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CAMA Working Paper 34/2019
May 2019

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JEL Classification

C51, C58, Q41, Q47

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ISSN 2206-0332

Residual Shape Risk on Natural Gas Market with Mixed Jump Diffusion*

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1 Introduction

A liberalization of the European gas markets under the Directive 2003/55/EC (EC 2003) led to a fragmentation of natural gas supply chain. One of many consequences of this "unbundling" process (Stern & Rogers 2017) is a current move towards the situation when company supplying the retail customers is not able to purchase natural gas directly from its parent company, but it has to purchase gas on the open market. However the granularity of monthly baseloads available on the open market does not allow for perfect hedging of daily deliveries of natural gas to final consumers. This introduces a new source of commodity risk into natural gas markets. In this paper we therefore conceptualize this new subclass of commodity risk, which we call residual shape risk (RSR), and we empirically evaluate it on a real portfolio of a leading natural gas retail supplier in the Czech Republic.

Our RSR concept naturally fits into the risk management literature (Senior 1999) which standardly distinguishes three main categories; a credit risk, an operational risk, and a market risk. The market risk category is usually further divided into subcategories like interest rate risk, currency risk, commodity risk, etc. In this article we are concerned with commodity risk which arises with price movements of commodity on wholesale market. Hence the RSR belongs into commodity risk category.

The evaluation of RSR requires modelling of prices of evaluated commodity, in our application natural gas. The literature (Baum *et al.* 2018; Benth *et al.* 2008; Borovkova & Mahakena 2015; Brix *et al.* 2018; Cao *et al.* 2018; Gomez-Valle *et al.* 2017a;b; Hsu *et al.* 2017; Mason & Wilmot 2014; Mishra & Smyth 2016; Safarov & Atkinson 2017) shows that the gas prices and energy commodities in general have quite complex price distribution as compared to financial assets. Gas prices commonly depart from normality by exhibiting heavy tails and a leptokurtic shape (Benth *et al.* 2008). They also exhibit jumps (Cao *et al.* 2018; Ficura & Witzany 2016; Mason & Wilmot 2014), a time-varying volatility (Baum *et al.* 2018; Brix *et al.* 2018), and a mean reversion (Brix *et al.* 2018; Hsu *et al.* 2017). Moreover, they are affected by many other factors like storage, weather, seasonality and political events and decisions (Gomez-Valle *et al.* 2018).

As our financial risk metrics we use Value-at-Risk (VaR) and Expected Shortfall (ES). VaR is the financial loss that is not exceeded with probability $1 - \alpha$, where α is the confidence level. For discussion of VaR in the energy markets risk management see Andriosopoulos & Nomikos (2015). While VaR is a standard financial measure used by Basel II and Basel III financial regulatory framework, its limitation is that it does not provide any indication of how much may be lost if extreme tail events happen. Therefore it is usually complemented by a computation of Expected Shortfall, which is also alternatively known as Conditional Value at Risk (CVaR). ES (CVaR) measures the average of worst losses. The ES at level α is the expected return of a portfolio in α percent of worst cases (Baum *et al.* 2018). Obviously, for the computation of VaR and ES the selection of appropriate distribution is crucial (Baran & Witzany 2012; Hung *et al.* 2008; Khindanova & Atakhanova 2002).

In the rest of this paper we firstly define RSR and explain its calculation in the section 2. Then we continue with description of data and empirical evaluation of RSR on Czech natural gas market in the section 3. In the section 4 we summarize our results and conclude.

2 Conceptualization of Residual Shape Risk

2.1 Definition of Residual Shape Risk

As already mentioned before, the residual shape risk stems from insufficient liquidity of wholesale products for hedging shaped sales. Thus, it can be represented for natural gas markets as a weighted difference between forward and spot price of natural gas where the weight is the deviation of the daily volume around the volume hedged at the forward market. In the empirical section of this paper, we use volumetric hedging. It means that the volume purchased at forward market is an average volume in the period which corresponds to a length of standard future product. In the Czech market, which we use in the empirical section, the shortest standard product, which can be traded on forward market, is one month. Thus, we evaluate the RSR against a deviation of daily volumes from average volumes in particular months. Hence, we define the RSR as

$$RSR^p = \frac{\sum_{t=1}^{N}(V_t - \overline{V}_t^m)(S_t - F_t)}{\sum_{t=1}^{N} V_t}, \tag{1}$$

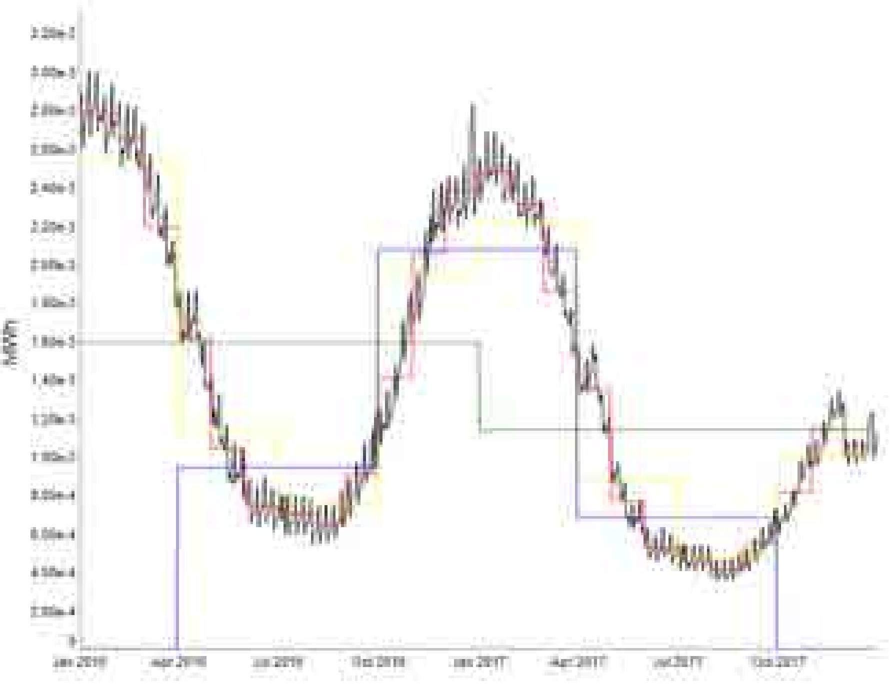
where index p denotes particular profile with a length of N. V_t is forecasted consumption volume in a day t. $\overline{V}_t^m$ is the average volume bought on monthly forward product market in month m, i.e monthly baseload. The second part of the product in the numerator is a difference of spot price at delivery S_t and forward price F_t. Unit of prices is EUR/MWh. We divide the numerator by the total volume of the profile in order to obtain profit or loss per MWh of an energy commodity. The forward price F_t is a weighted price of standard products consequently purchased and sold on forward market when shaping the profile. We shape the profile successively as individual products become liquid.

For better understanding now we look at one month separately. We sketch how F_t is obtained for days in August in the equation below, denoted as F_t^8. When customer uses natural gas for heating, August is usually month with the lowest consumption within the year. Such profile would be similar to the one in the Figure 1. Thus, this month would have to be hedged by a purchase of the yearly baseload firstly. Say at time t_1. Again $\overline{V}^Y$ represents average volume in the year. Then when summer product becomes liquid in time t_2, summer months would be rehedged by sell of difference between average year volume and average summer volume. The volume would be sold for $F_{t_2}^S$. Subsequently, retailer would rehedge again when third quarter becomes liquid in time t_3 by the difference between average volume in summer and third quarter ($Q3$). Last rehedge would be done in order to end up with average consumption in August. Hence, F_t forward price is price weighted by volumes traded during hedging the profile.

$$
\begin{aligned}
F_t^8 =& \frac{F_{t_1}^Y \overline{V}^Y - F_{t_2}^S(\overline{V}^Y - \overline{V}^S) - F_{t_3}^{Q3}(\overline{V}^S - \overline{V}^{Q3}) - F_{t_4}^8(\overline{V}^{Q3} - \overline{V}^8)}{\overline{V}^Y - (\overline{V}^Y - \overline{V}^S) - (\overline{V}^S - \overline{V}^{Q3}) - (\overline{V}^{Q3} - \overline{V}^8)} \\
=& \frac{\overline{V}^Y(F_{t_1}^Y - F_{t_2}^S) + \overline{V}^S(F_{t_2}^S - F_{t_3}^{Q3}) + \overline{V}^{Q3}(F_{t_3}^{Q3} - F_{t_4}^8) + \overline{V}^8 F_{t_4}^8}{\overline{V}^8}
\end{aligned} \tag{2}
$$

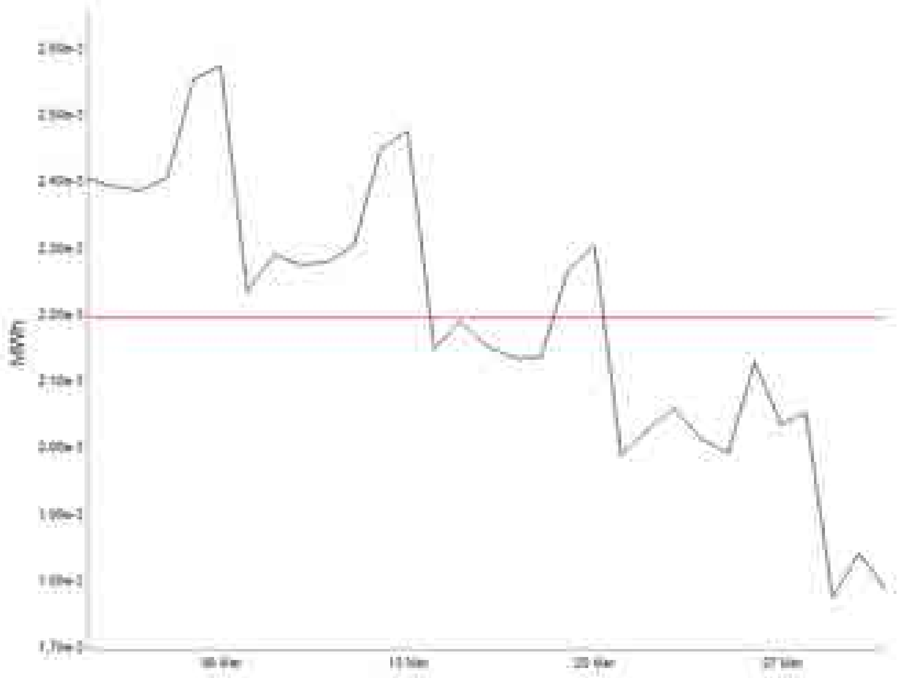
Note that every contract becomes liquid in different point in time, thus once yearly contract is purchased forward price fluctuates until the next shorter contract becomes liquid. Hence, the F_t price for day t is affected by every forward contract, which contains that day. We can see the sketch of hedging the profile in the Figure 1. Since we are dealing with a retail supply of an energy commodity, we always want to have an average volume in the corresponding period, i.e. we hedge the profile volume neutral. The sum of residual positions is zero. We repeat this algorithm as individual blocks become liquid until we hedge profile by monthly baseloads. Hence, we obtain a forward price for every day t which is a weighted price of these successive purchases and sales at the forward market.

Figure 1: Two years consumption profile with baseload products.



We determine the RSR as a profit and loss distribution. This means that positive value of the RSR is actually loss. Whenever additional volume has to be purchased on the spot market and the spot price is higher then the forward price was, loss occurs. This means that in formula (1) $V_t > \overline{V_t}^m$ and $S_t > F_t$. Similarly, when both inequalities are opposite, i.e. when there is a long position in the particular gas day and the weighted forward price is higher than the spot price, the loss occurs again. Loosely speaking, given the higher realized spot price, higher volume was purchased on the forward market than it should have been. Contrary, whenever these two differences have opposite sign a gain is obtained. For example, the residual position was purchased for the spot price that was lower than the forward price. For better understanding, observe the Figure 2. It shows shape of a consumption in March along with purchased monthly baseload product. The y-axis shows daily consumption as a percentage of a yearly consumption. A supplier has a short position in the first half of the month and if spot price exceeds the forward price, he incurs loss. On the contrary, at the end of the month there is a long position and if the spot price is higher than forward one, a supplier incurs gain.

Figure 2: Consumption in March hedged by monthly baseload.



We derive forward prices as expected spot prices at delivery. Hence, we expect the difference between them to be zero. Nevertheless, the RSR will not be zero in general. One would have to hedge the profile value neutral against expected spot prices. As we will use volumetric hedge, we do not expect the loss distribution to have exactly zero mean. Lastly, to determine the size of this risk we use Monte Carlo simulation of spot prices. In this way the hedge strategy described above is realized on simulated price paths to obtain the RSR loss distribution. We employ conventional five percent Value at Risk measure along with the Expected Shortfall measure, which captures the tail distribution better.

2.2 Calculation of Residual Shape Risk

First step of the RSR estimation is to model spot prices of an evaluated commodity, in our case natural gas. We follow approach suggested in Benth *et al.* (2008). We model the spot price dynamics with Ornstein-Uhlenbeck (OU) process using a sum of Gaussian and two compound Poisson processes to create a more complex jump mixed diffusion process. This induces desired leptokurtic shape for price innovations. As a second step, forward prices are then derived as expected spot prices at delivery using characteristics of spot price processes. In the third step we choose an appropriate risk evaluation metrics and apply it to a particular consumer profile.

2.3 Spot Model

Benth *et al.* (2008) presents a general form of geometric stochastic process for energy commodity spot price. However, we choose rather simple model specification for the purpose of our analysis. We include into our model one normal and one jump Ornstein-Uhlenbeck process. The model is described by the following equations.

$$\ln S(t) = \ln \Lambda(t) + X(t) + Y(t), \tag{3}$$

with OU processes $X(t)$ and $Y(t)$ following dynamics

$$dX(t) = -\alpha X(t)dt + \sigma dB(t), \tag{4}$$

and

$$dY(t) = -\alpha Y(t)dt + dI(t). \tag{5}$$

The term $\Lambda(t)$ in the first equation represents continuously differentiable deterministic seasonal function. Equations (4) and (5) represent normal and jump part of the price process, respectively. Such model is mean reverting process. We assume to have constant speeds of mean reversion common for the diffusion $B(t)$ and the jump $I(t)$ part. The coefficient α is the mean reverting parameter. Such parameter determines how fast the spot price reverts towards its seasonal mean $\Lambda(t)$. σ represents standard deviation of the normal innovations $B(t)$. The model further assumes that $B(t)$ and $I(t)$ are independent. This model specification is an extension of the Schwartz one-factor model (Schwartz 1997) obtained by including jumps.

The process of logarithmic price (3) has dynamics

$$d\ln S(t) = d\ln\Lambda(t) - (\ln S(t) - \ln\Lambda(t))dt + \sigma dB(t) + dI(t). \tag{6}$$

2.4 Forward Model

The fundamental pricing formula for forward contracts, which leads to arbitrage free relation between the spot and forward is presented in the equation (7). It says that under some probability measure Q, the forward price at a time t with a delivery at a time τ is an expected spot price at delivery given the information we have at the time t.

$$f(t,\tau) = \mathbb{E}_Q[S(\tau) \mid \mathcal{F}_t]. \tag{7}$$

We will not go into theoretical details here as they are thoroughly described in Benth *et al.* (2008). However, similar relation can be derived for a price of gas future contract, which delivers energy between time τ_1 and τ_2. The relation to spot price is similar. The price of a contract settled at the end of the delivery period is an average of expected spot prices within that period.

$$F(t,\tau_1,\tau_2) = \mathbb{E}_Q[\int_{\tau_1}^{\tau_2} \frac{1}{\tau_2-\tau_1} S(u)du \mid \mathcal{F}_t]. \tag{8}$$

Moreover, Benth *et al.* (2008) shows that under certain condition the future price can be derived from prices of forwards which deliver on particular days in the delivery period. It means that future price is an average price of these forward prices it our daily granularity market. The relation is show in the formula (9).

$$F(t,\tau_1,\tau_2) = \int_{\tau_1}^{\tau_2} \frac{1}{\tau_2-\tau_1} f(t,u)du. \tag{9}$$

When we take our specification of the spot price process in equation (3), the expectation in (7) becomes (Benth *et al.* (2008))

$$f(t,\tau) = \Lambda(\tau)\Theta(t,\tau,0)\exp(e^{-\alpha(\tau-t)}X(t) + e^{-\alpha(\tau-t)}Y(t)), \tag{10}$$

where $\Theta(t,\tau,0)$ is given as

$$\ln\Theta(t,\tau,0) = \psi(t,\tau,-ie^{-\alpha(\tau-t)}) + \frac{1}{2}\sigma^2\int_t^\tau (e^{-\alpha(\tau-u)})^2 du. \tag{11}$$

with $\psi(-ic)$ being the logarithm of the moment generating function of increment processes $Y(t)$, also called a cumulant function. The integral in the second part of the expression (11) represents cumulant function for Brownian motion. The zero in the function $\Theta(t,\tau,0)$ means that we are deriving expectation with market price of risk equal to zero, thus under equivalent measure $\mathbb{P}$. The expression in (10) can be rewritten then as

$$f(t,\tau) = \Lambda(\tau)\Theta(t,\tau,0)\left(\frac{S(t)}{\Lambda(t)}\right)^{e^{-\alpha(\tau-t)}}. \tag{12}$$

3 Empirical Evaluation

3.1 Data

For the purpose of our empirical analysis we choose the Czech OTE price index. "OTE, a.s." is the Czech electricity and gas market operator, which organizes day-ahead market. The OTE price index is a daily index, which represents country-wide price of natural gas at a particular gas day and we treat it as a spot price index as the gas day is the lowest granularity of the Czech natural gas market. The OTE index is derived as a weighted average of trades executed on the Intra-Day Gas market, where weights are volumes of particular trades.

The time series of our leading example ranges between 1.1.2016 and 31.12.2017. It consists of 731 daily observations. The data were downloaded from the web page of the market operator, the OTE. These data are published every day and they are publicly available[1]. Summary statistics of observed natural gas prices are presented in the Table 1.

Further, we employ data about consumers portfolio of a leading Czech gas retail supplier for a period between 1.1.2016 and 31.12.2017. It is used as a consumption profile for the purpose of our analysis. The profile consists of initial forecasts of consumption for households and small entrepreneurs. These forecasts are made using so-called standardized load profiles (SLP). SLP represents typical shape of annual consumption for particular group of customers. It predicts customers consumption based on expected temperatures and days in a week. These diagrams are thoroughly described in Novak *et al.* (2017). Our profile consists of mixture of different types of SLPs.

Table 1: Data Summary - OTE spot price index

Statistic	N	Mean	St. Dev.	Min	Max
S	731	15.930	2.580	11.180	23.030

In order not to disclose a size of the portfolio we scale the volume to unity according

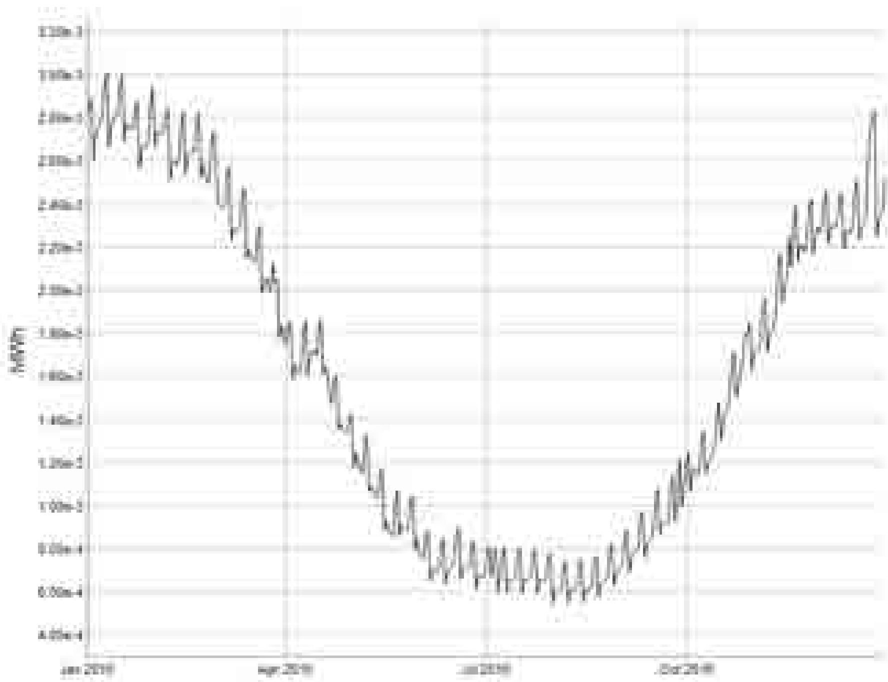
[1]Data available at OTE web page http://www.ote-cr.cz/statistika/rocni-zprava.

to (13):

$$c_d = \frac{V_d}{\sum_{d=1}^{N} V_d}. \tag{13}$$

We do this for both years. Thus, for each day we obtain a value that represents percentage of total year consumption on that day. An example of a consumption profile is presented in the Figure 3. We can see characteristic profile shape with higher consumption in winter along with a weekly pattern.

Figure 3: Yearly consumption profile.



3.2 Model Estimation

This section describes consecutive steps we take in order to appropriately estimate our model. We start with an analysis of an observed gas spot prices, where we obtain their distributional properties. Based on the analysis we fit a jump mixed diffusion process, which we use for the Monte Carlo (MC) simulation of spot and forward prices afterwards. Lastly, we use these simulations for an evaluation of the RSR over customer portfolio of a real company.

3.2.1 Spot Price Analysis

First, we look at a development of logarithmic spot prices. As there are no negative or zero prices in our sample, we can take logarithm of every price. Logarithmic prices are chosen deliberately in order to impose geometric nature of the model, which restricts simulation to generate non-negative prices. The development of logarithmic spot prices is presented in the Figure 4. From the visual inspection of the plot, we expect the price to follow some kind of seasonality and trend. It is a common feature of energy prices to exhibit some periodicity, natural gas in particular. Natural gas is widely used for heating. Thus, it tends to be more expensive in the winter when a demand increases.

Figure 4: Time series of logarithmic spot price.

We also check for outliers as they would affect fitted cyclical and mean function. To detect possible outliers we look at the differenced time series. In our case such series becomes series of log returns. In order to determine whether the observation is an outlier we adopt approach described by Benth *et al.* (2008). An observation is deemed to be outlying when it is lower than $Q_1 - 3 \times IQR$, or greater than $Q_3 + 3 \times IQR$, where Q_1 and Q_3 are lower and upper quartile, respectively. The IQR is interquartile range, defined as difference between the upper and the lower quartile. Following this rule six observation were found to be outlying. Hence, we replaced them with an average of preceding and following observation. The mean function is then estimated on this new trimmed time series.

There are few evident cycles to search for in the energy sector. As mentioned above we expect to find some form of yearly cycle, which represents an inclination of gas wholesale prices to grow in the winter and stay lower in the summer. However, we make use of the Fourier transform to precisely determine all possible cycles in the data. Following this approach, two signals were found. Firstly, the most important cycle was determined to be yearly as expected. Secondly, we found cycle with half year periodicity. The second cycle could be attributed to injecting of natural gas into underground storages during summer. Storage companies incur some additional costs when they have to reverse the flow in storage, i.e. to switch between injecting and withdrawing. Such situation can happen during high temperature days when there is a high demand for electricity to power air-conditions and gas fired power plants have to be switched on. Hence, it can cause temporal increase in spot prices. We use relationship defined in the equation (14) for modelling trend and seasonal components of spot prices.

$$ln\Lambda(t) = a_0 + a_1 t + a_2 cos(2\pi t/365)) + a_3 cos(2\pi t/182.5) \tag{14}$$

Table 2: Estimated coefficients of the mean level function

	Estimate	Std. Error	t value	Pr(>|t|)
$\widehat{a_0}$	2.566	0.006	412.289	0
$\widehat{a_1}$	0.001	0.00001	35.167	0
$\widehat{a_2}$	0.105	0.004	23.928	0
$\widehat{a_3}$	0.056	0.004	12.666	0

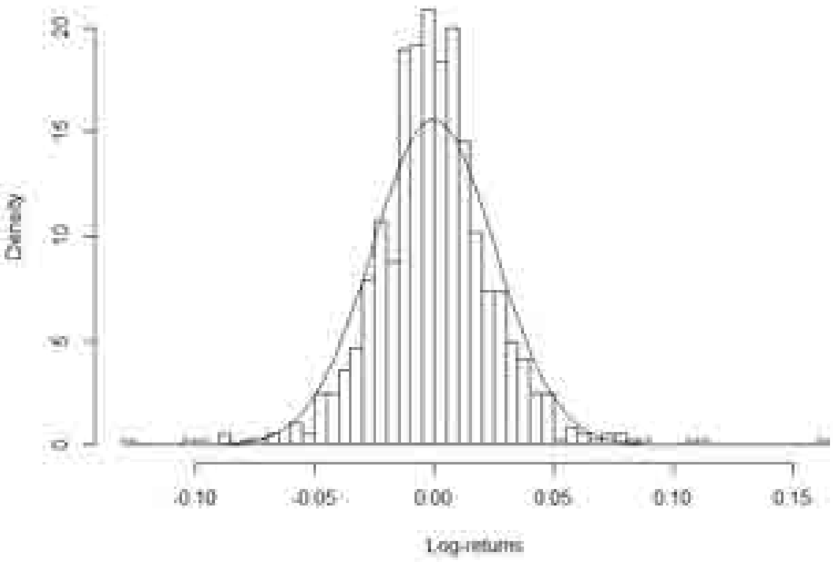
The linear time trend is often added to energy price series as a measure of inflation in price level. The cosines of year and half year component coincide in winter making the yearly cycle approximately 3 times stronger than the other one. The results are presented in the Table 2. We also estimated the effect of quarterly and weekly cycles, however they appeared to be insignificant. The fit of mean level function on logarithmic spot price is shown in the Figure 5.

Figure 5: Fitted mean level function on logarithmic prices.

By subtracting the mean level function from the original time series of logarithmic spot prices we obtain detrended and deseasonalized prices, which are ready for subsequent analysis. Further, when we look at the histogram of returns of this "detrended" series in the Figure 6, we see that it departs from normality. The histogram shows leptokurtic shape, i.e. higher probability of observation close to mean and heavier tails than normal distribution. This is a common feature of energy prices (Dukhanina *et al.* 2018; Hsu *et al.* 2017).

The Shapiro-Wilk normality test rejects the null hypothesis of normal distribution at one percent significance level. If we assume that the spot prices follow the geometric Brownian motion, then any test should reject null hypothesis of stationarity as the discrete time approximation of the Brownian motion is the AR(1) process with coefficient equal to unity, i.e. a random walk. Hence, we apply the Dickey-Fuller test to deseasonalized log prices which tests null hypothesis of unit root. The test yields statistic of -3.898. It corresponds to rejection of the null at roughly one percent confidence level. Also,

Figure 6: Histogram of log returns from detrended spot prices.



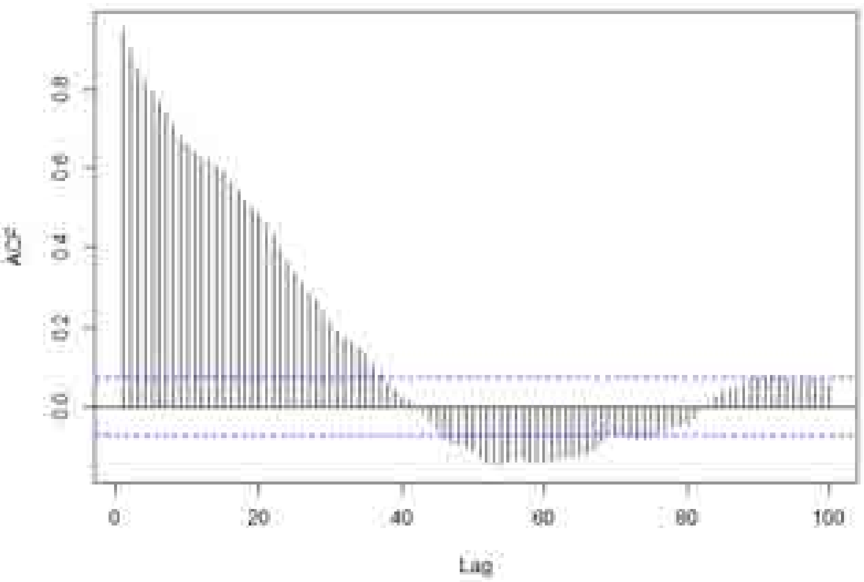
the result of the test suggests that the autoregression coefficient is less then one. This behaviour is expected from the mean-reverting process. Therefore, we continue with an analysis of the autocorrelation function (ACF) and the partial autocorrelation function (PACF).

The ACF of deseasonalized logarithmic spot prices suggests that the underlying process has long memory. Explanation of the departures from normality (observed in the typical financial distribution, also in the Figure 6) using memory mechanisms is generally treated in Stadnik (2014). The ACF is plotted in the Figure 7. Further, the PACF indicates that the prices follow AR(1) process as one can see in the Figure 8. Even though, AR(1) does not allow for long memory, we continue with model defined in (15) as it implies an analytically tractable formula for forward price. In order to evaluate RSR, we do not need very complex model. Hence,

$$z_t = \rho z_{t-1} + e_t \tag{15}$$

where the e_t is assumed to be an i.i.d. process.

Figure 7: The ACF of deseasonalized logarithmic spot prices with 95 % confidence bounds.



The data have daily granularity, thus we shift from a continuous to a discrete time. Then we let $Z(t) = X(t) + Y(t)$. Therefore $Z(t) = \ln S(t) - \ln \Lambda(t)$. From the dynamics of OU processes $X(t)$ and $Y(t)$ defined in equations (4) and (5) we have

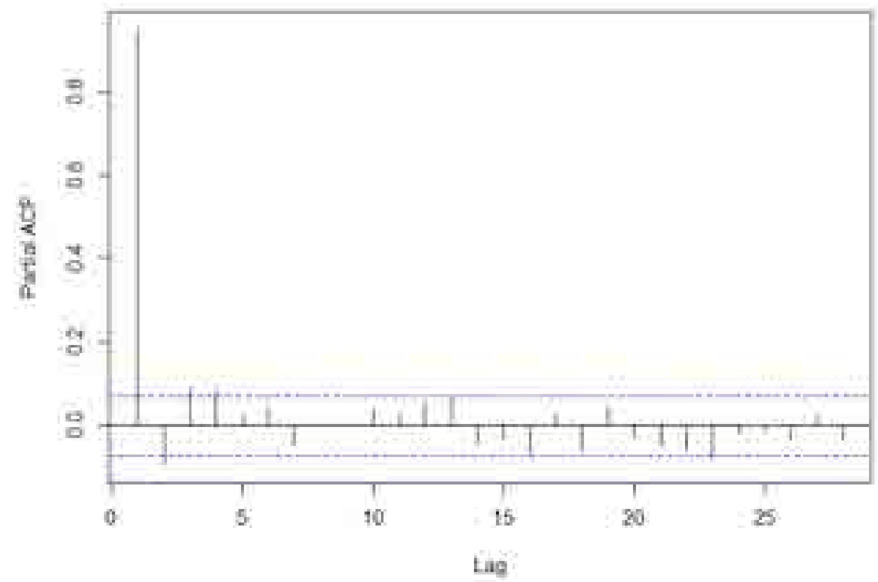
$$dZ(t) = -\alpha Z(t)dt + \sigma dB(t) + dI(t). \tag{16}$$

To achieve discrete approximation we add $Z(t)$ to both side of the equation and we get daily increments as

$$Z(t) \approx (1-\alpha)Z(t-1) + \sigma \Delta B(t) + \Delta I(t). \tag{17}$$

Note that $\Delta B(t)$ is a daily increment of Brownian motion i.e. $B(t) - B(t-1)$. Similarly, $\Delta I(t)$ in an increment of the jump process $I(t) - I(t-1)$. Both increments of Brownian motion and general Lévy process are i.i.d. sequences of random variable. Hence, their sum $\sigma\Delta B(t) + \Delta I(t)$ is also an i.i.d. Then we see that our discrete approximation corresponds to the AR process in (15) with $\rho = 1 - \alpha$ and $e_t = \sigma\Delta B(t) + \Delta I(t)$.

Figure 8: The PACF of deseasonalized logarithmic spot prices with 95 % confidence bounds.



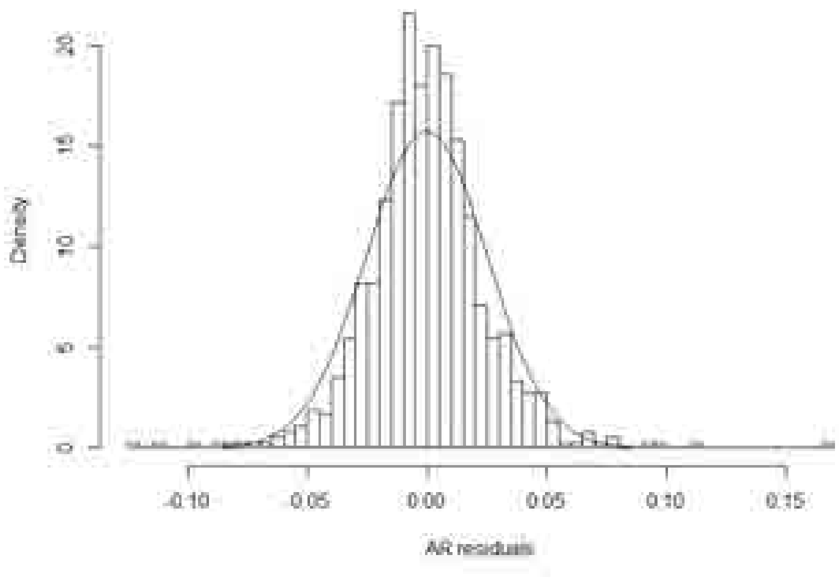
Hence, we continue by fitting AR(1) model on deseasonalized logarithmic spot prices $Z(t)$, defined in the equation (17), to determine speed of mean reversion. Thus, the estimated speed of mean reversion is $\widehat{\alpha} = 1 - \widehat{\rho} = 1 - 0.955 = 0.045$.

Hence, the spot prices revert rather slowly towards the seasonal level. The fitted AR(1) model is presented in the Table 3. Finally we look at the distribution of obtained residuals. Their histogram is depicted in the Figure 9 along with fitted normal distribution density curve. We see that the increment process indeed reveals leptokurtic shape. Therefore, we try to account for non-normal features of residuals by decomposing them into a mixed diffusion process.

Table 3: Fitted AR(1) model.

	Dependent variable:
$\widehat{\rho}$	0.955***
	(0.011)
Log Likelihood	1,647.015
σ^2	0.001
Akaike Inf. Crit.	−3,290.029
Note:	*p<0.1; **p<0.05; ***p<0.01

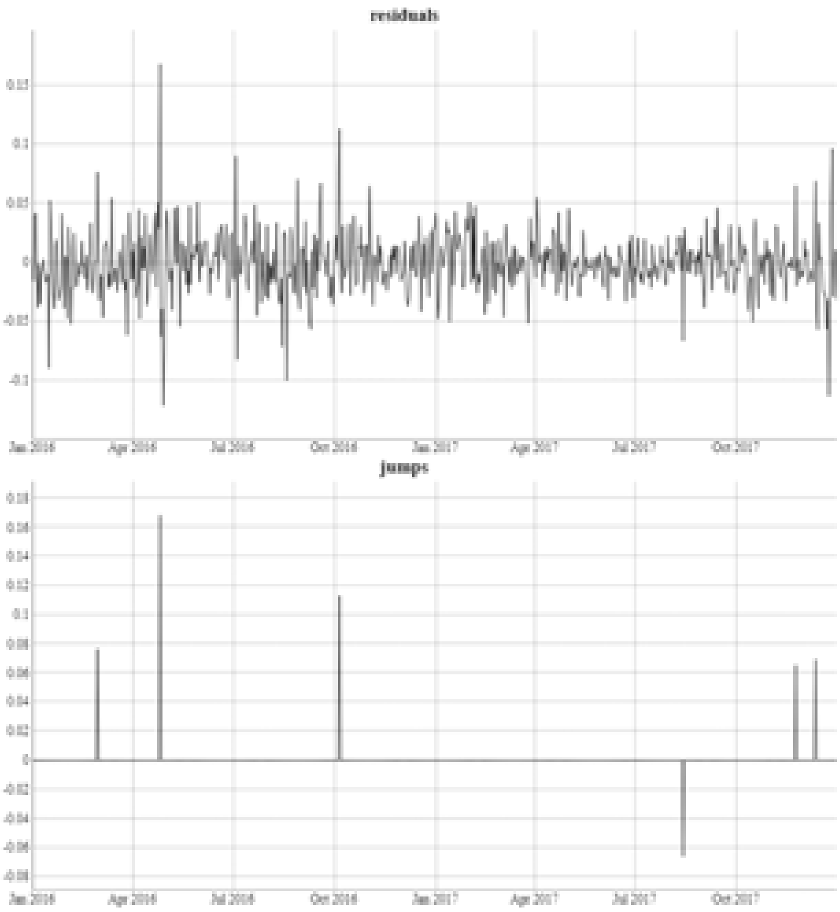
Figure 9: The histogram of obtained residuals from AR(1) model fit with normal density curve.



3.2.2 Residuals as a Jump Mixed Diffusion Process

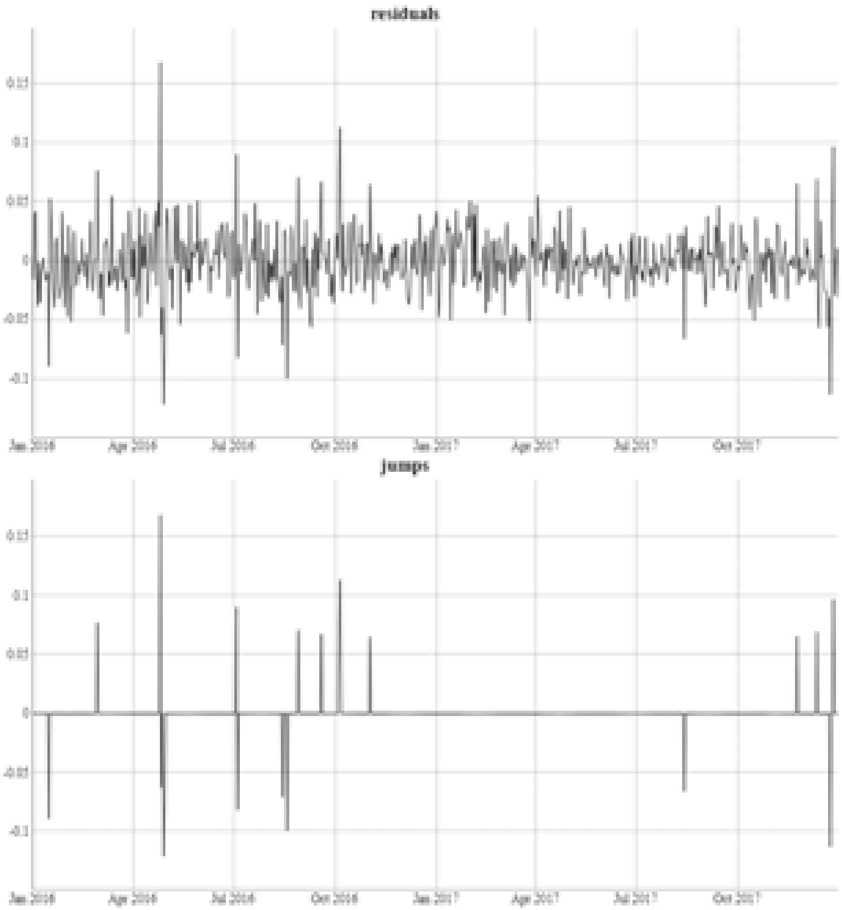
Now we decompose residuals into two separate processes: a Gaussian process and a jump process. We choose the window size of 16 days for deriving a bipower variation, an estimator of instantaneous volatility as recommended by Lee & Mykland (2007). The test indicates six jumps at five percent significance level. The comparison of jumps with series of residuals is presented in the Figure 10. Indeed we estimated jumps at places where one would expect them. However, when we look at the series of residuals, there are observations which look more like jumps than like normal innovations. For instance, the second estimated jump is followed by a return of approximately the same negative size. It is not determined to be jump at any choice of K not even at ten percent significance level, though.

Figure 10: Residuals (top) and estimated jumps (bottom) following Lee & Mykland (2007).



Therefore we apply different jump detection approach suggested by Clewlow & Strickland (2000). Under this alternative approach 20 jumps are detected. The recursive filtering algorithm determines observation to be a jump when it deviates more than three standard deviations from the mean and replaces it with a median of the sample then. The procedure is repeated until no new jumps are found. In our case the algorithm converges after five iterations. The estimated jumps are presented in the Figure 11. Using this approach (Clewlow & Strickland 2000) we are able to eliminate large jumps that follow one after another. It is primarily appreciable in the beginning of the series.

Figure 11: Residuals (top) and estimated jumps (bottom) following Clewlow & Strickland (2000).



We estimate standard deviation of filtered residuals after every iteration. The estimates are presented in the Table 4 along with number of detected jumps. After last iteration the estimated standard deviation becomes estimate of the σ from the normal part of the stochastic process. The $\widehat{\sigma}$ is 0.0204. Our estimate is approximately five times lower than estimated value of 0.1 for the National Balancing Point (NBP) day ahead prices in Steele (2010).

Hence, the Czech market appears to be calmer. It may be due to the fact that the UK has relatively limited gas storage capacity compared to the Czech Republic. The Czech storage capacity ranks among the highest in Europe. A total storage capacity represents approximately 40 percent of total annual gas consumption in the Czech Republic. Moreover, withdrawal capacity of Czech gas storages should cover higher demand during days with low temperatures (Zaplatilek 2015). Therefore the lack of extreme price movements at the Czech market may be caused by this high gas storage capacity.

Table 4: Summary of the jump filtering algorithm.

Iteration	Detected jumps	Std. deviation
1	11	0.0218
2	17	0.0208
3	19	0.0205
4	20	0.0204
5	20	0.0204

The arrival of a new information which causes price to jump is usually modelled as a compound Poisson process. Before we define such process, it is convenient to look at the distribution of jump sizes. In a literature they are often assumed to come from the normal distribution. However, when we look at the histogram of estimated jump sizes depicted in the Figure 12, it looks that the normal distribution might not be a good choice in our case. Benth *et al.* (2008) propose to look at positive and negative jumps separately. Their histograms are presented in the Figure 13. Indeed, the histograms suggest that the exponential distribution may fit the data better. Therefore, we define Lévy process $I(t)$ rather as a sum of two jump process as

$$I(t) = I^{+}(t) + I^{-}(t). \tag{18}$$

Hence, we have two compound Poisson processes defined as

$$I^{\pm}(t) = \sum_{k=1}^{N^{\pm}(t)} J_k^{\pm}, \tag{19}$$

where $N^{\pm}(t)$ are Possion processes with intensities $\lambda^{\pm}$. The sequences $J_k^{\pm}$ are assumed to be exponentially distributed i.i.d. random variables which represent jump sizes with average size $m_J^{\pm}$. Thus, $J_k^{\pm} \sim Exp(\frac{1}{m_J^{\pm}})$.

A possible reasonable alternative specification would be to assume that residuals come from the NIG distribution, which is a family of flexible distributions that includes skewed and fat-tailed distributions. The normal distribution is a special case of the NIG distribution.

Table 5: Estimated jump sizes and intensities.

Estimate	Positive jumps	Negative jumps
$\widehat{\lambda}$	0.0151	0.0123
$\widehat{m_J}$	0.0870	0.0865

The estimated parameters of the Lévy process defined in the equation 19 are summarized in the Table 5. Out of 20 jumps, 11 turned out to be positive, thus the estimated intensity of the positive jump arrival is slightly higher than the negative one. Still, the intensities are almost the same and imply that approximately five negative and positive jumps arrive during the year. An estimated average jump size is slightly higher for positive jumps.

Figure 12: Histogram of estimated jump sizes.

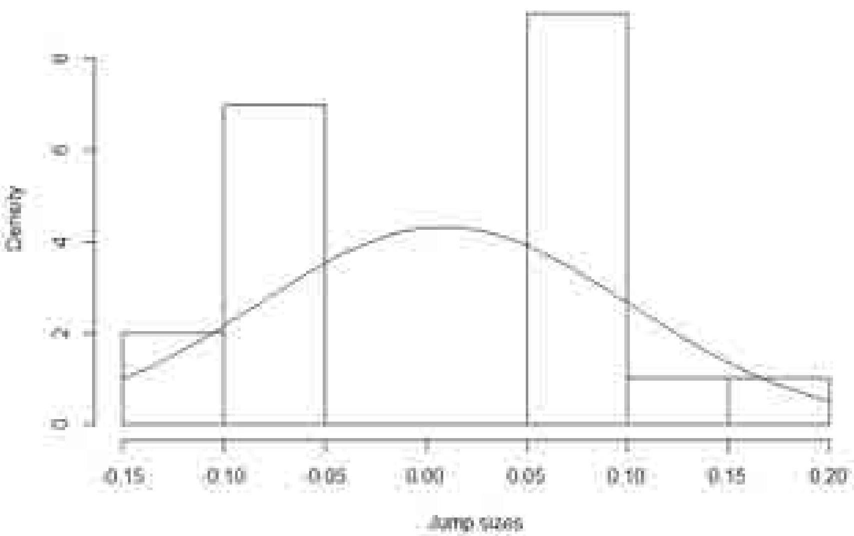
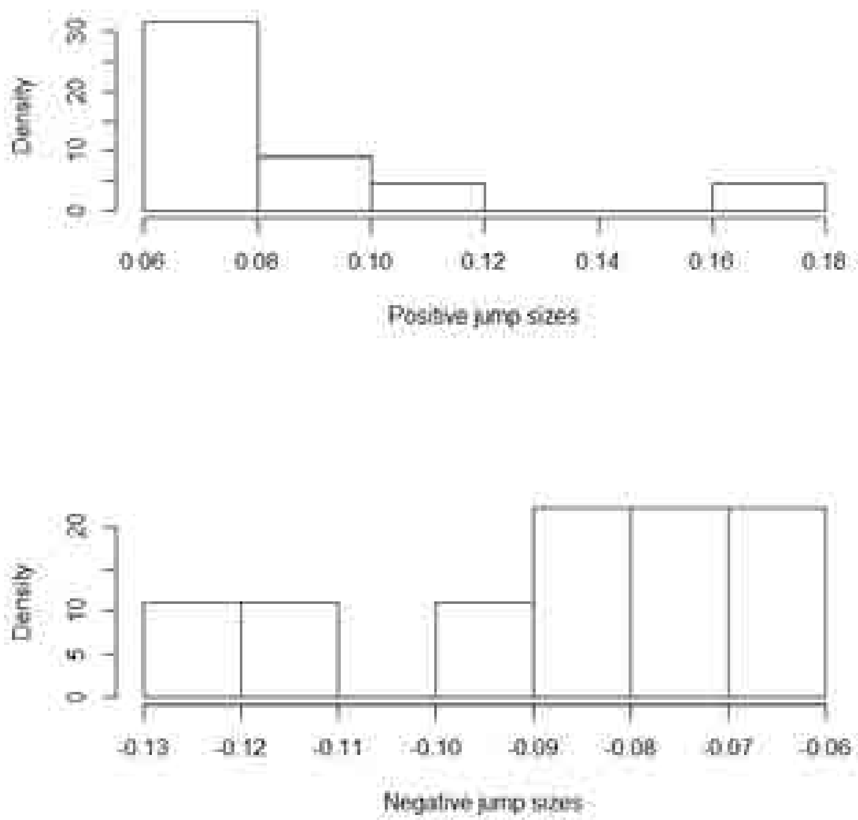


Figure 13: Histogram of estimated jump sizes.



The compound Poisson process is convenient for pricing forwards as it has simple cumulant function. A derivation of the function is shown in Benth *et al.* (2008). It is defined as

$$\psi_{I^\pm}(-iz) = \lambda^\pm(e^{-\ln(1-m_J^\pm z)} - 1) = \lambda^\pm\left(\frac{1}{1-m_J^\pm z} - 1\right), \tag{20}$$

where $\lambda^\pm$ is an intensity of new information arrival. When we input this expression into the equation (11) we get

$$\ln\Theta(t,\tau,0) = \int_t^\tau \frac{1}{2}\sigma^2 z^2 + \lambda^+\left(\frac{1}{1-m_J^+ z} - 1\right) + \lambda^-\left(\frac{1}{1-m_J^- z} - 1\right) du \tag{21}$$

where $z = e^{-\alpha(\tau-u)}$. Hence, having estimated all desired coefficients of the spot dynamics, we are able to price yearly, quarterly, and monthly gas futures contracts now.

3.2.3 Consumption Profiles

Finally, as we constructed a model for forward and spot prices, we are able to evaluate the RSR. Obviously, we need some consumption profile from which the residual position can be derived because the RSR depends on the shape of the profile. Hence, the real shape of Czech leading natural gas selling firm portfolio is used here. The portfolio consists of forecasted households consumption between years 2016 and 2017. We scale the shape of the portfolio to unity in order not to reveal its size as this information is confidential. It is not a problem because we evaluate the RSR per MWh, so just the shape is crucial for our calculations. We will refer to this portfolio as a profile from now on.

We hedge the profile with standard baseload products. Prices of these products were referred to as futures prices in the previous sections. The Table 6 presents all relevant gas futures contracts available at the Power Exchange Central Europe (PXE) throughout years 2016 and 2017. It provides the start and the end of the delivery, a length, and a liquidity for every product. We also present the first day when the product can be traded, i.e. becomes liquid. The time ranges between 0 and 731. When a product is liquid in time 0 it means it became liquid before the beginning point of our time horizon (January 1, 2016). We relate the time notation with respect to a start of the hedging strategy. Thus, we start to hedge one day before the first delivery day. We do this for simplicity of the algorithm. However, one can start earlier. For instance, when the yearly baseloads become liquid as these products are traded longest time before the delivery. Our strategy is to re-hedge in the first day when shorter product becomes liquid. These days correspond to days in the column adj_first_day. The residual position derived from the consumption profile is depicted in the Figure 14.

Figure 14: The residual position derived from consumption profile.

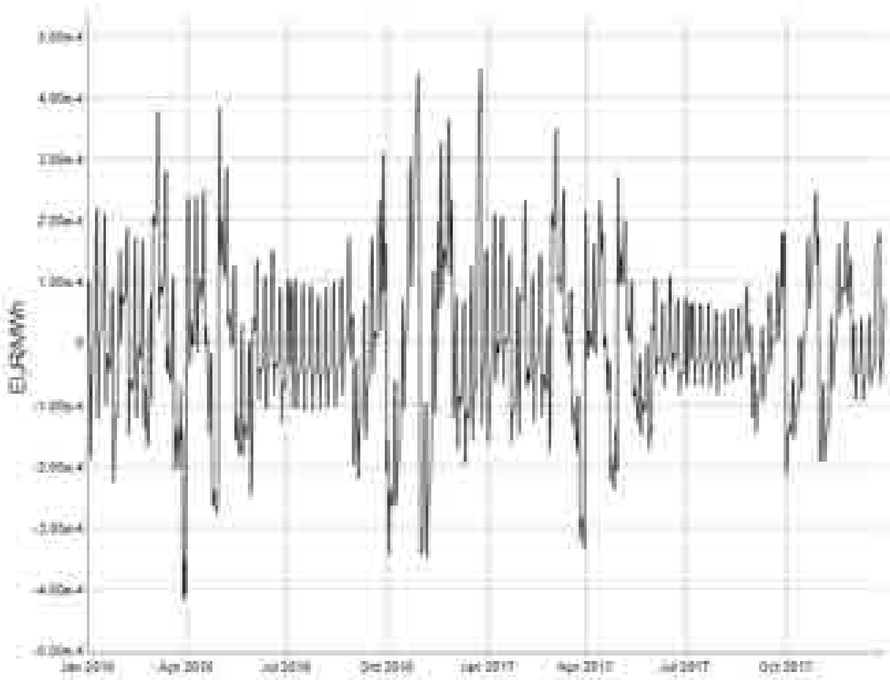


Table 6: Summary of liquid wholesale standard products - futures.

Product	Type	Length	Start	End	liquid_Ahead	adj_first_day
M1-16	M	31	1	31	92	0
M2-16	M	29	32	60	92	0
M3-16	M	31	61	91	90	0
M4-16	M	30	92	121	91	1
M5-16	M	31	122	152	90	32
M6-16	M	30	153	182	92	61
M7-16	M	31	183	213	91	92
M8-16	M	31	214	244	92	122
M9-16	M	30	245	274	92	153
M10-16	M	31	275	305	92	183
M11-16	M	30	306	335	92	214
M12-16	M	31	336	366	91	245
M1-17	M	31	367	397	92	275
M2-17	M	28	398	425	92	306
M3-17	M	31	426	456	90	336
M4-17	M	30	457	486	90	367
M5-17	M	31	487	517	89	398
M6-17	M	30	518	547	92	426
M7-17	M	31	548	578	91	457
M8-17	M	31	579	609	92	487
M9-17	M	30	610	639	92	518
M10-17	M	31	640	670	92	548
M11-17	M	30	671	700	92	579
M12-17	M	31	701	731	91	610
Q1-16	Q	91	1	91	365	0
Q2-16	Q	91	92	182	366	0
Q3-16	Q	92	183	274	366	0
Q4-16	Q	92	275	366	366	0
Q1-17	Q	90	367	456	366	1
Q2-17	Q	91	457	547	365	92
Q3-17	Q	92	548	639	365	183
Q4-17	Q	92	640	731	365	275
Sum-16	S	183	91	273	548	0
Win-16	S	182	274	455	548	0
Sum-17	S	183	456	638	548	0
Cal-16	Y	366	1	366	730	0
Cal-17	Y	365	367	731	731	0

3.2.4 RSR Results

As a final step, we evaluate the RSR by Monte Carlo simulation. We derive 25,000 simulations of spot price paths defined by discrete version of formula (3):

$$S(t) = \Lambda(t)\exp(Z(t)) \tag{22}$$

where

$$Z(t) = (1-\alpha)Z(t-1) + \sigma\Delta B(t) + \Delta I^{+}(t) + \Delta I^{-}(t) \tag{23}$$

and $\Lambda(t)$ is exponential of function defined in equation (14).

The calculations were made in the R Studio with seed 1. Once realizations of spot prices are derived, the algorithm "walks" through the hedge strategy path 25,000 times while storing a result of the RSR costs after every path calculated by expression (1). The calculation takes about one hour of computer processing unit time. The simulated RSR profit and loss per MWh distribution is presented in Figure 15. Moreover, we present one series of simulated RSR in the Figure 16.

Figure 15: The simulated RSR profit and loss per MWh with jump mix-diffusion errors.

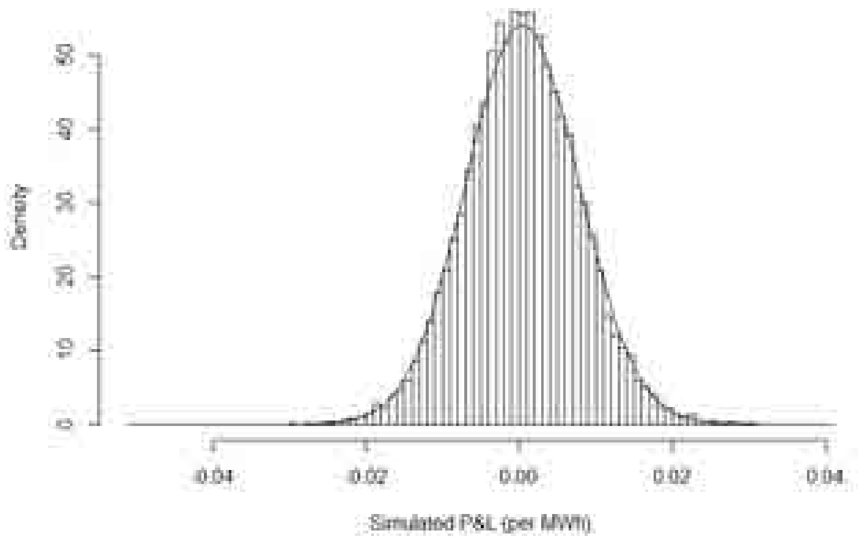
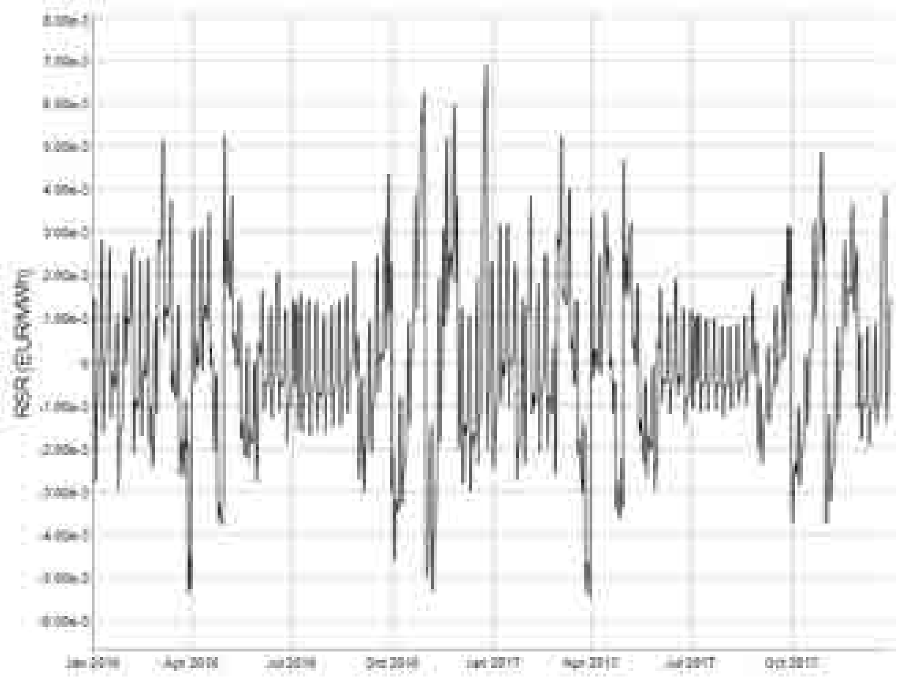


Figure 16: Simulated series of RSR.



The Table 7 shows descriptive statistics of simulated RSR profit and loss distributions. The VaR and the ES are 0.013 and 0.016 EUR/MWh respectively for our leading illustrative year 2016/2017. The ES for mixed-diffusion process is approximately 0.1 percent of average spot prices in our sample, which is 15.93 EUR/MWh. However the important benchmark for retail energy supplier is not a price but his profit margin. Under an assumption of 10 percent profit margin, we obtain the RSR for ES on the level of approximately 1 percent of profit margin.

Table 7: Comparison of RSR distribution statistics.

Statistic	2014/2015	2016/2017	2017/2018
mean	0.000	0.000	0.000
st. dev.	0.012	0.007	0.013
$VaR_{0.05}$	0.020	0.013	0.021
$ES_{0.05}$	0.027	0.016	0.029

Since spot prices were relatively calm in 2016/2017 period, our results for this leading illustrative year provide a conservative estimation of RSR . In order to show RSR for a more volatile periods we also estimated parameters of spot price model on data from 2014/2015 and 2017/2018. Estimates are presented in the Table 8. In both periods we detected more than 30 jumps. In period 2014/2015 the jump intensities increased approximately by 35 and 78 percent for positive and negative jumps respectively. Average jump sizes increased by 20 percent. Estimates for period 2017/2018 are higher by 45 and 90 percent for positive and negative jumps respectively. The mean reversion parameter stayed approximately the same for period 2014/2015, but increased by 16 percent in period 2017/2018. Development of spot prices in 2018 was more dramatic and especially a few days in the end of February, when price jumped by tens of percent up and down, influenced the parameters significantly. One can see in the Table 7 that simulated RSR increased by almost 1 euro cent. It is what we expect as the RSR greatly depends on the underlying spot price process.

Table 8: Comparison estimated spot price parameters.

Estimate	2014/2015	2016/2017	2017/2018
$\widehat{\lambda}^+$	0.0205	0.0151	0.0219
$\widehat{\lambda}^-$	0.0219	0.0123	0.0233
$\widehat{m_J}^+$	0.1088	0.0870	0.1310
$\widehat{m_J}^-$	0.0973	0.0865	0.1195
$\widehat{\alpha}$	0.0440	0.0450	0.0522
$\widehat{\sigma}$	0.0197	0.0204	0.0181

We should take into account that an energy supply is very competitive business everywhere, including Czech Republic. Almost every half a year some Czech gas supply company is going bankrupt. The Czech market operator "OTE, a.s." registered 69 natural gas suppliers by the end of May 2018 [2]. It appears that margins are low and probably a lot of companies operate close to zero economic profit. In such environment the supplier should be aware about even relatively low risk like the RSR, as we estimated. Nevertheless, a trading activity on energy markets grows every year and energy markets become more linked up. For instance, during hot summer when people switch on air conditioners

[2]Suppliers with more then 100 points of delivery - http://www.ote-cr.cz/statistika/mesicni-zprava-plyn/pocty-opm-dodavatelu

and there is low water levels in hydro power pumped plants additional capacity has to be added into the system. This capacity is usually covered by gas fired power plants, which can be dispatched quickly. Hence, higher demand for electricity may cause higher demand for natural gas and affect prices accordingly. Moreover, the power sector shifts towards more volatile, decentralized, renewable power sources. Naturally, it affects electricity prices, but it may also affect natural gas prices. Usually extreme energy prices are positively correlated with extreme weather conditions. With the climate change, natural gas spot prices may become more volatile. Hence, the RSR may become more relevant in the near future.

3.3 Hedging by Storage

Our set up of RSR concept assumes that retail suppliers do not have access to sufficiently flexible costless storage technology and hence need to balance the demand shortage/surplus at the spot market every day. If a portfolio manager of retail supplier company has available storage facility which would be costless (or sufficiently cheap) and sufficiently technologically flexible, he could use short-term storage to shift volumes from days when he is long to days with short position which immediately follows and vice versa. Also, the long-term storage may be reasonable for some period of time with such a technology.

Firstly, consider the issue of the storage cost in comparison with RSR implied cost. If retail manager wanted to use a storage to mitigate the risk, the price he would pay for such service would be at most the value of such risk. For a comparison with RSR empirically estimated on Czech data in our article we use the relevant storage opportunity costs of "innogy Gas Storage, s.r.o." that operates six out of eight underground natural gas storages present in the Czech Republic. These six storages are united into one virtual storage. Hence, innogy's price list for injection and withdrawal capacity is decisive. Its official web page states that minimum price for its service would be at least 0.5 EUR/MWh[3]. Our results in the previous section show that the magnitude of RSR on

[3] https://www.innogy-gasstorage.cz/en/media/Price_list.pdf

the relevant market is at the magnitude of about 1 or 2 Euro cents per MWh. Therefore the price (or opportunity cost) of gas storage is approximately 25 times higher than our highest estimate of RSR of 0.02 EUR/MWh.

Secondly, even if the storage would be financially viable, retailer would need to consider technical capabilities of the storage facility. The relevant capabilities for every underground gas reservoir are its injection and withdrawal curves. These curves represent amount of gas which can be injected or taken out of the storage during a day. Such curve depends on the actual amount of gas present in the storage. A realistic example of these curves is presented in the Appendix. Thus, the portfolio manager of retail supplier would need to compare these curves with his residual position to verify, whether the facility is flexible enough to cover the position. Moreover, there is usually some extra time needed to switch between injecting and withdrawing regimes, which brings additional restriction on the shape of the profile. In case the injection and withdrawal curves would match with the customer's (or portfolio) profile the RSR would be mitigated completely. In terms of RSR such situation would be equivalent to a flexible contract with producer, when retailer has a possibility to differ from contracted volume.

4 Conclusions

As a result of European energy sector liberalization activities at the energy supplier business, the last unit in energy supply chain, became close to activities of portfolio managers and traders on the financial markets. Moreover, as the market introduces more standard products including financial futures, wholesale energy markets become attractive for speculative traders. In a such environment, the energy managers had to adopt risk measuring metrics usually used in the financial sector. With a growing use of flat baseload products used for hedging of prices of short sales to final consumers the difference of forward and spot prices weighted by volume the residual position, which we term residual shape risk (RSR), appeared as a new concept which was missing in the previous more integrated gas markets. While we introduce, motivate and illustrate the RSR on the natural gas market, this concept may be applied for similar energy commodities too.

In order to evaluate RSR, first the dynamics of natural gas spot prices is estimated and subsequently forward prices are derived as expected spot prices at delivery using characteristics of the underlying spot pricess process. The RSR is then evaluated on an appropriate shape (profile) of retail energy supplier portfolio. Using volumetric hedge by hedging the profile with standard products that are liquid in a given time, the profit and loss distribution caused by the RSR is obtained. This distribution is derived by Monte Carlo simulation of spot price paths and by applying the volumetric hedging strategy on them. As a last step five percent VaR and ES financial risk metrics for these distributions are obtained.

In our illustrative conservative (for a 2016/2017 period of low volalitity) example of RSR for leading Czech natural gas retail supplier we obtained VaR and ES values of 0.013 and 0.016 EUR/ MWh, respectively. This means that even when looking at the tail of the distribution, we do not predict loss higher than 2 Euro cents per MWh in extreme cases. However, for the periods 2014/2015 and 2017/2018 when prices become more volatile estimates almost double. The RSR approximately corresponds to 1 percent of profit margin of natural gas retail supplier. This means that while it is not negligible risk, it is definitely not of a first order of importance for natural gas supplier and it does not call for a need to change current business policies and practices.

Since the Czech natural gas spot prices have lower volatility than British National Balancing Point (NBP) prices our empirical results are conservative in the sense of leading to a low value of RSR. In the environment with more volatile energy spot prices RSR would be higher and therefore more important for business decision making, possibly leading to changes in hedging practices. In particular with the increased RSR, the value neutral hedging, leading to zero expected value of RSR, could be more attractive to energy suppliers than currently used volumetric hedging.

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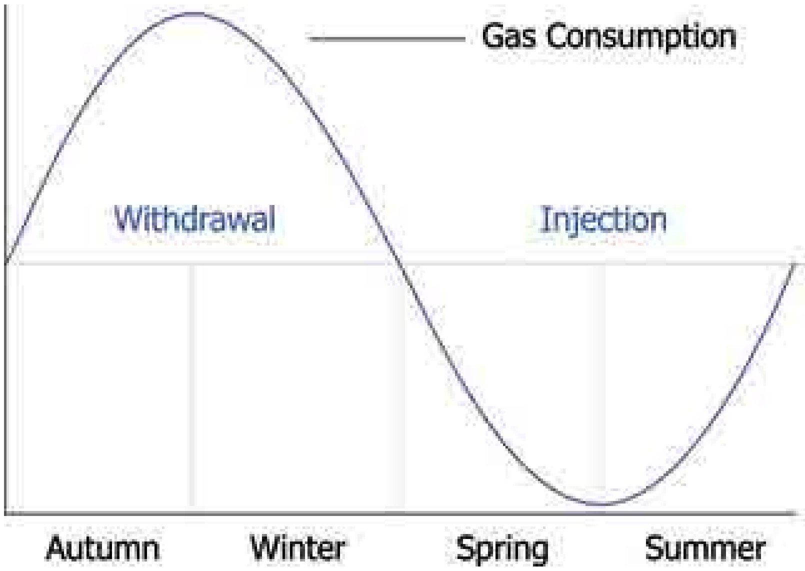
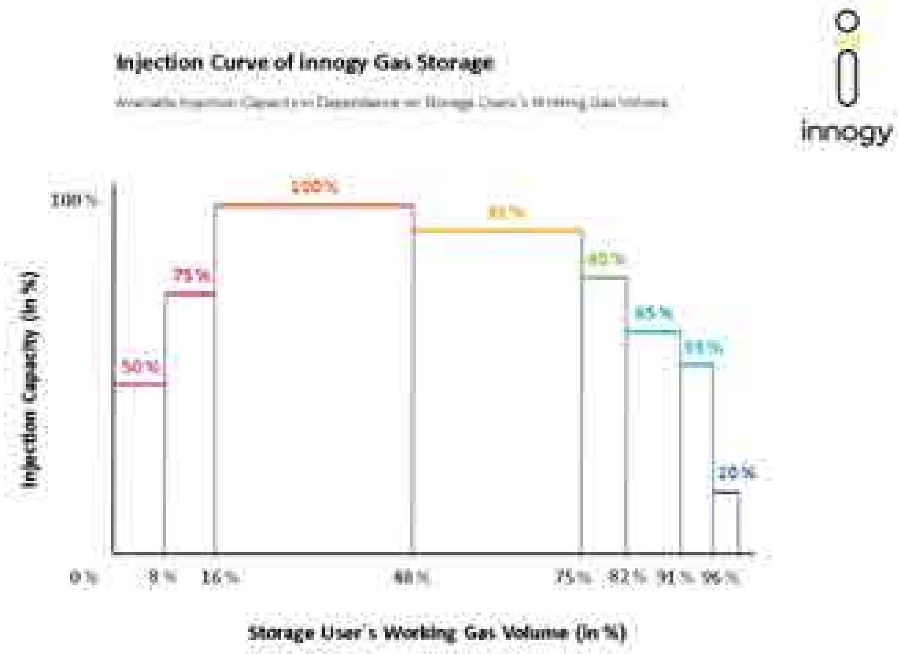
Appendix

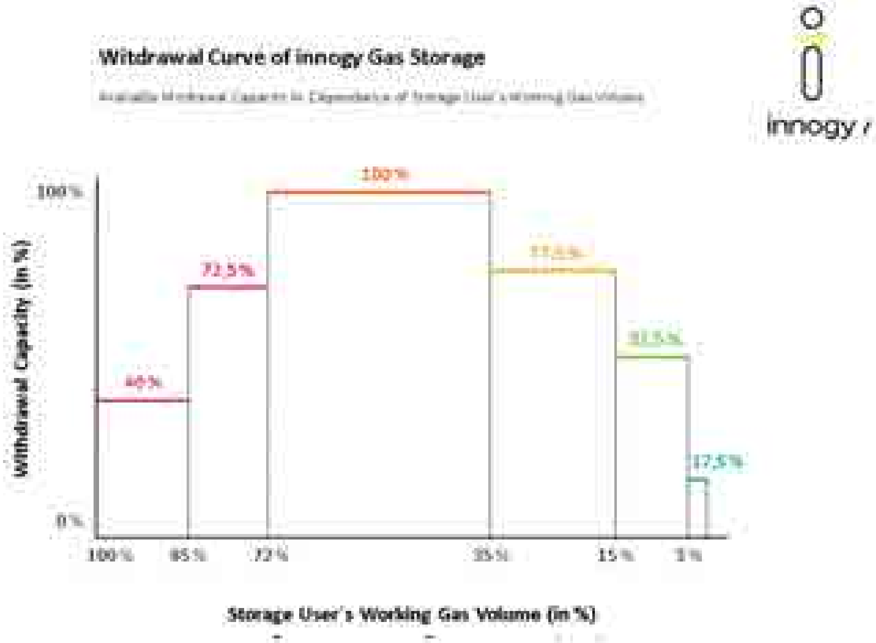
Figure 17: Seasonal utilization of natural gas storage.



Source: innogy Gas Storage s.r.o.



Source: innogy Gas Storage s.r.o.



Source: innogy Gas Storage s.r.o.