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Keywords

Quasi-hyperbolic discounting, Monetary policy, Time-consistency

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Monetary Policy when Preferences are Quasi-Hyperbolic*

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Abstract

We study discretionary monetary policy in an economy where economic agents have quasi-hyperbolic discounting. We demonstrate that a benevolent central bank is able to keep inflation under control for a wide range of discount factors. If the central bank, however, does not adopt the household's time preferences and tries to discourage early-consumption and delayed-saving, then a marginal increase in steady state output is achieved at the cost of a much higher average inflation rate. Indeed, we show that it is desirable from a welfare perspective for the central bank to quasi-hyperbolically discount by more than households do. Welfare is improved because this discount structure emphasizes the current-period cost of price changes and leads to lower average inflation. We contrast our results with those obtained when policy is conducted according to a Taylor-type rule.

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*All remaining errors are ours.

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1 Introduction

A large body of experimental evidence suggests that consumers discount the future more in the short-term than they do in the long-term (Ainslie, 1992). This phenomenon, captured by the notion that households have hyperbolic discounting, suggests that consumers desire instant gratification (Harris and Laibson, 2001) and that they value mechanisms that enable them to better exercise self-control and/or to constrain their future selves (Strotz 1956; Laibson 1997). When they discount the future hyperbolically, households value savings for the future income and insurance that they provide, yet cannot resist splurging a little on consumption today while planning to save for the future tomorrow. If they recognize that this behavior will repeat itself day after day, leading them to over-consume and under-save, then today's household will have an incentive to purchase illiquid assets in order to constrain themselves from over-consuming tomorrow. In principle, the same time-inconsistent behavior applies to other intertemporal decisions, such as the purchase of durable goods, and it can be applied to price-setting, capital accumulation, and inventory management decisions, where the firm is operating for the benefit of its hyperbolic equity-holders.

Although hyperbolic discounting features importantly in behavioral economics (Wilkinson and Klaes, 2017), there are relatively few instances of hyperbolic discounting appearing in general equilibrium macroeconomic contexts. Where hyperbolic discounting is considered it invariably appears in the form of quasi-hyperbolic discounting, which combines the usual geometric discounting with a separate factor that discounts all future periods relative to today (Phelps and Pollak, 1968; Laibson, 1997). Studies that have considered quasi-hyperbolic discounting in macroeconomic models have largely concentrated on the stochastic growth model and focused on the possibility of multiple equilibria arising through strategic interaction between the household and its future self (Krusell and Smith, 2003; Maliar and Maliar, 2005, 2006a). Applications of quasi-hyperbolic discounting include Krusell, Kuruşçu, and Smith (2002), who show that the solution to the planner's problem delivers lower welfare than the competitive equilibrium when households have quasi-hyperbolic discounting, and Graham and Snower (2013), who examine a sticky-wage New Keynesian model and demonstrate that quasi-hyperbolic discounting can overturn the Friedman rule. In Graham and Snower's model households prefer positive inflation because it erodes the real wage over time, leading them to work relatively less today and relatively more in the (quasi-hyperbolically discounted) future. Maliar and Maliar (2006b) build on Angeletos, Laibson, Repetto, Tobacman, and Weinberg (2001) and study a neoclassical growth model with heterogeneous households facing idiosyncratic labor productivity shocks and a borrowing constraint. They find that quasi-hyperbolic discounting has a large impact on the income distribution. Maeda (2018) extends Krusell and Smith (2002) to a monetary economy with a cash-in-advance constraint and shows that this constraint on cash-holdings prevents households from over-consuming in equilibrium and, when the government can only control money growth and not taxes, leads to the Friedman rule holding.

In this paper we examine quasi-hyperbolic discounting in a New Keynesian business cycle model and we explore the implications this form of discounting has for how the central bank should conduct monetary policy. The model is one in which monopolistically competitive firms employ capital and labor to produce goods and who set prices subject to Rotemberg (1982) adjustment costs. Households consume goods and supply labor and they have a portfolio of bonds and equities in which to save. In our benchmark scenario, the central bank conducts monetary policy optimally under discretion. Although the model is standard in many respects, quasi-hyperbolic discounting introduces important complications because the household's decision problem is no longer time-consistent. These complications are compounded by the fact that monetary policy is conducted with discretion. Most dynamic stochastic general equilibrium models with quasi-hyperbolic discounting must be solved numerically, which can be challenging because the strategic interactions between households and their future-selves can give rise to multiple equilibria (Krusell and Smith, 2003). We avoid the indeterminacy associated with log-linearization (Maliar and Maliar, 2006a) by solving our nonlinear model using a global solution method and we obtain a unique stable equilibrium by computing the interior solution to a system of generalized Euler equations (as recommended in Maliar and Maliar, 2005). Although the presence of sticky prices and optimal policymaking greatly complicates our model, we obtain considerable simplification by imposing symmetry on household and firm behavior in equilibrium, thereby precluding equilibria that exhibit heterogeneity.

Unlike previous studies that have focused largely on the effect that quasi-hyperbolic discounting has on consumption, saving, and labour supply, we focus on its implications for how the central bank should conduct monetary policy. In the absence of an efficient subsidy to offset the monopolistic distortion, discretionary monetary policy gives rise to both an inflation bias and a stabilization bias. We quantify the impact that the household's quasi-hyperbolic discounting has on how monetary policy is conducted and quantify the magnitude of the discretionary inflation bias. Next, we allow the central bank to also have quasi-hyperbolic discounting and examine the implications the central bank's discounting has for monetary policy. Lastly, we ask whether it is desirable for the central bank to be benevolent, i.e., whether it is desirable for the central bank to quasi-hyperbolically discount the future by more, less, or at the same rate as households. We contrast our results for discretionary policymaking with those from a Taylor-type rule.

We obtain five main results. First, consistent with previous studies, we find that quasi-hyperbolic households over-consume and under-save in equilibrium, leading to a capital stock that is smaller than it would be if households discounted geometrically. Second, although discretionary monetary policy continues to result in positive average inflation, because the central bank tries to use inflation surprises to raise output (discretionary inflation bias), the size of the discretionary inflation bias is somewhat smaller when households have quasi-hyperbolic discounting. This result emerges because firms make their pricing and production decisions to maximize their equity-value. Because it is costly to change prices and their equity-holders have quasi-hyperbolic discounting, firms choose to make smaller price changes in response to shocks and to spread price-changes out over time. Allowing the central bank to have quasi-hyperbolic discounting operates in a qualitatively similar way, and also leads to a smaller inflation bias. Third, we show that not only is it desirable for the central bank to have quasi-hyperbolic discounting, but that it should discount by more than households do. By doing so average inflation is lowered and becomes closer to the Ramsey optimal rate of zero, raising household welfare. This result parallels Rogoff (1985), who showed that discretionary outcomes could be improved by appointing an optimally conservative central banker that cares more about stabilizing inflation than society. With quasi-hyperbolic discounting the central bank cares relatively more about costly prices changes (inflation) in the present, leading it to behave as if it cares more about stabilizing inflation than society does. Fourth, with quasi-hyperbolic discounting households receive a pecuniary and a non-pecuniary return to owning stocks (or capital). For even small amounts of quasi-hyperbolic discounting the non-pecuniary component can be big, leading to a large total return that spills over to the return on bonds. Fifth, outcomes generated by the Taylor rule often differ greatly from the optimal discretionary policy. From a welfare perspective, greater hyperbolic discounting by households leads to greater inefficiency of the Taylor rule.

The remainder of the paper is organized as follows. In the following section we present our model, outline the decision problems for households and firms and discuss the first-order conditions that emerge in a symmetric equilibrium. Section 3 describes the central bank's decision problem and presents the generalized Euler equations associated with optimal discretionary policy. Section 4 focuses on interest rates and asset prices, illustrating how these are determined when agents have quasi-hyperbolic discounting. Section 5 presents the model's benchmark parameterization. Section 6 presents our main simulation results. Section 7 looks at policy delegation, examining the relationship between the household's and the central bank's discount rates. Section 8 concludes. Appendices contain derivations of the model's equilibrium conditions under different assumptions regarding capital's ownership, illustrate the solution strategy, and present results on numerical accuracy.

2 The model

The economy is populated by households, firms, and a government. Households supply labor and consume a bundle of differentiated goods. Households can save through purchasing (risk-free one-period nominal) bonds and stocks, earning income from their wealth and from working. Unlike many business cycle models, the households in our model have hyperbolic preferences (Laibson, 1997)—applying different discount factors at different points in time. Drawing on Phelps and Pollak (1968) and Laibson (1997), we approximate hyperbolic discounting by the quasi-hyperbolic discounting sequence $\{1, \beta\theta, \beta\theta^2, \beta\theta^3, ...\}$, where $\theta \in (0,1)$ reflects the usual geometric discounting and β allows short-term payoffs to be discounted more or less heavily relative to geometric discounting. If $\beta \in (0,1)$, then the short-run discount rate is higher than the long-run discount rate; the opposite is true if $\beta > 1$.

We assume that firms own the capital stock—whose initial level was financed through a stock issuance—and that firms finance capital's accumulation over time through retained earnings. The labor market is perfectly competitive, however firms produce differentiated goods that are aggregated and sold to households. Constraining a firm's pricing decision is a Rotemberg-style (Rotemberg, 1982) quadratic cost to changing prices. The government consists primarily of a central bank that is assumed to conduct policy under discretion by setting the nominal return on the bond in order to maximize household welfare. We also consider the case where monetary policy is conducted according to a Taylor-type rule.

Although our main analysis is conducted on the basis that firms own the capital stock, we could alternatively have assumed that households own the capital stock and that they rent it to firms in a perfectly competitive rental market. We show in Appendices A and B that both ownership structures are equivalent, even when households quasi-hyperbolically discount the future.

2.1 Households

There is a unit-measure of identical infinitely-lived households who derive utility from consumption and leisure. The representative household's expected discounted lifetime utility from period

t onward is given by

$$\mathcal{U}_t = \mathrm{E}_t \left[u_t + \beta \left(\theta u_{t+1} + \theta^2 u_{t+2} + \theta^3 u_{t+3} + ... \right) \right], \tag{1}$$

where u_t represents the instantaneous, or momentary, utility obtained in period t, E_t denotes the mathematical expectation operator conditional upon period-t information, and the parameters satisfy $\theta \in (0,1)$ and $\beta > 0$. Equation (1) distinguishes between the rate at which households discount the utility obtained in period $t+1$ relative to period t, which is given by $\beta\theta$, from the rate at which they discount the utility obtained in period $t+k$ relative to period $t+k-1$ ($k > 1$), which is given by θ. Following (Krusell and Smith, 2003), equation (1) represents a form of quasi-hyperbolic, or quasi-geometric, discounting. Notice that when $\beta = 1$ the standard case of geometric discounting is restored while when $\beta \neq 1$ there is Strotz-style (Strotz, 1956) time inconsistency embedded in household preferences. In the case that $\beta < 1$, households are more impatient today than they are in the future and vice-versa when $\beta > 1$.

We assume that momentary utility is described by the additively-separable function

$$u_t = u(c_t, h_t) = \frac{c_t^{1-\sigma} - 1}{1-\sigma} - \chi \frac{h_t^{1+\upsilon}}{1+\upsilon}, \tag{2}$$

where h_t represents hours worked and c_t is an aggregate good formed as a Dixit-Stiglitz bundle (Dixit and Stiglitz, 1977) of differentiated goods

$$c_t = \left[\int_0^1 c_t(j)^{\frac{\varepsilon_t - 1}{\varepsilon_t}} dj \right]^{\frac{\varepsilon_t}{\varepsilon_t - 1}}, \tag{3}$$

where $c_t(j)$ denotes goods purchased from the j'th firm and the elasticity of substitution between goods satisfies $\varepsilon_t > 1$, $\forall\ t$. In equation (2), the parameters are assumed to satisfy $\sigma > 0$, $\upsilon > 0$, and $\chi > 0$.

Expressed in terms of aggregate goods, the household's real flow-budget-constraint is

$$c_t + \frac{b_{t+1}}{1+R_t} + Q_t s_{t+1} = w_t h_t + \frac{b_t}{1+\pi_t} + Q_t s_t \left(1 + r_t^s\right),$$

where R_t is the net nominal interest rate, w_t is the real wage rate, π_t is the aggregate good's inflation rate, Q_t is the relative price of stocks, b_t is the real value of non-state-contingent nominal bonds, s_t is the number of stocks, and r_t^s is the dividend yield. With the aggregate consumption good produced according to equation (3), the demand for the j'th firm's good, $j \in [0,1]$, is

$$c_t(j) = \left(\frac{P_t(j)}{P_t} \right)^{-\varepsilon_t} c_t,$$

with the price of the aggregate good given by

$$P_t = \left[\int_0^1 P_t(j)^{1-\varepsilon_t} dj \right]^{\frac{1}{1-\varepsilon_t}}.$$

We assume that the representative household cannot precommit to future plans. With the economy's state vector summarized by the vector $\mathbf{Z}_t$, the state variables for the household's problem are b_t, s_t, and $\mathbf{Z}_t$. Adopting the apparatus of a recursive competitive equilibrium, we formulate the household's decision problem through the following Lagrangian, which will be extremized with respect to $\{c_t, h_t, b_{t+1}, s_{t+1}, \lambda_t\}$,

$$\mathcal{U}(b_t, s_t, \mathbf{Z}_t) = \left[\begin{array}{c} \frac{c_t^{1-\sigma}-1}{1-\sigma} - \chi \frac{h_t^{1+\upsilon}}{1+\upsilon} + \beta\theta \mathrm{E}_t \left[U(b_{t+1}, s_{t+1}, \mathbf{Z}_{t+1}) \right] \\ +\lambda_t \left(\begin{array}{c} w(\mathbf{Z}_t) h_t + \frac{b_t}{1+\pi(\mathbf{Z}_t)} + Q(\mathbf{Z}_t) s_t (1 + r^s(\mathbf{Z}_t)) \\ -c_t - \frac{b_{t+1}}{1+R(\mathbf{Z}_t)} - Q(\mathbf{Z}_t) s_{t+1} \end{array} \right) \end{array} \right], \tag{4}$$

taking the equilibrium law-of-motion for $\mathbf{Z}_t$ as given. In equation (4) the continuation value $U(b_{t+1}, s_{t+1}, \mathbf{Z}_{t+1})$ satisfies the recursion

$$U(b_t, s_t, \mathbf{Z}_t) = \left[\begin{array}{c} \frac{c_t^{1-\sigma}-1}{1-\sigma} - \chi \frac{h_t^{1+\upsilon}}{1+\upsilon} + \theta \mathrm{E}_t \left[U(b_{t+1}, s_{t+1}, \mathbf{Z}_{t+1}) \right] \\ +\lambda_t \left(\begin{array}{c} w(\mathbf{Z}_t) h_t + \frac{b_t}{1+\pi(\mathbf{Z}_t)} + Q(\mathbf{Z}_t) s_t (1 + r^s(\mathbf{Z}_t)) \\ -c_t - \frac{b_{t+1}}{1+R(\mathbf{Z}_t)} - Q(\mathbf{Z}_t) s_{t+1} \end{array} \right) \end{array} \right].$$

We close our description of the household's problem by noting that the elasticity of substitution between goods is stochastic, with $\varepsilon_t = \varepsilon e^{\zeta_t}$ and ζ_t obeying

$$\zeta_{t+1} = \rho_\zeta \zeta_t + \epsilon_{\zeta t+1},$$

with $\rho_\zeta \in (0,1)$ and $\epsilon_{\zeta t} \sim i.i.d.\ N\left(0, \sigma_\zeta^2\right)$. The elasticity shock, ζ_t, is common to all firms and forms one element in the economy's state vector, $\mathbf{Z}_t$.

2.2 Firms

There is a unit-continuum of monopolistically competitive firms. The j'th firm, $j \in [0,1]$, owns capital, $k_t(j)$, and employs labour, $h_t(j)$, using both inputs to produce their output, $y_t(j)$, according to the Cobb-Douglas production function

$$y_t(j) = e^{a_t} k_t(j)^\alpha h_t(j)^{1-\alpha}, \tag{5}$$

where $\alpha \in (0,1)$ and a_t is an aggregate technology shock that obeys

$$a_{t+1} = \rho_a a_t + \epsilon_{at+1},$$

with $\rho_a \in (0,1)$ and $\epsilon_{at} \sim i.i.d.\ N\left(0, \sigma_a^2\right)$. The aggregate technology, a_t, is another element in the economy's state vector, $\mathbf{Z}_t$.

The firm's capital evolves over time according to the law-of-motion

$$k_{t+1}(j) = (1-\delta) k_t(j) + i_t(j),$$

where the depreciation rate, $\delta \in [0,1]$, is common to all firms. The aggregate capital stock, K_t, is the final element in the economy's state vector, $\mathbf{Z}_t$.

Firms face a Rotemberg-style (Rotemberg, 1982) price adjustment cost, where the adjustment-cost is governed by $\omega \geq 0$. Each period every firm chooses how much labor to employ, how much investment to undertake, and the price at which to sell their good in order to maximize its equity-value. Profits are paid to the firm's equity-holders (households) in the form of a dividend.

After substituting the production function (equation 5) into the profit function (and dropping the j-index for notational convenience), the decision problem for the representative firm can be written recursively in the form

$$\mathcal{W}(k_t, p_{t-1}, \mathbf{Z}_t) = \max_{\{p_t, k_{t+1}\}} \left[\begin{array}{c} p_t^{1-\varepsilon_t} Y(\mathbf{Z}_t) - w(\mathbf{Z}_t) \left(\frac{p_t^{-\varepsilon_t} Y(\mathbf{Z}_t)}{e^{a_t} k_t{}^{\alpha}} \right)^{\frac{1}{1-\alpha}} - (k_{t+1} - (1-\delta) k_t) \\ -\frac{\omega}{2} \left(\frac{p_t}{p_{t-1}} (1+\pi(\mathbf{Z}_t)) - 1 \right)^2 Y(\mathbf{Z}_t) \\ +\beta\theta \mathrm{E}_t \left[\frac{C(\mathbf{Z}_{t+1})^{-\sigma}}{C(\mathbf{Z}_t)^{-\sigma}} W(k_{t+1}, p_t, \mathbf{Z}_{t+1}) \right] \end{array} \right], \quad (6)$$

taking the equilibrium law-of-motion for $\mathbf{Z}_t$ as given, where $C(\mathbf{Z}_t)$ denotes aggregate consumption, $Y(\mathbf{Z}_t)$ denotes aggregate output, and p_t denotes the firm's price relative to the aggregate good's price. Complementing equation (6) is the following recursive expression for the firm's continuation value

$$W(k_t, p_{t-1}, \mathbf{Z}_t) = \left[\begin{array}{c} p_t^{1-\varepsilon_t} Y(\mathbf{Z}_t) - w(\mathbf{Z}_t) \left(\frac{p_t^{-\varepsilon_t} Y(\mathbf{Z}_t)}{e^{a_t} k_t{}^{\alpha}} \right)^{\frac{1}{1-\alpha}} - (k_{t+1} - (1-\delta) k_t) \\ -\frac{\omega}{2} \left(\frac{p_t}{p_{t-1}} (1+\pi(\mathbf{Z}_t)) - 1 \right)^2 Y(\mathbf{Z}_t) \\ +\theta \mathrm{E}_t \left[\frac{C(\mathbf{Z}_{t+1})^{-\sigma}}{C(\mathbf{Z}_t)^{-\sigma}} W(k_{t+1}, p_t, \mathbf{Z}_{t+1}) \right] \end{array} \right].$$

2.3 Equilibrium conditions and aggregation

In our model all households and all firms are identical and they are of unit mass. We focus our attention on symmetric equilibria for which aggregation across agents implies $k_t = K_t$, $c_t = C_t$, $h_t = H_t$, $b_t = B_t$, and $s_t = S_t$, where capital letters indicate aggregate quantities. The bonds and stocks that are traded among households are assumed to be in zero-net-supply and fixed-net-supply, respectively, so we have $B_t = 0$, $\forall\ t$ and $S_t = 1$, $\forall\ t$, where our normalization that stocks equal 1 is without loss of generality.

We examine the household's decision problem in Appendix A.1. There we show that after aggregating across households the first-order conditions for a symmetric equilibrium from the household's problem can be written as

$$
\begin{aligned}
C_t^{-\sigma} w_t &= \chi H_t^{v}, && (7)\\
\frac{C_t^{-\sigma}}{1+R_t} &= \beta\theta \mathrm{E}_t \left[\frac{C_{t+1}^{-\sigma}}{1+\pi_{t+1}} \right], && (8)\\
Q_t C_t^{-\sigma} &= \beta\theta \mathrm{E}_t \left[C_{t+1}^{-\sigma} Q_{t+1} \left(1 + r_{t+1}^{s}\right)\right]. && (9)
\end{aligned}
$$

Equation (7) is an intra-temporal optimality condition for which the quasi-hyperbolic discounting parameter does not enter. Which is to say that the household's quasi-hyperbolic discounting does not change the trade-off that it faces when making its labor-leisure choice. The same cannot be said for equations (8) and (9), which are intertemporal optimality conditions associated with saving through purchasing bonds and stocks, respectively. For these saving-decisions, the quasi-hyperbolic discounting alters the rate at which household's discount the future relative to today. To the extent that $\beta < 1$, quasi-hyperbolic discounting serves to increase the compensation that households require in order to defer consumption.

Turning to the firm's decision problem, we show in Appendix A.2 that after aggregating across firms the first-order conditions for a symmetric equilibrium can be expressed as

$$C_t^{-\sigma} = \beta\theta \mathrm{E}_t \left[C_{t+1}^{-\sigma} \left(r_{t+1}^{k} + 1 - \delta + \frac{(1-\beta)}{\beta} \mathcal{K}_K(\mathbf{Z}_{t+1}) \right) \right], \tag{10}$$

$$\pi_t \left(1+\pi_t\right) = \frac{(1-\varepsilon_t)}{\omega} + \frac{\varepsilon_t x_t}{\omega} + \beta\theta \mathrm{E}_t \left[\frac{C_{t+1}^{-\sigma}}{C_t^{-\sigma}} \frac{Y_{t+1}}{Y_t} \pi_{t+1} \left(1+\pi_{t+1}\right) \right], \tag{11}$$

$$r_t^k = \alpha x_t \frac{Y_t}{K_t}, \tag{12}$$

$$w_t = (1-\alpha)\, x_t \frac{Y_t}{H_t}, \tag{13}$$

where x_t represents real marginal costs, r_t^k represents the shadow real rental rate of capital, and $\mathcal{K}_K\left(\mathbf{Z}_t\right)$ is the derivative of the decision rule for next-period's capital, $K_{t+1} = \mathcal{K}\left(\mathbf{Z}_t\right)$, with respect to K_t. Equations (12) and (13) are intra-temporal conditions that simply define capital's shadow rental rate and the real wage and do not depend on the household's quasi-hyperbolic discounting. Equation (11) is the economy's Phillips curve. The structure of the Phillips curve is affected by the quasi-hyperbolic discounting, but only to the extent that it changes the rate at which next-period's outcomes are discounted relative to today. The household's quasi-hyperbolic discounting does not have a larger effect on the Phillips curve's structure because we are focusing on a symmetric equilibrium in which all firms set the same price, which means that in equilibrium the relative goods-price for all firms always equals one.

The household's quasi-hyperbolic discounting does, however, impact equation (10), which characterizes the firm's intertemporal decision about capital accumulation and takes the form of a consumption-Euler equation, much like equations (8) and (9). Interestingly, in equation (10) quasi-hyperbolic discounting manifests itself in two ways. First, quasi-hyperbolic discounting changes the rate at which firms discount next-period relative to today, changing the compensation that the firm requires to be enticed to purchase an additional unit of capital rather than pay households a higher dividend. Second, quasi-hyperbolic discounting adds a term involving the derivative $\mathcal{K}_K(\mathbf{Z}_{t+1})$. This additional term, which disappears when $\beta = 1$, says that when making its capital decision, the firm takes into account how the acquisition of an additional unit of capital today changes next-period's capital-acquisition decision, an effect that arises because the firm's equity holders do not have time-invariant preferences. If the household owns the capital stock, then this term, $\mathcal{K}_K(\mathbf{Z}_{t+1})$, arises in the consumption-Euler equation for the capital decision as households use capital accumulation to constrain their future-selves. While the (shadow) rental rate represents a pecuniary return that households receive through owning stocks the derivative term, $\mathcal{K}_K(\mathbf{Z}_{t+1})$, represents a non-pecuniary return.

In addition to these first-order conditions, aggregating across firms and households gives us the aggregate production function

$$Y_t = e^{a_t} K_t^{\alpha} H_t^{1-\alpha},$$

the resource constraint

$$K_{t+1} = (1-\delta) K_t - C_t + \left(1 - \frac{\omega}{2}\pi_t^2\right) Y_t,$$

and the following expression for the dividend yield, r_t^s, which accounts for the pecuniary return on owning stocks

$$Q_t r_t^s = \left(1 - x_t - \frac{\omega}{2}\pi_t^2\right) Y_t + r_t^k K_t - \left(K_{t+1} - (1-\delta) K_t\right). \tag{14}$$

Equation (14) says that the dividend yield rises with an increase in the shadow rental rate of capital, r_t^k, and with a reduction in real marginal costs, x_t, or inflation, π_t.

3 Central bank

We assume that the central bank shares the household's momentary utility function and that it also has quasi-hyperbolic preferences, which is to say that we allow the central bank's discount factors, γ and ξ, to potentially differ from the household's, β and θ. Further, we assume that the central bank does not have access to a commitment technology and that it conducts policy under discretion. With monetary policy conducted under discretion, and with the central bank possessing quasi-hyperbolic preferences, the central bank's decision problem can be summarized by the Bellman equation

$$\mathcal{V}(\mathbf{Z}_t) = \max_{\{\pi_t\}} \left(\frac{C_t^{1-\sigma} - 1}{1-\sigma} - \chi \frac{H_t^{1+\upsilon}}{1+\upsilon} + \gamma\xi \mathrm{E}_t \left[V(\mathbf{Z}_{t+1}) \right] \right),$$

where the continuation value can be expressed recursively in the form

$$V(\mathbf{Z}_t) = \frac{C_t^{1-\sigma} - 1}{1-\sigma} - \frac{\chi}{1+\upsilon} H_t^{1+\upsilon} + \xi \mathrm{E}_t \left[V(\mathbf{Z}_{t+1}) \right],$$

subject to the constraints

$$\begin{aligned}
C_t^{-\sigma} &= \theta \mathrm{E}_t \left[L(\mathbf{Z}_{t+1}) \right], && (15) \\
\pi_t (1+\pi_t) C_t^{-\sigma} Y_t &= \frac{\varepsilon_t}{\omega} \left(x_t + \frac{1-\varepsilon_t}{\varepsilon_t} \right) C_t^{-\sigma} Y_t + \theta \mathrm{E}_t \left[M(\mathbf{Z}_{t+1}) \right], && (16) \\
\left(1 - \frac{\omega}{2} \pi_t^2 \right) Y_t &= C_t + K_{t+1} - (1-\delta) K_t, && (17) \\
Y_t &= e^{a_t} K_t^{\alpha} H_t^{1-\alpha}. && (18)
\end{aligned}$$

Among these four constraints, two are forward-looking: equations (15) and (16). In each of these forward-looking constraints we have introduced an auxiliary variable, $L(\mathbf{Z}_t)$ and $M(\mathbf{Z}_t)$, respectively, which are defined according to

$$\begin{aligned}
L(\mathbf{Z}_t) &= C_t^{-\sigma} \left(\beta \left(\alpha x_t \frac{Y_t}{K_t} + 1 - \delta \right) + (1-\beta) \mathcal{K}_K(\mathbf{Z}_t) \right), \\
M(\mathbf{Z}_t) &= \beta \pi_t (1+\pi_t) C_t^{-\sigma} Y_t.
\end{aligned}$$

Making these auxiliary variables functions of the economy's state in the central bank's decision problem reflects the assumption that policy is set with discretion. Specifically, while able to influence the economy's aggregate state, the discretionary central bank is unable to use policy to influence the process by which private-agents form expectations and must take the functions $L(\mathbf{Z}_t)$ and $M(\mathbf{Z}_t)$ as given when formulating policy.

It is notable from equations (15)—(18) that the key constraints on the central bank's policy decision are the production technology, the resource constraint, the Phillips curve, and the consumption-Euler equation associated with the optimal capital decision. The consumption-Euler equations associated with bonds (equation 8) and stocks (equation 9) are not binding constraints, but simply serve to determine equilibrium outcomes for R_t and Q_t, with r_t^s determined by equation (14).

The central bank's decision problem is treated in Appendix C, where we show that the first-order conditions for the optimal discretionary policy are

$$\begin{aligned}
\frac{\partial}{\partial C_t} &: \quad C_t^{-\sigma} + \frac{\sigma\chi}{\upsilon+\alpha}\frac{H_t^{1+\upsilon}}{C_t} - \phi_{1t}\left(1+\sigma\frac{1-\alpha}{\upsilon+\alpha}\left(1-\frac{\omega}{2}\pi_t^2\right)\frac{Y_t}{C_t}\right) - \phi_{2t}\sigma C_t^{-\sigma-1} \\
&\quad -\sigma\frac{1+\upsilon}{\alpha+\upsilon}\phi_{3t}\left(\frac{(1-\varepsilon_t)+\varepsilon_t x_t}{\omega} - \pi_t(1+\pi_t)\right)C_t^{-\sigma-1}Y_t = 0, &(19)\\
\frac{\partial}{\partial \pi_t} &: \quad -\phi_{3t}(1+2\pi_t)C_t^{-\sigma} - \phi_{1t}\omega\pi_t = 0, &(20)\\
\frac{\partial}{\partial x_t} &: \quad -\frac{\chi}{\upsilon+\alpha}\frac{H_t^{1+\upsilon}}{x_t} + \phi_{1t}\frac{1-\alpha}{\upsilon+\alpha}\left(1-\frac{\omega}{2}\pi_t^2\right)\frac{Y_t}{x_t} \\
&\quad +\phi_{3t}\left(\frac{\varepsilon_t x_t}{\omega} + \frac{1-\alpha}{\upsilon+\alpha}\frac{(1-\varepsilon_t)+\varepsilon_t x_t}{\omega} - \frac{1-\alpha}{\upsilon+\alpha}\pi_t(1+\pi_t)\right)C_t^{-\sigma}\frac{Y_t}{x_t} = 0, &(21)\\
\frac{\partial}{\partial K_{t+1}} &: \quad -\frac{\gamma\xi\alpha\chi}{\upsilon+\alpha}\mathrm{E}_t\left[\frac{H_{t+1}^{1+\upsilon}}{K_{t+1}}\right] + \xi\mathrm{E}_t\left[\phi_{1t+1}\left(\alpha\frac{1+\upsilon}{\upsilon+\alpha}\left(1-\frac{\omega}{2}\pi_{t+1}^2\right)\frac{Y_{t+1}}{K_{t+1}} + 1-\delta\right)\right] \\
&\quad +\xi\alpha\frac{1+\upsilon}{\upsilon+\alpha}\mathrm{E}_t\left[\phi_{3t+1}\left(\frac{(1-\varepsilon_t)+\varepsilon_t x_{t+1}}{\omega} - \pi_{t+1}(1+\pi_{t+1})\right)\frac{Y_{t+1}}{K_{t+1}}C_{t+1}^{-\sigma}\right] \\
&\quad -\xi(1-\gamma)\mathrm{E}_t\left[\left(C_{t+1}^{-\sigma} + \frac{\sigma\chi}{\upsilon+\alpha}\frac{H_{t+1}^{1+\upsilon}}{C_{t+1}}\right)\mathcal{C}_K(\mathbf{Z}_{t+1})\right] \\
&\quad +\frac{\xi(1-\gamma)\chi}{\upsilon+\alpha}\mathrm{E}_t\left[\frac{H_{t+1}^{1+\upsilon}}{x_{t+1}}\mathcal{X}_K(\mathbf{Z}_{t+1})\right] \\
&\quad -\phi_{2t}\theta\mathrm{E}_t\left[L_K(\mathbf{Z}_{t+1})\right] + \phi_{3t}\theta\mathrm{E}_t\left[M_K(\mathbf{Z}_{t+1})\right] - \phi_{1t} = 0. &(22)
\end{aligned}$$

where

$$\begin{aligned}
H_t &= \left(\left(\frac{1-\alpha}{\chi}\right)e^{a_t}x_t K_t^{\alpha}C_t^{-\sigma}\right)^{\frac{1}{\upsilon+\alpha}}, &(23)\\
Y_t &= \left(\left(\frac{1-\alpha}{\chi}\right)^{1-\alpha}e^{(1+\upsilon)a_t}x_t^{1-\alpha}K_t^{\alpha(1+\upsilon)}C_t^{-\sigma(1-\alpha)}\right)^{\frac{1}{\upsilon+\alpha}}, &(24)\\
L(\mathbf{Z}_t) &= C_t^{-\sigma}\left(\beta\left(\alpha e^{a_t}x_t\frac{Y_t}{K_t} + 1-\delta\right) + (1-\beta)\mathcal{K}_K(\mathbf{Z}_t)\right), &(25)\\
M(\mathbf{Z}_t) &= \beta\pi_t(1+\pi_t)C_t^{-\sigma}Y_t. &(26)
\end{aligned}$$

As a counterpoint, we also solve the model for the case where monetary policy is conducted according to the following Taylor-type rule

$$1+R_t=\frac{1+\overline{\pi}}{\beta\theta}\left(\frac{1+\pi_t}{1+\overline{\pi}}\right)^{\phi_\pi}\left(\frac{Y_t}{Y_{t-1}}\right)^{\phi_y}, \tag{27}$$

where $\overline{\pi}$ represents the inflation target, $\phi_\pi > 1$ and $\phi_y > 0$. Following Fernández-Villaverde, et al. (2015) and Dennis (2018), this Taylor rule has the central back setting the nominal interest rate in response to movements in inflation and real output growth.

4 Interest rates and the return on capital

From the household's optimal bond-holding decision, the net nominal interest rate, R_t, is governed by the Euler equation

$$\frac{1}{1+R_t}=\beta\theta\mathrm{E}_t\left[\frac{C_{t+1}^{-\sigma}}{C_t^{-\sigma}}\frac{1}{1+\pi_{t+1}}\right],$$

where the effect of the household's quasi-hyperbolic discounting is seen to cause the future to be discounted more sharply, raising the equilibrium interest rate on average. We can also compute the shadow return on a risk-free real bond, r_t, which must satisfy

$$\frac{1}{1+r_t}=\beta\theta\mathrm{E}_t\left[\frac{C_{t+1}^{-\sigma}}{C_t^{-\sigma}}\right],$$

From the firm's decision problem the shadow rental rate of capital is given by

$$r_t^k=\alpha x_t\frac{Y_t}{K_t},$$

where household's quasi-hyperbolic discounting has indirect-effects through the economy's real allocation. Where the shadow rental rate of capital represents the pecuniary return that households receive from owning stocks, the total net return they receive, r_t^{cap}, satisfies

$$1+r_t^{cap}=r_t^k+1-\delta+\frac{(1-\beta)}{\beta}\mathcal{K}_K(\mathbf{Z}_t). \tag{28}$$

According to equation (28), the total gross return on capital, $1+r_t^{cap}$, is the sum of two components: the gross pecuniary return, $r_t^k+1-\delta$, and the gross non-pecuniary return, $\frac{(1-\beta)}{\beta}\mathcal{K}_K(\mathbf{Z}_t)$. As we will see below, even for relatively small amounts of quasi-hyperbolic discounting the non-pecuniary component can be large.

5 Parameterization

We assume that a period in the model corresponds to one quarter of a year and parameterize the model to this frequency. We set the household's (geometric) discount factor, θ, to 0.99, which in the absence of quasi-hyperbolic discounting implies a steady state annual real interest rate of about 4 percent. As is common, we assume log-utility with respect to consumption, i.e. $\sigma = 1$, and we set the relative weight on the disutility of labor, χ, equal to 1. The Frisch labor supply elasticity, ν, is set equal to 1, which is consistent with a host of studies, including Fernández-Villarerde, Guerrón-Quintana, Kuester, and Rubio-Ramírez (2015), Guerrieri and Iacoviello (2017), and Chetty, Guren, Manoli, and Weber (2011), but smaller than Gust, Herbst, López-Salido, and Smith (2017) and Gavin, Keen, Richter, and Throckmorton (2015), who set this elasticity to 2 and 3, respectively. As Fernández-Villarerde, et al, (2015) comment, a lower value for ν (implying a higher labor-supply elasticity) is generally more appropriate for models that do not differentiate between the intensive and extensive margins.

In the production technology, values for α generally range from about 0.3 (Guerrieri and Iacoviello, 2017) to 0.40 (Cooley and Prescott, 1995). We set α equal to 0.33, in line with Gavin, et al, (2015) and Sala, Söderström, and Trigari (2008). In the capital accumulation equation, we set the depreciation rate, δ, to 0.025, which implies that capital depreciates at a 10 percent annualized rate. We set the steady state elasticity of substitution between goods, ε, to 11, implying a steady state mark-up of 10 percent. This value for ε has been used previously in a range of studies, including Krause, López-Salido, and Lubik (2008a) and Dennis (2018), and is consistent with the findings of Basu and Fernald (1997). Other recent studies have set ε to 6 (Christiano, Eichenbaum, and Evans, 2005) or 21 (Fernández-Villarerde et al, 2015; Krause, López-Salido, and Lubik, 2008b), implying much larger and much smaller steady-state markups, respectively, however we found that these values gave implausible values for steady state inflation. Turning to the price adjustment parameter, ω, we set it to 100, consistent with Gust, et al, (2017). In a log-linearized environment, this value for ω makes the Rotemberg model quantitatively similar to a Calvo model where the average frequency of price adjustment equals one year. Elsewhere in the literature, Gavin, et al, (2015) estimate ω to be 59.1, Ireland (2001) estimates it to be about 80, while the estimates in Gertler, Sala, and Trigari (2008) and Sala, Söderström, and Trigari (2008) imply a value closer to 150.

There are two shocks in the model, those to aggregate technology, a_t, and the elasticity of

Table 1: Benchmark Parameterization

Parameter	Value	Parameter	Value	Parameter	Value
θ	0.99	α	0.33	ρ_a	0.95
σ	1.0	ε	11.0	ρ_ζ	0.85
χ	1.0	ω	100.0	σ_a	0.008
ν	1.0	δ	0.0025	σ_ζ	0.06
β	1.0				

substitution among goods, ζ_t. As is common, these shocks are assumed to follow AR(1) processes:

$$\begin{aligned} a_{t+1} &= \rho_a a_t + \epsilon_{at+1}, \quad \epsilon_{at} \sim i.i.d.\ N\left(0, \sigma_a^2\right), \\ \zeta_{t+1} &= \rho_\zeta \zeta_t + \epsilon_{\zeta t+1}, \quad \epsilon_{\zeta t} \sim i.i.d.\ N(0, \sigma_\zeta^2). \end{aligned}$$

For the aggregate technology shock, we follow convention (see Faia (2009) and the references therein) and set the persistence parameter, ρ_a, to 0.95 and the standard deviation for the technology innovation, σ_a, to 0.008. For the elasticity of substitution shock, the estimates vary across the literature. Gertler, Sala, and Trigari (2008) estimate ρ_ζ and σ_ζ to be 0.81 and 0.008, Smets and Wouters (2007) estimate them to be 0.89 and 0.1, while Ichiue, Kurozumi, and Sunakawa (2013) estimate them to be 0.7 and 0.05. We set ρ_ζ and σ_ζ to 0.85 and 0.06, respectively, implying that 90 percent of the distribution for ε_t lies in the interval $[9.1, 13.3]$.

In our benchmark model the central bank's discount factor and its quasi-hyperbolic discount factor are assumed to be the same as for the household, implying that the central bank is benevolent. We summarize our benchmark parameterization in Table 1.

For the simulations based on the Taylor-type rule, equation (27), we assume $\pi = 2.5$, $\phi_\pi = 1.5$, and $\phi_y = 0.5/4$.

6 Results

In this section we present simulation results for a range of different model specifications. We begin with the benchmark model in section 6.1 in which households have geometric discounting and the central bank is benevolent, sharing the household's discount factors. In section 6.2 we allow households to have quasi-hyperbolic discounting while maintaining the assumption that the central bank shares the household's discount factors. Section 7 treats the case where households and the central bank have quasi-hyperbolic discounting and their discount factors are not equal.

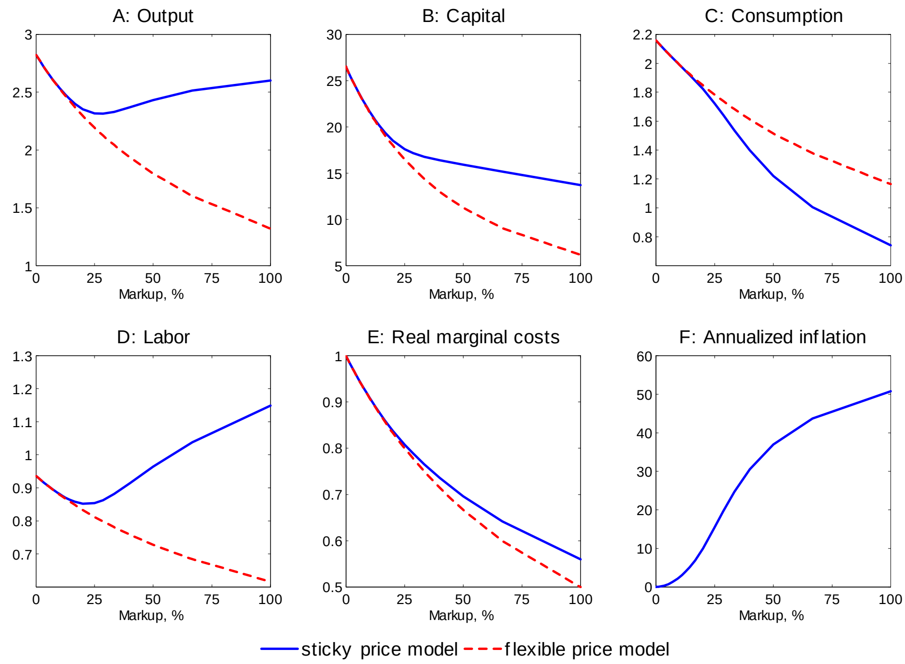

Figure 1: Steady state as function of the price markup.

6.1 The benchmark model

Before analyzing the stochastic economy it is useful to examine the effect that monopolistic competition has on production, consumption, and inflation, in a deterministic environment. Switching the model's stochastic elements off, the effect of varying ε on the model's nonstochastic steady state outcomes, through its consequences for the price markup, are presented in Figure 1. To better interpret the effects of monopolistic competition, we also report in Figure 1 the steady state results for the flex-price version ($\omega = 0$) of the model. For this exercise, we assume monetary policy is conducted under discretion.

The effect that the price markup has on steady state inflation is shown in Figure 1, panel F. When prices are costly to change and there is no production subsidy in place to offset the monopolistic distortion a higher markup leads to higher inflation, with annualized inflation reaching exceedingly high levels as the markup approaches 100 percent. The inflation that occurs as the markup rises is a product of the discretionary central bank's behavior. With monopolistic competition generating inefficiently low output, the central bank lowers the nominal interest rate in order to stimulate demand and raise output. But to meet higher demand for their good firms need to employ more workers, which boosts the demand for labour and pushes up the nominal wage and nominal marginal costs. Facing higher nominal marginal costs firms raise prices, causing inflation. As the markup gets bigger the central bank's efforts to stimulate aggregate demand intensify, giving rise to higher steady state inflation.

Because there are costs to changing prices, non-zero inflation has real costs. These real costs are illustrated in panels A—E through the difference between the solid line, representing the sticky-price model, and the dashed line, representing the flex-price model. Looking at the behavior of the flex-price model, as the markup increases output (panel A), capital (panel B), consumption (panel C), labour (panel D), and real marginal costs (panel E) all decrease monotonically. The higher markup is associated with firms having greater market power and leads to lower production. Lower production means less demand for capital and labour and also leads to declines in consumption and investment. The fact that real marginal costs decrease as the markup increases (panel E) simply reflects the increase in profits associated with firms having greater market power.

When the price markup is not too large, the steady state behavior of the sticky price model is similar to that for the flex-price model. However, as the markup becomes increasingly large important differences between the two models emerge. These differences are driven by the magnitude of inflation and with the output lost due to price-adjustment costs. Specifically, as the markup gets bigger, in order to partly offset the output lost due to price-adjustment costs, firms in the sticky-price model increase their production levels in order to maintain their profitability. As a result, the demand for capital and labour rises in the sticky-price model relative to the flex-price model.[1] Thus, unlike for the flex-price model, where output and labour decline monotonically, in the sticky-price model an increase in the markup causes output and labour to rise, following an initial fall. For a given markup, steady state output, capital, and labour are all higher in the sticky-price model than in the flex-price model, but this is not the case for consumption, which suffers as goods are devoted to covering price-adjustment costs and to supporting the capital stock. It is also worth noting that real marginal costs are higher in the sticky-price model than the flex-price model, indicating that inflation and the resulting price-adjustment costs have an adverse impact on profits.

Turning to the stochastic model, Table 2 reports the mean (standard deviation in brackets) of the stationary distributions for the sticky-price economy under both discretion (column 1) and the Taylor rule (column 2) to those for the flex-price economy (column 3). Comparing the sticky-price and flex-price economies, the main effect of sticky prices is to generate a positive

[1] See also the discussion in Ascari and Rossi (2012).

Table 2: Characteristics of the Stationary Distribution

		Sticky prices		Flexible prices
		Discretion	Taylor rule	
		(1)	(2)	(3)
Output	Y	2.540 [0.104]	2.539 [0.103]	2.534 [0.104]
Capital	K	21.734 [0.936]	21.722 [0.929]	21.664 [0.935]
Consumption	C	1.992 [0.062]	1.991 [0.062]	1.993 [0.062]
Investment	I	0.543 [0.053]	0.543 [0.053]	0.542 [0.053]
Labour	H	0.881 [0.010]	0.881 [0.010]	0.880 [0.011]
Real wage	w	1.756 [0.063]	1.755 [0.063]	1.753 [0.064]
Real marginal cost	x	0.909 [0.008]	0.909 [0.011]	0.909 [0.010]
Annualized inflation	π	2.580 [0.527]	2.519 [0.253]	–
Household welfare	$\mathcal{U}$	29.957 [0.989]	29.960 [0.992]	30.163 [0.988]
Nominal interest rate	R	6.782 [0.497]	6.720 [0.660]	4.097 [0.399]
Real interest rate	r	4.097 [0.384]	4.098 [0.411]	4.097 [0.399]
Rental rate	r^k	4.098 [0.427]	4.098 [0.431]	4.098 [0.447]
Return on capital	r^{cap}	4.098 [0.427]	4.098 [0.431]	4.098 [0.447]

Note: Statistics calculated using 10^6 simulated observations; standard deviations in brackets.

inflation rate (an inflation bias) when policy is conducted under discretion, consistent with Figure 1. With the (stochastic) price markup averaging just over 10 percent, the discretionary central bank's efforts to offset the monopolistic distortion results in higher inflation and a higher nominal interest rate.

Figure 2 plots impulse responses for technology shocks under both discretionary policy (solid lines) and the Taylor-rule policy (dashed lines) in the model with sticky prices. Looking at the responses under discretion, a positive technology shock raises the productivity of capital and labour, causing firms to demand more of these inputs, which raises the quantities of capital (panel B) and labour (panel E) traded and increases the real wage (panel F) and the real interest rate (panel J). With more capital and labour employed for production, real output rises (panel A) and the resulting increase in households' real income boosts consumption (panel C). Real marginal costs (panel G) are little-changed by the shock because the productivity increase is captured by higher factor prices. Because real marginal costs are little-affected, firms face minimal pressure to change prices, so inflation too is little-changed by the shock (panel H). As a consequence, monetary policy responds to the shock largely by accommodating it. The higher real return on capital boosts the real return on bonds and the central bank responds by allowing the nominal interest rate to rise in line with the higher real interest rate. Qualitatively, the results for the Taylor-rule policy are very similar to the discretionary policy, however it is noticeable that the discretionary policy leads to a much smaller inflation response, at the cost of greater movement in labour (panel E), the real wage (panel F) and output (panel A) when the shock hits.

Turning to Figure 1, under discretionary policymaking, a positive shock to the elasticity of substitution among goods leads to a decline in the markup, which has a direct negative impact on inflation (panel H). Greater competition among firms causes output to rise (panel A) and leads to greater demand for capital (panel B) and labour (panel E). Consumption rises (panel C) as a consequence of higher real income. Increased demand for capital and labour causes the real wage to rise (panel F) and this in turn causes real marginal costs to increase (panel G). Although real marginal costs have gone up, because there is greater competition among firms prices actually fall and inflation goes down (panel H). However, with greater costs and lower prices, firms profitability is adversely affected, which is reflected in a lower stock price. The central bank responds to the shock by lowering the nominal interest rate, but by less than the decline in inflation, allowing the real interest rate to rise and bring the real return on bonds into line with the higher real return on capital. The greatest differences between the discretionary policy and the Taylor rule policy can be seen in the behavior of inflation (panel H), which falls under discretion and rises under the Taylor rule. But this differences in behavior translates into a relatively small difference in the real interest rate (panel J) and the behavior of the real economy is qualitatively similar for the two policies.

6.2 Quasi-hyperbolic discounting

The previous section assumed that household's and the central bank used geometric discounting to discount the future. In this section we examine the effect that quasi-hyperbolic discounting by households and the central bank has for macroeconomic and financial outcomes. When introducing quasi-hyperbolic discounting, we impose $\theta = \xi$ and $\gamma = \beta$ (so that household's and the central bank discount symmetrically) to maintain the assumption that the central bank is benevolent. We consider cases where $\beta \neq \gamma$ in section 7.

In line with the standard mechanism discussed in Krusell and Smith (2003) and elsewhere,

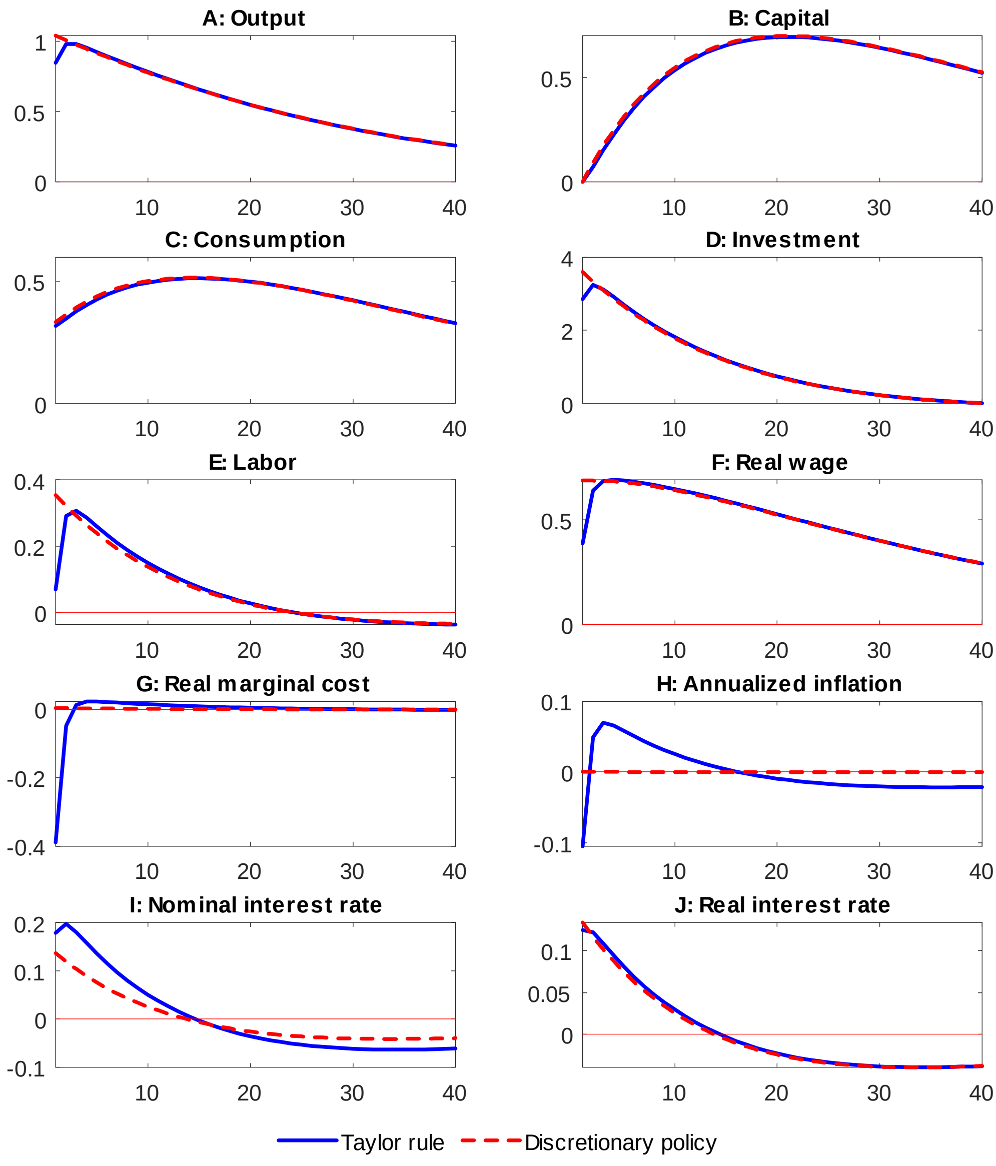

Figure 2: Responses to a technology shock under discretion and the Taylor-rule

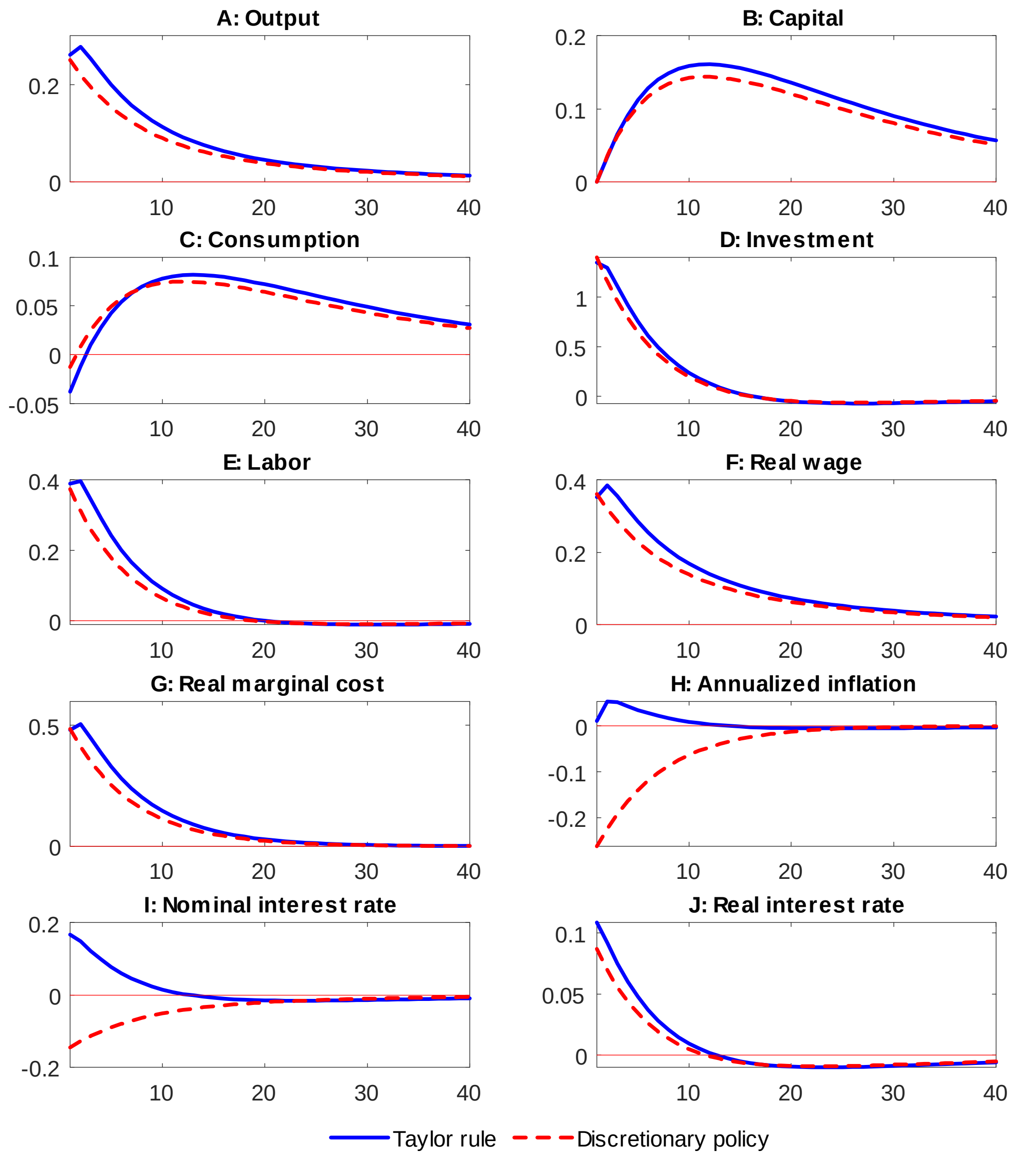

Figure 3: Responses to a price-elasticity shock under discretion and the Taylor rule

greater quasi-hyperbolic discounting results in households increasing their current consumption and reducing their current saving. As a result, less capital accumulation takes place and output, capital, and consumption are all lower on average. The effect is quantitatively substantial, as illustrated in Table 3.

Focusing on the model's stationary distribution, Table 3 shows the mean (standard deviations in parentheses) outcomes for the model's key macroeconomic and financial variables for different values of $\beta = \gamma$, allowing policy to be conducted either under discretion (columns (1)—(4)) or according to a Taylor rule (columns (5)—(8)). Looking at average outcomes, the table shows that as greater quasi-hyperbolic discounting takes place ($\beta = \gamma$ get smaller)—biasing household and central bank decision-making toward the present—output falls. Specifically, lowering $\beta = \gamma$ from 1.0 to 0.9 causes output to decline by approximately 10 percent.[2] Although greater quasi-hyperbolic discounting causes output, capital, consumption, labour, and the real wage to fall there are important differences in how each of these variables is affected. For example, although lowering $\beta = \gamma$ from 1.0 to 0.9 causes output to fall by 10.02 percent, capital falls by much more (24.55 percent) and labour falls by much less (1.84 percent). Labour does not decline to the same extent as output because households sacrifice some leisure in order to prevent a large decline in consumption. As a consequence, consumption falls by 6.02 percent, considerably less than output. The large decline in capital combined with a smaller decline in labour means that the capital-labour ratio goes down, and with relatively less capital, labour's productivity diminishes and real wages go down (by 7.77 percent).

Looking at real marginal costs, Table 3 shows that greater quasi-hyperbolic discounting causes real marginal costs to rise slightly under both discretionary policy and the Taylor-rule policy. The effect that quasi-hyperbolic discounting has on real marginal costs is related to the decline that firms face in the demand for their good, which causes them to lower their price markup. To understand the impact quasi-hyperbolic discounting has on inflation for the discretionary policy, note that quasi-hyperbolic discounting implies that costs to changing prices today are weighted more heavily than those to changing prices in the future. As a consequence, when responding to shocks firms find it beneficial to spread price changes out over time, making smaller price changes in the current period and deferring the remaining price change (and its associated cost) to the future. With smaller price changes taking place today, greater quasi-hyperbolic discounting acts somewhat like an increase in price rigidity. From the central bank's perspective, with

[2] Cutting β from 1.0 to 0.7 causes output to fall by about 30 percent, suggesting a linear relationship between the percent by which β falls and the percent by which output falls.

Table 3: Stationary Distribution as a Function of the Quasi-Hyperbolic Discount factor

		Discretion				Taylor rule			
Discounting	$\beta = \gamma$	1.00	0.99	0.95	0.90	1.00	0.99	0.95	0.90
		(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Output	Y	2.540	2.516	2.415	2.286	2.539	2.509	2.387	2.231
		[0.104]	[0.103]	[0.099]	[0.094]	[0.103]	[0.102]	[0.097]	[0.091]
Capital	K	21.734	21.181	19.009	16.398	21.722	21.049	18.437	15.367
		[0.936]	[0.916]	[0.837]	[0.740]	[0.929]	[0.903]	[0.799]	[0.674]
Consumption	C	1.992	1.981	1.935	1.872	1.991	1.978	1.922	1.842
		[0.062]	[0.062]	[0.062]	[0.061]	[0.062]	[0.062]	[0.061]	[0.060]
Investment	I	0.543	0.530	0.475	0.410	0.543	0.526	0.461	0.384
		[0.053]	[0.052]	[0.048]	[0.044]	[0.053]	[0.051]	[0.047]	[0.041]
Labour	H	0.881	0.880	0.873	0.865	0.881	0.879	0.871	0.867
		[0.010]	[0.009]	[0.009]	[0.008]	[0.010]	[0.010]	[0.010]	[0.010]
Real wage	w	1.756	1.743	1.690	1.619	1.755	1.739	1.674	1.587
		[0.063]	[0.063]	[0.062]	[0.060]	[0.063]	[0.063]	[0.061]	[0.059]
Real marginal costs	x	0.909	0.910	0.912	0.915	0.909	0.910	0.912	0.915
		[0.008]	[0.008]	[0.008]	[0.007]	[0.011]	[0.011]	[0.011]	[0.011]
Annualized inflation	π	2.580	2.559	2.478	2.385	2.519	2.519	2.520	2.522
		[0.527]	[0.520]	[0.493]	[0.462]	[0.253]	[0.254]	[0.258]	[0.267]
Household welfare	$\mathcal{U}$	29.957	29.270	26.432	22.685	29.960	29.180	25.924	21.457
		[0.989]	[0.980]	[0.942]	[0.895]	[0.992]	[0.981]	[0.937]	[0.881]
Nominal interest rate	R	6.782	11.139	30.969	62.443	6.720	11.098	31.026	62.664
		[0.497]	[0.516]	[0.608]	[0.760]	[0.660]	[0.692]	[0.847]	[1.111]
Real interest rate	r	4.097	8.367	27.804	58.660	4.098	8.368	27.804	58.660
		[0.384]	[0.403]	[0.489]	[0.634]	[0.411]	[0.432]	[0.531]	[0.700]
Rental rate	r^k	4.098	4.342	5.413	7.019	4.098	4.392	5.713	7.756
		[0.427]	[0.433]	[0.460]	[0.502]	[0.431]	[0.440]	[0.480]	[0.542]
Return on capital	r^{cap}	4.098	8.369	27.805	58.662	4.098	8.368	27.805	58.618
		[0.427]	[0.447]	[0.543]	[0.704]	[0.431]	[0.453]	[0.558]	[0.740]

Note: Statistics calculated using 10^6 simulated observations; standard deviations in brackets.

quasi-hyperbolic discounting operating similarly to an increase in price rigidity, it calculates that smaller inflation surprises are sufficient to boost output to the efficient level. In equilibrium, then, greater quasi-hyperbolic discounting leads to less inflation.

Turning to the financial variables, the most pronounced and obvious effect of quasi-hyperbolic discounting is to raise the real returns on capital and bonds. With quasi-hyperbolic discounting shifting demand from future- to current-consumption the relative price of current-consumption rises causing the pecuniary return on capital, as reflected in the (shadow) rental rate of capital, to rise. In addition, greater quasi-hyperbolic discounting increases greatly the non-pecuniary return on capital, which causes the (net) total return on capital, r^{cap}, to balloon. With households substituting between stocks and bonds (which do not offer a non-pecuniary return because they are in zero-net-supply) based on their total return, the rise in r^{cap} leads to a commensurate rise in the real interest rate.

6.2.1 Impulse responses

Although it is clear from Table 3 that quasi-hyperbolic discounting has an important impact on average outcomes, here we focus on dynamics. We compute impulse response functions for the discretionary response to technology shocks (Figure 4) and price-elasticity shocks (Figure 5) while allowing the extent of the quasi-hyperbolic discounting to vary.

Looking first at the responses to technology shocks, Figure 4 reveals that it is the financial variables that quasi-hyperbolic discounting affects most. The solid lines in Figure 4 correspond to $\beta = \gamma = 1$, the baseline case displayed in Figure 2. With quasi-hyperbolic discounting causing households to discount the entire future relative to today, increased quasi-hyperbolic discounting leads to an increased focus on today's consumption and leisure. Accordingly, relative to the baseline case, consumption (panel C) rises by more and labour (panel E) rises by less in response to the technology shock. Higher technology boosts the demand for labour, and with the supply of labour increasing by less relative to the baseline case, the real wage (panel F) rises by more, which pushes up real marginal costs (panel G). Because real marginal costs increase by more with quasi-hyperbolic discounting than they do for the baseline case, the firm's production costs are higher and their profitability is lower. At the same time, the real interest rate (panel J) rises by more than the baseline case, due to the increased demand for current consumption relative to future consumption. The discretionary policy response is to increase the nominal interest rate (panel I) by more than the baseline case, primarily due to the higher real interest rate.

Figure 5 shows how quasi-hyperbolic discounting alters the model's dynamic behavior follow-

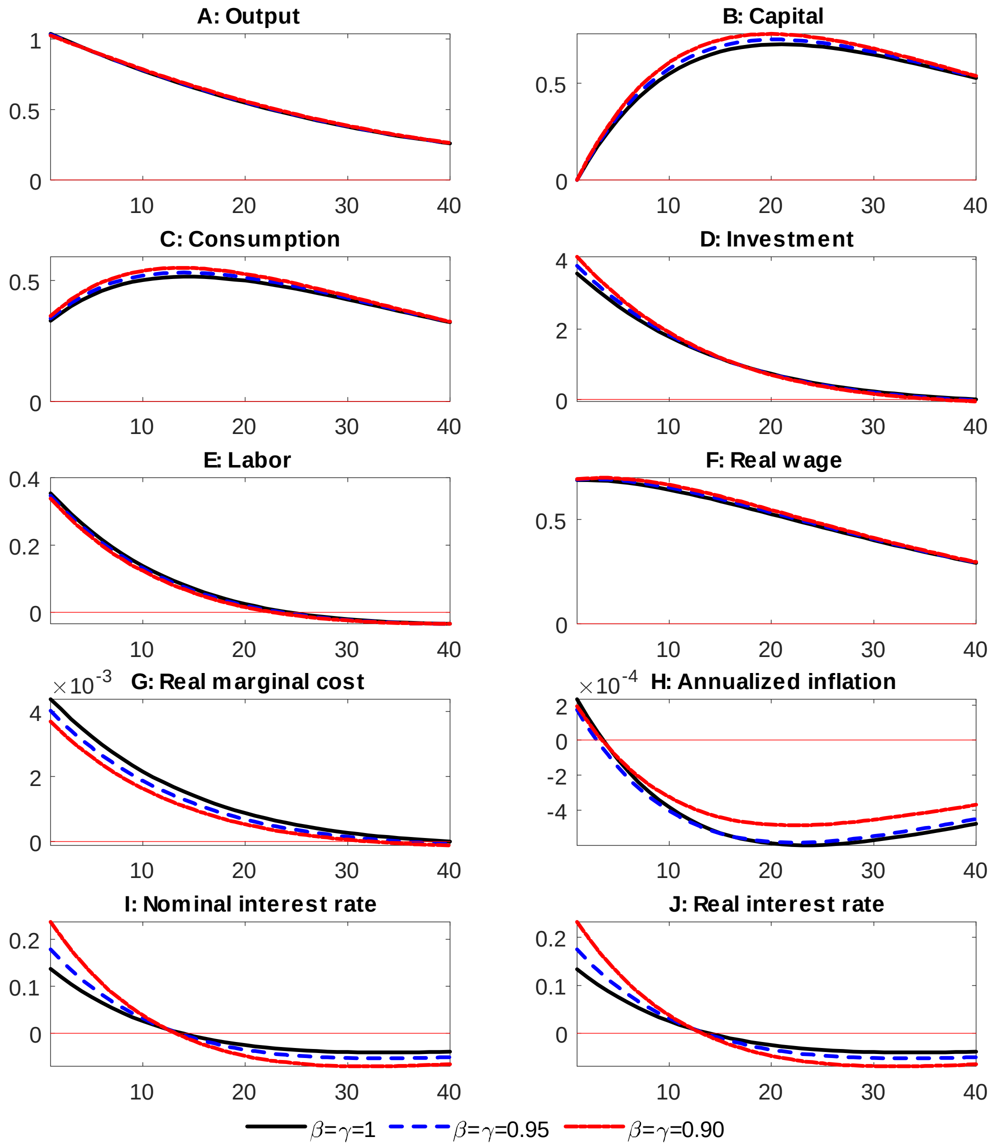

Figure 4: Responses to a technology shock with quasi-geometric discounting, $\beta = \gamma$, and discretionary policy

ing price-elasticity shocks. Relative to the baseline case in which $\beta = \gamma = 1$ (solid lines), with quasi-hyperbolic discounting the impulse responses are (generally) a little more muted. Households value leisure and consumption more today relative to the future, so labour (panel E) rises by less following the shock and consumption rises by more (panel C). Similarly, quasi-hyperbolic discounting makes firms want to defer costly price changes, so inflation (panel H) falls by less than the baseline case. The variables for which the effects of quasi-hyperbolic discounting are most pronounced are the real interest rate (panel J) and the nominal return on bonds (panel I). Quasi-hyperbolic discounting makes all of these variables more sensitive to the price-elasticity shock because it changes the relative demand for current consumption such that a bigger change in the relative price of consumption (the real interest rate) is required to induce households to defer consumption.

Although quasi-hyperbolic discounting affects the dynamic behavior of the macroeconomic variables, Figures 4 and 5 reveal that its greatest impact is on asset returns. This finding is consistent with the simulation results in Table 3, which show that the volatilities of asset returns and asset prices rise importantly as quasi-hyperbolic discounting increases.

7 Policy delegation

In the previous section we allowed the central bank to have quasi-hyperbolic preferences, but we restricted its discount factors to equal those of the representative household. This restriction forced the discretionary central bank to be benevolent, i.e., to conduct policy under discretion in order to maximize the welfare of the representative household. Here, we allow the central bank's quasi-hyperbolic discounting to differ from the representative household. We do this exercise for two reasons. First, by allowing the central bank's discounting to differ from the household's we can assess the degree to which the central bank's quasi-hyperbolic discounting affects economic outcomes. Second, because policy is being conducted under discretion and discretion is suboptimal, it is possible that the government should optimally delegate monetary policy to a central banker whose discounting differs from the household. If this is the case, then a related question is whether the central bank should discount the future by more or less than the household.

Table 4 summarizes equilibrium outcomes when the household and the central bank have different discount factors, assuming monetary policy si conducted under discretion. Beginning with the discretionary-policy case, comparing columns (1) and (2) we see that the central bank's quasi-hyperbolic discounting ($\gamma = 0.9$) causes it to conduct monetary policy in order to encourage

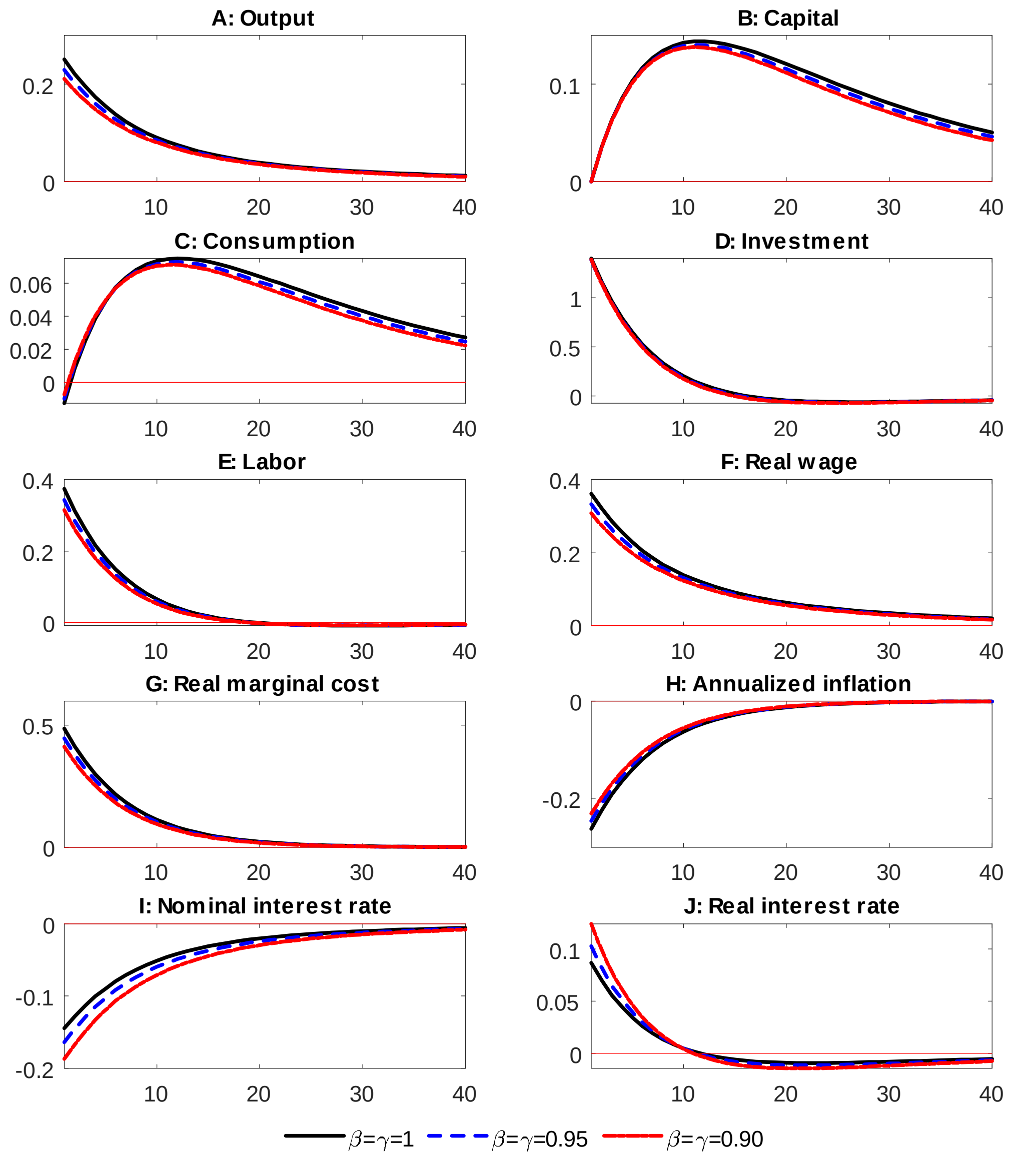

Figure 5: Responses to a price-elasticity shock with quasi-geometric discounting, $\beta = \gamma$.

greater consumption and leisure and discourage inflation. Because the central bank places greater emphasis on the present relative to the future, monetary policy is used to encourage households to bring consumption and leisure forward in time while also shifting price changes (which are costly) to the future, where they are discounted more heavily. With greater consumption and leisure taking place, investment and capital fall slightly, which leads to a decline in output. For the financial variables, the real returns on assets are barely affected while the stock price rises.

Where columns (1) and (2) allow us to identify what happens when the central bank has greater quasi-hyperbolic discounting than households (which are not quasi-hyperbolic discounters for that comparison), columns (3) and (4) relate to the opposite comparison: in column (3) both households and the central bank have quasi-hyperbolic discounting whereas in column (4) only the household does. As a consequence, relative to column (3), in column (4) the central bank uses monetary policy to encourage households to defer consumption and leisure, while bringing forward price changes, which raises inflation. In this particular case, the household's labour supply response is large, which increases output and permits consumption to actually rise. Although allowing the central bank's discounting to differ from the household's has effects on real variables, these effects are relatively small. However, the effects on nominal variables are larger and quantitatively significant.

Importantly, one consequence of the central bank's quasi-hyperbolic discounting is to raise household welfare. Household welfare is higher in column (2) than in column (1) and in column (3) than in column (4). In other words, it is desirable from a welfare perspective for the central bank to have quasi-hyperbolic discounting even if household's do not. This finding parallels other situations where distorting the central bank's objectives can raise welfare when monetary policy is conducted under discretion. For example, Dennis (2014) showed that having monetary policy conducted by a discretionary central bank with risk-sensitive preferences could improve welfare (lower loss) because the risk-sensitivity rendered feasible policies that were otherwise infeasible. Here, the central reason why the central bank's quasi-hyperbolic discounting raises household welfare is that it emphasizes the current-period cost of changing prices, in much the same way as greater price rigidity or greater concern for price changes. The outcome is less volatile inflation and an average inflation rate that is lower, closer to zero. Due to the greater emphasis placed on inflation appointing a central banker with quasi-hyperbolic discounting is similar to appointing a conservative central banker (Rogoff, 1985).

Finally, we note from Table 4 that the finding that the central bank's quasi-hyperbolic discounting can raise household welfare relies on the economy's steady state being inefficient. If

Table 4: The Effect of the Central Bank's Quasi-Hyperbolic Discounting

		Discretion			
Household	β	1.00	1.00	0.90	0.90
Central bank	γ	1.00	0.90	0.90	1.00
		(1)	(2)	(3)	(4)
Output	Y	2.540 [0.104]	2.536 [0.104]	2.286 [0.094]	2.302 [0.095]
Capital	K	21.734 [0.936]	21.681 [0.934]	16.398 [0.740]	16.592 [0.746]
Consumption	C	1.992 [0.062]	1.993 [0.062]	1.872 [0.061]	1.876 [0.061]
Investment	I	0.543 [0.053]	0.542 [0.053]	0.410 [0.044]	0.415 [0.044]
Labour	H	0.881 [0.010]	0.880 [0.010]	0.865 [0.008]	0.869 [0.008]
Real wage	w	1.756 [0.063]	1.754 [0.064]	1.619 [0.060]	1.630 [0.060]
Real marginal cost	x	0.909 [0.008]	0.909 [0.009]	0.915 [0.007]	0.919 [0.006]
Inflation	π	2.580 [0.527]	0.703 [0.312]	2.385 [0.462]	4.015 [0.633]
Household welfare	$\mathcal{U}$	29.957 [0.989]	30.155 [0.988]	22.685 [0.895]	22.555 [0.895]
Nominal interest rate	R	6.782 [0.497]	4.828 [0.388]	62.443 [0.760]	65.029 [0.947]
Real interest rate	r	4.097 [0.384]	4.097 [0.390]	58.660 [0.634]	58.660 [0.622]
Rental rate	r^k	4.098 [0.427]	4.098 [0.435]	7.019 [0.502]	7.016 [0.490]
Return on capital	r^{cap}	4.098 [0.427]	4.098 [0.435]	58.662 [0.704]	58.662 [0.687]

Note: Statistics calculated using 10^6 simulated observations; standard deviations in brackets.

we were to introduce a production subsidy (financed by a lump-sum tax) to make the economy's steady state efficient, then there would be no discretionary inflation bias. In that case, the decline in inflation generated by the central bank's quasi-hyperbolic discounting would drive drive inflation away from zero, which would lower household welfare.

To explore more fully whether it is desirable for the central bank to discount the future at a rate that differs from households, Table 5 reports the optimal value for the central bank's quasi-hyperbolic discount factor γ^* and the household's welfare level at this point (with the standard deviation for welfare given in square brackets) as a function of the household quasi-hyperbolic discount factor, β. Several interesting and important results are apparent from Table 5. First, it is desirable for the central bank's quasi-hyperbolic discounting to be stronger than that of the household ($\gamma^* < \beta$). Second, even when the household does not have quasi-hyperbolic discounting

Table 5: Impact of Quasi-Geometric Discounting on Household Welfare

β	γ^*	Welfare
0.90	0.865	22.6818 [0.8910]
0.92	0.865	24.2281 [0.9098]
0.94	0.865	25.7494 [0.9285]
0.96	0.866	27.2448 [0.9472]
0.98	0.868	28.7130 [0.9660]
1.00	0.873	30.1468 [0.9846]

($\beta = 1$), the central bank should ($\gamma^* < 1$). Third, the optimal value for γ^* is relatively insensitive to changes to β.

8 Conclusion

In this paper we study the conduct of discretionary monetary policy in an economy where economic agents have quasi-hyperbolic discounting. Households gain utility through consumption and leisure and save by purchasing bonds and equities. With the exception of the goods market, which is characterized by monopolistic competition and Rotemberg-prices, all other markets are assumed to be perfectly competitive. As is well-known, by weighting the present more than the future, relative to geometric discounting, quasi-hyperbolic discounting has important implications for the equilibrium return on savings, and hence on the capital stock and the level of production. However, in a model where there are costs to changing prices, quasi-hyperbolic discounting also has important consequences for the inflation rate.

With the central bank conducting monetary policy optimally under discretion, we show that the household's quasi-hyperbolic discounting changes the economy's average inflation rate, with greater quasi-hyperbolic discounting giving rise to lower average inflation. The economy's average inflation rate declines because the household's greater emphasis on the present (relative to the future) strengthens the incentive for firms to spread price-changes out over time, benefiting their equity-holders by making smaller price changes today and shifting the remaining price-change to the future (when it is discounted more heavily). This qualitative mechanism continues to hold when the economy's steady state is efficient, although its magnitude in reduced.

Our model also allows the central bank to have quasi-hyperbolic discounting, and for its discounting to differ from the household. We show that a benevolent central bank—one that shares household's preferences—is able to keep steady state inflation under control for a wide range of discount factors. If the central bank, however, does not adopt the household's time discounting and tries to discourage early consumption and delayed saving, then the resulting equilibrium produces only a small increase in output while generating a substantial rise in inflation. Indeed, we show that it is optimal for the central bank to (quasi-hyperbolically) discount the future more heavily than the household, and that doing so decreases inflation and increases welfare.

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A Appendix: The model where firms own capital

This section presents our benchmark model in which firms own the capital stock. We present an alternative formulation in which household's own the capital stock in Appendix B. The equivalence of these two formulations can be seen by comparing Appendices A.3. and B.3. The central bank's decision problem is presented in Appendix C.

A.1 Household's problem

The household's decision problem is described by the Lagrangian

$$\mathcal{U}(b_t, s_t, \mathbf{Z}_t) = \min_{\{\lambda_t\}} \max_{\{c_t, h_t, b_{t+1}, s_{t+1}\}} \left[\begin{array}{c} \frac{c_t^{1-\sigma}-1}{1-\sigma} - \chi \frac{h_t^{1+\upsilon}}{1+\upsilon} + \beta\theta \mathrm{E}_t \left[U(b_{t+1}, s_{t+1}, \mathbf{Z}_{t+1}) \right] \\ +\lambda_t \left(\begin{array}{c} w(\mathbf{Z}_t) h_t + \frac{b_t}{1+\pi(\mathbf{Z}_t)} + Q(\mathbf{Z}_t) s_t (1 + r^s(\mathbf{Z}_t)) \\ -c_t - \frac{b_{t+1}}{1+R(\mathbf{Z}_t)} - Q(\mathbf{Z}_t) s_{t+1} \end{array} \right) \end{array} \right],$$

where

$$U(b_t, s_t, \mathbf{Z}_t) = \left[\begin{array}{c} \frac{c_t^{1-\sigma}-1}{1-\sigma} - \chi \frac{h_t^{1+\upsilon}}{1+\upsilon} + \theta \mathrm{E}_t \left[U(b_{t+1}, s_{t+1}, \mathbf{Z}_{t+1}) \right] \\ +\lambda_t \left(\begin{array}{c} w(\mathbf{Z}_t) h_t + \frac{b_t}{1+\pi(\mathbf{Z}_t)} + Q(\mathbf{Z}_t) s_t (1 + r^s(\mathbf{Z}_t)) \\ -c_t - \frac{b_{t+1}}{1+R(\mathbf{Z}_t)} - Q(\mathbf{Z}_t) s_{t+1} \end{array} \right) \end{array} \right]. \quad (29)$$

The aggregate state vector, $\mathbf{Z}_t$, contains ζ_t, a_t, and K_t, and its equilibrium law-of-motion is taken as given. The first-order conditions with respect to c_t, h_t, b_{t+1}, and s_{t+1} are

$$\frac{\partial \mathcal{U}\left(b_t, s_t, \mathbf{Z}_t\right)}{\partial c_t} \quad : \quad c_t^{-\sigma} - \lambda_t = 0, \tag{30}$$

$$\frac{\partial \mathcal{U}\left(b_t, s_t, \mathbf{Z}_t\right)}{\partial h_t} \quad : \quad -\chi h_t^{\upsilon} + \lambda_t w_t = 0, \tag{31}$$

$$\frac{\partial \mathcal{U}\left(b_t, s_t, \mathbf{Z}_t\right)}{\partial b_{t+1}} \quad : \quad -\frac{\lambda_t}{1+R_t} + \beta\theta \mathrm{E}_t\left[U_b\left(b_{t+1}, s_{t+1}, \mathbf{Z}_{t+1}\right)\right] = 0, \tag{32}$$

$$\frac{\partial \mathcal{U}\left(b_t, s_t, \mathbf{Z}_t\right)}{\partial s_{t+1}} \quad : \quad -\lambda_t Q_t + \beta\theta \mathrm{E}_t\left[U_s\left(b_{t+1}, s_{t+1}, \mathbf{Z}_{t+1}\right)\right] = 0. \tag{33}$$

In equilibrium, the decision rules for bonds, stocks, labor and consumption take the form

$$b_{t+1} = \mathcal{B}\left(b_t, s_t, \mathbf{Z}_t\right), \tag{34}$$

$$s_{t+1} = \mathcal{S}\left(b_t, s_t, \mathbf{Z}_t\right), \tag{35}$$

$$h_t = \mathcal{H}\left(b_t, s_t, \mathbf{Z}_t\right), \tag{36}$$

$$c_t = \mathcal{C}\left(b_t, s_t, \mathbf{Z}_t\right). \tag{37}$$

We now substitute equations (34)—(37) into equation (29) and differentiate the resulting identity with respect to b_t and s_t to get

$$U_b\left(b_t, s_t, \mathbf{Z}_t\right) = c_t^{-\sigma}\left(\frac{1}{1+\pi_t} + \frac{1-\beta}{\beta}\left(\frac{\mathcal{B}_b\left(b_t, s_t, \mathbf{Z}_t\right)}{1+R_t} + Q_t \mathcal{S}_b\left(b_t, s_t, \mathbf{Z}_t\right)\right)\right), \tag{38}$$

$$U_s\left(b_t, s_t, \mathbf{Z}_t\right) = c_t^{-\sigma}\left(Q_t\left(1+r^s{}_t\right) + \frac{1-\beta}{\beta}\left(\frac{\mathcal{B}_s\left(b_t, s_t, \mathbf{Z}_t\right)}{1+R_t} + Q_t \mathcal{S}_s\left(b_t, s_t, \mathbf{Z}_t\right)\right)\right). \tag{39}$$

Substituting equations (38) and (39) into equations (31)—(33), using equation (30) to eliminate the Lagrange multiplier, and aggregating across the unit-mass of identical households gives

$$C_t^{-\sigma} w_t = -\chi H_t^{\upsilon}, \tag{40}$$

$$\frac{C_t^{-\sigma}}{1+R_t} = \theta \mathrm{E}_t\left[C_{t+1}^{-\sigma}\left(\frac{\beta}{1+\pi_{t+1}} + (1-\beta)\left(\begin{array}{c}\frac{\mathcal{B}_B\left(B_{t+1}, S_{t+1}, \mathbf{Z}_{t+1}\right)}{1+R_{t+1}} \\ +Q_{t+1}\mathcal{S}_B\left(B_{t+1}, S_{t+1}, \mathbf{Z}_{t+1}\right)\end{array}\right)\right)\right] \tag{41}$$

$$C_t^{-\sigma} Q_t = \theta \mathrm{E}_t\left[C_{t+1}^{-\sigma}\left(\begin{array}{c}\beta Q_{t+1}\left(1+r^s{}_{t+1}\right) \\ +(1-\beta)\left(\begin{array}{c}\frac{\mathcal{B}_S\left(B_{t+1}, S_{t+1}, \mathbf{Z}_{t+1}\right)}{1+R_{t+1}} \\ +Q_{t+1}\mathcal{S}_S\left(B_{t+1}, S_{t+1}, \mathbf{Z}_{t+1}\right)\end{array}\right)\end{array}\right)\right], \tag{42}$$

where C_t and H_t represent aggregate consumption and labor, respectively. Finally, with bonds in zero-net-supply ($B_t = 0\ \forall\ t$) and stocks in fixed-net-supply ($S_t = 1\ \forall\ t$, where this normalization is without loss of generality), we have that $\mathcal{B}_B\left(B_{t+1}, S_{t+1}, \mathbf{Z}_{t+1}\right) = \mathcal{B}_S\left(B_{t+1}, S_{t+1}, \mathbf{Z}_{t+1}\right) = \mathcal{S}_B\left(B_{t+1}, S_{t+1}, \mathbf{Z}_{t+1}\right) = \mathcal{S}_S\left(B_{t+1}, S_{t+1}, \mathbf{Z}_{t+1}\right) = 0$, and equations (40)—(42) simplify to equations (7)—(9) in the main text. The fact that these derivatives all equal zero simply means that households cannot use the accumulation of bonds and/or stocks to constrain their future selves.

A.2 Firm's problem

To formulate the representative firm's decision problem, we first substitute the production function and the demand function for the firm's good into its profit function. With these substitutions, the firm's decision problem takes the form

$$\mathcal{W}\left(k_t,p_{t-1},\mathbf{Z}_t\right)=\max_{\{p_t,k_{t+1}\}}\left[\begin{array}{c} p_t^{1-\varepsilon_t}Y\left(\mathbf{Z}_t\right)-w\left(\mathbf{Z}_t\right)\left(p_t^{-\varepsilon_t}Y\left(\mathbf{Z}_t\right)e^{-a_t}k_t^{-\alpha}\right)^{\frac{1}{1-\alpha}} \\ -\left(k_{t+1}-\left(1-\delta\right)k_t\right)-\frac{\omega}{2}\left(\frac{p_t}{p_{t-1}}\left(1+\pi\left(\mathbf{Z}_t\right)\right)-1\right)^2Y\left(\mathbf{Z}_t\right) \\ +\beta\theta\mathrm{E}_t\left[\frac{C(\mathbf{Z}_{t+1})^{-\sigma}}{C(\mathbf{Z}_t)^{-\sigma}}W\left(k_{t+1},p_t,\mathbf{Z}_{t+1}\right)\right]\end{array}\right],$$

where

$$W\left(k_t,p_{t-1},\mathbf{Z}_t\right)=\left[\begin{array}{c} p_t^{1-\varepsilon_t}Y\left(\mathbf{Z}_t\right)\left(1-\tau\right)-w\left(\mathbf{Z}_t\right)\left(p_t^{-\varepsilon_t}Y\left(\mathbf{Z}_t\right)e^{-a_t}k_t^{-\alpha}\right)^{\frac{1}{1-\alpha}} \\ -\left(k_{t+1}-\left(1-\delta\right)k_t\right)-\frac{\omega}{2}\left(\frac{p_t}{p_{t-1}}\left(1+\pi\left(\mathbf{Z}_t\right)\right)-1\right)^2Y\left(\mathbf{Z}_t\right) \\ +\theta\mathrm{E}_t\left[\frac{C(\mathbf{Z}_{t+1})^{-\sigma}}{C(\mathbf{Z}_t)^{-\sigma}}W\left(k_{t+1},p_t,\mathbf{Z}_{t+1}\right)\right]\end{array}\right], \tag{43}$$

and where the aggregate state is $\mathbf{Z}_t=\left[\begin{array}{ccc}\zeta_t & a_t & K_t\end{array}\right]'$ and its equilibrium law-of-motion is taken as given.

The first-order conditions can be written as

$$\frac{\partial\mathcal{W}\left(k_t,p_{t-1},\mathbf{Z}_t\right)}{\partial k_{t+1}}:\quad -1+\beta\theta\mathrm{E}_t\left[\frac{C_{t+1}^{-\sigma}}{C_t^{-\sigma}}W_k\left(k_{t+1},p_t,\mathbf{Z}_{t+1}\right)\right]=0, \tag{44}$$

$$\begin{aligned}\frac{\partial\mathcal{W}\left(k_t,p_{t-1},\mathbf{Z}_t\right)}{\partial p_t}:\quad & \left(1-\varepsilon_t\right)p_t^{-\varepsilon_t}Y_t+\frac{\varepsilon_t}{1-\alpha}w_tp_t^{-\varepsilon_t\left(\frac{\alpha}{1-\alpha}\right)}\left(Y_te^{-a_t}k_t^{-\alpha}\right)^{\frac{1}{1-\alpha}} \\ & -\omega\left(\frac{p_t}{p_{t-1}}\left(1+\pi_t\right)-1\right)Y_t\frac{1+\pi_t}{p_{t-1}}+\beta\theta\mathrm{E}_t\left[\frac{C_{t+1}^{-\sigma}}{C_t^{-\sigma}}W_p\left(k_{t+1},p_t,\mathbf{Z}_{t+1}\right)\right]=0.\end{aligned} \tag{45}$$

In order to find $W_k\left(k_t,p_{t-1},\mathbf{Z}_t\right)$ and $W_p\left(k_t,p_{t-1},\mathbf{Z}_t\right)$ we substitute the solution

$$\begin{aligned} k_{t+1} &= \mathcal{K}\left(k_t,p_{t-1},\mathbf{Z}_t\right), \\ p_t &= \mathcal{P}\left(k_t,p_{t-1},\mathbf{Z}_t\right), \end{aligned}$$

into equation (43) and differentiate the resulting identity with respect to k_t and p_{t-1}. From the first-order conditions we obtain

$$\begin{aligned} W_k\left(k_t, p_{t-1}, \mathbf{Z}_t\right) &= \frac{\alpha}{1-\alpha} w_t \frac{h_t}{k_t} + 1 - \delta + \frac{1-\beta}{\beta} \mathcal{K}_k\left(k_t, p_{t-1}, \mathbf{Z}_t\right) \\ &\quad + \frac{1-\beta}{\beta} \left(\begin{array}{c} \omega\left(\frac{p_t}{p_{t-1}}\left(1+\pi_t\right)-1\right) Y_t \frac{1+\pi_t}{p_{t-1}} \\ -\left(1-\varepsilon_t\right) p_t^{-\varepsilon_t} Y_t - \frac{\varepsilon_t}{1-\alpha} \frac{w_t}{p_t} h_t \end{array} \right) \mathcal{P}_k\left(k_t, p_{t-1}, \mathbf{Z}_t\right), \end{aligned} \tag{46}$$

$$\begin{aligned} W_p\left(k_t, p_{t-1}, \mathbf{Z}_t\right) &= \omega\left(\frac{p_t}{p_{t-1}}\left(1+\pi_t\right)-1\right) \frac{p_t}{p_{t-1}^2}\left(1+\pi_t\right) Y_t + \frac{1-\beta}{\beta} \mathcal{K}_p\left(k_t, p_{t-1}, \mathbf{Z}_t\right) \\ &\quad + \frac{1-\beta}{\beta} \left(\begin{array}{c} -\left(1-\varepsilon_t\right) p_t^{-\varepsilon_t} Y_t - \frac{\varepsilon_t}{1-\alpha} \frac{w_t}{p_t} h_t \\ +\omega\left(\frac{p_t}{p_{t-1}}\left(1+\pi_t\right)-1\right) Y_t \frac{1+\pi_t}{p_{t-1}} \end{array} \right) \mathcal{P}_p\left(k_t, p_{t-1}, \mathbf{Z}_t\right). \end{aligned} \tag{47}$$

We next substitute equations (46) and (47) into equation (44) and (**??**), and aggregate across firms. In a symmetric equilibrium in which all firms set the same price, so that the price of their good relative to that of the aggregate goods always equals one, this aggregation implies $\mathcal{P}_P\left(\mathbf{Z}_t\right) = \mathcal{P}_K\left(\mathbf{Z}_t\right) = \mathcal{K}_P\left(\mathbf{Z}_t\right) = 0$. To understand why aggregation implies $\mathcal{P}_P\left(\mathbf{Z}_t\right) = \mathcal{P}_K\left(\mathbf{Z}_t\right) = \mathcal{K}_P\left(\mathbf{Z}_t\right) = 0$, notice that if one firm sets the individual price above (below) the aggregate price so that $\mathcal{P}_P\left(\mathbf{Z}_t\right) \neq 0$ then all firms would do the same and the relative price would not equal one, which is inconsistent with the definition of the economy's aggregate price. Further, because the optimal relative price equals to one, it does not vary with the level of aggregate capital, so $\mathcal{P}_K\left(\mathbf{Z}_t\right) = 0$. Lastly, the fact that the optimal relative price always equals one means that $\mathcal{K}_P\left(\mathbf{Z}_t\right) = 0$. As a consequence, after aggregation we get

$$C_t^{-\sigma} = \beta\theta \mathrm{E}_t\left[C_{t+1}^{-\sigma}\left(\frac{\alpha}{1-\alpha} w_{t+1} \frac{H_{t+1}}{K_{t+1}} + 1 - \delta + \frac{1-\beta}{\beta} \mathcal{K}_K\left(\mathbf{Z}_{t+1}\right)\right)\right], \tag{48}$$

and

$$\pi_t\left(1+\pi_t\right) = \frac{1-\varepsilon_t}{\omega} + \frac{\varepsilon_t}{\omega(1-\alpha)} \frac{w_t H_t}{Y_t} + \beta\theta \mathrm{E}_t\left[\frac{C_{t+1}^{-\sigma}}{C_t^{-\sigma}} \frac{\pi_{t+1}\left(1+\pi_{t+1}\right) Y_{t+1}}{Y_t}\right], \tag{49}$$

respectively.

Now, let us define real marginal costs, x_t, the shadow real rental rate of capital, r_t^k, and the real wage, w_t, according to

$$x_t = \frac{1}{1-\alpha} \frac{w_t H_t}{Y_t},$$

$$r_t^k = \alpha x_t \frac{Y_t}{K_t}, \tag{50}$$

$$w_t = (1-\alpha) x_t \frac{Y_t}{H_t}, \tag{51}$$

then equations (48) and (49) become

$$1 = \beta\theta \mathrm{E}_t \left[\frac{C_{t+1}^{-\sigma}}{C_t^{-\sigma}} \left(\alpha \frac{x_{t+1}Y_{t+1}}{K_{t+1}} + 1 - \delta + \frac{1-\beta}{\beta} \mathcal{K}_K \left(\mathbf{Z}_{t+1} \right) \right) \right], \tag{52}$$

$$\pi_t \left(1+\pi_t\right) = \frac{\varepsilon_t}{\omega} \left(x_t + \frac{1-\varepsilon_t}{\varepsilon_t} \right) + \beta\theta \mathrm{E}_t \left[\frac{C_{t+1}^{-\sigma} Y_{t+1}}{C_t^{-\sigma} Y_t} \pi_{t+1} \left(1+\pi_{t+1}\right) \right]. \tag{53}$$

Equations (50) and (51) correspond to equations (12) and (13) in the main text and equations (52) and (53) correspond to equations (10) and (11) in the main text.

Finally, we note that aggregate profits distributed to households through dividends are given by

$$Q_t r_t^s = \left(1 - x_t - \frac{\omega}{2}\pi_t^2\right) Y_t + r_t^k K_t - \left(K_{t+1} - (1-\delta) K_t\right),$$

which corresponds to equation (14) in the main text.

A.3 Private sector equations

Collecting all of the first-order conditions from Appendices A.1 and A.2 together, and rearranging, we get

$$\begin{aligned}
C_t^{-\sigma} w_t &= \chi H_t^{\upsilon}, \\
\frac{C_t^{-\sigma}}{1+R_t} &= \beta\theta \mathrm{E}_t \left[\frac{C_{t+1}^{-\sigma}}{1+\pi_{t+1}} \right], \\
C_t^{-\sigma} Q_t &= \beta\theta \mathrm{E}_t \left[C_{t+1}^{-\sigma} \left(Q_{t+1} + \left(1 - x_{t+1} - \frac{\omega}{2}\pi_{t+1}^2\right) Y_{t+1} + r_{t+1}^k K_{t+1} - I_{t+1} \right) \right], \\
I_t &= K_{t+1} - (1-\delta) K_t, \\
C_t + K_{t+1} &= Y_t + (1-\delta) K_t - \frac{\omega}{2}\pi_t^2 Y_t, \\
C_t^{-\sigma} &= \beta\theta \mathrm{E}_t \left[C_{t+1}^{-\sigma} \left(r_{t+1}^k + 1 - \delta + \frac{1-\beta}{\beta} \mathcal{K}_K \left(\mathbf{Z}_{t+1} \right) \right) \right], \\
\pi_t \left(1+\pi_t\right) &= \frac{\varepsilon_t}{\omega} \left(x_t + \frac{1-\varepsilon_t}{\varepsilon_t} \right) + \beta\theta \mathrm{E}_t \left[\frac{C_{t+1}^{-\sigma} Y_{t+1}}{C_t^{-\sigma} Y_t} \pi_{t+1} \left(1+\pi_{t+1}\right) \right], \\
Y_t &= e^{a_t} K_t^{\alpha} H_t^{1-\alpha}, \\
r_t^k &= \frac{\alpha}{1-\alpha} w_t \frac{H_t}{K_t}, \\
w_t &= (1-\alpha) x_t \frac{Y_t}{H_t}.
\end{aligned}$$

B Appendix: The model where household's own capital

Here we consider an alternative version of the model in which household's rather than firms own the capital stock. With households owning the capital stock we assume that there is a perfectly competitive market in which firms can rent the capital from households.

B.1 Household's problem

With household's owning the capital stock their optimization problem becomes

$$\mathcal{U}(k_t, b_t, s_t, \mathbf{Z}_t) = \min_{\{\lambda_t\}} \max_{\{c_t, h_t, k_{t+1t}, b_{t+1}, s_{t+1}\}} \left[\begin{array}{c} \frac{c_t^{1-\sigma}-1}{1-\sigma} - \chi \frac{h_t^{1+\upsilon}}{1+\upsilon} + \beta\theta \mathrm{E}_t \left[U(k_{t+1}, b_{t+1}, s_{t+1}, \mathbf{Z}_{t+1}) \right] \\ +\lambda_t \left(\begin{array}{c} \left(1-\delta+r^k(\mathbf{Z}_t)\right) k_t + \frac{b_t}{1+\pi(\mathbf{Z}_t)} + w(\mathbf{z}_t) h_t \\ +Q(\mathbf{Z}_t) s_t (1+r^s(\mathbf{Z}_t)) \\ -c_t - \frac{b_{t+1}}{1+R(\mathbf{Z}_t)} - k_{t+1} - Q(\mathbf{Z}_t) s_{t+1} \end{array} \right) \end{array} \right],$$

with the continuation value given recursively by

$$U(k_t, b_t, s_t, \mathbf{Z}_t) = \left[\begin{array}{c} \frac{c_t^{1-\sigma}-1}{1-\sigma} - \chi \frac{h_t^{1+\upsilon}}{1+\upsilon} + \theta \mathrm{E}_t \left[U(k_{t+1}, b_{t+1}, s_{t+1}, \mathbf{Z}_{t+1}) \right] \\ +\lambda_t \left(\begin{array}{c} \left(1-\delta+r^k(\mathbf{Z}_t)\right) k_t + \frac{b_t}{1+\pi(\mathbf{Z}_t)} + w(\mathbf{Z}_t) h_t \\ +Q(\mathbf{Z}_t) s_t (1+r^s(\mathbf{Z}_t)) \\ -c_t - \frac{b_{t+1}}{1+R(\mathbf{Z}_t)} - k_{t+1} - Q(\mathbf{Z}_t) s_{t+1} \end{array} \right) \end{array} \right]. \tag{54}$$

The first-order conditions with respect to c_t, h_t, k_{t+1}, b_{t+1}, and s_{t+1} can be written as

$$\frac{\partial \mathcal{U}(k_t, b_t, s_t, \mathbf{Z}_t)}{\partial c_t} : c_t^{-\sigma} - \lambda_t = 0, \tag{55}$$

$$\frac{\partial \mathcal{U}(k_t, b_t, s_t, \mathbf{Z}_t)}{\partial h_t} : -\chi h_t^{\upsilon} + \lambda_t w_t = 0, \tag{56}$$

$$\frac{\partial \mathcal{U}(k_t, b_t, s_t, \mathbf{Z}_t)}{\partial k_{t+1}} : -c_t^{-\sigma} + \beta\theta \mathrm{E}_t \left[U_k(k_{t+1}, b_{t+1}, s_{t+1}, \mathbf{Z}_{t+1}) \right] = 0, \tag{57}$$

$$\frac{\partial \mathcal{U}(k_t, b_t, s_t, \mathbf{Z}_t)}{\partial b_{t+1}} : -\frac{c_t^{-\sigma}}{1+R_t} + \beta\theta \mathrm{E}_t \left[U_b(k_{t+1,} b_{t+1}, s_{t+1}, \mathbf{Z}_{t+1}) \right] = 0, \tag{58}$$

$$\frac{\partial \mathcal{U}(k_t, b_t, s_t, \mathbf{Z}_t)}{\partial s_{t+1}} : -c_t^{-\sigma} Q_t + \beta\theta \mathrm{E}_t \left[U_s(k_{t+1,} b_{t+1}, s_{t+1}, \mathbf{Z}_{t+1}) \right] = 0. \tag{59}$$

In order to find $U_k(k_t, b_t, s_t, \mathbf{Z}_t)$, $U_b(k_t, b_t, s_t, \mathbf{Z}_t)$, $U_s(k_t, b_t, s_t, \mathbf{Z}_t)$, we note that the solution we seek will give us the decision rules

$$\begin{aligned} k_{t+1} &= \mathcal{K}(k_t, b_t, s_t, \mathbf{Z}_t), \\ b_{t+1} &= \mathcal{B}(k_t, b_t, s_t, \mathbf{Z}_t), \\ s_{t+1} &= \mathcal{S}(k_t, b_t, s_t, \mathbf{Z}_t), \\ h_t &= \mathcal{H}(k_t, b_t, s_t, \mathbf{Z}_t), \end{aligned}$$

which we substitute into equation (54) and differentiate the resulting identity with respect to k_t, b_t, and s_t. From the resulting derivatives, and employing equations (55)—(59), we obtain

$$\begin{aligned} U_k(k_t,b_t,s_t,\mathbf{Z}_t) &= c_t^{-\sigma}\left(1-\delta+r_t^k+\frac{1-\beta}{\beta}\left(\begin{array}{c}\mathcal{K}_k(k_t,b_t,s_t,\mathbf{Z}_t)+\frac{\mathcal{B}_k(k_t,b_t,s_t,\mathbf{Z}_t)}{1+R_t}\\ +Q_t\mathcal{S}_k(k_t,b_t,s_t,\mathbf{Z}_t)\end{array}\right)\right), && (60)\\ U_b(k_t,b_t,s_t,\mathbf{Z}_t) &= c_t^{-\sigma}\left(\frac{1}{1+\pi_t}+\frac{1-\beta}{\beta}\left(\begin{array}{c}\mathcal{K}_b(k_t,b_t,s_t,\mathbf{Z}_t)+\frac{\mathcal{B}_b(k_t,b_t,s_t,\mathbf{Z}_t)}{1+R_t}\\ +Q_t\mathcal{S}_b(k_t,b_t,s_t,\mathbf{Z}_t)\end{array}\right)\right), && (61)\\ U_s(k_t,b_t,s_t,\mathbf{Z}_t) &= c_t^{-\sigma}\left(Q_t(1+r_t^s)+\frac{1-\beta}{\beta}\left(\begin{array}{c}\mathcal{K}_s(k_t,b_t,s_t,\mathbf{Z}_t)+\frac{\mathcal{B}_s(k_t,b_t,s_t,\mathbf{Z}_t)}{1+R_t}\\ +Q_t\mathcal{S}_s(k_t,b_t,s_t,\mathbf{Z}_t)\end{array}\right)\right). && (62)\end{aligned}$$

With bonds in zero-net-supply ($B_t=0\ \forall\ t$) and stocks in fixed-net-supply ($S_t=1\ \forall\ t$), we have $\mathcal{B}_B(K_t,B_t,S_t,\mathbf{Z}_t)=\mathcal{B}_S(K_t,B_t,S_t,\mathbf{Z}_t)=\mathcal{S}_B(K_t,B_t,S_t,\mathbf{Z}_t)=\mathcal{S}_S(K_t,B_t,S_t,\mathbf{Z}_t)=\mathcal{K}_B(K_t,B_t,S_t,\mathbf{Z}_t)=\mathcal{K}_S(K_t,B_t,S_t,\mathbf{Z}_t)=0$, so substituting equations (60)—(62) into equations (56)—(59), aggregating across households, and using equation (55) to eliminate the Lagrange multiplier gives

$$\begin{aligned} C_t^{-\sigma}w_t &= \chi H_t^{\upsilon},\\ C_t^{-\sigma} &= \beta\theta \mathrm{E}_t\left[C_{t+1}^{-\sigma}\left(r_{t+1}^k+1-\delta+\frac{1-\beta}{\beta}\mathcal{K}_K(K_{t+1},B_{t+1},S_{t+1},\mathbf{Z}_{t+1})\right)\right],\\ \frac{C_t^{-\sigma}}{1+R_t} &= \beta\theta \mathrm{E}_t\left[\frac{C_{t+1}^{-\sigma}}{1+\pi_{t+1}}\right],\\ C_t^{-\sigma}Q_t &= \beta\theta \mathrm{E}_t\left[C_{t+1}^{-\sigma}Q_{t+1}\left(1+r_{t+1}^s\right)\right].\end{aligned}$$

B.2 Firm's problem

The firm's decision problem takes the form

$$\mathcal{W}(p_{t-1},\mathbf{Z}_t)=\max_{\{p_t,k_t\}}\left[\begin{array}{c}p_t^{1-\varepsilon_t}Y(\mathbf{Z}_t)-w(\mathbf{Z}_t)\left(p_t^{-\varepsilon_t}Y(\mathbf{Z}_t)e^{-a_t}k_t^{-\alpha}\right)^{\frac{1}{1-\alpha}}-r^k(\mathbf{Z}_t)k_t\\ -\frac{\omega}{2}\left(\frac{p_t}{p_{t-1}}(1+\pi(\mathbf{Z}_t))-1\right)^2Y(\mathbf{Z}_t)+\beta\theta\mathrm{E}_t\left[\frac{C_{t+1}^{-\sigma}}{C_t^{-\sigma}}W_{t+1}(p_t,\mathbf{Z}_{t+1})\right]\end{array}\right],$$

where the firm's continuation value satisfies

$$W(p_{t-1},\mathbf{Z}_t)=\left[\begin{array}{c}p_t^{1-\varepsilon_t}Y(\mathbf{Z}_t)-w(\mathbf{Z}_t)\left(p_t^{-\varepsilon_t}Y(\mathbf{Z}_t)e^{-a_t}k_t^{-\alpha}\right)^{\frac{1}{1-\alpha}}-r^k(\mathbf{Z}_t)k_t\\ -\frac{\omega}{2}\left(\frac{p_t}{p_{t-1}}(1+\pi(\mathbf{Z}_t))-1\right)^2Y(\mathbf{Z}_t)+\beta\theta\mathrm{E}_t\left[\frac{C_{t+1}^{-\sigma}}{C_t^{-\sigma}}W_{t+1}(p_t,\mathbf{Z}_{t+1})\right]\end{array}\right]. \quad (63)$$

The first-order conditions can be written as

$$\begin{aligned} \frac{\partial\mathcal{W}(p_{t-1},\mathbf{Z}_t)}{\partial k_t} &: \quad \frac{\alpha}{1-\alpha}w_t\frac{h_t}{k_t}-r_t^k=0, && (64)\\ \frac{\partial\mathcal{W}(p_{t-1},\mathbf{Z}_t)}{\partial p_t} &: \quad (1-\varepsilon_t)p_t^{-\varepsilon_t}Y_t+\frac{\varepsilon_t}{1-\alpha}w_t p_t^{-\varepsilon_t\left(\frac{\alpha}{1-\alpha}\right)}\left(Y_t e^{-a_t}k_t^{-\alpha}\right)^{\frac{1}{1-\alpha}}\\ &\quad -\omega\left(\frac{p_t}{p_{t-1}}(1+\pi_t)-1\right)Y_t\frac{(1+\pi_t)}{p_{t-1}}+\beta\theta\mathrm{E}_t\left[\frac{C_{t+1}^{-\sigma}}{C_t^{-\sigma}}W_p(p_t,\mathbf{Z}_{t+1})\right]. && (65)\end{aligned}$$

In order to find $W_p(p_{t-1}, \mathbf{Z}_t)$ we substitute the decision rules

$$\begin{aligned} k_t &= \mathcal{K}(p_{t-1}, \mathbf{Z}_t), \\ p_t &= \mathcal{P}(p_{t-1}, \mathbf{Z}_t), \end{aligned}$$

into (63) and differentiate the resulting identity with respect to p_{t-1}. We then use the first-order conditions, equations (64) and (65), to obtain

$$\begin{aligned} W_p(p_{t-1}, \mathbf{Z}_t) &= \omega \left(\frac{p_t}{p_{t-1}} (1 + \pi_t) - 1 \right) \frac{p_t}{p_{t-1}^2} (1 + \pi_t) Y_t - \frac{1-\beta}{\beta} \frac{\varepsilon_t}{1-\alpha} \frac{w_t}{p_t} h_t \mathcal{P}_p(p_{t-1}, \mathbf{Z}_t) \\ &- \frac{1-\beta}{\beta} (1 - \varepsilon_t) p_t^{-\varepsilon_t} Y_t \mathcal{P}_p(p_{t-1}, \mathbf{Z}_t) \\ &+ \frac{1-\beta}{\beta} \omega \left(\frac{p_t}{p_{t-1}} (1 + \pi_t) - 1 \right) Y_t \frac{(1+\pi_t)}{p_{t-1}} \mathcal{P}_p(p_{t-1}, \mathbf{Z}_t). \end{aligned} \tag{66}$$

Substituting equation (66) into equations (64) and (65) and aggregating across firms, which in a symmetric equilibrium where all firms set the same price, implies $\mathcal{P}_p(p_t, \mathbf{Z}_{t+1}) = 0$, yields

$$\pi_t (1 + \pi_t) = \frac{1 - \varepsilon_t}{\omega} + \frac{\varepsilon_t}{\omega} x_t + \beta\theta \mathrm{E}_t \left[\frac{C_{t+1}^{-\sigma} Y_{t+1}}{C_t^{-\sigma} Y_t} (\pi_{t+1} (1 + \pi_{t+1})) \right],$$

where

$$w_t = (1 - \alpha) x_t \frac{Y_t}{H_t}.$$

Finally, the dividends distributed to households are given by

$$Q_t r_t^s = Y_t (1 - \tau) - w_t H_t - r_t^k K_t - \frac{\omega}{2} \pi_t^2 Y_t.$$

B.3 Private sector equations

Collecting all of the first-order conditions together, and rearranging, we get

$$
\begin{aligned}
C_t^{-\sigma} w_t &= \chi H_t^{\upsilon}, \\
\frac{C_t^{-\sigma}}{1+R_t} &= \beta\theta \mathrm{E}_t\left[\frac{C_{t+1}^{-\sigma}}{1+\pi_{t+1}}\right], \\
C_t^{-\sigma} Q_t &= \beta\theta \mathrm{E}_t\left[C_{t+1}^{-\sigma}\left(Q_{t+1}+\left(1-x_{t+1}-\frac{\omega}{2}\pi_{t+1}^2\right)Y_{t+1}\right)\right], \\
C_t + K_{t+1} &= Y_t + (1-\delta)K_t - \frac{\omega}{2}\pi_t^2 Y_t, \\
C_t^{-\sigma} &= \beta\theta \mathrm{E}_t\left[C_{t+1}^{-\sigma}\left(r_{t+1}^k + 1 - \delta + \frac{1-\beta}{\beta}\mathcal{K}_K\left(\mathbf{Z}_{t+1}\right)\right)\right], \\
\pi_t(1+\pi_t) &= \frac{\varepsilon_t}{\omega}\left(x_t + \frac{1-\varepsilon_t}{\varepsilon_t}\right) + \beta\theta \mathrm{E}_t\left[\frac{C_{t+1}^{-\sigma} Y_{t+1}}{C_t^{-\sigma} Y_t}\pi_{t+1}(1+\pi_{t+1})\right], \\
Y_t &= e^{a_t} K_t^{\alpha} H_t^{1-\alpha}, \\
r_t^k &= \frac{\alpha}{1-\alpha} w_t \frac{H_t}{K_t}, \\
w_t &= (1-\alpha)\frac{x_t Y_t}{H_t},
\end{aligned}
$$

which are equivalent to the equations reported in Appendix A.3 that were obtained under the assumption that firm's own the capital stock.

C Appendix: Discretionary policy

The decision problem facing the discretionary policymaker is summarized by the Bellman equation

$$
\mathcal{V}(\mathbf{Z}_t) = \max_{\{C_t, H_t, Y_t, x_t, K_{t+1}, \pi_t\}} \left(\frac{C_t^{1-\sigma}-1}{1-\sigma} - \chi\frac{H_t^{1+\upsilon}}{1+\upsilon} + \gamma\xi \mathrm{E}_t\left[V(\mathbf{Z}_{t+1})\right]\right), \tag{67}
$$

which is subject to the constraints

$$
\begin{aligned}
C_t^{-\sigma} &= \theta \mathrm{E}_t\left[L(\mathbf{Z}_{t+1})\right], && (68) \\
\left(\pi_t(1+\pi_t) + \frac{\varepsilon_t}{\omega}\left(x_t + \frac{1-\varepsilon_t}{\varepsilon_t}\right)\right) Y_t C_t^{-\sigma} &= \theta \mathrm{E}_t\left[M(\mathbf{Z}_{t+1})\right], && (69) \\
\left(1-\frac{\omega}{2}\pi_t^2\right) Y_t &= C_t + K_{t+1} - (1-\delta)K_t, && (70) \\
H_t &= \left(\frac{1-\alpha}{\chi} x_t Y_t C_t^{-\sigma}\right)^{\frac{1}{1+\upsilon}}, && (71) \\
Y_t &= e^{a_t} K_t^{\alpha} H_t^{1-\alpha}, && (72)
\end{aligned}
$$

with the continuation value satisfying the recursion

$$
V(\mathbf{Z}_t) = \frac{C_t^{1-\sigma}-1}{1-\sigma} - \frac{\chi}{1+\upsilon} H_t^{1+\upsilon} + \xi \mathrm{E}_t\left[V(\mathbf{Z}_{t+1})\right].
$$

Because monetary policy is conducted under discretion, the central bank cannot influence how private sector expectations are formed, a restriction imposed by introducing the auxillary variables $L(\mathbf{Z}_t)$ and $M(\mathbf{Z}_t)$, which are defined according to

$$\begin{aligned} L(\mathbf{Z}_t) &= C_t^{-\sigma}\left[\beta\left(\alpha x_t\frac{Y_t}{K_t}+1-\delta\right)+(1-\beta)\,\mathcal{K}_K(\mathbf{Z}_t)\right], \\ M(\mathbf{Z}_t) &= \beta C_t^{-\sigma}Y_t\pi_t\,(1+\pi_t)\,. \end{aligned}$$

After substituting equations (71) and (72) into equations (67)—(70), the central bank's decision problem can be expressed in terms of the Lagrangian

$$\mathcal{V}(\mathbf{Z}_t)=\left[\begin{array}{c} \frac{C_t^{1-\sigma}-1}{1-\sigma}-\chi\frac{\left(\frac{1-\alpha}{\chi}x_te^{a_t}K_t^{\alpha}C_t^{-\sigma}\right)^{\frac{1+\upsilon}{\upsilon+\alpha}}}{1+\upsilon}+\gamma\xi\mathrm{E}_t\left[V(\mathbf{Z}_{t+1})\right] \\ -\phi_{1t}\left(C_t+K_{t+1}-(1-\delta)\,K_t-\left(1-\frac{\omega}{2}\pi_t^2\right)e^{a_t}K_t^{\alpha}\left(\frac{1-\alpha}{\chi}x_te^{a_t}K_t^{\alpha}C_t^{-\sigma}\right)^{\frac{1-\alpha}{\upsilon+\alpha}}\right) \\ -\phi_{2t}\left(\theta\mathrm{E}_t\left[L(\mathbf{Z}_{t+1})\right]-C_t^{-\sigma}\right) \\ +\phi_{3t}\left(\begin{array}{c} \frac{(1-\varepsilon_t)(1-\tau)+\varepsilon_tx_t}{\omega}C_t^{-\sigma}e^{a_t}K_t^{\alpha}\left(\frac{1-\alpha}{\chi}x_te^{a_t}K_t^{\alpha}C_t^{-\sigma}\right)^{\frac{1-\alpha}{\upsilon+\alpha}} \\ +\theta\mathrm{E}_t\left[M(\mathbf{Z}_{t+1})\right]-\pi_t\,(1+\pi_t)\,C_t^{-\sigma}e^{a_t}K_t^{\alpha}\left(\frac{1-\alpha}{\chi}x_te^{a_t}K_t^{\alpha}C_t^{-\sigma}\right)^{\frac{1-\alpha}{\upsilon+\alpha}} \end{array}\right) \end{array}\right], \tag{73}$$

where ϕ_{1t}, ϕ_{2t}, and ϕ_{3t}, represent the Lagrange multipliers on the three remaining constraints. Now, differentiating equation (73) with respect to K_{t+1}, C_t, π_t, and x_t, the first-order conditions are

$$\frac{\partial\mathcal{V}(\mathbf{Z}_t)}{\partial K_{t+1}} : \gamma\xi\mathrm{E}_t\left[V_K(\mathbf{Z}_{t+1})\right]-\phi_{2t}\theta\mathrm{E}_t\left[L_K(\mathbf{Z}_{t+1})\right]+\phi_{3t}\theta\mathrm{E}_t\left[M_K(\mathbf{Z}_{t+1})\right]-\phi_{1t}=0, \tag{74}$$

$$\begin{aligned} \frac{\partial\mathcal{V}(\mathbf{Z}_t)}{\partial C_t} &: C_t^{-\sigma}+\frac{\sigma\chi}{\upsilon+\alpha}\frac{H_t^{1+\upsilon}}{C_t}-\phi_{1t}\left(1+\sigma\frac{1-\alpha}{\upsilon+\alpha}\left(1-\frac{\omega}{2}\pi_t^2\right)\frac{Y_t}{C_t}\right)-\phi_{2t}\sigma C_t^{-\sigma-1} \\ &- \sigma\frac{1+\upsilon}{\alpha+\upsilon}\phi_{3t}\left(\frac{\varepsilon_t}{\omega}\left(x_t+\frac{1-\varepsilon_t}{\varepsilon_t}\right)-\pi_t\,(1+\pi_t)\right)C_t^{-\sigma-1}Y_t=0, \\ \frac{\partial\mathcal{V}(\mathbf{Z}_t)}{\partial \pi_t} &: -\phi_{3t}\,(1+2\pi_t)\,C_t^{-\sigma}-\phi_{1t}\omega\pi_t=0, \\ \frac{\partial\mathcal{V}(\mathbf{Z}_t)}{\partial x_t} &: -\frac{\chi}{\upsilon+\alpha}H_t^{1+\upsilon}x_t^{-1}+\phi_{1t}\left(\frac{1-\alpha}{\upsilon+\alpha}\left(1-\frac{\omega}{2}\pi_t^2\right)\frac{Y_t}{x_t}\right) \\ &+ \phi_{3t}\left(\frac{\varepsilon_tx_t}{\omega}+\frac{1-\alpha}{\upsilon+\alpha}\frac{\varepsilon_t}{\omega}\left(x_t+\frac{1-\varepsilon_t}{\varepsilon_t}\right)-\frac{1-\alpha}{\upsilon+\alpha}\pi_t\,(1+\pi_t)\right)C_t^{-\sigma}Y_tx_t^{-1}=0. \end{aligned}$$

To progress further we must find $V_K(\mathbf{Z}_t)$. The solution provides the decision rules

$$\begin{aligned} x_t &= \mathcal{X}(\mathbf{Z}_t), \\ C_t &= \mathcal{C}(\mathbf{Z}_t), \\ \pi_t &= \pi(\mathbf{Z}_t), \\ K_{t+1} &= \mathcal{K}(\mathbf{Z}_t), \end{aligned}$$

which we substitute into equation (73) giving the identity

$$V(\mathbf{Z}_t)=\left[\begin{array}{c}\frac{\mathcal{C}(\mathbf{Z}_t)^{1-\sigma}-1}{1-\sigma}-\chi\frac{\left(\frac{1-\alpha}{\chi}e^{a_t}\mathcal{X}(\mathbf{Z}_t)K_t^{\alpha}\mathcal{C}(\mathbf{Z}_t)^{-\sigma}\right)^{\frac{1+\upsilon}{\upsilon+\alpha}}}{1+\upsilon}+\xi\mathrm{E}_t\left[V\left(\zeta_{t+1},a_{t+1},\mathcal{K}(\mathbf{Z}_t)\right)\right]\\ -\phi_{1t}\left(\begin{array}{c}\mathcal{C}(\mathbf{Z}_t)+\mathcal{K}(\mathbf{Z}_t)-(1-\delta)K_t\\ -\left(1-\frac{\omega}{2}\pi(\mathbf{Z}_t)^2\right)\left(\frac{1-\alpha}{\chi}e^{\frac{1+\upsilon}{1-\alpha}a_t}\mathcal{X}(\mathbf{Z}_t)K_t^{\alpha\frac{1+\upsilon}{1-\alpha}}\mathcal{C}(\mathbf{Z}_t)^{-\sigma}\right)^{\frac{1-\alpha}{\upsilon+\alpha}}\end{array}\right)\\ -\phi_{2t}\left(\theta\mathrm{E}_t\left[L\left(\zeta_{t+1},a_{t+1},\mathcal{K}(\mathbf{Z}_t)\right)\right]-\mathcal{C}(\mathbf{Z}_t)^{-\sigma}\right)\\ +\phi_{3t}\left(\begin{array}{c}\frac{(1-\varepsilon_t)(1-\tau)+\varepsilon_t\mathcal{X}(\mathbf{Z}_t)}{\omega}\mathcal{C}(\mathbf{Z}_t)^{-\sigma}\left(\frac{1-\alpha}{\chi}e^{\frac{1+\upsilon}{1-\alpha}a_t}\mathcal{X}(\mathbf{Z}_t)K_t^{\alpha\frac{1+\upsilon}{1-\alpha}}\mathcal{C}(\mathbf{Z}_t)^{-\sigma}\right)^{\frac{1-\alpha}{\upsilon+\alpha}}\\ +\theta\mathrm{E}_t\left[M\left(\zeta_{t+1},z_{t+1},\mathcal{K}(\mathbf{Z}_t)\right)\right]\\ -\pi(\mathbf{Z}_t)\left(1+\pi(\mathbf{Z}_t)\right)\left(\frac{1-\alpha}{\chi}e^{\frac{1+\upsilon}{1-\alpha}a_t}\mathcal{X}(\mathbf{Z}_t)K_t^{\alpha\frac{1+\upsilon}{1-\alpha}}\mathcal{C}(\mathbf{Z}_t)^{-\sigma}\right)^{\frac{1-\alpha}{\upsilon+\alpha}}.\end{array}\right)\end{array}\right]. \tag{75}$$

Then, differentiating equation (75) with respect to K_t yields

$$\begin{aligned}V_K(\mathbf{Z}_t) &= \left(C_t^{-\sigma}+\frac{\sigma\chi}{\upsilon+\alpha}\frac{H_t^{1+\upsilon}}{C_t}\right)\mathcal{C}_K(\mathbf{Z}_t)-\frac{\chi}{\upsilon+\alpha}\frac{H_t^{1+\upsilon}}{x_t}\mathcal{X}_K(\mathbf{Z}_t)\\ &- \frac{\alpha\chi}{\upsilon+\alpha}\frac{H_t^{1+\upsilon}}{K_t}+\xi\mathrm{E}_t\left[V_K(\mathbf{Z}_{t+1})\right]\mathcal{K}_K(\mathbf{Z}_t)\\ &- \phi_{1t}\left(\begin{array}{c}\mathcal{K}_K(\mathbf{Z}_t)+\mathcal{C}_K(\mathbf{Z}_t)-(1-\delta)+\sigma\frac{1-\alpha}{\upsilon+\alpha}\left(1-\frac{\omega}{2}\pi_t^2\right)\frac{Y_t}{C_t}\mathcal{C}_K(\mathbf{Z}_t)+\omega\pi_tY_t\pi_K(\mathbf{Z}_t)\\ -\frac{1-\alpha}{\upsilon+\alpha}\left(1-\frac{\omega}{2}\pi_t^2\right)\frac{Y_t}{x_t}\mathcal{X}_K(\mathbf{Z}_t)-\alpha\frac{\upsilon+1}{\upsilon+\alpha}\left(1-\frac{\omega}{2}\pi_t^2\right)\frac{Y_t}{K_t}\end{array}\right)\\ &- \phi_{2t}\left(\theta\mathrm{E}_t\left[L_K(\mathbf{Z}_{t+1})\right]\mathcal{K}_K(\mathbf{Z}_t)+\sigma C_t^{-\sigma-1}\mathcal{C}_K(\mathbf{Z}_t)\right)\\ &+ \phi_{3t}\left(\begin{array}{c}\theta\mathrm{E}_t\left[M_K(\mathbf{Z}_{t+1})\right]\mathcal{K}_K(\mathbf{Z}_t)-(1+2\pi_t)C_t^{-\sigma}Y_t\pi_K(\mathbf{Z}_t)\\ -\alpha\frac{\upsilon+1}{\upsilon+\alpha}\left(\pi_t(1+\pi_t)-\frac{\varepsilon_t}{\omega}\left(x_t+\frac{1-\varepsilon_t}{\varepsilon_t}\right)\right)\frac{Y_t}{K_t}C_t^{-\sigma}\\ -\sigma\frac{\upsilon+1}{\alpha+\upsilon}\left(\frac{\varepsilon_t}{\omega}\left(x_t+\frac{1-\varepsilon_t}{\varepsilon_t}\right)-\pi_t(1+\pi_t)\right)C_t^{-\sigma-1}Y_t\mathcal{C}_K(\mathbf{Z}_t)\\ -\left(\frac{1-\alpha}{\upsilon+\alpha}\left(\pi_t(1+\pi_t)-\frac{\varepsilon_t}{\omega}\left(x_t+\frac{1-\varepsilon_t}{\varepsilon_t}\right)\right)-\frac{\varepsilon_tx_t}{\omega}\right)\frac{Y_t}{x_t}C_t^{-\sigma}\mathcal{X}_K(\mathbf{Z}_t)\end{array}\right),\end{aligned}$$

and using equations (74)—(75) to simplify we get

$$\begin{aligned}V_K(\mathbf{Z}_t) &= \left(1-\frac{1}{\gamma}\right)\left(C_t^{-\sigma}+\frac{\sigma\chi}{\upsilon+\alpha}\frac{H_t^{1+\upsilon}}{C_t}\right)\mathcal{C}_K(\mathbf{Z}_t)-\left(1-\frac{1}{\gamma}\right)\frac{\chi}{\upsilon+\alpha}\frac{H_t^{1+\upsilon}}{x_t}\mathcal{X}_K(\mathbf{Z}_t)\\ &- \frac{\alpha\chi}{\upsilon+\alpha}\frac{H_t^{1+\upsilon}}{K_t}+\frac{1}{\gamma}\phi_{1t}\left(\alpha\frac{1+\upsilon}{\upsilon+\alpha}\left(1-\frac{\omega}{2}\pi_t^2\right)\frac{Y_t}{K_t}+1-\delta\right)\\ &- \frac{\alpha}{\gamma}\frac{1+\upsilon}{\upsilon+\alpha}\phi_{3t}\left(\pi_t(1+\pi_t)-\frac{\varepsilon_t}{\omega}\left(x_t+\frac{1-\varepsilon_t}{\varepsilon_t}\right)\right)\frac{Y_t}{K_t}C_t^{-\sigma}.\end{aligned} \tag{76}$$

After substituting equation (76) back into equation (74), the system of first-order conditions for the discretionary optimization problem can be written as

$$
\begin{aligned}
\frac{\partial}{\partial C_t} &: \quad C_t^{-\sigma}+\frac{\sigma\chi}{\upsilon+\alpha}\frac{H_t^{1+\upsilon}}{C_t}-\phi_{1t}\left(1+\sigma\frac{1-\alpha}{\upsilon+\alpha}\left(1-\frac{\omega}{2}\pi_t^2\right)\frac{Y_t}{C_t}\right)-\phi_{2t}\sigma C_t^{-\sigma-1} \\
&- \quad \sigma\frac{1+\upsilon}{\alpha+\upsilon}\phi_{3t}\left(\frac{\left(1-\varepsilon_t\right)\left(1-\tau\right)+\varepsilon_t x_t}{\omega}-\pi_t\left(1+\pi_t\right)\right)C_t^{-\sigma-1}Y_t=0, \qquad (77)\\
\frac{\partial}{\partial \pi_t} &: \quad -\phi_{3t}\left(1+2\pi_t\right)C_t^{-\sigma}-\phi_{1t}\omega\pi_t=0, \qquad (78)\\
\frac{\partial}{\partial x_t} &: \quad -\frac{\chi}{\upsilon+\alpha}\frac{H_t^{1+\upsilon}}{x_t}+\phi_{1t}\frac{1-\alpha}{\upsilon+\alpha}\left(1-\frac{\omega}{2}\pi_t^2\right)\frac{Y_t}{x_t} \\
&+ \quad \phi_{3t}\left(\frac{\varepsilon_t x_t}{\omega}+\frac{1-\alpha}{\upsilon+\alpha}\frac{\left(1-\varepsilon_t\right)\left(1-\tau\right)+\varepsilon_t x_t}{\omega}-\frac{1-\alpha}{\upsilon+\alpha}\pi_t\left(1+\pi_t\right)\right)C_t^{-\sigma}\frac{Y_t}{x_t}=0, \qquad (79)\\
\frac{\partial}{\partial K_{t+1}} &: \quad -\frac{\gamma\xi\alpha\chi}{\upsilon+\alpha}\mathrm{E}_t\left[\frac{H_{t+1}^{1+\upsilon}}{K_{t+1}}\right]+\xi\mathrm{E}_t\left[\phi_{1t+1}\left(\alpha\frac{1+\upsilon}{\upsilon+\alpha}\left(1-\frac{\omega}{2}\pi_{t+1}^2\right)\frac{Y_{t+1}}{K_{t+1}}+1-\delta\right)\right] \\
&+ \quad \xi\alpha\frac{1+\upsilon}{\upsilon+\alpha}\mathrm{E}_t\left[\phi_{3t+1}\left(\frac{\varepsilon_{t+1}}{\omega}\left(x_{t+1}+\frac{1-\varepsilon_{t+1}}{\varepsilon_{t+1}}\right)-\pi_{t+1}\left(1+\pi_{t+1}\right)\right)\frac{Y_{t+1}}{K_{t+1}}C_{t+1}^{-\sigma}\right] \\
&- \quad \xi\left(1-\gamma\right)\mathrm{E}_t\left[\left(C_{t+1}^{-\sigma}+\frac{\sigma\chi}{\upsilon+\alpha}\frac{H_{t+1}^{1+\upsilon}}{C_{t+1}}\right)\mathcal{C}_K(\mathbf{Z}_{t+1})\right] \\
&+ \quad \frac{\xi\left(1-\gamma\right)\chi}{\upsilon+\alpha}\mathrm{E}_t\left[\frac{H_{t+1}^{1+\upsilon}}{x_{t+1}}\mathcal{X}_K(\mathbf{Z}_{t+1})\right] \\
&- \quad \phi_{2t}\theta\mathrm{E}_t\left[L_K(\mathbf{Z}_{t+1})\right]+\phi_{3t}\theta\mathrm{E}_t\left[M_K(\mathbf{Z}_{t+1})\right]-\phi_{1t}=0. \qquad (80)
\end{aligned}
$$

where

$$
\begin{aligned}
H_t &= \left(\left(\frac{1-\alpha}{\chi}\right)e^{a_t}x_t K_t^{\alpha}C_t^{-\sigma}\right)^{\frac{1}{\upsilon+\alpha}}, \qquad (81)\\
Y_t &= \left(\left(\frac{1-\alpha}{\chi}\right)^{1-\alpha}e^{(1+\upsilon)a_t}x_t^{1-\alpha}K_t^{\alpha(1+\upsilon)}C_t^{-\sigma(1-\alpha)}\right)^{\frac{1}{\upsilon+\alpha}}, \qquad (82)\\
L(\mathbf{Z}_t) &= C_t^{-\sigma}\left(\beta\left(e^{a_t}x_t\alpha K_t^{\alpha-1}H_t^{1-\alpha}+1-\delta\right)+\left(1-\beta\right)\mathcal{K}_K(\mathbf{Z}_t)\right), \qquad (83)\\
M(\mathbf{Z}_t) &= \beta\pi_t\left(1+\pi_t\right)C_t^{-\sigma}Y_t. \qquad (84)
\end{aligned}
$$

Equations (77)—(84) correspond to equations (19)—(26) in the main text.

D Appendix: Numerical solution

To solve the central bank's optimal policy problem, described by equations (15)—(26) in the main text, it is convenient to rewrite them more compactly as

$$
\begin{aligned}
0 &= C_t^{-\sigma} + \frac{\sigma\chi}{\upsilon+\alpha}\frac{H_t^{1+\upsilon}}{C_t} - \phi_{1t}\left(1 + \sigma\frac{1-\alpha}{\upsilon+\alpha}\left(1-\frac{\omega}{2}\pi_t^2\right)\frac{Y_t}{C_t}\right) - \phi_{2t}\sigma C_t^{-\sigma-1} \\
&\quad -\sigma\frac{1+\upsilon}{\alpha+\upsilon}\phi_{3t}\left(\frac{\left(1-\varepsilon e^{\zeta_t}\right)+\varepsilon e^{\zeta_t}x_t}{\omega} - \pi_t\left(1+\pi_t\right)\right)C_t^{-\sigma-1}Y_t, && (85) \\
0 &= -\phi_{3t}\left(1+2\pi_t\right)C_t^{-\sigma} - \phi_{1t}\omega\pi_t, && (86) \\
0 &= -\frac{\chi}{\upsilon+\alpha}\frac{H_t^{1+\upsilon}}{x_t} + \phi_{1t}\frac{1-\alpha}{\upsilon+\alpha}\left(1-\frac{\omega}{2}\pi_t^2\right)\frac{Y_t}{x_t} && (87) \\
&\quad +\phi_{3t}\left(\frac{\varepsilon e^{\zeta_t}x_t}{\omega} + \frac{1-\alpha}{\upsilon+\alpha}\frac{\left(1-\varepsilon e^{\zeta_t}\right)+\varepsilon e^{\zeta_t}x_t}{\omega} - \frac{1-\alpha}{\upsilon+\alpha}\pi_t\left(1+\pi_t\right)\right)C_t^{-\sigma}\frac{Y_t}{x_t} \\
0 &= \mathcal{D}_{t+1} - \phi_{2t}\theta\mathcal{L}_{K,t+1} + \phi_{3t}\theta\mathcal{M}_{K,t+1} - \phi_{1t} && (88) \\
0 &= C_t^{-\sigma} - \theta\mathcal{L}_{t+1}, && (89) \\
0 &= \left(\pi_t\left(1+\pi_t\right) + \frac{\left(\varepsilon e^{\zeta_t}-1\right)-\varepsilon e^{\zeta_t}x_t}{\omega}\right)Y_tC_t^{-\sigma} - \theta\mathcal{M}_{t+1}, && (90) \\
0 &= \left(1-\frac{\omega}{2}\pi_t^2\right)Y_t - C_t - K_{t+1} + \left(1-\delta\right)K_t, && (91)
\end{aligned}
$$

where

$$
\begin{aligned}
H_t &= \left(\left(\frac{1-\alpha}{\chi}\right)e^{a_t}x_tK_t^{\alpha}C_t^{-\sigma}\right)^{\frac{1}{\upsilon+\alpha}}, && (92) \\
Y_t &= \left(\left(\frac{1-\alpha}{\chi}\right)^{1-\alpha}e^{(1+\upsilon)a_t}x_t^{1-\alpha}K_t^{\alpha(1+\upsilon)}C_t^{-\sigma(1-\alpha)}\right)^{\frac{1}{\upsilon+\alpha}}, && (93)
\end{aligned}
$$

and

$$
\begin{aligned}
\mathcal{L}_{t+1} &= \mathrm{E}_t\left[L((\zeta_{t+1}, a_{t+1}, K_{t+1})\right], \\
\mathcal{M}_{t+1} &= \mathrm{E}_t\left[M(\zeta_{t+1}, a_{t+1}, K_{t+1})\right], \\
\mathcal{D}_{t+1} &= \mathrm{E}_t\left[D(\zeta_{t+1}, a_{t+1}, K_{t+1})\right], \\
\mathcal{L}_{K,t+1} &= \mathrm{E}_t\left[L_K(\zeta_{t+1}, a_{t+1}, K_{t+1})\right], \\
\mathcal{M}_{Kt+1} &= \mathrm{E}_t\left[M_K(\zeta_{t+1}, a_{t+1}, K_{t+1})\right],
\end{aligned}
$$

with the definitions

$$
\begin{aligned}
L(\zeta_t, a_t, K_t) &\equiv C_t^{-\sigma}\left(\beta\left(e^{a_t}x_t\alpha K_t^{\alpha-1}H_t^{1-\alpha}+1-\delta\right)+(1-\beta)\,\mathcal{K}_K(\zeta_t,a_t,K_t)\right), & (94)\\
M(\zeta_t, a_t, K_t) &\equiv \beta\pi_t\,(1+\pi_t)\,C_t^{-\sigma}Y_t, & (95)\\
D(\zeta_t, a_t, K_t) &\equiv \xi\phi_{1t}\left(\alpha\frac{1+\upsilon}{\upsilon+\alpha}\left(1-\frac{\omega}{2}\pi_t^2\right)\frac{Y_t}{K_t}+1-\delta\right)-\frac{\gamma\xi\alpha\chi}{\upsilon+\alpha}\frac{H_t^{1+\upsilon}}{K_t}\\
&\quad +\xi\alpha\frac{1+\upsilon}{\upsilon+\alpha}\phi_{3t}\left(\frac{\left(1-\varepsilon e^{\zeta_t}\right)+\varepsilon e^{\zeta_t}x_t}{\omega}-\pi_t\,(1+\pi_t)\right)\frac{Y_t}{K_t}C_t^{-\sigma}\\
&\quad -\xi\,(1-\gamma)\left(C_t^{-\sigma}+\frac{\sigma\chi}{\upsilon+\alpha}\frac{H_t^{1+\upsilon}}{C_t}\right)\mathcal{C}_K(\zeta_t,a_t,K_t)\\
&\quad +\frac{\xi\,(1-\gamma)\,\chi}{\upsilon+\alpha}\frac{H_t^{1+\upsilon}}{x_t}\mathcal{X}_K(\zeta_t,a_t,K_t). & (96)
\end{aligned}
$$

Equations (85)—(91) are a system of seven equations containing seven unknowns: six control variables, C_t, π_t, x_t, ϕ_{1t}, ϕ_{2t}, and ϕ_{3t}, and one future state variable, K_{t+1}. We solve this nonlinear system on a set of nodes constructed for the state variables whose domain given by $\zeta \in [\zeta_{\min}, \zeta_{\max}]$, $a \in [a_{\min}, a_{\max}]$, and $K \in [K_{\min}, K_{\max}]$. We compute a set of Gauss-Chebyshev nodes, $\mathbf{Z} = \{\zeta_k, a_j, K_i; k = 1...N_\zeta, j = 1...N_a, i = 1...N_K\}$, for the state space $[\zeta_{\min}, \zeta_{\max}] \times [a_{\min}, a_{\max}] \times [K_{\min}, K_{\max}]$ and use a three-dimensional Chebyshev polynomial to approximate the unknown functions.[3]

Using $\mathbf{Z}_{k,j,i} \in \mathbf{Z}$ to denote a particular grid point, our solution algorithm can be summarized as follows:

Step 1. Initialize arrays for $H_t^{(0)}$, $Y_t^{(0)}$, $\pi_t^{(0)}$, $\phi_{1t}^{(0)}$, $\phi_{2t}^{(0)}$, and $\phi_{3t}^{(0)}$, to store solution outcomes.

Step 2. Conjecture initial state-contingent functions for $K_{t+1}^{(0)} = \mathcal{K}^{(0)}(\mathbf{Z}_{k,j,i})$, $C_t^{(0)} = \mathcal{C}^{(0)}(\mathbf{Z}_{k,j,i})$, $x_t^{(0)} = \mathcal{X}^{(0)}(\mathbf{Z}_{k,j,i})$, $L_t^{(0)} = L^{(0)}(\mathbf{Z}_{k,j,i})$, $M_t^{(0)} = M^{(0)}(\mathbf{Z}_{k,j,i})$, and $D_t^{(0)} = D^{(0)}(\mathbf{Z}_{k,j,i})$ at each grid point $\mathbf{Z}_{k,j,i} \in \mathbf{Z}$.

Step 3. At iteration n, approximate the functions $\mathcal{K}^{(n)}$, $\mathcal{C}^{(n)}$, $\mathcal{X}^{(n)}$, $L^{(n)}$, $M^{(n)}$, and $D^{(n)}$ using three-dimensional Chebyshev polynomials whose weights are computed using Chebyshev-regression. Approximate the derivatives $L_K^{(n)}$, $M_K^{(n)}$, $\mathcal{K}_K^{(n)}$, $\mathcal{X}_K^{(n)}$, and $\mathcal{C}_K^{(n)}$ by differentiating the corresponding polynomial.

Step 4. At each grid point, $\mathbf{Z}_{k,j,i} \in \mathbf{Z}$:

[3] A similar approach is discussed in Maliar and Maliar (2006), Anderson, Kim, and Yun, 2010), and Maliar and Maliar (2005).

Step 4.1. Compute the conditional expectations: $\mathcal{L}_{t+1}^{(n)}$, $\mathcal{M}_{t+1}^{(n)}$, $\mathcal{D}_{t+1}^{(n)}$, $\mathcal{L}_{K,t+1}^{(n)}$, and $\mathcal{M}_{K,t+1}^{(n)}$ using Gauss-Hermite quadrature.

Step 4.2. Solve equations (85)—(91) using a nonlinear solver and use the solution to update $K_{t+1}^{(n+1)}(\mathbf{Z}_{k,j,i})$, $C_t^{(n+1)}(\mathbf{Z}_{k,j,i})$, $x_t^{(n+1)}(\mathbf{Z}_{k,j,i})$, $\pi_t^{(n+1)}(\mathbf{Z}_{k,j,i})$, $\phi_{1t}^{(n+1)}(\mathbf{Z}_{k,j,i})$, $\phi_{2t}^{(n+1)}(\mathbf{Z}_{k,j,i})$, and $\phi_{3t}^{(n+1)}(\mathbf{Z}_{k,j,i})$.

Step 4.3. Update $H_t^{(n+1)}(\mathbf{Z}_{k,j,i})$, $Y_t^{(n+1)}(\mathbf{Z}_{k,j,i})$, $L_t^{(n+1)}(\mathbf{Z}_{k,j,i})$, $M_t^{(n+1)}(\mathbf{Z}_{k,j,i})$, and $D_t^{(n+1)}(\mathbf{Z}_{k,j,i})$ using equations (92)—(96).

Step 5. Compute the distance

$$\begin{aligned}\Upsilon = & \left\|K_{t+1}^{(n+1)} - K_{t+1}^{(n)}\right\|_\infty + \left\|C_t^{(n+1)} - C_t^{(n)}\right\|_\infty + \left\|x_t^{(n+1)} - x_t^{(n)}\right\|_\infty + \left\|\pi_t^{(n+1)} - \pi_t^{(n)}\right\|_\infty \\ & + \left\|\phi_{1t}^{(n+1)} - \phi_{1t}^{(n)}\right\|_\infty + \left\|\phi_{2t}^{(n+1)} - \phi_{2t}^{(n)}\right\|_\infty + \left\|\phi_{3t}^{(n+1)} - \phi_{3t}^{(n)}\right\|_\infty \\ & + \left\|L_t^{(n+1)} - L_t^{(n)}\right\|_\infty + \left\|M_t^{(n+1)} - M_t^{(n)}\right\|_\infty + \left\|D_t^{(n+1)} - D_t^{(n)}\right\|_\infty .\end{aligned}$$

If Υ is greater than the given tolerance (we use $1e-6$), then increment the iteration counter, n, and return to Step 3. Otherwise, stop.

We used the following parameters in this algorithm. For the state space, we set the domain $\zeta \in [-3\sigma_\zeta, 3\sigma_\zeta]$, $a \in [-3\sigma_z, 3\sigma_z]$, and $K \in [5, 35]$. We used a grid with 15 nodes for capital and 7 nodes each for technology and the elasticity of substitution. Each function was approximated with a Chebyshev polynomial of order 4 for ζ, 4 for a, and 14 for capital. Conditional expectations were computed using Gauss-Hermite quadrature with 5 points for each shock.

The same algorithm was used to compute Taylor rule policy. We set capital's domain to $K \in [15, 30]$, and output's domain to $Y \in [1.5, 3.2]$. We used a grid on output with 9 nodes and a Chebyshev polynomial of order 4 to approximate functions. All other parameters were identical to those in the model of discretionary policy.

Table D1 reports the Euler-equation residuals for certain combinations of β and γ. To compute them we split the domain for capital into 200 uniform points and those for technology and the elasticity of substitution into 50 uniform points, and computed the residuals of the consumption Euler equation at each point on this grid. We found the key determinant for accuracy to be the order of the Chebyshev polynomial for capital. When this order was below

14 there was a noticeable decline in accuracy.

Table D1: Numerical accuracy: Consumprion-Euler residuals

		Discretionary policy			Taylor-type rule	
Discount factor HH	β	1.00	0.90	0.90	1.00	0.90
Discount factor CB	γ	1.00	0.90	1.00	–	–
Maximum		1.5e-06	1.3e-06	1.1e-06	4.2e-07	5.4e-07
Mean		4.6e-07	4.0e-07	3.9e-07	1.4e-07	1.9e-07
Median		4.3e-07	3.5e-07	3.5e-07	1.2e-07	1.6e-07

To compute the stochastic steady state we used 10^6 random draws. We followed Potter (2000) to compute the nonlinear impulse responses.