Hence, it is clear that not all λ_i's equal ± 1, but for all the eigenvalues, we have $||\lambda_i|| = 1$, $i = 1,\dots,n$. It is also clear that orthogonal matrix A only has eigenvalues ± 1 if and only if it is symmetric.

NOTE

1. Excellent solutions also have been proposed by R.W. Farebrother, G. Trenkler, and by K.M. Abadir and K. Hadri, the posers of the problem.

REFERENCE

Horn, R.A. & C.R. Johnson (1985) *Matrix Analysis.* New York: Cambridge University Press.

96.5.4. *On the Bias of Standard Errors of the LS Residual under Nonnormal Errors*—Solution,[1] proposed by Offer Lieberman, Aman Ullah, and Robert Breunig. Let us write

$$s_u = (s_u^2)^{1/2} = (s_u^2 - \sigma^2 + \sigma^2)^{1/2} = \sigma\left(1 + \frac{s_u^2 - \sigma^2}{\sigma^2}\right)^{1/2}$$

$$\simeq \sigma\left[1 + \frac{1}{2}\left(\frac{s_u^2 - \sigma^2}{\sigma^2}\right) - \frac{1}{8}\left(\frac{s_u^2 - \sigma^2}{\sigma^2}\right)^2\right] \qquad \textbf{(8)}$$

up to $O_p(n^{-1})$, where $s_u^2 - \sigma^2 = O_p(n^{-1/2})$. Taking expectations on both sides of (8) and using $Es_u^2 = \sigma^2$ (see (11) below) we get $O(n^{-1})$,

$$E(s_u - \sigma) = -\frac{1}{8\sigma^3} E(s_u^2 - \sigma^2)^2. \qquad \textbf{(9)}$$

Further, up to $O(n^{-3/2})$, we get

$$E\{(\hat{V}(b_j))^{1/2}\} - (V(b_j))^{1/2} = (E\{s_u - \sigma\})a_{jj}^{1/2}. \qquad \textbf{(10)}$$

To obtain $E(s_u^2 - \sigma^2)^2$ under nonnormal errors (2), we note that (see Ullah and Srivastava, 1994, eq. 2.18)

$$E(u'Mu) = \sigma^2 \operatorname{tr}(M) = (n-k)\sigma^2$$
$$E(u'Mu)^2 = \sigma^4[\gamma_2 \operatorname{tr}(\dot{M}) + (n-k)(n-k+2)]. \qquad \textbf{(11)}$$

Thus, $Es_u^2 = \sigma^2$ and

$$V(s_u^2) = E(s_u^2 - \sigma^2)^2 = Es_u^4 - \sigma^4 = \frac{1}{(n-k)^2} E(u'Mu)^2 - \sigma^4$$
$$= \frac{2\sigma^4}{(n-k)}\left[1 + \gamma_2 \frac{\operatorname{tr}(\dot{M})}{2(n-k)}\right], \qquad \textbf{(12)}$$

which is the result in (6). Next, substituting (12) in (9) provides the result in (4). Further, substituting (9) and (12) in (10) provides the result in (5). Note that σ^4 in (4) and (5) should be read as σ^3.

Finally, observe that $\operatorname{tr}(\dot{M}) = n - 2k + O(1/n)$. Substituting this in (2) and taking the limit as $n \to \infty$, we get (7).

NOTE

1. The editor is grateful to O. Leiberman for pointing out that σ^4 in equations (4) and (5) of Problem 96.5.4 should be σ^3.

REFERENCE

Ullah, A. & V.K. Srivastava (1994) Moments of the ratio of quadratic forms in non-normal variables with econometric examples. *Journal of Econometrics* 62, 129–141.

96.5.5. *Linear Combinations of Stationary Processes*—Solution.[1] Two solutions have been proposed independently by Geert Dhaene and by Gordon C.R. Kemp, which are published below. Each contains an interesting derivation.

1. Solution proposed by Geert Dhaene. Linear combinations of (covariance) stationary processes are not always (covariance) stationary. To show this, let $\{u_t\}$ and $\{v_t\}$ be two nondegenerate i.i.d. processes that, moreover, are mutually independent and identical. Further, define $\{w_t\}$ by

$$w_t = \begin{cases} u_t & \text{if } t \text{ is even} \\ v_t & \text{otherwise.} \end{cases}$$

Clearly, $\{u_t\}$ and $\{w_t\}$ are both i.i.d. and thus stationary (and mutually identical but not independent). The linear combination $\{u_t - w_t\}$, however, is not (covariance) stationary because the subsequence $\{u_{2t+1} - w_{2t+1}\}$ is nondegenerate, whereas the shifted subsequence $\{u_{2t} - w_{2t}\}$ is zero.

2. Solution proposed by Gordon C.R. Kemp. The following simple example demonstrates that arbitrary linear combinations of two or more (covariance) stationary processes need not be (covariance) stationary. Let $\{\epsilon_n\}_{-\infty}^{\infty}$ be a sequence of i.i.d. $N(0,\sigma^2)$ random variables, with $\sigma^2 > 0$, and let $X_n = \epsilon_n$ and $Y_n = (-1)^n \epsilon_n$ for $n = 0, \pm 1, \pm 2, \ldots$. Clearly, both $\{X_n\}_{-\infty}^{\infty}$ and $\{Y_n\}_{-\infty}^{\infty}$ are also i.i.d. $N(0,\sigma^2)$ and so are (covariance) stationary processes. Now, define $Z_n = (X_n + Y_n)$ for all $n = 0, \pm 1, \pm 2, \ldots$; then

$$Z_n = [1 + (-1)^n]\epsilon_n = \begin{cases} 2\epsilon_n & \text{if } n \text{ is even,} \\ 0 & \text{if } n \text{ is odd.} \end{cases}$$

Hence, $\operatorname{var}(Z_n) = 4\sigma^2$ if n is even, whereas $\operatorname{var}(Z_n) = 0$ if n is odd; clearly, as the variance of Z_n is not invariant to changes in n, the process $\{Z_n\}_{-\infty}^{\infty}$ is not (covariance) stationary.

However, an arbitrary linear combination of two (or more) jointly covariance stationary processes will itself be covariance stationary. Thus, let $\{u_{1,n}\}_{-\infty}^{\infty}, \ldots, \{u_{p,n}\}_{-\infty}^{\infty}$ be jointly covariance stationary processes so that $E(u_{i,n}) = \mu_i$ and