Increasing Returns and Economic Efficiency*

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Abstract
This paper argues that, from the viewpoint of Pareto optimality, the industries with increasing returns are under-expanded relative to those without increasing returns and those with higher degrees of increasing returns are under-expanded relative to those with lower degrees. Ignoring administrative and indirect (such as rent-seeking) costs, subsidies on goods produced under conditions of (high degrees of) increasing returns financed by taxes on goods produced under non-increasing and lower increasing returns may increase efficiency. This argument is related to the original Pigovian case for encouraging industries with decreasing costs of production (Section 1), demonstrated for the general case (Section 3) and illustrated for a specific case (Section 4), after providing an existence proof of average-cost pricing general equilibrium (Section 2).

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1. Introduction

The proofs of the existence of a competitive equilibrium typically depends on the convexity of the production possibility sets. However, the presence of increasing returns may make the set non-convex. It is also well-known that increasing returns may give rise to certain efficiency problems in a market economy (e.g. Arrow 1987, 2000, Dall'Olio & Vohra 2001, Guesnerie 1975, Heal 1999, Quinzii 1992b, Villar 1996, Vohra 1994). However, in the real world, increasing returns are important at the levels of the individual, the firm, the industry, the economy, and globally. Increasing returns are widespread for a number of reasons. First, there are learning costs in producing virtually any good and the related better skills through practice. Next, there are usually some degrees of indivisibilities giving rise to sizable fixed costs. A shop has to be of some minimum size and office space has to be rented for a certain minimum period. Machines come in certain sizes. Arrow (1995) explains how the relevance of information and knowledge in production makes increasing returns prevalent. (See also Wilson 1975, Radner & Stiglitz 1984, Arthur 1994, Arrow et.al. 1998. On empirical evidence for increasing returns, see Ades & Glaeser 1999, Antweiler & Trefler 2002, Fubgleton 2003, McCombie et al. 2003, Kwack & Sun 2005.) A very common situation is for a firm to have a high fixed cost and relatively low and roughly constant (or even decreasing, due to, e.g. the economies of bulk purchase) marginal costs, making the average cost curve a sharply decreasing rectangular hyperbola, becoming flatter only at very high levels of output. Such a firm loves to sell more at any given price above MC. In contrast, a perfect competitor refuses to sell more at the given price. (At the profit-maximizing equilibrium, the MC curve cuts the horizontal price line from below, making the additional costs of producing more exceeding the additional revenue generated.) If you asked 1,000 business persons whether they want to sell more at the given (prevailing) price, I bet you will get no less than 990 positive answers! Perfect competition prevails in no more than 1% of the cases!

The problems caused by increasing returns may be seen even using concepts familiar to first-year undergraduates. If the producer is a perfect competitor in the factor market, he takes input prices as given. Increasing returns thus makes his average-cost curve downward sloping. With a horizontal demand curve for his product, he will increase output indefinitely, at least until either increasing returns no longer apply or until he is no longer a perfect competitor. The assumption of perfect competition thus does not allow the continued existence of increasing returns.

Nearly a century ago, some economists (in particular, Pigou 1912) advocated the taxation/subsidization of goods with increasing/decreasing costs of production or with upward/downward sloping supply curves to increase overall surplus. However, Pigou referred to competitive industries, where in the absence of real external effects, efficiency should rule. As Young
(1913) pointed out in his review of Pigou’s book, the upward-sloping supply curve involves higher rents for the relevant factors of production, involving the transfer of purchasing power. Further discussion (on costs and returns reprinted in AEA 1952) revealed some problems in Pigou’s analysis. Pigou used an example of a non-congested, wide but uneven road and a congested, narrow but well-surfaced road to illustrate the over-use of the narrow road with increasing costs. Knight (1924) correctly pointed out that this is due to the failure of pricing the congested road. With optimal pricing (in the absence of pricing costs, optimal both for the private owner of the road and for social efficiency), no overuse will be involved. This is not surprising to modern economists taught with the Pareto optimal nature of a competitive equilibrium in the absence of unaccounted external effects. More than half a century ago when environmental consciousness was not high, economists could confidently conclude that the ‘departure of the economist’s free competition from the ideal of social costs is in fact negligible for external economies’ (Ellis & Fellner 1943, p.511). However, even if we ignore external costs like environmental disruption, the undisputed widespread existence of increasing returns still makes a competitive equilibrium non-existent (in the sense NOT that the economy remains competitive but equilibrium does not exist but that the economy cannot remain perfectly competitive), not to mention the issue of its Pareto optimality. Thus, if we apply Pigou’s argument not for the case of perfect competition, the verdict is different.

In the presence of increasing returns, since a firm finds its average cost to be downward sloping over the whole relevant range, it will expand output indefinitely as long as it is a price taker. The expansion will eventually make price-taking behavior no longer relevant. We may then have the monopolistic restriction of output, with price above marginal cost (which equals marginal revenue). This will lead to under-production if all other sectors are perfectly competitive. What is the situation if monopolistic competition is prevalent? Dixit & Stiglitz’ (1977) pioneering analysis shows that no general conclusion can be made. The more specific models of Heal (1980, 1999) show that the combination of imperfection competition and increasing returns leads to the over-serving of large markets and under-serving of small markets. (See also Spence 1976 and Lancaster 1979 on optimal product variety.) This is consistent with the simple partial-equilibrium intuition that may be illustrated with a downward-sloping average cost curve. To abstract away from problems arising from monopolistic power, assume free entry/exit such that the price of each good equals average cost in long-run equilibria.

Even so, it can be shown (next section) that, from the viewpoint of Pareto optimality, the industries with increasing returns are under-expanded relative to those without increasing returns and those with higher degrees of increasing returns are under-expanded relative to those with lower degrees (thus going beyond Pigou, even if we re-interpret or re-apply Pigou’s case to non-perfect

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1 They show that, for the case of a constant elasticity of substitution between different goods, the market equilibrium under free entry/exit coincides with the (non-negative profit) constrained optimum but no general conclusion is possible with non-constant elasticity.
competition), at least if we start from a position of average-cost pricing equilibrium as is likely to be the case for long-run equilibrium under free entry. Subsidies on goods produced under conditions of (high degrees of) increasing returns financed by taxes on goods produced under non-increasing and lower increasing returns may increase efficiency (ignoring other effects like second-best and externalities that may offset increasing returns as well as ignoring administrative and indirect costs such as rent-seeking activities).\footnote{The issue of equality is also ignored. This issue is less relevant if we accept the argument for treating a dollar as a dollar on specific issues, leaving equality to be pursued by the general tax/transfer policy (Ng 1984). On some special interaction of equity and efficiency issues in the presence of increasing returns, see Brown & Heal 1979, Ng 1985, Vohra 1992.}

The intuition why the basic point is valid is not difficult to see. With a falling average cost curve, even pricing at average cost (in contrast to a possibly much more restrictive policy of pricing at marginal revenue = marginal cost) involves pricing at above marginal cost (as MC is below AC when AC is decreasing), hence implying under-production. (In contrast to Pigou’s case of perfect competition, the higher/lower costs here refer to actual resource costs rather than changes in the prices/rents of inputs.) This partial-equilibrium intuition remains true in a general-equilibrium framework unless offset by other considerations such as the second-best interrelationship of being very complementary/substitutable to another good of opposite/similar efficiency considerations that cannot be dealt with directly. That this is true can be seen by taking all other goods as a composite good, making the partial equilibrium analysis also a general equilibrium one. On the other hand, apart from applications to certain areas like international trade (e.g. Kemp 1984, Helpman & Krugman 1985), the more sophisticated analysis of increasing returns in the recent decades (e.g. papers collected in Heal 1999) focuses more on the existence of general equilibria and in particular, the existence and Pareto optimality of marginal-cost pricing equilibria.

Nevertheless, the inclination of economists against intervention needs not be mistaken. First, while increasing returns are prevalent, we or the government may not know which industries are subject to higher degrees of increasing returns than others. If we thus consider a model where all goods have the same degree of increasing returns, then the market equilibrium cannot be improved upon by taxes and subsidies, given the infeasibility of taxing leisure. Secondly, imposing taxes/subsidies on goods with high degrees of increasing/decreasing returns would open up a floodgate of rent-seeking activities that are likely to consume an enormous amount of resources many times the gain of the optimal taxes/subsidies. Perhaps it is optimal to continue to pretend that increasing returns do not exist.

2. Existence of Average-Cost Pricing Equilibria with Increasing Returns

In this section, we show the existence of an average-cost pricing equilibrium under the conditions of (possible existence of) increasing returns. This demonstration is based on a standard neoclassical
model by Brown & Heal (1983) but generalized to any numbers of individuals, goods, and resources. (On the existence of general equilibrium with increasing returns, see, e.g., Bonnisseau & Cornet 1988, Bonnisseau & Meddeb 1999, Brown et al. 1986, Quinzii 1992a.)

The set of individuals is taken as given and resources (or factors), including labor, are in fixed supply. This is not really restrictive, as we may view leisure as a good and produced one to one using labor time. Denoting the numbers of individuals, goods (subsuming services), and resources as I, G, and R respectively, we have

\[(2.1) \quad R_1^r + R_2^r + \ldots + R_I^r = R^r; \quad r = 1, \ldots, R \quad \text{(individual ownership of resources)}\]

where \(R_i^r\) is the amount of resource \(r\) owned by individual \(i\).

Each individual has a conventional utility function

\[(2.2) \quad U^i = U^i(G^i_1, G^i_2, \ldots, G^i_G); \quad i = 1, \ldots, I \quad \text{(individual utility functions)}\]

where \(G^i_g\) is the amount of good \(g\) consumed by individual \(i\).³

Denoting the prices of goods and resources as \(P_g\) and \(W_r\) respectively, we may write the budget constraint of individuals as

\[(2.3) \quad \sum_{g=1}^{G} P_g G^i_g = Y^i; \quad i = 1, \ldots, I \quad \text{(individual budget constraints)}\]

Each individual may receive her income \(Y^i\) from supplying the resources (including labor) owned to the business firms and/or from receiving shares (given) of profits of firms.

\[(2.4) \quad Y^i = \sum_{r=1}^{R} W_r R^i_r + \sum_{g=1}^{G} S^i_g (P_g G^i_g - \sum_{r=1}^{R} W_r R^i_r)\]

where \(S^i_g\) is the share of individual \(i\) in the profit of the firm producing good \(g\). Each individual is taken to be small relative to the whole economy and hence takes the prices of goods and resources as well as her profits shares (relatively and absolutely) as given and maximizes (2.2) subject to (2.3) and (2.4) (the latter two equations may be combined into one), giving first-order conditions:

\[(2.5) \quad \frac{\partial U^i}{\partial G^i_g} / \frac{\partial U^i}{\partial G^i_G} = \frac{P_g}{P_G}; \quad g = 1, \ldots, G-1; \quad i = 1, \ldots, I \quad \text{(utility maximization)}\]

These conditions of MRS (marginal rates of substitution) = price ratios, together with the budget constraints above, determine the individual demand functions for the various goods.

Each firm has a production function or equivalently a cost function with conventional properties except that increasing returns are allowed:

\[(2.6) \quad C^g = C^g(W_1, \ldots, W_R, G_g); \quad g = 1, \ldots, G \quad \text{(cost functions)}\]

The demand for resources by each firm is derived from cost minimization subject to output constraint:

³ That some of each good is needed may also be included to ensure an interior solution and the use of the conventional first-order conditions. Also, points like the no explicit treatment of joint production are also implicit.
(2.7) \[
\frac{\partial G_g}{\partial R_{rg}} / \frac{\partial R_g}{\partial G_r} = \frac{W_r}{W_s} ; r, s = 1, \ldots, R; g = 1, \ldots, G
\] (cost minimization)

(2.8) \[
G_g(R_{1g}, \ldots, R_{Rg}) = G_g; g = 1, \ldots, G
\] (production functions)

where \(R_{rg}\) is the amount of resource \(r\) used in the production of good \(g\). The production functions and resource constraints define the production possibility set and the corresponding production possibility frontier (super-surface) for the economy. This set, while conventional in other aspects (it thus remains homeomorphic to a convex and compact set; conventional assumptions like divisibility in goods and factors, etc. remain), need not be convex (due to the allowance for increasing returns).

While we allow for increasing returns in the production of goods and hence do not require perfect competition in all the product markets, we continue to assume perfect competition in the resource markets, i.e. we do not consider monopolistic power in the employment of factors. Thus, we still have the tangency condition between the marginal rates of technical substitution and the resource price ratios which is common to all firms, as specified in (2.7).

Whether a good is produced under the condition of perfect competition or monopolistic competition with free entry, or a public utility constrained to just break even, we have average-cost pricing at the long-run equilibrium:

(2.9) \[
P_g = C_g(W_1, \ldots, W_R, G_g)/G_g; g = 1, \ldots, G
\] (average-cost pricing)

With average-cost pricing from free entry (as in the Dixit-Stiglitz model), the tangency point of the average-cost curve with the demand curve also entails the point of profit maximization as all other points entails negative profits. In all three cases (perfect competition, monopolistic competition, constrained public utility), (2.9), together with (2.7) and (2.8), describe the situation where each firm is in an average-cost pricing equilibrium.

General equilibrium is defined as a set of relative product prices \(P_i/P_G\), a set of relative resource prices \(W_i/W_R\); product demands \(G_i^g; i = 1, \ldots, I; g = 1, \ldots, G\); and output level \(G_g; g = 1, \ldots, G\) such that all individuals are at utility maximization equilibrium and all firms are at average-cost pricing equilibrium described above and all markets clear.

(2.10) \[
G^1_g + \ldots + G^I_g = G_g; g = 1, \ldots, G
\] (product market clearance)

(2.11) \[
R_{r1} + \ldots + R_{rg} = R_r; r = 1, \ldots, R
\] (resource market clearance)

**Theorem 1 (Brown-Heal generalized):** A productively efficient average-cost pricing equilibrium exists.

The general proof of this theorem is not dependent on the graphical presentation for the special case of just two goods for two dimensional illustration to improve understanding of the general proof. Due to the (possible) presence of increasing returns, the production possibility set need not be convex. However, with the traditional assumptions including divisibility of goods, the set remain homeomorphic (topologically equivalent) to a convex set. Similarly, its outer boundary, the production possibility frontier (PPF) also remains homeomorphic to a convex and compact set. Take
any point \( G^0 \) on the PPF. Take the set of resource prices consistent with cost-minimization production at this point and of product prices consistent with average-cost pricing at this point (\( G^0 \)). Locate the consumption (or product demand) point \( G^1 \) of consumer utility maximization at this set of prices (and the corresponding budget constraints). In general, \( G^1 \) and \( G^0 \) are different points and \( G^1 \) needs not be on the PPF. Construct a ray from the origin to pass through \( G^1 \). The intersection of this ray and the PPF is unique as the PPF is downward-sloping and the ray is upward-sloping. Call this intersection point \( G^2 \). This completes the description of the transformation of any point \( G^0 \) on the PPF to another point \( G^2 \) on the PPF. Under the traditional set of assumptions (including divisibility of goods and continuity of preferences), this transformation is continuous; any point arbitrarily close to \( G^0 \) will be transformed (mapped) into a point arbitrarily close to \( G^2 \). Since the PPF is homeomorphic to a convex and compact set and since the transformation (mapping) is continuous and map the set into itself, from Brouwer’s fixed point theorem, there exist a fixed point that maps into itself. In other words, there exists a point \( G^* \) on the PPF where \( G^0 \) and \( G^2 \) are the same point, i.e. \( G^* = G^0 = G^1 \). (See Figure 2.) By construction, this point \( G^* \) meets the requirements of profit maximization and cost minimization for the firms and utility maximization for the consumers with average-cost pricing for the firms. It is thus an AC-pricing general equilibrium and it is productively efficient as it is on the PPF. A general proof not confined to the two dimensional illustration follows.
**Proof:** In our model, while the production possibility set for the economy need not be convex, the production possibility frontier (super-surface) is topologically equivalent to a compact and convex set. From Brouwer’s fixed point theorem, any continuous mapping/transformation of a set that is homeomorphic (topologically equivalent) to a compact and convex set into itself possess a fixed point. Thus, any continuous mapping of the production possibility frontier (PPF) has a fixed point that maps into itself.

Consider the following continuous mapping $\Phi$ of PPF into PPF: $(G_1, \ldots, G_g)^0 \to (W_1/W_R, \ldots, W_{R-1}/W_R)^0$, $(P_1, \ldots, P_G)^0 \to (G_1, \ldots, G_G)^1 \to (G_1, \ldots, G_G)^2$, where

(i) $(G_1, \ldots, G_g)^0$ is an arbitrary point on the PPF.

(ii) $(W_1/W_R, \ldots, W_{R-1}/W_R)^0$ is the set of relative resource prices determined by the common (to all firms using the same pair of resources) marginal rates of technical substitution as specified in (2.7) above at the point $G^G(\mathbf R_1 G, \ldots, \mathbf R_R G) = G^0 G_i G_G$; $g = 1, \ldots, G$, i.e. the same point as $(G_1, \ldots, G_g)^0$.

(iii) $(P_1, \ldots, P_G)^0$ is the set of product prices determined in accordance to the average cost of producing each good $g$, i.e. $P^0 G_g = C^G W_1, \ldots, W_R, G_g/G_G$; $g = 1, \ldots, G$, at the given production levels given by $(G_1, \ldots, G_g)^0$.

(iv) $(G_1^d, \ldots, G_G^d)^1$ is the market demand for the various goods, i.e. $G^d G_g = G^{1 G}_g + \ldots, G^{1 G}_G$; $g = 1, \ldots, G$, where each $G^{1 G}_g$ is the individual utility-maximization quantity of $g$ good demanded by individual $i$ at the set of product prices $(P_1, \ldots, P_G)^0$ and resource prices $(W_1/W_R, \ldots, W_{R-1}/W_R)^0$.
3. The Efficiency of Encouraging Goods with High Degrees of Increasing Returns

Next, it will be shown that, from the viewpoint of Pareto optimality, goods with increasing returns are under-produced relative to those without increasing returns and those with higher degrees of increasing returns are under-produced relative to those with lower degrees. Subsidies on goods produced under conditions of (high degrees of) increasing returns financed by taxes on goods produced under non-increasing and lower increasing returns may increase efficiency. These results apply even though the production of all goods (including those with increasing returns) are assumed to be produced at prices equalling the average costs of production. This is first shown for generally in this section and next for a specific case in the next section. For the general case, two alternative methods are used, the first establishing a Pareto improvement and the second establishing positive net benefits using a reasonable cost-benefit analysis.

First, from an average-cost pricing equilibrium as established in the previous section, we have, as a condition for market equilibrium,

\[ P_g = A_g \]

where \( P_g \) and \( A_g \) are respectively the (producer) price and average cost of good \( g \).

For simplicity, as in the previous section, we abstract from the monopsonistic power of producers, i.e., we assume factor-price taking producers. (The absence of monopsonistic power is less unrealistic than that of perfect competition in the output markets.) This allows us to define the (local) degree of increasing returns to scale \( I_g \) (for any good \( g \)) as the percentage reduction in the average cost due to a one percentage increase in the output, or

\[ I_g \equiv \left( \frac{\partial A_g}{\partial G_g} \right) \frac{G_g}{A_g} \]

where \( G_g \) is the output level of good \( g \). This definition of the degree of increasing returns describes a situation of constant/decreasing returns as one having a zero/negative degree of increasing returns, as intuitively agreeable. This is convenient as it allows us to compare two goods in terms of their...
degrees of increasing returns, covering all cases of increasing, constant, and decreasing returns. This definition may be applied either to increasing returns at the firm or industry level, depending on the interpretation of \( G_g \). Since we are mainly concerned with firm-level increasing returns here, \( G_g \) is taken as the output level of firm \( g \). (With increasing returns, equilibrium implies a downward-sloping demand curve. Hence it is more appropriate to regard each firm as producing a distinct good, making the distinction of the firm and the industry no longer relevant.)

Each consumer maximizes a utility function

\[
U^i = U^i(G^i_1, G^i_2, \ldots, G^i_G); \ i = 1, \ldots, I
\]

subject to a budget constraint

\[
\sum_{g=1}^{G} P_g G_g^i = 0,
\]

where some of the \( g \)th ‘goods’ could be the negative amounts of (labour) services performed. (This notational convention differs from that of the previous section, but the essential substance does not differ.) The first-order conditions for an interior solution to this maximization problem are

\[
U^i_g / U^i_h = P_g / P_h; \ i = 1, \ldots, I; \ g, h = 1, \ldots, G,
\]

where \( U^i_g \equiv \partial U^i / \partial G^i_g \) is the marginal utility of good \( g \) to individual \( i \). Eq. 4.5 specifies the equality of (absolute) MRS (marginal rate of substitution) for any pair of goods with the price ratio. The second-order conditions are assumed satisfied; similarly for the following maximization problem.

We turn now to the conditions for Pareto optimality (or alternatively optimality in accordance to a Paretian social welfare function). Maximizing the utility of any individual given those of others and subject to the production possibility constraint

\[
F(G_1, \ldots, G_G) = 0,
\]

where \( G_g = \sum_{i=1}^{I} G_g^i \) for each \( g \), the first-order conditions are

\[
U_g^i / U_h^i = F_g / F_h; \ i = 1, \ldots, I; \ g, h = 1, \ldots, G,
\]

where \( U_g^i / U_h^i \) is the marginal rate of substitution (equalling the ratio of marginal utilities) and \( F_g / F_h \) is the marginal rate of transformation between \( g \) and \( h \) and the latter equals the ratio of the marginal costs of producing \( g \) and \( h \), \( M_g / M_h \). (The absence of external effects is assumed.) From (3.1) and (3.5), we have, for a market equilibrium,

\[
U_g^i / U_h^i = A_g / A_h; \ i = 1, \ldots, I; \ g, h = 1, \ldots, G.
\]

By definition, \( A_g \equiv C_g / G_g \) where \( C_g \) is the total costs of producing good \( g \). Thus, from (3.2) and from simple differentiation, we have, for all \( g \),

\[
I_g \equiv - (\partial A_g / \partial G_g) G_g / A_g = 1 - M_g / A_g,
\]

where \( M_g \equiv \partial C_g / \partial G_g \) is the marginal cost of producing good \( g \).

From (3.9), we have at any point

\[
I_g > I_h \text{ iff } A_g / A_h > M_g / M_h.
\]
Let us now start from any market equilibrium point \( P \). From (3.8) and (3.10), we have for this point \( P \),

\[
(3.11) \quad \text{MRS}_{gh} > \frac{M_g}{M_h} \text{ iff } I_g > I_h.
\]

At this point \( P \), either the economy is on its transformation curve or production possibility frontier (PPF) or it is inside it. If it is inside it, it is obvious that a Pareto improvement could be made by producing more of some or all goods, including good \( g \). Thus, Theorem 2A below obviously holds. Thus, let us take the case where the economy is on its PPF. (The existence of increasing returns does not rule out productive efficiency in the sense of producing at the frontier.) In the absence of external effects, the (absolute) slope of the production possibility frontier in the \( g/h \) plane (i.e. holding all goods other than \( g \) and \( h \) unchanged) equals \( \frac{M_g}{M_h} \) (with good \( g \) on the horizontal axis and good \( h \) on the vertical). Thus, from (3.11), if good \( g \) has a higher degree of increasing returns than good \( h \), the (absolute) slope of the PPF at \( P \) is less steep than the (absolute) MRS\(_{gh} \) (slope of the indifference curve which is tangent to the common price line) for all individuals who consume both \( g \) and \( h \). Thus, if good \( g \) has a higher degree of increasing returns than good \( h \), the market-equilibrium MRS\(_{gh} \) is larger than the slope of PPF. Given that both \( g \) and \( h \) are goods (not bads), both the indifference curve and the PPF at \( P \) are downward sloping. Given the usual assumptions about preferences (including local non-satiation), a movement down the (downward-sloping) transformation curve involving a larger amount of good \( g \) and a smaller amount of good \( h \) must lead to a point of higher indifference curve/surface. We thus have,

**Theorem 2A:** At an average-cost pricing market equilibrium, if the degree of increasing returns for good \( g \) is larger than that for good \( h \), a point of higher efficiency (Pareto-superior) could be reached by increasing the output of \( g \) and decreasing the output of \( h \) (holding the consumption/production of other goods unchanged).

Note that this proposition is applicable between any two goods of different degrees of increasing returns, including two of both decreasing returns. Efficiency could be increased by increasing/decreasing the good of less/more decreasing returns. It is obvious that, for such two goods, it may be even more efficient to decrease production of both of them and increase production of another (set of) good(s) that has even higher degree of increasing returns than both these two goods, if it exists. The validity of Theorem 2A partly depends on the assumed absence of external effects. Obviously, if good \( g \) happens to have a large amount of external costs, this may outweigh the efficiency consideration based on increasing returns. The validity of Theorem 2A also partly depends on the abstraction away of possible second-best offsetting effects. Holding the consumption/production of other goods unchanged allows us to benefit from the divergence between the MRS and MRT of the relevant pair of goods. If important inefficiencies exist in other goods, the variation in the quantities of these goods as a response to the change in the relative quantities of goods \( g \) and \( h \) may cause indirect effects that may be reinforcing or offsetting to the direct effects.
When we deal with situations in the real economy where it may not be possible to abstract away such second-best complications, it remains true that the efficiency gain of Theorem 2A may still be taken as applicable for the benchmark case. Alternatively stated, in the absence of specific knowledge on how the second-best consideration affects the result, the expected gain is still positive in accordance to the principle of third best (Ng 1979/1983, Sec.9.3). When the specific knowledge is available, we may then combine them to assess the net result. In actual application, what is more worrisome is the likely inefficiencies of incorrect identification, rent-seeking activities, corruption, and so on. Thus, unless the divergences involved are very large, a laissez-faire policy may still be optimal.

It is tempting but incorrect to make the following inference. Comparing the conditions for Pareto optimality (PO) specified in (3.7) and those for market equilibrium (ME) in (3.8), we have

\[(3.12) \quad \text{MRS}_{gh}^{\text{PO}} > \text{MRS}_{gh}^{\text{ME}} \iff \frac{M_g}{M_h} > \frac{A_g}{A_h}.\]

From (3.12) and (3.10), it might then be concluded that

\[(3.13) \quad I_g > I_h \iff \text{MRS}_{gh}^{\text{ME}} > \text{MRS}_{gh}^{\text{PO}}.\]

This inference is incorrect since each of the various inequalities applies to the same point while ME and PO are different points. As we move from a ME to a PO point, the values of goods other than g and h may be changed and this change may (though not necessarily so) affect the validity of (3.13). In particular, suppose there is another good k, which has even a higher degree of increasing returns than g (or some other efficiency reasons like external benefits, if not assumed absent, calling for its increased production). The movement from the original market equilibrium point P to the Pareto-optimal point Q may then involve a much increased output of good k. If k is a close Edgeworth-complement of g, the much higher value of good k may make MRS$_{gh}^{\text{PO}}$ larger than MRS$_{gh}^{\text{ME}}$ (all in absolute terms) and hence invalidating (3.13).

Next, a cost-benefit analysis is used to show that, ignoring the administrative, transitional, and indirect (e.g. rent-seeking) costs of making the change, the aggregate (over individuals) net benefits of increasing the output of a good (denoted H) with a higher degree of increasing returns made possible by reducing that (L) with a lower degree is positive. In our analysis where external effects and second-best complications (which may go either way) are absent/ignored, the aggregate net benefits of the above change by a marginal amount are measured by: the marginal value of good H times $\Delta G_H$ – the marginal costs of good H times $\Delta G_H$ – (the aggregate marginal value of good L times $\Delta G_L$ – the marginal costs of good L times $\Delta G_L$), i.e. the absolute value of the change in good L), or

\[(3.14) \quad \text{NB} = V_H \Delta G_H - M_H \Delta G_H - [V_L \Delta G_L - M_L \Delta G_L] \]

where NB = aggregate net benefit of the above change, V = marginal value, M = marginal cost. Since the change has to be feasible, the marginal costs of good H times $\Delta G_H$ must equal the marginal costs of good L times $\Delta G_L$.

\[(3.15) \quad M_H \Delta G_H = M_L \Delta G_L \]
Substitute (3.15) and the value of $\Delta G_H$ from (3.15) into (3.14), yielding

$$(3.16) \quad NB = [V_H M_L \mid \Delta G_L / M_H] - (V_L \mid \Delta G_L)$$

With average-cost pricing and individual utility maximization, we have marginal value = price = average cost. Thus, replacing marginal value $V$ by average cost $A$, (3.16) becomes

$$(3.17) \quad NB = (A_H - A_L) M_L \mid \Delta G_L$$

Since $M_L \mid \Delta G_L$ is positive, the aggregate net benefits of the above change $NB$ are positive if and only if $A_H / M_H - A_L / M_L$ is positive. From (3.9) above, a good $H$ has a higher degree of increasing returns than another good $L$ if and only if $A_H / M_H > A_L / M_L$. Thus, (3.17) is positive. We thus have

**Theorem 2B:** At an average-cost pricing market equilibrium, if the production/consumption of a good with a lower degree of increasing returns is decreased to allow for a corresponding increase in the production/consumption of another good with a higher degree of increasing returns, holding the consumption/production of other goods unchanged, the aggregate net benefits of the change is positive.

From Theorems 2A and 2B, we may state the following

**Corollary:** A necessary condition for an average-cost pricing market equilibrium to be Pareto optimal is for every good to have the same degree of increasing returns.

However, as noted above, leisure may be regarded as a good that each individual may produce from labor time at the rate of one to one, i.e. with constant returns. Thus, in the presence of increasing returns in the production of some goods, an average-cost pricing market equilibrium cannot be Pareto-optimal, as at least leisure will be over-consumed in comparison to those goods with increasing returns, unless offset by other considerations like environmental disruption not considered in the above analysis, as traditional.

**4. A Specific Case Illustrated**

As this is just an illustrative specific case, let us opt for simplicity by considering the case of a Cobb-Douglas utility function with three goods (with good $Z$ being leisure/labour) and simple production functions with no intermediate inputs. The representative (atomistic) individual maximizes

$$(4.1) \quad U = \ln x + \ln y + \ln z$$

subject to the budget constraint

$$(4.2) \quad (p - s)x + (q + t)y = 3 - z$$
where \( x \) and \( y \) are the amounts of the two goods consumed, \( p \) and \( q \) are the producer prices of these two goods, \( s \) and \( t \) are the per unit subsidy/tax on \( X \) and \( Y \), leisure/labour \( Z \) is used as the *numeraire*, and the amount of time endowment is normalized to equal 3 to simplify calculation without affecting the substantive result. The maximization gives the following demand functions

\[
(4.3) \quad x = 1/(p-s); \quad y = 1/(q+t); \quad z = 1.
\]

On the supply side, let the production functions of the two goods be

\[
(4.4) \quad x = z^2; \quad y = z^2;
\]

to represent the specific case where good \( X \) is subject to large increasing returns and \( Y \) to constant returns with respect to the amount of labour (only input) used. Average-cost pricing then ensures that the relevant supply functions are

\[
(4.5) \quad x = 1/p^2; \quad q = 1.
\]

The equation of supply and demand at each market gives us the equilibrium prices and quantities:

\[
(4.6) \quad x = 1/(p-s); \quad p = [1 + (1-4s)^{1/2}] / 2; \quad q = 1; \quad y = 1/(1+t).
\]

The requirement of government budget balance necessitates

\[
(4.7) \quad sx = ty.
\]

Substituting \( x \) (after substituting out \( p \)) and \( y \) from (4.6) into (4.7), we have

\[
(4.8) \quad t = 2s/[1+(1-4s)^{1/2} - 4s].
\]

Substitute \( t \) from (4.8) into the expression for \( y \) in (4.6) and the resulting values for \( x \) and \( y \), and also \( z = 1 \) (from Eq.4.3) into (4.1), we have \( U \) only as a function of \( s \). We may then examine how \( U \) varies with \( s \) and conclude accordingly. However, mathematically it is more convenient to get the same conclusions by the following alternative method.

Starting from a situation without any subsidy and tax (i.e. \( s = t = 0 \)), we have \( x = y = z = 1 \), and \( U = 0 \). Then consider a situation of \( s = 0.025 \) and \( t = 0.027046276 \) (which satisfy the requirement of budget balance, as may be verified). We then have, \( p = 0.97434165, \quad x = 1.05336156, \quad y = 0.97366596, \quad \text{and} \quad U = 0.0786735 \) which is an increase from the previous case of no subsidy/tax. However, we could do better in finding out the optimal \( s \) and \( t \) that satisfy budget balance and maximize \( U \). From (4.2) – (4.5), we may derive the feasible values of \( x \) and \( y \) as given by

\[
(4.9) \quad y = 2 - x^{1/2}.
\]

Substituting \( y \) from (4.9) and \( z = 1 \) into (4.1), we may express \( U \) as a function of \( x \) only.

\[
(4.10) \quad U = \ln x + \ln (2 - x^{1/2})
\]

Maximizing this function with respect to \( x \), we have the optimal value of \( x \) as \( x = 1.77777\ldots \) and the corresponding value of \( y \) (from Eq. 4.9) as \( y = 0.6666\ldots \). With \( z = 1 \), this give the optimised value of \( U \) as approximately 0.17. We may then go back to ask for the values of \( s \) and \( t \) that will both meet the budget balance and achieve these values for \( x \) and \( y \), obtaining,

\[
(4.11) \quad s = 0.1875; \quad t = 0.5
\]
as the optimal tax/subsidy. Of course, these high optimal values of tax/subsidy depends on the unrealistically high degree of increasing returns in the production of X which is not meant to be descriptive.

The above depends on the ability to identify which goods have higher degrees of increasing returns. In the absence of this knowledge or if all goods have the same degree of increasing returns, could taxes/subsidies be used to encourage more production to tap the increasing returns? If we rule out lump-sum taxes and taxes on leisure as impracticable and confine to the normal type of taxes/subsidies on consumption (or production) and income, and require the government to observe the balanced budget requirement, it is not difficult to show that no improvement is possible through taxes/subsidies. The intuitive explanation is that a subsidy on consumption (work) has to be financed by a tax on income (consumption) which exactly offsets the subsidy.

References


HEAL, Geoffrey (1999), *The Economics of Increasing Returns*, Edward Elgar.


