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Keywords

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JEL Classification

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Inverse J effect of economic growth on fertility: a model of gender wages and maternal time substitution

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Abstract

This paper presents a model where economic growth, via growth in female wages relative to male wages, encourages households to raise paid female labor supply and have more children by substituting child care for maternal time. A threshold logarithm per capita output, above which fertility decline reverses, depends on subsidized child care, maternity pay, and the value placed on children and maternal time spent rearing children. The predictions explain recent evidence and identify cross country differences in gender wages, family policy and willingness to substitute maternal time in childrearing as important factors in an inverse J-shaped effect of economic growth on fertility. The analysis is robust to the introduction of education and cost sharing among children in child rearing. Economies of scale in child rearing reduces the threshold logarithm of per capita output. Demand for child quality continues to rise with wages despite fertility decline reversal.

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Introduction

An emerging empirical debate has considered whether the negative effect of economic growth on fertility reverses to a positive at advanced stages of development. This paper has developed an economic model where the effect on fertility of economic growth via gender wages depends on family policy and parental willingness to substitute purchased child care for maternal time. The purpose has been to explore how and whether further economic growth reverses fertility decline at advanced stages of development.

In the main, existing models of endogenous fertility have predicted that economic growth causes fertility decline by raising the cost of rearing children. Parents value children, but children come at a cost. Economic growth may reduce fertility by raising female wages relative to male wages and hence the opportunity cost of maternal time used in child rearing (Barro and Becker 1988; Galor and Weil 1996) or by raising the relative rate of return to investing in education per child (Becker et al. 1990).

The similarities in levels of education and differences in policies to help families combine female employment and child rearing across high income Organization for Economic Cooperation and Development (OECD) countries (Luci-Greulich and Thevenon 2014) have suggested that models in which rising female relative wages affect fertility will have had implications for the current debate. The practical benefit of having developed a theoretical model capable of explaining how rising female relative wages could contribute to a turnaround in fertility decline is illustrated in Figures 1 and 2 which show trends in gender pay gaps and fertility for a sample of high income English-speaking, Asian and Nordic countries.

Figure 1 depicts the evolution of the gender pay gap in four OECD countries between 1975 and 2015. In the mid-1970s, the gender pay gap was around 40% in Japan, the United Kingdom and the United States and approximately 28% in Finland. In both the United Kingdom and the United States, the gap steadily declined between the mid-1970s and the early 2010s, although the improvement was

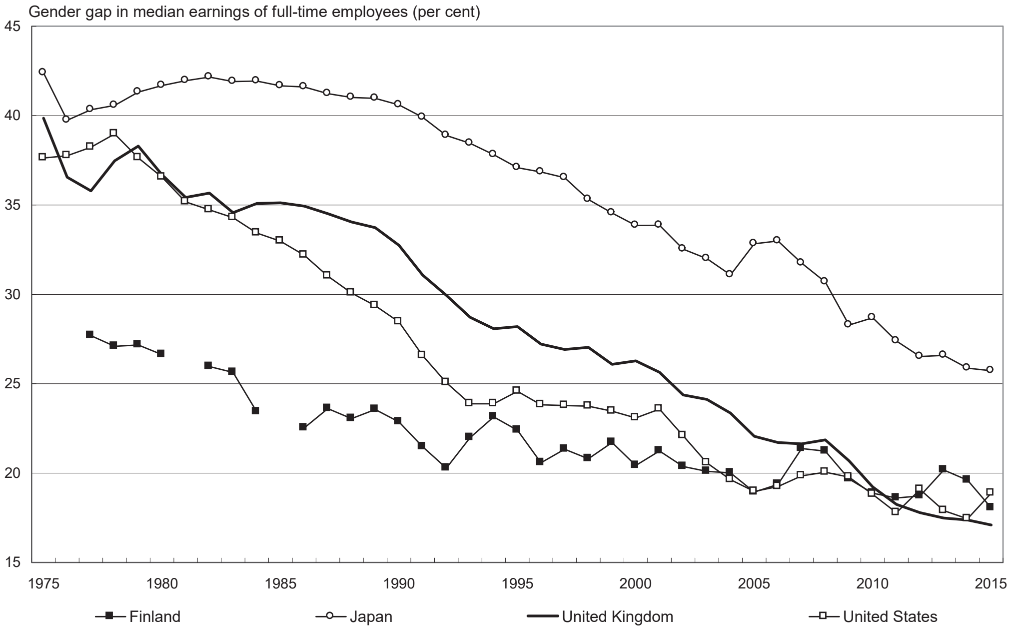

Figure 1: Gender gap in median earnings of full-time employees, selected OECD countries, 1975-2015. *Source: OECD (2017)*

larger in the United States relative to the United Kingdom between the mid-1980s and the mid-2000s. By 2015, the gender pay gap in both countries was around half that seen in 1975. In Japan, the gender pay gap remained high until the late 1980s. Thereafter, Japan witnessed practically unbroken, albeit moderate, decreases until 2004. By 2015, Japan faced the third highest gender pay gap in the OECD. In Finland, the gender pay gap has stabilized over the last decade or so. Nevertheless, Finland's gender pay gap in 2015 was at least one third smaller than it was in 1977.

Following a long period of decline, many OECD countries have witnessed a recent rebound in fertility (Luci-Greulich and Thevenon 2014; OECD 2017). The average total fertility rate[1] (TFR) across the OECD declined from 2.7 children per woman in 1970 to below replacement level of 2.1 since 1982, bottomed out at 1.6 in 2002 and then edged up (OECD 2017). Figure 2 shows trends in the TFR for the same countries as in Figure 1. The United Kingdom and the United States witnessed

[1] The period total fertility rate indicates the average births per woman in her lifetime if she were to experience, throughout her lifetime, the current age-specific fertility rates observed in a given year.

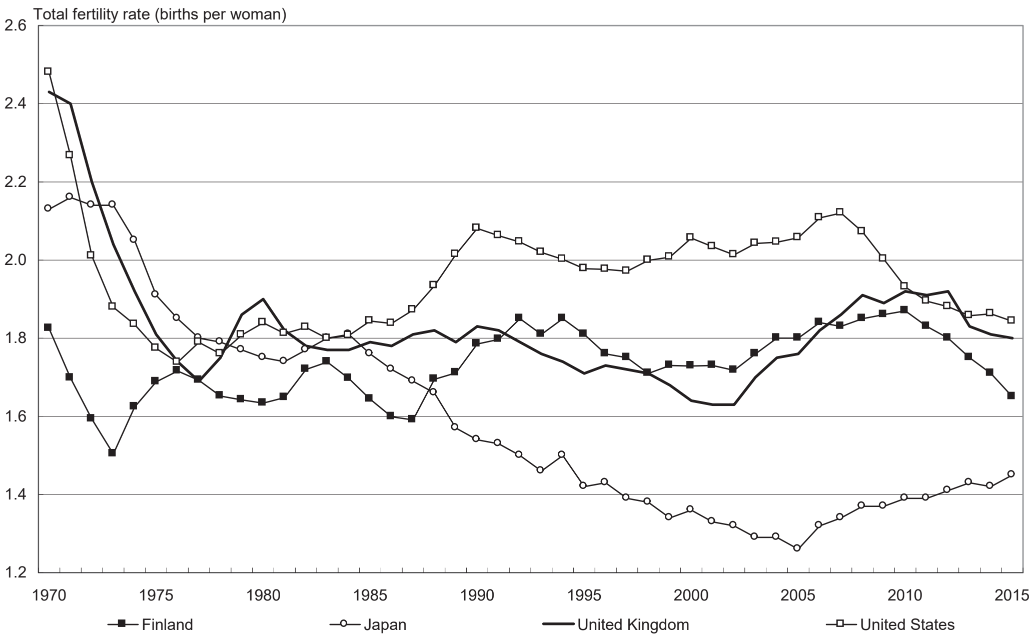

Figure 2: Total fertility rate, selected OECD countries, 1970-2015. *Source: OECD (2017)*

similar declines in the TFR during the 1970s. Since 2002, the TFR increased by approximately 0.3 children per woman in the United Kingdom up to 2012. In the United States, the TFR increased by a similar amount between the mid-1980s and early 1990s, and oscillated near replacement level until the late-2000s. In Finland, the decline in TFR started early and was less evident in the 1970s. Finland's TFR rebounded from around 1.6 in the late 1980s and oscillated around 1.8 since the mid-1990s. In contrast, Japan's TFR continued to decline until 2004.

Intuitively, closing the gender wage gap has had opposing effects on household fertility. On the one hand, children have become more affordable. On the other hand, children have become more costly if unpaid maternal time was used in child rearing. The effect of rising female wages on the child rearing cost and thus price of children would have depended on the intensity with which maternal time was used. Consider the effect of a 10% rise in female wages if unpaid maternal time was the sole child rearing input, as in Galor and Weil (1996). If female wages rose by 10%, the cost of rearing children would have also risen by 10%. However, household wages would have risen by less than 10%. If the negative effect of increased cost outweighed the positive effect of increased affordability of children then fertility would have declined.

Empirical evidence has supported competing income and substitution effects on the demand for children. For 27 European countries, Hondroyiannis (2010) has found a positive income effect from a rise in per capita GDP and negative substitution effect from a rise in real wages. Salamaliki (2017) has found evidence of a positive income effect on the birth rate in Greece. A priori, fertility decline could have reversed with rising female relative wages if the positive income effect of a less than 10% rise in household wages outweighed the negative substitution effect. This could have occurred where the income elasticity of demand for children was high or the rise in cost of child rearing was less than 10%.

A priori reasoning has suggested that paid maternity leave and substitution to purchased child care could have mitigated the rising cost of child rearing which has underpinned fertility decline. Apps and Rees (2004) and Martinez and Iza (2004) have shown that closing the gender wage gap could raise fertility when households can substitute purchased child care for maternal time.[2]

The price of child care has been constant in these models, as commonly assumed in endogenous fertility models where child care has been a substitute for maternal time (Hirazawa and Yakita 2009), and maternal time used in child rearing has been unpaid. However, evidence has suggested that subsidies to contain rising child care prices and paid maternity leave have influenced whether a country at an advanced stage of development has had low fertility or close to replacement fertility (Thevenon 2011).

In contrast to the existing literature, the model presented in this paper has analyzed the effect on fertility of an increase in the level and logarithm of per capita output, while having incorporated the role of child care affordability, paid maternity leave and inherent preference to use maternal time in childrearing. We have provided

[2] Apps and Rees (2004) were concerned with the effects of taxation and family payments rather than economic growth. Martinez and Iza (2004) analyzed the evolution of capital per person and fertility over time where maternal time and child care were perfect substitutes.

a theoretical basis for observing a reversal in the relationship between fertility and logarithm of per capita output. Existing theoretical models have predicted that fertility must decline over time with economic growth due to an inevitable trade-off between unpaid time spent rearing children and paid work. In contrast, we have shown that whether the effect of economic growth on fertility reverses from a negative to a positive depends on the gender wage gap, family policy and cultural norms, reflected in the value placed on having children and substitutability of child care for maternal time in rearing children. We have further extended our model to incorporate parental value from the quality of children, where education is regarded as investment in quality, and cost sharing among children in child rearing. A particularly interesting prediction of the extended analysis has been that the child quantity-quality trade off need not apply once fertility decline has reversed.

Review of related empirical literature

Using cross country and longitudinal data for more than 100 countries, Myrskyla et al. (2009) found evidence of an inverse J-shaped association between Human Development Index (HDI)[3] and TFR, where the negative association reversed to a positive as HDI approached 0.95. The threshold HDI value was initially identified from a cross country plot of logarithmic scaled HDI and TFR in 2005. Furuoka (2009) contested this approach and, using a threshold regression analysis, found that the negative effect on TFR of an increase in the level of HDI weakened at a HDI of 0.77, but did not reverse.

Luci-Greulich and Thevenon (2014) estimated TFR as a quadratic function of the logarithm of per capita output across OECD countries. Their estimated threshold per capita Gross Domestic Product (GDP) of around $30,000[4] in year 2005 Purchasing Power Parity (PPP) US dollars corresponded to Myrskyla et al.'s (2009) HDI threshold of 0.95. Lacalle-Calderon et al. (2017), using a new sample selection and quantile regression approach for OECD and non-OECD countries, confirmed

[3] HDI is a composite index of life expectancy, education and logarithm of per capita output.

[4] Herein, dollar values are in US currency.

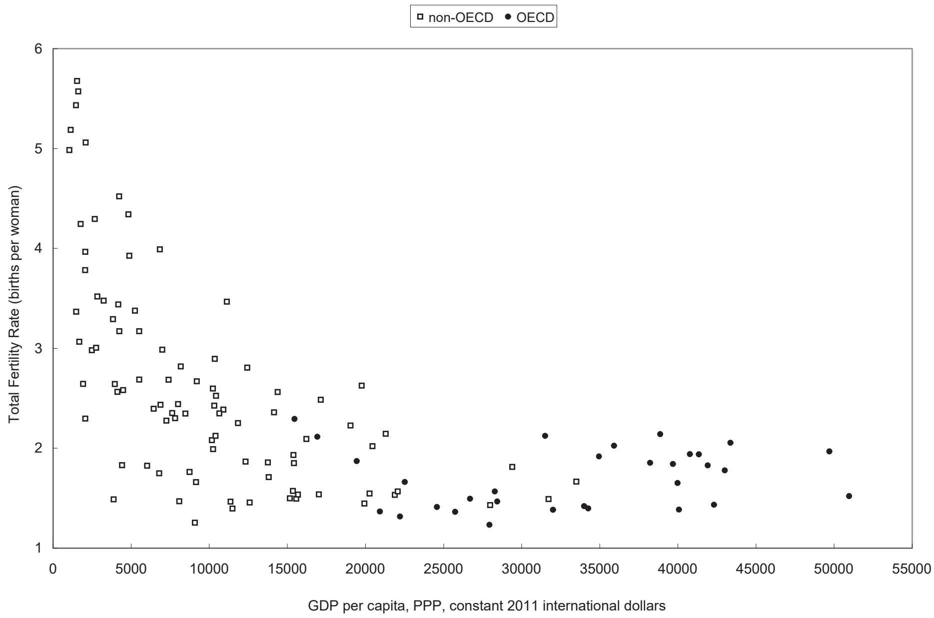

Figure 3: Fertility and Level of Real Gross Domestic Product (GDP) per capita in OECD countries, 2005-2014. *Source: World Bank (2016)*

the existence of an inverse J-shaped relationship between TFR and logarithm of per capita GDP but found that the inverse J-shape depended also on the fertility level.

Figures 3 and 4 depict the significance of logarithmic scaling of per capita output. While Figure 3 shows no discernible association across OECD countries between TFR and level of per capita GDP, a polynomial trend line in Figure 4 suggests a possible U-shaped association across OECD countries between TFR and logarithm of per capita GDP around 10.3.

Empirical studies have provided compelling evidence that supports an inverse J effect of economic growth, measured as an increase in the logarithm of per capita GDP, on TFR. Cross-country regression analyses, after controlling for birth postponement and country-specific effects across OECD countries, found evidence that the effect of an increase in the logarithm of per capita GDP on TFR reversed at logarithm per capita GDP in year 2005 constant US$ around 10.39 (Luci-Greulich and Thevenon 2014). A longitudinal regression analysis of 18 high-income countries from 1970 to 2011 found evidence of a threshold logarithm per capita GDP in year 2005 constant

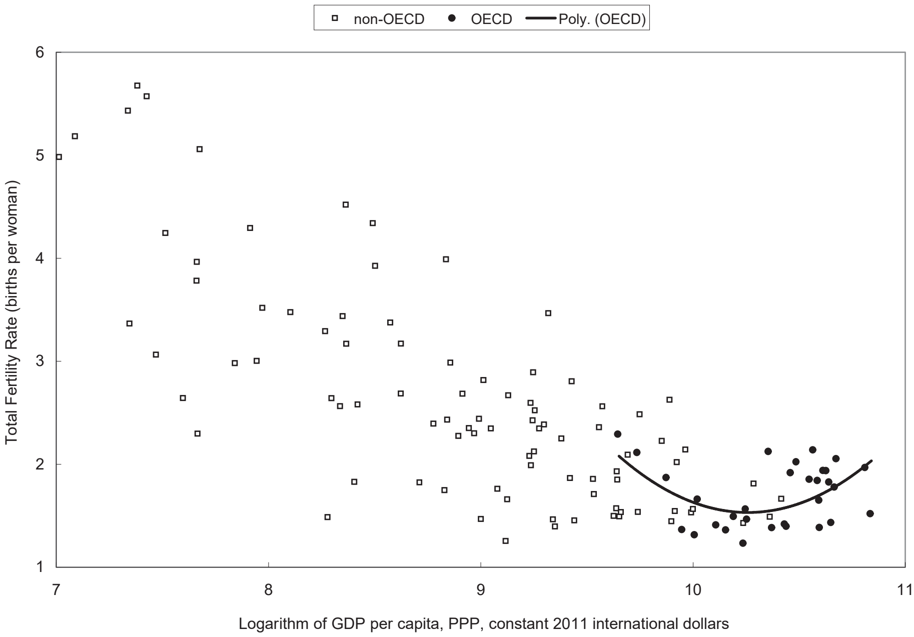

Figure 4: Fertility and Logarithm of Real Gross Domestic Product (GDP) per capita in OECD countries, 2005-2014. *Source: World Bank (2016)*

US$ around 10.38 (Dominiak et al. 2015). Furuoka (2010) employed a threshold regression analysis for the United States from 1960 to 2007 and found evidence that TFR decline reversed at a threshold logarithm of real per capita GDP around 10.01.

More than an emerging U-shaped relationship, Figure 4 reveals two groups of OECD countries with per capita GDP above $30,000 in year 2011 PPP, but with either low fertility or close to replacement fertility. To elaborate, consider Finland, Japan and the United Kingdom with average logarithm of per capita GDP for the period of approximately 10.44, 10.55 and 10.46, respectively. Despite similar logarithm of per capita GDP, the average TFR for the period in Finland and the United Kingdom was much higher than in Japan.

Existing literature has shown that factors other than economic growth contributed to fertility decline reversal and thus whether an OECD country was above the U-shaped curve in Figure 4, as in the case of Finland and the United Kingdom, or below, as in the case of Japan. McDonald (2000) and Luci-Greulich and Thevenon (2014) found that family assistance in the form of paid maternity leave and affordable child care have been important in helping families combine female employment and child rearing. Moreover, Lacalle-Calderon et al. (2017) found that the inverse J-shaped effect of economic growth on fertility has been conditioned by other factors, such as female employment, gender equality or specific cultural norms. We have incorporated these themes in the economic model developed here.

Model of household child rearing and fertility choice

The model presented herein is an economic description of household decisions regarding the number of children and the substitution of purchased child care for maternal time in child rearing. The representative household is headed by a man and woman with joint consumption and utility for the purpose of analyzing gender wage effects.

While couples derive satisfaction from having children, they are constrained by the household budget, meaning household income limits total spending on children and other consumption. There are two aspects to household decision making. First, the couple chooses the mix of maternal time and purchased child care to rear a given number of children. This gives the child rearing cost which determines spending on children in the budget constraint. Second, the household chooses the number of children that maximizes utility subject to the budget constraint.

Economic growth raises mean female wages relative to male wages

The rise in average female wages relative to male wages with growth in per capita output features in the growth models of Galor and Weil (1996) and Martinez and Iza (2004). These models assume that men are endowed with skilled labor and physical labor, whereas women are endowed with skilled labor. Economic growth raises the marginal product of skilled labor but does not affect the marginal product of physical labor. The stylized assumption of ex-ante gender differences in labor endowments yield two predictions. First, female wages are lower than male wages for a given level of per capita output. Second, growth in per capita output raises female and male wages, but female wages rise proportionately more. Thus, growth in per capita output closes but does not eliminate the gender wage gap. In this paper, we retain these two predictions because advanced economies face gender wage gaps which are closing, while abstracting from the stylized assumption of different labor endowments.

The reduced form equations are given by

$$\ln \tilde{w}_f < \ln w_f < \ln w_m \tag{1a}$$

$$\frac{d \ln w_f}{d \ln y} > \frac{d \ln w_m}{d \ln y} > 0 \tag{1b}$$

where w_f and w_m denote the net of tax average real wage paid to women and men, respectively, and y denotes real output per capita. Women's time spent rearing children is subsidized at the rate of maternity pay, m, where $\tilde{w}_f = (1 - m)w_f$. Thus, $\tilde{w}_f$ is the opportunity cost of maternal time per child, which is reduced if women receive maternity pay. The rate of growth or proportionate change in per capita output, female wages and male wages, is given by $d \ln y$, $d \ln w_f$ and $d \ln w_m$, respectively.

Household child care and maternal time use depends on relative costs

Each man and woman is endowed with a unit of time. Men allocate their unit time endowment to the paid labor force. The household allocates women's time to rearing children when the opportunity cost in terms of foregone wages is lower for women than it is for men. Let n denote the number of children. To raise children, women may employ a fraction of their time endowment, $\hat{z} \in (0, 1)$. Female labor supply to the paid workforce is $(1 - \hat{z}n)$, which is decreasing in the total time spent rearing children, $\hat{z}n$, but does not depend linearly on the number of children.

The household may raise the woman's paid labor supply by purchasing child care input, $\hat{x}$, which is subsidized at the rate β where $\tilde{p}_x = (1 - \beta) p_x$ and p_x is the unsubsidized price of child care. The total cost of rearing children is

$$\tilde{w}_f z + \tilde{p}_x x \tag{2}$$

where $z = \hat{z}n$ and $x = \hat{x}n$ denote the total maternal time input and total purchased child care, respectively.

The production function for child rearing is of Constant Elasticity of Substitution (CES) form

$$n = [\alpha z^{\rho} + (1-\alpha)\, x^{\rho}]^{\frac{1}{\rho}}\,; \quad -\infty \leq \rho \leq 1 \tag{3}$$

where ρ determines the elasticity of substitution between maternal time and purchased child care, given by $\sigma = 1/(1-\rho)$, and α is a distributional weight which indicates the relative value placed on the inputs in child rearing $(0 < \alpha < 1)$. This functional form allows explicit analysis without imposing strict assumptions regarding the substitutability of maternal time for purchased child care.

The parameters of equation (3) can be explained intuitively. The CES form contains various assumptions about the substitutability between child rearing inputs as special cases, depending on the value of the parameter ρ. For example, in the special case where $\rho = 1$, maternal time and child care are perfect substitutes and the production function is linear, $n = \alpha z + (1-\alpha)\, x$. In this case, the representative household would use either maternal time and no purchased child care $(z > 0, x = 0)$ or purchased child care and no maternal time $(z = 0, x > 0)$, depending on whether $\tilde{w}_f < \tilde{p}_x$ or $\tilde{w}_f > \tilde{p}_x$, respectively. In the general case where $\rho < 1$, the representative household would use a combination of maternal time and child care. The representative household's choice of child rearing inputs depends on the input prices, value placed on using maternal time, reflected by the parameter α. A high value of α would suggest that child rearing is inherently intensive in maternal time due to social and cultural factors.

A representative household chooses the mix of maternal time and purchased child care to rear a given number of children. That is, the household chooses x and z to minimize (2) subject to (3). The input demand for maternal time and purchased child care per child is, respectively,

$$\hat{z}^*\left(\tilde{w}_f,\tilde{p}_x,\alpha\right) = \left[\left(\frac{\tilde{w}_f}{\alpha^{1/\rho}}\right)^{\frac{\rho}{\rho-1}} + \left(\frac{\tilde{p}_x}{(1-\alpha)^{1/\rho}}\right)^{\frac{\rho}{\rho-1}}\right]^{-\frac{1}{\rho}} \left(\frac{\tilde{w}_f}{\alpha}\right)^{\frac{1}{\rho-1}} \tag{4a}$$

$$\hat{x}^*\left(\tilde{w}_f,\tilde{p}_x,\alpha\right) = \left[\left(\frac{\tilde{w}_f}{\alpha^{1/\rho}}\right)^{\frac{\rho}{\rho-1}} + \left(\frac{\tilde{p}_x}{(1-\alpha)^{1/\rho}}\right)^{\frac{\rho}{\rho-1}}\right]^{-\frac{1}{\rho}} \left(\frac{\tilde{p}_x}{1-\alpha}\right)^{\frac{1}{\rho-1}} \tag{4b}$$

where an increase in the net female wage, $\tilde{w}_f$, reduces demand for maternal time, an increase in the subsidized child care price, $\tilde{p}_x$, reduces demand for child care and an increase in the preference for maternal time in child rearing, α, increases demand for maternal time and reduces demand for child care. Substituting from equation (4) for $z = \hat{z}n$ and $x = \hat{x}n$ in the total cost of rearing children, equation (2), yields the child rearing cost function,

$$g\left(\tilde{w}_f,\tilde{p}_x\right)n = \left[\left(\frac{\tilde{w}_f}{\alpha^{1/\rho}}\right)^{\frac{\rho}{\rho-1}} + \left(\frac{\tilde{p}_x}{(1-\alpha)^{1/\rho}}\right)^{\frac{\rho}{\rho-1}}\right]^{\frac{\rho-1}{\rho}} n \tag{5}$$

where $g\left(\tilde{w}_f,\tilde{p}_x\right)$ can be thought of as the price of child rearing per number of children.

Representative household utility takes the following form

$$V = \gamma \ln n + (1-\gamma)\ln c \tag{6}$$

where c denotes consumption, the price of which is normalized to 1. With c as the numeraire, p_x is the price of purchased child care relative to the price of c. The household derives utility directly from the number of children, where $\gamma \in (0,1)$ captures the relative value placed on children. The natural log function, featured in the existing literature, means that parents have diminishing marginal utility and derive utility from both children and consumption.

The potential income of the household is the product of wages and potential labor supply, $w_f + w_m$. The household budget constraint is therefore

$$w_f + w_m = g\left(\tilde{w}_f, \tilde{p}_x\right) n + c \tag{7}$$

where $g\left(\tilde{w}_f, \tilde{p}_x\right) n$ is spent on children and c is spent on consumption. An increase in the logarithm of per capita output, $\ln y$, affects the household budget in two respects. First, the potential income of the household increases. Second, the child rearing price per pair of children comprises foregone female wages which increase.

A representative household chooses the mix of spending on children and consumption given the constraint on household income. That is, the household chooses n and c to maximize (6) subject to (7), yielding

$$c = (1-\gamma)\left(w_f + w_m\right) \tag{8a}$$

$$n = \frac{\gamma\left(w_f + w_m\right)}{g\left(\tilde{w}_f, \tilde{p}_x\right)} \tag{8b}$$

which tells us that households increase their spending on consumption as wages increase. However, the effect of an increase in wages on demand for children is ambiguous.

Effect of wages on substitution of child care for maternal time

Growth in the net male wage has an unambiguously positive effect on fertility. For small changes, the derivative of the logarithm approximates the percentage change. Thus, the change in fertility in response to *growth* in the net male wage is

$$\frac{\partial n}{\partial \ln w_m} = \frac{\gamma w_m(y)}{g\left(\tilde{w}_f(y), \tilde{p}_x, \alpha\right)} > 0 \tag{9}$$

Intuitively, growth in the after tax male wage has a positive income effect on the demand for children.

Fertility is also increasing in the *level* of the net male wage,

$$\frac{\partial n}{\partial w_m} = \frac{\gamma}{g\left(\tilde{w}_f(y), \tilde{p}_x\right)} > 0 \tag{10}$$

where the magnitude of the partial derivative diminishes as y increases because the child rearing price per pair of children, $g(\tilde{w}_f(y), \tilde{p}_x)$ is increasing in y. This contrasts equation (9), where the partial derivative is both increasing in $w_m(y)$ and decreasing in $g(\tilde{w}_f(y), \tilde{p}_x)$.

Thus, whereas the effect of an increase in the level of wages on fertility diminishes at higher levels of per capita output, an increase in the logarithm of wages continues to affect fertility. This result also holds for female wages, since $\partial n / \partial \ln w_i = w_i(y) \partial n / \partial w_i; i = f, m$. We summarize this discussion with the following remark.

While growth, measured as an increase in the logarithm, of wages affects fertility, the effect of an increase in the level of wages on fertility diminishes as the level of per capita output increases.

In contrast to equation (9), the effect of growth in the net female wage on fertility could be U-shaped. From equation (8b),

$$\frac{\partial n}{\partial \ln w_f} = \frac{[\gamma - (1-m) z^*(\tilde{w}_f(y), \tilde{p}_x, \alpha)] w_f(y)}{g(\tilde{w}_f(y), \tilde{p}_x)} \lesseqgtr 0 \tag{11}$$

Intuitively, growth in the after tax female wage has a positive income effect and competing negative substitution effect on the demand for children. The relative strength of the substitution effect depends on the rate of maternity pay and total maternal time used in child rearing. At low levels of $\ln y$, the substitution effect dominates and an increase in $\ln w_f$ causes an initial decline in fertility. The substitution effect weakens at higher levels of $\ln y$ because the corresponding higher $\ln w_f$ raises the opportunity cost of maternal time. This reduces the child rearing input demand for total maternal time. The income effect dominates and fertility now rises.

An increase in the logarithm of female wages raises female labor supply, but does not necessitate fertility decline. Households may choose to have more children by substituting purchased child care for maternal time in child rearing.

The supply of female labor to the paid workforce is $1 - \hat{z}^*(\tilde{w}_f, \tilde{p}_x) n$. If maternal time input per unit, $\hat{z}$, were fixed, a rise in the supply of female labor to the paid workforce would imply a decrease in n. However, the per unit input demand for maternal time, $\hat{z}^*(\tilde{w}_f, \tilde{p}_x, \alpha)$, falls as w_f rises. A rise in n brought about by growth in w_f is therefore consistent with a fall in total maternal time used to rear children, $\hat{z}^*(\tilde{w}_f, \tilde{p}_x)\, n$, and a rise in the supply of female labor to the paid workforce. This explains recent evidence which suggests that countries with rising female employment are likely to experience a fertility rebound (Luci-Greulich and Thevenon 2014) and that paid maternity leave and affordable child care are important in reconciling women's labor force participation and child rearing (Thevenon 2011).

Referring to (11), there is a threshold logarithm female wage: $z^*(\tilde{w}_f(\bar{y}), \tilde{p}_x, \alpha) = \gamma/(1-m)$. Beyond this threshold, growth in female wages can reverse fertility decline by reducing the total maternal time spent in child rearing so that the income effect dominates and fertility rises. All else equal, the higher the price of purchased child care and the lower the rate of maternity pay and relative utility weight for children, the higher the threshold logarithm of female wages.

There is a threshold logarithm of female wages, below (above) which growth in female wages induces fertility decline (upturn). The threshold is increasing in the subsidized price of purchased child care, $\tilde{p}_x$, and the value placed on maternal time spent rearing children, α, and decreasing in the value placed on children, γ, and the rate of maternity pay, m.

Intuitively, a low value placed on having children due for instance to cultural factors diminishes the positive income effect on demand for children. A lower rate of maternity pay raises maternal time's share of total child rearing costs and strengthens the negative substitution effect on demand for children. Similarly, a higher price of purchased child care raises the input of total maternal time when child care and maternal time are substitutes, thereby reducing the demand for children. This suggests that a threshold across countries may be sensitive to cultural factors and policy.

Thus, fertility rises with growth in male wages, but fertility initially declines and then rebounds with growth in female wages if the logarithm of female wages exceeds a threshold value. Furthermore, growth in per capita output raises female wages proportionately more than male wages. We herein bring together these predictions to analyze the aggregate effect of growth in per capita output on fertility.

Threshold logarithm of per capita output

The total derivative of n with respect to $\ln y$ is

$$\frac{dn}{d\ln y} = \frac{\partial n}{\partial \ln w_m}\frac{d\ln w_m}{d\ln y} + \frac{\partial n}{\partial \ln w_f}\frac{d\ln w_f}{d\ln y} \begin{cases} < 0 & \text{if } \ln y < \ln \bar{y} \\ \\ > 0 & \text{if } \ln y > \ln \bar{y} \end{cases} \tag{12}$$

where $\partial n/\partial \ln w_m > 0$ and $\partial n/\partial \ln w_f \lesseqgtr 0$. The prediction that female wages rise proportionately more than male wages, $d\ln w_f/d\ln y > d\ln w_m/d\ln y > 0$, means that the change in fertility due to growth in female wages, $\partial n/\partial \ln w_f$, receives greater weight. Growth in male wages raises fertility. The aggregate effect of growth in per capita output on fertility depends on whether fertility declines or rises as female wages grow and the proportionate rise in female wages relative to males wages.

There is a threshold logarithm female wage which switches the effect on demand for children from negative to positive. Once the logarithm female wage surpasses this threshold, growth in per capita output must raise fertility because demand for children rises with not only male wages, but also female wages. However, growth in per capita output may raise fertility even if the logarithm female wage is below its threshold. If the growth in female wages is relatively weak, then less weight is given to the negative effect on fertility. Thus, the threshold logarithm of per capita output at which fertility rebounds is sensitive to the growth in female wages relative to male wages.

Child quality and economies of scale in rearing children

The above analysis assumes that the cost of child rearing is a linear function of the number of children and that utility is derived directly from the number of children. This section relaxes these assumptions by considering that the cost of child rearing is a non-linear function of the number of children due to economies of scale and parents care about children's welfare and success which depends on child quality as well as quantity. The extended model incorporates child quality, cost of education and child rearing economies of scale to demonstrate the generalizability of our predictions.

To allow for cost sharing among children, the total cost of rearing children is

$$(\tilde{w}_f \hat{z} + \tilde{p}_x \hat{x})\, n^{\phi}, \quad 0 < \phi \leq 1 \tag{13}$$

where the special case $\phi = 1$ yields equation (2). The analysis herein relaxes this assumption to consider child rearing economies of scale, whereby a second child need not reduce by half the benefit that the first child derives from each hour of care.

A representative household minimizes (13) subject to (3), which yields the input demand for maternal time and purchased child care in equation (4). When $0 < \phi < 1$, total maternal time and total purchased child care used to rear children, $z = \hat{z}n^{\phi}$ and $x = \hat{x}n^{\phi}$, respectively, are non-linear functions of n, as is the child rearing cost function

$$g(\tilde{w}_f, \tilde{p}_x)\, n^{\phi} = \left[\left(\frac{\tilde{w}_f}{\alpha^{1/\rho}} \right)^{\frac{\rho}{\rho-1}} + \left(\frac{\tilde{p}_x}{(1-\alpha)^{1/\rho}} \right)^{\frac{\rho}{\rho-1}} \right]^{\frac{\rho-1}{\rho}} n^{\phi} \tag{14}$$

where the total cost per child, $g(\tilde{w}_f, \tilde{p}_x)\, n^{\phi-1}$ diminishes as the number of children increases. Female labor supply to the paid workforce is $1 - \hat{z}n^{\phi}$.

Taking the lead of Becker et al. (1990), we allow parents to care not only about the number of children (quantity), but also about their education (quality). Consider that a representative household's utility function is given by $U = (nq)^{\gamma}\, c^{1-\gamma}$ where nq denotes the aggregate human capital of children which may proxy success or well-being. Since a natural logarithm of this function represents the same preferences, we may write the representative household's utility as

$$V = \gamma \ln n + \gamma \ln q + (1 - \gamma) \ln c \tag{15}$$

where q denotes the education that each child receives.

The budget constraint facing a representative household is

$$w_f + w_m = g\left(\tilde{w}_f, \tilde{p}_x\right) n^{\phi} + E + c \tag{16}$$

where E is the amount spent educating children. Following Jones and Vollrath (2013), E can be translated into units of education per child, $q = E + \bar{q}$, where $\bar{q}$ captures an inherent amount of education children acquire even if not formally schooled. We find this approach to introducing the cost of education useful for the analysis here which focuses on the inverse J effect of economic growth via rising female relative wages on fertility.

A representative household maximizes (15) subject to (16), yielding

$$c = \frac{(1-\gamma)\left(w_f + w_m + \bar{q}\right)}{1 + \frac{\gamma}{\phi}} \tag{17a}$$

$$q = \frac{\gamma\left(w_f + w_m + \bar{q}\right)}{1 + \frac{\gamma}{\phi}} \tag{17b}$$

$$n = \left[\frac{\gamma\left(w_f + w_m + \bar{q}\right)}{g\left(\tilde{w}_f, \tilde{p}_x\right)(\gamma + \phi)}\right]^{1/\phi} \tag{17c}$$

where (17b) and (17c) tell us that growth in net wages unambiguously raises spending on education per child, whereas fertility may follow a U-shaped pattern of initial decline which then reverses. Child quality is inversely correlated with fertility. However, the inverse correlation breaks down at a threshold logarithm of female wages, beyond which households increase both the number of children and spending on education per child.

From equations (17b) and (17c),

$$\frac{\partial q}{\partial \ln w_f} = \frac{\gamma}{1+\frac{\gamma}{\phi}} w_f(y) > 0 \tag{18a}$$

$$\frac{\partial n}{\partial \ln w_f} = \frac{\left[1-\left(1+\frac{\phi}{\gamma}\right)(1-m)\, z^*\left(\tilde{w}_f(y), \tilde{p}_x, \alpha, \phi\right)\right] w_f(y)\, n}{\phi\left(w_f(y)+w_m(y)+\bar{q}\right)} \lesseqgtr 0 \tag{18b}$$

Intuitively, growth in net female wages has a negative substitution effect on demand for children which dominates a positive income effect, causing an initial decline in fertility. As parents derive utility from the aggregate human capital of their offspring, they trade child quantity for quality. This represents the demographic transition marked by fertility decline and rising education. However, the negative substitution effect on demand for children weakens at a higher logarithm female wage because the high opportunity cost of maternal time reduces demand for maternal time in child rearing. The income effect dominates and fertility rises, which together with a rise in child quality yields higher parental utility.

Referring to (18b), there is a threshold logarithm female wage corresponding to

$$z^*\left(\tilde{w}_f(y), \tilde{p}_x, \alpha, \phi\right) = \frac{\left(\gamma/\left(\gamma+\phi\right)\right)^{1/\phi}}{(1-m)}$$

above which growth in female wages reverses fertility decline by reducing total time spent rearing children so that the income effect dominates. The threshold is increasing in the value of ϕ. Thus, economies of scale in child rearing, captured by $0 < \phi < 1$, lowers the threshold logarithm female wage above which growth in female wages induces fertility upturn. Intuitively, less total maternal time and total purchased child care is used to rear any given number of children, which weakens the negative substitution effect of growth in female wages and thus demand for children increases at a lower threshold logarithm female wage.

Conclusion

The question of whether the negative effect of economic growth on fertility reverses to a positive across advanced economies is one of social and economic importance.

Recent evidence suggests a cross country threshold level of per capita output at which fertility rebounds. This paper explains how the threshold depends on logarithmic scaling and cross country differences in key variables which affect fertility choice.

The model of household fertility and child rearing choice presented in this paper explains that economic growth may reverse fertility decline through growth in female wages relative to male wages. Fertility is increasing in household wages and the value placed on children, and decreasing in the price of rearing children, which increases in female wages net of maternity pay, the subsidized price of child care and the value placed on maternal time spent rearing children. Growth in male wages raises fertility, but fertility declines and may then rebound with growth in female wages because rising foregone female wages encourage couples to reconcile paid female employment with having children by substituting between child care and maternal time to minimize the cost of rearing children. Our theoretical explanation for how growth in female relative wages may reverse fertility decline is robust to relaxing the assumptions that parents derive utility directly from the number of children rather than indirectly from the aggregate human capital of their offspring and that total maternal time and total purchased child care are linear functions of the number of children.

The analysis yields the following predictions. First, while the effect of the level of wages on fertility diminishes as per capita output increases, the logarithm of wages continues to affect fertility. Second, economic growth may reverse fertility decline beyond a threshold logarithm of per capita output that depends on the proportionate growth in female and male wages. Third, in response to an increase in the logarithm of female wages, paid female labor supply rises, but the negative effect on demand for children becomes positive at a threshold logarithm of female wages, which is increasing in the subsidized price of purchased child care and decreasing in the rate of maternity pay and the preference for children. Fourth, child quality is inversely correlated with fertility while growth in female relative wages induces fertility decline, although demand for child quality may continue to rise after fertility decline reverses. Finally, economies of scale, whereby the total cost of child rearing increases at a diminishing rate with the number of children, lower the threshold logarithm female wage above which growth in female wages induces fertility.

The first prediction explains why empirical studies which use the logarithm of per capita output find evidence of a reversal in the effect on fertility at advanced stages of development. The second prediction suggests that the relative growth in female and male wages is an important consideration for future empirical research. The third prediction explains why OECD countries with rising female employment rates also experience fertility rebound and that the threshold logarithm level of per capita output may vary across countries due to child care prices, policies that contain child rearing costs and cultural differences. The fourth prediction fits the demographic transition marked by fertility decline and rising education with economic development and suggests a possible breakdown in the inverse correlation between fertility and child quality at advanced stages of development. The final prediction explains how cost sharing among children could contribute to persistent fertility decline in OECD countries with very low fertility and earlier onset of fertility rebound from relatively high rates in other OECD countries.

A comparison of family policies and fertility trends across OECD countries supports the prediction that the threshold logarithm level of per capita output may vary due to policies that help reconcile female labor force participation with child rearing. Luci and Thevenon (2012) provide a country typology that compares child care enrolment and public spending on family benefits, including parental leave and child care subsidies, with the OECD average. Nordic countries outperform other OECD countries in providing comprehensive support to working parents with children under school age. English-speaking countries provide comparatively less in-kind support, while financial support is greater but targeted at low-income families. By contrast, public spending on family benefits in Asian countries with the lowest fertility is below the OECD average. The variation in gender wage gaps in combination with policies across Anglophone, Nordic and Asian OECD countries corresponds with fertility trends in the United States and the United Kingdom, Finland and Japan, respectively.

The model predictions also identify cultural differences as an important element of fertility rebound. With respect to the preference for children, cultural differences could include government controls such as the one-child policy in China and religious beliefs which restrict abortions and contraceptive use. The average total fertility rate in countries with restrictive abortion policies is approximately 40% higher than in countries with liberal policies and countries such as Ireland where approximately 65% of women use contraception, compared with 84% in the United Kingdom, have near replacement fertility (United Nations, 2014).

Regarding inherent willingness to substitute for maternal time, cultures which preserve an outdated male breadwinner model where women must choose between child bearing and work risk a low fertility trap. The assumption in our model that female wages are on average lower than male wages captures the real world gender wage gap and does not suggest that women by default should choose between child rearing and work. On the contrary, we explain that paid maternity leave and substitution of maternal time for subsidized child care offer female-male couples a means by which they can reconcile women's careers and family, thereby enabling society to reverse fertility decline.

This study provides a comprehensive analysis of how gender wages, family policy, cultural willingness to substitute for maternal time, child quality and childrearing economies of scale influence the J-shaped effect of economic growth on fertility. Nonetheless, the study has limitations with respect to the role of endogenous family policy and trends in one-parent, same-gender parents and childless couples over time.

As with gender pay gaps, policies have changed over time. Family policy and parents' inherent willingness to substitute for maternal time may be endogenous in the sense that they are influenced by the overall fertility rate of society. We may reasonably intuit, for instance, that Finland's earlier onset of decline in the total fertility rate to a low of 1.5 in 1974 helped motivate family friendly policies.

In focusing on how growth in female relative wages could reverse fertility decline, we do not deny the importance of emerging demographic trends, such as one-parent families, same-gender two-parent families and couples who may not desire children. While the proportion of children living in households with two parents is more than 80% in most OECD countries, the rate is less than 70% in the United States (OECD, 2017). The higher portion of sole-parents in the United States has interesting implications for the role of policies to contain child rearing costs in fertility decline reversal. While the United States lacks paid parental leave, public spending on family benefits targets low-income families including sole-parents and thus could help explain fertility decline reversal. Moreover, as this paper has highlighted the importance of policies to help working female-male couples in reversing fertility decline as gender pay gaps close, policies should evolve to contain child rearing costs for same-gender couples. Similarly, the high proportion of childless couples in OECD countries with very low fertility raises the question of whether couples choose not to have children because they cannot reconcile work with childrearing or because they do not derive utility from children as an interesting direction for future research.

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