Australian National University

Crawford School of Public Policy

CAMA

Centre for Applied Macroeconomic Analysis

Changes in the Inflation Target and the Comovement between Inflation and the Nominal Interest Rate

CAMA Working Paper 30/2019
March 2019

Yunjong Eo
School of Economics, The University of Sydney
Centre for Applied Macroeconomic Analysis, ANU

Denny Lie
School of Economics, The University of Sydney
Centre for Applied Macroeconomic Analysis, ANU

Abstract

Does raising an inflation target require increasing the nominal interest rate in the short run? We answer this question using a standard New Keynesian model with rich backward-looking elements. We first analytically show that the short-run comovement between inflation and the nominal interest rate is less likely to be positive, all else equal, as the monetary authority reacts more aggressively to the deviation of inflation from its target or as more backward-looking elements are incorporated into the model. Meanwhile, features of the model that enhance forward-looking behavior, such as partial price indexation to the inflation target or a lower degree of price rigidity, are shown to help increase the likelihood of positive comovement. However, we find that this so-called Neo-Fisherism is most likely to hold even with a significant degree of backward-lookingness in the model, unless the monetary authority reacts to inflation in an extremely aggressive manner, close to strict inflation targeting. In addition, we estimate New Keynesian models of the U.S. economy and confirm our results that the U.S. economy exhibits Neo-Fisherism: raising the inflation target necessitates a short-run increase in the nominal interest rate. This finding is robust to empirically-plausible parameterizations of the model and to the specification of price indexation to the inflation target in firms' price-setting process.

Keywords

Neo-Fisherism, inflation expectations, a Taylor-type rule, strict inflation targeting, hybrid NKPC

JEL Classification

E12, E32, E58, E61

Address for correspondence:

(E) cama.admin@anu.edu.au

ISSN 2206-0332

Changes in the Inflation Target and the Comovement between Inflation and the Nominal Interest Rate*

Yunjong Eo† Denny Lie‡

March 25, 2019

Abstract

Does raising an inflation target require increasing the nominal interest rate in the short run? We answer this question using a standard New Keynesian model with rich backward-looking elements. We first analytically show that the short-run comovement between inflation and the nominal interest rate is less likely to be positive, all else equal, as the monetary authority reacts more aggressively to the deviation of inflation from its target or as more backward-looking elements are incorporated into the model. Meanwhile, features of the model that enhance forward-looking behavior, such as partial price indexation to the inflation target or a lower degree of price rigidity, are shown to help increase the likelihood of positive comovement. However, we find that this so-called Neo-Fisherism is most likely to hold even with a significant degree of backward-lookingness in the model, unless the monetary authority reacts to inflation in an extremely aggressive manner, close to strict inflation targeting. In addition, we estimate New Keynesian models of the U.S. economy and confirm our results that the U.S. economy exhibits Neo-Fisherism: raising the inflation target necessitates a short-run increase in the nominal interest rate. This finding is robust to empirically-plausible parameterizations of the model and to the specification of price indexation to the inflation target in firms' price-setting process.

JEL Classification: E12; E32; E58; E61;

Keywords: Neo-Fisherism; inflation expectations; a Taylor-type rule; strict inflation targeting; hybrid NKPC

---

*First Draft: July 2018. We thank Saroj Bhattarai, Jinill Kim, Mariano Kulish, Jae Won Lee, James Morley, Dilhan Perera, Dan Rees, and conferences and seminars participants at University of Sydney, Society for Nonlinear Dynamics and Econometrics Conference, Asian Meeting of Econometric Society, and Sydney Macro Reading Group Workshop. Andrew Gaffney provided valuable research assistance.

†School of Economics, The University of Sydney, NSW 2006, Australia; Centre for Applied Macroeconomic Analysis, The Australian National University; Email: yunjong.eo@sydney.edu.au

‡School of Economics, The University of Sydney, NSW 2006, Australia; Centre for Applied Macroeconomic Analysis, The Australian National University; Email: denny.lie@sydney.edu.au

1 Introduction

1.1 Overview

Since the global financial crisis (GFC) and the subsequent Great Recession, nominal interest rates in the U.S. and other developed economies have been persistently lower than before the GFC. Inflation rates, however, have also continued to be low in these economies and in some inflation-targeting economies, have been below the targets. This low inflation, low interest rate environment presents a challenge for central banks. When the short-term nominal interest (policy) rate is low, the central bank's ability to use its conventional monetary policy tool and cut its policy rate during a recession or an economic downturn is more limited. As shown by Kiley and Roberts (2017), in such an environment the frequency and length of hitting the effective lower bound (ELB) on the nominal interest rate are higher, and this may lead to poorer economic performance associated with inflation and economic activity being more volatile and systematically falling short of their desirable levels. To alleviate these concerns, several alternative policy frameworks have been proposed. One such framework is for an inflation-targeting central bank to simply raise its inflation target, as proposed by Blanchard, Dell'Ariccia and Mauro (2010), Ball (2014), and Krugman (2014), among others.[1]

Raising the inflation target, especially in a low interest rate environment, in turn poses a substantive policy question: Does raising the target entail an increase in the nominal interest rate? This question has an important policy implication because if a higher inflation target entails a reduction in the nominal rate, policy implemented to avoid the ELB may, in fact, result in hitting the ELB. The answer to this question is relatively clear in the long run. From the Fisher equation,

$$i_t = E_t\pi_{t+1} + r_t \tag{1}$$

where i_t is the nominal interest rate, r_t is the real interest rate, and $E_t\pi_{t+1}$ is the one-period ahead expected inflation rate. The nominal interest rate and expected inflation, and

[1]Two other notable alternative frameworks that have been proposed are price-level targeting (e.g., Gaspar, Smets and Vestin (2010), Bernanke (2017), and Williams (2017)) and nominal-income targeting (e.g., McCallum and Nelson (1999), Frankel (2013), and Williams (2016)). For other studies on changing the inflation target, see Williams (2016), Rosengren (2018), and Summers, Wessel and Murray (2018).

hence actual inflation, move together one-for-one in the long run insofar as the classical dichotomy holds, i.e., the long-run real interest rate is independent of nominal variables and is solely determined by macroeconomic fundamentals such as the discount rate and long-run output growth. Building on this long-run relationship, Cochrane (2016) and Williamson (2016) argue that a central bank can raise inflation even in the short run by setting a higher interest rate consistent with an inflation target. They dub this property Neo-Fisherism.

In the short run, however, the answer is not so clear-cut. The presence of nominal frictions such as price and wage rigidities complicates the short-run relationship between the inflation target, inflation, and the nominal interest rate. The comovement between inflation and the nominal interest rate may break down, as nominal shocks (e.g., an increase in the inflation target) have short-run effects on the real interest rate.

1.2 Main findings and contribution

In this paper, we investigate the Neo-Fisherian property, which we define, following Garín, Lester and Sims (2018), as a positive comovement between the nominal interest rate and inflation conditional on a change in the inflation target. We do this first within a prototypical New Keynesian model, where the closed-form analytical solution is readily available, and then in a more general model with rich backward-looking elements, estimated to the U.S. economy. Our main findings and contribution are summarized as follows.

First, we investigate the relationship between Neo-Fisherism and the monetary policy stance as well as the deep parameters in the prototypical New Keynesian model involving forward-looking and backward-looking elements. We show that as the monetary authority reacts more aggressively to the deviation of inflation from its target, inflation and the nominal interest rate are less likely to comove positively following an increase in the inflation target. In addition, backward-looking (forward-looking) elements make them less (more) likely to positively comove in response to the inflation target shock.

However, we find that the model is most likely to exhibit Neo-Fisherism for a range of compelling structural parameter values unless the monetary authority reacts to inflation in an extremely aggressive manner, close to strict inflation targeting. This is true in spite of the presence of rich backward-looking elements in the model.

The intuition behind this finding can be understood from the Fisher equation (1). When the target is raised, the real interest rate decreases contemporaneously irrespective of the value of the inflation reaction coefficient in the policy rule. When the central bank reacts extremely aggressively to the inflation deviation, e.g., under strict inflation targeting, inflation and expected inflation are largely stabilized around the target level. It follows then, from (1), that the nominal interest rate is less likely to increase in the short run, in line with the reduction in the real rate. Allowing for backward-looking elements in the model also enhances the likelihood of a contemporaneous decrease in the nominal interest rate, as these elements reduce the change in expected inflation and cause a larger decrease in the real interest rate following an increase in the inflation target. On the other hand, when the inflation reaction coefficient is low, e.g., a Taylor-rule coefficient of 1.5, agents expect inflation to be less stabilized, which implies that expected inflation jumps more following an increase in the inflation target. This in turn enhances the possibility of short-run comovement between inflation and the nominal interest rate. Here, the increase in expected inflation is sufficiently high to counteract the decrease in the real interest rate. We find that the upper bound of the inflation reaction coefficient that guarantees Neo-Fisherian results is considerably larger than most compelling values found in the literature for the U.S. and other economies. Thus, New Keynesian models with typical parameterizations of a Taylor-type rule and other standard model equations considered in the literature will most likely exhibit Neo-Fisherism.

Second, we confirm our first main finding using an estimated New Keynesian model of the U.S. economy. We allow for rich backward-looking elements in the model, such as habit formation in consumption, price indexation to past inflation, and interest-rate smoothing. Our results show that the U.S. economy exhibits Neo-Fisherism over an empirically-plausible range of parameter values. Despite the presence of rich backward-looking elements, the positive comovement between inflation and the nominal interest rate is only reversed for an implausibly-large value of the inflation reaction coefficient in the policy rule. Conditional on our estimates of the policy rule parameters, even maximum price indexation to past inflation or full habit formation in consumption cannot break down Neo-Fisherism.

Third, we consider a more generalized price indexation scheme in which firms explicitly take into account the current inflation target in their price-setting process. We adopt this indexation specification from Fève, Matheron and Sahuc (2010), who find gradual changes in the inflation target have been a major driving force of business cycle fluctuations in the euro area. This specification leads to a generalized New Keynesian Phillips curve (NKPC) where current inflation depends directly on the inflation target, in addition to past inflation, expected inflation, and the output gap. This indexation scheme enhances the forward-looking effect following a change in the inflation target and increases the possibility of Neo-Fisherism. We compare the model with this generalized NKPC to an otherwise conventional New Keynesian model of the U.S. economy using Bayesian model selection procedures and find that the U.S. data strongly support the model with the generalized NKPC. Our first and second findings, however, do not depend on this indexation scheme and the generalized NKPC specification.[2]

1.3 Related literature

Our paper is closely related to several recent studies in the literature.

We contribute to resolving the discrepancy between theoretical and empirical findings on Neo-Fisherism in New Keynesian models. For example, Ireland (2007), Cogley, Primiceri and Sargent (2010), Castelnuovo (2012), and Uribe (2018) find that a highly persistent or a permanent change in the inflation target leads to a short-run positive comovement between inflation and the nominal interest rate, based on estimated models for the U.S. economy with rich backward-looking elements. Garín, Lester and Sims (2018) meanwhile argue that a modest, empirically plausible degree of backward-looking behavior in the NKPC (through "rule-of-thumb" price setters) can eliminate Neo-Fisherism using a strict inflation targeting rule, even when the monetary authority raises the inflation target almost permanently. Our first main finding provides an answer on why they reach different conclusions. As we discussed previously, the strict inflation-targeting rule considered in Garín, Lester and Sims (2018) overstates the role of the backward-looking component in the NKPC in breaking down Neo-Fisherism and understates the role of the forward-looking effect in inflation expectations formation.

[2] Regarding the robustness of our results in respect of the indexation to the inflation target, see Proposition 4 in Section 3.2 for the analytical result and Section 4.5 for the empirical result.

In addition, the generalized NKPC considered in our paper is closely related to the specification of the NKPC in Uribe (2018). He assumes that inflation and the nominal interest rate are both cointegrated with a permanent monetary policy shock. These specifications of the permanent monetary policy shock and the NKPC are similar to a special case of our model in which the inflation target shifts permanently and prices are fully indexed to the inflation target (i.e., when firms are not allowed to optimally adjust their prices in a Calvo (1983) manner).[3] Based on his estimated model, Uribe (2018) finds that a permanent monetary policy shock leads to an immediate increase in the nominal interest rate, inflation, and output and to a decline in the real rate, both in a structural VAR model and a New Keynesian DSGE model. Those responses of the key macroeconomic variables are similar to our findings in the special case outlined above. Our paper, however, shows that this positive comovement could hold even under a non-permanent shift in the inflation target and with a substantial degree of backward-lookingness in the NKPC. Further to this, we also look at the impact that monetary policy stance has on this comovement.

The finding that strict inflation targeting understates the role of forward-looking elements in inflation expectations formation is also consistent with the implications of Bhattarai, Lee and Park (2014*a*), who consider a purely forward-looking New Keynesian model. They find that inflation almost always overshoots changes in the inflation target for plausible parameterizations in the literature. Our findings show that allowing for backward-looking components in the model under strict inflation targeting may alter their conclusion. However, regardless of the existence of backward-looking components, the monetary policy stance affects inflation expectations formation in the same way. The inflation overshooting identified by Bhattarai, Lee and Park (2014*a*) is equivalent to the positive comovement between inflation and the nominal interest rate in our paper, given that the nominal interest rate reacts positively to the inflation gap—the difference between inflation and its target—in the policy rule. More importantly, they show that a stronger reaction to inflation in a Taylor-type rule decreases the response of inflation, implying that Neo-Fisherian results are less likely.

[3] See Section 2.1 for the generalized NKPC specification.

1.4 Organization

The rest of this paper is organized as follows. Section 2 presents a prototypical New Keynesian model and introduces a generalized NKPC where firms explicitly take into account the inflation target in their price-setting process. Section 3 analytically studies the relationship between Neo-Fisherism and monetary policy as well as several key structural parameters of the model. Section 4 considers a more general model estimated to the U.S. economy, which we use to investigate the short-run comovement between the inflation target, inflation, and the nominal interest rate for a range of empirically plausible parameter values. Section 5 concludes.

2 The prototypical New Keynesian model

We present a prototypical New Keynesian model along the line of the textbook model in Galí (2015). This simple model has a rich enough propagation mechanism for our purpose and it allows us to derive a closed-form analytical solution in Section 3. In particular, we consider a generalized version of a New Keynesian Phillips curve (NKPC) in which firms explicitly take into account the change in the inflation target when setting their prices.[4] This generalized NKPC produces a stronger forward-looking effect, which enlarges the region of the parameter space in which raising the inflation target necessitates a short-run increase in the nominal interest rate. We first introduce the generalized NKPC and then complete the model with an IS curve and two different monetary policy rules: strict inflation targeting and a Taylor-type rule.

2.1 A generalized New Keynesian Phillips curve with an inflation-target adjustment

As in Calvo (1983) and Yun (1996), only a $(1-\theta) \in [0, 1)$ fraction of the firms are allowed to optimally adjust their prices at any given period. Similar to Fève, Matheron and Sahuc

[4]Section 4 provides empirical support for this specification in the U.S. economy. Using a similar specification, Fève, Matheron and Sahuc (2010) find that gradual changes in the inflation target have been a major driving force of business cycle fluctuations in the euro area.

(2010), firms that are not allowed to adjust optimally, with probability θ, simply index their prices to a weighted average of past gross inflation Π_{t-1}, the gross inflation target at time t Π_t^*, and steady-state gross inflation $\bar{\Pi}$:

$$P_t(i) = P_{t-1}(i)\Pi_t^{*\varrho}\left(\bar{\Pi}^{1-\tau}\Pi_{t-1}^{\tau}\right)^{1-\varrho} \tag{2}$$

where $\varrho \in [0, 1]$ is interpreted as the degree of indexation to the current inflation target, τ is associated with the degree of indexation to the first lag of inflation as in Christiano, Eichenbaum and Evans (2005). There are two relevant special cases of the price indexation mechanism in (2). First, when $\tau = 0$ it is equivalent to the price-setting mechanism with indexation to the weighted average of the inflation target and the steady-state inflation rate, and there is no dependence of past inflation in the NKPC. Second, when $\varrho = 0$, we have the standard hybrid NKPC resulting from the indexation mechanism with the steady-state inflation rate and past inflation. We will discuss how the inflation target evolves over time later when we discuss monetary policy.

Each optimizing firm i chooses an identical optimal nominal price, $\tilde{P}_t$, to maximize the expected discounted sum of profits

$$\sum_{s=0}^{\infty} Q_{t,t+s}\theta^s \left[\tilde{P}_t\Pi_{t+s}^{*\varrho}\Pi_{t+s-1}^{\tau(1-\varrho)}\left(\bar{\Pi}^{(1-\tau)(1-\varrho)}\right)^s Y_{t+s}(i) - W_{t+s}(i)L_{t+s}(i)\right] \tag{3}$$

where $Q_{t,t+s}$ is the nominal stochastic discount factor between t and $t+s$.

The resulting first-order condition of the firms' optimal pricing problem and the associated aggregate-price level equation based on the indexation rule (2) make up the pricing block of the model. Taking the first-order approximation of these equations around the steady state leads to a generalized version of the New Keynesian Phillips curve (NKPC) equation[5]:

$$\pi_t - \tau(1-\varrho)\pi_{t-1} - \varrho\pi_t^* = \beta E_t\left[\pi_{t+1} - \tau(1-\varrho)\pi_t - \varrho\pi_{t+1}^*\right] + \kappa y_t \tag{4}$$

[5] The indexation rule in (2) implies that we allow for non-zero steady-state inflation in the log-linearization.

where

$$\kappa = \frac{(1-\theta\beta)(1-\theta)(\sigma+\eta)}{\theta},$$

π_t denotes inflation deviation from its steady state, and y_t denotes the output gap, defined as the log deviation of output from its natural level.[6] The slope of the NKPC κ is a function of structural parameters. Here, $\beta \in (0,1)$ is the discount factor, $\sigma > 0$ is the inverse elasticity of intertemporal substitution, and $\eta \geq 0$ is the inverse Frisch elasticity of labor supply. Equivalently, (4) can be written as

$$\pi_t = \gamma_b \pi_{t-1} + \gamma_f E_t \pi_{t+1} + \tilde{\kappa} y_t + \delta \left(\pi_t^* - \beta E_t \pi_{t+1}^* \right) \tag{5}$$

where

$$\begin{aligned}
\gamma_b &\equiv \frac{\tau(1-\varrho)}{1+\beta\tau(1-\varrho)}, \\
\gamma_f &\equiv \frac{\beta}{1+\beta\tau(1-\varrho)}, \\
\delta &\equiv \frac{\varrho}{1+\beta\tau(1-\varrho)}, \\
\tilde{\kappa} &\equiv \frac{\kappa}{1+\beta\tau(1-\varrho)}.
\end{aligned}$$

When $\tau = 0$ the NKPC above reduces to a purely forward-looking version,

$$\pi_t - \varrho \pi_t^* = \beta E_t \left[\pi_{t+1} - \varrho \pi_{t+1}^* \right] + \kappa y_t. \tag{6}$$

Additional details on the derivation of (4) are presented in Appendix A.

[6] When firms follow the full indexation scheme with $\varrho = 1$ in (2) and the inflation target shifts permanently (i.e., $\phi_{\pi^*} = 1$ in (8)), the NKPC in (4) collapses to $\widehat{\pi}_t = \beta E_t \widehat{\pi_{t+1}} + \kappa y_t$ where $\widehat{\pi}_t = \pi_t - \pi_t^*$, which is the NKPC specification in Uribe (2018). See equation (15) in Uribe (2018) for more details. Note that Uribe (2018) assumes that inflation and the nominal interest rate are cointegrated with a permanent monetary policy shock. Thus, the permanent monetary policy shock considered in Uribe (2018) is equivalent to the permanent shock to the inflation target in our paper. This becomes clear when we consider the Fisher equation in the long-run because raising the nominal interest rate permanently implies in fact raising the inflation target permanently (which should be equal to expected inflation in the long run), given that the long-run real interest rate is solely determined by macroeconomic fundamentals.

2.2 IS curve and monetary policy

The log-linearized version of the New Keynesian model yields the following representation for the IS curve:

$$y_t = \frac{1}{1+h}E_t y_{t+1} + \frac{h}{1+h}y_{t-1} - \sigma^{-1}\frac{1-h}{1+h}(i_t - E_t\pi_{t+1}), \tag{7}$$

where i_t is the nominal interest rate deviation from its steady state and $h \in [0, 1)$ is the internal habit parameter as in Fuhrer (2000).[7] Setting $h = 0$ in (7) results in a standard forward-looking IS curve.

In terms of monetary policy, the inflation target is adjusted as follows:

$$\pi_t^* = \phi_{\pi^*}\pi_{t-1}^* + \epsilon_{\pi^*,t} \tag{8}$$

where π_t^* is the inflation target deviation from steady-state inflation, $0 < \phi_{\pi^*} \leq 1$, and $\epsilon_{\pi^*,t} \neq 0$ when the central bank newly adjusts the inflation target.[8] When $\phi_{\pi^*} = 1$, the inflation target is adjusted permanently, and it is equivalent to shifting its long-run target (steady-state inflation). We consider two different types of monetary policy: (i) strict inflation targeting and (ii) a Taylor-type rule.

Under strict inflation targeting, the monetary policy authority conducts monetary policy in such a way to set inflation to its target:

$$\pi_t = \pi_t^*. \tag{9}$$

Under a Taylor-type rule, the authority adjusts the nominal interest rate according to

$$i_t = \psi_\pi(\pi_t - \pi_t^*).^{9} \tag{10}$$

[7] For now, without any loss of generality, we assume away the technology shock and the preference shock.

[8] This autoregressive specification follows that considered in Cogley, Primiceri and Sargent (2010), Del Negro, Giannoni and Schorfheide (2015), and Bhattarai, Lee and Park (2016).

[9] This interest-rate rule is assumed to respond to the inflation gap only to be comparable to the strict inflation targeting rule. This specification also helps to find an analytical solution when allowing for a backward-component in the NKPC. However, we will consider a Taylor-type rule in which the nominal interest rate responds to the output gap in addition to the inflation gap in Section 4.

Note that strict inflation targeting is a special case of this Taylor-type rule in that the policymaker puts a high weight on inflation such that $\psi_\pi \to \infty$ in (10). Finally, structural parameters are collected in $\Theta = (\phi_{\pi^*}, \psi_\pi, \psi_y, \varrho, \tau, \theta, \sigma, h, \beta, \eta)$.

3 Analytical results based on the prototypical NK model

In this section we analytically show the relationship between Neo-Fisherism and the model's key structural parameters. We first use a strict inflation-targeting rule, which permits a closed-form solution of the model even with the presence of backward-looking elements. We then consider a Taylor-type rule and examine how the comovement between inflation and the nominal interest rate depends on the inflation gap reaction coefficient. Our analytical result will show that strongly reacting to the inflation gap amplify the effect of backward-looking elements of the model on breaking down Neo-Fisherism.

3.1 Monetary policy rule 1: Strict inflation targeting

We first examine how the backward-looking component in the NKPC reduces the possibility of Neo-Fisherism. To focus on this component, we consider a purely forward-looking IS curve by setting $h = 0$ in (7), which results in

$$y_t = E_t y_{t+1} - \sigma^{-1}(i_t - E_t \pi_{t+1}). \tag{11}$$

We will take into account the backward-looking component in the IS curve later. Thus, the model consists of the generalized NKPC (4), the IS curve (11), the evolution of the inflation target (8), and the strict inflation-targeting rule (9). Since inflation always increases in the inflation target under strict inflation targeting, we only need to check the response of the nominal interest rate. The solution for the nominal interest rate is given by

$$i_t = \Psi_0(\Theta)\pi_t^* + \Psi_1(\Theta)\pi_{t-1}^* \tag{12}$$

where $\Psi_0(\Theta)$ and $\Psi_1(\Theta)$ are functions of the structural parameters Θ.[10] Using this solution, we analytically characterize the responses of the nominal interest rate conditional on changes in the inflation target with respect to the structural parameters such as τ, ϱ, ϕ_{π^*}, θ, and σ as follows.

Proposition 1 *Under strict inflation targeting* (9) *with the generalized hybrid NKPC* (4) *and the IS curve* (11)*, the model is more likely to exhibit a comovement between inflation and the nominal interest rate conditional on changes in the inflation target (Neo-Fisherism) as*

(i) the degree of the backward-lookingness in the NKPC is weaker (i.e., τ in (4) *gets smaller),*

(ii) prices are more indexed to the inflation target in price-setting (i.e., ϱ in (4) *gets larger), and*

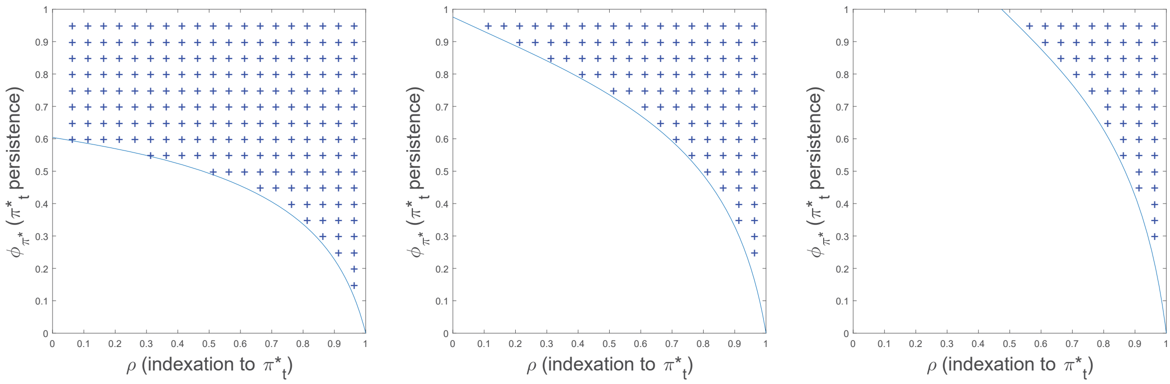
(iii) the change in the inflation target is more persistent (i.e., ϕ_{π^*} in (8) *gets larger).*

Proof. See Appendix B. ■

What is the intuition behind this proposition? Consider first the effect of the backward-looking parameter, τ. Suppose the economy is initially in the steady state and the monetary authority decides to increase the inflation target. Under strict inflation targeting, current inflation is completely stabilized at its target. The hybrid NKPC (5) shows that current inflation is determined by the weighted sum of expected inflation, past inflation, the output gap, and the inflation target. Given expected inflation, a larger value of τ puts more weight on past inflation, which remains at its steady-state level and is unaffected by the inflation-target shock. This in turn requires a larger increase in the output gap to satisfy the given increase in inflation through the NKPC. A larger increase in the output gap is associated with a larger reduction in the real interest rate through the IS curve (11). From the Fisher equation, it follows then a large enough value of τ may lead to a reduction in the nominal interest on impact and result in breaking down Neo-Fisherism. A similar intuition on the

[10] See Lemma 1 in Appendix B for the solution.

Figure 1: The Neo-Fisherian region in the (ϱ, ϕ_{π^*}) parameter space under strict inflation targeting for different degrees of backward-lookingness in the NKPC



(a) $\tau = 0.00$ (b) $\tau = 0.25$ (c) $\tau = 0.50$

Note: The sign of '+' indicates a pair of (ϱ, ϕ_{π^*}) values associated with the positive comovement between inflation and the nominal interest rate conditional on a change in the inflation target. The structural parameter τ is related to the extent to which firms take into account past inflation in setting their prices. See the hybrid NKPC in (4). We set $\beta = 0.99$, $\sigma = 1$, $\eta = 1$, and $\theta = 0.7$.

relationship between backward-looking element in the NKPC and Neo-Fisherism is discussed in Garín, Lester and Sims (2018), under the same strict inflation targeting rule.

On the other hand, as ϱ gets larger, firms are relatively more forward-looking in their price-setting. This results in an NKPC with a higher degree of forward-lookingness, i.e. lower γ_b and higher γ_f and δ in (5). All else equal, the output gap does not have to increase as much to satisfy the NKPC, and hence, the required reduction in the real interest rate in the IS curve is smaller. The presence of price indexation to the inflation target thus enlarges the parameter space associated with Neo-Fisherism. The comovement between inflation and the nominal interest rate is more likely as ϱ gets larger. Notice that the intuition above depends on the extent of the jump in expected inflation. This is where the value of ϕ_{π^*} comes into play. As the inflation target is more persistent, expected inflation jumps higher for a given increase in the target, which raises the possibility of Neo-Fisherism.[11]

To numerically check the impact of these structural parameters on the Neo-Fisherian

[11]Under strict inflation targeting, $E_t\pi_{t+1} = E_t\pi^*_{t+1} = \phi_{\pi^*}\pi^*_t$. Hence, the jump in expected inflation for a given increase in the inflation target only depends on the value of ϕ_{π^*}. Under a Taylor-type rule, however, expected inflation also depends on the values of τ and ϱ, i.e., on the degree of backward- or forward-lookingness in the economy.

relationship, we calibrate the model with $\theta = 0.7$, $\beta = 0.99$, $\sigma = 1$, and $\eta = 1$ throughout this section, unless noted otherwise. These parameter values are associated with the slope of the NKPC $\kappa = 0.26$. Figure 1 presents the Neo-Fisherian regions for $\tau = 0$, 0.25, and 0.50, across the parameter space of (ϱ, ϕ_{π^*}). Thus, Figure 1 shows whether a set of values for structural parameters $(\tau, \varrho, \phi_{\pi^*})$ is associated with a positive comovement between inflation and the nominal interest rate. The other parameters are set to the same values as noted before. Comparing the three panels in the figure, we confirm the results in Proposition 1. In particular, Panel (c) under $\tau = 0.50$ shows that the Neo-Fisherian region shrinks drastically in comparison to Panel (a) under $\tau = 0$. When $\tau = 0.50$, even a nearly permanent increase in the inflation target ($\phi_{\pi^*} \approx 1$) cannot ensure Neo-Fisherism unless the indexation parameter to the inflation target ϱ is roughly greater than 0.5. Thus, under strict inflation targeting, the effect of τ appears to be significant in determining the response of the nominal interest rate on impact to a change in the inflation target.[12]

Proposition 2 *Under strict inflation targeting* (9) *with the generalized hybrid NKPC* (4) *and the IS curve* (11)*, the model is more likely to exhibit a comovement between inflation and the nominal interest rate conditional on changes in the inflation target (Neo-Fisherism) as*

(i) the slope of the NKPC is steeper (i.e., κ in (4) *gets larger) and*

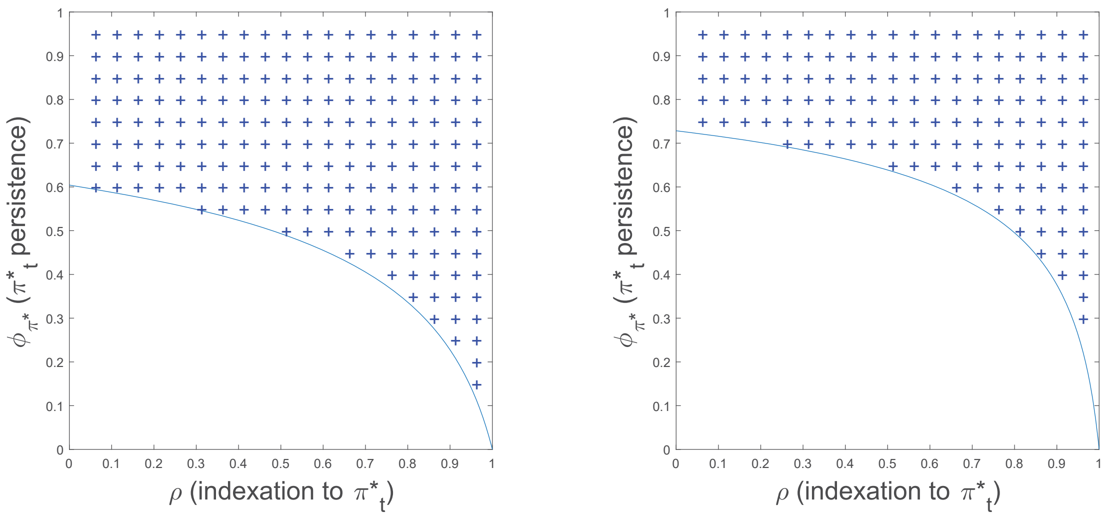
(ii) the elasticity of intertemporal substitution gets higher (i.e., $1/\sigma$ in (11) *gets larger).*

Proof. See Appendix B. ■

Proposition 2 shares a similar intuition and implication to Proposition 1. As the slope of the NKPC gets steeper (i.e. prices are more flexible), inflation is more sensitive to the output gap. In addition, the output gap is more sensitive to the real interest rate as the elasticity of intertemporal substitution gets higher. Therefore, the increase in the inflation target requires a smaller decrease in the real interest rate (given expected inflation), making the model more likely to exhibit Neo-Fisherism.

[12] Note that the coefficient on π^*_{t-1} in (12) increases in τ and hence, this lagged effect can alter the shape of the impulse response of the nominal interest rate conditional on the inflation target shock over time. See Lemmas 1 and 3 in Appendix B.

Figure 2: The Neo-Fisherian region in the (ϱ, ϕ_{π^*}) parameter space under strict inflation targeting for different slopes of the NKPC



(a) $\theta = 0.70$ ($\kappa = 0.26$) (b) $\theta = 0.80$ ($\kappa = 0.10$)

Note: The sign of '+' indicates a pair of (ϱ, ϕ_{π^*}) values associated with the positive comovement between inflation and the nominal interest rate conditional on a change in the inflation target. The Calvo parameter θ is inversely related to the slope of the NKPC κ in (4). We set $\beta = 0.99$, $\sigma = 1$, and $\eta = 1$.

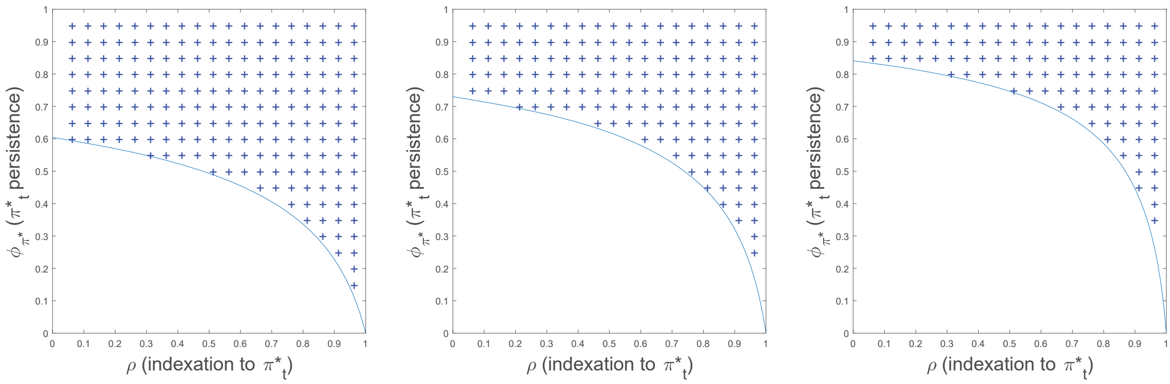
Figure 2 graphically illustrates the positive comovement for the parameter space of (ϱ, ϕ_{π^*}) with two different values of the Calvo parameter θ.[13] Under $\theta = 0.7$ (benchmark), Neo-Fisherism requires a value of ϕ_{π^*} greater than 0.6 when $\varrho = 0$. However, as ϱ increases, the region of the parameter space associated with Neo-Fisherism expands massively. We then consider an alternative value of $\theta = 0.8$, which implies a flatter NKPC slope of $\kappa = 0.10$.[14] As suggested by Proposition 2, the flatter NKPC slope shrinks the Neo-Fisherian region. Similar to the benchmark case of $\theta = 0.7$, however, the model is more likely to exhibit Neo-Fisherism as the value of ϱ increases.

Our results and intuition from the hybrid NKPC case imply that allowing for a backward-looking component in the IS curve as in (7) may also limit the parameter space associated with Neo-Fisherism. To focus on this effect, we set $\tau = 0$ and arrive at the following

[13] We set $\tau = 0$ in generating the figure to solely focus on the effect of the slope of the NKPC.

[14] The flattening of the Phillips curve in the U.S. and other advanced economies since the early 1980s has been documented in various studies, for example, Roberts (2006), Kuttner and Robinson (2010), and Blanchard (2016).

Figure 3: The Neo-Fisherian region in the (ϱ, ϕ_{π^*}) parameter space under strict inflation targeting for different degrees of backward-lookingness in the IS curve



(a) $h = 0.00$ (b) $h = 0.25$ (c) $h = 0.50$

Note: The sign of '+' indicates a pair of (ϱ, ϕ_{π^*}) values associated with the positive comovement between inflation and the nominal interest rate conditional on a change in the inflation target. The structural parameter h is the degree of habit formation in consumption. We set $\beta = 0.99$, $\sigma = 1$, $\eta = 1$, and $\theta = 0.7$.

proposition.

Proposition 3 *Under strict inflation targeting* (9) *with the generalized NKPC* (6) *and the hybrid IS curve* (7)*, the model is more likely to exhibit a comovement between inflation and the nominal interest rate conditional on changes in the inflation target (Neo-Fisherism) as the degree of the backward-lookingness in the IS curve is weaker (i.e., h in* (7) *gets smaller).*

Proof. See Appendix B. ■

Figure 3 confirms the proposition with three different values of $h = 0.0, 0.25, 0.50$ and shows that the Neo-Fisherian region shrinks as the degree of the backward-lookingness in the IS curve gets stronger.

3.2 Monetary policy rule 2: a Taylor-type rule

We now consider a Taylor-type rule (10), which is a more realistic setting to study the possibility of Neo-Fisherism. In particular, we examine the interaction between the inflation reaction coefficient in the Taylor-type rule with the backward-looking component in the NKPC. Our findings under strict inflation targeting in the previous section show that the backward-looking component in the NKPC appears to be more important than that in the IS curve in reducing the Neo-Fisherian region. Thus, this section focuses on the backward-looking component in the NKPC, but we will confirm our findings numerically in a more general setting based on the estimated model of the postwar U.S. economy in the next section. The inflation reaction coefficient ψ_π is restricted to be greater than one to ensure equilibrium determinacy and less than infinity to distinguish a Taylor-type rule in this section from strict inflation targeting.[15]

Under the Taylor-type rule (10), the solutions for inflation and the nominal interest rate are given by

$$\pi_t = \Phi_0(\Theta)\pi_t^* + \Phi_1(\Theta)\pi_{t-1}, \tag{13}$$

$$i_t = \Gamma_0(\Theta)\pi_t^* + \Gamma_1(\Theta)\pi_{t-1}, \tag{14}$$

where the coefficients $\Phi_0(\Theta)$, $\Phi_1(\Theta)$, $\Gamma_0(\Theta)$, and $\Gamma_1(\Theta)$ are all functions of the structural parameters Θ.[16] Based on these analytical solutions, we arrive at the following proposition.

Proposition 4 *Under the Taylor type-rule* (10) *with the IS curve* (11) *and the generalized hybrid NKPC* (4)*,*

(i) inflation always increases in the inflation target,

(ii) the model is least likely to exhibit a comovement between inflation and the nominal interest rate conditional on changes in the inflation target (Neo-Fisherism) under strict inflation targeting compared to a Taylor-type rule, all else equal, and

(iii) (i) and (ii) still hold when $\varrho = 0$.

Proof. See Appendix B. ■

Proposition 4 above has an important implication for our assessment on the importance of the backward-looking component in the NKPC. It indicates that assuming strict inflation

[15] Bhattarai, Lee and Park (2014*b*) analytically show that the Taylor principle, under which the nominal interest rate reacts to more than one-for-one to in ation in the long-run, is a necessary and sufficient condition for determinacy in the New Keynesian model with backward-looking elements. Thus, the lower bound for ψ_π is set to one in our analysis. For details on the Taylor principle, see Bullard and Mitra (2002).

[16] See Lemma 2 in Appendix B for the solution.

targeting may overstate the role of the backward-looking component in breaking down Neo-Fisherism and understate the possibility of Neo-Fisherism. Furthermore, as strict inflation targeting is a special case of the Taylor-type rule when $\psi_\pi \to \infty$, Proposition 4 implies that Neo-Fisherism can be less likely as ψ_π gets larger. We will confirm this implication numerically in the next section.

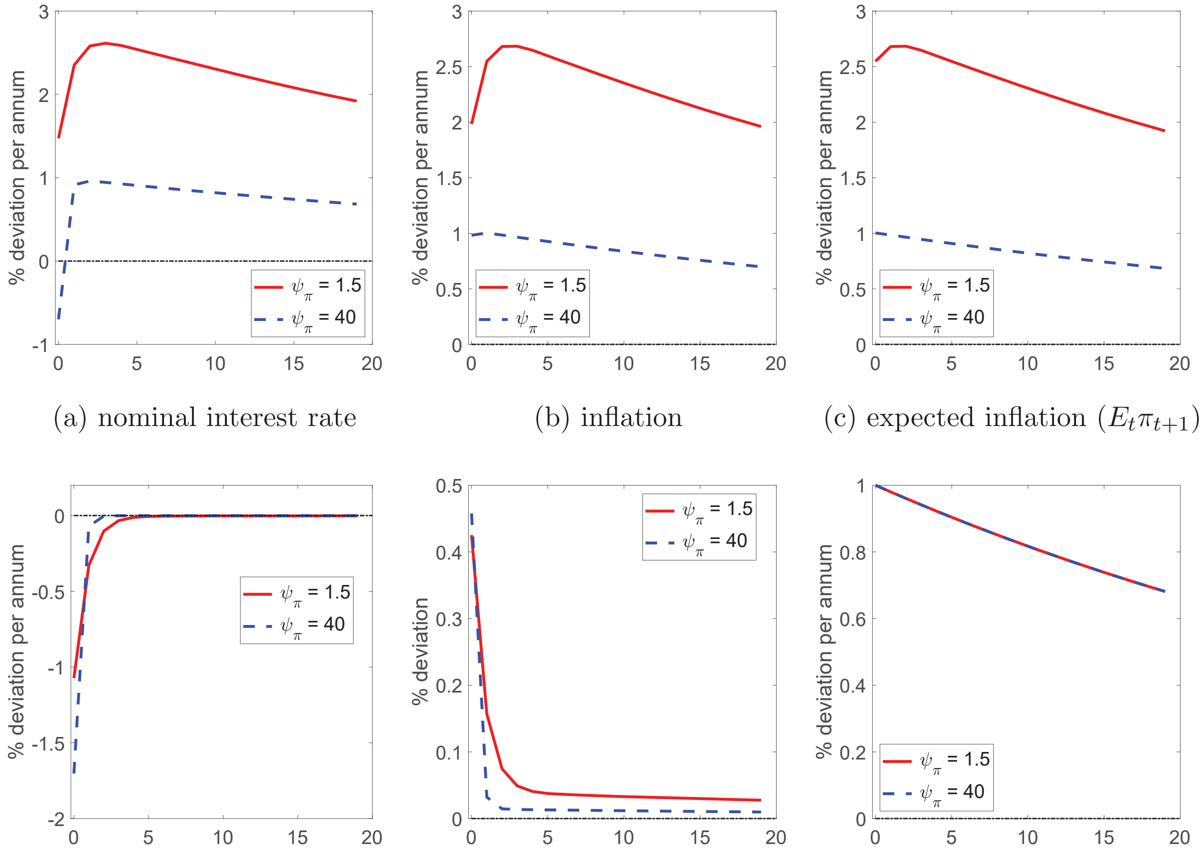
The intuition behind Proposition 4 is as follows. Under strict inflation targeting, inflation and expected inflation are completely stabilized at the inflation target level. Under a Taylor-type rule, however, agents expect inflation to be less stabilized, i.e., expected inflation and inflation jump more following the increase in the inflation target. All else equal, and given the Fisher equation, this higher jump in expected inflation makes a contemporaneous increase in the nominal interest rate more likely and increases the possibility of Neo-Fisherism. Note that the comovement also depends on the response of the real interest rate. Following the increase in the inflation target, the real interest rate decreases contemporaneously, both under strict inflation targeting and a Taylor-type rule. The occurrence of Neo-Fisherism thus depends on whether the increase in expected inflation is high enough to counteract the decrease in the real interest rate.

We confirm our intuition using the impulse response functions to an inflation target shock for two different values of inflation reaction coefficient, ψ_π. The benchmark case assumes the standard Taylor-rule coefficient of $\psi_\pi = 1.5$ as in Taylor (1993), while we set a higher value of $\psi_\pi = 40$ in the second case to mimic strict inflation targeting. We set $\varrho = 0$ to shut down the effect of the indexation scheme to the inflation target and further assume $\tau = 0.5$. This large value of τ implies a degree of backward-lookingness in the NKPC that is close to the highest value for the U.S. economy reported in Galı and Gertler (1999).[17] The persistence parameter of the inflation target shock is set to $\phi_{\pi^*} = 0.98$, which is our posterior mean estimate for the U.S. economy presented in the next section. Other parameters are set as follows: $\beta = 0.99$, $\sigma = 1$, $\eta = 1$, and $\theta = 0.7$.

Panel (a) of Figure 4 shows that in spite of the presence of a significant degree of backward-looking component in the NKPC, the nominal interest rate increases on impact

[17] The degree of backward-lookingness in our NKPC (γ_b) is 0.33, when $\tau = 0.5$, $\rho = 0$, and $\beta = 0.99$. This is in line with the upper estimate of γ_b in Galı and Gertler (1999) — see their Table 2.

Figure 4: Impulse response functions to an inflation target shock for different values of the inflation reaction coefficient



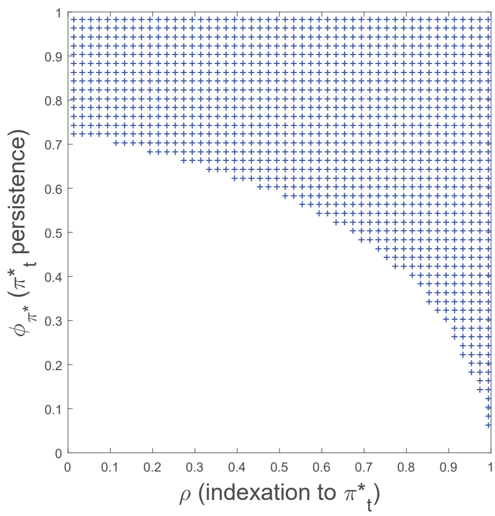
(a) nominal interest rate (b) inflation (c) expected inflation ($E_t\pi_{t+1}$)

(d) real interest rate (e) output gap (f) inflation target

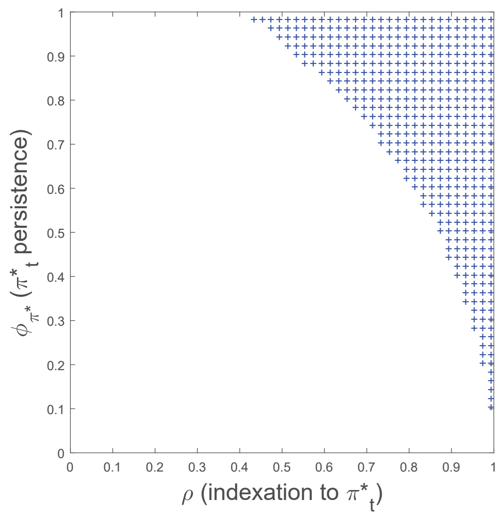
Note: The figure plots the impulse response functions of selective variables to a 1% per annum inflation target shock for different values of the inflation reaction coefficient in a Taylor-type rule ψ_π. The high value of $\psi_\pi = 40$ is chosen to mimic strict inflation targeting. Other parameters are set to $\tau = 0.5$, $\varrho = 0$, $\phi_{\pi^*} = 0.98$, $\beta = 0.99$, $\sigma = 1$, $\eta = 1$, and $\theta = 0.7$.

when $\psi_\pi = 1.5$. On the other hand, when $\psi_\pi = 40$, raising the inflation target necessitates a contemporaneous decrease in the nominal interest rate. Panels (b) and (c) of Figure 4 confirm that both expected inflation and inflation increase larger when $\psi_\pi = 1.5$ compared to the case when $\psi_\pi = 40$. Here, expected inflation rises by 2.5% on impact following a 1% increase in the inflation target, while it increases almost one-to-one (0.98%) with the inflation target when $\psi_\pi = 40$. Notice that the real interest rate decreases by less on impact for smaller ψ_π, which also contributes to the occurrence of Neo-Fisherism when $\psi_\pi = 1.5$.

Figure 5: The Neo-Fisherian region in the (ϱ, ϕ_{π^*}) parameter space under a Taylor-type rule for different values of ψ_π



(a) $\psi_\pi = 1.5$



(b) $\psi_\pi = 40$

Note: The sign of '+' indicates a pair of (ρ, ϕ_{π^*}) values associated with the positive comovement between inflation and the nominal interest rate conditional on a change in the inflation target. The parameter ψ_π is the reaction coefficient to the inflation gap in the Taylor-type rule. The high value of $\psi_\pi = 40$ is chosen to mimic strict inflation targeting. Other parameters are set to $\tau = 0.5$, $\beta = 0.99$, $\sigma = 1$, $\eta = 1$, and $\theta = 0.7$.

In sum, the nominal interest rate under $\psi_\pi = 1.5$ increases by 1.4% on impact, but under $\psi_\pi = 40$ it decreases by about 0.7%.

The implication that strict inflation targeting may understate the forward-looking effect is consistent with the finding in Bhattarai, Lee and Park (2014*a*), who consider a purely forward-looking New Keynesian model with the same monetary policy rule as ours. They find that inflation almost always overshoots the inflation target for reasonable parameterizations in the literature. More importantly, they show that a stronger reaction to inflation in a Taylor-type rule decreases the response of inflation to the inflation target shock.[18] While their model lacks any backward-looking component, the key intuition is applicable to our analytical findings. In a way, strict inflation targeting understates the role of the forward-looking element and overstates the role of the backward-looking element in the NKPC in forming inflation expectations.

[18]See Propositions 1 and 2 in Bhattarai, Lee and Park (2014*a*) for more details.

We next examine how the inflation reaction coefficient ψ_π affects the Neo-Fisherism region across the parameter space of (ϱ, ϕ_{π^*}), using the same values of ψ_π above. Other parameter values are set as noted previously. Figure 5 shows that Neo-Fisherian region is markedly smaller under $\psi_\pi = 40$. In contrast, under the standard Taylor-rule coefficient of $\psi_\pi = 1.5$, even when there is no indexation to the inflation target ($\varrho = 0$), there is a positive comovement between inflation and the nominal interest rate as long as the persistence of the inflation-target shock is roughly greater than 0.7. This corresponds to a half-life of the inflation-target shock of only 1.9 quarters for the model to exhibit Neo-Fisherism, even with a substantial degree of backward-lookingness in the NKPC.

4 Comovement between inflation and the nominal interest rate in the U.S. economy

Our analytical finding in the previous section that a reasonably-parameterized, prototypical New Keynesian model with a hybrid NKPC and a Taylor-type rule is likely to exhibit Neo-Fisherism naturally raises a question: Does the finding still apply when the model is extended to include richer backward-looking components such as habit formation and interest-rate smoothing? In this section, we answer this question using an estimated New Keynesian model of the U.S. economy.

4.1 Model and structural parameter values

The model considered in this section consists of (i) IS curve with habit formation in (7), (ii) the generalized NKPC with backward- and forward-looking elements in (4), (iii) the inflation-target adjustment in (8), and (iv) a Taylor-type rule with interest-rate smoothing,

$$i_t = \phi_i i_{t-1} + (1 - \phi_i) \left[\psi_\pi(\pi_t - \pi_t^*) + \psi_y y_t\right] + \epsilon_{i,t}, \tag{15}$$

where $\epsilon_{i,t}$ is the monetary policy shock.

The model is fitted to the U.S. data with the time unit of one quarter using four observables: GDP per capita, CPI inflation, the federal funds rate, and the ten-year inflation

Table 1: U.S. economy: Model Parameters and Comparison

	Unrestricted (Benchmark)	Restricted ($\varrho = 0$)	Description
β	0.99	0.99	Quarterly discount rate
σ	1.60	1.60	Preference parameter
h	0.64	0.65	Habit formation
κ	0.11	0.11	NKPC slope
τ	0.26	0.05	Indexation to past inflation
ϱ	0.76	0.00	Indexation to the inflation target
ψ_π	1.46	1.45	Inflation coefficient in the interest rate rule
ψ_y	0.42	0.42	Output coefficient in the interest rate rule
ϕ_i	0.90	0.90	Smoothing parameter in the interest rate rule
ϕ_{π^*}	0.98	0.98	Persistence of the inflation target shock
$\ln p(Y\|\mathcal{M})$	-444.12	-446.25	Log marginal likelihood
$Pr(\mathcal{M}\|Y)$	0.89	0.11	Posterior model probability

Note: The values for the structural parameters except for β are from the posterior mean estimates for the U.S. economy with the sample period of 1982:Q1 to 2009:Q2.

expectations from the Blue Chip Economic Indicators survey and the Survey of Professional Forecasters. Del Negro and Schorfheide (2005) and Del Negro, Giannoni and Schorfheide (2015) show that including the ten-year inflation expectations helps track historical movements in the time-varying inflation target over time because it contains information on low-frequency inflation movements. The estimated model has four economic shocks, including the inflation-target shock.[19] More details about the model and the Bayesian estimation procedure are contained in Appendix C.

In the benchmark model, we consider the general case of $\varrho \in [0, 1]$. We will later consider a restricted model with $\varrho = 0$ in the estimation to assess if our findings are robust to the specification of the indexation scheme to the inflation target in the NKPC (4) in Section 4.5. Note that the calculated posterior model probabilities and log marginal likelihoods for the two different models presented in Table 1 show that the benchmark model without the restriction of ϱ is strongly supported by the data compared to the restricted model with $\varrho = 0$. For example, the posterior model probability for the benchmark model is 0.89 while

[19]The four shocks are a monetary policy shock added to the Taylor-type rule, an inflation target shock to the evolution of the inflation target, a cost-push shock added to the generalized NKPC, and a preference shock added to the IS curve. The sample period for the estimation ranges from 1982:Q4 to 2009:Q2, excluding the passive monetary policy period and the Volcker-disinflation period as shown in Lubik and Schorfheide (2004) and the zero-lower bound period, in which unconventional monetary policy has been conducted.

that for the restricted model with $\varrho = 0$ is 0.11 only.

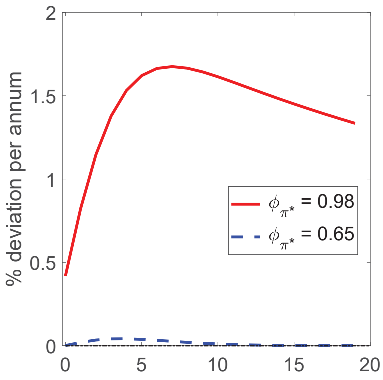
Table 1 presents the structural parameter values. We calibrate the quarterly discount rate β to 0.99. The values of other parameters are set based on the posterior mean estimates. The habit formation parameter and the inverse elasticity of intertemporal substitution are $h = 0.64$ and $\sigma = 1.60$, respectively. The parameter values for the NKPC in (4) are $\tau = 0.26$, $\rho = 0.76$, and $\kappa = 0.11$. These values imply that firms' price-setting behavior is governed by the indexation weights of 0.06 for past inflation, 0.18 for the steady-state inflation rate, and 0.76 for the inflation target. The low value of κ is associated with the flattening Phillips curve in the U.S. economy, which has been documented in various studies, e.g., Roberts (2006), Kuttner and Robinson (2010), and Blanchard (2016), and Proposition 2 in this paper shows that the flat NKPC reduces the possibility of Neo-Fisherism. The Taylor-rule coefficients are $\psi_\pi = 1.46$, $\psi_y = 0.42$, and $\phi_i = 0.90$. The inflation reaction coefficient ψ_π is within the range of compelling values described in Schmitt-Grohé and Uribe (2007) and also consistent with the estimates found in the literature. The inflation target persistence parameter is estimated to be $\phi_{\pi^*} = 0.98$, which is largely consistent with the value for the postwar U.S. economy set in various studies in the literature (e.g. Cogley, Primiceri and Sargent (2010), Del Negro, Giannoni and Schorfheide (2015), and Bhattarai, Lee and Park (2016)).

4.2 Impulse response functions to inflation-target adjustment

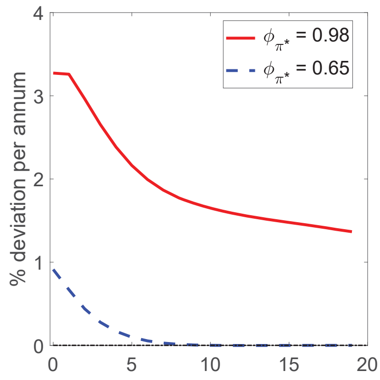
To assess how changes in the inflation target affect various variables in the model, Figure 6 plots the impulse responses to a 1% per-annum inflation target shock. In addition to the estimated $\phi_{\pi^*} = 0.98$, we also consider the case of $\phi_{\pi^*} = 0.65$, which corresponds to the cut-off value—the lower bound of ϕ_{π^*}—for the model to exhibit Neo-Fisherism. All other parameter values are set as in Table 1.

Under the benchmark parameterization with $\phi_{\pi^*} = 0.98$, inflation increases on impact by more than 3% per annum in response to a 1% increase in the target. This positive inflation gap—the gap between inflation and the inflation target—is due to a higher expected inflation, caused by the persistent (but temporary) increase in the inflation target. Inflation remains elevated well above the initial target even after 20 quarters, and so does the nominal interest rate. Associated with the increase in the inflation target, the monetary authority needs to

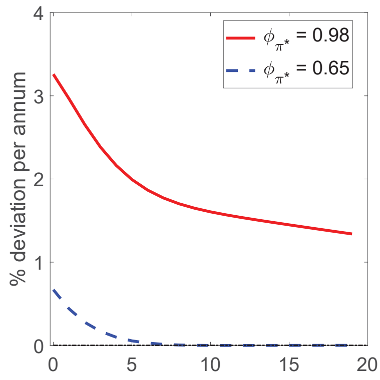
Figure 6: Impulse responses to an inflation target shock for different values of ϕ_{π^*}



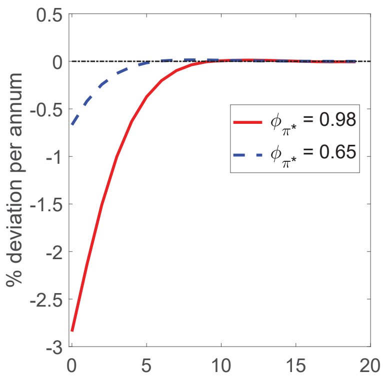
(a) nominal interest rate



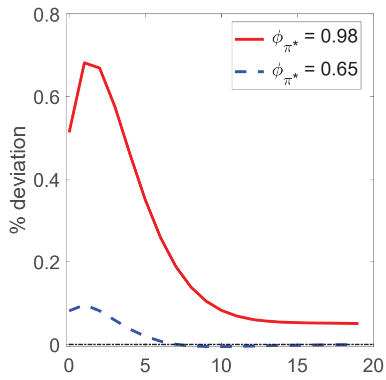
(b) inflation



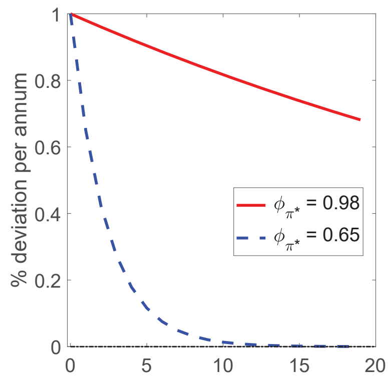
(c) expected inflation ($E_t \pi_{t+1}$)



(d) real interest rate



(e) output gap



(f) inflation target

Note: This figure plots the impulse responses of selective variables to a 1% per annum inflation target shock for different values of inflation target persistence, ϕ_{π^*}. The value of $\phi_{\pi^*} = 0.98$ is the posterior mean for the U.S. economy and the value of $\phi_{\pi^*} = 0.65$ is a counterfactual value which ensures Neo-Fisherism with parameterization of the U.S. economy except for ϕ_{π^*}. All other parameter values are set as in the benchmark model of the U.S. economy presented in Table 1.

contemporaneously raise the nominal interest rate by about 0.43% per annum on impact. Despite a prolonged period of higher nominal interest rates, higher expected inflation is associated with only a relatively-short period of lower real interest rates, resulting in a comparable period of higher output levels.[20]

[20]In Appendix C.3, we show that despite a same positive response of the nominal interest rate on impact, an inflation target shock is not equivalent to a contractionary monetary policy shock in a Taylor-type rule. Under a contractionary monetary policy shock, the increase in the nominal interest rate is associated with a *higher* real interest rate and *lower* inflation and output level.

When $\phi_{\pi^*} = 0.65$ instead, the nominal interest rate is unchanged on impact and it increases slightly in the next several quarters. In this case, a much lower increase in expected inflation leads to inflation to increase by 0.9% per annum only on impact. The half-life of the inflation target shock when $\phi_{\pi^*} = 0.65$ is about 1.6 quarters only. Thus, as long as the half-life of the inflation target shock is greater than 1.6 quarters, the monetary authority needs to increase the nominal interest (policy) rate in order to raise the inflation target.

The impulse responses based on the estimated model thus support our finding that the U.S. economy is most likely to exhibit Neo-Fisherism. Raising the inflation target would cause a large increase in expected inflation, resulting in a short-run positive comovement between inflation and the nominal interest rate.

4.3 Neo-Fisherism, the Taylor rule, and the degree of backward- or forward-lookingness

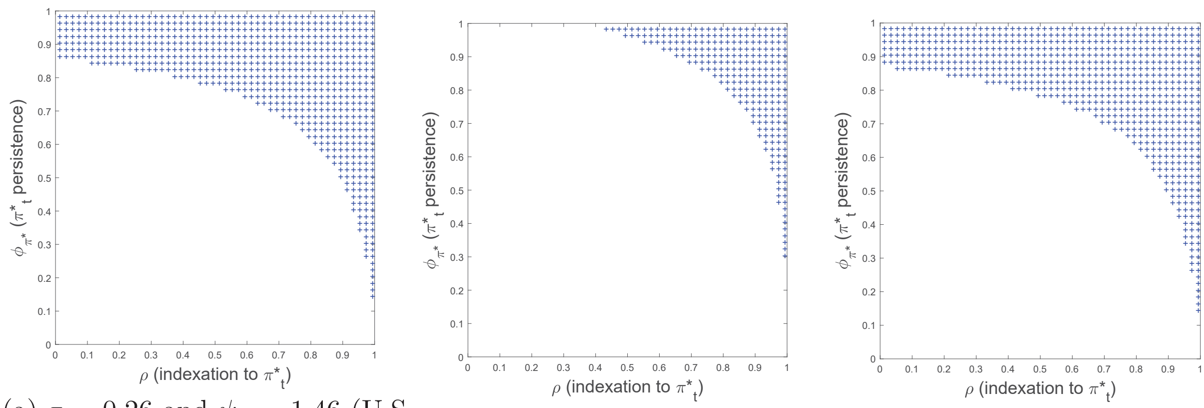
We now study how the comovement between inflation and the nominal interest rate is affected by monetary policy stance, i.e., the inflation reaction coefficient ψ_π, and the degree of backward- or forward-lookingness in this more general model.[21]

4.3.1 The Taylor-rule inflation reaction coefficient

Figure 7 plots the Neo-Fisherian region in the parameter space of (ρ, ϕ_{π^*}) for the benchmark parameterization in Panel (a) and for $\psi_\pi = 40$ in Panel (b). All other parameters, including the past indexation parameter τ, are set to the parameter values in the benchmark model presented in Table 1. Panel (b) can be thought of as an approximation to the strict inflation targeting rule with $\psi_\pi \to \infty$. Comparison of Panels (a) and (b) of Figure 7 clearly demonstrates that the model under the Taylor-type rule with $\psi_\pi = 1.46$ in Panel (a) exhibits Neo-Fisherism for a much wider range of values of ρ and ϕ_{π^*}. Here, even when there is no indexation to the inflation target ($\rho = 0$), the comovement occurs as long as $\phi_{\pi^*} = 0.85$ or larger. When $\psi_\pi = 40$, a higher inflation target necessitates a contemporaneous decrease in

[21]We focus only on the effect of the inflation reaction coefficient ψ_π in the Taylor-type rule because, as shown in Appendix D, a stronger reaction to the output marginally increases the possibility of Neo-Fisherism, especially relative to the effect of ψ_π.

Figure 7: The Neo-Fisherian region in the (ρ, ϕ_{π^*}) parameter space under a Taylor-type rule



(a) $\tau = 0.26$ and $\psi_\pi = 1.46$ (U.S. estimates)

(b) $\tau = 0.26$ and $\psi_\pi = 40$

(c) $\tau = 0.50$ and $\psi_\pi = 1.46$

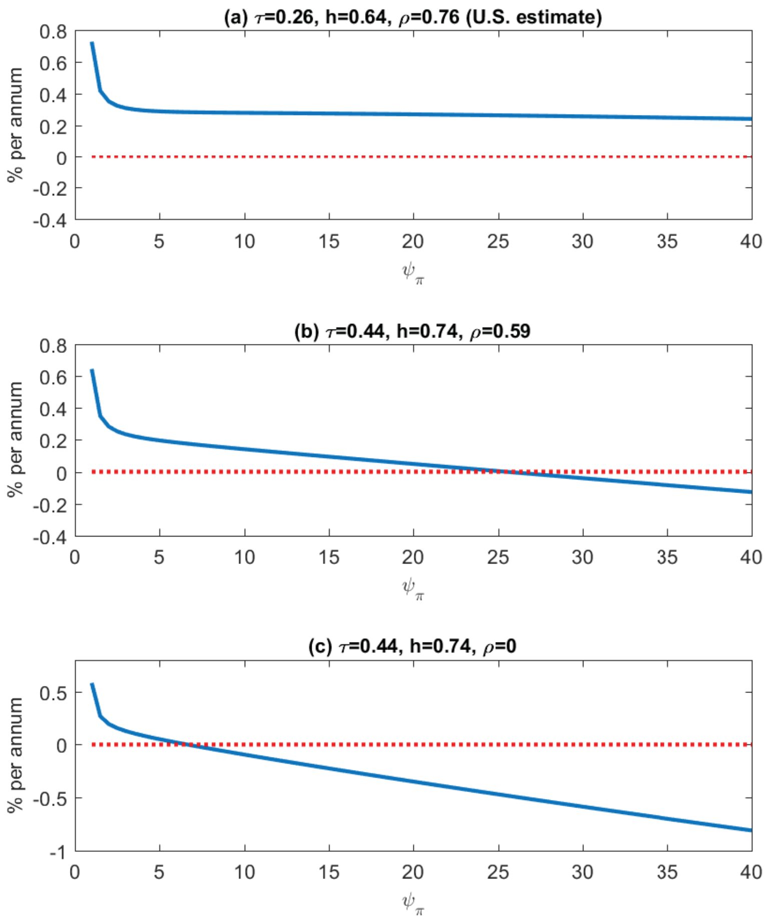
Note: The sign of '+' indicates a pair of (ρ, ϕ_{π^*}) values associated with the positive comovement between inflation and the nominal interest rate conditional on a change in the inflation target. All other parameter values are set as in the benchmark model presented in Table 1.

the nominal interest rate regardless of the persistence of the inflation target, for any value of ρ lower than 0.43.

In Panel (c) of Figure 7, we instead depict the Neo-Fisherian region under the assumption of $\tau = 0.50$, with all other parameter values, including ψ_π, set as in the benchmark model presented in Table 1. In the alternative parameterization, $\tau = 0.50$ constitutes a high degree of indexation to past inflation and is above the 90th percentile of our estimate for the U.S. economy. Comparing Panel (c) to Panel (a), we observe that the Neo-Fisherian region is only marginally smaller when $\tau = 0.50$ than under the benchmark parameterization with $\tau = 0.26$. Even when there is no indexation to the inflation target ($\rho = 0$), the comovement occurs as long as $\phi_{\pi^*} \geq 0.88$. This is in stark contrast to the result in Panel (b) under the appoximate-strict inflation targeting ($\psi_\pi = 40$), discussed previously. Thus, an extremely high value of the inflation reaction coefficient in the Taylor-type rule exaggerates the effect of backward-looking elements on the responses of the nominal interest rate on impact.

Further to this, Figure 8 plots the contemporaneous responses of the nominal interest rate to an inflation target shock over $\psi_\pi \in (1, 40]$ for various combinations of values of τ, h, and ρ. As shown in Panel (a), the model exhibits Neo-Fisherism for all values of

Figure 8: Contemporaneous response of i_t to an inflation-target shock for various combinations of values of τ, h, and ρ



Notes: This figure plots the contemporaneous (period-0) response of the nominal interest rate, i_t, to a 1% per annum inflation target shock for various combinations of values of the degree of indexation to past infaltion, τ, the habit parameter, h, and the degree of indexation to current inflation target, ρ, with all other parameter values set as in the benchmark model of U.S. economy presented in Table 1.; Panel (a) the benchmark parametrization, based on posterior mean estimates for the U.S. economy presented in Table 1; Panel (b) the case where we set τ and h to their 90th percentiles and ρ to its 10th percentile of the posterior distributions; Panel (c) the case where we set τ and h to their 90th percentiles of the posterior distribution and $\varrho = 0$

$\psi_\pi \in (1, 40]$ under the benchmark parameterization in Table 1. In Panel (b), we consider an alternative combination of parameter values of τ, h, and ρ that represents the most *empirically-unfavorable* case for the model to exhibit Neo-Fisherism. Here, the values of $\tau = 0.44$ and $h = 0.74$ are the 90th percentile of the posterior distributions of the parameters, while the value of $\rho = 0.59$ is based on the 10th percentile. See Appendix 4 for posterior distributions in detail. Panel (b) shows that Neo-Fisherism occurs for any value of ψ_π less than 26. At the extreme, even when we further set $\rho = 0$ to eliminate the effect of indexation to the inflation target on inflation and its expectations as depicted in Panel (c), Neo-Fisherism occurs as long as $\psi_\pi \leq 7$, which is the case for all known, empirically-plausible estimates for the U.S. economy.[22]

4.3.2 The degree of backward- or forward-lookingness

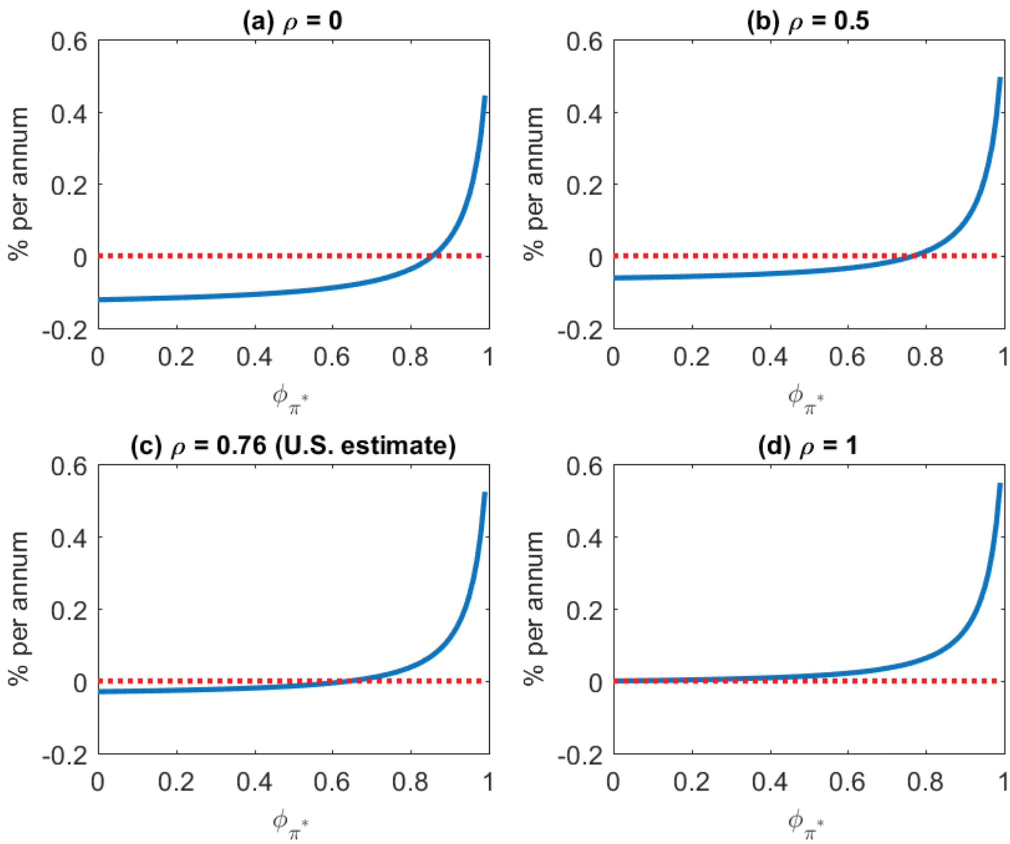
Next, we assess how the comovement between inflation and the nominal interest rate is affected by different values of ρ, τ, and h, in a manner similar to the analytical study in the previous section. All three parameters affect the degree of backward-lookingness or forward-lookingness of the model. As shown in Figure 6 and in the analytical study, an inflation-target shock generates a hump-shaped nominal interest rate response. This implies that if the response of the nominal interest rate to the shock is positive on impact, the model would exhibit Neo-Fisherism. For this reason, the analysis below focuses on the contemporaneous responses of the nominal interest rate to the inflation-target shock. In generating all the results below, we assume a 1% per-annum shock.

Indexation to the inflation target Figure 9 plots the contemporaneous response of the nominal interest rate i_t as a function of ϕ_{π^*}, the parameter governing the inflation-target persistence, for a range of values of ρ, the degree of price indexation to the inflation target. We set all other parameter values to the benchmark values reported in Table 1. The contemporaneous response of i_t increases everywhere in ϕ_{π^*} across different values of ρ. When $\rho = 0$, the response is positive for any $\phi_{\pi^*} > 0.86$, where the half-life of the inflation target shock is 4.6 quarters only. The monetary authority is thus not required to maintain a higher inflation target for an extended period for the economy to exhibit Neo-Fisherism even without an indexation to the inflation target. As the degree of indexation to the inflation target gets higher, the cut-off value decreases monotonically.[23] For example, at the posterior mean estimate of $\rho = 0.76$, the cut-off value is $\phi_{\pi^*} = 0.65$. This result is

[22] In addition, Schmitt-Grohé and Uribe (2007) argue that a value of ψ_π higher than 3 would be difficult for the monetary authority to communicate to the public.

[23] Since the model is linear, the size of the inflation-target shock is immaterial for the cut-off value.

Figure 9: Contemporaneous response of i_t to an inflation-target shock as a function of ϕ_{π^*} and ρ

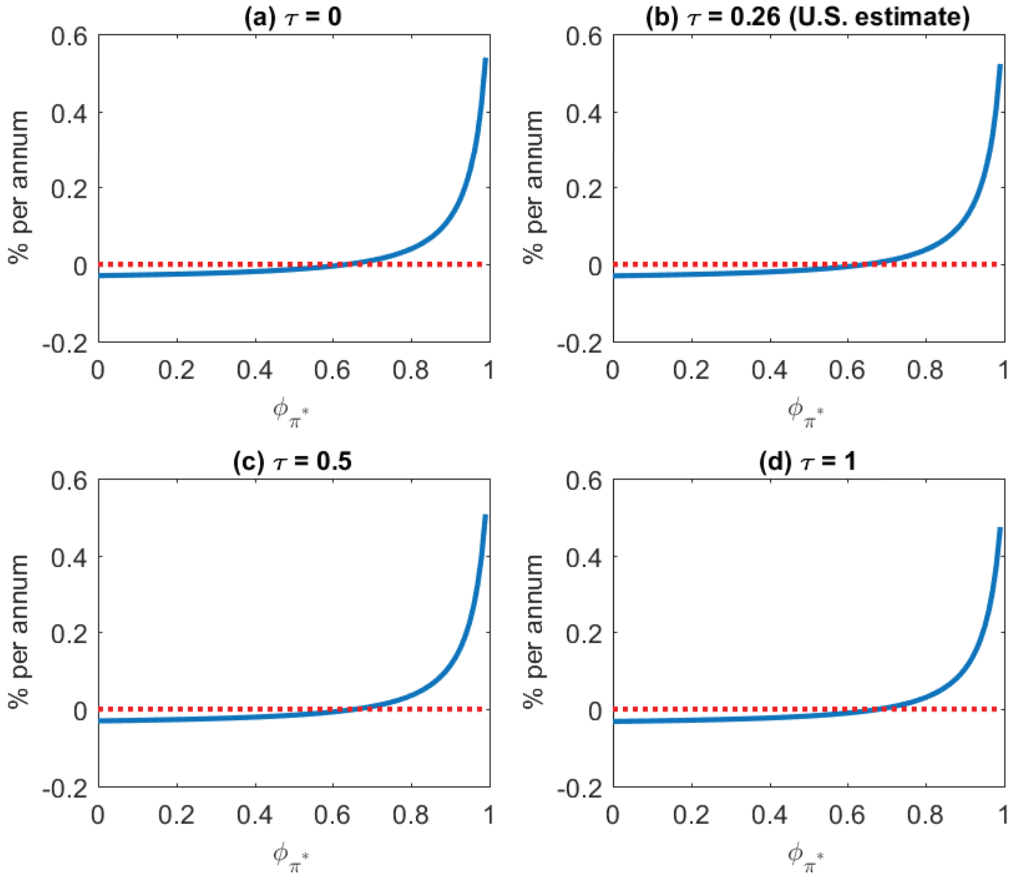


Note: This figure plots the contemporaneous (period-0) response of the nominal interest rate, i_t, to a 1% per annum inflation target shock for various values of inflation target persistence, ϕ_{π^*}, and degree of indexation to inflation target, ρ. All other parameter values are set as in the benchmark model of the U.S. economy presented in Table 1.

consistent with our previous finding that the Neo-Fisherian region in the (ρ, ϕ_{π^*}) parameter space expands as firms take into account more the inflation-target adjustment in their pricing mechanism. When the degree of indexation to the inflation target is at the highest at $\rho = 1$, the contemporaneous response of i_t is everywhere non-negative.

Indexation to past inflation Figure 10 plots the contemporaneous response of i_t as a function of ϕ_{π^*}, for a range of values of τ, the degree of indexation to past inflation, instead. As τ increases, the Neo-Fisherism cut-off value also increases. Hence, the result depicted in Figure 10 is consistent with the analytical finding in the previous section and

Figure 10: Contemporaneous response of i_t to an inflation-target shock as a function of ϕ_{π^*} and τ



Note: This figure plots the contemporaneous (period-0) response of the nominal interest rate, i_t, to a 1% per annum inflation target shock for various values of inflation target persistence, ϕ_{π^*}, and degree of indexation to past inflation, τ. All other parameter values are set as in the benchmark model of the U.S. economy presented in Table 1.

in Garín, Lester and Sims (2018): it is more likely for the nominal interest rate to decrease on impact as the coefficient on the backward-looking inflation component in the NKPC gets larger. Differently, however, we find that there is no possible value of τ in which the model does not exhibit Neo-Fisherism regardless of the persistence of the inflation target. Garín, Lester and Sims (2018) find that, under strict inflation targeting, once the parameter governing the fraction of rule-of-thumb price-setters is greater than 0.15—a modest degree of backward-lookingness—their model does not exhibit Neo-Fisherism for any value of ϕ_{π^*}.[24] Further to this, as shown in Figure 10, the cut-off value only marginally increases from about $\phi_{\pi^*} = 0.64$ when $\tau = 0$ (no indexation to past inflation), to about 0.69 when $\tau = 1$ (full indexation to past inflation). This result indicates that once we consider a Taylor-type rule with reasonable reaction coefficients and price indexation to the inflation target, the backward-looking element in the NKPC plays a markedly smaller role in determining whether the model exhibits Neo-Fisherism.

Habit formation With regard to the habit parameter, h, we confirm the insight from the analytical study, as shown by Figure 11. As the degree of habit formation increases, the Neo-Fisherian cut-off value in terms of ϕ_{π^*} parameter increases, indicating that the comovement becomes less likely. In spite of this, however, the cut-off value only increases from 0.50 when $h = 0.1$ to 0.76 when $h = 0.9$. Hence, similar to the past indexation parameter, the role of habit formation appears to be marginal at best in this more general, estimated model.

4.4 The relative importance of monetary policy and indexation to the inflation target

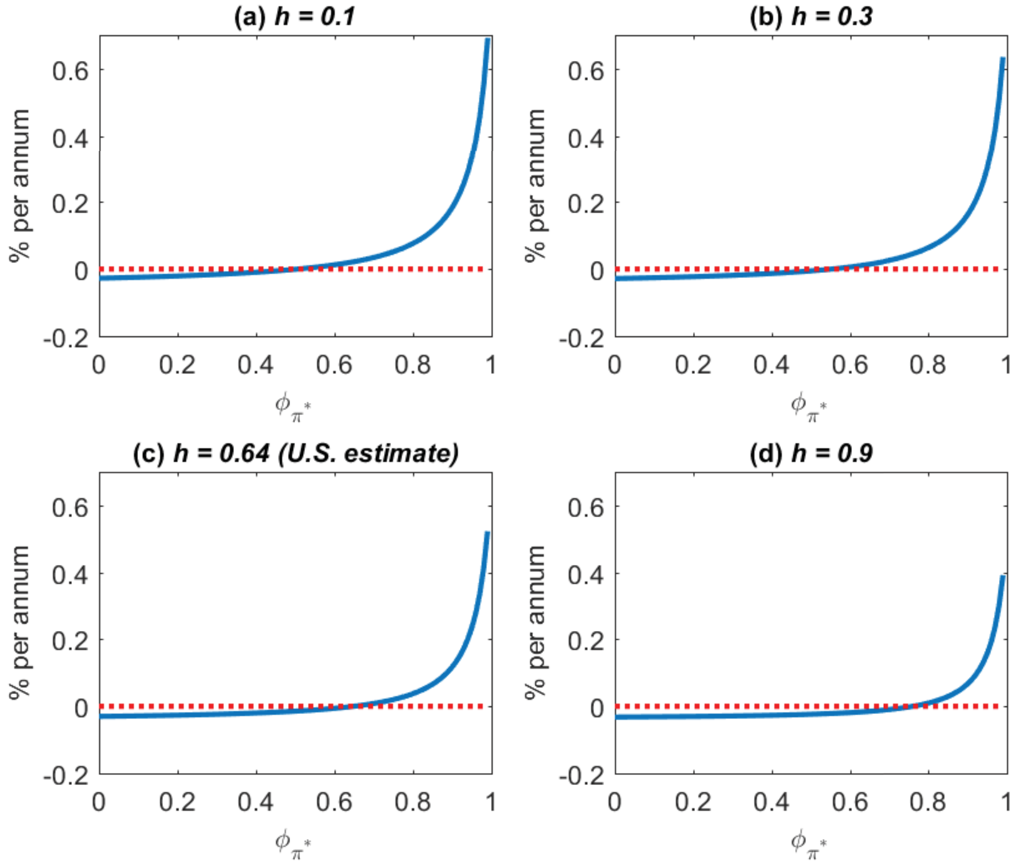
Having established the significance of the inflation reaction coefficient in the Taylor-type rule and the indexation to inflation target in generating a comovement between the inflation target, inflation, and the nominal interest rate, we now assess their relative importance. Particularly, we are interested in their relative effects on the role of the backward-looking parameters, τ and h.

Figure 12 now plots the Neo-Fisherian region in the (ρ, ψ_π) parameter space instead.[25] In the benchmark U.S. estimates with $\tau = 0.26$ depicted in Panel (a), we find that even when

[24] While we use a different modeling mechanism from that in Garín, Lester and Sims (2018) to incorporate the backward-looking component into the NKPC, the two mechanisms — the rule-of-thumb price-setters mechanism of Galı and Gertler (1999) and the indexation mechanism of Christiano, Eichenbaum and Evans (2005) used for our analysis — share a parallel. For example, we find that the case where ω, the fraction of rule-of-thumb price-setters, is 0.25 (shown in the top right panel of Figure 4 in Garín, Lester and Sims (2018)) is similar in terms of direction and magnitude to the case of $\tau = 0.5$ under the indexation mechanism of Christiano, Eichenbaum and Evans (2005). Hence, $\omega = 0.15$ corresponds to a value of $\tau < 0.5$, which can be classified as a modest degree of backward-lookingness.

[25] The top left-hand corner in the figure thus corresponds to the specific case considered in Garín, Lester and Sims (2018), with zero indexation to the inflation target and under strict inflation targeting.

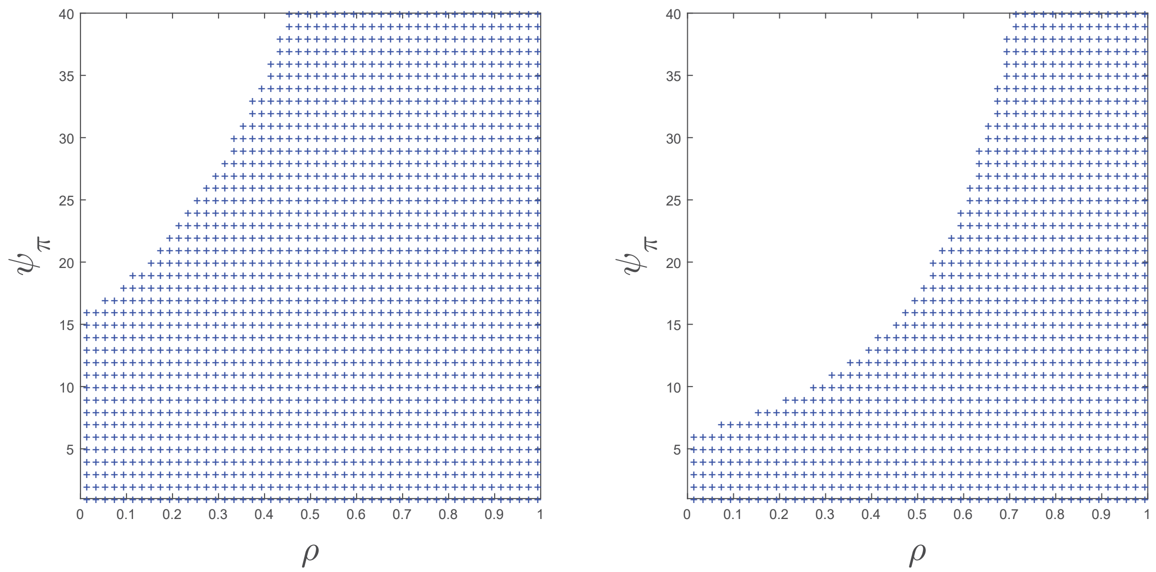
Figure 11: Contemporaneous response of i_t to an inflation-target shock as a function of ϕ_{π^*} for various values of h



Note: This figure plots the contemporaneous (period-0) response of the nominal interest rate, i_t, to a 1% per annum inflation target shock for various values of inflation target persistence, ϕ_{π^*}, and the habit parameter, h. All other parameter values are set as in the benchmark model of the U.S. economy presented in Table 1.

there is zero indexation to the inflation target, ψ_π needs to be at least 16.5 to break down Neo-Fisherism. This value of $\psi_\pi \geq 16.5$ is unlikely to be chosen for U.S. monetary policy and in fact, for any other economy engaging in flexible inflation targeting. On the flip side, the positive comovement occurs for any value of $\psi_\pi > 1$ as long as $\rho \geq 0.43$. Further, Panel (b) with $\tau = 0.5$ shows that even when $\rho = 0$, it requires $\psi_\pi \geq 7$ to break down the positive comovement in this more backward-looking environment. The results presented in Figure 12 indicate that the inflation reaction coefficient in the Taylor-type rule plays a more important role than the indexation to the inflation target in generating Neo-Fisherism and dampening the effects of the backward-looking components such as indexation to past inflation.

Figure 12: The Neo-Fisherian region in the (ρ, ψ_π) parameter space under a Taylor-type rule



(a) $\tau = 0.26$ (U.S. estimate)   (b) $\tau = 0.50$

Note: The sign of '+' indicates a pair of (ρ, ψ_π) values associated with the positive comovement between inflation and the nominal interest rate conditional on a change in the inflation target. All other parameter values are set as in the benchmark model of the U.S. economy presented in Table 1.

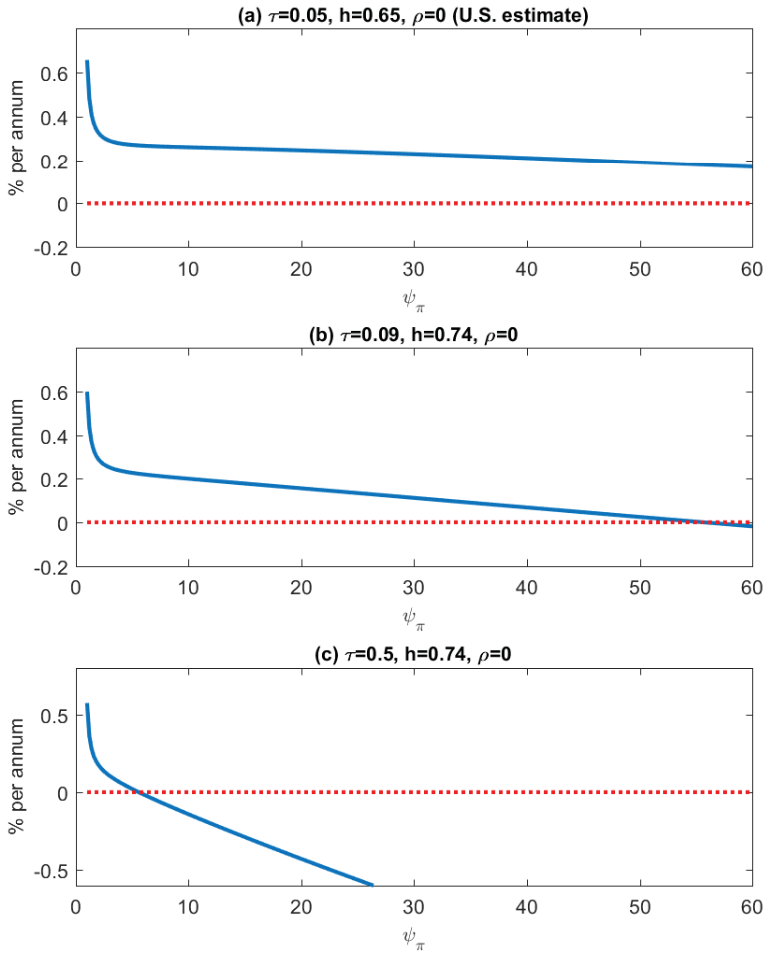
4.5 Robustness: the restricted model with $\varrho = 0$

Although the U.S. data strongly support the benchmark model without the restriction on ϱ compared to the restricted model with $\varrho = 0$, we examine if the Neo-Fisherian implication for the U.S. economy relies on the specification of the indexation scheme in the NKPC.[26] We redo the analysis in Figure 8 under the restricted model with $\varrho = 0$. Here, the structural parameter values are set based on the posterior means when we estimate the model with the restriction of $\varrho = 0$. The resulting parameter values are reported in the column for Restricted ($\varrho = 0$) of Table 1. Additional details on the estimation are contained in Appendix 4.

Figure 13 plots the contemporaneous response of the nominal interest rate under this alternative parameterization. Panel (a) of Figure 13 shows that Neo-Fisherism occurs for all considered values of $\psi_\pi \in [1, 60]$. When we instead set the values of τ and h to their 90th

[26] See the log marginal likelihood and posterior model probability comparison between the two alternative models in Table 1.

Figure 13: Restricted model with $\varrho = 0$: Contemporaneous response of i_t to an inflation-target shock for various combinations of values of τ and h



Note: This figure plots the contemporaneous (period-0) response of the nominal interest rate, i_t, to a 1% per annum inflation target shock for various combinations of values of the degree of indexation to past inflation, τ and the habit parameter, h, when the model is estimated with the restriction of $\varrho = 0$ (no indexation to the inflation target). All other parameter values are set as in the restricted model with $\varrho = 0$ of the U.S. economy (posterior means) presented in Table 1.; Panel (a) the parameterization based on the posterior means for the U.S. economy; Panel (b) the case where we set τ and h to their 90th percentiles of the posterior distributions; Panel (c) the case where we set h to its 90th percentile and $\tau = 0.5$, which is much greater than the 90th percentile of the posterior distribution ($\tau = 0.09$); This parameterization would be least favorable combination of the values of τ and h based on the U.S. data.

percentiles of the posterior distributions in Panel (b), ψ_π still needs to be larger than 56 to reverse the positive comovement. This cut-off value of ψ_π is larger than the corresponding case in Panel (b) of Figure 8 because the backward-looking indexation parameter, τ, is estimated to be much smaller in this restricted case. In Panel (c), we set $\tau = 0.5$—well above the 90th percentile of the posterior distribution—while keeping all other parameter values as in the column for Restricted ($\varrho = 0$) of Table 1. In this much more backward-looking environment, the positive comovement occurs as long as $\psi_\pi \leq 5.5$. This range of values is still within all reasonable estimates of ψ_π for the U.S. economy.

In Appendix E, we also redo the analysis of the effect of various values of τ, h, and ϕ_{π^*} on Neo-Fisherism as depicted in Figures 10 and 11, based on the restricted model with $\varrho = 0$. While the Neo-Fisherian cut-off value in terms of the persistence parameter ϕ_{π^*} is now larger in each considered case, it continues to be largely insensitive to different values of τ and h and is still below our estimate of ϕ_{π^*}. All of these indicate that the specification of partial price indexation to the inflation target is not crucial for our finding of the likely presence of Neo-Fisherism in the U.S. economy.

5 Conclusion

In this paper, we investigate whether raising an inflation target requires increasing the nominal interest rate in the short run in a standard New Keynesian model with rich backward-looking elements.

We find that the short-run comovement between inflation and the nominal interest rate conditional on changes in the inflation target is less likely to be positive, all else equal, as the monetary authority reacts more aggressively to the deviation of inflation from its target or as more backward-looking elements are incorporated into the model. Meanwhile, features of the model that enhance forward-looking behavior, such as partial price indexation to the inflation target and a lower degree of price rigidity, are shown to increase the likelihood of the positive comovement.

However, we show that this so-called Neo-Fisherism is likely to hold even with a significant degree of backward-lookingness in the model and a high degree of price rigidity, unless the monetary authority reacts to inflation in an extremely aggressive manner, close to strict inflation targeting.

Our estimated New Keynesian model indicates that the U.S. economy exhibits Neo-Fisherism: raising the inflation target necessitates a short-run increase in the nominal interest rate. This finding is robust to empirically-plausible parameterizations of the model and to the specification of the price indexation to the inflation target in firms' price-setting process.

References

Ball, Laurence. 2014. "The Case for a Long-Run Inflation Target of Four Percent." International Monetary Fund Working Paper 1492.

Bernanke, Ben S. 2017. "Monetary policy in a new era." *Peterson Institute*.

Bhattarai, Saroj, Jae Won Lee, and Woong Yong Park. 2014*a*. "Inflation dynamics: The role of public debt and policy regimes." *Journal of Monetary Economics*, 67: 93–108.

Bhattarai, Saroj, Jae Won Lee, and Woong Yong Park. 2014*b*. "Price indexation, habit formation, and the Generalized Taylor Principle." *Journal of Economic Dynamics and Control*, 48: 218–225.

Bhattarai, Saroj, Jae Won Lee, and Woong Yong Park. 2016. "Policy regimes, policy shifts, and US business cycles." *Review of Economics and Statistics*, 98(5): 968–983.

Blanchard, Olivier. 2016. "Where to from Here? The Phillips Curve: Back to the 60s?" *The American Economic Review*, 106(5): 31–34.

Blanchard, Olivier, Giovanni Dell'Ariccia, and Paolo Mauro. 2010. "Rethinking Macroeconomic Policy 1." *Rethinking Macroeconomic Policy*.

Bullard, James, and Kaushik Mitra. 2002. "Learning about monetary policy rules." *Journal of Monetary Economics*, 49(6): 1105–1129.

Calvo, Guillermo A. 1983. "Staggered prices in a utility-maximizing framework." *Journal of Monetary Economics*, 12(3): 383–398.

Castelnuovo, Efrem. 2012. "Testing the Structural Interpretation of the Price Puzzle with a Cost-Channel Model." *Oxford Bulletin of Economics and Statistics*, 74(3): 425–452.

Christiano, Lawrence J, Martin Eichenbaum, and Charles L Evans. 2005. "Nominal rigidities and the dynamic effects of a shock to monetary policy." *Journal of Political Economy*, 113(1): 1–45.

Cochrane, John H. 2016. "Do Higher Interest Rates Raise or Lower Inflation?" *Working paper, February.*

Cogley, Timothy, Giorgio E Primiceri, and Thomas J Sargent. 2010. "Inflation-gap persistence in the US." *American Economic Journal: Macroeconomics*, 2(1): 43–69.

Del Negro, Marco, and Frank Schorfheide. 2005. "Policy predictions if the model does not fit." *Journal of the European Economic Association*, 3(2-3): 434–443.

Del Negro, Marco, Frank Schorfheide, Frank Smets, and Rafael Wouters. 2007. "On the fit of new Keynesian models." *Journal of Business & Economic Statistics*, 25(2): 123–143.

Del Negro, Marco, Marc P Giannoni, and Frank Schorfheide. 2015. "Inflation in the great recession and new keynesian models." *American Economic Journal: Macroeconomics*, 7(1): 168–96.

Fève, Patrick, Julien Matheron, and Jean-Guillaume Sahuc. 2010. "Inflation target shocks and monetary policy inertia in the euro area." *The Economic Journal*, 120(547): 1100–1124.

Frankel, Jeffrey. 2013. "Nominal-GDP targets, without losing the inflation anchor." *Is Inflation Targeting Dead?*, 90.

Fuhrer, Jeffrey C. 2000. "Habit formation in consumption and its implications for monetary-policy models." *American Economic Review*, 90(3): 367–390.

Galí, Jordi. 2015. *Monetary policy, inflation, and the business cycle: an introduction to the new Keynesian framework and its applications.* Princeton University Press.

Galı, Jordi, and Mark Gertler. 1999. "Inflation dynamics: A structural econometric analysis." *Journal of Monetary Economics*, 44(2): 195–222.

Garín, Julio, Robert Lester, and Eric Sims. 2018. "Raise Rates to Raise Inflation? Neo-Fisherianism in the New Keynesian Model." *Journal of Money, Credit and Banking*, 50.

Gaspar, Vitor, Frank Smets, and David Vestin. 2010. "Is time ripe for price level path stability?" *Challenges in Central Banking: The Current Institutional Environment and Forces Affecting Monetary Policy*, 21–51.

Ireland, Peter N. 2007. "Changes in the Federal Reserve's inflation target: Causes and consequences." *Journal of Money, Credit and Banking*, 39(8): 1851–1882.

Kiley, Michael T, and John M Roberts. 2017. "Monetary policy in a low interest rate world." *Brookings Papers on Economic Activity*, 2017(1): 317–396.

Krugman, Paul. 2014. "Inflation targets reconsidered." *Navigating Monetary Policy in the New Normal 8*, 110.

Kuttner, Ken, and Tim Robinson. 2010. "Understanding the flattening Phillips curve." *The North American Journal of Economics and Finance*, 21(2): 110–125.

Lubik, Thomas A, and Frank Schorfheide. 2004. "Testing for indeterminacy: an application to US monetary policy." *The American Economic Review*, 94(1): 190–217.

McCallum, Bennett T, and Edward Nelson. 1999. "Nominal income targeting in an open-economy optimizing model." *Journal of Monetary economics*, 43(3): 553–578.

Roberts, John M. 2006. "Monetary Policy and Inflation Dynamics." *International Journal of Central Banking*, 2(3).

Rosengren, Eric S. 2018. "Reviewing monetary policy frameworks: remarks at a Forum on the Federal Reserve's Inflation Target." In *the Hutchins Center on Fiscal and Monetary Policy, the Brookings Institution, Washington, DC*.

Schmitt-Grohé, Stephanie, and Martin Uribe. 2007. "Optimal simple and implementable monetary and fiscal rules." *Journal of Monetary Economics*, 54(6): 1702–1725.

Summers, Lawrence H, David Wessel, and John David Murray. 2018. "Rethinking the Fed's 2 percent inflation target." In *A report from the Hutchins Center on Fiscal & Monetary Policy at Brookings*. Hutchings Center on Fiscal & Monetary Policy.

Taylor, John B. 1993. "Discretion versus policy rules in practice." *Carnegie-Rochester Conference Series on Public Policy*, 39: 195–214.

Uribe, Martín. 2018. "The Neo-Fisher Effect: Econometric Evidence from Empirical and Optimizing Models." National Bureau of Economic Research.

Williams, John C. 2016. "Monetary Policy in a Low R-star World." *FRBSF Economic Letter*, 23.

Williams, John C. 2017. "Preparing for the Next Storm: Reassessing Frameworks and Strategies in a Low R-star World." *FRBSF Economic Letter*, 13.

Williamson, Stephen. 2016. "Neo-Fisherism: A Radical Idea, or the Most Obvious Solution to the Low-Inflation Problem?" *The Regional Economist, FRBSL.*

Yun, Tack. 1996. "Nominal price rigidity, money supply endogeneity, and business cycles." *Journal of Monetary Economics*, 37(2): 345–370.

Appendices

A Derivation of the generalized hybrid New Keynesian Phillips curve

In this appendix, we provide the derivation of the generalized hybrid NKPC in (5). The NKPC (6) is a special case when $\tau = 0$.

The indexation mechanism for firms that are not allowed to optimally adjust their prices is given by

$$P_t(i) = P_{t-1}(i)\Pi_t^{*\varrho}\left(\bar{\Pi}^{1-\tau}\Pi_{t-1}^{\tau}\right)^{1-\varrho}.$$

We first derive the log-linear approximation of the aggregate price level. Let X_t be the optimal nominal price at time t chosen by firms that are allowed to adjust their prices optimally, which occurs with probability $(1-\theta)$. Assuming a CES aggregation with elasticity ε, the aggregate price level is given by

$$P_t^{1-\varepsilon} = (1-\theta)X_t^{1-\varepsilon} + \theta\left(\Pi_t^{*\varrho}\left(\bar{\Pi}^{1-\tau}\Pi_{t-1}^{\tau}\right)^{1-\varrho}P_{t-1}\right)^{1-\varepsilon}$$

or

$$1 = (1-\theta)x_t^{1-\varepsilon} + \theta\left(\Pi_t^{*\varrho}\left(\bar{\Pi}^{1-\tau}\Pi_{t-1}^{\tau}\right)^{1-\varrho}\Pi_t^{-1}\right)^{1-\varepsilon} \tag{A.1}$$

where $x_t \equiv X_t/P_t$ is the real optimal reset price. Log-linearizing (A.1) around the long-run steady state and rearranging lead to

$$\hat{x}_t = \frac{\theta}{1-\theta}\left[\hat{\pi}_t - \tau(1-\varrho)\hat{\pi}_{t-1} - \varrho\hat{\pi}_t^*\right] \tag{A.2}$$

where $\hat{\pi}_t$ and $\hat{\pi}_t^*$ denote the deviation of inflation and the inflation target from their long-run steady-state value. To arrive at (A.2), we utilize the steady-state conditions $\bar{x} = 1$, based on (A.1), and $\Pi^* = \bar{\Pi}$.

Next, we derive the expression for the optimal reset price. The first-order condition of the firms' optimal nominal price problem can be expressed as

$$E_t \sum_{j=0}^{\infty} \theta^j Q_{t,t+j} Y_{t+j} P_{t+j} \Psi_{tj}^{1-\varepsilon} \left(X_t - \frac{\varepsilon}{\varepsilon - 1} MC_{t+j} \Psi_{tj}^{-1} \right) = 0 \tag{A.3}$$

where Y_t is the aggregate output, MC_t is the nominal marginal cost, and $Q_{t,t+j}$ is the nominal discount factor from time t to time $t+j$. The variable Ψ_{tj} enters the CES demand function for any good i, $Y_{t+j}(i) = Y_{t+j} \left(\frac{P_{t+j}(i)\Psi_{tj}}{P_{t+j}} \right)$, with

$$\Psi_{tj} = \begin{cases} 1 & \text{if} \quad j = 0 \\ \prod_{k=0}^{j-1} \Pi_{t+k}^{*\varrho} \left(\bar{\Pi}^{1-\tau} \Pi_{t+k-1}^{\tau} \right) & \text{if} \quad j \geq 1 \end{cases}$$

The expression (A.1) can be recursively written as

$$X_t = \frac{\varepsilon}{\varepsilon - 1} \frac{D_{1,t}}{D_{2,t}}$$

with

$$D_{1,t} = Y_t P_t^{\varepsilon-1} MC_t + \theta\beta E_t \left[\left(\Pi_{t+1}^* \right)^{-\varrho\varepsilon} \left(\bar{\Pi}^{1-\tau} \Pi_t^{\tau} \right)^{-(1-\varrho)\varepsilon} D_{1,t+1} \right],$$

$$D_{2,t} = Y_t P_t^{\varepsilon-1} + \theta\beta E_t \left[\left(\Pi_{t+1}^* \right)^{\varrho(1-\varepsilon)} \left(\bar{\Pi}^{1-\tau} \Pi_t^{\tau} \right)^{(1-\varrho)(1-\varepsilon)} D_{2,t+1} \right].$$

where β is the subjective discount factor. Dividing these recursive equations by appropriate deflators and rearranging lead to the following recursive expression for the real optimal price:

$$x_t = \frac{\varepsilon}{\varepsilon - 1} \frac{\tilde{D}_{1,t}}{\tilde{D}_{2,t}}, \tag{A.4}$$

$$\tilde{D}_{1,t} = mc_t + \theta\beta E_t \left[\left(\Pi_{t+1}^* \right)^{-\varrho\varepsilon} \left(\bar{\Pi}^{1-\tau} \Pi_t^{\tau} \right)^{-(1-\varrho)\varepsilon} \left(\Pi_{t+1} \right)^{\varepsilon} \tilde{D}_{1,t+1} \right], \tag{A.5}$$

and

$$\tilde{D}_{2,t} = 1 + \theta\beta E_t \left[\left(\Pi_{t+1}^* \right)^{\varrho(1-\varepsilon)} \left(\bar{\Pi}^{1-\tau} \Pi_t^{\tau} \right)^{(1-\varrho)(1-\varepsilon)} \left(\Pi_{t+1} \right)^{\varepsilon-1} \tilde{D}_{2,t+1} \right], \tag{A.6}$$

where $\tilde{D}_{1,t} \equiv D_{1,t}/(Y_t P_t^{\varepsilon})$ and $D_{2,t} \equiv D_{2,t}/(Y_t P_t^{\varepsilon-1})$. We can log-linearize (A.4), (A.5), and

(A.6) around the steady state, resulting in

$$\hat{x}_t = \widehat{D}_{1,t} - \widehat{D}_{2,t}, \tag{A.7}$$

$$\begin{aligned} \widehat{D}_{1,t} &= (1-\theta\beta)\widehat{mc}_t + \theta\beta\varepsilon E_t\left[\hat{\pi}_{t+1} - \tau(1-\varrho)\hat{\pi}_t - \varrho\hat{\pi}^*_{t+1}\right] \\ &\quad +\theta\beta E_t\widehat{D}_{1,t+1}, \end{aligned} \tag{A.8}$$

and

$$\begin{aligned} \widehat{D}_{2,t} &= \theta\beta(\varepsilon-1)E_t\left[\hat{\pi}_{t+1} - \tau(1-\varrho)\hat{\pi}_t - \varrho\hat{\pi}^*_{t+1}\right] \\ &\quad +\theta\beta E_t\widehat{D}_{2,t+1}. \end{aligned} \tag{A.9}$$

Combining (A.7), (A.1), and (A.1) with the expression in (A.2) and rearranging yield

$$\hat{\pi}_t = \gamma_b\hat{\pi}_{t-1} + \gamma_f E_t\hat{\pi}_{t+1} + \tilde{\lambda}\widehat{mc}_t + \varrho\delta\left(\hat{\pi}^*_t - \beta E_t\hat{\pi}^*_{t+1}\right) \tag{A.10}$$

where

$$\begin{aligned} \gamma_b &\equiv \frac{\tau(1-\varrho)}{1+\beta\tau(1-\varrho)}, \\ \gamma_f &\equiv \frac{\beta}{1+\beta\tau(1-\varrho)}, \\ \delta &\equiv \frac{1}{1+\beta\tau(1-\varrho)}, \\ \text{and } \tilde{\lambda} &\equiv \frac{\lambda}{1+\beta\tau(1-\varrho)} = \frac{(1-\theta\beta)(1-\theta)}{\theta(1+\beta\tau(1-\varrho))}. \end{aligned}$$

Finally, to write the real marginal cost $\widehat{mc}_t$ in terms of the output gap $\hat{y}_t$ (defined as the deviation of output from its flexible-price level), we assume that the household's utility function depends on consumption, C_t, and labor effort, N_t, and is given by $u(C_t, N_t) = \frac{C_t^{1-\sigma}-1}{1-\sigma} - \frac{N_t^{1+\eta}}{1+\eta}$. Also, the production function is $Y_t = A_t N_t$, so that the real marginal cost is $mc_t = w_t/A_t$, where A_t and w_t denote the technology shock and real wage, respectively. This implies that $\widehat{mc}_t = (\sigma+\eta)\hat{y}_t$ (see Galí (2015), chapter 3 for more details).

B Proofs

Lemma 1 *Under strict inflation targeting* (9) *with the generalized hybrid NKPC* (4) *and the IS curve* (11), *the solution is given by*

$$\pi_t = \pi_t^*, \tag{B.1}$$

$$y_t = \Psi_0^y(\Theta)\pi_t^* + \Psi_1^y(\Theta)\pi_{t-1}^*, \text{ and} \tag{B.2}$$

$$i_t = \Psi_0(\Theta)\pi_t^* + \Psi_1(\Theta)\pi_{t-1}^*. \tag{B.3}$$

Proof. The model is given by

$$y_t = E_t y_{t+1} - \sigma^{-1}(i_t - E_t\pi_{t+1}), \tag{B.4}$$

$$\pi_t = \gamma_f E_t\pi_{t+1} + \gamma_b\pi_{t-1} + \tilde{\kappa}y_t + \mu\pi_t^*, \tag{B.5}$$

and

$$\pi_t = \pi_t^* \tag{B.6}$$

where $\mu = \varrho\,(1 - \beta\phi_{\pi^*})\,/\,(1 + \beta\tau(1 - \varrho))$. Note that $E_t\pi_{t+1}^* = \phi_{\pi^*}\pi_t^*$.

It is straightforward to find the solution for inflation as $\pi_t = \pi_t^*$ from (B.6). Then substitute it into the NKPC (B.5) as

$$\pi_t^* = \gamma_f\phi_{\pi^*}\pi_t^* + \gamma_b\pi_{t-1}^* + \tilde{\kappa}y_t + \mu\pi_t^*. \tag{B.7}$$

Rearrange (B.7) and collect terms related to y_t, π_t^*, and π_{t-1}^*. It leads to the solution for the output gap as

$$\begin{aligned} y_t &= \frac{1 - \gamma_f\phi_{\pi^*} - \mu}{\tilde{\kappa}}\pi_t^* - \frac{\gamma_b}{\tilde{\kappa}}\pi_{t-1}^* \\ &= \frac{1 + \beta\tau(1-\varrho) - \beta\phi_{\pi^*} - \varrho\,(1 - \beta\phi_{\pi^*})}{\kappa}\pi_t^* - \frac{\tau(1-\varrho)}{\kappa}\pi_{t-1}^* \\ &= \Psi_0^y\pi_t^* + \Psi_1^y\pi_{t-1}^* \end{aligned} \tag{B.8}$$

where

$$\Psi_0^y = \frac{(1-\varrho)(1-\beta\phi_{\pi^*}+\beta\tau)}{\kappa} \text{ and } \Psi_1^y = -\frac{\tau(1-\varrho)}{\kappa}.$$

Substitute (B.8) into the IS curve (B.4) for y_t and $E_t y_{t+1}$ as

$$\Psi_0^y \pi_t^* + \Psi_1^y \pi_{t-1}^* = \Psi_0^y \phi_{\pi^*}\pi_t^* + \Psi_1^y \pi_t^* - \sigma^{-1} i_t + \sigma^{-1}\phi_{\pi^*}\pi_t^*. \tag{B.9}$$

Rearrange (B.9) and collect terms related to i_t, π_t^*, and π_{t-1}^* as

$$\begin{aligned} i_t &= \sigma\Psi_0^y\phi_{\pi^*}\pi_t^* + \sigma\Psi_1^y\pi_t^* + \phi_{\pi^*}\pi_t^* - \sigma\Psi_0^y\pi_t^* - \sigma\Psi_1^y\pi_{t-1}^* \\ &= \left(\phi_{\pi^*} + \sigma\Psi_0^y\phi_{\pi^*} - \sigma\Psi_0^y + \sigma\Psi_1^y\right)\pi_t^* - \sigma\Psi_1^y\pi_{t-1}^* \\ &= \left[\phi_{\pi^*} - \frac{(1-\phi_{\pi^*})(1-\varrho)(1-\beta\phi_{\pi^*}+\beta\tau)}{\sigma^{-1}\kappa} - \frac{\tau(1-\varrho)}{\sigma^{-1}\kappa}\right]\pi_t^* + \frac{\tau(1-\varrho)}{\sigma^{-1}\kappa}\pi_{t-1}^* \\ &= \Psi_0(\Theta)\pi_t^* + \Psi_1(\Theta)\pi_{t-1}^* \end{aligned} \tag{B.10}$$

where

$$\Psi_0(\Theta) = \left[\phi_{\pi^*} - \frac{(1-\varrho)(1-\phi_{\pi^*})(1-\beta\phi_{\pi^*})}{\sigma^{-1}\kappa} - \frac{\tau(1-\varrho)\{1+\beta(1-\phi_{\pi^*})\}}{\sigma^{-1}\kappa}\right] \tag{B.11}$$

$$\Psi_1(\Theta) = \frac{\tau(1-\varrho)}{\sigma^{-1}\kappa}. \tag{B.12}$$

■

Proof of Proposition 1

Proof. Strict inflation targeting (9) implies that inflation always increases as the inflation target increases. Thus, to verify the positive comovement between inflation and the nominal interest rate conditional on changes in the inflation target, we just need to show the nominal interest rate increases in the inflation target. The closed-form solution for the nominal interest rate is given by (B.10) in Lemma 1 as

$$\begin{aligned} i_t = &\left[\phi_{\pi^*} - \frac{(1-\varrho)(1-\phi_{\pi^*})(1-\beta\phi_{\pi^*})}{\sigma^{-1}\kappa} - \frac{\tau(1-\varrho)\{1+\beta(1-\phi_{\pi^*})\}}{\sigma^{-1}\kappa}\right]\pi_t^* \\ &+\frac{\tau(1-\varrho)}{\sigma^{-1}\kappa}\pi_{t-1}^*. \end{aligned} \tag{B.13}$$

We confirm the increase of the nominal interest rate in response to the increase of the inflation target by checking the coefficient on π_t^* only, because the coefficient on π_{t-1}^* is always greater than or equal to zero. The coefficient on π_t^* in (B.13) can be positive or negative. Thus, the necessary and sufficient condition for the positive comovement is equivalent to the positive coefficient on π_t^*:

$$\phi_{\pi^*} - (1-\varrho)\frac{(1-\phi_{\pi^*})(1-\beta\phi_{\pi^*})}{\sigma^{-1}\kappa} - \frac{\tau(1-\varrho)}{\sigma^{-1}\kappa}\{1+\beta(1-\phi_{\pi^*})\} > 0. \tag{B.14}$$

It is straightforward to show that the coefficient on π_t^* increases (i.e., the necessary and sufficient condition is more likely to hold) as τ gets smaller or ϱ and ϕ_{π^*} get larger. ■

Proof of Proposition 2

Proof. Similar to proof for Proposition 1, it is straightforward to show that the coefficient on π_t^* in (B.14) increases (i.e., the necessary and sufficient condition is more likely to hold) as κ and $1/\sigma$ get larger. ■

Proof of Proposition 3

Proof. Similar to the approach to finding the solution for the model with the hybrid NKPC (see Lemma 1), the solution for the nominal interest rate is given by

$$i_t = \left[\phi_{\pi^*} - (1-\varrho)(1-\frac{\phi_{\pi^*}}{1+h})\frac{1+h}{1-h}\frac{(1-\beta\phi_{\pi^*})}{\sigma^{-1}\kappa}\right]\pi_t^* + (1-\varrho)\frac{h}{1-h}\frac{(1-\beta\phi_{\pi^*})}{\sigma^{-1}\kappa}\pi_{t-1}^*. \tag{B.15}$$

The coefficient on π_{t-1}^* is always positive or equal to zero and so the model exhibits Neo-Fisherism as long as the coefficient on π_t^* is positive. It is straightforward to show that as h gets larger the coefficient on π_t^* is less likely to increase for a given increase in the inflation target. ■

Lemma 2 *The solution for i_t to the New Keynesian Model with the IS curve (11), the hybrid NKPC (4), and the Taylor-type rule (10) is given by*

$$i_t = \Gamma_0\pi_t^* + \Gamma_1\pi_{t-1} \tag{B.16}$$

where

$$\Gamma_0 = \frac{1}{\lambda_2 - \phi_{\pi^*}} \frac{1}{\lambda_3 - \phi_{\pi^*}} \frac{\psi_\pi \kappa \sigma^{-1}}{\beta} \times$$
$$\left\{ \phi_{\pi^*} - (1-\varrho) \frac{(1-\phi_{\pi^*})(1-\beta\phi_{\pi^*})}{\kappa\sigma^{-1}} - \frac{\tau(1-\varrho)}{\kappa\sigma^{-1}} \{1 + \beta(1-\phi_{\pi^*})\} + \frac{\tau(1-\varrho)}{\kappa\sigma^{-1}} \left(\frac{\lambda_2 + \lambda_3 - \phi_{\pi^*}}{\lambda_2\lambda_3} \right) \right\},$$
$$\Gamma_1 = \lambda_1 \psi_\pi$$

and λ_1, λ_2, and λ_3 are eigenvalues associated with the model solution such that

$$\begin{aligned} |\lambda_1| < 1 &< |\lambda_2| \leq |\lambda_3|, \\ \lambda_1\lambda_2 + \lambda_2\lambda_3 + \lambda_3\lambda_1 &= \frac{1 + \gamma_b + \tilde{\kappa}\sigma^{-1}\psi_\pi}{\gamma_f} > 0, \\ \lambda_1\lambda_2\lambda_3 &= \frac{\gamma_b}{\gamma_f} > 0, \\ \lambda_1 + \lambda_2 + \lambda_3 &= \frac{1 + \gamma_f + \tilde{\kappa}\sigma^{-1}}{\gamma_f} > 0. \end{aligned}$$

Proof. The model is given by

$$y_t = E_t y_{t+1} - \sigma^{-1} \left(i_t - E_t \pi_{t+1} \right), \tag{B.17}$$

$$\pi_t = \gamma_f E_t \pi_{t+1} + \gamma_b \pi_{t-1} + \tilde{\kappa} y_t + \mu \pi_t^*, \tag{B.18}$$

and

$$i_t = \psi_\pi (\pi_t - \pi_t^*) \tag{B.19}$$

where $\mu = \varrho\left(1 - \beta\phi_{\pi^*}\right) / \left(1 + \beta\tau(1-\varrho)\right)$ and other structural parameters are defined as those in Section 2.

We want to collapse the three equations in the model above into a single equation for i_t. Push (B.19) one period ahead and take expectations of it in time t (i.e., E_t) as

$$E_t i_{t+1} = \psi_\pi E_t \left(\pi_{t+1} - \pi_{t+1}^* \right). \tag{B.20}$$

Then substitute (B.20) into the IS curve (B.17) to remove $E_t\pi_{t+1}$ as

$$y_t = E_t y_{t+1} - \sigma^{-1} E_t \left(i_t - \frac{1}{\psi_\pi} i_{t+1} - \pi^*_{t+1} \right). \tag{B.21}$$

Use a lag operator L for $E_t x_{t+1} = L^{-1} E_t x_t$, (B.18) and (B.21) are expressed as

$$E_t(1 - \gamma_f L^{-1} - \gamma_b L)\pi_t = \tilde{\kappa} y_t + \mu \pi^*_t \tag{B.22}$$

and

$$E_t(1 - L^{-1}) y_t = -\sigma^{-1} E_t \left(i_t - \frac{1}{\psi_\pi} i_{t+1} - \pi^*_{t+1} \right). \tag{B.23}$$

Now, substitute (B.19) and (B.23) into (B.22) as

$$\begin{aligned} E_t(1 - \gamma_f L^{-1} - \gamma_b L) \left(\frac{1}{\psi_\pi} i_t + \pi^*_t \right) &= -E_t \tilde{\kappa} \sigma^{-1} (1 - L^{-1})^{-1} \left(i_t - \frac{1}{\psi_\pi} i_{t+1} - \pi^*_{t+1} \right) + \mu \pi^*_t \\ &= -E_t \tilde{\kappa} \sigma^{-1} (1 - L^{-1})^{-1} \left((1 - \frac{1}{\psi_\pi} L^{-1}) i_t - \pi^*_{t+1} \right) + \mu \pi^*_t. \end{aligned} \tag{B.24}$$

Multiply both sides of (B.24) by $(1 - L^{-1})$ as

$$\begin{aligned} &E_t(1 - L^{-1})(1 - \gamma_f L^{-1} - \gamma_b L) \left(\frac{1}{\psi_\pi} i_t + \pi^*_t \right) = -\tilde{\kappa} \sigma^{-1} E_t \left((1 - \frac{1}{\psi_\pi} L^{-1}) i_t - \pi^*_{t+1} \right) \\ &+ E_t(1 - L^{-1}) \mu \pi^*_t. \end{aligned} \tag{B.25}$$

Rearrange (B.25) and collect terms related to i_t and π^*_t, respectively as

$$\begin{aligned} & E_t \left[\left\{ \frac{1 + \gamma_b - \gamma_b L - (1 + \gamma_f) L^{-1} + \gamma_f L^{-2}}{\psi_\pi} + \tilde{\kappa} \sigma^{-1} \left(1 - \frac{1}{\psi_\pi} L^{-1} \right) \right\} i_t \right] \\ = \; & E_t \left[\left\{ -1 - \gamma_b + \gamma_b L + (1 + \gamma_f) L^{-1} - \gamma_f L^{-2} + \tilde{\kappa} \sigma^{-1} L^{-1} + \mu - \mu L^{-1} \right\} \pi^*_t \right]. \end{aligned} \tag{B.26}$$

The LHS of (B.26) is given by

$$
\begin{aligned}
& E_t\left[\left\{\frac{1+\gamma_b-\gamma_b L-(1+\gamma_f)L^{-1}+\gamma_f L^{-2}}{\psi_\pi}+\tilde{\kappa}\sigma^{-1}\left(1-\frac{1}{\psi_\pi}L^{-1}\right)\right\}i_t\right] \\
= & \frac{1}{\psi_\pi}E_t\left[\left\{1+\gamma_b-\gamma_b L-(1+\gamma_f)L^{-1}+\gamma_f L^{-2}+\tilde{\kappa}\sigma^{-1}\psi_\pi-\tilde{\kappa}\sigma^{-1}L^{-1}\right\}i_t\right] \\
= & \frac{1}{\psi_\pi}E_t\left[\left\{(1+\gamma_b+\tilde{\kappa}\sigma^{-1}\psi_\pi)-\gamma_b L-(1+\gamma_f+\tilde{\kappa}\sigma^{-1})L^{-1}+\gamma_f L^{-2}\right\}i_t\right] \\
= & \frac{\gamma_f}{\psi_\pi}E_t\left[\left\{\frac{1+\gamma_b+\tilde{\kappa}\sigma^{-1}\psi_\pi}{\gamma_f}-\frac{\gamma_b}{\gamma_f}L-\frac{1+\gamma_f+\tilde{\kappa}\sigma^{-1}}{\gamma_f}L^{-1}+L^{-2}\right\}i_t\right] \\
= & \frac{\gamma_f}{\psi_\pi}E_t\left[(1-\lambda_1 L)(\lambda_2-L^{-1})(\lambda_3-L^{-1})i_t\right].
\end{aligned}
$$

The last line in (B.27) uses the following lag operator expression

$$
(1-\lambda_1 L)(\lambda_2-L^{-1})(\lambda_3-L^{-1})=(\lambda_1\lambda_2+\lambda_2\lambda_3+\lambda_3\lambda_1)-\lambda_1\lambda_2\lambda_3 L-(\lambda_1+\lambda_2+\lambda_3)L^{-1}+L^{-2}. \tag{B.27}
$$

In short, eigenvalues λ_1, λ_2, and λ_3 are such that

$$
\lambda_1\lambda_2+\lambda_2\lambda_3+\lambda_3\lambda_1 = \frac{1+\gamma_b+\tilde{\kappa}\sigma^{-1}\psi_\pi}{\gamma_f}>0, \tag{B.28}
$$

$$
\lambda_1\lambda_2\lambda_3 = \frac{\gamma_b}{\gamma_f}>0, \tag{B.29}
$$

$$
\lambda_1+\lambda_2+\lambda_3 = \frac{1+\gamma_f+\tilde{\kappa}\sigma^{-1}}{\gamma_f}>0, \tag{B.30}
$$

and

$$
|\lambda_1|<1<|\lambda_2|\leq|\lambda_3| \tag{B.31}
$$

to ensure determinacy. Also, because γ_b, γ_f, $\tilde{\kappa}$, ψ_π, and σ are all positive, $\lambda_1\lambda_2+\lambda_2\lambda_3+\lambda_3\lambda_1>0$, $\lambda_1\lambda_2\lambda_3>0$, and $\lambda_1+\lambda_2+\lambda_3>0$.

The RHS of (B.26) is given by

$$
\begin{aligned}
& E_t\left[\left\{-1-\gamma_b+\gamma_b L+(1+\gamma_f)L^{-1}-\gamma_f L^{-2}+\tilde{\kappa}\sigma^{-1}L^{-1}+\mu-\mu L^{-1}\right\}\pi_t^*\right] \\
= & E_t\left[\left\{(\mu-1-\gamma_b)+(1+\gamma_f-\mu+\tilde{\kappa}\sigma^{-1})L^{-1}-\gamma_f L^{-2}\right\}\pi_t^*\right]+\gamma_b\pi_{t-1}^* \\
= & \left\{(\mu-1-\gamma_b)+(1+\gamma_f-\mu+\tilde{\kappa}\sigma^{-1})\phi_{\pi^*}-\gamma_f\phi_{\pi^*}^2\right\}\pi_t^*+\gamma_b\pi_{t-1}^* \qquad \text{(B.32)}
\end{aligned}
$$

because $E_t\pi^*_{t+1} = E_t L^{-1}\pi^*_t = \phi_{\pi^*}\pi^*_t$.

Combine (B.27) for the LHS of (B.26) and (B.32) for the RHS of (B.26) as

$$\begin{aligned}
& \frac{\gamma_f}{\psi_\pi} E_t\left[(1-\lambda_1 L)(\lambda_2 - L^{-1})(\lambda_3 - L^{-1}) i_t\right] \\
= & \left\{(\mu - 1 - \gamma_b) + (1 + \gamma_f - \mu + \tilde{\kappa}\sigma^{-1})\phi_{\pi^*} - \gamma_f \phi^2_{\pi^*}\right\}\pi^*_t + \gamma_b \pi^*_{t-1}. \qquad \text{(B.33)}
\end{aligned}$$

Multiply both sides of (B.33) by $(\lambda_2 - L^{-1})^{-1}(\lambda_3 - L^{-1})^{-1}$ as

$$\begin{aligned}
& \frac{\gamma_f}{\psi_\pi} E_t\left[(1-\lambda_1 L) i_t\right] \\
= & E_t\left[(\lambda_2 - L^{-1})^{-1}(\lambda_3 - L^{-1})^{-1}\left\{(\mu - 1 - \gamma_b) + (1 + \gamma_f - \mu + \tilde{\kappa}\sigma^{-1})\phi_{\pi^*} - \gamma_f \phi^2_{\pi^*}\right\}\pi^*_t\right] \\
& + E_t\left[(\lambda_2 - L^{-1})^{-1}(\lambda_3 - L^{-1})^{-1}\gamma_b \pi^*_{t-1}\right]. \qquad \text{(B.34)}
\end{aligned}$$

The RHS of (B.34) can be rearranged as

$$\begin{aligned}
& E_t\left[(\lambda_2 - L^{-1})^{-1}(\lambda_3 - L^{-1})^{-1}\left\{(\mu - 1 - \gamma_b) + (1 + \gamma_f - \mu + \tilde{\kappa}\sigma^{-1})\phi_{\pi^*} - \gamma_f \phi^2_{\pi^*}\right\}\pi^*_t\right] \\
& + E_t\left[(\lambda_2 - L^{-1})^{-1}(\lambda_3 - L^{-1})^{-1}\gamma_b \pi^*_{t-1}\right] \\
= & E_t\left[(\lambda_3 - L^{-1})^{-1}\frac{1}{\lambda_2 - \phi_{\pi^*}}\left\{(\mu - 1 - \gamma_b) + (1 + \gamma_f - \mu + \tilde{\kappa}\sigma^{-1})\phi_{\pi^*} - \gamma_f \phi^2_{\pi^*} + \frac{\gamma_b}{\lambda_2}\right\}\pi^*_t\right] \\
& + E_t\left[(\lambda_3 - L^{-1})^{-1}\frac{1}{\lambda_2}\gamma_b \pi^*_{t-1}\right] \\
= & \frac{1}{\lambda_2 - \phi_{\pi^*}}\frac{1}{\lambda_3 - \phi_{\pi^*}}\left\{(\mu - 1 - \gamma_b) + (1 + \gamma_f - \mu + \tilde{\kappa}\sigma^{-1})\phi_{\pi^*} - \gamma_f \phi^2_{\pi^*} + \frac{\gamma_b}{\lambda_2} + \frac{\gamma_b}{\lambda_3} - \frac{\phi_{\pi^*}\gamma_b}{\lambda_2\lambda_3}\right\}\pi^*_t \\
& + \frac{1}{\lambda_2}\frac{1}{\lambda_3}\gamma_b \pi^*_{t-1}
\end{aligned}$$

because

$$\begin{aligned}
E_t(\lambda_2 - L^{-1})^{-1}\pi^*_t &= E_t \frac{1}{\lambda_2}\left(1 + \frac{1}{\lambda_2}L^{-1} + \frac{1}{\lambda_2^2}L^{-2} + \cdots\right)\pi^*_t \\
&= E_t \frac{1}{\lambda_2}\left(\pi^*_t + \frac{1}{\lambda_2}\pi^*_{t+1} + \frac{1}{\lambda_2^2}\pi^*_{t+2} + \cdots\right) = \frac{1}{\lambda_2}\left(\pi^*_t + \frac{\phi_{\pi^*}}{\lambda_2}\pi^*_t + \frac{\phi^2_{\pi^*}}{\lambda_2^2}\pi^*_t + \cdots\right) \\
&= \frac{1}{\lambda_2}\frac{1}{1 - \phi_{\pi^*}/\lambda_2}\pi^*_t = \frac{1}{\lambda_2 - \phi_{\pi^*}}\pi^*_t \qquad \text{(B.35)}
\end{aligned}$$

and similarly

$$E_t(\lambda_2 - L^{-1})^{-1}\pi^*_{t-1} = E_t\frac{1}{\lambda_2}\left(1 + \frac{1}{\lambda_2}L^{-1} + \frac{1}{\lambda_2^2}L^{-2} + \cdots\right)\pi^*_{t-1} = \frac{1}{\lambda_2}\pi^*_{t-1} + \frac{1}{\lambda_2}\frac{1}{\lambda_2 - \phi_{\pi^*}}\pi^*_t.$$

The derivations above also apply to expanding for $(\lambda_3 - L^{-1})^{-1}$. Note that (B.31) ensures $|\phi_{\pi^*}/\lambda_2| < 1$ and $|\phi_{\pi^*}/\lambda_3| < 1$.

Finally, the solution is given by

$$\begin{aligned}
& i_t - \lambda_1 i_{t-1} \\
= & \frac{\psi_\pi}{\gamma_f}\frac{1}{\lambda_2 - \phi_{\pi^*}}\frac{1}{\lambda_3 - \phi_{\pi^*}}\left\{(\mu - 1 - \gamma_b) + (1 + \gamma_f - \mu + \tilde{\kappa}\sigma^{-1})\phi_{\pi^*} - \gamma_f\phi_{\pi^*}^2 + \frac{\gamma_b}{\lambda_2} + \frac{\gamma_b}{\lambda_3} - \frac{\phi_{\pi^*}\gamma_b}{\lambda_2\lambda_3}\right\}\pi^*_t \\
& + \frac{\psi_\pi}{\gamma_f}\frac{1}{\lambda_2}\frac{1}{\lambda_3}\gamma_b\pi^*_{t-1} \\
= & \frac{1}{\lambda_2 - \phi_{\pi^*}}\frac{1}{\lambda_3 - \phi_{\pi^*}}\frac{\psi_\pi}{\beta}\left\{\kappa\sigma^{-1}\phi_{\pi^*} - (1 - \varrho)(1 - \phi_{\pi^*})(1 + \beta\tau - \beta\phi_{\pi^*})\right. \\
& \left. + \tau(1 - \varrho)\left(\frac{\lambda_2 + \lambda_3 - \phi_{\pi^*}}{\lambda_2\lambda_3} - 1\right)\right\}\pi^*_t + \frac{1}{\lambda_2}\frac{1}{\lambda_3}\frac{\psi_\pi\tau(1 - \varrho)}{\beta}\pi^*_{t-1}.
\end{aligned} \tag{B.36}$$

Substitute $i_{t-1} = \psi_\pi(\pi_{t-1} - \pi^*_{t-1})$ into (B.36) and rearrange it as

$$\begin{aligned}
i_t = & \lambda_1\psi_\pi(\pi_{t-1} - \pi^*_{t-1}) + \frac{1}{\lambda_2 - \phi_{\pi^*}}\frac{1}{\lambda_3 - \phi_{\pi^*}}\frac{\psi_\pi\kappa\sigma^{-1}}{\beta} \times \\
& \left\{\phi_{\pi^*} - (1 - \varrho)\frac{(1 - \phi_{\pi^*})(1 - \beta\phi_{\pi^*}) + \{1 + \beta(1 - \phi_{\pi^*})\}\tau}{\kappa\sigma^{-1}} + \frac{(1 - \varrho)\tau}{\kappa\sigma^{-1}}\left(\frac{\lambda_2 + \lambda_3 - \phi_{\pi^*}}{\lambda_2\lambda_3}\right)\right\}\pi^*_t \\
+ & \frac{1}{\lambda_2}\frac{1}{\lambda_3}\frac{\psi_\pi\tau(1 - \varrho)}{\beta}\pi^*_{t-1} \\
= & \lambda_1\psi_\pi\pi_{t-1} + \frac{1}{\lambda_2 - \phi_{\pi^*}}\frac{1}{\lambda_3 - \phi_{\pi^*}}\frac{\psi_\pi\kappa\sigma^{-1}}{\beta} \times \\
& \left\{\phi_{\pi^*} - (1 - \varrho)\frac{(1 - \phi_{\pi^*})(1 - \beta\phi_{\pi^*}) + \{1 + \beta(1 - \phi_{\pi^*})\}\tau}{\kappa\sigma^{-1}} + \frac{(1 - \varrho)\tau}{\kappa\sigma^{-1}}\left(\frac{\lambda_2 + \lambda_3 - \phi_{\pi^*}}{\lambda_2\lambda_3}\right)\right\}\pi^*_t
\end{aligned} \tag{B.37}$$

Thus, we show that the solution for i_t is expressed as

$$i_t = \Gamma_0\pi^*_t + \Gamma_1\pi_{t-1} \tag{B.38}$$

where

$$
\begin{aligned}
\Gamma_0 &= \frac{1}{\lambda_2 - \phi_{\pi^*}} \frac{1}{\lambda_3 - \phi_{\pi^*}} \frac{\psi_\pi \kappa \sigma^{-1}}{\beta} \times \\
&\left\{ \phi_{\pi^*} - (1-\varrho) \frac{(1-\phi_{\pi^*})(1-\beta\phi_{\pi^*}) + \{1+\beta(1-\phi_{\pi^*})\}\tau}{\kappa\sigma^{-1}} + \frac{(1-\varrho)\tau}{\kappa\sigma^{-1}} \left(\frac{\lambda_2 + \lambda_3 - \phi_{\pi^*}}{\lambda_2\lambda_3} \right) \right\} \\
\Gamma_1 &= \lambda_1 \psi_\pi. \qquad \text{(B.39)}
\end{aligned}
$$

■

Lemma 3 *The solution for* π_t *to the New Keynesian Model with the IS curve* (B.17)*, the NKPC* (B.18)*, and the Taylor-type rule* (B.19) *is given by*

$$
\pi_t = \Phi_0 \pi_t^* + \Phi_1 \pi_{t-1} \qquad \text{(B.40)}
$$

where

$$
\begin{aligned}
\Phi_0 &= \frac{1}{(\lambda_2 - \phi_{\pi^*})(\lambda_3 - \phi_{\pi^*})} \frac{\psi_\pi \kappa \sigma^{-1}}{\beta} \left[1 + \varrho \frac{\beta(1-\phi_{\pi^*})(1-\beta\phi_{\pi^*})}{\psi_\pi \kappa \sigma^{-1}} \right] \qquad \text{(B.41)} \\
\Phi_1 &= \lambda_1 \qquad \text{(B.42)}
\end{aligned}
$$

and λ_1*,* λ_2*, and* λ_3 *are eigenvalues associated with the model solution in Lemma 2.*

Proof. We find the solution for π_t similar to the approach in Lemma 2. Substitute (B.18) into the IS curve (B.17) to remove i_t as

$$
y_t = E_t y_{t+1} - \sigma^{-1} E_t \left(\psi_\pi (\pi_t - \pi_t^*) - \pi_{t+1} \right). \qquad \text{(B.43)}
$$

Use a lag operator as

$$
E_t(1 - L^{-1}) y_t = E_t \left(-\sigma^{-1}\psi_\pi + \sigma^{-1} L^{-1} \right) \pi_t + \sigma^{-1} \psi_\pi \pi_t^*. \qquad \text{(B.44)}
$$

Then, we use a lag operator for the hybrid NKPC and substitute (B.44) into it as

$$\begin{aligned} E_t(1-\gamma_f L^{-1}-\gamma_b L)\pi_t &= \tilde{\kappa} y_t + \mu\pi_t^* \\ &= \tilde{\kappa} E_t\left[(1-L^{-1})^{-1}\left(-\sigma^{-1}\psi_\pi + \sigma^{-1}L^{-1}\right)\pi_t + (1-L^{-1})^{-1}\sigma^{-1}\psi_\pi\pi_t^*\right] + \mu\pi_t^*. \end{aligned} \tag{B.45}$$

Rearrange (B.45) and collect terms related to π_t and π_t^*, respectively as

$$E_t\left[(1-\gamma_f L^{-1}-\gamma_b L) - \tilde{\kappa}(1-L^{-1})^{-1}\left(-\sigma^{-1}\psi_\pi + \sigma^{-1}L^{-1}\right)\right]\pi_t = E_t\left[\tilde{\kappa}(1-L^{-1})^{-1}\sigma^{-1}\psi_\pi + \mu\right]\pi_t^* \tag{B.46}$$

Multiply both sides of (B.46) by $(1-L^{-1})$ as

$$\begin{aligned} E_t\left[(1-L^{-1})(1-\gamma_f L^{-1}-\gamma_b L) - \tilde{\kappa}\left(-\sigma^{-1}\psi_\pi + \sigma^{-1}L^{-1}\right)\right]\pi_t &= E_t\left[\tilde{\kappa}\sigma^{-1}\psi_\pi + (1-L^{-1})\mu\right]\pi_t^* \\ E_t\left[\left(1+\gamma_b+\tilde{\kappa}\sigma^{-1}\psi_\pi\right) - \left(1+\gamma_f+\tilde{\kappa}\sigma^{-1}\right)L^{-1} - \gamma_b L + \gamma_f L^{-2}\right]\pi_t &= E_t\left[\tilde{\kappa}\sigma^{-1}\psi_\pi + \mu\left(1-\phi_{\pi^*}\right)\right]\pi_t^* \end{aligned}$$

We express the solution using the eigenvalues λ_1, λ_2, and λ_3 as

$$\begin{aligned} E_t(1-\lambda_1 L)(\lambda_2 - L^{-1})(\lambda_3 - L^{-1})\pi_t &= \frac{\tilde{\kappa}\sigma^{-1}\psi_\pi + \mu\left(1-\phi_{\pi^*}\right)}{\gamma_f}\pi_t^* \\ (1-\lambda_1 L)\pi_t &= E_t\frac{\tilde{\kappa}\sigma^{-1}\psi_\pi + \mu\left(1-\phi_{\pi^*}\right)}{\gamma_f}(\lambda_2 - L^{-1})^{-1}(\lambda_3 - L^{-1})^{-1}\pi_t^* \\ (1-\lambda_1 L)\pi_t &= \frac{\tilde{\kappa}\sigma^{-1}\psi_\pi + \mu\left(1-\phi_{\pi^*}\right)}{\gamma_f}\frac{1}{\lambda_2-\phi_{\pi^*}}\frac{1}{\lambda_3-\phi_{\pi^*}}\pi_t^* \end{aligned} \tag{B.47}$$

The solution for π_t is then given by

$$\pi_t = \lambda_1\pi_{t-1} + \frac{1}{\lambda_2-\phi_{\pi^*}}\frac{1}{\lambda_3-\phi_{\pi^*}}\frac{\psi_\pi\kappa\sigma^{-1}}{\beta}\left[1+\varrho\frac{\beta(1-\phi_{\pi^*})(1-\beta\phi_{\pi^*})}{\psi_\pi\kappa\sigma^{-1}}\right]\pi_t^*. \tag{B.48}$$

■

Lemma 4 $0 < \lambda_1 < 1$.

Proof. We start with Lemma 2. It shows $|\lambda_1| < 1$ to ensure determinacy in (B.31). We will further show $\lambda_1 > 0$. Suppose $\lambda_1 < 0$. Since $\lambda_1\lambda_2\lambda_3 > 0$ from (B.29) and $|\lambda_1| < 1$, it is required that $\lambda_2\lambda_3 < 0$. We know that $\lambda_1\lambda_2 + \lambda_2\lambda_3 + \lambda_3\lambda_1 = \lambda_1(\lambda_2+\lambda_3) + \lambda_2\lambda_3 > 0$

from (B.28) . Since $\lambda_1 < 0$ and $\lambda_2\lambda_3 < 0$, it is required that $\lambda_2 + \lambda_3 < 0$. However, it contradicts the fact that $\lambda_1 + \lambda_2 + \lambda_3 > 0$ from (B.30). In addition, since $\lambda_1\lambda_2\lambda_3 > 0$, it is required that $\lambda_1 \neq 0$. Therefore, it must be $\lambda_1 > 0$. In combination with Lemma 2, we can show that $0 < \lambda_1 < 1$. ■

Lemma 5 $\lambda_2 > 1$ *and* $\lambda_3 > 1$.

Proof. We again start with Lemma 2. Because $\lambda_1 + \lambda_2 + \lambda_3 = \frac{1+\gamma_f+\tilde{\kappa}\sigma^{-1}}{\gamma_f} = 1 + \frac{1+\tilde{\kappa}\sigma^{-1}}{\gamma_f} > 1$ and $0 < \lambda_1 < 1$, it is required that $\lambda_2 + \lambda_3 > 0$. In addition, because $\lambda_1\lambda_2\lambda_3 > 0$ and $0 < \lambda_1 < 1$, it is required that $\lambda_2\lambda_3 > 0$. Then, because $\lambda_2 + \lambda_3 > 0$ and $\lambda_2\lambda_3 > 0$, it is required that $\lambda_2 > 0$ and $\lambda_3 > 0$. In combination with $|\lambda_1| < 1 < |\lambda_2| \leq |\lambda_3|$, we can show that $\lambda_2 > 1$ and $\lambda_3 > 1$. ■

Lemma 6 *As* $\psi_\pi \to \infty$, $\lambda_1 \to 0$ *and* $\lambda_1\psi_\pi \to \frac{\tau(1-\varrho)}{\sigma^{-1}\kappa}$.

Proof. Note that the solution for strict inflation targeting with the hybrid NKPC (i.e., $\psi_\pi \to \infty$) is given by $\pi_t = \pi_t^*$. From (B.40) in Lemma 2, it requires that $\lim_{\psi_\pi\to\infty} \Phi_1 = 0$ and $\lim_{\psi_\pi\to\infty} \Phi_0 = 1$. Note that $\Phi_1 = \lambda_1$. Therefore, $\lim_{\psi_\pi\to\infty} \lambda_1 = 0$.

Now, we consider $\lim_{\psi_\pi\to\infty} \Phi_0 = 1$. Note that $\Phi_0 = \frac{1}{(\lambda_2-\phi_{\pi^*})(\lambda_3-\phi_{\pi^*})}\frac{\psi_\pi\kappa\sigma^{-1}}{\beta}\left[1+\varrho\frac{\beta(1-\phi_{\pi^*})(1-\beta\phi_{\pi^*})}{\psi_\pi\kappa\sigma^{-1}}\right]$. Because $\lim_{\psi_\pi\to\infty} \Phi_0 = 1$ and $\lim_{\psi_\pi\to\infty}\left[1+\varrho\frac{\beta(1-\phi_{\pi^*})(1-\beta\phi_{\pi^*})}{\psi_\pi\kappa\sigma^{-1}}\right] = 1$, it requires that

$$\lim_{\psi_\pi\to\infty} \frac{1}{(\lambda_2-\phi_{\pi^*})(\lambda_3-\phi_{\pi^*})}\frac{\psi_\pi\kappa\sigma^{-1}}{\beta} = 1$$

. We can show that

$$\begin{aligned}
& \frac{1}{\lambda_2-\phi_{\pi^*}}\frac{1}{\lambda_3-\phi_{\pi^*}}\frac{\psi_\pi\kappa\sigma^{-1}}{\beta} \\
= & \frac{1}{\lambda_2\lambda_3-\phi_{\pi^*}(\lambda_2+\lambda_3)+\phi_{\pi^*}^2}\frac{\psi_\pi\kappa\sigma^{-1}}{\beta} \\
= & \frac{1}{\frac{\tau(1-\varrho)}{\lambda_1\beta}-\phi_{\pi^*}(\frac{1+\beta\tau(1-\varrho)}{\beta}-\lambda_1)+\phi_{\pi^*}^2}\frac{\psi_\pi\kappa\sigma^{-1}}{\beta} \\
= & \frac{\lambda_1\psi_\pi}{\frac{\tau(1-\varrho)}{\beta}-\lambda_1\phi_{\pi^*}\frac{1+\beta\tau(1-\varrho)}{\beta}+\lambda_1^2\phi_{\pi^*}+\lambda_1\phi_{\pi^*}^2}\frac{\kappa\sigma^{-1}}{\beta}.
\end{aligned} \tag{B.49}$$

Therefore, $\lim_{\psi_\pi \to \infty} \lambda_1 \psi_\pi = \frac{\tau(1-\varrho)}{\sigma^{-1}\kappa}$. We will confirm theses conditions in the case of the solutions for i_t in Lemma 7. ■

Proof of Proposition 4

Proof. We first show that inflation increases in the inflation target. In Lemma 3, Φ_1 is always positive because $\Phi_1 = \lambda_1$ and $0 < \lambda_1 < 1$ from Lemma 4. In addition, because $\lambda_2 > 1$ and $\lambda_3 > 1$ in Lemma 5, Φ_0 is always positive. Thus, inflation increases when the monetary authority raises the inflation target.

Now, we consider the response of the nominal interest rate to the change in the inflation target. Because inflation always increases in the inflation target and so should the nominal interest rate. Lemma 2 shows that the solution for i_t to the New Keynesian model is give by

$$i_t = \Gamma_1 \pi_{t-1} + \Gamma_0 \pi_t^*$$

where

$$\begin{aligned}
\Gamma_1 &= \psi_\pi \lambda_1 \\
\Gamma_0 &= \frac{1}{\lambda_2 - \phi_{\pi^*}} \frac{1}{\lambda_3 - \phi_{\pi^*}} \frac{\psi_\pi \kappa \sigma^{-1}}{\beta} \times \\
&\quad \left\{ \phi_{\pi^*} - (1-\varrho) \frac{(1-\phi_{\pi^*})(1-\beta\phi_{\pi^*}) + \{1+\beta(1-\phi_{\pi^*})\}\tau}{\kappa\sigma^{-1}} + \frac{(1-\varrho)\tau}{\kappa\sigma^{-1}} \left(\frac{\lambda_2 + \lambda_3 - \phi_{\pi^*}}{\lambda_2\lambda_3} \right) \right\}.
\end{aligned}$$

Because $0 < \lambda_1 < 1$ in Lemma 4, Γ_1 is always positive. In order for the model to exhibit the comovement between inflation and the nominal interest rate Γ_0 should be positive. Thus, we will find the condition for $\Gamma_0 > 0$. Lemma 5 for $\lambda_2 > 1$ and $\lambda_3 > 1$ implies that the scaling factor in Γ, $\frac{1}{\lambda_2 - \phi_{\pi^*}} \frac{1}{\lambda_3 - \phi_{\pi^*}} \frac{\psi_\pi \kappa \sigma^{-1}}{\beta}$, is always positive. Therefore, the sign of Γ_0 depends on the sign of

$$\Omega = \phi_{\pi^*} - \frac{(1-\varrho)(1-\phi_{\pi^*})(1-\beta\phi_{\pi^*})}{\kappa\sigma^{-1}} - \frac{\tau(1-\varrho)}{\kappa\sigma^{-1}} \{1+\beta(1-\phi_{\pi^*})\} + \frac{\tau(1-\varrho)}{\kappa\sigma^{-1}} \left(\frac{\lambda_2 + \lambda_3 - \phi_{\pi^*}}{\lambda_2\lambda_3} \right). \tag{B.50}$$

We want to find how ψ_π affects the role of the backward-looking component in breaking down Neo-Fisherism. Note that eigenvalues λ_2 and λ_3 are functions of the model parameters including τ and ψ_π and

$$\begin{aligned}
\frac{\lambda_2+\lambda_3-\phi_{\pi^*}}{\lambda_2\lambda_3} &= \frac{\frac{1+\gamma_f+\tilde{\kappa}\sigma^{-1}}{\gamma_f}-\lambda_1-\phi_{\pi^*}}{\frac{\gamma_b}{\gamma_f\lambda_1}} \\
&= \frac{(1+\gamma_f+\tilde{\kappa}\sigma^{-1})\lambda_1-\gamma_f\lambda_1^2-\gamma_f\phi_{\pi^*}\lambda_1}{\gamma_b} \\
&= \frac{(1+\beta\tau(1-\varrho)+\beta+\kappa\sigma^{-1})\lambda_1-\beta\lambda_1^2-\beta\phi_{\pi^*}\lambda_1}{\tau(1-\varrho)} \\
&= \frac{\beta\lambda_1(1-\lambda_1)+\{\beta\tau(1-\varrho)+\kappa\sigma^{-1}+(1-\phi_{\pi^*}\beta)\}\lambda_1}{\tau(1-\varrho)} \\
&= \frac{\lambda_1\{\beta(1-\lambda_1)+\beta\tau(1-\varrho)+\kappa\sigma^{-1}+(1-\phi_{\pi^*}\beta)\}}{\tau(1-\varrho)}. \qquad \text{(B.51)}
\end{aligned}$$

Because $0<\lambda_1<1$,

$$\frac{\lambda_2+\lambda_3-\phi_{\pi^*}}{\lambda_2\lambda_3}>0. \qquad \text{(B.52)}$$

It implies that as $\frac{\lambda_2+\lambda_3-\phi_{\pi^*}}{\lambda_2\lambda_3}$ gets smaller, Ω in (B.50) gets smaller.

In addition, from the fact that $lim_{\psi_\pi\to\infty}\lambda_1=0$ in Lemma 6 and the equation in (B.51), we can show that

$$lim_{\psi_\pi\to\infty}\frac{\lambda_2+\lambda_3-\phi_{\pi^*}}{\lambda_2\lambda_3}=0. \qquad \text{(B.53)}$$

Therefore, under strict inflation targeting Ω is the smallest and the model is the least likely to exhibit Neo-Fisherism.

It is straightforward to show that setting $\varrho=0$ does not alter the results. ■

Lemma 7 *As $\psi_\pi\to\infty$, the solution for i_t in the model with the Taylor-type rule approaches that in the model with strict inflation targeting.*

Proof. Now, we confirm our conditions in the case of the solutions for i_t. We will show $\lim_{\psi_\pi\to\infty}\Gamma_1=\Psi_1$ and $\lim_{\psi_\pi\to\infty}\Gamma_0=\Psi_0$. Because $\Gamma_1=\lambda_1\psi_\pi$ and $\Psi_1=\frac{\tau(1-\varrho)}{\sigma^{-1}\kappa}$, it is straightforward to show that $\lim_{\psi_\pi\to\infty}\Gamma_1=\Psi_1$ from Lemma 6. Further, because

$$\begin{aligned}
\Gamma_0 &= \frac{1}{\lambda_2-\phi_{\pi^*}}\frac{1}{\lambda_3-\phi_{\pi^*}}\frac{\psi_\pi\kappa\sigma^{-1}}{\beta} \\
&\times\left\{\phi_{\pi^*}-(1-\varrho)\frac{(1-\phi_{\pi^*})(1-\beta\phi_{\pi^*})}{\kappa\sigma^{-1}}-\frac{\tau(1-\varrho)}{\kappa\sigma^{-1}}\{1+\beta(1-\phi_{\pi^*})\}+\frac{\tau(1-\varrho)}{\kappa\sigma^{-1}}\left(\frac{\lambda_2+\lambda_3-\phi_{\pi^*}}{\lambda_2\lambda_3}\right)\right\},
\end{aligned}$$

$\lim_{\psi_\pi \to \infty} \frac{1}{(\lambda_2 - \phi_{\pi^*})(\lambda_3 - \phi_{\pi^*})} \frac{\psi_\pi \kappa \sigma^{-1}}{\beta} = 1$, and $\lim_{\psi_\pi \to \infty} \frac{\lambda_2 + \lambda_3 - \phi_{\pi^*}}{\lambda_2 \lambda_3} = 0$, it is straightforward to show that $\lim_{\psi_\pi \to \infty} \Gamma_0 = \Psi_0$.

■

C The New Keynesian model of the U.S. economy

In this appendix, we introduce a New Keynesian DSGE model, which is estimated for the U.S. economy in Section 4 and explain how the model is estimated based on the U.S data.

C.1 Model

This model involves three main equations; an hybrid IS equation, a hybrid NKPC, and a Taylor-type monetary policy rule with interest-rate smoothing. The model also includes the evolution of the inflation target and has four exogenous economic shocks: a preference shock, a cost-push shock, a monetary policy shock, and an inflation target shock.

The log-linearized version of the model for estimation is as follows.

$$y_t = \frac{1}{1+h} E_t y_{t+1} + \frac{h}{1+h} y_{t-1} - \frac{1-h}{(1+h)\sigma} [i_t - E_t \pi_{t+1}] + \eta_t, \tag{C.1}$$

$$[\pi_t - \tau(1-\varrho)\pi_{t-1} - \varrho\pi_t^*] = \beta E_t \left[\pi_{t+1} - \tau(1-\varrho)\pi_t - \varrho\pi_{t+1}^*\right] + \kappa y_t + \mu_t, \tag{C.2}$$

$$i_t = \phi_i i_{t-1} + (1-\phi_i)\left[\psi_\pi(\pi_t - \pi_t^*) + \psi_y y_t\right] + \epsilon_{i,t}, \tag{C.3}$$

$$\eta_t = \phi_\eta \eta_{t-1} + \epsilon_{\eta,t}, \tag{C.4}$$

$$\mu_t = \phi_\mu \mu_{t-1} + \epsilon_{\mu,t}, \tag{C.5}$$

$$\pi_t^* = \phi_{\pi^*} \pi_{t-1}^* + \epsilon_{\pi^*,t}, \tag{C.6}$$

where x_t denote the percentage deviation of a variable X_t from its steady-state $\bar{X}$.

The hybrid IS curve (C.1) is the intertemporal Euler equation with habit formation in consumption and is derived from the households' optimization problem where σ^{-1} denotes the intertemporal substitution elasticity and h denotes the degree of consumption habit. The preference shock η_t in (C.1) follows the AR(1) process in (C.4). The hybrid NKPC in (C.2) is produced from a continuum of monopolistically competitive firms' profit maximization problem where κ is the slope of the Phillips curve, τ is the indexation to past inflation, and ϱ is the indexation to the inflation target. The quarterly discount rate β is calibrated to 0.99. The cost-push shock μ_t in (C.2) evolves according to the univariate AR(1) process in (C.5). The monetary authority follows a Taylor-type rule by adjusting the nominal interest rate in response to deviations of inflation from its target and output from its flexible price level and is assumed to smooth the policy rate. The monetary policy shock $\epsilon_{i,t}$ captures the unexpected deviation from the policy rule. The inflation target π_t^* follows the AR(1) process as in (C.6).

In order to construct a likelihood function, we utilize the Kalman filter given a state-space form of measurement and transition equations. The measurement equations link observables to unobservable state variables in the transition equation system, which is given by the solution to the model.

The measurement equations take the form:

$$Y_t = y_t \tag{C.7}$$

$$INF_t = 4\bar{\pi}^Q + 400\pi_t \tag{C.8}$$

$$FFR_t = 4\bar{\pi}^Q + 4\bar{r}^Q + 400 i_t \tag{C.9}$$

$$INF_{t+1,t+40} = 4\bar{\pi}^Q + 400\left(\frac{1}{40}\sum_{j=1}^{40} E_t \pi_{t+j}\right) \tag{C.10}$$

where Y_t is the HP-filtered log of GDP per capita, FFR_t is the annualized Federal funds rate, INF_t is the annualized quarter to quarter CPI inflation rate, and $INF_{t+1,t+40}$ is the annualized average expected CPI inflation rate over the next 10 years.

The estimation is based on postwar U.S. quarterly data from 1982:Q1 to 2009:Q2, excluding the passive monetary policy period and the Volcker-disinflation period as shown in Lubik and Schorfheide (2004) and the zero-lower bound period. Following Del Negro et al. (2007) and Del Negro, Giannoni and Schorfheide (2015), we include the 10-year CPI inflation expectations as an observable in the estimation of our model to track low-frequency fluctuations in inflation, which are associated with the time-varying inflation target. The

Table C.1: Estimates of structural parameters

	Prior			Posterior					
				Benchmark			Restricted ($\varrho = 0$)		
Parameter	Density	Mean	SD	Mean	10th	90th	Mean	10th	90th
$\bar{\pi}^Q$	Normal	0.50	0.20	0.6609	0.5322	0.8273	0.6479	0.5160	0.8213
$\bar{r}^Q$	Normal	0.50	0.20	0.3920	0.2657	0.5147	0.3941	0.2626	0.5216
σ	Gamma	1.00	0.40	1.6016	1.2081	1.9990	1.6033	1.2091	2.0134
h	Beta	0.50	0.10	0.6448	0.5531	0.7414	0.6477	0.5559	0.7425
κ	Gamma	0.50	0.10	0.1099	0.0753	0.1375	0.1078	0.0737	0.1351
τ	Beta	0.50	0.25	0.2603	0.0002	0.4419	0.0549	0.0019	0.0873
ϱ	Beta	0.50	0.25	0.7566	0.5915	0.9987	0.0000	–	–
ψ_π	Normal	1.70	0.30	1.4571	1.0930	1.7406	1.4531	1.0723	1.7179
ψ_y	Normal	0.30	0.20	0.4223	0.2786	0.5603	0.4242	0.2728	0.5619
ϕ_i	Beta	0.60	0.15	0.9017	0.8844	0.9200	0.9018	0.8851	0.9218
ϕ_μ	Beta	0.60	0.15	0.6871	0.6099	0.7868	0.6880	0.6096	0.7864
ϕ_η	Beta	0.60	0.15	0.3396	0.2311	0.4364	0.3280	0.2233	0.4288
ϕ_{π^*}	Beta	0.60	0.15	0.9816	0.9749	0.9891	0.9818	0.9750	0.9895
$100\sigma_\eta$	Inv.G	0.20	0.50	0.3697	0.3191	0.4172	0.3750	0.3211	0.4236
$100\sigma_\mu$	Inv.G	0.20	0.50	0.1590	0.1270	0.1873	0.1577	0.1242	0.1839
$100\sigma_i$	Inv.G	0.20	0.50	0.1657	0.1499	0.1799	0.1666	0.1498	0.1813
$100\sigma_{\pi*}$	Inv.G	0.030	0.05	0.0272	0.0177	0.0354	0.0286	0.0183	0.0377

10-year CPI inflation expectations data from the Survey of Professional Forecasters are available from 1991:Q4 and so we construct the data set prior to 1991:Q4 using the 10-year CPI inflation expectations from the Blue Chip Economic Indicators survey as in Del Negro, Giannoni and Schorfheide (2015). Both 10-year CPI inflation expectations are obtained from the Philadelphia Fed's Real-Time Data Research Center. Note that the Blue Chip forecasts were taken twice a year and we linearly interpolate missing observations prior to 1991:Q4. The parameters $\bar{\pi}^Q$ and $\bar{r}^Q$ are related to the steady-states of the model economy as follows.

$$\bar{\pi}^Q = 100\bar{\pi}, \quad \bar{r}^Q = 100\bar{r}.$$

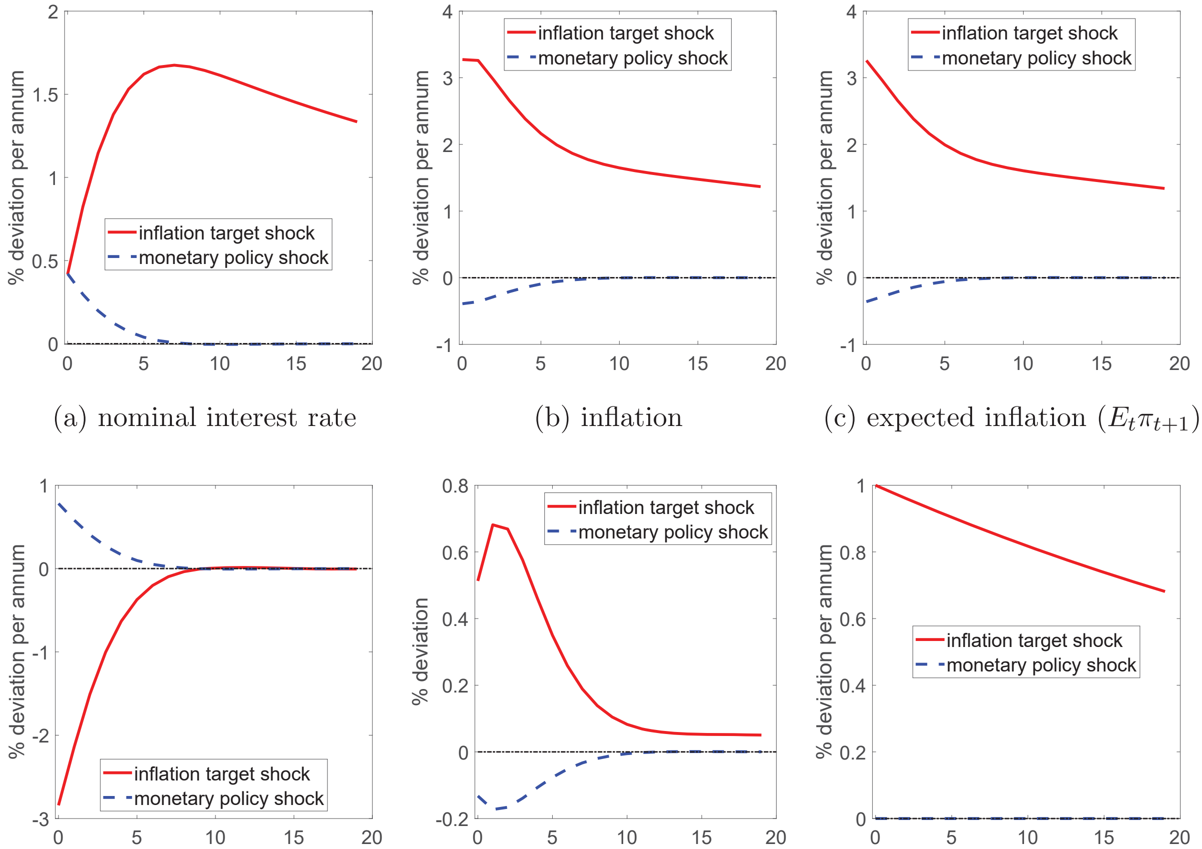
C.2 Estimation

Table C.1 summarizes the prior distribution that we use for the estimation of the model presented in the previous section. Our key parameters are those related to backward-looking elements. We use the prior of a Beta distribution with mean 0.50 and standard deviation 0.25 for h, τ, and ϱ so that it is reasonably diffuse and close to a Uniform prior on (0,1). The prior for σ is a Gamma distribution with mean 1.0 and standard deviation 0.40 so that the implied elasticity of intertemporal substitution from the prior is around one. The prior for κ is a Gamma distribution with mean 0.50 and standard deviation 0.10. The standard deviation of the innovation to the inflation target ($\sigma_{\pi*}$) governs the drift of the inflation target, and we adopt an Inverse Gamma distribution with mean 0.030 and standard deviation 0.05. We also choose priors for reaction coefficients to inflation and the output gap in a Taylor-type rule with a Normal distribution with mean 1.7 and standard deviation 0.30 for ψ_{π} and a Normal distribution with mean 0.3 and standard deviation 0.20 for ψ_{y}. These priors for the monetary policy rule are set to be consistent with those of the previous literature (e.g. Cogley, Primiceri and Sargent (2010) and Del Negro, Giannoni and Schorfheide (2015)). For all persistence parameters including ϕ_{π^*}, we use a Beta prior with mean 0.60 and standard deviation 0.15. For the standard deviations of the other shocks, we choose fairly disperse priors of an Inverse Gamma distribution with mean 0.20 and standard deviation 0.50. The prior mean for the quarterly steady-state inflation rate ($\bar{\pi}^{Q}$) is set to 0.50 with standard deviation 0.20 so that it is consistent with the inflation target of 2%. The prior mean for the steady-state real rate ($\bar{r}^{Q}$) is also set to 0.50 with standard deviation 0.20. We report the posterior means and the 10th and 90th percentiles of the posterior distributions of all the estimated model parameters in Table C.1.

C.3 Impulse responses to an inflation-target shock vs. to a monetary-policy shock

In Figure C.1, we jointly plot the impulses responses to an inflation-target shock and to a monetary-policy shock. The figure shows that an inflation-target shock is not equivalent to a conventional monetary-policy shock in a Taylor-type rule. While a contractionary monetary-policy shock similarly calls the monetary authority to raise the nominal interest rate, the responses of output, inflation, and the real interest rates are all markedly different. Here, a comparable increase in the nominal interest rate under a contractionary monetary-policy

Figure C.1: Impulse response functions to an inflation-target shock and a contractionary monetary-policy shock



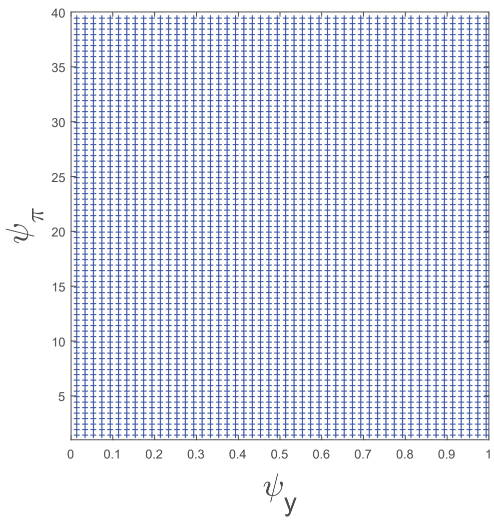
(a) nominal interest rate (b) inflation (c) expected inflation ($E_t \pi_{t+1}$)

(d) real interest rate (e) output gap (f) inflation target

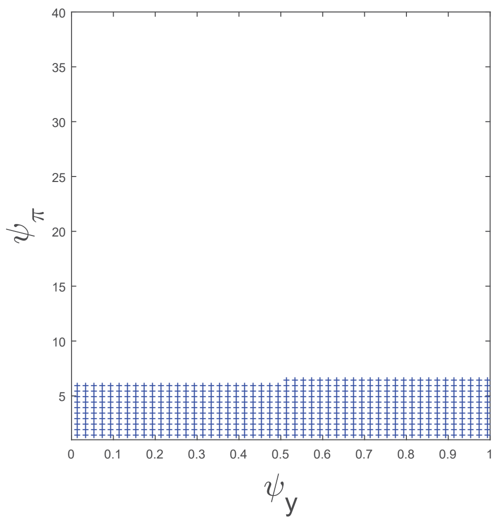
Note: This figure plots the impulse response functions to (i) a 1% inflation target shock and (ii) a contractionary monetary policy shock whose magnitude is set to generate the same response of i_t on impact (i.e., period 0) as that on impact to the 1% inflation target shock. All parameter values are set as in the benchmark model of the U.S. economy presented in Table 1.

shock raises the real interest rate and lowers inflation and output.

Figure D.1: The Neo-Fisherian region in the (ψ_y, ψ_π) parameter space under a Taylor-type rule



(a) $\varrho = 0.76$, $\tau = 0.26$ (U.S. estimates)



(b) $\varrho = 0$, $\tau = 0.50$

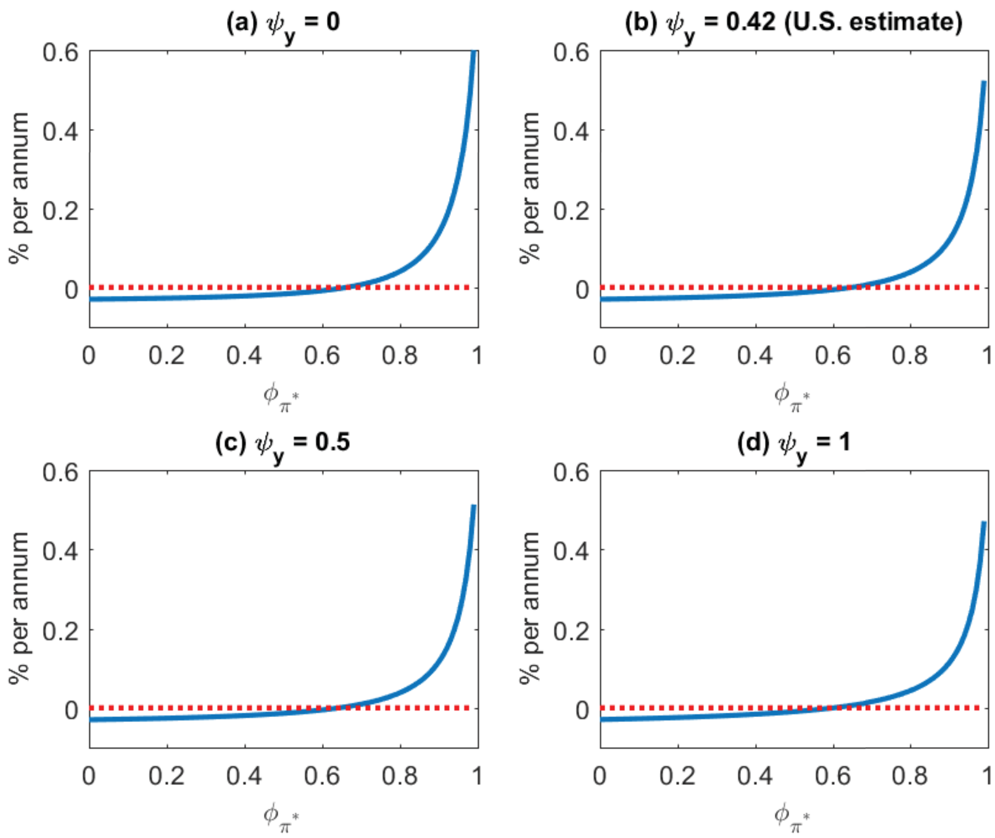
Note: The sign of '+' indicates a pair of (ψ_y, ψ_π) values associated with the positive comovement between inflation and the nominal interest rate conditional on a change in the inflation target. Except for the stated values of ϱ and τ in each subfigure, all parameter values are set as in the benchmark model of the U.S. economy presented in Table 1.

D The effect of the output coefficient in the Taylor rule (ψ_y) on the comovement

In this appendix, we investigate how the comovement between the inflation target, inflation, and the nominal interest rate varies with the Taylor-rule output feedback coefficient, ψ_y.

Figure D.1 plots the Neo-Fisherian region in the (ψ_y, ψ_π) parameter space under the benchmark parameterization based on the estimates for the U.S. economy in Panel (a) and under a counterfactual parameterization in which we remove the indexation to the inflation target and allow for a higher degree of backward-looking indexation with $\varrho = 0$, $\tau = 0.50$ in Panel (b). Under the benchmark parameterization, we have a positive comovement for any $\psi_\pi \in (1, 40]$ and $\psi_y \in [0, 1]$. In Panel (b) with a high degree of backward-looking indexation, the Neo-Fisherian cut-off value in terms of ψ_π value only marginally increases. This increase suggests that the comovement is more likely as the central bank more strongly responds

Figure D.2: Contemporaneous response of i_t to an inflation-target shock as a function of ϕ_{π^*} for various values of ψ_y



Note: This figure plots the contemporaneous (period-0) response of the nominal interest rate, i_t, to a 1% per annum inflation target shock for various values of inflation target persistence, ϕ_{π^*}, and the Taylor-rule output feedback coefficient, ψ_y. All other parameter values are set as in the benchmark model of the U.S. economy presented in Table 1.

to output fluctuations (higher ψ_y), i.e. as it runs a less strict or more flexible inflation targeting policy. Both panels, nevertheless, show that for the range of plausible values of ψ_y ($\psi_y \in [0, 1]$), the output feedback coefficient in the Taylor rule is not an important determinant of the comovement between the inflation target, inflation, and the nominal interest rate compared to the inflation feedback coefficient.

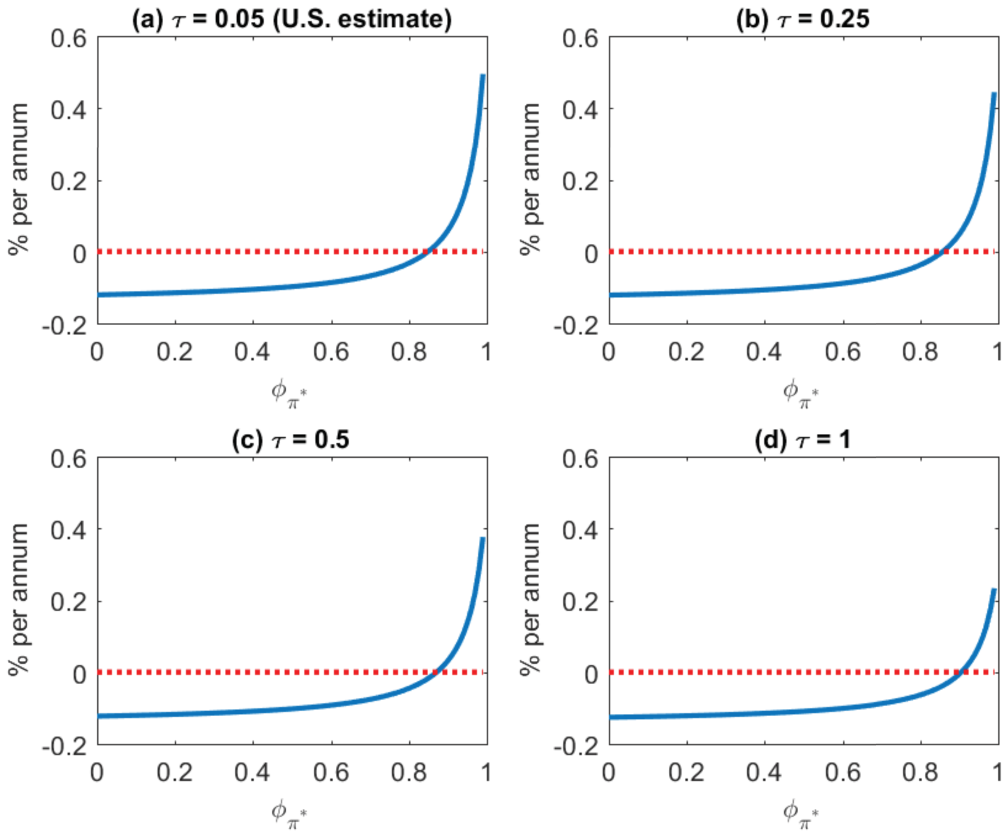
As a further check, Figure D.2 plots the contemporaneous response of i_t as a function of ϕ_{π^*}, for four different values of ψ_y. The Neo-Fisherian cut-off value in terms of ϕ_{π^*} value is again largely insensitive to various plausible values of ψ_y. Here, the cut-off value only decreases from 0.66 when $\psi_y = 0$ to 0.60 when $\psi_y = 1$.

E More results on the restricted model of the U.S. Economy ($\varrho = 0$)

In this appendix, we present additional results based on the alternative parameterization, where we restrict $\varrho = 0$ in the estimation. See the last three columns of Table C.1 for the resulting parameter estimates. We use the posterior means as parameter values to generate the subsequent figures.

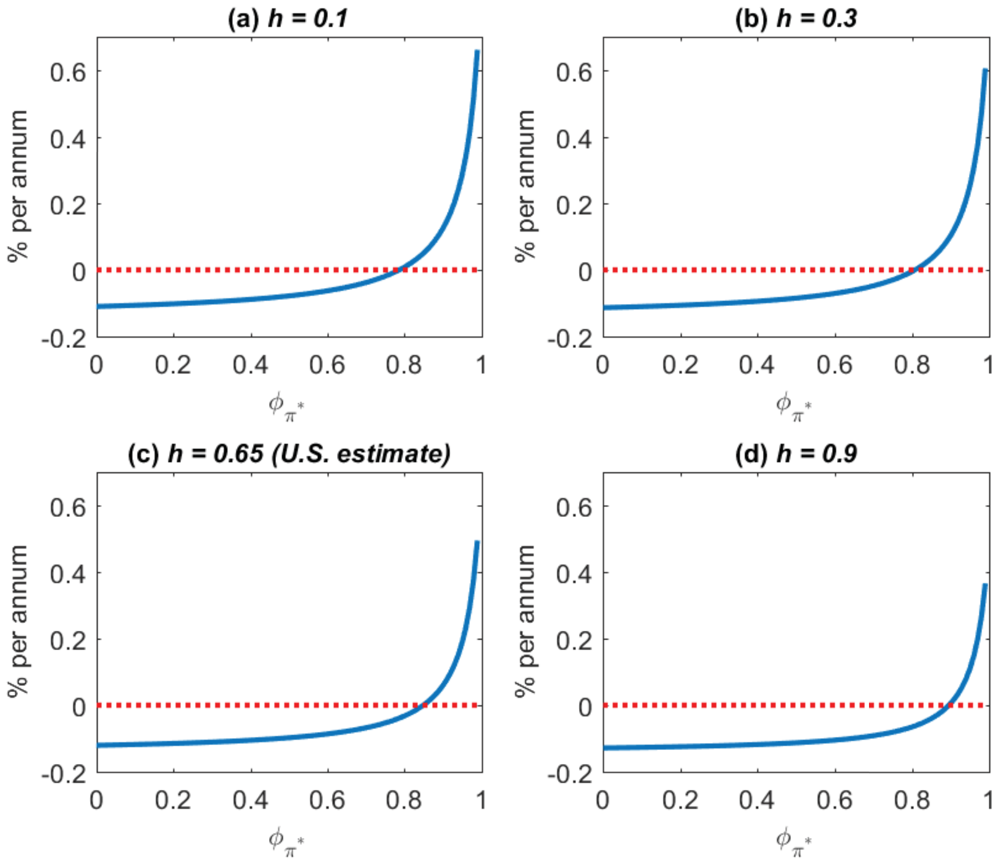
Figure E.1 and Figure E.2 redo the analysis in Figure 10 and Figure 11, respectively. In both figures, the Neo-Fisherian cut-off value in terms of the persistence parameter ϕ_{π^*} continues to be largely insensitive to different values of τ and h. These additional results support the notion that the specification of a partial price indexation to the inflation target does not alter our conclusion that the U.S. economy is most likely to exhibit Neo-Fisherism.

Figure E.1: Contemporaneous response of i_t to an inflation-target shock as a function of τ and ϕ_{π^*} (Restricted model with $\varrho = 0$)



Note: This figure plots the contemporaneous (period-0) response of the nominal interest rate, i_t, to a 1% per annum inflation target shock for various values of the degree of indexation to past inflation, τ, and the inflation target persistence, ϕ_{π^*}. All other parameter values are set as in the restricted model with $\varrho = 0$ of the U.S. economy presented in Table 1.

Figure E.2: Contemporaneous response of i_t to an inflation-target shock as a function of h and ϕ_{π^*} (Restricted model with $\varrho = 0$)



Note: This figure plots the contemporaneous (period-0) response of the nominal interest rate, i_t, to a 1% per annum inflation target shock for various values of the habit parameter, h, and the inflation target persistence, ϕ_{π^*}. All other parameter values are set as in the restricted model with $\varrho = 0$ of the U.S. economy presented in Table 1.