Australian National University

Crawford School of Public Policy

CAMA

Centre for Applied Macroeconomic Analysis

Population and house prices in the United Kingdom

CAMA Working Paper 21/2018
May 2018

Creina Day
Centre for Applied Macroeconomic Analysis, Crawford School of Public Policy, ANU

Abstract

Real house prices rise in the United Kingdom amid growing concern of an impending correction. The rate of household formation has increased with strong population growth, due to elevated rates of natural increase and net migration, and lack of growth in average household size, due to a rise in single-person households with population ageing. This paper presents an overlapping generations model of housing, endogenous labour, savings and growth to analyse the effect of an increase in the household formation rate and speculative demand under rational expectations on house prices in a general equilibrium. We find that real house prices rise over time if the rate of household formation outstrips the rate of housing supply, but do not follow a speculative bubble path in the long run. The results explain why the upward trend in real house prices reflects market fundamentals and has continued despite population ageing as the number of working and retired households grows relative to the number of older people seeking to sell.

Keywords

JEL Classification

Address for correspondence:

(E) cama.admin@anu.edu.au

ISSN 2206-0332

This is a version of the following article: Day, C. (2018), Population and house prices in the United Kingdom. Scottish Journal of Political Economy, vol. 65, issue 2, May 2018, pp. 127-141, which has been published in final form. (http://online.wiley.com/doi/10.1111/sjpe.12166)

Population and house prices in the United Kingdom

Creina Day

CAMA, Australian National University (creina.day@anu.edu.au)

Abstract

Real house prices rise in the United Kingdom amid growing concern of an impending correction. The rate of household formation has increased with strong population growth, due to elevated rates of natural increase and net migration, and lack of growth in average household size, due to a rise in single-person households with population ageing. This paper presents an overlapping generations model of housing, endogenous labour, savings and growth to analyse the effect of an increase in the household formation rate and speculative demand under rational expectations on house prices in a general equilibrium. We find that real house prices rise over time if the rate of household formation outstrips the rate of housing supply, but do not follow a speculative bubble path in the long run. The results explain why the upward trend in real house prices reflects market fundamentals and has continued despite population ageing as the number of working and retired households grows relative to the number of older people seeking to sell.

I. Introduction

A baby is born every forty seconds and somebody dies every fifty seconds, on average, in the United Kingdom. There is a net gain from overseas of one immigrant every two minutes and fifty seconds. As a result, the overall population increases by more than one thousand persons daily, and recently surpassed 65.35 million persons residing in 27.1 million households. While young people seeking to enter the housing market have faced steep price rises, housing has become an increasingly important source of wealth for older people when they seek to exit the market. This paper explains why the number of new households has grown with population in the United Kingdom and develops an overlapping generations model to analyse the role of strong population growth and speculative demand in rising house prices.

The OECD recently warned that house prices appear overvalued but continue to rise in Australia, Canada, New Zealand and the United Kingdom, posing a risk of impending sharp corrections (OECD, 2016). Figure 1 depicts the rise in real house prices since the mid 2000s in these countries. Real house prices in the United Kingdom increased significantly in the decade prior to the global financial crisis in late 2008, fell in 2009 and subsequently recovered. During the same period, population in the United Kingdom grew at the highest rate in 50 years.[1] However, it is the number of new households rather than the population growth per se that contributes to housing market demand.

In this paper, we explain why strong population growth has translated to higher

[1] The average annual population growth rate between the 1990s and the 2000s more than doubled, up from 0.28% to 0.64%, and is projected to reach 0.71% this decade. Population growth averaged 0.61% per annum during the 1960s baby boom (Office for National Statistics, 2017).

rates of household formation in the United Kingdom. We develop an optimising intertemporal model of housing, endogenous labour supply, savings and economic growth to explain how higher rates of household formation can contribute to rising house prices and whether a speculative bubble may emerge as a long run equilibrium outcome. In doing so, we shed light on recent concern that the United Kingdom faces a correction in house prices.

[Figure 1 about here]

Asset bubbles can arise when assets are purchased in anticipation that they can be resold at a higher price to another investor who will buy them for the same reason. Arbitrage between assets and physical capital requires that bubbles grow at the rate of return on capital. Thus, eventually the value of the bubble will be too large relative to the economy. This appears to rule out a bubble until one considers that the economy itself is growing as new households form. If the rate of return on capital is less than the rate of household formation, the economy will grow faster than asset prices, enabling a bubble to exist.

Housing is an intrinsically useful asset, serving a dual role as an investment vehicle and a durable good which households consume. Whether a bubble on an intrinsically useful asset exists in equilibrium depends on whether the asset pays a dividend which grows at a rate less than the economy. On the one hand, strong growth in the number of new households entering the housing market over time provides a necessary condition for a bubble. On the other hand, the usefulness of housing as a consumer durable implies a user cost which in turn implies conditions on the rate at which prices appreciate in a general equilibrium model.

This paper formalises the intuition for the roles of household formation and user cost of housing within an overlapping generations framework where people live for two periods. As households during the first period, people work and consume leisure, save, purchase and consume real estate. The second period marks the end of the household, as older people sell housing and use the capital gains on housing and interest on savings to fund consumption, including residence in non-private dwellings, such as care homes or nursing homes. House prices are endogenously determined by a market where demand grows relative to supply as new households form over time. The user cost associated with consumption of housing falls as future prices increase relative to current prices.

Existing models predict that population affects real house prices via age structure. Mankiw and Weil (1989) develop a partial equilibrium intertemporal model of the housing market to predict a significant decline in real house prices due to population ageing over the twenty years to 2007 in the United States. Poterba (1991) finds inconclusive evidence that population age structure explains the failure of United States real house prices to fall in the 1980s, as the user cost of housing suggests they should have, and points to speculation in the absence of rational expectations as an alternative explanation. Garino and Sarno (2004) develop a three period overlapping generations model of housing demand and find evidence of two rational bubbles in United Kingdom house prices, 1983 to 2002, the latter still ongoing at the end of their sample period. In contrast to the existing theoretical literature, the analysis in this paper[2] considers how speculative demand and population growth, via household

[2] Existing models provide a partial equilibrium analysis where income is an exogenous variable. For the purpose of this paper, which focuses on the role of household formation and speculative

formation, affects real house prices.

Recent research for the United Kingdom[3] sheds light on why population ageing need not reduce real house prices and whether rises in house prices could outstrip rises in income. Chen et al (2012) find, given that propensities to form households differ between age groups, projected population ageing to 2035 is not likely to be a major determinant of house prices in Scotland. Bell and Rutherford (2012) find that the combined effects of population ageing and the trend towards receiving long-term care at home will maintain the level of housing demand above what it otherwise would be in the United Kingdom, resulting in excess demand in the housing market by 2030. Miles (2012) find that rises in house prices become more likely to outstrip rises in average income at higher population density.

This paper contributes to the theoretical literature in three respects. First, we explain how the rate of household formation is increasing in population growth and decreasing in average household size, which in turn has not risen as the number of single-person households increases with population ageing. Second, we provide a partial equilibrium analysis of rising house prices due to market fundamentals, including growth in the number of households, and speculative demand under rational expectations. Third, we develop an overlapping generations model of economic growth to analyse how rising house prices persist when the rate of household formation outstrips supply and whether a speculative bubble could exist in a general

demand in sustaining rising real house prices in the long run, we develop a general equilibrium model.

[3]Mankiw and Weil (1989) led to a large body of empirical research into the effect of population ageing on house prices for several countries, including Canada and the United States (see, for example, Engelhardt and Poterba (1991), Ermisch (1996) and Levin et al. (2009)).

equilibrium.

II. Population and the household formation rate

Market demand for housing depends on several factors, including real household wages, current and expected future house prices, and the number of households seeking to buy. Underlying market pressure arises as new households form and thus the number of households seeking to enter the market outstrips the number exiting. The role of market fundamentals and speculation in determining house prices is modelled in the following section. Here, we explain the source of recent increases in the rate of household formation.

We start with a simple decomposition of the number of households, N, into three components

$$N = Pop \times \frac{Popr}{Pop} \div \frac{Popr}{N} \tag{1}$$

where Pop is the total population, $Popr$ is the population living in private residential dwellings and $Popr/N$ measures the average household size. The rate at which new households form depends on the interaction between growth in population and average household size since $Popr/Pop$ remains steady over time.

From (1), the rate of household formation, n, is given by

$$n = f - d + m - g\left(\frac{Popr}{N}\right) \tag{2}$$

where $g\,(Pop) = f - d + m$ decomposes the growth rate in population, where $f - d$ is (crude birth rate - crude death rate)/10, which gives the rate of natural increase in percentage form, and m is the rate of net overseas migration in percentage form, and $g\,(Popr/N)$ is the growth rate in average household size.

[Figure 2 about here]

Figure 2 depicts the two components of population change, natural increase and net overseas migration. Since the mid 2000s, significant increases in net migration and higher natural increase have contributed to increasing population. Rising immigration underpins recent increases in net migration. The direct effect of net migration has increased the population by more than 250,000 per year on average since 2004, which is approximately 50,000 more people per year than natural increase for the same period. The higher rate of natural increase is attributed to a rise in the birth rate. Births began to rise in the mid 2000s, peaking at 813,000 in 2012. The long run trend in the number of deaths is more stable than the number of births.

The two sources of population growth place upward pressure on housing demand. All else equal, demand for housing is increasing in the birth rate since children require housing. Immigrants tend to be aged 20 to 35. The group of the population aged 20 to 35 in 2015 increased in size when compared with the group of the population aged 10 to 25 in 2005. Such a change is generated by adding to the population through immigration. A significant rise in net overseas migration increases the working age population seeking to enter the housing market.

Referring to equation (2), higher population growth need not raise the household formation rate if there is offsetting growth in average household size. As the total fertility rate began to decline in the 1970s, so did average household size. Referring to Figure 3, average household size continued to fall in the 1980s, but has remained relatively constant since the 2000s.

[Figure 3 about here]

Increased population growth and lack of growth in average household size have fuelled higher growth in the number of households since the mid 2000s. Presently, around 30 per cent of households contain one person, compared with only 17 per cent in 1971. The prevalence of single-person households partly reflects ageing of the population. Persons aged 65 to 74 living alone saw a statistically significant increase of 22 per cent over the past decade. Growth in the number of households, all else equal, generates an increase in housing demand.

People are not only living longer, they are choosing to live in their own home as long as practicable. The population aged 85 and over residing in private dwellings has increased during the past decade with improvements in health. Recent government policy initiatives to help older people live at home longer may also boost future numbers of people receiving support at home. Thus, population ageing need not translate to an increase in housing supply relative to demand.

III. MODEL OF HOUSING, ENDOGENOUS LABOUR, SAVINGS AND GROWTH

Consider an economy in which the production function for final output is given by

$$Y_t = AK_t^{\alpha} L_t^{1-\alpha} \tag{3}$$

where A denotes the level of technology, which we normalise to 1, K_t denotes physical capital and L_t denotes labour in period t. The production of output per unit of labour, $y_t \equiv Y_t / L_t$, is therefore

$$y_t = k_t^{\alpha} \tag{4}$$

where k_t denotes the physical capital to labour ratio. Profit maximisation and competitive markets imply that factors are paid their marginal product, so that

$$1 + r_t = \alpha k_t^{\alpha - 1} \tag{5a}$$

$$w_t = (1 - \alpha) k_t^{\alpha} \tag{5b}$$

where r_t is the rate of return on capital in period t and w_t is the real wage in period t.

In this stylised two period model, people reside in housing and thus live as households in period t. Older people are not counted as households when they sell housing in period $t+1$. Each household is endowed with a unit of time in period t, of which they choose to supply l_t as labour for paid work, $0 < l_t < 1$. Endogenous labour supply in period t captures the feature that households could be working in paid employment or retired.

For a representative household in period t, lifetime utility is

$$U = (1 - \gamma - \delta) \ln c_{1t} + \gamma \ln h_t + \delta \ln (1 - l_t) + \beta \ln c_{2t+1} \tag{6}$$

where c_{1t} is consumption when a household in period t, c_{2t+1} is consumption when elderly in period $t+1$, h_t is the amount of housing purchased in period t and $(1 - l_t)$ is leisure time in period t, and $\beta = 1/(1+\rho)$ is the discount rate with a constant time preference parameter, ρ. The first period budget constraint is

$$c_{1t} + p_t h_t + s_t = w_t l_t \tag{7}$$

where p_t is the price of housing purchased in period t and s_t is savings which is stored in the other asset of the economy, physical capital. The second period budget constraint is

$$c_{2t+1} = (1 + r_{t+1})\, s_t + p_{t+1} h_t \tag{8}$$

where p_{t+1} is the price of housing sold and r_{t+1} is the rate of return on physical capital in period $t+1$. Equations (7) and (8) give the lifetime budget constraint

$$w_t = c_{1t} + \frac{c_{2t+1}}{(1 + r_{t+1})} + \pi_t h_t + w_t\,(1 - l_t) \tag{9}$$

where $\pi_t = p_t - p_{t+1}/\,(1 + r_{t+1})$ is the user cost of housing.

The maximisation of (6) subject to (9) gives

$$\begin{aligned}
c^*_{1t} &= (1 - \gamma - \delta)\,\frac{(1+\rho)}{(2+\rho)} w_t && \text{(10a)}\\
c^*_{2t+1} &= \frac{(1+r_{t+1})}{(2+\rho)} w_t && \text{(10b)}\\
h^*_t &= \frac{\gamma}{\pi_t}\frac{(1+\rho)}{(2+\rho)} w_t && \text{(10c)}\\
l^*_t &= l^* = 1 - \delta\frac{(1+\rho)}{(2+\rho)} && \text{(10d)}
\end{aligned}$$

which is optimal consumption when a household, optimal consumption when elderly, demand for housing and optimal labour supply, respectively. The user cost of real estate, π_t, is endogenous as the price of housing is determined by the market for housing so that market demand coincides with supply.

Let us distinguish between aggregate labour supply, L_t, and the number of households, N_t, at time t. We have $L_t = l^* N_t$, where the household's labour supply, l^*, is given by (10d), which is a constant. This is due to the combined effect of logarithmic utility and time structure of the model where there is no inherited financial wealth in the first period and no labour income when elderly in the second period. Intuitively, the negative substitution effect and negative pure income effect of an increase in the real wage on demand for leisure are exactly offset by a positive wealth effect.

Some authors use this two period specification as a model of endogenous retirement (Heijdra, 2009). As the preference for leisure, δ, increases, optimal leisure time, $(1 - l^*)$, and thus portion of households who are retired in the first period rises. We find the approach useful for the analysis here because it allows us to distinguish labour supply in the production function from the number of households and model a market for housing where growth in the number of households relative to the number of elderly who no longer consume housing determines growth in the number of buyers relative to the number of sellers over time.

At time t, households purchase housing from the elderly at the market price, p_t. Thus, market demand is

$$h_t^D = \frac{\gamma}{\pi_t} \frac{(1+\rho)}{(2+\rho)} w_t N_t \tag{11}$$

where N_t is the number of households at time t. The stock of housing available for purchase may grow over time at the rate of government land release, g, from an initial amount of $\bar{h}$.[4] Thus, market supply is

$$h_t^S = \bar{h}(1+g)N_{t-1} \tag{12}$$

where N_{t-1} is the number of households at time $t-1$, who are now elderly selling housing at time t. The number of households grows over time at the rate n, so that $N_t = (1+n)N_{t-1}$.

Thus, the market price for housing is

$$p_t = \frac{p_{t+1}}{(1+r_{t+1})} + \gamma\frac{(1+\rho)}{(2+\rho)}\frac{(1+n)}{\bar{h}(1+g)}w_t \tag{13}$$

which is increasing in the rate of household formation because there are $(1+n)$ as many buyers as there are sellers and decreasing in the rate of land release because there is $(1+g)$ more housing land available. The market price is increasing in wages, which in turn increase with capital per household according to equation (5b). The current price is also increasing in the discounted future price. These properties of equation (13) are summarised in the following remark.

Remark 1 *The fundamental market price of housing is increasing in the discounted future price, real wages and rate of household formation, and decreasing in housing land supply.*

Intuitively, household demand for housing is downward sloping or decreasing in the user cost of housing, which in turn is increasing in the current price and

[4] Glaeser et al (2008) argue that markets with more elastic supply experience fewer housing bubbles. Extending the model to include price elasticity of supply is an interesting direction for future research.

decreasing in the discounted future price. Increasing wages boost household demand because housing is a normal good. The stock of housing is depleted by an increase in the number of households seeking to buy relative to the number of older persons seeking to sell, putting pressure on the market price to rise. However, government land release replenishes the stock of housing land, relieving upward pressure on the market price.

Speculative bubble in partial equilibrium

We may solve explicitly for p_t in partial equilibrium under the assumption of rational expectations. From equation (13),

$$p_t = \frac{1}{E_t a_{t+1}} E_t p_{t+1} + c w_t \tag{14}$$

where $E_t a_{t+1} = (1 + E_t r_{t+1})$ and $c = \gamma \left[(1+\rho) \,/\, (2+\rho)\right] / \left[(1+n)/\bar{h}(1+g)\right]$.

Referring to the appendix, using the law of iterated expectations and solving recursively yields

$$p_t = \prod_{i=0}^{T} \frac{1}{E_t a_{t+i}} p^h_{t+T} + c \sum_{i=0}^{T-1} \prod_{i=0}^{T-1} E_t w_{t+i} \frac{a_t}{E_t a_{t+i}} \tag{15}$$

where T is the time horizon of the household. The term $\prod_{i=0}^{T} E_t \frac{1}{a_{t+i}} p_{t+T}$ captures expected appreciation which is central to the existence of a bubble.

Referring to the appendix,

$$p_t = c \sum_{i=0}^{\infty} \prod_{i=0}^{\infty} E_t w_{t+i} \frac{a_t}{E_t a_{t+i}} \tag{16}$$

is a possible solution, which gives the price of housing as the present discounted value of expected future income. However, relaxing the condition that the expected house prices will not explode too fast, equation (14) has a general solution of the form

$$p_t = p_t^* + b_t \tag{17}$$

where p_t^* denotes the fundamental solution.

Substituting for p_t and $E_t p_{t+1}$ from equation (17) in equation (14) implies

$$p_t^* + b_t = \frac{1}{E_t a_{t+1}} E_t p_{t+1}^* + \frac{1}{E_t a_{t+1}} E_t b_{t+1} + c w_t \tag{18}$$

which, using the definition of p_t^* and substituting for p_t^* and $E_t p_{t+1}^*$ reduces to

$$b_t = \frac{1}{E_t a_{t+1}} E_t b_{t+1} \Leftrightarrow E_t b_{t+1} = b_t E_t a_{t+1} \tag{19}$$

where $|a_{t+1}| > 1 \Leftrightarrow (1 + r_{t+1}) > 1$. While p_t^* is the fundamental solution, b_t is called a speculative bubble. This finding is summarised in the following remark.

Remark 2 *A speculative bubble in house prices may exist under rational expectations in partial equilibrium.*

In general equilibrium, however, there are conditions other than equation (14) that must be satisfied by a speculative bubble solution. We therefore analyse the dynamics of the economy and whether a housing bubble is possible in general equilibrium.

Dynamic system

The capital to labour ratio at period t can be written as

$$k_t \equiv \frac{K_t}{L_t} = \frac{K_t}{l^* N_t} \equiv \frac{\bar{k}_t}{l^*} \tag{20}$$

where $\bar{k}_t \equiv K_t/N_t$ is capital per household. With endogenous labour supply, the number of households (N_t) no longer coincides with the amount of labour used in production $(l^* N_t)$. By redefining the capital to labour ratio as $k_t \equiv \bar{k}_t/l^*$, however, the expressions for the real interest rate and real wage are still as in equations (5a) and (5b). Unlike k_t which depends on labour supply choice, l^*, in period t, $\bar{k}_t$ is a predetermined variable, which means the stock in period t accumulates from a predetermined stock in period $t-1$. It is therefore useful to express the dynamics in terms of $\bar{k}$.

Goods market equilibrium implies that the capital stock at period $t+1$ is determined by the aggregate savings of households at the end of period t,

$$K_{t+1} = s_t^* N_t \tag{21}$$

where $N_t = N_{t+1}/(1+n)$. From equations (7) and (10a), optimal savings is

$$s_t^* = \frac{(\beta+\gamma)}{(1+\beta)} w_t - p_t h_t \tag{22}$$

where $(\beta+\gamma)\,/\,(1+\beta) = (1+\gamma\,(1+\rho))\,/\,(2+\rho)$.[5]

[5] From equations (8) and (10b), $s_t^* = 1/\,(2+\rho)\,w_t - (p_{t+1}h_t)\,/\,(1+r_{t+1})$, which is equivalent to equation (22), given equation (10c).

The dynamics of the economy can be summarised through the equation of motion for physical capital per household. Substituting from equations (5b) and (22), using $k_t \equiv \bar{k}_t / l^*$, in equation (21),

$$\bar{k}_{t+1} = \frac{(\beta + \gamma)}{(1 + \beta)} \frac{(1 - \alpha)}{(1 + n)} \left(\frac{\bar{k}_t}{l^*} \right)^\alpha - \frac{p_t h_t}{(1 + n)} \tag{23}$$

where $p_t h_t = (p_t^* + b_t) h_t$ gives the aggregate value of housing, including a possible bubble. Substituting from equation (10d) in equation (23) gives

$$(1 + n)\bar{k}_{t+1} = \frac{(\beta + \gamma)}{(1 + \beta)^{1-\alpha}} \frac{(1 - \alpha)}{(1 + \beta - \delta)^\alpha} \left(\bar{k}_t \right)^\alpha - p_t h_t. \tag{24}$$

In a stationary equilibrium of this model, $\bar{k}_{t+1} = \bar{k}_t = \bar{k}$. If $\bar{k}$ is constant in equation (23), then ph must be constant in a steady state equilibrium and satisfy

$$ph = \frac{(\beta + \gamma)}{(1 + \beta)^{1-\alpha}} \frac{(1 - \alpha)}{(1 + \beta - \delta)^\alpha} \bar{k}^\alpha - (1 + n)\bar{k} \tag{25}$$

where $\left[(\beta + \gamma) / (1 + \beta)(1 + \beta - \delta) \right] (1 - \alpha) \bar{k}^\alpha$ is savings out of household wages. Intuitively, the aggregate value of housing equals net savings.

Referring to equation (12), the amount of housing available to be traded in the market, h_t, changes over time at the rate $(1 + g)/(1 + n)$. For the aggregate value of housing, ph, to be constant, the price of housing must change over time at the rate $(1 + n)/(1 + g)$. Thus, if h_t falls over time because $n > g$ then p rises proportionally in a steady state equilibrium. This finding is summarised in the following remark.

Remark 3 *In a steady state equilibrium, the aggregate value of housing is constant.*

If the quantity of housing to be traded falls over time because the rate of household formation outstrips the rate of supply then house prices will rise proportionally over time.

We have shown that house prices rise over time if $n > g$ at the rate $(1+n)/(1+g)$ in a steady state equilibrium. Substituting for $p_{t+1} = ((1+n)/(1+g))\, p_t$, the user cost of housing in steady state would be

$$\pi_t = p_t - \frac{((1+n)/(1+g))\, p_t}{(1+r)} \tag{26}$$

which must be strictly positive, otherwise demand for housing would be unbounded. Thus, the condition $(1+r) > (1+n)/(1+g)$ must be satisfied in a steady state equilibrium. This condition does not rule out dynamic inefficiency $(r < n)$ since $(1+n) > (1+r) > (1+n)/(1+g)$ is possible.

Consider an economy with initial capital per household k_0, where $f^{'}(k_0) < n$, and house prices comprise a bubble component that grows at the rate of return on capital, which is $1 + r_0 = f^{'}(k_0)$. The user cost of housing, based on house prices with fundamental value, p_t^*, and a bubble component, b_t, is positive since $(1+r_0) > (1+n)/(1+g)$. If h_t falls at the rate $(1+g)/(1+n)$ and b_t rises at $(1+r_0) > (1+n)/(1+g)$, then the aggregate value of the bubble component, $b_t h_t$, rises. The equation of motion can be expressed as

$$b_t h_t = \frac{(\beta+\gamma)}{(1+\beta)^{1-\alpha}} \frac{(1-\alpha)}{(1+\beta-\delta)^{\alpha}} k_t^{\alpha} - (1+n)k_{t+1} - p_t^* h_t \tag{27}$$

where the aggregate value of the bubble equals net savings minus the aggregate fundamental value of housing.

Intuitively, a housing bubble could emerge in transitional dynamics to soak up excess savings. The initial capital per household is dynamically inefficient. There is excess savings for investment in physical capital in the sense that the rate of return on capital is less than the rate of household formation. In this case, rising house prices could comprise a bubble component growing at the rate of return on capital which exceeds the rate at which house prices would rise in equilibrium due to the rate of household formation exceeding the rate of housing supply.

Referring to equation (27), a rise in $b_t h_t$ implies that capital per household will gradually fall over time. Thus, the rate of return on capital would rise over time, addressing dynamic inefficiency in savings for capital per household. Real wages would fall over time, placing downward pressure on demand for housing, which would oppose the upward pressure on fundamental prices due to the rate of household formation outstripping the rate of supply. The bubble component, growing at a rising rate of return on capital, would increasingly contribute to rising house prices over time. However, the economy would not converge to a steady state in which a bubble in overall house prices exists, as summarised in the following remark.

Remark 4 *A speculative bubble where house prices grow at the rate of return on capital could not exist in a steady state equilibrium.*

A situation where house prices grow at the rate $(1+r)$ could not persist in a steady state equilibrium because the aggregate value of housing must be constant, which in turn requires that house prices also grow at the rate $(1+n)/(1+g)$. Referring to equation (26), the user cost of housing must be positive, which rules out a steady state equilibrium where house prices grow at the rate $(1+r) = (1+n)/(1+g)$. Otherwise, the user cost of housing would be zero and housing demand would be undefined.

IV. Conclusion

Real house prices continue to rise amid growing concern of an impending sharp correction in speculative demand. The theoretical model presented in this paper explains how higher rates of household formation, driven by strong population growth, can underpin rising real house prices and why a speculative bubble cannot be sustained in a long run equilibrium.

The prediction in existing theoretical models that real house prices decline due to population ageing is contentious. In contrast to the existing literature, we observe that the rate of household formation has increased with strong population growth, due to higher net migration, and lack of growth in average household size. The latter is attributed to population ageing whereby the number of sole-person households rises as more retired people remain in their own home longer.

This paper analyses the effects of growth in the number of households and speculative demand on real house prices: first, in a partial equilibrium intertemporal model of household choice and the housing market under rational expectations; and then, in a general equilibrium overlapping generations model of endogenous labour supply, savings, housing and economic growth.

Endogenous labour supply allows for households to work and retire during the first period of the overlapping generations model. The elderly are not counted as households when they sell housing in the second period. Within this structure, the number of households grows with increasing net migration and retirees who consume housing, relative to the number of older sellers in a market for housing. The novel contribution is a concise theoretical analysis that implies an ageing population, via the number of retired households, can contribute to rising house prices and informs government policy regarding the rate of housing land release.

The analysis predicts that:

1. The fundamental market price of housing is increasing in the rate of household formation, real wages and discounted future price, and decreasing in the rate of housing land supply.

2. A speculative price bubble growing at the rate of return on capital may emerge under rational expectations in a partial equilibrium.

3. In the steady state equilibrium of an overlapping generations model of economic growth, real house prices rise over time if the rate of household formation outstrips the rate of housing land supply, but cannot follow a speculative bubble path.

The results in this paper suggest that a long run trend of rising real house prices reflects market fundamentals rather than speculative demand. Forward looking agents may bid up the market price in anticipation of expected capital gains. However, as an intrinsically useful asset, housing has a user cost measured as the current price minus the future price discounted by the rate of return on capital, which is endogenously determined in a general equilibrium. A speculative bubble growing at the rate of return on capital would imply a non-positive user cost and thus unbounded demand for housing.

Some interesting implications arise. The elevated rate of household formation in the United Kingdom, an economy witnessing a marked and long lasting rise in real house prices since the 2000s, has increased the number of potential entrants to the housing market. In a steady state equilibrium, where the rate of return on capital and real wage are endogenously determined, market demand for housing increases relative to supply. In the model, real house prices stabilise in the long run if the rate of housing land supply equals the rate of household formation. Ensuring that the rate of housing land release keeps pace with the rate of household formation may therefore help relieve upward pressure on real house prices in the long run.

The model does not suggest an optimal rate of housing land release with respect to the normative question of whether governments should influence real house prices, because externalities associated with housing affordability and market corrections do not feature in the model. Rather, the analysis suggests that governments seeking to relieve upward pressure on real house prices consider elevated rates of household formation due to the combined effect of immigration-led population growth and tendency for retired households to remain in housing longer when deciding the rate of housing land release.

In the wake of the Brexit vote, net migration to the United Kingdom has fallen in March 2017 to its lowest level in three years. The sensitivity of the model implications to a fall in net migration is therefore worth considering. The key question is whether the rate of household formation, which our analysis identifies can underpin rising real house prices, would fall. During the past decade, immigration-led population growth fuelled growth in the number of households as the profiles of immigrant and retired households have had offsetting effects on average household size. While population growth may fall with net migration, average household size could also decline as retired households predominate. Growth in the number of households would outstrip population growth and thus the model implications hold.

REFERENCES

BELL, D. and RUTHERFORD A. (2012). Long-term care and the housing market. *Scottish Journal of Political Economy*, **59**, pp. 543-563.

CHEN, Y., GIBB, K., LEISHMAN, C. and WRIGHT, R. E. (2012). The impact of population ageing on house prices: A micro-simulation approach. *Scottish Journal of Political Economy*, **59**, pp. 523-542.

ERMISCH, J. (1996). The demand for housing and population ageing: micro-econometric evidence. *Economica*, 63, pp. 384-404.

ENGELHARDT, G. V. and POTERBA, J. M. (1991). House prices and demographic change: Canadian evidence. *Regional Science and Urban Economics*, **21**, pp. 539-46.

HEIJDRA, B. (2009). *Foundations of Modern Macroeconomics.* Oxford University Press: Oxford.

LEVIN, E., MONTAGNOLI, A. and WRIGHT, R. E. (2009). Demographic change and the housing market: evidence from a comparison of Scotland and England. *Urban Studies*, 46, pp. 27-43.

GARINO, G. and SARNO, L. (2004). Speculative bubbles in U.K. house prices: Some new evidence. *Southern Economic Journal*, **70**, pp. 777-795.

Federal Reserve Bank of Dallas. (2016). International House Price Dataset.

MANKIW, N. G. and WEIL, D. (1989). The baby boom, the baby bust and the housing market. *Regional Science and Urban Economics,* **19**, pp. 235-58.

MILES, D. (2012). Population density, house prices and mortgage design. *Scottish Journal of Political Economy*, **59**, pp. 444-466.

OECD (2016). *Global Economic Outlook November 2016.* OECD, Paris.

Office for National Statistics (2016). *Overview of the UK Population: February 2016.* London.

Office for National Statistics (2017). *Overview of the UK Population: March 2017.* London.

POTERBA, J. M. (1991). House price dynamics: the role of tax policy and demography. *Brookings Papers on Economic Activity,* **2**, pp. 143-203.

Appendix

Derivation of equation (15)

Writing equation (14) at time $t+1$ and taking the expectations of both sides conditional on information at time t yields

$$E_t p_{t+1} = \frac{1}{1 + E_t\left(E_{t+1} r_{t+2}\right)} E_t\left(E_{t+1} p_{t+2}\right) + cE_t w_{t+1}$$

Using the law of iterated expectations,

$$E_t p_{t+1} = E_t \frac{1}{a_{t+2}} E_t p_{t+2} + c E_t w_{t+1}$$

Replacing in (14) gives

$$p_t = E_t \frac{1}{a_{t+1}} E_t \frac{1}{a_{t+2}} E_t p_{t+2} + E_t \frac{1}{a_{t+1}} c E_t w_{t+1} + c w_t$$

Solving recursively up to time T,

$$\begin{aligned} p_t &= E_t \frac{1}{a_{t+1}} ... E_t \frac{1}{a_{t+T}} E_t p_{t+T} \\ &\quad + c E_t \frac{1}{a_{t+1}} ... E_t \frac{1}{a_{t+T-1}} c E_t w_{t+T-1} \\ &\quad + ... + E_t \frac{1}{a_{t+1}} c E_t w_{t+1} + c w_t \end{aligned}$$

which using summation and product notation, gives

$$p_t = \prod_{i=0}^{T} E_t \frac{1}{a_{t+i}} p_{t+T} + c \sum_{i=0}^{T-1} \prod_{i=0}^{T-1} E_t w_{t+i} \frac{E_t \frac{1}{a_{t+i}}}{\frac{1}{a_t}}$$

The transversality condition

$$\lim_{T \to \infty} \prod_{i=0}^{T} E_t \frac{1}{a_{t+i}} p_{t+T} = 0$$

means the expectation will not explode too fast. Assuming this condition, as $T \to \infty$,

$$p_t = c \sum_{i=0}^{\infty} \prod_{i=0}^{\infty} E_t w_{t+i} \frac{E_t \frac{1}{a_{t+i}}}{\frac{1}{a_t}}$$

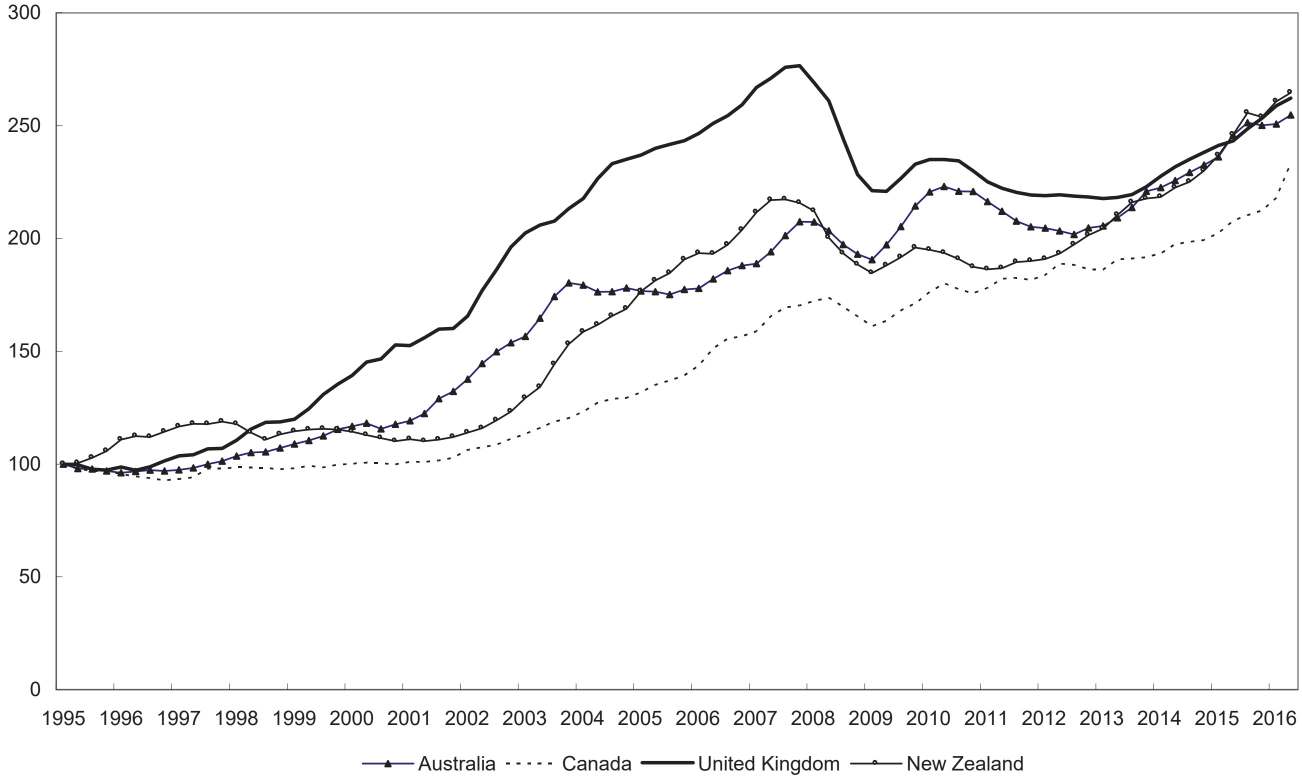

FIGURE 1

Real House Price Index, 1995-2016

Source: Federal Reserve Bank of Dallas (2016)

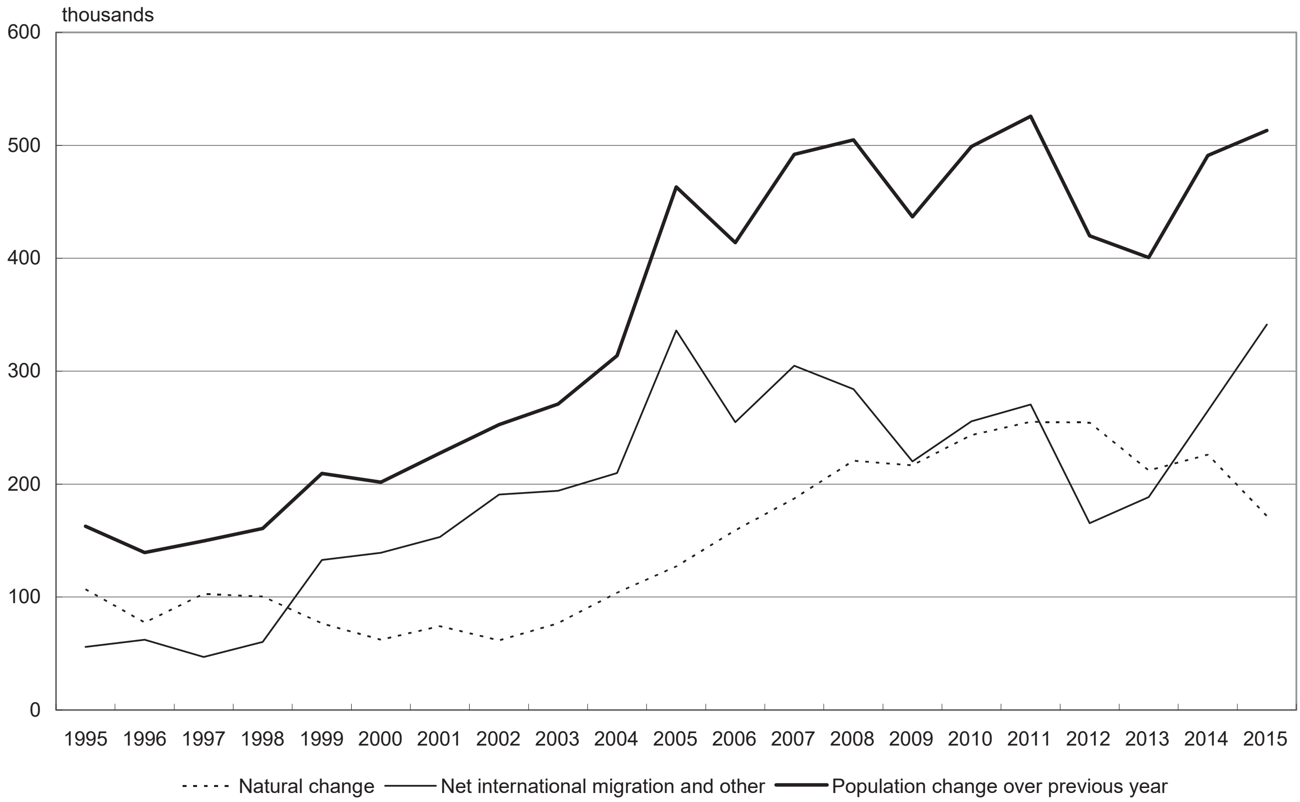

FIGURE 2

Contributions to Estimated Population, (change over previous year), United Kingdom, 1995-2015

Source: Office for National Statistics (2016)

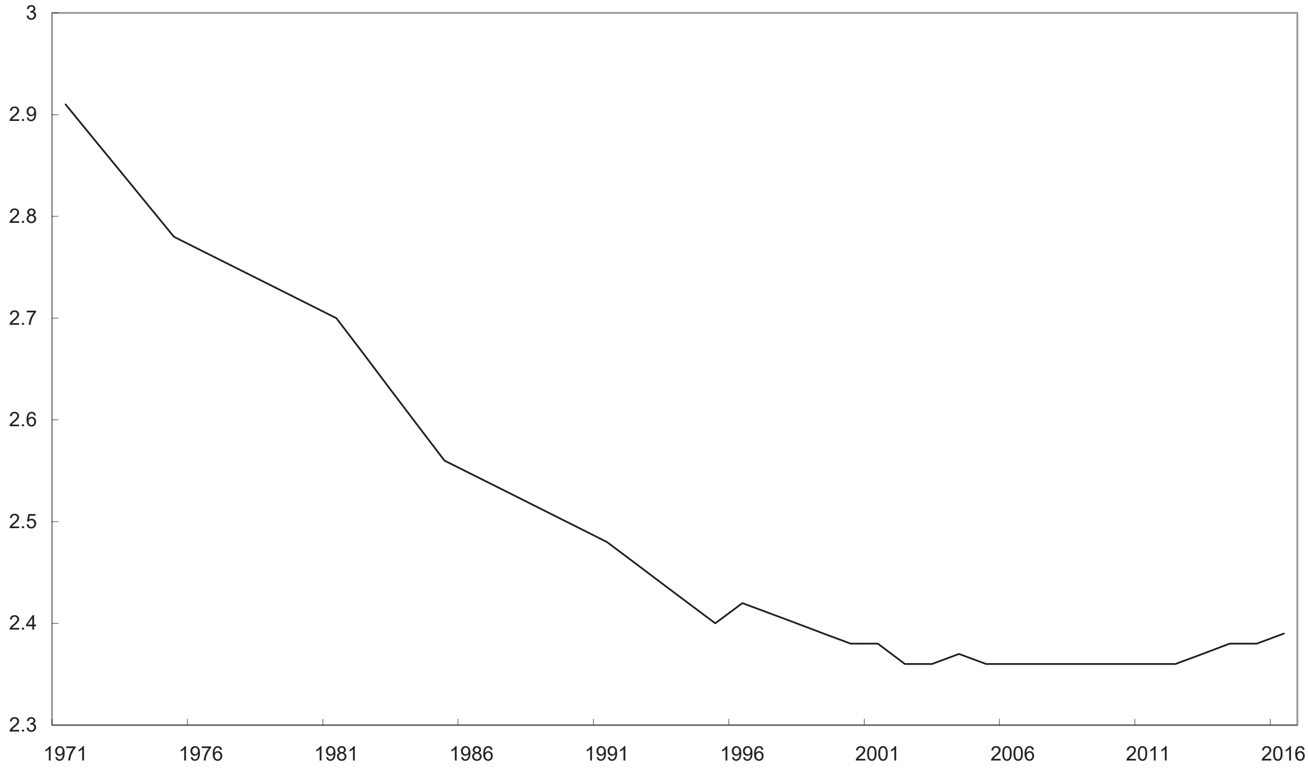

FIGURE 3

Average Household Size, Great Britain,1971-2016

Source: Office for National Statistics, General Lifestyle Survey