Show that the ML estimators $\hat{\mu}(y)$ and $\hat{\alpha}(y)$ are equivariant estimators of μ and α, that is, $\hat{\mu}(y^*) = d\hat{\mu}(y) + k$ and $\hat{\alpha}(y^*) = d\hat{\alpha}(y)$, for any data transformation, $y_i^* = dy_i + k$, $d > 0$ is a scalar, and k is a real vector.

96.5.3. *Roots of an Orthogonal Matrix*, proposed by Karim M. Abadir and Kaddour Hadri. Derive the eigenvalues of an orthogonal matrix **A** by showing that they need not be restricted to be ± 1 only.

96.5.4. *On the Bias of Standard Errors of the LS Residual and the Regression Coefficients under the Nonnormal Errors*, proposed by Aman Ullah and Robert Breuning. Let us consider the linear regression model

$$y = X\beta + u, \tag{1}$$

where y is an $n \times 1$ vector, X is an $n \times k$ matrix of nonstochastic elements, β is a $k \times 1$ vector of regression coefficients, and u is an $n \times 1$ disturbance vector whose elements are i.i.d. such that, for $i = 1, \dots, n$,

$$Eu_i = 0, \qquad Eu_i^2 = \sigma^2, \qquad Eu_i^3 = \gamma_1 \sigma^3, \qquad Eu_i^4 = (\gamma_2 + 3)\sigma^4, \tag{2}$$

where γ_1 and γ_2 are Pearson's measure of skewness and kurtosis of the distribution. Let

$$b = (X'X)^{-1}X'y, \qquad s_u^2 = \frac{1}{n-k}\, u'Mu; \qquad M = I - X(X'X)^{-1}X' \tag{3}$$

and $\hat{V}(b) = s_u^2(X'X)^{-1}$ be the estimators of β, σ^2, and $V(b)$, respectively. Assuming $X'X = O(n)$ show that, up to $O(n^{-1})$,

$$\text{Bias}(s_u) = E(s_u) - \sigma = -\frac{1}{8\sigma^4}\, V(s_u^2) \tag{4}$$

and, up to $O(n^{-3/2})$,

$$\text{Bias}\{(\hat{V}(b_j))^{1/2}\} = E\{(\hat{V}(b_j))^{1/2}\} - (V(b_j))^{1/2} = -\frac{1}{8\sigma^4}\, V(s_u^2) a_{jj}^{1/2}, \tag{5}$$

where $j = 1, \dots, k$, a_{jj} is the diagonal element of $(X'X)^{-1}$, and

$$V(s_u^2) = \frac{2\sigma^4}{(n-k)} \left[1 + \frac{\gamma_2 \operatorname{tr}(\dot{M})}{2(n-k)} \right]; \tag{6}$$

$\dot{M}$ is the matrix of the squared elements of M.

Further, show that

$$\frac{V(s_u^2)}{V(s_u^2)_N} = 1 + \frac{\gamma_2}{2} \tag{7}$$

as $n \to \infty$, where $V(s_u^2)_N$ is the $V(s_u^2)$ under normality ($\gamma_2 = o$).