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This paper provides a framework to endogenize rates of return for risk-free bonds and risky capital in an overlapping generation model. The rate of return on capital is endogenized by introducing idiosyncratic production shocks to avoid computation challenges associated with aggregate production shocks in the literature. The framework enables the interaction between financial markets and macroeconomic conditions in a production economy. Based on this framework, the paper first examines life-cycle portfolio choice without demographic change, and illustrates that several factors such as borrowing costs, labor income and production risk play important roles in life-cycle portfolios. The paper then investigates the impacts of population aging on macroeconomic conditions, life-cycle behaviors and financial market structures. The results show that population aging leads to higher capital-labor ratios, and reduces the rates of return on both assets. The bond market shrinks significantly, and capital decreases if the fertility rate declines but increases if the mortality rate declines, leading to structural change in financial markets. The impacts on life-cycle variables are quite different in the fertility and mortality cases particularly at the late stage of life.

Keywords

Demographic change, portfolio choice, financial market structure, risk premium, idiosyncratic production shock, overlapping generation model.

JEL Classification

J11, G11, C63, C68, E21, E23

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Demographic Impacts on Life Cycle Portfolios and Financial Market Structures*

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Abstract

This paper provides a framework to endogenize rates of return for risk-free bonds and risky capital in an overlapping generation model. The rate of return on capital is endogenized by introducing idiosyncratic production shocks to avoid computation challenges associated with aggregate production shocks in the literature. The framework enables the interaction between financial markets and macroeconomic conditions in a production economy. Based on this framework, the paper first examines life-cycle portfolio choice without demographic change, and illustrates that several factors such as borrowing costs, labor income and production risk play important roles in life-cycle portfolios. The paper then investigates the impacts of population aging on macroeconomic conditions, life-cycle behaviors and financial market structures. The results show that population aging leads to higher capital-labor ratios, and reduces the rates of return on both assets. The bond market shrinks significantly, and capital decreases if the fertility rate declines but increases if the mortality rate declines, leading to structural change in financial markets. The impacts on life-cycle variables are quite different in the fertility and mortality cases particularly at the late stage of life.

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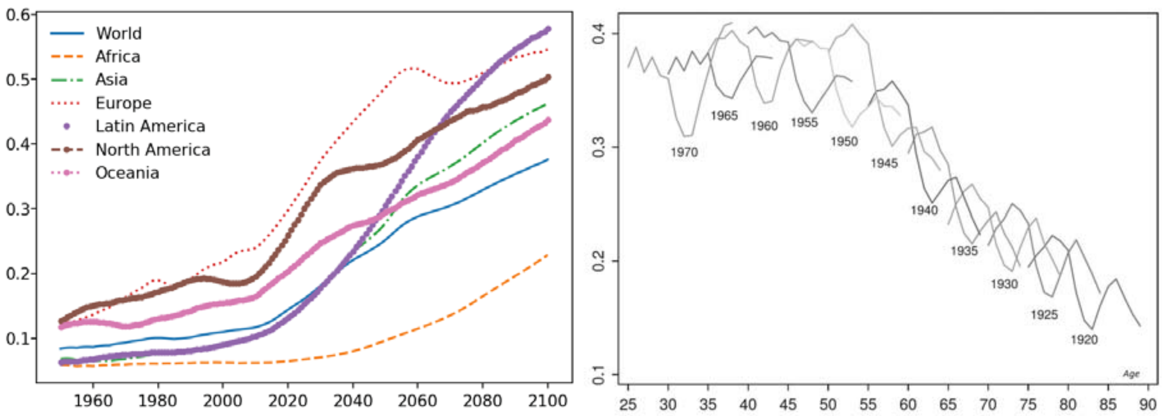
1 Introduction

The world has been experiencing dramatic change in demographic structure since the 1950s due to declining fertility rates and increasing life expectancy. This change has been driving the world towards aging, with the elderly dependency ratio increasing fast all around the world in the next several decades (Figure 1a). The demographic transition has important impacts on, among others, financial markets such as asset prices, asset returns, portfolio allocations and international capital flows (Kent et al. 2006). Most studies along this line investigate the demographic impacts on asset prices and returns because financial investment is the main avenue of financing retirement consumption. There is much less research on demographic effects on financial market structures, as driven by age-dependent household saving and asset allocation. But this is also important as many theoretical models in finance suggest that portfolio choice exhibits strong life-cycle patterns. This life-cycle pattern implies that the accelerating population aging is expected to have important impacts on financial market structures and rates of return on different assets.

Figure 1: Population and Portfolio Structure

(a) Elderly Dependency Ratio (b) Stock Share in Financial Wealth



Sources: (a) United Nations World Population Prospects 2019; (b) Fagereng et al. (2017) Figure 3.

Empirical studies on household portfolios mainly focus on two issues: (1) participation rates in stock markets across ages, and (2) shares of stocks and bonds in financial wealth across ages. Most of the studies, which are based on household survey data, find that participation in stock markets tends to follow a life-cycle pattern but the shares of stocks and bonds in financial wealth tend to vary little with age (see, e.g., Poterba & Samwick 1997, Poterba 2001, 2004, Guiso et al. 2002). However, Fagereng et al. (2017) argue that it is problematic to use cross-sectional household survey data to infer age patterns of portfolio choice.[1] Instead, they draw on a large sample of

[1]Fagereng et al. (2017) provide three reasons: (1) inferences about age patterns of portfolios based on cross-sectional data must be drawn from comparisons of portfolio holdings of individuals at different ages, rather than of the same individual as the age varies; (2) most studies ignore the fact that the stock share is only defined for stock market participants and that participation in stock markets is an endogenous choice; (3) the evidence is based on household surveys, which are notoriously prone to measurement problems, and measurement problems are likely to be correlated with age.

Norwegian household tax records, and find that the share of stocks in financial wealth also shows strong life-cycle patterns (Figure 1b).

Theoretically, portfolio choice has been widely studied as a central topic in finance (see Curcuru et al. 2010 for a review). One strand of the literature examines the life-cycle effects on portfolio choice. In a seminal paper, Samuelson (1969) concludes that optimal portfolio shares do not change with age in a model with the absence of labor income, frictionless markets, and independent and identically distributed returns on financial assets. However, Merton (1971) shows that, with deterministic labor income, individuals capitalize lifetime labor income at the risk-free interest rate and treat the capitalized human wealth as an addition to the financial wealth, which increases the share of risky assets in portfolios compared to the case without labor income. As human wealth decreases over time in a finite horizon, the share of risky assets tends to decline over time. Their models have been extended in many ways such as incorporating market incompleteness, asset market transaction costs, alternative preference assumptions, housing investment, pension systems, etc. Most models reach a broad consensus that portfolio choice has a strong life-cycle pattern that households rebalance their portfolios away from risky assets to risk-free assets when they are aging given the level and risk of labor income change over the life cycle. This theoretical consensus is consistent with the empirical findings based on the Norwegian data.

Methodologically, all those studies on portfolio choice in finance assume exogenous rates of return on different types of assets. On the other hand, many studies in macroeconomics investigate the impacts of demographic change in general equilibrium life-cycle models assuming an aggregate asset (capital) market with an endogenous rate of return. The disconnection between the two streams of studies misses the interaction between financial markets and macroeconomic conditions particularly in the context of demographic change. To our best knowledge, only a few studies incorporate different types of assets with endogenous rates of return in general equilibrium life-cycle models (Brooks 2000, Hasanhodzic & Kotlikoff 2013, Reiter 2015, and Bonnar et al. 2016). These studies endogenize the risky rate of return on capital by introducing aggregate production shocks, but it is well known that such models and, more generally, all heterogeneous agent models with aggregate shocks, pose computational challenges.

This paper examines demographic impacts on life-cycle portfolio choice and financial market structure in an overlapping generation (OLG) model with endogenous rates of return on risky and risk-free assets. Our contribution is four-fold. First, the paper endogenizes the rate of return on risky assets in a production economy by introducing idiosyncratic shocks in the spirit of Angeletos & Calvet (2006) instead of aggregate shocks to avoid computation complexity. Undiversifiable investment risks are pervasive in economic activities. In the US, privately-held firms account for about half of production, employment and corporate equity, and represent more than half the financial wealth of rich households (Carroll 2000). In developing economies, privately-held firms usually overwhelmingly dominate publicly-held firms, so the lack of risk diversification is more pervasive. Even for publicly-held firms, although production risks are diversified across shareholders, there is no doubt that shareholders often earn heterogeneous rates of return in stock markets due to different portfolios.

Second, the paper provides a new framework to illustrate life-cycle portfolio patterns in a general equilibrium model with endogenous rates of return, which extends the literature on portfolio choice in finance. The paper shows that the portfolio pattern in a production economy is consistent with the finance literature, but the model also allows us to examine how the production side of an economy affects portfolio choice.

Third, the model enables the interaction between financial markets and macroeconomic conditions. It allows demographic change to affect portfolio choice through changing macroeconomic variables including rates of return on different assets, and also captures aggregate effects of individual portfolio choice on macroeconomic variables. By disaggregating the financial market, this paper extends the macroeconomic literature that examines demographic impacts on an aggregate financial market. The paper shows that population aging reduces the rates of return on assets, which is consistent with the macroeconomic literature. Several recent studies find that demographic change in advanced economies can explain significant fractions of the decline in the real interest rate in the last several decades (Carvalho et al. 2016, Gagnon et al. 2016, Fujita & Fujiwara 2016, Lisack et al. 2017, Sudo & Takizuka 2018). The paper also demonstrates that population aging can affect the structure of financial markets. The bond market shrinks, and the capital market either decrease or increase depending on whether population aging is driven by a fertility decline or a mortality decline (a longer life span).

Fourth, the model also serves as a new framework to endogenize risk premia in a production economy. A long literature builds life-cycle models to endogenize rates of return on assets in an exchange economy in order to explain the risk premium puzzle (Mehra & Prescott 1985). For example, Constantinides et al. (2002) endogenize the rates of return on risk-free and risky assets in an exchange economy and show that the introduction of borrowing constraints on young generations can increase the risk premium. Mehra (2008) provides a comprehensive review on the studies of risk premia. Our model introduces borrowing costs in a quadratic polynomial form, and generates a risk premium of 2.27 percent in the benchmark scenario which is higher than many studies.

The remainder of this paper is organized as follows. Section 2 introduces the model, and solves households' two-step decisions (portfolio choice and consumption-saving), as well as firm's decisions under idiosyncratic production shocks. Section 3 characterizes the equilibrium conditions, followed by the model calibration in Section 4. Section 5 introduces a computation algorithm for the model, and Section 6 presents the results of various baseline and aging simulations. Section 7 concludes.

2 The Model

2.1 Demographics

Time is discrete and goes forever, i.e., $t = 0, 1, 2, ...$ Each period there are J generations in the economy, indexed by $j = 1, ..., J$. The measure of each generation aged j in period t is denoted by N_t^j, and each generation consists of infinite households indexed by $i \in [0, N_t^j]$. The total measure of all generations is given as

$$N_t = \sum_{j=1}^{J} N_t^j$$

Each period the new generation grows at a rate g_t (referred to as the fertility rate), i.e.,

$$N_t^{j=1} = (1 + g_t)N_{t-1}^{j=1}$$

Each existing generation moves forward by one period and lives up to J periods. A generation survives from age j to age $j+1$ with a probability of ξ^{j+1} conditional on their living at age j. All households die at the end of age J if they survive until age J, i.e., $\xi^{J+1} = 0$. Thus,

$$N_{t+1}^{j+1} = \xi_{t+1}^{j+1} N_t^j, j \leq J - 1$$

Total population N_t grows at a rate n_t, i.e.,

$$N_t = (1 + n_t)N_{t-1}$$

where the growth rate is related to the fertility rate as

$$n_t = \frac{N_t}{N_{t-1}} - 1 = \frac{\sum_{j=1}^{J} N_t^j}{\sum_{j=1}^{J} N_{t-1}^j} - 1$$

It follows that $n_t = g_t$ in a steady state with a constant fertility rate. Each household works in periods $j = 1, ..., J_R - 1$, and retires and receives pensions in periods $j = J_R, ..., J$.

To investigate the aging effects, we compare two scenarios: a baseline with a constant demographic structure, and an aging scenario with an aging demographic structure. We consider a constant fertility and growth rate in each scenario, so the relative population share of each cohort, m_t^j, can be written as

$$m_t^j = (1 + n_t)^{1-j} \tag{1}$$

To separate the effect of demographic structure change from the effect of total population change, we normalize m_t^j in each scenario such that

$$\sum_{j=1}^{J} m_t^j = J \tag{2}$$

Therefore, m_t^j represents the weight on each cohort given the same total population size. To consider the pure effect of a change in total population, we can scale per-capita variables by the population size.

2.2 Assets

There are two types of assets in this economy: risk-free bonds and risky capital. Bonds serve as a financial instrument like a banking system through which households can borrow and lend from each other. If households supply bonds, they borrow from others; if households hold bonds, they lend to others. The bond market is competitive, so the risk of bonds is completely diversified. However, we assume there are borrowing costs in the bond market, which is often the case in the finance literature. Households accumulate capital through direct investment in their own firms, and there is no capital market, so households are faced with heterogeneous risky rates of return on capital, which are driven by idiosyncratic production shocks. This approach models heterogeneous rates of return either on investment in privately held firms where households directly hold capital, or on investment in stock markets where households indirectly hold capital.

2.3 Timing

The timing of events in this economy is as follows:

(1) At the beginning of each period, a new generation is born with labor endowment but no financial assets, and existing households hold (positive or negative) bonds and capital stock through their decisions from last period.

(2) Production shocks are realized.

(3) Firms hire labor from the labor market, and wage is determined.

(4) Production occurs.

(5) At the end of each period, workers receive pre-determined risk-free bond income, state-dependent capital income, and after-tax labor income, while retirees receive bonds income, capital income and pensions.

(6) All households make decisions on how to allocate their resources between consumption, bonds and capital.

(7) Death shocks are realized. i.e., an household either survives over this period or dies with debts or bequests (bequests hereafter). All accidental bequests of households who die this period are equally distributed among all survivors.

2.4 Households

Households supply labor inelastically in a perfect labor market until they retire, and run their own firms with self-accumulated capital, and borrow or lend in the bond market. They make portfolio choice and consumption-savings decisions to maximize their life-time utility.

2.4.1 Preferences

The period utility of a household in period t is represented by the following function with constant relative risk aversion (CRRA)

$$u\left(c_t^{j,i}\right) = \frac{(c_t^{j,i})^{1-1/\gamma}}{1-1/\gamma} \tag{3}$$

where $c_t^{j,i}$ is consumption of a household aged j indexed i in period t, and γ represents the intertemporal elasticity of substitution. Households have standard time-separable preferences as

$$U_t^j = u(c_t^j) + \beta\xi^{j+1}E_t(U_{t+1}^{j+1}) \tag{4}$$

where $0 < \beta < 1$ is the subjective discount factor, and ξ^{j+1} is the survival probability of each cohort into the next period.

2.4.2 Budget Constraints

In period t, a working household aged j indexed i supplies labor inelastically and receives labor income given the wage rate w_t and pays labor taxes at the rate of τ_t. The labor productivity depends on age j and follows a deterministic profile e^j. A retired household aged $j (j \geq J_R)$ receives pension income which is a constant fraction κ of the labor income in the last working period.

Each period households can invest in two assets: risk-free bonds and risky capital. Total financial wealth is denoted by $a_t^{j,i}$, of which a fraction $\omega_t^{j,i}$ is capital $k_t^{j,i}$, and the rest is bonds $b_t^{j,i}$, i.e.,

$$a_t^{j,i} = b_t^{j,i} + k_t^{j,i}, \quad \omega_t^{j,i} = \frac{k_t^{j,i}}{a_t^{j,i}}$$

Assume that households are faced with borrowing costs when they borrow from the bond market, and the borrowing costs follow a quadratic polynomial function of the bond share $(\omega - 1)$ and total assets a, i.e.,

$$\delta_B(\omega, a) = \begin{cases} 0, & \omega \leq 1 \\ \left(\frac{\eta_1}{2}(\omega - 1)^2 + \eta_2(\omega - 1)\right) a, & \omega > 1 \end{cases}$$

where $\eta_1, \eta_2 > 0$ are constant coefficients. This follows Hasanhodzic & Kotlikoff (2013) and Chen & Mangasarian (1996) who assume borrowing costs to be increasing in the bond share and to be scalable with respect to total assets.

Denote capital investment by $\chi_t^{j,i}$, and assume capital does not depreciate. The capital stock evolves as

$$k_{t+1}^{j+1,i} = k_t^{j,i} + \chi_t^{j,i}$$

Assume that total assets cannot be negative, $a_t^{j,i} \geq 0$, which implies that household borrowings must have capital assets of equivalent value as collateral. We also assume $\omega_t^{j,i} \geq 0$ to restrict households from short-selling capital.

The rates of return on the two assets are $r_t^{j,i}$ and r_t^b respectively, where $r_t^{j,i}$ depends on idiosyncratic production shocks $A_t^{j,i}$. The weighted average rate of return on total financial wealth is

$$R_t^{j,i} = 1 + r_t^b + \omega_t^{j,i}\left(r_t^{j,i} - r_t^b\right)$$

As households are subject to the probability of death, they may die with accidental bequests. We assume that the bequests of all agents who die at the end of each period are aggregated and then distributed equally among all survivors, i.e.,

$$q_t \sum_{j=1}^{J} m_t^j = \sum_{j=1}^{J} \left[(1 - \xi^{j+1}) a_{t+1}^{j+1} m_t^j\right] \tag{5}$$

where q_t denotes average bequest each worker receives.

Therefore, the budget constraint of a household aged j indexed i in period t is

$$c_t^{j,i} + a_{t+1}^{j+1,i} = R_t^{j,i} a_t^{j,i} + (1 - \tau_t) w_t^{j,i} e^j + p_t^j + q_t - \delta_B(\omega_t^{j,i}, a_t^{j,i}) \tag{6}$$

where

$$p_t^j = \begin{cases} 0, & j < j_R \\ \kappa w_t \cdot e^{j=j_R-1}, & j \geq j_R \end{cases}$$

2.5 Household Problem

Denote the set of the state variables for household i aged j at time t by $z_t^{j,i} = \{j, i, a_t^{j,i}, \omega_t^{j,i}, A_t^{j,i}\}$. For simplicity, we will drop household index i in the rest of the paper unless necessary. The household problem is

$$\begin{aligned} V(z_t^j) &= \max_{\{c_t^j, a_{t+1}^{j+1}, \omega_{t+1}^{j+1}\}} u(c_t^j) + \beta\xi_{t+1}\,\mathbb{E}\left[V(z_{t+1}^{j+1})\right] \\ s.t.\quad & c_t^j + a_{t+1}^{j+1} = R_t^j a_t^j + (1-\tau_t)w_t^j e^j + p_t^j + q_{t+1} - \delta_B(\omega_t^j, a_t^j) \\ & R_t^j = 1 + r_t^b + \omega_t^j\left(r_t^k - r_t^b\right) \\ & q_t \sum_{j=1}^{J_R-1} m_t^j = \sum_{j=1}^{J}\left[(1-\xi_t^j)a_t^j m_t^j\right] \\ & (\omega_{t+1}^{j+1} - 1)a_{t+1}^{j+1} \leq L \end{aligned} \tag{7}$$

where household borrowing is constrained by a natural limit which equals the expected present value of entire future labor income as

$$L = \mathbb{E}\left[\sum_{k=j+1}^{J} \frac{(1-\tau_{t+k-j})w_{t+k-j}^k e^k + p_{t+k-j}^k}{\prod_{i=1}^{k-j}(1+r_{t+i}^b)}\right]$$

This problem is similar to the classic portfolio problem studied by Samuelson (1969) and Merton (1969).

To simplify the notation, we sum up all available resources of a household in each period t into a cash-on-hand variable as

$$X_t^j(z_t^j) = R_t^j(a_t^j + q_t) + (1-\tau_t)w_t^j e^j + p_t^j - \delta_B(\omega_t^j, a_t^j) \tag{8}$$

We divide this problem into two sub-problems: a portfolio choice problem and a consumption-saving problem. We first solve the optimal investment structure given any level of total assets, and then solve the optimal consumption-saving decision.

2.5.1 Portfolio Choice

Given any level of next-period asset a_{t+1}, the choice of next-period investment structure only affect the next-period cash-on-hand, X_{t+1}^{j+1}, and expected value function $\mathbb{E}\left[V(j+1, X_{t+1}^{j+1})\right]$. Hence, for every next-period asset a_{t+1}, we can determine optimal investment structure $\omega_{t+1}^{j+1}(j, a_{t+1}^{j+1})$ by solving

$$\begin{aligned} Q(j, a_{t+1}^{j+1}) &= \max_{\{\omega_{t+1}^{j+1}\geq 0\}} \mathbb{E}\left[V(j+1, X_{t+1}^{j+1})\right] \\ s.t.\quad X_{t+1}^{j+1} &= R_{t+1}^{j+1}a_{t+1}^{j+1} + (1-\tau_{t+1})w_{t+1}^{j+1}e^{j+1} + p_{t+1}^{j+1} + q_{t+2} - \delta_B(\omega_{t+1}^{j+1}, a_{t+1}^{j+1}) \end{aligned} \tag{9}$$

The first-order condition with respect to ω_{t+1}^{j+1} yields

$$\frac{\partial Q(j, a_{t+1}^{j+1})}{\partial \omega_{t+1}^{j+1}} = \mathbb{E}\left[\frac{\partial V(j+1, X_{t+1}^{j+1})}{\partial X_{t+1}^{j+1}}\frac{\partial X_{t+1}^{j+1}}{\partial \omega_{t+1}^{j+1}}\right] = 0$$

The envelope theorem implies

$$\frac{\partial V(j+1, X_{t+1}^{j+1})}{\partial X_{t+1}^{j+1}} = \frac{du(c_{t+1}^{j+1})}{dc_{t+1}^{j+1}} = (c_{t+1}^{j+1})^{-1/\gamma}$$

From the definition of the cash-on-hand, we have

$$\frac{\partial X_{t+1}^{j+1}}{\partial \omega_{t+1}^{j+1}} = \left[r_{t+1}^{j+1} - r_{t+1}^{b} + \mathbb{1}\{\omega_{t+1}^{j+1} > 1\} * \left(\eta_1(\omega_{t+1}^{j+1} - 1) + \eta_2\right)\right] a_{t+1}^{j+1}$$

The first-order condition reduces to

$$\mathbb{E}\left\{\left[r_{t+1}^{j+1} - r_{t+1}^{b} + \mathbb{1}\{\omega_{t+1}^{j+1} > 1\} * \left(\eta_1(\omega_{t+1}^{j+1} - 1) + \eta_2\right)\right] a_{t+1}^{j+1} \left(c_{t+1}^{j+1}\right)^{-1/\gamma}\right\} = 0 \tag{10}$$

Given a certain level of a_{t+1}^{j+1}, as all households share the same production shock, the expectation removes the household heterogeneity, so the portfolio choice is independent of production shocks.

2.5.2 Consumption-Saving Decision

Households take into account the optimal investment structure $\omega_{t+1}^{j+1}(j, a_{t+1})$ when they make decisions on consumption c_t^j and savings a_{t+1}^{j+1} by maximizing

$$\begin{aligned} V(j, X_t^j) = &\max_{\{c_t^j, a_{t+1}^{j+1}\}} u(c_t^j) + \beta \xi^{j+1} Q(j, a_{t+1}^{j+1}) \\ s.t. \quad & X_t^j = c_t^j + a_{t+1}^{j+1} \end{aligned} \tag{11}$$

We can write the Lagrangian as

$$\mathcal{L} = \max_{\{c_t^j, a_{t+1}^{j+1}\}} u(c_t^j) + \beta \xi^{j+1} Q(j, a_{t+1}^{j+1}) + \lambda\left(X_t^j - c_t^j - a_{t+1}^{j+1}\right)$$

The first-order conditions with respect to c_t and a_{t+1} are respectively

$$(c_t^j)^{-1/\gamma} = \lambda$$

$$\beta \xi^{j+1} \frac{\partial Q(j, a_{t+1}^{j+1})}{\partial a_{t+1}^{j+1}} = \lambda$$

where

$$\frac{\partial Q(j, a_{t+1}^{j+1})}{\partial a_{t+1}} = \mathbb{E}\left[R_{t+1}^{j+1}\left(c_{t+1}^{j+1}\right)^{-1/\gamma}\right]$$

Combining the above two conditions yields the optimality condition as

$$(c_t^j)^{-1/\gamma} = \beta \xi^{j+1} \mathbb{E}\left[R_{t+1}^{j+1}\left(c_{t+1}^{j+1}\right)^{-1/\gamma}\right] \tag{12}$$

Together with the budget constraint, we can solve a_{t+1}^{j+1} and c_t^j for all states $\{j, i, X\}$.

2.6 Firms

Each household aged j indexed i runs their own firm also indexed by (j, i). All firms employ labor from a competitive labor market but accumulate their own capital to produce homogeneous goods in a Cobb-Douglas production function with stochastic productivity as

$$y_t^{j,i} = A_t^{j,i}(k_t^{j,i})^{\alpha}(l_t^{j,i})^{1-\alpha} \tag{13}$$

where $y_t^{j,i}$ denotes the output of an individual firm and $A_t^{j,i}$ is the productivity which is log-normally distributed, i.e., $\ln\left(A_t^{j,i}\right) \sim N(\mu_A, \sigma_A^2)$, and is independent and identically distributed across firms and over time.

Given w_t, the labor demand of each firm is derived from the first-order condition as

$$l_t^{j,i} = \left(A_t^{j,i}\frac{1-\alpha}{w_t}\right)^{1/\alpha} k_t^{j,i} \tag{14}$$

The capital income of each household is defined as

$$\pi_t^{j,i} = y_t^{j,i} - w_t l_t^{j,i}$$

The rate of return on capital is then

$$r_t^{j,i} = \frac{\pi_t^{j,i}}{k_t^{j,i}} = \frac{y_t^{j,i} - w_t l_t^{j,i}}{k_t^{j,i}}$$

Combining the above four equations yields

$$r_t^{j,i} = \left(A_t^{j,i}\right)^{1/\alpha} \alpha \left(\frac{1-\alpha}{w_t}\right)^{(1-\alpha)/\alpha} \tag{15}$$

Given all firms share the same production shock, we can conclude that the expected rate of return on capital is equal across firms at the beginning of each period.

2.7 Government

The government is simple in the model. The only role is to operate a pay-as-you-go pension system by collecting labor income taxes and distributing tax revenues to retirees as pensions. The pension is uniform across all retirement periods and is calculated as a fixed proportion, κ, of the last period labor income

$$p_t^j = \kappa w_t e^{j_R - 1}, j = J_R, ..., J$$

To balance the pension system, the government sets the tax rate τ_t to equate total pension benefits and contributions such that

$$\tau_t = \frac{\sum_{j=J_R}^{J} m_t^j p_t^j}{\sum_{j=1}^{J_R-1} m_t^j w_t e^j} \tag{16}$$

3 Equilibrium Conditions

Definition 1. Given the initial state of the economy, a recursive equilibrium is a set of policy functions $\{c(z_t), a(z_t), \omega(z_t)\}$ for the households, a set of input choices $\{l_t(z_t)\}$ for the firms, a set of prices $\{w_t, r_t(z_t), r_t^k\}$ such that

(1) Aggregate and individual behaviors are consistent: individual consumption, effective labor supply, individual labor demand, capital stock and investment sum up to their aggregate counterparts. In addition, all individual borrowing costs sum up to the aggregate borrowing cost.

$$C_t = \sum_{j=1}^{J} \int_{i=0}^{N_t^j} c_t^{j,i} \tag{17}$$

$$L_t^S = \sum_{j=1}^{J_R} \int_{i=0}^{N_t^j} e^j \tag{18}$$

$$L_t^D = \sum_{j=1}^{J_R} \int_{i=0}^{N_t^j} l_t^{j,i} \tag{19}$$

$$K_t = \sum_{j=1}^{J} \int_{i=0}^{N_t^j} k_t^{j,i} \tag{20}$$

$$I_t = \sum_{j=1}^{J} \int_{i=0}^{N_t^j} \chi_t^{j,i} \tag{21}$$

$$\Delta_t = \sum_{j=1}^{J} \int_{i=0}^{N_t^j} \delta_B(\omega_t^{j,i}, a_t^{j,i}) \tag{22}$$

(2) Given prices (w_t, r_t^b, r_t), the policy functions $c(z_t)$, $a(z_t)$ and $\omega(z_t)$ solve the household problem (7).

(3) The government budget balances.

$$\tau_t \sum_{j=1}^{J_R-1} m_t^j w_t e^j = \sum_{j=J_R}^{J} m_t^j p_t^j \tag{23}$$

(4) All markets clear.

- The goods market clears:

$$Y_t = C_t + I_t + \Delta_t \tag{24}$$

- The bond market clears:

$$\sum_{j=1}^{J} \int_{i=0}^{N_t^j} b_t^{j,i} = 0 \tag{25}$$

- The labor market clears:

$$L_t^D = L_t^S \tag{26}$$

Borrowing costs in the bond market is a dead-weight loss in the economy. We assume that aggregate bond supply equals aggregate bond demand, and incorporate aggregate borrowing costs into the goods market clearing condition, assuming that borrowers pay the borrowing costs in terms of goods.[2]

(5) The aggregate capital stock evolves over time as

$$\xi_{t+1} K_{t+1} = K_t + I_t \tag{27}$$

Definition 2. A steady state of the economy is an equilibrium path on which prices, wages, tax rates and all individual variables are constant over time and aggregate variables all grow at the rate that is the sum of the population growth rate and the productivity growth rate.

4 Calibration

Table 1 summarizes the values for economic and demographic parameters in this study. We follow standard calibration practices and parameter values based on the US data and studies.

Households who are born directly into the labor force work for 10 periods (or 50 years) and live for 14 periods until an actual age of 90. They are subject to age-dependent mortality rates which are calculated based on the US population data over the period of 2015-2020 from the United Nations World Population Prospects 2019. The annual discount factor is calibrated to 0.96 so as to match the empirical capital-output ratio of around 3, so β is assigned 0.815 in our model as one period represents five years. The relative risk aversion coefficient takes a standard value of 2 from the macroeconomic literature.

The capital share in production has a standard value of 0.36. The technology level is calibrated to 1.57 such that wage is normalized to around one. The variance of the log-normally distributed idiosyncratic production shock is assigned 0.8 percent which corresponds to an annual standard deviation of 14 percent in a normal term. It is empirically close to the annual standard deviation of about 15 percent for S&P500.

For the borrowing cost function, we choose 0.1 for the linear term coefficient and 0.08 for the quadratic term coefficient to generate slowly but exponentially increasing borrowing costs and also to restrict the aggregate borrowing costs below 1 percent of the output in the economy.

[2] Alternatively, we can incorporate aggregate borrowing costs into the bond market clearing condition where the difference between aggregate bond supply and demand equals aggregate borrowing costs.

Table 1: Parameter Values

Description	Parameter	Value
Demographics		
- Number of age cohorts	J	14
- Retirement age	J_R	10
Household preferences		
- Discount factor	β	0.815
- Risk aversion parameter	$1/\gamma$	2
Production parameters		
- Capital share in production	α	0.36
- Technology level	A	1.57
- Idiosyncratic productivity risk	σ_A^2	0.008
Borrowing cost function		
- Quadratic term	η_1	0.08
- Linear term	η_2	0.1

5 Computation

The general idea of our computation follows the Gauss-Seidel method. The solution algorithm of OLG models such as Auerbach & Kotlikoff (1987) starts with guesses about aggregate variables such as aggregate capital and labor, and then calculate the interest rate and wage based on the marginal products of capital and labor.[3] Given the interest rate and wage, one can solve household problems and aggregate individual decisions to update aggregate capital and labor supplies until the goods market clears. Most of these models assume a representative firm in a competitive environment, so there is a simple aggregate factor demand function and it is straightforward to calculate the interest rate and wage given aggregate capital and labor.

In our model, however, several features increase the computation complexity. First, there are infinite individual firms and we cannot update wage by calculating the marginal product of labor at the aggregate level. Second, there is a bond market, and both bond demand and supply occur at the individual level, indicating there are no simple aggregate demand or supply function and we cannot calculate the rate of return on bonds from either an aggregate demand or supply function. Third, households accumulate their own capital and there is no capital market and hence no common rate of return on capital due to idiosyncratic production shocks.

As we cannot calculate wage and bond returns from either an aggregate demand or supply function, our strategy is to update wage and bond returns separately, and thus our algorithm consists of two-tier iterations. The outer iteration uses the bisection method to solve for a bond return that clears the bond market. The inner iteration also uses the bisection method to solve for a wage rate that clears the labor market, given aggregate variables and the bond return. More specifically, our computation proceeds as follows:

(1) **Initialization.** Calculate demographic composition. Discretize stochastic pro-

[3]Since aggregate capital and labor supplies are used in the production function, this implicitly imposes the clearing conditions for capital and labor markets.

ductivity $A_t^{j,i}$ and other state variables. Make initial guesses of aggregate variables $\{K_t, B_t, L_t^D, L_t^S, I_t, Y_t\}$. Initialize price variables including wage, the bond return, and the tax rate.

(2) **Capital return update.** Calculate capital returns given wage based on (15).

(3) **Portfolio choice.** Given wage, the bond return, the capital returns and the tax rate, we solve for optimal portfolio choice $\omega_{t+1}^{j+1}(j, a_{t+1}^{j+1})$ for each cohort backwards given next-period assets a_{t+1}^{j+1} based on (10).

(4) **Consumption decision.** Given optimal portfolio choice, we solve for optimal consumption and savings $a_{t+1}^{j+1}(j, X_{t+1}^{j+1})$ and $c_t^j(j, X_t^j)$ given cash-on-hand states based on (12).

(5) **Distribution.** Calculate the distribution of population across different states based on policy functions $a_{t+1}^{j+1}(j, X_{t+1}^{j+1})$, $c_t^j(j, X_t^j)$ and $\omega_{t+1}^{j+1}(j, a_{t+1}^{j+1})$ for each cohort.

(6) **Aggregation.** Update aggregate variables based on individual decisions and population distributions.

(7) **Wage update.** Update wage with the bisection method.

(8) **Labor market.** Check if the labor market clears. If $|L^D - L^S| < \sigma$, go to next step. Otherwise, go to Step 10.

(9) **Bond return update.** Update bond returns with the bisection method.

(10) **Government pension rate.** Calculate the budget-balancing tax rate.

(11) **Bequest.** Calculate accidental bequests based on (5).

(12) **Bond market.** Check if the bond market clears. If $|B| < \sigma$, we stop. Otherwise, go to Step 2.

The inner iteration consists of Steps 2-8 which update wage and clear the labor market, and the outer iteration consists of Steps 2-12 which update bond returns and clear the bond market. When the bond and labor markets clear, the goods market also clears based on Walras's law.

6 Results

We first solve for a steady state in a baseline without demographic change. In the baseline, we consider different values for several key parameters and illustrate how the parameters affect portfolio choice as well as other life-cycle and macroeconomic variables. We then solve for a steady state in an aging scenario. We consider three aging cases: a decrease in the fertility rate, a decrease in the mortality rate (or an increase in the survival rate), and a combination of the two cases. We will show that a fertility decline and a mortality decline have different impacts although they both lead to population aging.

6.1 Baseline

To examine the roles of several key parameters in portfolio choice as well as other life-cycle and macroeconomic variables, we consider the borrowing costs in the bond market, the labor income share in the production function, the production risk, and the risk aversion degree in the preferences. Therefore, we consider five cases in the baseline. Our benchmark case (B1) includes standard borrowing costs, a standard labor income share, a standard risk aversion degree, and a standard production risk. In each other case, we change only one parameter to undertake a comparative static study relative to the benchmark case. Other cases include (B2) no borrowing costs; (B3) a higher labor income share where the capital share in production, α, decreases from 0.36 to 0.30; (B4) a higher risk aversion degree where the relative risk aversion coefficient increases from 2 to 2.5; and (B5) a higher production risk with a 50 percent increase in the standard deviation of the production risk.

We first compare the macroeconomic variables in various cases, and then present the life-cycle patterns of income, consumption, portfolio choice and production, and then discuss risk premia.

6.1.1 Macroeconomic Variables

Table 2 presents the values of aggregate variables in five cases. The wage in the benchmark case (B1) is normalized to one. In the benchmark, as there are borrowing costs in the bond market, a small fraction of output (0.7 percent) evaporates as a dead-weight loss in the economy.

Comparing the case of no borrowing costs (B2) to the benchmark, the bond supply and demand significantly increase, but this does not much affect aggregate variables. Without borrowing costs, young workers would borrow more to invest in capital. The rise in bond demand increases the rate of return on bonds, and reduces the risk premium (the impact on the risk premium will be discussed in Section 6.1.3). Therefore, old cohorts reduce their capital holdings and increase bond supply. The overall effect on capital is quite small. The main impact is that the distribution of capital over cohorts becomes much flatter with a much weaker life-cycle pattern.

In the case of a high labor income share (B3), production becomes more labor intensive. Given the same labor supply, the capital stock decreases, resulting in lower output. This change also leads to lower capital per worker, which has a negative effect on the marginal product of labor. However, a higher share of labor income in the production implies a higher share of output is paid to workers, which has a positive effect on the marginal product of labor. In our numerical example, the positive effect dominates the negative one, leading to a higher wage.

If households are more risk averse (B4), they prefer to hold less capital, resulting in lower output. Given the fixed labor supply, capital per worker is lower and hence the marginal product of labor decreases, resulting in lower wage.

In response to a higher production risk (B5), as households consumption over retirement heavily depends on capital income, they would hold less capital and supply more bonds after they retire. The increase of bond supply pushes down the rate of return, so young workers would borrow more and increase their investment on capital. Our example shows that the increase of workers' capital holdings dominates the reduction of retirees' capital, resulting in higher capital stock. This further leads to higher output and wage.

Table 2: Aggregate variables

Description	Variable	B1	B2	B3	B4	B5
Output	Y	28.77	28.84	27.82	27.02	29.00
			(0.2%)	*(-3.3%)*	*(-6.1%)*	*(0.8%)*
Consumption	C	28.63	28.84	27.69	26.88	28.83
			(0.7%)	*(-3.3%)*	*(-6.1%)*	*(0.7%)*
Borrowing Costs	Δ	0.14	0.00	0.13	0.14	0.17
			(-100%)	*(-7.1%)*	*(0)*	*(21.4%)*
Capital	K	18.00	18.12	15.94	15.12	18.12
			(0.7%)	*(-11.4%)*	*(-16.0%)*	*(0.7%)*
Bond Demand (Supply)	B	1.52	4.61	1.51	1.56	1.85
			(203.3%)	*(-0.7%)*	*(2.6%)*	*(21.7%)*
Labor Demand (Supply)	L	18.23	18.23	18.23	18.23	18.23
			(0)	*(0)*	*(0)*	*(0)*
Capital-Labor Ratio	K/L	0.99	0.99	0.87	0.83	0.99
			(0.7%)	*(-11.4%)*	*(-16%)*	*(0.7%)*
Wage	w	1.01	1.01	1.07	0.95	1.02
			(0.2%)	*(5.8%)*	*(-6.1%)*	*(0.8%)*
Pension Tax Rate	τ	0.18	0.18	0.18	0.18	0.18
			(0)	*(0)*	*(0)*	*(0)*

*Values in brackets show percentage deviations of variables from the benchmark case.

6.1.2 Life-Cycle Variables

Figure 2a presents life-cycle labor income (and pension income). Labor income increases until retirement due to an improvement in labor productivity over the life cycle. Pension income is earned thereafter. The differences in life-cycle labor income across cases are completely attributable to the differences in the wage given labor supply is inelastic.

Figure 2b presents life-cycle consumption which shows a hump-shaped pattern. Our model yields a real interest rate higher than the subjective discount factor, which implies that households are sufficiently patient to tilt up their consumption path over time when they are young. However, as households age and the survival rate decreases, they become impatient and prefer immediate to future consumption, resulting in a decrease in consumption at the final stage of life. In the case of a higher labor income share (B3), as the rate of return decreases, households reduce their savings. Because of lower rates of return, future consumption becomes cheaper relative to present consumption, so households consume more when they are young and less when they are old compared to the benchmark case. If households are more risk-averse (B4), consumption shifts downward across all cohorts because households receive less income due to lower wage. When there is a higher production risk (B5), there is not much change in the life-cycle consumption pattern as there are no significant changes in price variables from the benchmark.

Figure 2: Labor Income and Consumption

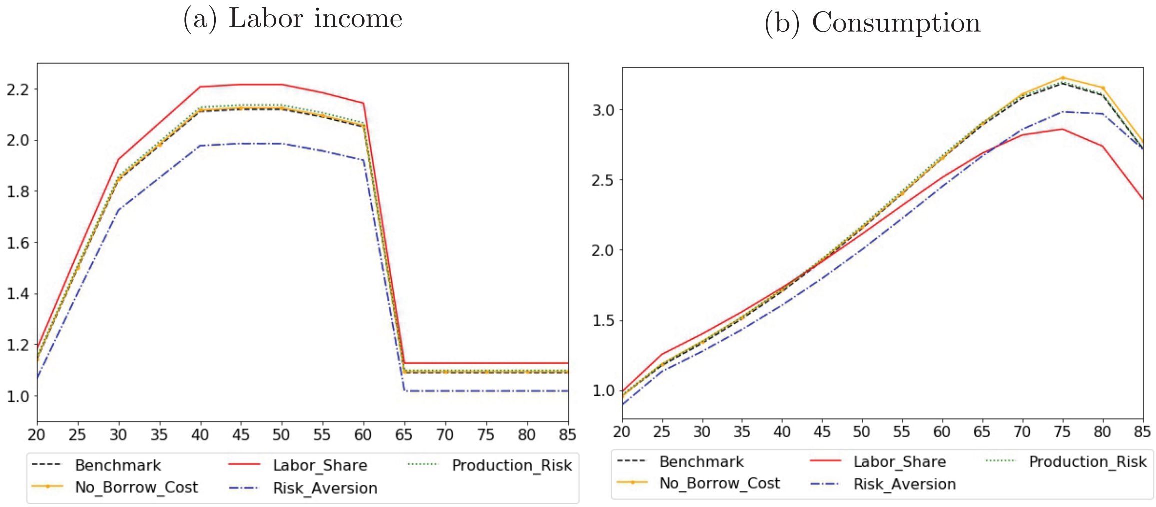
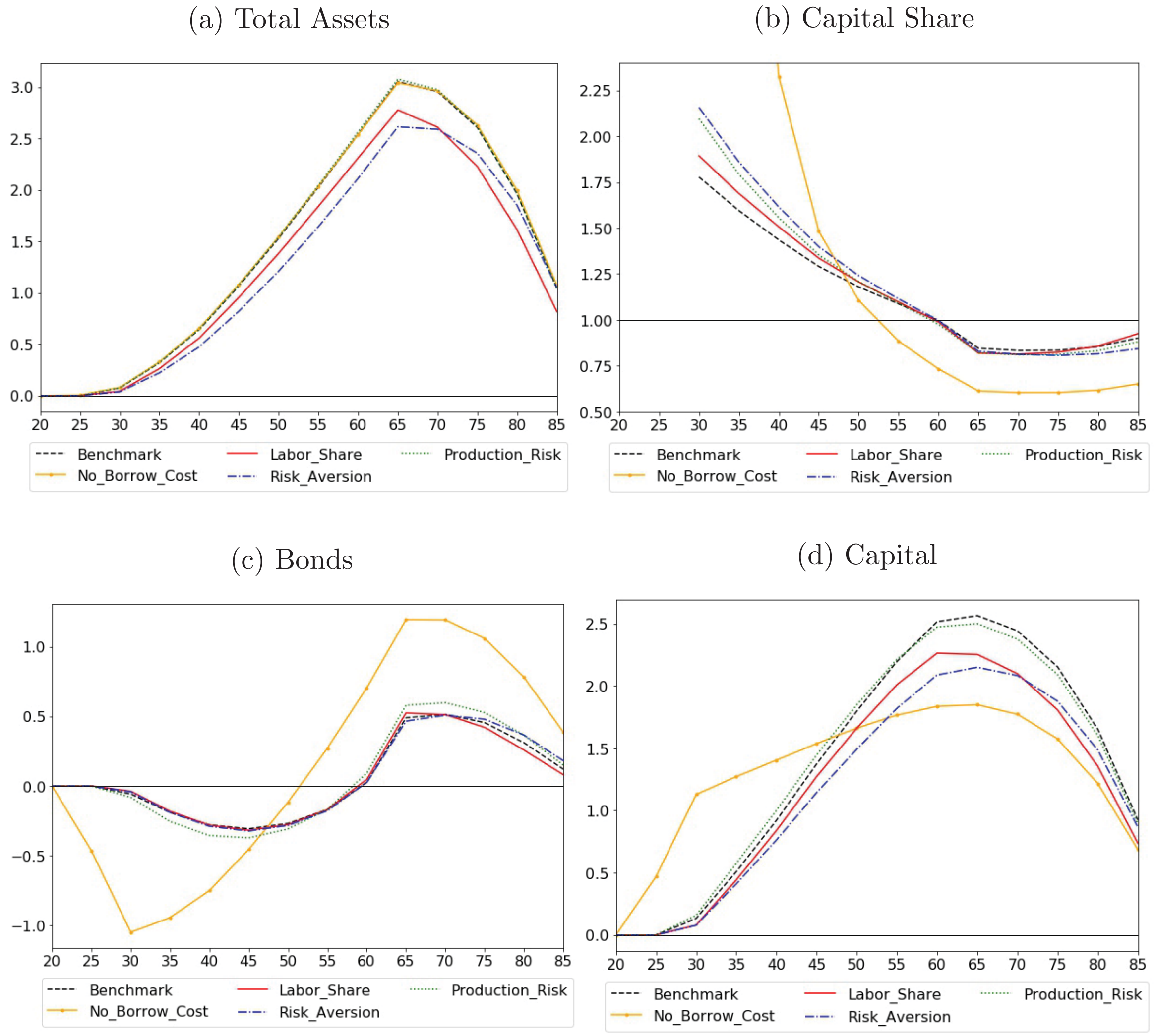


Figure 3a shows a typical hump-shaped asset accumulation pattern that peaks toward retirement due to household life-cycle motives to smooth consumption. The speed of accumulation and de-accumulation of assets is determined by portfolio returns, the discount factor, as well as the survival probability. The assets are completely diminished at the end of life without bequest motives. The differences in the asset profile across cases are driven by the differences in the labor income profile and the consumption profile.

Figure 3b presents the share of capital in total assets. If the share is above one, households borrow in the bond market and supply bonds. If the share is below one, households lend in the bond market and hold bonds. The portfolio choice shows a strong life-cycle pattern that households rebalance their portfolios away from capital to bonds over time. This is consistent with the theoretical finding in the finance literature. Given a constant level of relative risk aversion, households would adjust their portfolios to keep the relative risk constant over time. With more resources derived from risk-free human wealth, young cohorts increase their risk exposure by investing a large portion of portfolios in capital through borrowing from the bond market. As the households age, their human wealth decreases whereas their savings (and capital holdings) increases until retirement before decreasing. This change in composition between human wealth, bonds and capital holdings requires their portfolios to gradually shift from capital to bonds.

Bonds are supplied by young workers who borrows against their future income, and demanded by retirees who hold bonds to insure their financial income against risk (Figure 3c). On the other hand, capital has a hump-shaped resemblance of assets but is skewed less to the left as a result of leverage that increases capital holdings of younger cohorts (Figure 3d). The findings of portfolio choice is in line with Bonnar et al. (2016) and roughly match the empirical pattern of portfolio allocation.

Figure 3: Portfolio Choice



Households differ in portfolio choice and asset holdings in various cases particularly in the early stage of life. In the case of no borrowing costs (B2), young cohorts borrow as high as 60 times of their net asset value from the bond market and invest in capital for excess rates of return. Once we introduce borrowing costs in other cases, the capital share declines substantially and borrowing amounts become more realistic.

In the case of a higher labor income share (B3), households are willing to hold more capital when they are young. The rationale is as follows. The CRRA utility function implies that households would keep the share of capital in total wealth, η, constant over time, where

$$\eta = \frac{\omega * a}{a + h} \tag{28}$$

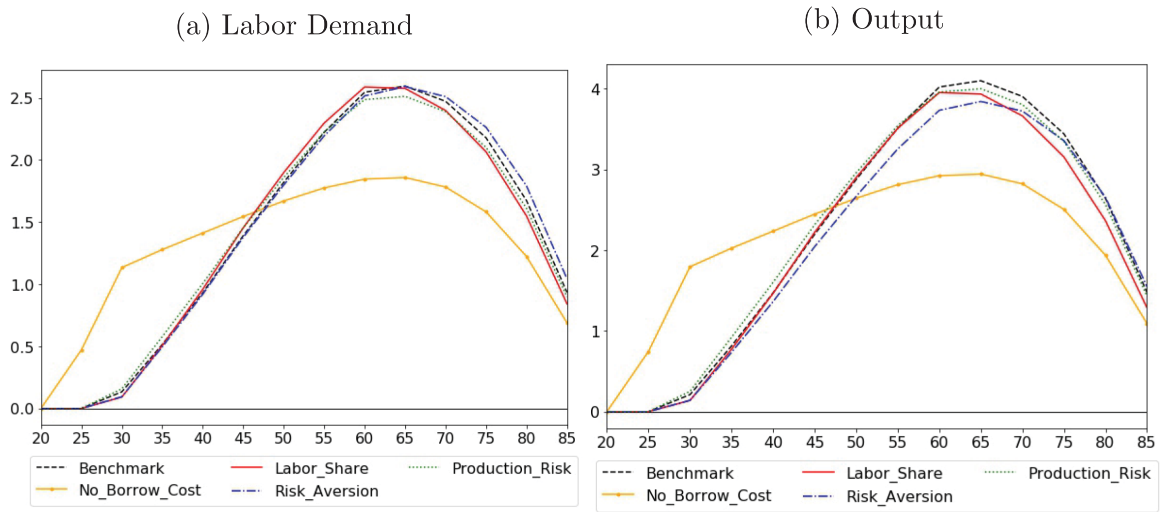
where h denotes human wealth which depends on wage and the interest rate. The capital share of young households increases for three reasons. First, young workers have higher human wealth and thus have smaller relative risk exposure given the same level of production capital, so they would increase the capital share in their portfolio to maintain a constant level of relative risk over time. This result is qualitatively consistent with the finding based on partial-equilibrium portfolio choice models with labor incomes in the finance literature. Second, there is a general-equilibrium effect in our model. When the labor income share increases, this also affects the production of the economy and hence the risk premium between the two assets. Our result shows that this slightly increases the risk premium, which is in favor of holding more capital. Third, due to lower returns, households would save less, so their human wealth accounts for a larger fraction in total wealth, which would reduce their level of relative risk exposure. To maintain a constant level of relative risk, households would increase their capital share. This further strengthens the first impact.

If there is higher risk aversion (B4), households require a lower capital share. In addition, there are two more effects that affect portfolio decisions. First, a lower wage rate and higher interest rate reduce human wealth, which require a smaller capital share to maintain a constant relative risk. Second, lower savings reduce the relative risk and require a higher capital share. In our case, the later effect dominates the former and results in a higher share of capital in young cohorts, even after taking into account the impact of higher risk aversion.

When there is higher production risk (B5), the risk premium must increase, and the portfolio choice would respond to the higher risk premium. In addition, wage and the interest rate are slightly higher, and thus also affect the risk level. The net effect is that households would reduce capital holdings after retirement and increase capital when they are young, resulting in a higher capital share in the early stage of life.

On the production side, capital within each firm is directly invested by each individual, so the hump-shaped profile of capital determines the optimal level of labor demand which together produce output (Figure 4a and 4b).

Figure 4: Firm's Labor Demand and Output



6.1.3 Risk Premium

Our model provides a framework to endogenize the risk premium, which is defined as the difference between the expected rate of return on capital and the risk-free rate on bonds. In the case without borrowing costs (B2), the risk premium has a small value of 0.2 percent, consistent with those generated in standard representative-agent general equilibrium models without market frictions.

The literature on risk premia suggests that several factors including borrowing constraints can contribute to a higher risk premium. In our model, we assume that the borrowing costs take a quadratic polynomial form. The linear term helps generate a large risk premium and the quadratic term imposes a high penalty for over-leveraging so that portfolio shares are empirically realistic. On the one hand, the borrowing costs reduce bond supply of younger cohorts, resulting in excess bond demand that increases the bond price and pushes down the bond return. On the other hand, the costs constrain borrowing amounts and decrease the investment in capital, resulting in a lower capital-labor ratio and a higher rate of return on capital. A larger gap between risk-free and risky rates of return leads to a higher risk premium.

As it is not central in our study to replicate empirical risk premia, we provide a framework that can accommodate various specifications such as hard borrowing constraints (Constantinides et al. 2002), transaction costs (Aiyagari & Gertler 1991), habit formation and capital adjustment costs (Jermann 1998). Our numerical example with quadratic polynomial borrowing costs generates a risk premium of 2.27 percent which, although still lower than the empirical value of about 6 percent, is higher than some studies with endogenous rates of return such as Hasanhodzic & Kotlikoff (2013) and Bonnar et al. (2016).

Table 3: Rates of Return on Assets

Rates of Return (% p.a.)	Variable	B1	B2	B3	B4	B5
Risk-Free Rate	r^b	9.23	11.26	8.17	10.47	9.13
			*(2.03)**	*(-1.06)*	*(1.24)*	*(-0.11)*
Risky Rate	$E(r)$	11.51	11.46	10.47	12.87	11.52
			(-0.05)	*(-1.03)*	*(1.36)*	*(0.01)*
Risk Premium	$E(r) - r^b$	2.27	0.20	2.30	2.40	2.40
			(-2.07)	*(0.03)*	*(0.12)*	*(0.12)*

*Values in brackets are percentage point deviations from the baseline scenario.

In addition to borrowing costs, our results show that other factors also have slight impacts on risk premia (Table 3). In the case of a higher labor share (B3), as production becomes more labor intensive, the capital stock decreases and all cohorts reduce their capital holdings. Capital per worker also decreases, which has a positive effect on the marginal product of capital. However, a higher share of labor income in the production function implies a lower share of output is paid to capital, which has a negative effect on the marginal production of capital. In our numerical example, the overall effect is negative, resulting in a lower rate of return on capital. As young workers reduce capital, they require less borrowings, pushing down bond supply. Old cohorts would substitute from capital to bonds, which pushes up bond demand. The shifts in bond supply and demand push down the rate on bonds. In our results, the rate on bonds decreases slightly more than the rate on capital, leading to a mild increase in the risk premium.

If households are more risk averse (B4), they require a higher risk premium to hold capital as a compensation for the same level of risk. Similarly, if there is a higher production risk (B5), households with same level of risk aversion will require higher capital return, leading to a higher risk premium.

6.2 Aging Scenarios

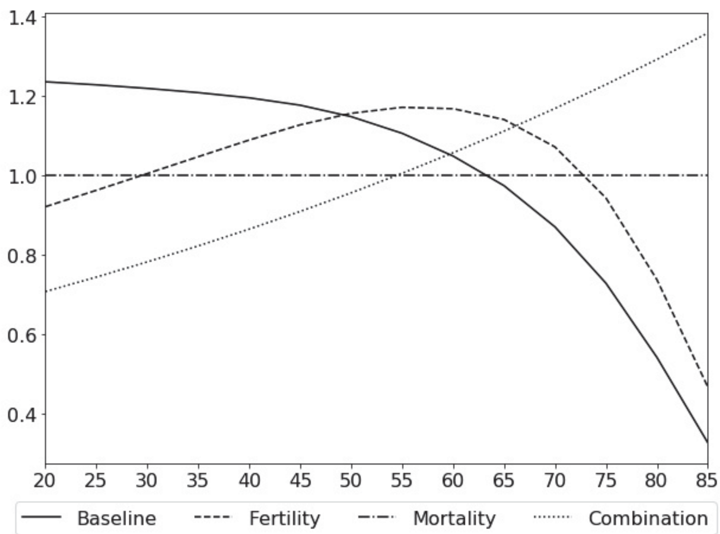
To examine the effect of demographic change on portfolio choice and macroeconomic variables, we examine three aging scenarios: (1) a fertility decline: the fertility rate declines by 1 percent every period (corresponding to every five years); (2) a mortality decline (or a longer life expectancy): the mortality rate declines to zero for all cohorts except the last one; (3) both fertility and mortality declines: a combination of the first two scenarios.

As fertility and/or mortality rates decline, total population and population structure both change. We separate the two effects by implementing the demographic shock in two steps. First, we keep total population constant in different scenarios but allow population structure to change, and this will capture the effect of demographic composition changes. Second, we then allow total population to change, and this will capture the effect of total population change.

6.2.1 Demographic Composition

In the fertility case, the demographic structure changes with less shares of young workers, more shares of old workers (aged 50-65), and also more shares of retirees in the economy (Figure 5). This change results in a hump-shaped demographic structure. If only mortality rates decrease, the shares of workers increase while the shares of retirees decrease, leading to a flatter demographic structure. In our extreme case, there is no death except the last cohort, so the demographic structure is completely flat. These two aging scenarios have both qualitatively and quantitatively different economic implications. When aging occurs from the decreases in both fertility and mortality rates, population structure becomes more concentrated in old cohorts.

Figure 5: Population Structure

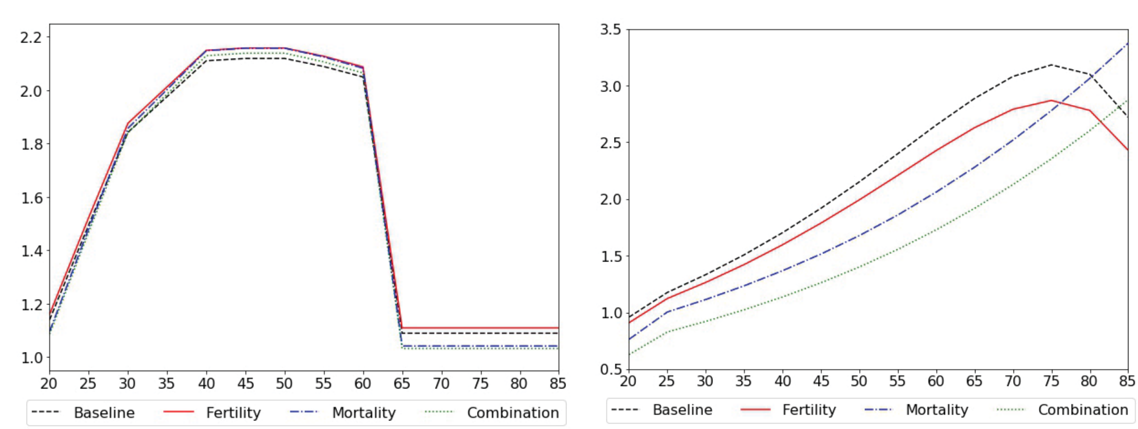


6.2.2 Life-Cycle Variables

In all three aging scenarios, aggregate effective labor supply declines more relative to aggregate capital, so wage increases and labor income also rises. Meanwhile, the marginal product of capital decreases, resulting in lower rates of return on capital.

Figure 6: Labor Income and Consumption

(a) Labor Income (b) Consumption

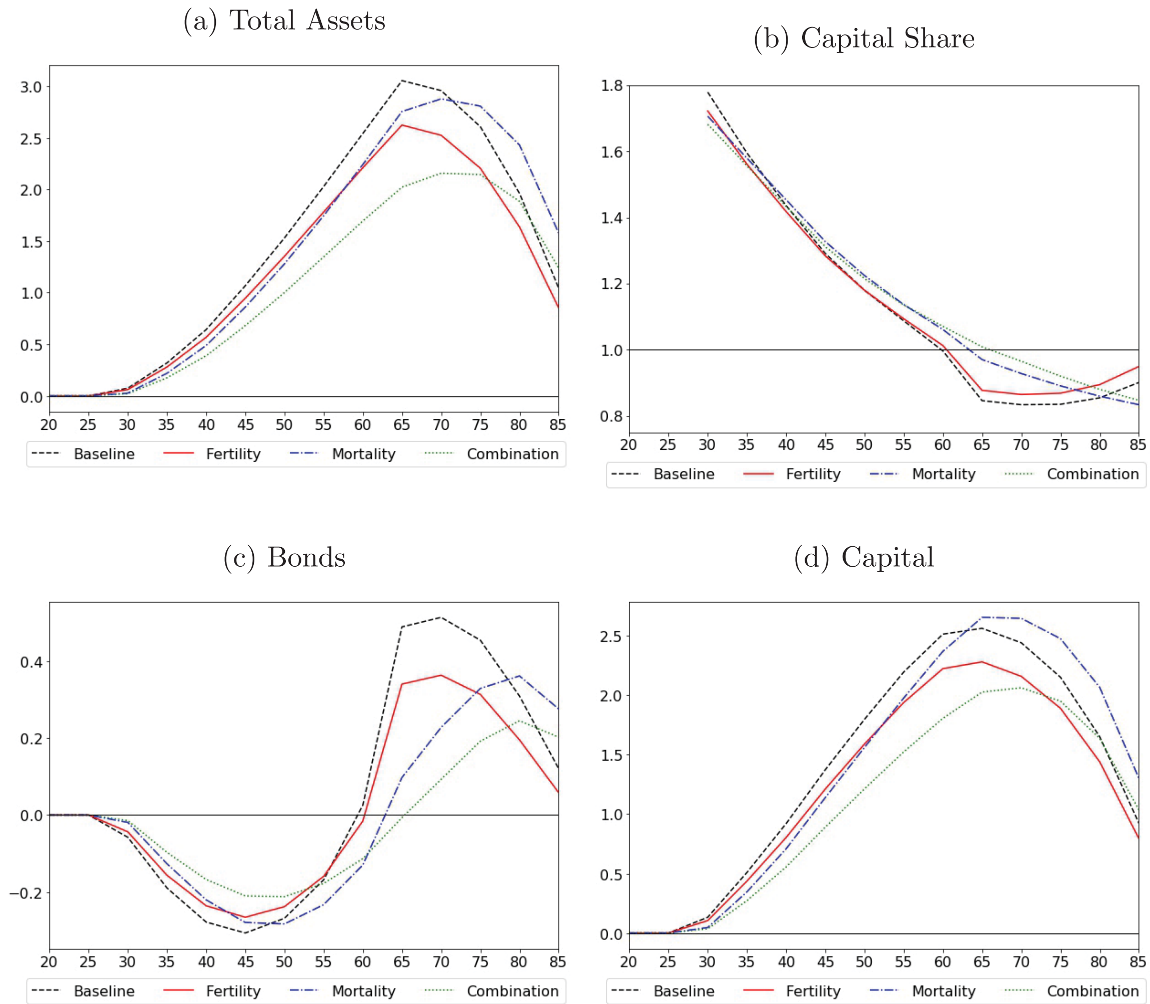


Although workers earn higher gross wage, the after-tax wage is lower because higher pension tax rates are required due to higher dependency ratios. The reduction in the net wage and and also in the interest rates (explained later) result in lower human wealth, which tends to push down consumption over the life cycle. In the fertility case, the consumption path is uniformly below the baseline consumption. In addition, with lower interest rates, current consumption becomes cheaper relative to future consumption, leading to a flatter consumption path over time. However, in the mortality (and the combined) case, consumption tilts up after retirement because the mortality rate declines and old cohorts become more patient to consume.

In terms of assets, in the fertility case, as human wealth declines, households hold less assets over the life cycle (Figure 7a). In the mortality case, the asset profile before retirement is similar to the fertility case, but it is much higher after retirement. This is because households become more patient in consumption and would save more for consumption in the late stage of life due to the mortality decline. The case of fertility and mortality declines captures the combined effect of the two cases.

All aging scenarios share a similar pattern in the portfolio profile to the baseline scenario. However, there is less capital share among young cohorts and less bond holdings among old cohorts, resulting in flatter portfolio profiles. As illustrated before, the change in portfolio choice is attributable to three factors: the risk premium, savings and human wealth. In all aging scenarios, labor supply shrinks and the capital-labor ratio increases, resulting in a lower rate of return on capital. Meanwhile, there is a higher share of old cohorts who demand bonds and a lower share of young cohorts who supply bond, pushing down the risk-free rate. The decrease in the risk-free rate is smaller as any change in the bond market is constrained by borrowing costs, dampening the pass-through of the rate change from the capital market to the bond market. Therefore, the risk premium becomes smaller in an aging economy with borrowing costs. In response to a lower risk premium, young cohorts would borrow less and reduce investment in capital. Meanwhile, old people would substitute away from capital to bonds. This results in lower capital holdings across all cohorts in the fertility case. However, in the mortality case, the portfolio pattern is different, with higher capital shares when households get old. As the mortality rate declines, households would increase consumption towards the end of their life, so they would hold more savings and prefer a higher capital share, which enables higher consumption in the late stage of life. In addition, the after-tax wage decreases, but the interest rate also decreases, so the change in human wealth is small and hence the impact on portfolio is also small. Our results show that the net effect on portfolio is small in the fertility case, but is moderate in the mortality case when households age.

Figure 7: Portfolio Choice

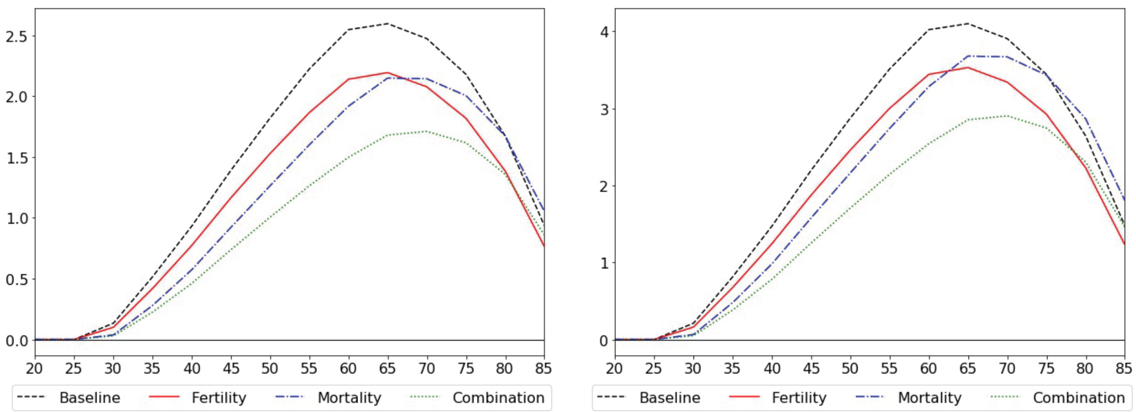


The change in the risk premium in the aging scenarios is attributable to the borrowing costs. If there are no borrowing costs, the risk premium hardly changes. This indicates that demographic change does not much influence the risk premium in the financial market in our model unless we introduce market frictions such as borrowing costs in the bond market. Intuitively, all financial markets must move together in response to an economy-wide shock if there are no financial frictions. In addition to borrowing costs, participation costs is another common market friction in the capital (or stock) market. Fagereng et al. (2017) show that participation costs in the stock market can significantly affect portfolio choice. Other assumptions may also have potential impacts on portfolio choice. For example, if households have age-dependent risk aversion, the life-cycle pattern would be quite different as we have shown that the capital share is sensitive to the risk aversion degree.

On the production side, the capital stock of households at each age determine their labor demand. In the low fertility case, all cohorts hold less capital, and thus require less labor at all ages. In the mortality case, young cohorts hold less capital while old cohorts hold more, resulting in less labor in early stages but more labor in late stages. Outputs of firms owned by each age cohort are also shifted downwards in the fertility case, which is due to declines in both labor and capital. In the mortality case, the output of firms by young cohorts decreases while that of old cohorts increases because they hold more capital at their late stage of life.

Figure 8: Firm's Labor Demand and Output

(a) Labor Demand (b) Output



6.2.3 Aggregate Variables

We first examine the effects of demographic composition changes on per-capita variables and then show the effects on aggregate variables when total population changes. The impacts on per-capita variables come from the impacts on life-cycle behaviors and the changes in demographic compositions. Table 4 presents per-capita variables in various aging scenarios.

In all aging scenarios, aggregate effective labor supply declines, so pre-tax wage increases but after-tax wage decreases because the pension tax rate must increase to support higher shares of retirees. Meanwhile, the marginal product of capital decreases, resulting in lower rates of return on capital. Young workers would borrow less and reduce investment in capital. Retirees would also reduce capital holdings and switch to bonds. The shifts in bond supply and demand push down the rate of return on bonds. Since young workers borrow less and the share of young workers decreases, the demand for bonds falls significantly. Due to the aggregate labor reduction, output falls and consumption also decreases.

Table 4: Per-Capita and Price Variables in Aging Scenarios

Description	Variable	Baseline	Fertility	Mortality	Fertility& Mortality
Output	Y	2.055	1.948	1.909	1.680
			(-5.2%)	*(-7.1%)*	*(-18.3%)*
Consumption	C	2.045	2.001	1.901	1.732
			(-2.1%)	*(-7.1%)*	*(-15.3%)*
Investment	I	0.00	-0.061	0.00	-0.059
			(NA)	*(NA)*	*(NA)*
Borrowing Costs	Δ	0.010	0.008	0.009	0.006
			(-20%)	*(-10%)*	(-40%)
Capital	K	1.286	1.260	1.379	1.195
			(-2.0%)	*(7.2%)*	*(-7.1%)*
Bond Demand (Supply)	B	0.109	0.092	0.094	0.070
			(-15.1%)	*(-13.8%)*	*(-35.2%)*
Labor Demand (Supply)	L	1.302	1.211	1.115	0.990
			(-7.0%)	*(-14.4%)*	*(-24.0%)*
Capital-Labor Ratio	K/L	0.99	1.04	1.24	1.21
			(1.9%)	*(8.4%)*	*(7.5%)*
Wage	w	1.01	1.03	1.10	1.09
			(1.9%)	*(8.4%)*	*(7.5%)*
Risk-Free Rate (% p.a.)	r^b	9.23	8.88	7.74	7.90
			(-0.35)	*(-1.50)*	*(-1.33)*
Risky Rate (% p.a.)	$E(r)$	11.51	11.13	9.97	10.12
			(-0.37)	*(-1.54)*	*(-1.38)*
Risk Premium (% p.a.)	$E(r) - r^b$	2.27	2.25	2.23	2.22
			(-0.02)	*(-0.04)*	*(-0.05)*
Pension Tax Rate (%)	τ	0.18	0.24	0.30	0.42
			(0.06)	*(0.13)*	*(0.24)*

*Values in brackets show percentage deviations for variables in levels and absolute deviations for variables in rates from the baseline scenario.

There are differences between the fertility and mortality cases. In the fertility case, capital decreases due to the labor reduction. But in the mortality case, capital increases because retirees save more through holding capital for more consumption over their retirement and also the share of retirees increases. As households as a whole increase capital, the rate of return on capital decreases further. The output still decreases due to the labor contraction, resulting in less consumption as well.

The aging scenarios have significant impacts on the financial market structure. In the baseline, the relative size of the bond market is 8.5 percent of the capital stock. In the fertility case, the capital stock declines by 2 percent while the bond market shrinks by 15.2 percent, so the relative size of the bond market reduces to 7.3 percent. In the mortality case, the capital stock declines by 7.2 percent while the bond market shrinks by 13.8 percent, resulting in a relative ratio of 6.8 percent. In the combined case, the relative size of the bond market further reduces to 5.9 percent. The change in the relative size is driven partly by the change in the life-cycle portfolio and partly by the change in the demographic composition.

If we allow total population to change, we can obtain the values of aggregate variables by scaling per-capita variables by the population size. To illustrate this, we assume that the aging population in each scenario occurs for 20 periods (or 100 years) and then evaluate how much aggregate variables change over 20 periods. In addition to the changes in per-capita variables, the changes in the population size clearly affect aggregate variables of the whole economy. Table 5 presents the values of aggregate variables when the population size changes in the aging scenarios. When the fertility rate decreases at a constant rate of 1 percent every period, the population in period 20 decreases to 12.11. However, if the mortality rates of all cohorts linearly decrease towards 0 over 20 periods, the population increases to 16.69 because old people live longer on average, and the life expectancy increases from 61.3 years in the baseline to 90 years. In the combined scenario, the population increases to 14.56 where the mortality effect dominates.

Table 5: Aggregate Variables in Aging Scenarios

Description	Variable	Baseline	Fertility	Mortality	Fertility& Mortality
Total Population	N	14	12.11	16.69	14.56
			(-13.5%)	*(19.2%)*	*(4.0%)*
Output	Y	28.77	23.59	31.87	24.46
			(-18.0%)	*(10.8%)*	*(-15.0%)*
Consumption	C	28.63	24.24	31.72	25.22
			(-15.3%)	*(10.8%)*	*(-11.9%)*
Investment	I	0.00	-0.74	0.00	-0.85
			(NA)	*(NA)*	*(NA)*
Borrowing Costs	Δ	0.14	0.10	0.14	0.09
			(-28.6%)	*(0)*	*(-35.7%)*
Capital	K	18.00	15.26	23.01	17.39
			(-15.2%)	*(27.8%)*	*(-3.4%)*
Bond Demand (Supply)	B	1.52	1.12	1.56	1.02
			(-26.6%)	*(2.7%)*	*(-32.7%)*
Labor Demand (Supply)	L	18.23	14.67	18.61	14.41
			(-19.5%)	*(2.1%)*	*(-20.9%)*

*Values in brackets show percentage deviations from the baseline scenario.

A decline in total population in the fertility scenario exacerbates the decline of per-capita variables. If the change in fertility rates is the main driver of demographic change, aggregate variables would decrease, and the economy would shrink. However, if the change in the mortality rate (or in the life expectancy) is the main contributor to demographic change, the increasing population size would mitigate the adverse impact of the aging demographic composition on aggregate variables. In our closed economy model, the population composition is much more interesting than the population size, but in open economies, the change in the population size is also important because it can change the relative size between economies, and hence change the landscape of the world economy, generating spillover effects across countries.

7 Conclusions

This paper provides a novel framework to endogenize rates of return for risk-free bonds and risky capital in an OLG model. The rate of return on capital is endogenized by introducing idiosyncratic production shocks to avoid computation challenges related to aggregate production shocks in the literature. The framework bridges the finance literature on portfolio choice that assumes exogenous rates of return and the macroeconomic literature on demographic change that mostly has one type of asset, enabling the interaction between financial markets and macroeconomic conditions. This framework is suitable for examining portfolio choice and risk premia in a production economy, and also for evaluating the impacts of demographic change on macroeconomic conditions and financial market structures.

The paper first examined life-cycle portfolio choice in a baseline without demographic change, showing that the patterns can be affected by borrowing costs, labor income levels, production risk, and risk aversion degree. Borrowing costs have significant impacts on portfolio choice over the life cycle. Without borrowing costs, young households would borrow much more in the bond market and increase investment in capital, pushing up the capital share in the asset allocation at the early stage of life. In our model, labor income is risk free and hence human wealth serves as a safe asset balancing relative risks in financial assets over time, so human wealth levels can affect life-cycle portfolio choice because individual human wealth declines over time. In addition, production risk and the risk aversion degree directly affect portfolio choice. The paper also examined risk premia in the baseline, and illustrated that borrowing costs significantly contribute to risk premia, and production risk and risk aversion also matter. In particular, the introduction of borrowing costs in a quadratic polynomial form generates life-cycle portfolio allocation that approximately resembles empirical data.

The paper further investigated the impacts of demographic change on life-cycle variables, macroeconomic variables and the structure of financial markets. We considered three demographic scenarios: a fertility decline, a mortality decline, and a combination of fertility and mortality declines. Each demographic scenario leads to changes in both population size and composition, so we separated the two effects in each scenario. The aging demographic composition results in lower labor supply and higher capital-labor ratio, pushing down the rates of return on both assets. Pre-tax wage increases but after-tax wage decreases because the pension tax rate increases in all scenarios. The bond market shrinks significantly in all scenarios, and capital decreases in the fertility case but increases in the mortality case, causing structural change in the financial market. Due to the labor reduction, per-capita output falls and consumption also decreases. In terms of life-cycle variables, the consumption profile is quite different in the fertility and mortality cases particularly at the late stage of life. To finance consumption over retirement, household savings are also different in the two cases, leading to different impacts in the bond market and in capital accumulation. But the life-cycle portfolio choice changes less significantly because the investment in bonds and capital tends to move together and the risk premium does not change much. In addition, the population size change does not affect life-cycle variables, per-capita variables and financial market structures, but it scales aggregate variables in the economy, which is more interesting in an open economy compared to a closed economy because it can change the landscape of different economies.

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