THE AUSTRALIAN NATIONAL UNIVERSITY

First Semester Final Examination 2012

Masters Microeconomics

IDEC8064

Writing period: 3 Hours duration
Study period: 30 Minutes duration
Permitted materials: English/Foreign Language Dictionary

You must attempt to answer all questions.
Total points: 100

All questions to be completed in the script book provided.

This exam has 6 pages.
1. Provide a **clear explanation** (using English and graphs where appropriate) for each of the following statements (5 points each):

(a) In a two-sector general equilibrium model, two things are clear: (a) full employment of capital and labour does not necessarily imply efficient levels of output, and (b) an increase in the demand and price of a good produced with relatively labour intensive technology will result in increasing costs and a long-run supply function that is positively sloped for that good.

(b) A common property equilibrium for a fishery is inefficient because fishing firms fail to take account of their actions on the overall stock of fish and the cost of fishing and all of the economic profit or economic rent due to the fishery (as a valuable input to production) is lost.

(c) The use of ‘individual transferable quotas’ combined with a total allowable catch (TAC) can result in efficiency gains as a result of the transfer of quota from high to low cost producers. This is generally preferable to the use of effort controls, which restrict a fishing inputs, and a TAC alone, since both result in ‘rent dissipation’ or lost profits.

(d) In consumer theory, a Hicksian compensated demand curve is always negatively sloped, but the Marshallian demand curve may have a positive or negative slope, and can be either steeper or less steep than the Hicksian compensated demand curve.

(e) The smaller the ‘rate of time preference’ \((\rho)\) and the lower the ‘inverse of the constant intertemporal rate of substitution’ \((\theta)\), the higher is the steady-state value of consumption and the faster the rate of growth in capital per unit of labour in transition to the steady-state.

(f) If an optimal dynamic model of fishing, assuming that intertemporal profits are maximized, if the initial stock of fish is less than its steady-state value, while initial harvest is greater than its steady state value, then the optimal transition to the steady-state must imply an immediate fall in harvest and a gradual increase in both the stock of fish and harvest.

2. For two inputs \(x_1\) and \(x_2\), assume that a firm has a basic production function given by

\[
Q = A x_1^{0.5} x_2^{0.25}
\]

and faces market-given exogenous input prices \(w = (w_1, w_2)\) and output price \(p\).
(a) Maximize profits and solve for optimal input demands \( x_1^*(p,w_1,w_2) \), \( x_2^*(p,w_1,w_2) \), profit maximizing output \( Q^*(p,w_1,w_2) \), and the maximum-value profit function \( \pi^*(p,w_1,w_2) \). (5 points)

(b) Minimize the cost of producing \( Q \) units of output, solving for the cost-minimizing input demands, \( \hat{x}_1(w_1,w_2,Q) \) and \( \hat{x}_2(w_1,w_2,Q) \), and construct the minimum-value cost function \( C^*(w_1,w_2,Q) \). (5 points)

(c) Calculate the marginal cost function \( MC(w_1,w_2,Q) = \partial C^*(w_1,w_2,Q) / \partial Q \), verify the relationship between marginal cost and a change in output and provide a clear explanation for this result. (5 points)

(d) Verify and provide a clear explanation for the following statement: “The minimum value cost function for this problem, \( C^*(w_1,w_2,Q) \), is concave in input price \( w_i \).” (5 points)

(e) Verify and provide a clear explanation for each of the following (5 points):

(i) \( \hat{x}_1(w_1,w_2,Q^*(p,w_1,w_2)) = x_1^*(p,w_1,w_2) \)

(ii) \( MC(w_1,w_2,Q^*(p,w_1,w_2)) = p \)

3. Assume a simple production function given by

\[
Q = F(x_1,x_2) = Ax_1^\alpha x_2^\beta
\]

and let \( \alpha = \beta = 0.5 \) so that constant returns to scale (CRS) holds.

(a) Derive the minimum value cost function for input prices \( w_1 \) and \( w_2 \) of the form

\[
C^* = C(w_1,w_2,Q)
\]

for this production function. (5 points)

(b) Assume that the data set under examination contains no information on each \( w_i \) but that income shares \( \varepsilon_1 = w_1 x_1 / Q \) and \( \varepsilon_2 = w_2 x_2 / Q \) are available. With constant returns to scale and all inputs variable derive a form of the cost function given by \( C^* = C(\varepsilon_1,\varepsilon_2,Q) \) that is equivalent to equation (2). (5 points)
4. Consider the following technical inefficiency model estimates drawn from the stochastic production frontier estimates for rice production in Vietnam:

<table>
<thead>
<tr>
<th>Technical Inefficiency Model</th>
<th>Coefficient</th>
<th>T-ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>Constant</td>
<td>-2.13</td>
<td>-2.95</td>
</tr>
<tr>
<td>Education (ED)</td>
<td>-0.74</td>
<td>-4.46</td>
</tr>
<tr>
<td>Plots (PLOTS)</td>
<td>0.25</td>
<td>2.96</td>
</tr>
<tr>
<td>Ratio of land with LUC (CERT)</td>
<td>-0.24</td>
<td>-3.48</td>
</tr>
<tr>
<td>Ratio of good quality land (QUAL)</td>
<td>-1.61</td>
<td>-4.65</td>
</tr>
<tr>
<td>Access to extension services (EXT)</td>
<td>-0.34</td>
<td>-4.43</td>
</tr>
<tr>
<td>Access to credit (CRE)</td>
<td>-0.10</td>
<td>-1.75</td>
</tr>
</tbody>
</table>

where ED is the level of education of the farmer, PLOTS is the average number of plots on a farm, CERT is the presence of a land use certificate, QUAL is a measure of soil quality and access to irrigation, EXT is access to extension services and CRE is access to farm credit.

(a) Provide explanations for the signs on each variable (i.e., what is the likely explanation for a negative sign on the education (ED) variable, a positive sign on the plots (PLOTS) variable, and a negative sign on the CERT, QUAL, EXT and CRE variables)? (5 points)

(b) What are the key policy issues and suggested policy responses that result from this estimate of technical inefficiency in rice production (i.e., what are the recommended actions for government policy makers in Vietnam)? (5 points)

5. For boundary conditions \( w(0) = w_0 \) and \( w(T) = b > 0 \), solve the following ‘life cycle’ consumption problem

\[
\max \int_0^T e^{-\rho t} \frac{c^{1-\theta}}{1-\theta} \, dt \quad s.t. \quad dw/dt = -c(t)
\]
for the optimal time paths $c^*(t)$ and $w^*(t)$ . (5 points)

6. Consider the following figure drawn from the paper on the bioeconomic model of the Western and Central Pacific tuna fishery:

Figure 2 Net present value of profit (2008 prices in US$ millions) of dynamic $B_{MEY}$ and business-as-usual transition paths.

for MEY ‘Maximum Economic Yield’ and BAU ‘Business as Usual’ in the fishery, over a fifty year horizon from the present.

Provide a clear explanation for: (a) why both curves (for the most part) slope downward over time? (b) what does the difference between MEY and BAU measure? (c) why is MEY less than BAU initially, but then MEY is greater than BAU throughout? (d) why is the time over which MEY is less than BAU so short? (5 points)

7. Consider the following basic utility function

$$u(x_1,x_2) = x_1x_2$$

for consumer goods $x_1$ and $x_2$. Assume a level of money or nominal income $M$ and let prices for the two consumer goods be given by $p_1$ and $p_2$.

(a) Derive both the Marshallian demands for goods, $x^*_1(p_1, p_2, M)$ and $x^*_2(p_1, p_2, M)$ and Hicksian compensated demands $\hat{x}_1(p_1, p_2, U)$ and $\hat{x}_2(p_1, p_2, U)$. (5 points)

(b) Using an indirect utility function $v(p_1, p_2, M)$ and minimum expenditure function $e(p_1, p_2, U)$, verify the following ‘duality identities’ (5 points):

$$v(p, e(p, u)) = u$$
$$e(p, v(p, M)) = M$$
\[ x^*(p,e(p,u)) = \dot{x}(p,u) \]

(c) Finally, evaluate the relevant partial derivatives and verify the Slutsky equation given by (5 points):

\[ \frac{\partial x_i^*}{\partial p_i} = \frac{\partial \dot{x}_i}{\partial p_i} - x_i \frac{\partial x_i^*}{\partial M} \]