{\epsilon}{c(1-s)} \right] \) relative to the gains from trade, \( (u-\omega) \), because \( \frac{1}{c(1-s)} \) is the average duration of inventory. The equation above thus simply states that the static high (low) price is chosen if the ratio is larger (smaller) than the
threshold, $\frac{1-s}{1+c(1-s)}$. A similar condition is obtained below for the interior solution for high-low pricing.

**Optimality of a high-low pricing policy: intuition**

In a static setting, the store cannot effectively price discriminate unless it knows the type of each customer. The only choice left is whether or not to price the good low enough to induce customers to buy to add to their inventory. Equation (3) suggests that this strategy is chosen if the share of customers without inventory is large. We also know that customers will not purchase the good to store if the price is above the reservation level, $W$, which is the maximum price they are willing to pay to store the good.

Once we allow the firm to price discriminate by changing the price from time to time, the firm can exploit the property that lower (higher) prices increase profits when the average inventory held by customers is low (high). We demonstrate through several steps that a type of high-low pricing policy emerges as the optimal policy of intertemporal price discrimination with menu cost. We describe the intuition behind this idea before formally proving the optimality of such a policy.

Suppose the store employs a high-low pricing policy in the following manner: a high price is posted for $[1, \tau_1] = T_H$, and a low price is posted for $[\tau_1 + 1, \tau_1 + \tau_2] = T_L$. Then the process is repeated. When the price is high, the customer will not buy the good to store up since zero net utility is received even if he or she immediately consumes the good upon purchase. This implies that when the price is high, inventories will be depleted gradually and the share of customers without inventory will increase. The low price must be low enough that customers are marginally induced to buy the good to store. The crucial question is whether or not such a reservation price is constant. Intuition suggests that such a price will not be constant if the store employs a high-low policy: toward the end of the low-price period, customers wishing to add to their inventory are willing to pay more for the good than they are at the beginning of the period. This is because customers get a strictly positive net utility from buying and consuming the good at the low price. The fact that such a valuable opportunity will soon end acts as an inducement to buy during this period.

These arguments suggest that the customer's willingness to pay for the inventory should be declining during the high-price period in anticipation of the low-price period ahead,
although such a shadow price is immaterial during the high-price period. A typical pricing pattern therefore starts (arbitrarily) with a high-price period during which no inventory is accumulated and existing inventory is gradually depleted by customers. Demand during the high-price period is confined to those without inventory who happen to shop on the day and also happen to get $u$ from consumption. At $\tau_1$, the price is at its lowest level. The price then gradually rises during the low-price period, mirroring the rising reservation price as inventory increases. The cycle ends with a sudden jump at $\tau_1 + \tau_2$ back to the high price. Then the cycle is repeated.

Now consider the role of menu cost. It is clear that if the menu cost is high, the cyclical pricing policy described above will not be optimal. Instead, either a permanently high or permanently low price will be chosen. As we gradually lower the menu cost, the first pattern that will emerge is one with a minimum number of price changes: that is, a simple high-low pricing policy in which the 'low' price is kept constant until the price is returned to the high level. As the menu cost falls, the number of price adjustments during the low-price period increases, mirroring more and more the price path without menu cost. As we will see shortly, however, if there is no menu cost, the optimal policy is infinitely frequent alternations between high and low prices, as this policy maximises the sustainable size of $\pi$ consistent with low pricing.

**Optimal pricing policy: formal analysis**

We now characterise the optimal high-low pricing policy described above by using a dynamic pricing model. We then demonstrate that the store optimally chooses such a policy given the optimal response of its customers. In a separate statistical appendix (available from the authors on request), we prove that the optimal policy starting from an arbitrary level of $\pi$ indeed converges to the optimal stationary policy analysed in this section.

**Customers’ optimal policy**

Each customer either buys to consume or to add to inventory. Since the price never exceeds the reservation utility level, $u$, customers always purchase the good for immediate consumption.

We denote by $V_i(t = 0, 1; t_1, \cdots, t_1 + t_2)$ the asset value of the consumption stream (net of purchasing and inventory holding costs) for customers with $i$ units of inventory in period $t$ during the cycle. We have:
\begin{align*}
V_t^1 &= -\varepsilon + c(1-s)\max[0, u-W_{t+1}] + cs \max[u-p_{t+1}, u-W_{t+1}, 0] + V_{t+1}^1 \\
V_t^0 &= s(1-c)\max[0, W_{t+1} - p_{t+1}] + cs \max[u-p_{t+1}, 0] + \max[W_{t+1} - p_{t+1}, 0] + V_{t+1}^0 \\
W_t &= V_t^1 - V_t^0
\end{align*}

where \( t+1 = 1, \text{if } t = t_1 + t_2 \). These equations can be understood as follows. First, the consumers with one unit of inventory already at hand will pay \( \varepsilon \) per period for holding that inventory. The consumer will consume one unit of inventory with probability \( c \). If the consumer does not shop that day, the inventory will be depleted and there will be a capital loss of \( W_t \). By construction, postponing consumption is never optimal and \( u - W_t \) is non-negative. If the consumer does shop that day, he or she can replenish the inventory (obtaining \( u - p_t \)) or deplete the inventory (obtaining \( u - W_t \)). Those without inventory consume the good only if they shop on the day, (obtaining \( u - p_t \)). Whether or not the consumer purchases one more unit for inventory depends upon the sign of \( W_t - p_t \). Denote by \( T_H \) (\( T_L \)) the period of the high (low) price. Recall that, by assumption:

\begin{align*}
p_t &= u \quad \text{for } t \in T_H \\
p_t &\leq W_t \quad \text{for } t \in T_L
\end{align*}

The latter property is obtained because \( W_t \) is the maximum price that consumers are willing to pay to store the good. We can use these pricing policies to rewrite equations (6) and (7). The results are:

\begin{align*}
V_t^1 &= -\varepsilon + c(u-W_{t+1}) + V_{t+1}^1 \quad \text{for } t \in T_H \\
V_t^1 &= -\varepsilon + c(1-s)(u-W_{t+1}) + cs(u-p_{t+1}) + V_{t+1}^1 \quad \text{for } t \in T_L \\
V_t^0 &= V_{t+1}^0 \quad \text{for } t \in T_H \\
V_t^0 &= s(1-c)(W_{t+1} - p_{t+1}) + cs(u-p_{t+1} + W_{t+1} - p_{t+1}) + V_{t+1}^0 \quad \text{for } t \in T_L
\end{align*}

These two pairs of equations summarise the intertemporal pricing constraint for the store, assuming that it is following a high-low pricing policy.
Optimal pricing policy without menu cost

We now consider the store’s optimal pricing policy. The analysis is limited to the choice of either a constant or a high-low pricing policy. The question of which of the two the store is likely to choose will be discussed later. For the time being, we assume that the firm pursues a variant of a high-low pricing policy. We assume that the cycle begins with the high-price period, followed by the low-price period.

As it turns out, the analysis without menu cost is not only simpler but also allows the nature of the optimal pricing policy to be seen far more clearly. Without menu cost, the firm can costlessly change prices provided that during the low-price periods, \( p_t \leq W_t \) is satisfied. Obviously the store sets the price such that:

\[
p_t = W_t \quad \text{for} \quad t \in T_L
\]

Substituting the equality for \( p_t \) in (6′) and (7′), and then subtracting (7′) from (6′) for both sides, we obtain:

\[
W_{t+1} = (1-c)^{-1} + \left\{W_t + \epsilon - cu\right\} \quad \text{for} \quad t \in T_H
\]

\[
W_{t+1} = \left[1 - c(1-s)\right]^{-1} \left\{W_t + \epsilon - c(1-s)u\right\} \quad \text{for} \quad t \in T_L
\]

The corresponding change in \( \pi_t \) for each period is given by:

\[
\pi_t = c + (1-c)\pi_{t-1} \quad \text{for} \quad t \in T_H
\]

\[
\pi_t = c(1-s) + \left[1 - c(1-s)\right] \pi_{t-1} \quad \text{for} \quad t \in T_L
\]

Denote by \( \bar{W} \) and \( \bar{\pi} \), respectively, the initial level of each variable at \( t = 0 \). We can solve the difference equations to obtain:

\[
W_t = W_H - \left(\frac{1}{1-c}\right)^t (W_H - \bar{W}) \quad \text{for} \quad t \in T_H
\]

\[
W_t = W_L + \eta^{-t} \left\{W_H - W_L - \left(\frac{1}{1-c}\right)^t (W_H - \bar{W})\right\} \quad \text{for} \quad t \in T_L
\]
As we limit our analysis to the stationary price policy, the price cycle must repeat itself after \((t_1 + t_2)\) periods. Thus we require:

\[
\pi_{t_1+t_2} = \pi \\
W_{t_1+t_2} = \bar{W}
\]

Using these terminal conditions, we obtain:

\[
\pi = 1 - (1 - AB)^{-1}\left(1 - B\right)\left(1 - \pi^*\right) \\
A \equiv (1 - c)^{t_1}, \; B \equiv \left\{1 - c(1 - s) - s\right\}^{t_2}
\]

\[
\bar{W} = (DE - 1)^{-1}\left\{E'(D - 1)W_H + (E - 1)W_L\right\} \\
D \equiv \left\{\frac{1}{1 - c}\right\}^{t_1} = A^{-1}, \; E \equiv \eta^{t_2} = \left\{\frac{1}{1 - c(1 - s)}\right\}^{t_2}
\]

Using these expressions, the equations in (10) are rewritten as follows:

\[
W_t = W_H - W_\Delta (DE - 1)^{-1}(E - 1)(1 + c)^{t_1} \text{ for } t \in T_H \\
W_t = W_L + \eta^{t_2}W_\Delta (DE - 1)^{-1}(D - 1) \text{ for } t \in T_L \\
W_\Delta = W_H - W_L \equiv \frac{s\epsilon}{c(1 - s)} > 0
\]
Note that \( W_t \) decreases during the high-price period in preparation for the expected price decline in the low-price period. By the same token, \( W_t \) increases during the low-price period, as customers anticipate an increase back to the high price. The time path of \( \pi_t \) is given by:

\[
\begin{align*}
\pi_t &= 1 - (1 - c)(1 - AB)^{-1}(1 - B)(1 - \pi^*) \quad \text{for } t \in T_H \\
\pi_t &= \pi^* + \left[ 1 - c(1 - s) - s \right]^{-1} (1 - AB)^{-1}(1 - A)(1 - \pi^*) \quad \text{for } t \in T_L
\end{align*}
\]

(15)

We can now compute the store’s net profit for a representative cycle. The total volume of sales is given by:

\[
Q_t = sc \pi_{t-1} \quad \text{for } t \in T_H \\
Q_t = s(c + \pi_{t-1}) \quad \text{for } t \in T_L
\]

(16)

We can use these equations to obtain the net profit for a representative cycle as follows:

\[
M = (u - \omega) sc \sum_{t=1}^{t} \pi_{t-1} + s \sum_{j=1}^{t} (W_{t+j} - \omega)(c + \pi_{t+j-1})
\]

(17)

We can substitute equations (14) and (15) for \( W_t \) and \( \pi_t \) in equation (17). After some computations and rearrangement of terms, we obtain:

\[
\frac{M}{s} = (u - \omega)ct_1 + (W_L - \omega)(c + \pi^*)t_2 \\
+ \left[ (W_L - \omega) - \{c(1 - s) + s\}(u - \omega) \right] \frac{(1 - \pi^*)}{(1 - s) + s} (1 - A)(1 - B) \\
+ \frac{W_{\Delta}(D - 1)(E - 1)(c + \pi^*)}{(DE - 1)(1 - s)} + \frac{W_{\Delta}(D - 1)(1 - \pi^*)(1 - A)(1 - F)}{(DE - 1)(1 - AB)s}
\]

(17')

\[
F \equiv BE = \left[ \frac{(1 - c(1 - s) - s)}{1 - c(1 - s)} \right]^{\gamma_s} < 1
\]
The optimal policy maximises net profit in each period:

\[
\left\{ t_1^*, t_2^* \right\} = \arg \max \left[ \frac{M}{t_1 + t_2} \right]
\]

(18)

As it turns out, without menu cost, the optimal policy is one of infinitely frequent price changes that minimise the deviation of \( \pi \) around a steady state. On the other hand, the ratio of \( t_1^* \) to \( t_2^* \) converges to a positive finite value. To establish the claim, we employ the following lemmas.

**Lemma 1.** Consider a function:

\[
\psi(x; a, b, \alpha, \beta) = \frac{\left(1-a^\alpha\right)\left(1-b^\beta\right)}{x\left(1-a^\alpha b^\beta\right)} , \quad 0 < a, b < 1, \quad \alpha, \beta > 0
\]

\( \psi(x) \) is monotonically decreasing in \( x \) in \( (0, \infty) \) and

\[
\psi_0(a, b, \alpha, \beta) = \lim_{x \to 0} \psi(x) = \frac{\alpha \beta \log(a) \log(b)}{\left(\alpha \log(a) + \beta \log(b)\right)}
\]

**Proof.** The result is immediate by using L'Hopital's rule twice: that is, by differentiating the denominator and the numerator twice and evaluating them at \( x = 0 \).

**Lemma 2.** Consider a function:

\[
\phi(x; a, b, \alpha, \beta) = \frac{\left(a^\alpha - 1\right)\left(b^\beta - 1\right)}{x\left(a^\alpha b^\beta - 1\right)} , \quad a, b > 1, \quad \alpha, \beta > 0
\]

\( \phi(x) \) is monotonically decreasing in \( x \) in \( (0, x) \) and

\[
\phi_0(a, b, \alpha, \beta) = \lim_{x \to 0} \phi(x) = \frac{\alpha \beta \log(a) \log(b)}{\left(\alpha \log(a) + \beta \log(b)\right)}
\]

**Proof.** Apply the same argument as in lemma 1.
Lemma 3. Consider a function:

\[
\xi(x; a, b, d, \varepsilon, \alpha, \beta) \equiv \frac{(1-a^\alpha) (d^\alpha b^\varepsilon - 1) \left[ (b^\varepsilon)^\beta \right]}{1-a^\alpha b^\varepsilon d^\alpha \beta x - 1} \quad x > 0 \quad a, b, d, \varepsilon, \alpha, \beta > 0, \quad b^\varepsilon < 1
\]

\(\xi(x)\) is monotonically decreasing in \(x\) in \((0, x]\) and

\[
\xi_0(a, b, d, \varepsilon, \alpha, \beta) = \lim_{x \to 0^+} \xi(x) = \frac{\alpha^2 \beta \log(a) \log(d) \log(b) \log(\varepsilon)}{-\alpha \log(a) + \beta \log(b) \log(\alpha) + \beta \log(\varepsilon)}
\]

Proof. Apply L'Hopital's rule thrice.

We now rewrite the maximand (17').

\[
\tilde{M} \equiv \frac{M}{sT} = (u - \omega) c \theta + (W_L - \omega) (c + \pi^*) (1 - \theta)
\]

\[
+ \frac{[(W_L - \omega) - c(l - s)(u - \omega) (l - \pi^*)]}{c(l - s) + s} \times \psi [T; 1 - c, 1 - c(l - s) - s, \theta, 1 - \theta]
\]

\[
+ \frac{W_A (c + \pi^*)}{c(l - s)} \times \phi [T; (1 - c) (1 - c(l - s)) \theta, 1 - \theta]
\]

\[
+ \frac{W_A (1 - \pi^*)}{s} \times \xi [T; 1 - c, 1 - c(l - s) - s, \frac{1}{1 - c} \frac{1}{1 - c(l - s)} \theta, 1 - \theta]
\]

where the following new variables are introduced:

\[
T \equiv t_1 + t_2
\]

\[
\theta \equiv \frac{t_1}{T}
\]
Using the first three lemmas, we obtain the following result.

**Lemma 4.** The optimal policy is obtained by setting \( T \to 0 \), and choosing the \( \theta \) that maximises:

\[
\tilde{M}(\theta) = (\mu - \omega)k\theta + (W_L - \omega)(c + \pi^*) (1 - \theta) + \frac{[W_L - \omega] - \{c(1-s) + s\} (\mu - \omega) (1 - \pi^* )}{c(1-s)} \\
\times \psi_0 [1 - c, 1 - c(1-s) - s, \theta, 1 - \theta] + \frac{W_A (c + \pi^*)}{c(1-s)} \phi_0 \left[ \frac{1}{1-c}, \frac{1}{1-c(1-s)}, \theta, 1 - \theta \right] \\
+ \frac{W_A (1 - \pi^*)}{s} \xi_0 \left[ 1 - c, 1 - c(1-s) - s, \frac{1}{1-c}, \frac{1}{1-c(1-s)}, \theta, 1 - \theta \right] 
\]

**Proof.** The proof is immediate from lemmas 1–3, since the first two terms of the maximand are independent from \( T \), whereas the remaining three terms are maximised by setting \( T \to 0 \).

Having converted the maximisation problem into the choice of \( \theta \), we next establish the following:

**Lemma 5.** The optimal policy entails the following choices:

- A static high-price policy, \( \theta^* = 1 \);
- A dynamic high-low policy, \( 0 < \theta^* < 1 \); or
- A static low-price policy, \( \theta^* = 0 \)

The value function \( \tilde{M} \) is applicable to all of these cases. In particular, we have:

\[
\tilde{M}(1) = (\mu - \omega)k \\
\tilde{M}(0) = (W_L - \omega)(c + \pi^*)
\]

Moreover, if the parameters \((c, s)\) are sufficiently small positive numbers, the following conditions characterise the partitions of the parameter space into each of the three types of optimal policy.
Proof. See the appendix available from the authors on request.

The conditions in equation (20) for an interior solution can be interpreted as follows. As shown above, the ratio \( R \) signifies the relative importance of the normalised inventory cost to the size of the surplus. Lemma 5 states that this ratio must lie within the interval given above in order for the store to choose the high-low pricing policy. If the ratio is too small, the store optimally sets the low price permanently, whereas if the ratio exceeds the upper limit, the store always chooses the high price. The upper and lower thresholds in equation (20) can be compared to the threshold for the static low- and high-price policies above. We have:

\[
R_L < R_S < R_U
\]  

(21)

Thus the static threshold lies in between the lower and the upper threshold for the interior solution of the high-low policy. Another property of high-low price policy is that the price level during the low-price period is higher than the static low price, \( W_L \). This can be confirmed from equation (10).

\[
W_L \geq \frac{D(E-1)W_L + (D-1)W_{HL}}{DE-1} > W_L
\]

The store can induce customers to purchase the good for inventory at a price higher than the reservation level under the static low-price policy precisely because the price discount is known to be temporary (a high-low pricing policy). As a result, the level of inventory is always lower than \( 1 - \pi^* = \frac{s}{c(1-s) + s} \), the level of inventory at the steady state under the static low-price policy.\(^{15}\)
Optimal pricing policy with menu cost

We now introduce the cost of changing prices. Then, each high-low pricing policy is characterized by \((k + 1)\)-tuple of positive integers denoted by:

\[
T = \{t_1, t_2^1, t_2^2, \ldots, t_2^k\} \quad \text{for} \quad k \geq 1
\]

where the firm adopts the following price policy:

\[
\begin{align*}
  p_t &= u & \text{for} & \quad t \in T_H \\
  p_t &= p_L^j & \text{for} & \quad t \in T_L^j \\
  T_H &= [1, t_1] \\
  T_L^j &= \left[ t_1 + \sum_{n=1}^{j-1} t_2^n + 1, t_1 + \sum_{n=1}^{j} t_2^n \right], \quad j = 1, 2, \ldots, k
\end{align*}
\]

The firm changes the price \((k + 1)\) times during the cycle and incurs \((k + 1)\sigma\) of menu costs. Since the cycle repeats itself, \(k + 1 \geq 2\). For the first \(t_1\) periods, the price is set at the reservation utility level, \(u\). At \(t = t_1 + 1\), the price is reduced and the low-price period begins. For the first \(t_2^1\) periods, the price is set at \(p_L^1\). Then the price is changed to \(p_L^2\), which lasts \(t_2^2\) periods, and so on, until the price is finally increased back to \(u\) after \(t_1 + \sum_{n=1}^{k} t_2^n\) periods.

The maximand with menu cost is given by:

\[
M = -(k + 1)\delta + (u - \omega)s\sum_{i=1}^{t_1} \pi_{i-1} + s \sum_{j=1}^{k} \sum_{m=t_0^j+1}^{t_0^{j+1}} (p_L^j - \omega)(c + \pi_{m-1})
\]

Lengthy computations are relegated to the statistical appendix. The derivation process is analogous to those without menu cost. The maximand is given by:
where $B_j$ and $B$ are given by:

$$B_j \equiv \{1 - c(1 - s) - s\}^{\frac{1}{2}}$$

$$B \equiv \{1 - c(1 - s) - s\}^{\frac{1}{2}} \equiv B_1 B_2 B_3 \ldots B_k,$$

and $p_j^L$ is given by:

$$p_j^L = \left[ c(1 - s) + s\lambda \right]^{-1}(\lambda - 1)\mu +$$

$$\left[ c(1 - s) + s\lambda \right]^{-1}\lambda\left[ c(1 - s) + s\right]^{\Theta_{ik} D - 1} \times$$

$$\left[ \Theta_{1j-1} (D - 1)W_H + \Theta_{0j-1} (\Theta_{ik} D - 1) - \Theta_{1j-1} D\Theta_{0k} \right] \mu$$

Then, the optimal solution is given by:
\[ \{ \mathcal{T}^*, k \} = \left\{ t^*_1, t^*_2, \ldots, t^*_k \right\} = \arg \max \left[ \frac{M}{t_1 + \sum_{j=1}^{k} t_j^*} \right] \]

Unfortunately, it is impossible to characterise the optimal solution. In the case of the interior solution (i.e., the high-low policy, rather than either the static high- or low-price policy), we have, however:

\[ M^*|_{\sigma = 0} > M^*|_{\sigma > 0} + \frac{(k + 1)\sigma}{T^*} \]

because the optimal solution strikes the balance between menu cost and the loss from setting the length of the price cycle too long (the optimal cycle length is zero). This implies that for a high-low policy to be optimal, it is necessary that:

\[ M^*|_{\sigma = 0} > \max \left[ \delta_c(u - \omega), s(W_L - \omega) \right] + \frac{(k + 1)\sigma}{T^*} \]

Therefore, for an interior solution with menu cost, the necessary condition is more stringent than the one without menu cost because, by construction, the necessary (and sufficient) condition without menu cost is given by setting \( \sigma = 0 \) above, in which case we recover equation (21). Hence we get:

\[ R_L < R_L|_{\sigma > 0} < R < R_U|_{\sigma > 0} < R_U \]

**Christmas bargain effects, sales promotions and the high-low price cycle**

In the base model, we assumed that the number of customers visiting the store each day is constant. As we show below, however, the data exhibit a large variation in the number of
visitors within a week, a month or a year. In this section we analyse the effect of deterministic variations of $s$.

The concentration of price markdowns and sales during heavy trading periods is noted by Warner and Barsky (1995), who collected a variety of retail data in the United States and found: ‘frequent markdowns in the intensive shopping period prior to Christmas, and a tendency for such sales to occur in weekends’ (p. 322). Our model can be modified to incorporate deterministic fluctuations in shopping probability. Such seasonality is common in retail stores: weekends are heavy shopping periods, so are the Christmas and New Year periods. Business is generally slow in mid-winter and the peak of summer. A store can time its price cycle to increase profit – lowering prices on heavy shopping days.

The underlying mechanism is again that of price discrimination. The high-low pricing in our model is a compromised form of price discrimination: instead of perfectly price discriminating between bargain hunters and other shoppers, retail stores lower prices when they judge the share of bargain hunters to be large. In our model, bargain hunters are those who buy the good to store rather than for immediate consumption. Timing the low-price period to heavy shopping days helps to increase the share of purchases by bargain hunters. Because the share of bargain hunters decreases gradually (decreases in $\pi$) as the low-price period continues, the store will want to time this period to heavy shopping days, thereby increasing sales to bargain hunters within a shorter period. As a result, there will be a lower proportion sold to non-bargain hunters (those buying for immediate consumption), enhancing the effectiveness of dynamic price discrimination. Setting a high price on heavy shopping days is a less profitable strategy because there will be fewer potential customers.

Because the store benefits from timing the low-price period to heavy shopping days, it has an incentive to generate an increase in customers by using advertisements and other sales promotion activities. If the store can attract a large number of bargain hunters, it can reduce the length of the low-price period and hence the volume sold at that low price to customers who are purchasing for immediate consumption.

To demonstrate this point, suppose that a retail store can pinpoint its sales promotion and advertising activities to increase the number of customers on a particular day during the price cycle from $s$ to $(1 + \gamma)$; $(\gamma > 0)$. Denote by $(\pi^*)$ the optimal policy corresponding to a particular configuration of parameters:
\[ P^* = \arg \max_{\theta^*} \left[ \frac{M(P; \Theta_0^*)}{\ell_1^* + \ell_2^*} \right] \]

\[
\Theta_0^* \equiv \{ \varrho_0, \epsilon_0, u_0, c_0, s_0, \sigma_0 \} \]

Consider:

\[
\Delta = \frac{M(P^*; \Theta_1^*)}{\ell_1^* + \ell_2^*} - \frac{M(P; \Theta_0^*)}{\ell_1^* + \ell_2^*}
\]

\[
\Theta_1^* \equiv \{ \varrho_0, \epsilon_0, u_0, c_0, \sigma_0, \{ \gamma \} \}
\]

\[
\{ s_t \} = s_0 \quad \text{if} \quad t \notin t_S
\]

\[
\{ s_t \} = (1 + \gamma) s_0 \quad \text{if} \quad t \in t_S, \quad \gamma > 0
\]

The increase in profit, measured by \( \Delta \), depends on the timing of the sale within the cycle. Hence we denote:

\[
\Delta = \Delta(t_S)
\]

**Lemma 6.** Suppose \( \ell_1^*, \ell_2^* > 0 \). That is, the optimal policy is high-low pricing. Then, \( \ell_1^* + 1 = \arg \max (\Delta(t_S)) \).

**Proof.** See the statistical appendix available from the authors on request.

One of the seven supermarket chains does adopt such a policy, designating day 20 of every month as a store-wide sale day. This chain advertises the sale day on television and in newspapers. On day 20 the price of curry paste is almost always marked down from the previous day, only to be increased on the next day, the 21st.

**Numerical examples of optimal pricing policy**

The optimal pricing policy with menu cost is highly non-linear, and conventional comparative static analysis is powerless to examine its characteristics. We therefore provide a variety of numerical examples to illustrate how changes in the parameters affect the pricing policy.\(^{18}\)
We start with the description of the benchmark equilibrium as shown in Table 2. In this benchmark case, we set: $u = 10$, $\omega = 5.11$, $s = 0.35$, $c = 0.19$, $\epsilon = 0.35$, and $\sigma = 0.05$. The optimal policy\(^1\) is a price cycle of 6.76 days, roughly one week, in which the high price, 10 yen, is set for the first 67 per cent of the cycle, or 4.53 days, then the price is cut to 7.3 yen. This first low-price period lasts 1.15 days, then the price is increased slightly to 7.47 yen, which lasts for 1.08 days, and the cycle ends with a return to the high price. Therefore, the optimal number of price changes, $k^*$, is two. The average discount during the low-price period is 26 per cent (the average price during the low-price period is 7.4 yen). The maximand (per unit of time) at the optimal policy is 1.03977. Compared with this value, the maximand is, respectively, 0.33 per cent, 0.52 per cent, 1.23 per cent, 2.07 per cent, and 10.64 per cent lower if $k = 1, 3, 4, 5$ and 0 (the static pricing policy). On the other hand, the maximand is 8.52 per cent smaller than the maximum value if the menu cost, $\sigma$, is zero.

Table 1 shows the comparative statics results obtained by varying one of the parameters and keeping the rest at the level of the benchmark case. The effects of a change in $s$ are shown in the second row of the Table 1 and they are quite intuitive. Both the cycle length and the average price duration are increasing in $s$. As the customer shops more frequently, the benefit of holding inventory declines. Hence, to induce customers to store inventory, the shops need

<table>
<thead>
<tr>
<th>Parameter</th>
<th>$s$</th>
<th>$c$</th>
<th>$\omega$</th>
<th>$\epsilon$</th>
<th>$\sigma$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$T$ cycle length</td>
<td>+2.59</td>
<td>-0.60</td>
<td>+3.16</td>
<td>+4.59</td>
<td>+1.13</td>
</tr>
<tr>
<td>$\frac{T}{k+1}$ price duration</td>
<td>+3.87</td>
<td>-0.80</td>
<td>+4.90</td>
<td>+6.31</td>
<td>+1.39</td>
</tr>
<tr>
<td>$\frac{T_H}{T}$ share of high price periods</td>
<td>+1.05</td>
<td>-0.36</td>
<td>+1.49</td>
<td>+1.32</td>
<td>+0.12</td>
</tr>
<tr>
<td>$M$ maximand</td>
<td>+0.65</td>
<td>+1.29</td>
<td>-1.44</td>
<td>-0.45</td>
<td>-0.05</td>
</tr>
<tr>
<td>$E(\nu)$ mean (time average)</td>
<td>+0.18</td>
<td>-0.22</td>
<td>+0.24</td>
<td>+0.18</td>
<td>+0.04</td>
</tr>
<tr>
<td>$E(\nu L</td>
<td>T_L)$ mean price in $T_L$</td>
<td>+0.09</td>
<td>+0.08</td>
<td>+0.10</td>
<td>-0.16</td>
</tr>
</tbody>
</table>

Notes: The figure in each cell shows the average percentage change of an endogenous variable (column) when one of the parameters (row) is increased by 1 per cent when the rest of the parameters are set at the benchmark values. In computing the figures, only the results from interior equilibria are used (we exclude equilibria that have static pricing policies as optimal).
to discount more during the low-price period and shorten the low-price period. Eventually, beyond the threshold value of $s$, it becomes optimal always to sell at the high price. Similarly, with $s$ below the threshold, it is optimal always to set the low price so that customers without inventory always purchase to store.

A similar logic applies for the effects of a change in consumption, $c$. An increase in $c$ decreases the inventory held by consumers and hence raises the reservation price of the inventory. As a result, the low-price period increases but the average discount during this period becomes smaller. Eventually, beyond the threshold value, the optimal strategy is a static low-price policy.

An increase in marginal cost ($\omega$) obviously raises the average price not only by shortening the length of the low-price period but also by raising the average price during the period.

If the cost of holding inventory, $\epsilon$, increases, the effect is quantitatively large. As a result, the store must set a lower price during the low-price period, and the consequent decline in profit reduces the length of this period. The effects on the price level and discounts are relatively small, and it is unclear whether the results shown in the Table 1 are robust.

In Table 2 we show how a change in the menu cost affects the optimal choice of $k$. The optimal policy is a longer price cycle and larger value of $k$ as $\sigma$ decreases (bottom row). Starting with the benchmark value of 0.05, the optimal policy changes from $k = 2$ to $k = 1$ at $\sigma = 0.091$ or larger. Reducing $\sigma$ further, the optimal policy changes to $k = 3$ at $\sigma = 0.0074$, and then to $k = 4$ at $\sigma = 0.00059$. Correspondingly, the cycle length (not shown in Table 3) gets shorter and shorter as $\sigma$ decreases. For example, at $\sigma = 0.24$ (the maximum value in our numerical example), the cycle length is 16.6 days, whereas it is less than 2.5 days when the optimal $k$ first becomes 3.

Note that $\sigma = 0.05$ corresponds to 5 per cent of the high price. These simulation results suggest that three or more price changes during the low-price period will be found only when the parameter values and the optimal policy are extreme; for example, extremely small menu costs (less than 1 per cent of the price tag) and an extremely short price cycle (less than three days). The results are consistent with our data, in that most price changes are between the regular high price and the regular low price; that is, a high-low pricing policy. The rest of the simulation results shown in Table 2 support the evidence that the high-low pricing policy is optimal. Aside from changes in menu cost, we did not find any parameter configuration in
which the optimal policy was more than three price changes. Frequent price changes are far more likely under either a static low-price or static high-price policy.

The data and preliminary empirical analysis

The data used in this paper is from the Nikkei Database and includes daily observations of prices and quantities sold of the two competing brands of curry paste at 18 different stores belonging to one of Japan's seven supermarket chains. Unfortunately, the stores were not located close enough to each other to allow an analysis of strategic interactions. The sample period is 1 July 1991 to 31 August 1996. During the period retail prices in Japan were stable and there was virtually no discernible ongoing or future inflation (the consumer price index rose by only 3.6 per cent between 1991 and 1996).

Price changes: frequencies, duration and store characteristics

Table 3 provides several key indicators that we use to characterise the pricing policy of the stores. The sample data exhibit an extreme number of price changes. The data gathered by Pesendorfer (1998) on the sales of two brands of ketchup sold at US supermarkets showed price durations were shorter when prices were lower. This is a characteristic shared by our data, but we registered even shorter durations – on average prices remained for less than 10 days and no store in the sample maintained prices for longer than 60 days even at the regular
Table 3  Summary statistics of price data: House brand

<table>
<thead>
<tr>
<th>Store No.</th>
<th>Price (yen)</th>
<th>Prange shares (per cent)</th>
<th>Price duration (days) at prange=</th>
<th>Chain No.</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>mean</td>
<td>min.</td>
<td>max.</td>
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</tr>
<tr>
<td>1 [1]</td>
<td>221</td>
<td>128</td>
<td>253</td>
<td>1.1</td>
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<tr>
<td></td>
<td>13.5</td>
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<td></td>
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<td>145</td>
<td>253</td>
<td>1.1</td>
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<td>22.3</td>
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<td>130</td>
<td>253</td>
<td>1.1</td>
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<td>11.4</td>
<td>47.8</td>
<td>8.8</td>
<td></td>
</tr>
<tr>
<td>4 [1]</td>
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<td>146</td>
<td>253</td>
<td>1.1</td>
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<td>6.9</td>
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<td>253</td>
<td>1.1</td>
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<td>6 [1]</td>
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<td>1.1</td>
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<td>12.6</td>
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<td>7 [1]</td>
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<tr>
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<td>12 [4]</td>
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<td>17 [7]</td>
<td>238</td>
<td>100</td>
<td>248</td>
<td>1.7</td>
</tr>
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<td></td>
<td>4.4</td>
<td>16.9</td>
<td>5.9</td>
<td></td>
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<tr>
<td>18 [7]</td>
<td>242</td>
<td>100</td>
<td>248</td>
<td>1.7</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>9</td>
<td>2</td>
<td></td>
</tr>
</tbody>
</table>

|          | 5.29        | 2.67                      | 6.79                             | 2.05      |
|          | 5.14        | 2.76                      | 6.82                             | 6.7       |
|          | 5.23        | 2.21                      | 8.08                             | 2.31      |
|          | 5.16        | 2.34                      | 7.09                             | 1.81      |
|          | 5.14        | 2.82                      | 6.96                             | 2.16      |
|          | 5.03        | 1.99                      | 6.65                             | 4.41      |
|          | 7.1         | 5.12                      | 6.33                             | 7.64      |
|          | 11.3        | 1.29                      | 21.71                            | 2         |
|          | 8.32        | 1.25                      | 15.92                            | 1         |
|          | 9.34        | 9.75                      | 7.75                             | 10.19     |
|          | 13.24       | 6.34                      | 23.21                            | 6.23      |
|          | 17.81       | 4.48                      | 28.97                            | 4.55      |
|          | 10.22       | 4.42                      | 12.04                            | 6.95      |
|          | 5.46        | 2.06                      | 8.29                             | 1.6       |
|          | 5.54        | 2.22                      | 8.88                             | 1.66      |
|          | 20.29       | 1.24                      | 17.13                            | 1.17      |
|          | 14.3        | 2.34                      | 9.76                             | 5.89      |
|          | 23.02       | 1.6                       | 12.77                            | 4.25      |
Table 3 (cont’d) summary statistics of price data: S&B brand

<table>
<thead>
<tr>
<th>Store No.</th>
<th>Price (yen)</th>
<th>Prange shares (per cent)</th>
<th>Price duration (days) at prange=</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>mean/min</td>
<td>max.</td>
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<tr>
<td>1 [1]</td>
<td>225/136</td>
<td>272</td>
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</tr>
<tr>
<td>2 [1]</td>
<td>227/138</td>
<td>272</td>
<td>4.7</td>
</tr>
<tr>
<td>3 [1]</td>
<td>227/128</td>
<td>272</td>
<td>5.8</td>
</tr>
<tr>
<td>4 [1]</td>
<td>227/145</td>
<td>272</td>
<td>6.5</td>
</tr>
<tr>
<td>5 [1]</td>
<td>226/138</td>
<td>272</td>
<td>8.2</td>
</tr>
<tr>
<td>6 [1]</td>
<td>226/99</td>
<td>272</td>
<td>7.9</td>
</tr>
<tr>
<td>7 [1]</td>
<td>226/148</td>
<td>272</td>
<td>7</td>
</tr>
<tr>
<td>8 [2]</td>
<td>238/138</td>
<td>278</td>
<td>4.2</td>
</tr>
<tr>
<td>10 [3]</td>
<td>213/100</td>
<td>368</td>
<td>30.4</td>
</tr>
<tr>
<td>12 [4]</td>
<td>202/128</td>
<td>228</td>
<td>13.6</td>
</tr>
<tr>
<td>13 [5]</td>
<td>245/128</td>
<td>283</td>
<td>17.9</td>
</tr>
<tr>
<td>14 [6]</td>
<td>203/148</td>
<td>268</td>
<td>12.3</td>
</tr>
<tr>
<td>15 [6]</td>
<td>205/134</td>
<td>268</td>
<td>14.5</td>
</tr>
<tr>
<td>16 [7]</td>
<td>261/100</td>
<td>268</td>
<td>0.5</td>
</tr>
<tr>
<td>17 [7]</td>
<td>249/100</td>
<td>268</td>
<td>20.1</td>
</tr>
<tr>
<td>18 [7]</td>
<td>243/100</td>
<td>268</td>
<td>20.9</td>
</tr>
</tbody>
</table>
high-price level. Slade (1998, 1999), reports that in 80 per cent of weekly observations, prices had not changed, which implies an unconditional mean duration of more than one month. Although both Pesendorfer and Slade indicate the existence of focal prices, our study shows not only the existence of focal prices but that such prices dominate: two focal prices made up most of the daily price observations. In the empirical analysis below, we exploit this finding and use the following representation of pricing patterns:

\[
\begin{align*}
prange_t &= 1 \text{ if } p_t < \text{pregLow} \\
prange_t &= 2 \text{ if } p_t = \text{pregLow} \\
prange_t &= 3 \text{ if } \text{pregLow} < p_t < \text{pregHigh} \\
prange_t &= 4 \text{ if } p_t = \text{pregHigh}
\end{align*}
\]

We investigate whether a pattern of price changes appears over time. For each store in the seven chains we thoroughly checked the relative frequency of price changes in both directions by day of the week, day of the month, by month, and on holidays. We found only one statistically significant regularity – that the seven stores belonging to chain 1 all reduced the prices of both brands on day 20 of each month, and increased them again on the 21st. This was because day 20 was a regular sale day, common to all the stores of this chain. Except for this pattern, we failed to detect any simple regularity in price changes, although we found mild seasonality in price changes over a year.20 Somewhat unexpectedly, as shown in Table 4, the prices of the two competing brands were significantly positively correlated within each store. The probability of a higher price was significantly larger when the price of the competing brand was also high, and vice versa. Although we found no simple pricing pattern, except in the stores running a sale on day 20, prices are far from random. Using a conventional probit model and survival analysis (results not shown), we find a robust and strongly negative dependence of price duration on price changes, even after controlling for different seasonal effects.

These findings are consistent with our modelling specifications in several ways. First, they strongly suggest that retailers, not wholesalers or manufacturers, set prices on their own initiative using the information available to them. Given the strong positive correlation between the prices of the two competing brands within each store, it is extremely unlikely that the suppliers of these brands are coincidentally adjusting prices. The evidence instead
indicates that the prices of the two brands reflect common factors relevant to the optimal pricing policy. The pattern we found on day 20 is a prime example, as this sale day is likely to be accompanied by significant promotion and advertising aimed at increasing the number of the customers visiting the store. The extremely high frequency of price changes provides further support for the theory that changes in the wholesale price are behind the pricing policy. The evidence also rejects the view that prices were being randomly set. As we will show, the results indicate that large day-to-day fluctuations in the number and the type of customers is the most likely reason for the pricing strategy adopted by the stores.

**‘Stylised’ facts and alternative models**

Frequent price changes without any apparent time trend or seasonality occur regularly across many grocery items sold in Japanese supermarkets. Recurrent price increases and decreases without accompanying changes in costs can be an optimal pricing policy only if there are intertemporal variations in demand. As pointed out by Slade (1998), many models obtaining price changes centre on the gradually decreasing negative effect of past prices on current demand. One reason that has been advanced is that a price lower than the ‘fair’ price will attract customers, and as goodwill or customer capital accumulates, current demand will also increase. Changes in tastes or habits and imperfect information are other reasons that have been put forward to explain the relationship between past prices and current demand. Because customers collect price information infrequently and at different times, the time since prices were last surveyed varies among customers. Some customers will shop because

<table>
<thead>
<tr>
<th>S&amp;B brand price</th>
<th>House brand price</th>
</tr>
</thead>
<tbody>
<tr>
<td>overall</td>
<td>range=4</td>
</tr>
<tr>
<td>price range=4</td>
<td>0.428</td>
</tr>
<tr>
<td>price range=3</td>
<td>0.095</td>
</tr>
<tr>
<td>price range=2</td>
<td>0.362</td>
</tr>
<tr>
<td>price range=1</td>
<td>0.115</td>
</tr>
</tbody>
</table>

Table 4 Correlations in pricing of the competing brands
of lower prices set in the past, resulting in a gradually decreasing negative dependence on the past price.

In models of store sales on the other hand, current demand depends positively on past prices. The longer it has been since the last sale, the larger is the pent-up demand for the sale.

Table 5 shows the results of the regressions relating current demand to past prices. There is no doubt that current demand for both brands is positively correlated with past prices (of one, two or four weeks ago). The regressions also show that the current price of the competing brand will have a large effect on demand for the good. The results decisively reject those of models that predict that current demand is negatively dependent on past prices.

**Dependence of current demand on the current price and its duration**

Although the positive dependence of demand on past prices is a characteristic shared by many (including our own) sales-based models, our model differs in predicting the effect of the duration of the high price on current demand during the high-price period. The dependence of current demand on the level of inventory, \( \pi \), implies that the volume sold monotonically declines during the low-price period as customers continue to build up inventory, but

<table>
<thead>
<tr>
<th>Price of rival brand in the past:</th>
<th>Average price in brand the past:</th>
<th>House brand price</th>
<th>S&amp;B brand price</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>current</td>
<td>past</td>
<td>rival brand</td>
</tr>
<tr>
<td>not included</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1 week</td>
<td>-0.030***</td>
<td>0.974-10^{-3***}</td>
<td>-0.022****</td>
</tr>
<tr>
<td>2 weeks</td>
<td>-0.030***</td>
<td>0.326-10^{-2***}</td>
<td>-0.022****</td>
</tr>
<tr>
<td>4 weeks</td>
<td>-0.031***</td>
<td>0.551-10^{-2***}</td>
<td>-0.022****</td>
</tr>
<tr>
<td>included</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1 week</td>
<td>-0.030***</td>
<td>0.954-10^{-3***}</td>
<td>0.141-10^{-2***}</td>
</tr>
<tr>
<td>2 weeks</td>
<td>-0.030***</td>
<td>0.321-10^{-2***}</td>
<td>0.104-10^{-2***}</td>
</tr>
<tr>
<td>4 weeks</td>
<td>-0.031***</td>
<td>0.542-10^{-2***}</td>
<td>0.691-10^{-3***}</td>
</tr>
</tbody>
</table>

Notes: All regressions are fixed-effect panel regressions and, aside from those shown in the table, they include the following variables: year, month, day of the week, holidays, the day before shops close, the day after shops close (all these are dummy variables), and cproxy, representing seasonal fluctuations in consumption; *** significant at the 1 per cent level, ** significant at the 5 per cent level, * significant at the 10 per cent level.
monotonically increases during the high-price period as inventory is depleted. In a nutshell, demand is negatively dependent on price duration when the price is low, and positively dependent on price duration when the price is high.

Table 6 shows the panel regression on the volume sold, incorporating the differential impacts of price durations. As the model in this paper predicts, price duration generally has a significantly negative effect on the volume sold, with only exception being the House brand in price range 3 (the coefficient is insignificant). At the regular high price (price range 4), price duration has a significantly positive effect on the amount sold in all but one regression. The price of the competing brand has a significantly positive effect on demand, as found in the preliminary regressions in Table 5. Recall the finding in Table 4 that the prices of the two competing brands are highly correlated. Taken together, these facts suggest that the prices of the two brands do respond to a common demand shock that is specific to the store and highly volatile.

<table>
<thead>
<tr>
<th></th>
<th>House brand</th>
<th></th>
<th>S&amp;B brand</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>regression 1</td>
<td>regression 2</td>
<td>regression 1</td>
<td>regression 2</td>
</tr>
<tr>
<td></td>
<td>coeff.</td>
<td>p-value</td>
<td>coeff.</td>
<td>p-value</td>
</tr>
<tr>
<td>prange1</td>
<td>80.99</td>
<td>[0.000]</td>
<td>87.34</td>
<td>[0.000]</td>
</tr>
<tr>
<td>prange2</td>
<td>21.84</td>
<td>[0.000]</td>
<td>25.64</td>
<td>[0.000]</td>
</tr>
<tr>
<td>prange3</td>
<td>11.94</td>
<td>[0.000]</td>
<td>14.5</td>
<td>[0.000]</td>
</tr>
<tr>
<td>duration*p1</td>
<td>–1.81</td>
<td>[0.000]</td>
<td>–1.7</td>
<td>[0.000]</td>
</tr>
<tr>
<td>duration*p2</td>
<td>–0.09</td>
<td>[0.002]</td>
<td>–0.22</td>
<td>[0.000]</td>
</tr>
<tr>
<td>duration*p3</td>
<td>0.097</td>
<td>[0.433]</td>
<td>0.032</td>
<td>[0.026]</td>
</tr>
<tr>
<td>duration*p4</td>
<td>0.046</td>
<td>[0.071]</td>
<td>0.062</td>
<td>[0.007]</td>
</tr>
<tr>
<td>rivalprange1</td>
<td>–</td>
<td>–</td>
<td>–6.43</td>
<td>[0.000]</td>
</tr>
<tr>
<td>rivalprange2</td>
<td>–</td>
<td>–</td>
<td>–3.51</td>
<td>[0.001]</td>
</tr>
<tr>
<td>rivalprange3</td>
<td>–</td>
<td>–</td>
<td>–4.86</td>
<td>[0.000]</td>
</tr>
<tr>
<td>( R^2 ) model</td>
<td>fixed effect panel regressions</td>
<td>0.357</td>
<td>0.377</td>
<td>0.244</td>
</tr>
</tbody>
</table>

Notes: Both regressions include the following variables: year, month, day of the week, holidays, the day before shops close, the day after shops close (all these are dummy variables), and \( cproxy \), representing seasonal fluctuations in consumption; \( rivalprange \) is a dummy variable that is unity in the case of the House (S&B) brand if the price of the S&B (House) brand is in the price range \( x (=1, 2 \text{ or } 3) \)
In summary, we found that neither a simple time-dependent rule nor an explanation based on imperfect information, the tastes of consumers or the amount of customer capital can account for the statistical regularities documented above. On the other hand, these data support our sales-based model, which is the only one that predicts differential effects of past prices on current demand.

The empirical analysis

The base model developed above is deterministic, giving rise to an infinite sequence of price cycles of identical length and pattern. As is clear from Figure 1, pricing patterns are in reality highly irregular and show large variations in length and magnitude.

We believe that the two most important temporal variations responsible for the fluctuations are: (a) changes in the number of customers during a price period; and (b) occasional but sharp changes (declines) in the wholesale price of curry paste. Information on the wholesale price is not available, however.

In order to take account of these effects, we use regression results for each store and brand. The specification of these regressions is identical to those of the panel regressions in Table 6. Since the regressions in Table 6 estimate the daily sales volume, not the number of visitors to the store each day, we need to filter out the effect on demand from variations in the number of customers. We therefore use the fitted value of the sales volume and subtract the estimated effects of: (1) those due to the price duration, as they reflect gradual changes in \( \pi \); and (2) the effect of dummy variables for the price range \( \text{prange} \), as they represent the average size of \( \pi \); and (3) the effect of \( \text{cproxy} \), as this is the proxy for temporal variations in \( c \). Although we have no data on wholesale prices, we learned through interviews that: (1) at the regular low price \( \text{prange2} \), stores intentionally avoid advertising the lower price, and the regular low price is not in response to a change in the wholesale price; (2) the short sales (that last at most three days) at prices below the regular low price \( \text{prangel} \) are in response to a reduction in the wholesale price, and are accompanied by promotion and advertising activities conducted by both the store and the wholesalers. These occasional sales are treated as responding to an increase in shoppers, \( s \). We therefore include a dummy variable for the lowest price because we expect that the dummy primarily represents increased customer awareness of the low price and the increase in \( s \). We ran the following regression for each of the store/brand observations (indexed by \( i \)).
Pacific Economic Papers

\[ \log(sales_i^t) = \alpha_0^t + \sum_k \alpha_{ik}^t \times dummy_{kt}^t + \alpha_2^t \times proxy_t + \sum_{j=1}^{4} [\alpha_{ij}^t \times prange_{jt}^t + \alpha_{ij}^t \times prange_{jt}^t \times duration_{jt}^t] + \alpha_{ij}^t \times \{duration_{jt}^t \}^2 + u_t^i \]

Denoting the estimated coefficients by \( ^{\hat{\cdot}} \), we obtain the measure for the number of shoppers as follows:

\[ \hat{s}_i^t = \hat{\alpha}_0^t + \sum_k \hat{\alpha}_{ik}^t \times dummy_{kt}^t + \alpha_{i1}^t \times prange_{jt}^t \]

We obtain two measures, depending on whether or not the terms in the parenthesis in the sales regression are included: we denote by type 1 [type 2] the measure without [with] the parenthesised term. In view of the theoretical model in this paper, the volume of sales changes over time as: (1) \( \pi \) changes; (2) \( c \) changes; (3) \( s \) changes for exogenous reasons (such as heavy shopping over weekends, less shopping in mid-winter); and (4) advertising and sales promotion changes \( s \). The variable \( \hat{s}_i^t \) is obtained by subtracting from the fitted value of sales volume, \( \log(sales_i^t) \) the effects of (1) and (2) in the regressions and incorporating the effects of (3) and (4). Finally, we normalised the variable by taking the anti-log and adjusting the average to unity.

**Empirical analysis of the demand shifts**

One of the key characteristics of our model is that the composition and size of the demand for the good depends on the price policy chosen by the store. Recall that demand is given by:

\[ Q_t = s_i (c_t + \pi_{t-1}) \quad \text{if} \quad t \in t_L \]
\[ Q_t = s_i c_t \pi_{t-1} \quad \text{if} \quad t \in t_H \]

(16')
where we now allow \( s \) and \( c \) to vary over time. Equation (16') can be solved for \( \pi \) to get:

\[
\pi_{t-1} = Q_t / s_t - c_t \quad \text{if} \quad t \in t_L
\]

\[
\pi_{t-1} = Q_t / (s_t c_t) \quad \text{if} \quad t \in t_H
\]

Recall also that the following equations determine the time path of \( \pi \).

\[
\Delta \pi = \pi_t - \pi_{t-1} = c_t (1 - \pi_{t-1}) \quad \text{if} \quad t \in t_H
\]

\[
\Delta \pi = c_t (1 - s_t) (1 - \pi_{t-1}) - s_t \pi_{t-1} \quad \text{if} \quad t \in t_L
\]

We can eliminate \( \pi \) from the system above to get:

\[
Q_{t+1} = \frac{s_{t+1}}{s_t} \left\{ 1 - c_t (1 - s_t) - s_t \right\} Q_t + s_{t+1} c_{t+1} \left\{ 1 - s_t \right\} \quad \text{if} \quad t \in t_L
\]

\[
Q_{t+1} = \frac{s_{t+1} c_{t+1}}{s_t c_t} (1 - c_t) Q_t + s_{t+1} c_{t+1} c_t \quad \text{if} \quad t \in t_H
\]

The pair of equations in (23) can be estimated using one of the non-linear estimation methods once we have data on \( Q_t, s_t, \) and \( c_t \). We have obtained the estimate \( \hat{\pi} \), except that the average of the sales volume is unknown. Similarly, our data on consumption, \( \text{cproxy}_t \), need an estimate of the average level. Finally, our data on the volume of sales need to be readjusted to incorporate the unknown number of customers (which was normalised to unity in the theoretical model above). Hence we have:

\[
s_{ik}^{t+1} = \lambda_s^{ik} \hat{s}_{ik}^{t} \equiv \lambda_s^{ik} \hat{s}_{ik}
\]

\[
c_{ik}^{t+1} = \lambda_c^{ik} \text{cproxy}_{ik}^{t} \equiv \lambda_c^{ik} \hat{c}_{ik}
\]

\[
Q_{ik}^{t+1} = \frac{\hat{Q}_{ik}^{t}}{X^{ik}}
\]

In the last equation, \( \hat{Q}_{ik}^{t} \) is the observed sales volume, and \( X^{ik} \) is the volume of regular customers for store \( i \), brand \( k \). Combining everything together we have for each store/brand case:
\[
\bar{Q}_{t+1} = \nabla_t [a_{11} \frac{\bar{y}_{t+1}}{\bar{y}_t} \bar{Q}_t - a_{12} \frac{\bar{y}_{t+1}}{\bar{y}_t} \bar{c}_t \bar{Q}_t + a_{13} \frac{\bar{y}_{t+1}}{\bar{y}_t} \bar{c}_t \bar{Q}_t - a_{14} \frac{\bar{y}_{t+1}}{\bar{y}_t} \bar{Q}_t + a_{15} X \bar{y}_{t+1} \{\mathcal{C}_t \}^2 - a_{16} X \bar{y}_{t+1} \{\mathcal{C}_t \}^2 \bar{c}_t] \\
+ (1 - \nabla_t) a_{21} \frac{\bar{y}_{t+1} \bar{c}_{t+1}}{\bar{y}_t \bar{c}_t} \bar{Q}_t - a_{22} \frac{\bar{y}_{t+1} \bar{c}_{t+1}}{\bar{y}_t \bar{c}_t} \bar{c}_t \bar{Q}_t + a_{23} X \bar{y}_{t+1} \bar{c}_{t+1} \bar{c}_t \]  
\tag{23'}
\]

where $\nabla$ is unity during the low-price period and zero otherwise. The set of restrictions are:

\[
\begin{align*}
    a_{11} &= a_{21} = 1 \\
    a_{12} &= a_{22} = \lambda_c \\
    a_{13} &= \lambda_y \lambda_c \\
    a_{14} &= \lambda_s \\
    a_{15} &= a_{23} = \lambda_y \lambda_c^2 \\
    a_{16} &= \lambda_s \lambda_c^2
\end{align*} \tag{24}
\]

The entire system given by equations (23') and (24) can be estimated by the maximum likelihood method for non-linear regressions. We used our estimate of the average number of shoppers to each store in the sample as the proxy for $X^{ik}$. This leaves two unknowns, $\lambda_c$ and $\lambda_y$, which we estimated using the maximum likelihood method. The results are shown in Table 7. Except for stores 8, 9, 13 and 16, selling the House brand, and store 16, selling the S&B brand, the estimated coefficients are positive and less than unity. Most of them are also highly significant. Since the estimation requires positive sales volumes for both the current and previous periods, we lose many observations in some store/brand combinations, and the regressions performed relatively poorly in those cases. Overall, these estimates provide us with intuitively reasonable magnitudes for the two key parameters, $\lambda_c$ and $\lambda_y$: the average probability of consumption ranges from 0.2 per cent to 1 per cent per day, whereas the average probability that a consumer will shop that day ranges between 0.1 per cent and 0.3 per cent.

**Ordered probit model of price changes**

The second important prediction of the model is that the pricing policy depends crucially upon the level of inventory, $\pi$. The probability of a price decline should be monotonically increasing
Table 7  Non-linear regressions on sales

<table>
<thead>
<tr>
<th>Chain No.</th>
<th>Store No.</th>
<th>$\lambda_c^{\text{type 1}}$</th>
<th>$\lambda_s^{\text{type 1}}$</th>
<th>$\lambda_c^{\text{type 2}}$</th>
<th>$\lambda_s^{\text{type 2}}$</th>
<th>N</th>
<th>$\bar{X}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>0.00921</td>
<td>0.102</td>
<td>0.0151</td>
<td>0.121</td>
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<td>4579</td>
</tr>
<tr>
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<td>2</td>
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<td>0.0118</td>
<td>0.344</td>
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<td>4336</td>
</tr>
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</tr>
<tr>
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<tr>
<td>2</td>
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<td>0.14</td>
<td>-0.00525**</td>
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<td>0.0084</td>
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<td>1277</td>
<td>3604</td>
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<td>0.00959</td>
<td>0.113</td>
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<td>2549</td>
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<td>7</td>
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<td>0.194**</td>
<td>-0.0169**</td>
<td>0.280**</td>
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<td>807</td>
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<td>0.00875**</td>
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<td>0.071**</td>
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<td>728</td>
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<td>\hat{\lambda}_s \ (\ddot{s})</td>
<td>\hat{\lambda}_c \ (\ddot{c})</td>
<td>\hat{\lambda}_s \ (\ddot{s})</td>
<td>N</td>
<td>\hat{\chi}</td>
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<td>0.0185</td>
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<td>0.00258</td>
<td>0.0205</td>
<td>0.0813</td>
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<td>0.00549</td>
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<td>0.0644*</td>
<td>-0.0160**</td>
<td>0.0308**</td>
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<tr>
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<td>0.13</td>
<td>0.00696</td>
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<td>881</td>
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<td>0.00814</td>
<td>0.171</td>
<td>0.00875</td>
<td>0.172</td>
<td>146</td>
<td>728</td>
</tr>
</tbody>
</table>

Notes: All regressions are estimated by the non-linear maximum likelihood method and include first-order serial correlation corrections. Based on standard errors computed from a heteroscedastic-consistent matrix, all the coefficients are significant at the 1 per cent level unless otherwise noted: *not significant at 1 per cent level but significant at 5 per cent, **not significant at 5 per cent level.
in \( \pi \), whereas the probability of a price increase should be monotonically decreasing in \( \pi \). This prediction can be examined in an ordered probit model:

\[
\Delta p = p_t - p_{t-1}
\]
\[
\Delta p > 0 \text{ if } \Psi > p^U
\]
\[
\Delta p = 0 \text{ if } p^D < \Psi < p^U
\]
\[
\Delta p < 0 \text{ if } \Psi < p^D
\]

\[
\Psi = \alpha_1 \pi + \alpha_2 \pi \text{Low} + \alpha_3 \text{Low} + \alpha_4 s^e + \text{dummies}
\]
\[
\alpha_1 < 0, \quad \alpha_2 > 0, \quad \alpha_3 > 0, \quad \alpha_4 < 0
\]

wherein Low is a dummy variable that equals zero whenever the price is below the regular high price (\( p^< \text{range} 4 \)), and \( s^e \) is the expected number of customers for the next two weeks. One of the two unknown threshold values – \( p^U \) and \( p^D \) – can be set arbitrarily at zero, and the remaining unknown and other parameters can be estimated. We expect \( \alpha_1 \) to be negative, as the store is more likely to lower the price when the share of customers without inventory is high. The likelihood of this occurring will be less if the price is already below the regular level (\( \alpha_2 > 0 \)). We cannot sign \( \alpha_3 \), because although a low current price reduces the probability of a further price decline, there may be a further decline in price if retailers know the likely number of shoppers in the future (see below). We expect \( \alpha_4 < 0 \) because the expected increase in shoppers should induce the store to reduce the price. We use the fitted value of the regression on the number of customers to construct the \( \pi \) series.\(^{21}\)

To test the model’s predictions, we first construct a time series for the unobservable variable, \( \pi_t \), by using the estimates of the two key parameters, \( s \) and \( c \), obtained above:

\[
\tilde{\pi}_t = \hat{\lambda}_s \tilde{s}_t
\]
\[
\tilde{c}_t = \hat{\lambda}_c \text{proxy}_t
\]
\[
\hat{\pi}_t = \tilde{c}_t + \left(1 - \tilde{c}_t\right) \tilde{\pi}_{t-1} \text{ for } t \in T_H
\]
\[
\hat{\pi}_t = \tilde{c}_t \left(1 - \tilde{c}_t\right) + \left(1 - \tilde{c}_t\right) \tilde{\pi}_{t-1} \text{ for } t \in T_L
\]

The equations above generate the time series for \( \pi \) based on its initial value. To eliminate the effect of the initial value of \( \pi \) on the series, we first arbitrarily picked an initial value and
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computed the empirical density of $\pi$. Then we chose the next initial value by drawing from the probability distribution. The process is continued until the probability distribution converges. Alternatively, we eliminated some portion of the early observations of $\pi$. The results of the probit analysis were similar irrespective of the method used.

Table 8 reports the ordered probit model given by equation (25). We report only the estimations based on the first method of eliminating the effect of the initial value of $\pi$. The results were obtained by pooling all observations for all the stores for which theoretically correct estimates of $s$ and $c$ were obtained (i.e., they are all positive and less than unity). The results are highly robust and the estimated coefficients have the correct signs and are significant at the 1 per cent level. In particular, the constructed series of $\pi$ is highly significant and shows that an increase in $\pi$ increases (lowers) the probability of a price decline (increase). We also find that the expected increase in the number of shoppers has a significantly positive (negative) impact on the probability of a price decline (increase).

### Table 8 Ordered probit model of price changes

<table>
<thead>
<tr>
<th></th>
<th>House brand</th>
<th></th>
<th>S&amp;B brand</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>coeff.</td>
<td>Pr ($\Delta p &gt; 0$)</td>
<td>Pr ($\Delta p &gt; 0$)</td>
<td>Pr ($\Delta p &gt; 0$)</td>
</tr>
<tr>
<td>$\hat{\pi}$</td>
<td>-2.05</td>
<td>0.3871</td>
<td>-0.00522</td>
<td>-0.3823</td>
</tr>
<tr>
<td>$\text{low} \times \hat{\pi}$</td>
<td>0.12</td>
<td>-0.0216</td>
<td>0.0003</td>
<td>0.214</td>
</tr>
<tr>
<td>$\text{low}$</td>
<td>-0.67</td>
<td>0.0744</td>
<td>-0.001</td>
<td>-0.0734</td>
</tr>
<tr>
<td>$\text{shopping intensity}$</td>
<td>-0.077</td>
<td>0.0136</td>
<td>-0.00018</td>
<td>-0.0134</td>
</tr>
<tr>
<td>$\text{store sale}$</td>
<td>-1.04</td>
<td>0.184</td>
<td>-0.0025</td>
<td>-0.181</td>
</tr>
<tr>
<td>$\text{store sale}+1$</td>
<td>0.95</td>
<td>-0.165</td>
<td>0.0022</td>
<td>0.163</td>
</tr>
<tr>
<td>$R^2$</td>
<td>0.086</td>
<td>0.093</td>
<td></td>
<td></td>
</tr>
<tr>
<td>no. of samples</td>
<td>21872</td>
<td>25820</td>
<td></td>
<td></td>
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<tr>
<td>log likelihood</td>
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<td>-11830</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Notes: (*) the right hand side is an index variable; 0 for price decrease, 1 for no change and 1 for price increase, respectively, from the previous day. All the estimated coefficients are correct signs and significant at the 1 per cent level. (1,2,3) are the estimated effect of an increase in each variable on the probability that $\Delta p < 0, = 0, > 0$, respectively. We use the estimated values of $\hat{s}_1$. See the explanations in the main text.
Cross-sectional variations in the estimated coefficients

The model presented in this paper has five free parameters \( \omega, \varepsilon, \sigma, s, \) and \( c \) and the sixth parameter, \( u \), can be used as a numeraire. We obtained estimates of \( s \) and \( c \) in the nonlinear model reported above. This leaves three parameters. Since we have no strong reason to believe that customers in different stores and/or customers buying different brands have different inventory costs, we treat \( \varepsilon \) as an unknown but common constant across brands and stores.

Next, given that scanner technology and inventory and logistical systems will be the same in all stores in our sample, we assume that the cost of changing prices, \( \sigma \), does not vary across brands or stores within the chain. Moreover, the menu cost should be independent from the amount of the product sold.\(^{24}\) Thus we presume that the menu cost per unit of customer is given by:

\[
\sigma_{jk}^i = \tilde{\sigma}_k^i / \tilde{O}_{jk}^i
\]  

wherein the menu cost \( \tilde{\sigma}_k^i \) is specific to chain \( k \) and \( \tilde{O}_{jk}^i \) is the average number of customers purchasing brand \( i \) at store \( j \), which we normalised to unity in the theoretical model.

Earlier in the paper, we obtained the following comparative statics results using numerical simulations:

\[
\log \left\{ E \left[ \bar{p}_{L\,jk}^i \right] \right\} = \alpha_0 + \alpha_1 \log \left( s_{jk}^i \right) + \alpha_2 \log \left( c_{jk}^i \right) + \alpha_3 \log \left( \omega_{jk}^i \right) - \alpha_4 \log \left( \sigma_{jk}^i \right) - \alpha_5 \log \left( \varepsilon \right)
\]

\[
\log \left\{ E \left[ T \frac{k+1}{T_{jk}} \right] \right\} = \beta_0 + \beta_1 \log \left( s_{jk}^i \right) - \beta_2 \log \left( c_{jk}^i \right) + \beta_3 \log \left( \omega_{jk}^i \right) + \beta_4 \log \left( \sigma_{jk}^i \right) + \beta_5 \log \left( \varepsilon \right)
\]

\[
\log \left\{ E \left[ \frac{T_H}{T} \right]_{jk} \right\} = \gamma_0 + \gamma_1 \log \left( s_{jk}^i \right) - \gamma_2 \log \left( c_{jk}^i \right) + \gamma_3 \log \left( \omega_{jk}^i \right) + \gamma_4 \log \left( \sigma_{jk}^i \right) + \gamma_5 \log \left( \varepsilon \right)
\]
wherein all the parameters (\(\alpha\), \(\beta\) and \(\gamma\)) are positive. Denote by \(\Delta\) log-differencing across stores and brands within a chain. Using equation (26) to substitute for \(\sigma^i_{jk}\), the above equations can be rewritten to obtain within-chain regressions.

\[
\Delta \log \left\{ E \left[ \bar{p}_{ij} \right]_{jk} \right\} = \alpha_0^k + \alpha_1 \Delta \log (\epsilon^i_{jk}) + \alpha_2 \Delta \log (\epsilon^i_{jk}) \\
+ \alpha_4 \Delta \log (\bar{Q}^i_{jk}) + u_1
\]  
(27a)

\[
\Delta \log \left\{ E \left[ \frac{T}{k+1} \right]_{jk} \right\} = \beta_0^k + \beta_1 \Delta \log (\epsilon^i_{jk}) - \beta_2 \Delta \log (\epsilon^i_{jk}) \\
- \beta_4 \Delta \log (\bar{Q}^i_{jk}) + u_2
\]  
(27b)

\[
\Delta \log \left\{ E \left[ \frac{T_H}{T} \right]_{jk} \right\} = \gamma_0^k + \gamma_1 \Delta \log (\epsilon^i_{jk}) - \gamma_2 \Delta \log (\epsilon^i_{jk}) \\
- \gamma_4 \log (\bar{Q}^i_{jk}) + u_3
\]  
(27c)

wherein chain-specific constants and error terms (\(u\)) represent the impact of the unobservable \(\Delta \omega^i_{jk}\). Note also that the \(\epsilon\)s are cancelled out in these within-chain regressions.

Table 9 shows the within-chain estimates of these parameters in equations (27a–c). The first column is the within-chain panel OLS (ordinary least squares) results and the second set of regression results are SUR (seemingly unrelated regressions) run for joint estimations of \(\alpha\), \(\beta\) and \(\gamma\). The results are broadly consistent with the results of the numerical examples, but most of the estimated coefficients for \(S\) and \(C\) are not statistically significant. Some of them are even the wrong signs. It should be noted, however, that none of the coefficients is both statistically significant and the wrong sign. Moreover, the coefficients on \(\log (\bar{Q}^i_j)\) are statistically significant and the signs are correct in all three regressions.
Conclusion

This paper developed a dynamic sales-based model and applied it to the pricing policy of Japanese supermarkets, finding that results were consistent and supportive of the model’s predictions. In particular, we found that the customers respond to price markdowns by accumulating merchandise as inventory, which is gradually consumed during the high-price period. This behaviour is confirmed by the regression results, which demonstrate that price duration has a strong negative (positive) dependence of sales volume during the low- (high) price period. We also estimated the determinants of the price changes and found that the proportion of customers without inventory has a statistically significant effect on price changes.

These facts are not easily reconcilable with the findings of other models. Our data do not support the prediction of the customer capital model that demand is negatively dependent
on past prices. In addition, although the duration of a sale is typically shorter than the
duration of a higher price, the sale does not appear to bring a burst of demand within that
period, as other models predict. These models do not examine the length of a sale period nor
when that period started. Random pricing models that focus on store sales are also at variance
with the evidence in this paper of a strong negative dependence of price changes on price
duration. Our model scored better than any of these other models but still leaves many
important questions unanswered.

One of the major issues not explored here is the relationship between prices of different
brands of a product within a store. As we noted earlier, statistical evidence indicates a
significant correlation between the pricing of two similar brands. It seems likely that
customers can easily switch between brands in response to markdowns in price of one of the
brands. It is not clear, however, how the incorporation of the substitution effect would alter
the findings of the model.

Another issue of interest is the effect sales promotions have on pricing. We found
evidence that prices were being coordinated with sales promotions but the results did not
identify how this interaction worked in practice. This paper can be considered as a first step
toward more comprehensive analysis of pricing and sales promotion. The results indicate that
supermarkets do induce fluctuations in the number of customers, and therefore the amount
sold, by employing sophisticated dynamic pricing and sales promotions strategies.

Notes

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Science. The authors benefited greatly from interviews conducted at the purchasing division
of chain 1. Chain 1 also provided data on the daily flow of the customers to individual stores
in our sample. Earlier versions of this paper were presented at The Australian National
University, the University of the Philippines (Dilman) the Thailand Development Research
Institute, Tohoku University and the Macroeconomic Workshop (Osaka, Japan). The usual
disclaimers apply.

1 Curry paste is generally sold in half-solid bars. Curry cooked with vegetables or meat
and served with rice is extremely popular in Japan. Around 80 billion yen worth of
curry paste is sold each year, enough for 1.5 billion curry and rice dishes. The two
brands chosen for our sample are the most popular sellers. Each brand is likely to
have less than a 10 per cent market share. The House brand chosen is one of five major
brands produced by this company. Each brand comes in three or four varieties
(typically distinguished by spiciness). House commands 40–50 per cent of the curry
paste market. Curry paste in Japan is similar to Campbell’s soup in the United States: you are bound to find at least a few packets in any Japanese house.

This number does not include the 182 days when none of the product was sold or when the store was closed.

For example, Slade (1998) found prices last on average for five weeks, compared with 5–25 days in our data (Table 3). In Aguirregabiria’s (1999) study of 534 brands, the average price duration was 1–2.3 months.

In Varian (1980) and Sobel (1984), the game is played among oligopolistic firms. The mixed strategy arises in a price-setting game because a random sale, if successful, generates supranormal profits. To maintain the equilibria, it is necessary that the success of the sales is randomly distributed so that the expected profit during the sale period is compatible with the profit during normal trading. The driving factor behind the model is the accumulation of shoppers who are willing to postpone consumption and wait for the sale. In Conlisk et al. (1984), the game is played between a monopolist and its customers. The high price is maintained for some time before it begins to decline gradually until all potential shoppers have been through the store by the end of the period with the lowest price. This pattern is then repeated. A gradual price decline is necessary to retain and accumulate a large number of shoppers.

This is in sharp contrast to the negative correlation between current demand and past prices found in the data that Slade (1998) uses. Moreover, highly frequent price changes, especially between the two focal prices, suggest that the factor underlying the price changes also occurs frequently. Fluctuations in customer capital is therefore an implausible explanation.

Highly frequent price changes could also be explained by fluctuations in store inventory, the explanation adopted by Aguirregabiria (1999). Circumstantial evidence strongly suggests dynamics in (S–s) inventory are not behind the price changes in our data. In the (S–s) inventory model, the lower and the upper bounds of the inventory are determined by trading off the lump-sum cost of replenishing inventory and the expected loss of profits from the reduction in stock. The retail stores in our sample have virtually unlimited access to stock, which can be ordered in small units and often arrives the next day.

According to a representative of a major curry manufacturer, most curry paste purchased will be consumed within two months and any packets left over are more likely to be disposed of than consumed. He noted, however, that curry paste stored at room temperature will last at least one year without deteriorating in quality.

Alternatively, markdowns can be modelled as dynamic price discrimination in the presence of heterogeneous consumers (shoppers and non-shoppers). Although a popular assumption, it is not employed in this paper for the following reasons. First of all, bargain hunters do buy large amounts to store at home, the very behaviour that we focus on in our specification. Our model incorporates this crucial feature without the heterogeneity assumption. The distinction between bargain hunters and non-bargain hunters is also problematic when we apply the model to our data: the crucial difference between the two is that bargain hunters are more patient (they have a low time discount) than non-bargain hunters. The highly frequent price markdowns exhibited and the fact that most grocery items are purchased regularly by households (e.g., at least once a month) make this type of specification unsuitable.
Our choice of model is dictated by the fact that the data do not cover any pair of retailers situated close enough to each other to allow tests of the role of strategic interactions among competitors. The model developed here could be extended to incorporate strategic interactions among retailers. In view of the comparison between the monopoly model in Conlisk et al. (1984) and the oligopoly model in Sobel (1984), we expect that the crucial property (a price cycle punctuated by periodic price markdowns) would remain valid in an extended model, although Sobel (1991) suggests a possible mixed strategy equilibria under such setting.

Allowing multiple units of inventory complicates the analysis and adds little to the results.

Again, a stochastic consumption pattern is a convenient shortcut to represent the difficulty of perfectly aligning consumption and shopping patterns.

The model can be easily extended to incorporate a positive time discount without changing the substance of the analysis.

The store can in principle follow a non-linear price strategy (discounting for volume) in order to price discriminate. We assume this strategy is not followed. In interviews, we learned that firms rarely offer discounts for bulk buying. Although scanner technology has substantially reduced the cost of price changes, discounting for volume is costly under this technology, as the store must prepare different packaging but still put the same number of bar codes through the register. Another reason to assume away volume discounting is that it is an imperfect measure: although consumers purchasing multiple units are necessarily those without inventory, the reverse is not true. Some customers buy only one unit to store, not for immediate consumption.

Since the choice of the time unit is arbitrary, we can choose a unit small enough (but still finite) so that parameters $\zeta$, $s$ and $\varepsilon$ are all small positive numbers.

From equation (12), it is immediate that $\bar{\pi} > \pi^*$. Moreover, using equation (11), we know that $\pi_t \geq \bar{\pi}$ for any $t$.

It is also likely that the share of bargain hunters actually increases, either because bargain hunters are better informed and/or because retailers advertise to entice bargain hunters to visit during the period. These factors (absent in our model) reinforce our argument that stores benefit from setting a low price during heavy shopping periods.

This observation was backed up by a director in charge of pricing at a national supermarket chain, who commented that sales are always accompanied by promotions including newspaper advertisements, in-house demonstrations, special displays and so on. Chevalier et al. (2000) find similar patterns of counter-cyclical markups for a wide variety of items sold at a large supermarket chain in the United States.

The computer program used for these numerical examples was MATLAB (version 5.3) and is available from Kenn Ariga upon request.

We ignore the integer constraints except for the choice of $k$. 
For example, take weekly fluctuations in the relative frequency of price changes: Tuesday is a popular day for a price change in the House brand, (11.2 per cent of price rises and 9.7 per cent price falls took place on this day), Friday is the least popular day for a price increase, while prices fell the least frequently on Thursdays. A very similar pattern was observed for the S&B brand: Tuesdays registered the highest frequency in both directions (8.4 per cent and 6.2 per cent), with Fridays the least popular day for an increase (4.4 per cent) and Thursdays the least popular for a decrease (4.6 per cent).

It is reasonable to assume that the store can perfectly predict the right-hand-side variables used to forecast the number of shoppers in the near future.

The results are virtually identical under alternative specifications of the initial value of \( \pi \) or with alternative estimators for \( \hat{s}_t \).

Although we do not know the sign of \( \alpha_3 \), a comparison of the regression results without \( s^c \) (not shown in Table 8) against those with \( s^c \) shows that \( \alpha_3 \) is always smaller in the absolute value for the latter cases. This is consistent with our expectation, as part of the predictive power of the current price on a future sale taking place is removed with the inclusion of \( s^c \).

There is no requirement that the price tag be attached to the merchandise. All the stores in our sample used scanners that were linked to the prices displayed on the shelves of each store. As shown in Levy et. al (1997), if stores were required to attach price tags to individual merchandise, the cost of changing prices would be significantly increased.

Lach and Tsiddon (1996) found significant within-store synchronisation in the price of wine but not in the price of meat products. Their findings are consistent with our model.

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