Australian
National
University

Crawford School of Public Policy

CAMA

Centre for Applied Macroeconomic Analysis

Search Complementarities, Aggregate Fluctuations, and Fiscal Policy

CAMA Working Paper 40/2022
May 2022

Jesús Fernández-Villaverde
University of Pennsylvania

Federico Mandelman
Federal Reserve Bank of Atlanta

Yang Yu
Shanghai Jiao Tong University

Francesco Zanetti*
University of Oxford
Centre for Applied Macroeconomic Analysis, ANU

Abstract

We document five novel facts about the role of search effort in forming trading relationships among firms by combining a variety of micro and macro datasets. These facts strongly suggest the presence of search complementarities. To study the implications of these facts for aggregate fluctuations, we build a dynamic general equilibrium model, disciplined by our new firm-level evidence on search effort. The model matches key aspects of the macro and micro data that have remained unaccounted for by standard models, including the time-varying bimodal distribution of output and the strong, nonlinear propagation of shocks. Also, changes to the volatility of shocks have nonlinear effects on macroeconomic fluctuations that advance a novel interpretation of the Great Moderation. Finally, we provide a new account of the state-dependent effects of fiscal policy.

Keywords

Search complementarities, aggregate fluctuations, macroeconomic volatility, government spending.

JEL Classification

C63, C68, E32, E37, E44, G12.

Address for correspondence:

(E) cama.admin@anu.edu.au

ISSN 2206-0332

Search Complementarities, Aggregate Fluctuations, and Fiscal Policy

Jesús Fernández-Villaverde
University of Pennsylvania

Federico Mandelman
Federal Reserve Bank of Atlanta

Yang Yu
Shanghai Jiao Tong University

Francesco Zanetti*
University of Oxford and CAMA

April 2022

Abstract

We document five novel facts about the role of search effort in forming trading relationships among firms by combining a variety of micro and macro datasets. These facts strongly suggest the presence of search complementarities. To study the implications of these facts for aggregate fluctuations, we build a dynamic general equilibrium model, disciplined by our new firm-level evidence on search effort. The model matches key aspects of the macro and micro data that have remained unaccounted for by standard models, including the time-varying bimodal distribution of output and the strong, nonlinear propagation of shocks. Also, changes to the volatility of shocks have nonlinear effects on macroeconomic fluctuations that advance a novel interpretation of the Great Moderation. Finally, we provide a new account of the state-dependent effects of fiscal policy.

Keywords: Search complementarities, aggregate fluctuations, macroeconomic volatility, government spending.

JEL classification: C63, C68, E32, E37, E44, G12.

*Correspondence: francesco.zanetti@economics.ox.ac.uk (Zanetti). We thank Davide Debortoli, Jan Eeckhout, Bob Hall, Ben Lester, Guido Menzio, Ernesto Pastén, Edouard Schaal, Shouyong Shi, Martin Uribe, Ivan Werning, the editor, several referees, and participants at multiple conferences and seminars for valuable comments and suggestions. Gorkem Bostanci provided outstanding research assistance. The views expressed in this paper are solely the responsibility of the authors and should not be interpreted as reflecting the views of the Federal Reserve Bank of Atlanta or the Federal Reserve System. Zanetti gratefully acknowledges financial support from the British Academy. The usual disclaimer applies.

1 Introduction

The production of goods and services in the value-added chains pervasive in modern economies results from trading relationships among firms. For instance, manufacturing an airplane requires thousands of specialized inputs, from carbon fiber reinforced thermoplastics (CFRPs) to advanced electronics. The planes, in turn, are used by airlines, which must link them with many inputs, like ticket reservation servers or onboard food and drinks, to deliver air travel services.

Forming these trading relationships requires search effort by firms. An airplane manufacturer must find a CFRP producer and a CFRP producer must find a buyer for its products. This search effort goes well beyond locating partners. We have in mind, among others, the effort by buyers in analyzing vendors (e.g., assessing the quality of CFRPs delivered by a new supplier and checking their suitability for proprietary production processes) and in completing contractual arrangements, certifications, and regulatory compliance procedures. For the suppliers, we have in mind the effort related to advertising and branding, participating in trade fairs, tendering offers, adapting production processes to buyer requirements, and setting up supply procedures to process and track orders. The ample space dedicated to these topics in operations management textbooks (e.g., Heizer et al., 2016, or Stevenson, 2018) proves how seriously practitioners take the building of trading relationships. Are practitioners right? What do the data say about the role of search effort in forming trading relationships? And what are the implications for aggregate fluctuations?

To answer these questions, we use the Occupational Employment Survey, the American Productivity and Quality Center and the FactSet Supply Chain Relationships databases, the BEA input-output tables, and the Center for Research in Security Prices and the Compustat Fundamentals Annual data. By being the first to combine these micro datasets –together with aggregate data– to study the link between search effort and trading relationships, we uncover five novel empirical facts.

Fact 1 is that a higher search effort by a firm forecasts a firm creating more trading relationships. Fact 2 is the positive correlation between a firm's trading relationships and its market value and sales. While our research design does not ascertain the causality behind these correlations, an intuitive interpretation of Facts 1 and 2 is that firms that exert greater search effort build more trading relationships and command greater market value and higher sales.

This interpretation is suggestive because these correlations persist even when we employ lagged values of our regressors as well as time and firm fixed effects.

Fact 3 is that the correlation between the increase in the search effort of a firm and the increase in trading relationships is stronger when the search effort in the industries connected with the firm is also higher. Fact 4 is the significant positive correlation in the increase of search efforts of interconnected industries. There are two natural interpretations of Facts 3 and 4. One interpretation is that firms' behavior is correlated because they are hit by aggregate shocks. But this interpretation is hard to reconcile with the observation that Facts 3 and 4 still appear, even when we purge, with a two-stage estimator, the search efforts of prospective suppliers from changes in the search efforts and economic conditions in the firm's industry. The second, more robust interpretation is that there are search complementarities in the data. That is, when a firm searches with more intensity for a partner, it is more profitable for potential partners to search with higher intensity (and conversely, when a firm searches with less intensity for a partner, it is best for potential partners to also search with lower intensity).

Finally, Fact 5 is that the micro correlations in Facts 1-4 are also present in the aggregate: output and intermediate inputs comove positively. Fluctuations in intermediate inputs account for 71% of the movements in gross industry output, and this contribution increases during recessions.

To account for these five facts and study their implications for aggregate fluctuations, we build a dynamic general equilibrium model, which we discipline with our new firm-level evidence on search effort. This model casts new light on some classic macroeconomic questions like the shape of aggregate fluctuations or the effects of fiscal policy. In the model, firms that produce intermediate and final goods must build long-lasting trading relationships to produce a final good by exerting costly effort. Motivated by Facts 3 and 4, we assume that the matching function among firms is supermodular even if it has (small) decreasing-returns-to-scale. Under mild conditions (compatible with our empirical evidence), supermodularity overcomes the congestion effects of many conventional search environments and generates search complementarities.

In terms of exogenous movements in fundamentals, households are subject to discount factor shocks, while firms experience productivity shocks. Since households own the firms in the economy, the discount factor shocks also affect how firms discount the future. Thus, the return from establishing a trading relationship between firms depends on fundamentals *and* on the search effort of potential trading partners.

The interaction between fundamentals and search effort defines three regions of state variable values: a region where there is a unique *passive* stage equilibrium (where firms search for partners in the current period with zero effort), a region where there is a unique *active* stage equilibrium (where firms search for partners in the current period with positive effort), and a region where both stage equilibria exist. In this case, we will assume that the economy stays in the same stage equilibrium as in the previous period: if yesterday firms did not search, today firms still do not search; if yesterday firms searched with positive effort, today firms still search. History dependence is a transparent equilibria selection device and a strong predictor of empirical behavior in coordination games similar to ours (see the classic findings in Van Huyck et al., 1990, 1991). Loosely speaking, search complementarities provide a microfoundation for what would appear, at first sight, to be increasing returns to matching à la Diamond (1982).

We close the model with a labor market where firms post job vacancies and fill them with workers from households in an off-the-shelf Diamond-Mortensen-Pissarides (DMP) frictional labor market. The DMP block gives us a framework to analyze unemployment and vacancies but, for simplicity, does not present search complementarities.

Quantitatively, our model has three key elements: the degree of search complementarity in the matching function, the coefficients of the search cost function, and the stochastic properties of the discount factor shock. We tightly discipline all three of them by calibrating our model to U.S. micro and macro observations. The degree of search complementarity in the matching function replicates the degree of search complementarities in Fact 3. Our parameterization of the search costs function is based on surveys of search effort from the American Productivity and Quality Center administered to 4,000 firms. The discount factor shock, following Hall (2017), matches the stochastic properties of the expected returns of the stock market index.

Given this calibration, our model matches key properties of the U.S. aggregate variables, including the autocorrelations and skewness of their distributions, endogenous movements in labor productivity, and a more realistic volatility of unemployment than standard business cycle models. Since firms post more vacancies in the active stage equilibrium, output is higher and unemployment is lower than in the passive stage equilibrium. Thus, aggregate shocks can induce large aggregate fluctuations by switching the economy between stage equilibria. If the model starts from the active stage equilibrium deterministic steady state, an adverse shock to the discount factor of 15.5% that makes households more impatient moves the system to the passive stage equilibrium, reducing output by roughly 16%. The drop in output is in the ballpark of the one observed for the U.S. in the financial crisis of 2008 measured as a deviation with respect to trend (between 2007.Q4 and 2014.Q4, output per capita fell 12.4% in the U.S. with respect to its post-war trend). Given our calibration, this is a low probability but not a rare event. Smaller shocks fail to move the economy away from the original stage equilibrium, and the subsequent dynamics are similar to those of conventional business cycle models. Also, we show how shocks to the discount factor –proxied by a broad range of indexes– are correlated with unemployment, the creation of trading relationships, and the volume of intermediate inputs and output.[1]

Interestingly, our model links nonlinearly the volatility of exogenous shocks with aggregate outcomes in two distinctive ways that are not present in other business cycle models. Since these two sharp and distinctive predictions of our model are also present in the data, they provide strong supportive evidence for our mechanism.

First, when the volatility of shocks is high, the distribution of output is bimodal, as the model switches between low and high search effort with high probability. Thus, we present a mechanism that accounts for the influential results by Adrian et al. (2019), who have documented how the empirical distribution of output has, indeed, switched between periods of uni- and bimodality.

Second, when the volatility of shocks is low, output is very persistant, as the economy rarely switches between low and high search effort. Hence, search complementarities can transform transitory negative shocks into protracted slumps. This is a key implication because, since the Great Moderation started in 1984, recessions have been more infrequent but also more persistent, particularly for unemployment (Liu et al., 2019). In comparison with standard business cycle models (which require an exogenous variation in the persistence of shocks or some form of hysteresis), our economy endogenously delivers this fact, allowing us to reconceive the aftermath of the financial crisis of 2008. According to our model, output remained below trend, and employment-to-population ratios were depressed for a decade because the economy in 2008 moved to a stage equilibrium with less search and did not abandon it even after the original adverse shocks evaporated. The long-lasting weak recovery from the financial crisis is a

[1] All our results come without adding expectational shocks as in Kaplan and Menzio (2016). We do not include them to focus more sharply on the interaction between shocks to fundamentals and search complementarities. For the same reason, we postpone for future research the study of non-Markov strategies by firms, alternative stage equilibrium selection devices, and limit cycles such as those in Beaudry et al. (2016, 2018, 2020).

displeasing consequence of the Great Moderation, not a refutation of it as often claimed.

Given the empirical success of our model, we can also use it to study fiscal policy. In our example above, a CFRP producer can supply an airplane manufacturer or provide materials for the construction of a new, seismic-resistant public school in California. If the government raises its expenses (modeled as more government-owned firms such as a new public school), the search incentives for private firms increase, and the economy can switch from a passive stage equilibrium to an active one. In this case, the fiscal multipliers can be as high as 1.59. On the other hand, if search effort is already high (or the fiscal expansion too small in a passive stage equilibrium), the fiscal multiplier will be as low as 0.25. Thus, our model provides an explanation for the strong state dependence of fiscal multipliers in the data documented by Auerbach and Gorodnichenko (2012), Owyang et al. (2013), and Ghassibe and Zanetti (2020).

Search complementarities are an instance of the strategic complementarities defined by Bulow et al. (1985). There is a long tradition in economics of linking strategic complementarities to aggregate fluctuations, going back to Diamond (1982), Weitzman (1982), Howitt (1985), and Diamond and Fudenberg (1989) and explored by Cooper and John (1988), Chatterjee et al. (1993), and Kaplan and Menzio (2016). Recent papers with strategic complementarities, but with mechanisms different from ours, include Schaal and Taschereau-Dumouchel (2018) (with complementarities in production capacity), Sterk (2016) (with complementarities created by the lost skills of unemployed workers), and Eeckhout and Lindenlaub (2018) (with complementarities between on-the-job search and vacancy posting by firms).

How does our paper add to this tradition? First, we address the lack of empirical firm-level evidence that has long afflicted the literature on strategic complementarities. Using new datasets, we document five novel facts about the search efforts of firms and the forming of trading relationships that strongly suggest the existence of strategic complementarities. Second, we study a dynamic equilibrium model of strategic complementarities tightly disciplined by observed data. Furthermore, this model matches key aspects of the macro and micro data, including the time-varying bimodal distribution of output, that have remained unaccounted for. Third, we show how the nonlinear effects of changes to the volatility of shocks on our economy can help us think differently about the Great Moderation. Finally, we provide a novel account of the state-dependent effects of fiscal policy.

2 Five facts about search effort and trading relationships

We document five new facts about the search effort of firms and trading relationships. The first four facts use microdata, while the last fact relies on aggregate data. Fact 1 is that more search effort by a firm (as measured by two proxies we build) forecasts a firm creating more trading relationships. Fact 2 is that more trading relationships are correlated with higher sales and market value. Facts 1 and 2, together, indicate that firms that increase their search effort are also firms that increase their sales and market value. Fact 3 is that the correlation between the increase in the search effort of a firm and the increase in trading relationships is stronger when the search effort in the industries with which the firm is connected is also higher. Fact 4 shows a significant positive correlation in the increase of search efforts of connected industries. Facts 3 and 4 can be accounted for parsimoniously by search complementarities among firms. Thus, Facts 3 and 4 motivate our theoretical model in Section 3 and provide empirical moments to discipline our calibration in Section 5. Fact 5 reports that the micro correlations in Facts 1-4 also hold in the aggregate: output and intermediate inputs co-move positively in the BEA data.

Fact 1: Search effort forecasts trading relationships

We construct two proxies for search efforts using alternative firm-level datasets. With these proxies, we show that searching activities absorb a substantial amount of firms' resources and changes in search efforts forecast an increase in the number of trading relationships.

Search effort proxy 1: We derive our first proxy for search efforts from the Occupational Employment Survey (OES) database constructed by the BLS, which reports yearly employment and wages at the 3-digit-NAICS industry level, including detailed occupation levels between 2003 and 2020. The database covers 1.1 million establishments and comprises 57% of jobs in the U.S. The BLS collects the employment and wage information from establishments in six semiannual panels in three consecutive years. Every six months, a new panel of data is added, and the oldest panel is dropped. For instance, if a firm is surveyed and added to the database in year t, the information on the firm's employment and wages will remain in the database until the firm is replaced with another firm in year $t + 3$. This updating method results in repeated cross-sectional data every three years.

Following Michaillat and Saez (2015), we approximate a firm k's search effort in matching with suppliers by the number of workers whose occupation is ordering, buying, purchasing, and procurement. Analogously, we approximate a firm k's effort in matching with customers by the number of workers whose occupation is advertising, marketing, sales, demonstration, and promotion. On average, firms allocate 1.4% and 1.9% of employment to these two types of search efforts, respectively.[2]

We start by measuring the employment involved in "buying" in industry i in period t by $emp_{i,t}^{buy}$ and employment involved in "selling" by $emp_{i,t}^{sell}$ (we do not have information on these variables at the firm's level, only at the industry level). Then, we define $\Delta\sigma_{i,t}^{buy} = \frac{ln\left(emp_{i,t}^{buy}\right) - ln\left(emp_{i,t-3}^{buy}\right)}{3}$ and $\Delta\sigma_{i,t}^{sell} = \frac{ln\left(emp_{i,t}^{sell}\right) - ln\left(emp_{i,t-3}^{sell}\right)}{3}$ as our measures of search efforts for buying and selling firms in each industry i, respectively.[3]

At the firm level, the change in employment in search activities is equal to the change at the industry level plus a firm-specific idiosyncratic component:

$$\Delta\sigma_{i,k,t}^{buy} = \Delta\sigma_{i,t}^{buy} + \Delta\widehat{\sigma}_{i,k,t}^{buy}, \tag{1}$$

and

$$\Delta\sigma_{i,k,t}^{sell} = \Delta\sigma_{i,t}^{sell} + \Delta\widehat{\sigma}_{i,k,t}^{sell}. \tag{2}$$

Since search efforts are measured at the industry level, and the changes in efforts at the firm level ($\Delta\widehat{\sigma}_{i,k,t}^{buy}$ and $\Delta\widehat{\sigma}_{i,k,t}^{buy}$) are unobserved, we assume that firm-level changes in efforts are orthogonal to the observed industry-level changes, i.e., $\left(\Delta\widehat{\sigma}_{i,k,t}^{buy}, \Delta\widehat{\sigma}_{i,k,t}^{sell}\right) \perp \left(\Delta\sigma_{i,t}^{buy}, \Delta\sigma_{i,t}^{sell}\right)$.

Search effort proxy 2: We derive a second proxy for search efforts using the American Productivity and Quality Center (APQC) database for 2018-2021. APQC surveys over 4,000 firms about their practice in sales, marketing, contracting, and procurement, and publicly discloses the most recent cross-section moments. The median spending on searching for suppliers at the firm level is about 1.4% of total revenue (coincidently, in the OES 1.4% of employment is involved in searching for suppliers). The median spending on searching for customers is about 7.5% of total revenue, which is higher than the measurement from the OES data derived from employment within the industry (1.9%). This result is unsurprising as many efforts in marketing and sales are outsourced to industries such as publishing and broadcasting.

[2]We exclude the retail and wholesale industries, since their trading relationships might not reflect the inter-firm cooperation in production that we aim to study.

[3]We use third log differences due to the BLS's data-updating method. First log differences underestimate the change in employment, as 2/3 of the firms do not update their information in two consecutive years.

Figure 1 shows the breakdown of spending on search activities for firms that expand lines of customers and suppliers (the non-personnel cost to carry out the procurement process includes systems cost, overhead cost, outsourced cost, and others). The advantage of the APQC dataset is the detailed measurements for a wide range of search costs, which we will use to calibrate the cost function in the theoretical model. The drawback of the APQC dataset is the lack of time dimension given the recent collection of data. Thus, while our proxy based on the APQC database provides a detailed cross-sectional measure of efforts in different categories, we will use our first proxy based on the OES database to study the changes in search efforts over time. Appendix A documents that our results below are robust to measuring search efforts using advertisement expenses as proposed by Hall (2014).

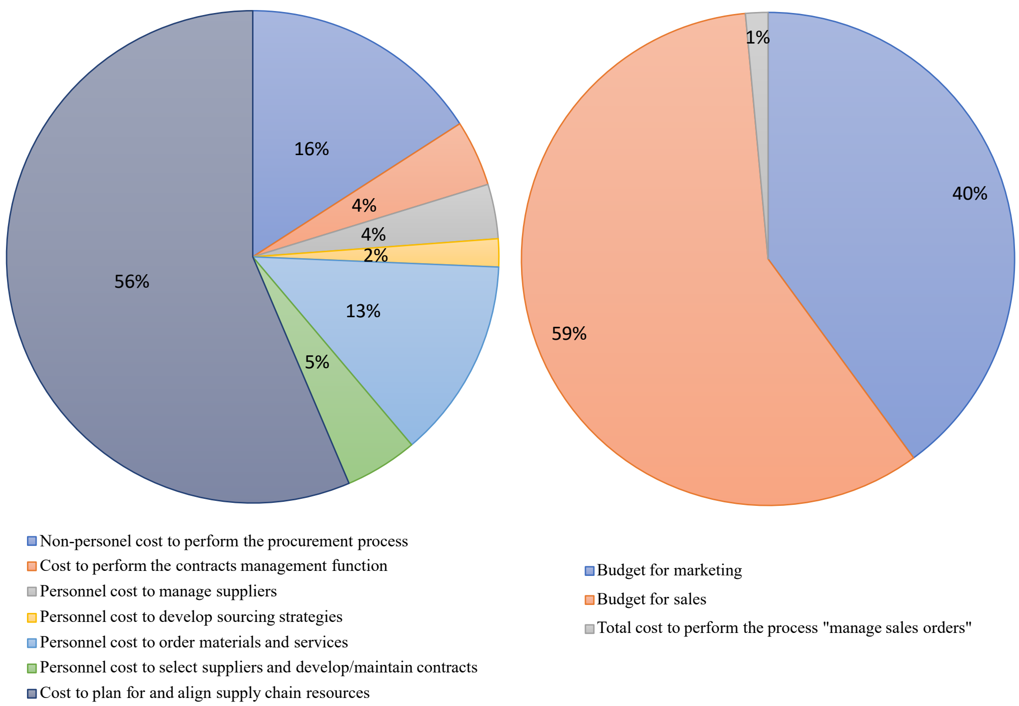


Figure 1: Spending on searching for suppliers and customers

Trading relationships: We obtain the number of trading relationships using FactSet Supply Chain Relationships data. FactSet collects firms' relationship information from public sources such as SEC 10-K annual filings, investor presentations, and press releases since 2003. Using the sample period 2003-2021, we obtain 289,239 distinct customer-supplier trading relationships. Most of the trading relationships are continuative, with only 15,522 (5%) of them experiencing a reinstatement from previously dissolved relationships.

The observed average duration of relationships is 3.5 years (Figure 2 plots the histogram for the duration of customer-supplier trading relationships). But since our sample ends in 2021, with many relationship still ongoing, 3.5 years is a downward-biased estimate of the true persistence of relationships. The number of supplier firms that sell intermediate goods to the customer firm k that operates in industry i, in year t is $n^{sup}_{i,k,t}$, while $n^{cus}_{i,k,t}$ is the number of customer firms that purchase intermediate goods from the supplier firm k that operates in industry i in year t.

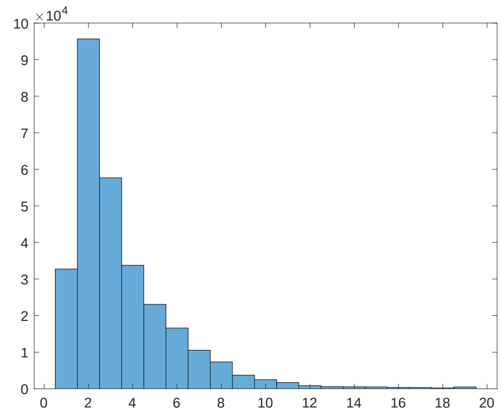


Figure 2: Histogram of duration (year) of trading relationships

Estimation: We now show that changes in our proxies for search efforts predict growth in the number of trading relationships. Since our proxies for search efforts are on firms operating in the U.S., we focus on the subset of U.S. firms within FactSet.

In particular, we estimate:

$$\Delta n^{sup}_{i,k,t} = \beta \Delta \sigma^{buy}_{i,k,t-p} + \chi_t + \gamma_{i,k} + \epsilon_{i,k,t}, \tag{3}$$

where $\Delta n^{sup}_{i,k,t}$ is the first difference in the number of supplier firms, $\Delta \sigma^{buy}_{i,k,t-p}$ is the change in buying effort at the firm level with p-years' lag, χ_t and $\gamma_{i,k}$ are year and firm fixed effects, and $\epsilon_{i,k,t}$ is the component of $\Delta n^{sup}_{i,k,t}$ orthogonal to changes in search efforts and fixed effects plus any measurement error. Using equation (1) to substitute out $\Delta \sigma^{buy}_{i,k,t-p}$ in equation (3) yields:

$$\Delta n^{sup}_{i,k,t} = \beta \Delta \sigma^{buy}_{i,t-p} + \chi_t + \gamma_{i,k} + \eta_{i,k,t}, \text{ with } \eta_{i,k,t} = \beta \Delta \widehat{\sigma}^{buy}_{i,k,t-p} + \epsilon_{i,k,t}. \tag{4}$$

Columns (1) and (2) in Table 1 show the estimation results for equation (4) for 0 and 1−year lags. The coefficient β for $\Delta \sigma^{buy}_{i,k,t-p}$ is positive and statistically significant for both lags (the estimated coefficient remains significant for lags up to $p = 7$). Thus, stronger search efforts predict faster matching with suppliers in the subsequent years. This persistent effect is consistent

Table 1: Search effort forecasts the number of relationships

	(1)	(2)	(3)	(4)
Dependent variable	Change of suppliers ($\Delta n^{sup}_{i,k,t}$)		Change of customers ($\Delta n^{cus}_{i,k,t}$)	
Year lag (p)	0	1	0	1
$\Delta\sigma^{buy}_{i,t-p}$	0.95***	1.07***		
	(0.17)	(0.17)		
$\Delta\sigma^{sell}_{i,t-p}$			0.55**	0.83***
			(0.26)	(0.29)
Time FE	Yes	Yes	Yes	Yes
Firm FE	Yes	Yes	Yes	Yes
R^2	0.26	0.23	0.13	0.12
Observations	24,346	24,844	26,610	26,925

Note: Yearly data 2003-2020. The dependent variables are the change in the number of suppliers ($\Delta n^{sup}_{i,k,t}$) and customers ($\Delta n^{cus}_{i,k,t}$) matched by each firm for Columns (1)-(2) and (3)-(4), respectively. Standard errors are in parentheses. ** and *** denote significance at the 5% and 1% level, respectively.

with the time-consuming formation of trading relationships.

Similarly, to check whether more intensive search efforts for selling forecast acquiring more customers, we estimate:

$$\Delta n^{cus}_{i,k,t} = \beta \Delta\sigma^{sell}_{i,k,t-p} + \chi_t + \gamma_{i,k} + \epsilon_{i,k,t}, \tag{5}$$

where $\Delta n^{cus}_{i,k,t}$ is the first difference in the number of customer firms and $\Delta\sigma^{sell}_{i,k,t-p}$ is the change in selling effort at the firm level with p-years' lag. As before, we use equation (2) to substitute out $\Delta\sigma^{sell}_{i,k,t-p}$ in equation (5), which yields:

$$\Delta n^{cus}_{i,k,t} = \beta \Delta\sigma^{sell}_{i,t-p} + \chi_t + \gamma_{i,k} + \eta_{i,k,t}, \text{ with } \eta_{i,k,t} = \beta \Delta\widehat{\sigma}^{sell}_{i,k,t-p} + \epsilon_{i,k,t}. \tag{6}$$

Columns (3) and (4) in Table 1 show the results for 0 and 1-year lags. As before, the coefficient β for $\Delta\sigma^{sell}_{i,k,t-1}$ is positive and statistically significant.

Fact 2: Trading relationships correlate with firm value and sales

Table 2 provides evidence for the positive correlation between trading relationships and a firm's economic fundamentals by regressing market value and sales over the number of trading relationships. The dependent variables are the market value and sales, obtained from the Center for Research in Security Prices (CRSP) and Compustat Fundamentals Annual data, respectively, and the regressions $ln\left(n^{sup}_{i,k,t}\right)$ and $ln\left(n^{cus}_{i,k,t}\right)$ are the log of the number of suppliers and customers constructed with FactSet data.

Columns (1) and (2) document that a 1% increase in the number of suppliers is associated with a 0.09% and 0.1% rise in market value and sales, respectively. Columns (3) and (4) in Table 2 show that a 1% increase in the number of customers is associated with a 0.1% and 0.07% rise in market value and sales, respectively.

Table 2: Match creation forecasts firm growth

	(1)	(2)	(3)	(4)
Dependent variable	Market value	Sales	Market value	Sales
$ln\left(n^{sup}_{i,k,t}\right)$	0.09***	0.10***		
	(0.01)	(0.01)		
$ln\left(n^{cus}_{i,k,t}\right)$			0.10***	0.07***
			(0.01)	(0.01)
Time FE	Yes	Yes	Yes	Yes
Firm FE	Yes	Yes	Yes	Yes
R^2	0.96	0.92	0.96	0.92
Observations	22,269	22,198	23,504	23,469

Note: Yearly data 2003-2020. Standard errors are in parentheses. *** denotes significance at the 1% level.

Fact 3: Search efforts in a firm and the connected industries are complements

Is the correlation between the search effort of a firm and the increase in trading relationships stronger when potential trading partners in connected industries search more actively? To answer this question, we identify each industry's supplier and customer industries using the BEA input-output tables, which report the use of intermediate input for 66 private industries in 3-digit NAICS. For each industry i, let $sup\,(i)$ be the set of supplier industries that sell intermediate goods to industry i. Adapting our previous notation, we denote with $\sigma^{sell}_{sup(i),t}$ the selling efforts in searching for customers in the suppliers' industry to industry i. Since each industry has multiple supplier industries, we measure the average search effort for industry i's supplier industries as the mean of these supplier industries' search efforts weighted by the value of intermediate goods that industry i purchases from them. Then, $\Delta\sigma^{sell}_{sup(i),t} = \sum_{j\in sup(i)} \omega_{i,j,t}\Delta\sigma^{sell}_{j,t}$, where $\omega_{i,j,t}$ is the fraction of the value of intermediate goods that industry i purchases from industry j, and $\Delta\sigma^{sell}_{j,t}$ is the first difference in the selling effort of industry j in searching for customers.

Analogously, we denote the buying effort of industry i's customer industries in searching for suppliers as $\sigma^{buy}_{cus(i),t}$, and compute $\Delta\sigma^{buy}_{cus(i),t} = \sum_{j\in cus(i)} \widehat{\omega}_{i,j,t}\Delta\sigma^{buy}_{j,t}$, where $\widehat{\omega}_{i,j,t}$ is the fraction of the value of intermediate goods that industry i sells to industry j and $\Delta\sigma^{buy}_{j,t}$ is the first difference in the buying effort of industry j in searching for suppliers.

Then, we estimate:

$$\Delta n^{sup}_{i,k,t} = \beta_1 \Delta\sigma^{buy}_{i,k,t-1} + \beta_2 \Delta\sigma^{buy}_{i,k,t-1} \times \Delta\sigma^{sell}_{sup(i),t-1} + \chi_t + \gamma_{i,k} + \epsilon_{i,k,t}, \quad (7)$$

where $\Delta\sigma^{buy}_{i,k,t-1} \times \Delta\sigma^{sell}_{sup(i),t-1}$ is the interaction term between the changes in the search effort of the firm and the changes in the search effort of its connected firms. According to regression (7), the marginal contribution of firm k's change in search effort to the relationship formation is equal to $\beta_1 + \beta_2 \times \Delta\sigma^{sell}_{sup(i),t-1}$. A positive value for β_2 indicates that firm k's change in search effort forecasts a stronger formation of new trading relationships conditional on a higher search effort in the supplier industries.

Using equation (1) to substitute out $\Delta\sigma^{buy}_{i,k,t-1}$ in equation (7) yields:

$$\Delta n^{sup}_{i,k,t} = \beta_1 \Delta\sigma^{buy}_{i,t-1} + \beta_2 \Delta\sigma^{buy}_{i,t-1} \times \Delta\sigma^{sell}_{sup(i),t-1} + \chi_t + \gamma_{i,k} + \eta_{i,k,t}, \quad (8)$$

with $\eta_{i,k,t} = \beta_2 \widehat{\Delta\sigma}^{buy}_{i,k,t-1} \times \Delta\sigma^{sell}_{sup(i),t-1} + \epsilon_{i,k,t}$.

Table 3: Search efforts are complements

	(1)	(2)	(3)	(4)
Dependent variable	Change of suppliers ($\Delta n^{sup}_{i,k,t}$)		Change of customers ($\Delta n^{cus}_{i,k,t}$)	
	One-stage	Two-stage	One-stage	Two-stage
$\Delta\sigma^{buy}_{i,t-1}$	1.17***	1.11***		
	(0.21)	(0.26)		
$\Delta\sigma^{buy}_{i,t-1} \times \Delta\sigma^{sell}_{sup(i),t-1}$	4.01***	5.99**		
	(1.66)	(2.42)		
$\Delta\sigma^{sell}_{i,t-1}$			1.50***	1.41***
			(0.32)	(0.31)
$\Delta\sigma^{sell}_{i,t-1} \times \Delta\sigma^{buy}_{cus(i),t-1}$			3.22***	6.08***
			(1.23)	(1.85)
Time FE	Yes	Yes	Yes	Yes
Firm FE	Yes	Yes	Yes	Yes
R^2	0.29	0.32	0.14	0.16
Observations	20,533	18,741	23,939	20,994

Note: Yearly data 2003-2020. The dependent variables are the change in the number of suppliers ($\Delta n^{sup}_{i,k,t}$) for columns (1) and (2) and of customers ($\Delta n^{cus}_{i,k,t}$) for columns (3) and (4). Standard errors are in parentheses. ** and *** denote significance at the 5% and 1% level, respectively.

Column (1) of Table (3) shows that β_2 is positive and statistically significant. This estimate is consistent with the hypothesis of supermodularity in the matching function: stronger search by the prospective suppliers makes a firm's search effort more productive.

Below, Fact 4 will document that $\Delta\sigma^{buy}_{i,t}$ and $\Delta\sigma^{sell}_{sup(i),t}$ are positively correlated. This is a natural manifestation of search complementarities. But the correlation can also be a byproduct of common shocks. If this is the case, $\Delta\sigma^{buy}_{i,t-1} \times \Delta\sigma^{sell}_{sup(i),t-1}$ will be positively correlated with $\left(\Delta\sigma^{buy}_{i,t-1}\right)^2$. Then, a $\beta_2 > 0$ in regression (8) may come from a missing quadratic term on $\Delta\sigma^{buy}_{i,t-1}$ rather than from search complementarities.

To address this concern, we conduct a two-stage exercise. In the first stage, we purge $\Delta\sigma^{sell}_{sup(i),t}$ from changes in effort in industry i ($\Delta\sigma^{buy}_{i,t}$), the influence of aggregate conditions in industry i ($y_{i,t}$), and an industry-specific fixed effect (α_i) by running:

$$\Delta\sigma^{sell}_{sup(i),t} = \alpha_i + \beta_i \Delta\sigma^{buy}_{i,t} + \kappa_i y_{i,t} + \Delta\varsigma^{sell}_{sup(i),t}. \tag{9}$$

Thus, the residual $\Delta\varsigma^{sell}_{sup(i),t}$ is the change in search efforts exerted by the suppliers of industry i that is orthogonal to industry j's changes in search efforts and the industry's economic conditions. In the second stage, we replace $\Delta\sigma^{sell}_{sup(i),t-1}$ with $\Delta\varsigma^{sell}_{sup(i),t-1}$ in equation (8).

Table 3 reports the results of our two-stage procedure in column (2). Since β_2 is positive and statistically significant, the heightened search efforts by prospective suppliers correlate with an increase in a firm's own search efforts, even when the search efforts of prospective suppliers are orthogonal to changes in the search efforts and economic conditions in the firm's industry.

Next, we examine whether the change in effort of the supplier firm forecasts a stronger formation of new trading relationships conditional on higher search effort in the customer industries by estimating:

$$\Delta n^{cus}_{i,k,t} = \beta_1 \Delta\sigma^{sell}_{i,k,t-1} + \beta_2 \Delta\sigma^{sell}_{i,k,t-1} \times \Delta\sigma^{buy}_{cus(i),t-1} + \chi_t + \gamma_{i,k} + \epsilon_{i,k,t}, \tag{10}$$

where $\Delta\sigma^{sell}_{i,k,t-1} \times \Delta\sigma^{buy}_{cus(i),t-1}$ is the interaction term between the change in search effort of firm k and the change in search effort of the customer industries.

Using equation (1) to substitute out $\Delta\sigma^{sell}_{i,k,t-1}$ in equation (10), we get:

$$\Delta n^{cus}_{i,k,t} = \beta_1 \Delta\sigma^{sell}_{i,k,t-1} + \beta_2 \Delta\sigma^{sell}_{i,t-1} \times \Delta\sigma^{buy}_{cus(i),t-1} + \chi_t + \gamma_{i,k} + \eta_{i,k,t}, \tag{11}$$

with $\eta_{i,k,t} = \beta_2 \widehat{\Delta\sigma}^{sell}_{i,k,t-1} \times \Delta\sigma^{buy}_{cus(i),t-1} + \epsilon_{i,k,t}$.

Column 3 in Table 3 shows the estimation results for regression (11). The coefficient β_2 is positive and statistically significant, which documents that heightened search by a prospective customer correlates with an increase in the firm's own search efforts. Column 4 in the table shows the results when we replace $\Delta\sigma^{buy}_{cus(i),t-1}$ with $\Delta\varsigma^{buy}_{cus(i),t-1}$ to purge the estimation from the effect of common shocks. Again, the results are consistent with the idea that the matching process among firms is supermodular in their search efforts.

Fact 4: Positive comovement of search efforts in connected industries

Next, we document the positive comovements of the search efforts between connected industries, which are consistent with the presence of strategic complementarities in search efforts among firms engaged in a trading relationship.

To study the comovement of search efforts in connected industries, we estimate for the customer industry:

$$\Delta\sigma^{buy}_{i,t} = \omega\Delta\sigma^{sell}_{sup(i),t} + \upsilon_i + \gamma_t + \epsilon_{i,t},$$

where $\Delta\sigma^{buy}_{i,t}$ is the change in search effort in industry i as a customer industry at period t, ω is our coefficient of interest, and $\Delta\sigma^{sell}_{sup(i),t}$ is the change in search effort of industry i's supplier industries. Controlling for firm fixed effects rules out the possibility that the comovements in search efforts are not a consequence of correlated shocks across firms.

The estimate for the coefficient ω is equal to 0.36 (column (1) of Table 4). Its significance at the 1% level is strong evidence that changes in the search efforts of supplier industries are positively correlated with the changes in search efforts of customer industries beyond the presence of common shocks.

Analogously, we estimate the equation for the supplier industry:

$$\Delta\sigma^{sell}_{i,t} = \omega\Delta\sigma^{buy}_{cus(i),t} + \upsilon_i + \gamma_t + \epsilon_{i,t}, \tag{12}$$

where $\Delta\sigma^{sell}_{i,t}$ is the change in the search effort of industry i as a supplier industry at period t, and $\Delta\sigma^{buy}_{cus(i),t}$ is the change in the search effort of industry i's customer industries. Table 4 shows (in column 3) that ω is positive and statistically significant, which confirms our previous result that changes in search effort are correlated between connected industries.

As with Fact 3, a possible complication with our findings could be the presence of shocks that are specific to each pair of connected industries and that cannot be removed by the time fixed effects. To address this concern, we use a two-stage procedure to purge the observed search efforts from the influence of common shocks. The first stage is characterized by equation (9) without $\Delta\sigma_{i,t}^{buy}$ as an independent variable. In the second stage, we replace the changes in search efforts with the residual changes in search efforts obtained from the first stage, and study the comovement in residual changes in search efforts that exclude the influence of common shocks. Column (2) in Table 4 shows a positive correlation between changes in search effort in connected industries even after excluding common shocks.

Table 4: Search efforts are positively correlated between connected industries

	(1)	(2)	(3)	(4)
Dependent variable	Buying effort ($\Delta\sigma_{i,t}^{buy}$)		Selling effort ($\Delta\sigma_{i,t}^{sell}$)	
	One-stage	Two-stage	One-stage	Two-stage
$\Delta\sigma_{sup(i),t}^{sell}$	0.36***	0.47***		
	(0.12)	(0.14)		
$\Delta\sigma_{cus(i),t}^{buy}$			0.15***	0.34***
			(0.05)	(0.12)
Time FE	Yes	Yes	Yes	Yes
Industry FE	Yes	Yes	Yes	Yes
R^2	0.59	0.63	0.17	0.17
Observations	754	751	745	663

Note: Yearly data 2003-2018. The dependent variables are buying effort ($\Delta\sigma_{i,t}^{buy}$) for columns (1) and (2) and selling effort ($\Delta\sigma_{i,t}^{sell}$) for columns (3) and (4). Standard errors are in parentheses. *** denotes significance at the 1% level.

We can apply the same two-stage exercise to equation (12). The estimation results are reported by Column (4) in Table 4, which verifies the positive correlation between changes in search effort in connected industries.

Fact 5: Positive comovement of output with intermediate inputs

Fact 5 is that output and intermediate inputs co-move in the fashion predicted by search complementarities. The BEA compiles a measure of gross output (O) equal to the sum of an industry's value added (VA) and intermediate inputs (II), i.e., $O = VA + II$. BEA data are annual over the period 1997-2015. Figure 3 plots the cyclical component of gross output (blue line), intermediate inputs (red line), and industry value added (yellow line) together with NBER-dated recession periods (grey bands). We extract the cyclical component of the variable using an HP filter. The figure reveals that fluctuations in intermediate inputs are more procyclical than those in output. The Great Recession witnessed a sharp fall in intermediate input and gross production across industries, while the value added remained more stable.

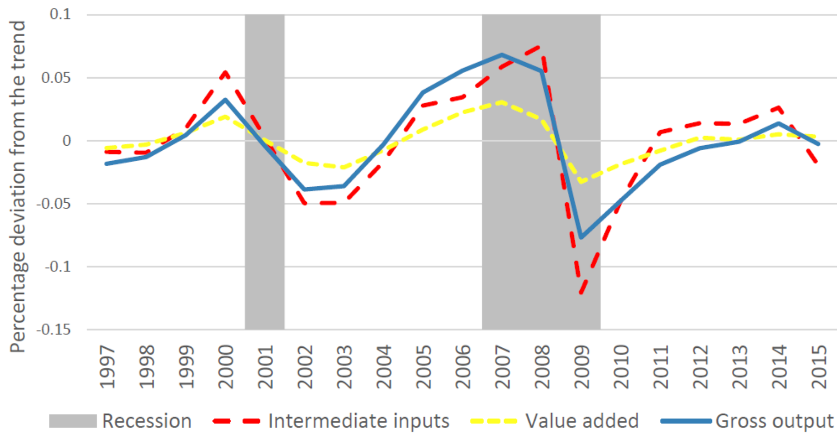


Figure 3: Intermediate inputs, value added, and gross output

To determine the relative contribution of value added and industry input to the overall volatility of gross output, we decompose the variance of the gross industrial output in terms of its covariance terms: $\text{Var}(O) = \text{Cov}(VA, O) + \text{Cov}(II, O)$. Using this identity, together with the definition $O = VA + II$, and plugging in observed data, we find that the contribution of industry inputs to movements in industrial gross output is $\frac{\text{Cov}(II, VA+II)}{\text{Var}(VA+II)} = 0.71$.

Thus, fluctuations in intermediate input account for 71% of the movements in gross industry output. This average contribution increases during recessions. For example, in 2008, industry intermediate input decreased by 1.9 trillion, making up 84% of the decline in gross industrial output (2.3 trillion).

3 A model with search complementarities

We build a dynamic general equilibrium model of business cycles that, by generating the five facts in Section 2, allows us to revisit the aggregate behavior of the economy (Section 6), is driven by plausible shocks (Section 7), and lets us investigate the nonlinear effect of the volatility of shocks (Section 8) and fiscal policy (Section 9).

To do so, we postulate a discrete-time model where firms in the intermediate-goods sector (I) and the final-goods sector (F) are connected through trading relationships. Firms will invest search effort in building these relationships because more effort leads to more relationships (Fact 1 in Section 2) and higher sales and profits (Fact 2). The matching function will be supermodular in the search effort of firms (even if it presents decreasing returns to scale). The supermodularity will create search complementarities that replicate Facts 3 and 4. Since we will deal with general equilibrium, including the presence of households, the model also captures Fact 5.

Households: There is a continuum of households of size 1. Households can either work one unit of time per period for a wage w or be unemployed and receive h utils of home production and leisure. Households do not have preferences for working –or searching for a job– in either sector $i \in \{I, F\}$ and receive the firms' profits.

Households are risk-neutral and discount the future by $\beta\xi_t$ per period, where $\beta < 1$ is a constant and ξ_t is a discount factor shock that follows $\log \xi_t = \rho_\xi \log \xi_{t-1} + \sigma_\xi \epsilon_{\xi,t}$, with $\rho_\xi \leq 1$ and $\epsilon_{\xi,t} \sim \mathcal{N}(0, 1)$. When $\xi_t > 1$, households are *more* patient than average and, conversely, when $\xi_t < 1$ households are *less* patient than average. Innovations to ξ_t encapsulate demographic shifts, movements in financial frictions, or fluctuations in risk tolerance. Cochrane (2011) and Hall (2016, 2017) provide evidence that those shocks are a central source of aggregate fluctuations. Since households own the firms, firms also employ $\beta\xi_t$ to discount future profits.

Labor matching: At the beginning of each period t, any willing new firm can post a vacancy in either sector $i \in \{I, F\}$ at the cost of χ per period to hire job-seeking households. Each firm posts a vacancy for one worker. Vacancies and job seekers meet in a DMP frictional labor market. Since this DMP block is standard, its only role is to provide a natural framework to discuss unemployment and vacancies without undue complexity.

Given $u_{i,t}$ unemployed households and $v_{i,t}$ posted vacancies in sector i, a constant-returns-to-scale matching technology $m(u_{i,t}, v_{i,t})$ determines the number of hires and vacancies filled in t. The new hires start working in $t + 1$. The job-finding rate, $\mu_{i,t} = m(u_{i,t}, v_{i,t})/u_{i,t} = \mu(\theta_{i,t})$, and the probability of filling a vacancy, $q_{i,t} = m(u_{i,t}, v_{i,t})/v_{i,t} = q(\theta_{i,t})$, are functions of each sector's labor market tightness ratio $\theta_{i,t} = v_{i,t}/u_{i,t}$. Then, $\mu'(\theta_{i,t}) > 0$ and $q'(\theta_{i,t}) < 0$.

At the end of each period t, existing jobs terminate at a rate δ and unfilled vacancies expire. The newly unemployed households are split evenly to search in each sector. Once an unemployed household is assigned to search in one sector, it is not allowed to search in another sector (given the symmetry across sectors and our calibration below, workers do not mind this restriction). Appealing to a law of large numbers, unemployment, $u_t = u_{I,t} + u_{F,t}$, evolves as:

$$u_{t+1} = u_t - \underbrace{\left[\mu_I\left(\theta_{I,t}\right)u_{I,t} + \mu_F\left(\theta_{F,t}\right)u_{F,t}\right]}_{\text{Job creation}} + \underbrace{\delta\left(1 - u_t\right)}_{\text{Job destruction}}. \tag{13}$$

Trading relationships: Once job vacancies are filled, a final-goods firm must form a trading relationship with an intermediate-goods firm to manufacture together, starting in $t+1$, the final good sold to households, which is also our numeraire. A technology with variable search effort governs inter-firm matching. Search effort is costly, but it increases the probability of forming a trading relationship. Variable search effort generates search complementarities since the optimal search effort by one firm will be (weakly) increasing in the number of firms searching in the opposite sector and their search effort (see below for details). This stylized matching summarizes more sophisticated inter-firm networks such as those in Jones (2013) and Acemoglu et al. (2012).

In a trading relationship, the intermediate-goods firm uses its worker to produce $y_{I,t} = z_t$, where z_t is the stochastic productivity that follows $\log z_t = \rho_z \log z_{t-1} + \sigma_z \epsilon_{z,t}$, with $\rho_z \leq 1$ and $\epsilon_{z,t} \sim \mathcal{N}(0,1)$. The final-goods firm takes $y_{I,t}$ and, employing its worker, transforms it one-to-one into the final good, $y_{F,t} = y_{I,t} = z_t$. If a firm fails to form a trading relationship in t, it produces no output and continues searching for a partner in $t+1$. At the end of each period, a constant fraction of existing trading relationships is destroyed because either the job is destroyed with probability δ, or the trading relationship fails at a rate $\widetilde{\delta}$.[4] In the former case, the firms dissolve. In the latter case, the firms become single firms, but the jobs survive.

Search effort: Building on Burdett and Mortensen (1980), the number of inter-firm matches is $M\left(\widetilde{n}_{F,t}, \widetilde{n}_{I,t}, \widetilde{\sigma}_{F,t}, \widetilde{\sigma}_{I,t}\right) = \left(\phi + (\psi + \widetilde{\sigma}_{F,t}^{0.5})(\psi + \widetilde{\sigma}_{I,t}^{0.5})\right) H\left(\widetilde{n}_{F,t}, \widetilde{n}_{I,t}\right)$, where $\widetilde{n}_{F,t}$ is the number of single firms in sector F with search effort, $\widetilde{\sigma}_{F,t}$; $\widetilde{n}_{I,t}$ and $\widetilde{\sigma}_{I,t}$ are the analogous variables for the I sector (because of random search, all firms within a sector search with the same intensity). The function $H(\cdot)$ has constant returns to scale and it is strictly increasing in both terms. We set up its units by choosing $H(1,1) = 1$. We will explain momentarily why we specify two different parameters, $\{\phi, \psi\} > 0$, and why we set the power of $\widetilde{\sigma}_{i,t}$ to 0.5.

Each firm optimally chooses $\widetilde{\sigma}_{i,t} \geq 0$ to trade off search cost and the profits from matching success. The cost of $\widetilde{\sigma}_{i,t}$ is:

$$c\left(\widetilde{\sigma}_{i,t}\right) = c_0 \widetilde{\sigma}_{i,t}^{0.5} + c_1 \frac{\widetilde{\sigma}_{i,t}^{(1+\nu)/2}}{1+\nu}, \tag{14}$$

where $c_0 > 0$ creates a concave cost tranche and $c_1 > 0$, with $\nu > 1$, a convex cost tranche.

[4] To simplify, in a trading relationship, the jobs in the intermediate-goods firm and the final-goods firm terminate simultaneously with probability δ or survive simultaneously with probability $1-\delta$. In single firms, the job destruction rate is also δ. We assume that $\delta + \widetilde{\delta} < 1$, and that the separation of job matches and trading relationships is a mutually exclusive event.

Given the inter-firm market tightness ratio $\widetilde{n}_F/\widetilde{n}_I$, the probability that a sector I firm will form a trading relationship with a sector F firm is:

$$\pi_I = \frac{M\left(\widetilde{n}_{F,t}, \widetilde{n}_{I,t}, \widetilde{\sigma}_{F,t}, \widetilde{\sigma}_{I,t}\right)}{\widetilde{n}_I} = \left(\phi + (\psi + \widetilde{\sigma}_{F,t}^{0.5})(\psi + \widetilde{\sigma}_{I,t}^{0.5})\right) H\left(\frac{\widetilde{n}_{F,t}}{\widetilde{n}_{I,t}}, 1\right), \tag{15}$$

and the probability that a sector F firm will form a trading relationship with a sector I firm is:

$$\pi_F = \frac{M\left(\widetilde{n}_{F,t}, \widetilde{n}_{I,t}, \widetilde{\sigma}_{F,t}, \widetilde{\sigma}_{I,t}\right)}{\widetilde{n}_F} = \left(\phi + (\psi + \widetilde{\sigma}_{F,t}^{0.5})(\psi + \widetilde{\sigma}_{I,t}^{0.5})\right) H\left(1, \frac{\widetilde{n}_{I,t}}{\widetilde{n}_{F,t}}\right). \tag{16}$$

In the symmetric equilibria where $\widetilde{n}_{F,t} = \widetilde{n}_{I,t}$, we have:

$$\pi_{F,t} = \pi_{I,t} = \phi + (\psi + \widetilde{\sigma}_{F,t}^{0.5})(\psi + \widetilde{\sigma}_{I,t}^{0.5}) = \phi + \psi^2 + \psi\widetilde{\sigma}_{F,t}^{0.5} + \psi\widetilde{\sigma}_{I,t}^{0.5} + \widetilde{\sigma}_{F,t}^{0.5}\widetilde{\sigma}_{I,t}^{0.5}. \tag{17}$$

We can see now why we specified two different parameters, $\{\phi, \psi\} > 0$. The parameter ψ determines the impact of $\widetilde{\sigma}_{i,t}$ on the matching probability (17) without considering any interaction with $\widetilde{\sigma}_{-i,t}$. Thus, ψ bounds the marginal return to searching from below when prospective partners search with zero effort, a mechanism that will govern the degree of supermodularity in the model. In comparison, ϕ is a scaling parameter, unrelated to $\widetilde{\sigma}_{i,t}$, that will allow us to match the average inter-firm matching probabilities in the data. This difference separately identifies ϕ and ψ.

Equation (17) has decreasing returns to scale on $\widetilde{\sigma}_{F,t}$ and $\widetilde{\sigma}_{I,t}$. Nonetheless, $\widetilde{\sigma}_{F,t}^{0.5}\widetilde{\sigma}_{I,t}^{0.5}$, the most relevant term for the quantitative analysis, is homogeneous of degree 1. Since we are looking for a microfoundation for the increasing returns to scale assumption in Diamond (1982) through the endogeneity of search effort, homogeneity of degree 1 is an intuitive baseline.[5]

To simplify notation, we define $\sigma_{i,t} = \widetilde{\sigma}_{i,t}^{0.5}$. Then, equation (17) becomes:

$$\pi_{F,t} = \pi_{I,t} = \phi + \left(\psi + \sigma_{F,t}\right)\left(\psi + \sigma_{I,t}\right). \tag{18}$$

Together with equation (14), this result implies that the net gain from searching can be negative, in which case the firm chooses $\sigma_i = 0$, or positive and the firm picks $\sigma_i > 0$.

A key implication of the existence of these two alternatives is that we have multiple stage (i.e., within period t) equilibria. One stage equilibrium is *passive*, with $\sigma_{I,t} = \sigma_{F,t} = 0$, low production, and high unemployment. The other stage equilibrium is *active*, with $\{\sigma_{I,t}, \sigma_{F,t}\} > 0$,

[5]Given our calibration in Section 5, equation (17) also has decreasing returns to scale if we express it in terms of the costs $c\left(\widetilde{\sigma}_{i,t}\right)$. However, the function is nearly homogeneous of degree 1 for all but very small levels of cost (and hence search effort). Again, this is a natural benchmark.

high production, and low unemployment. The expression "stage equilibrium," which we borrow from the literature on repeated games, highlights that we look at possible outcomes within one given period. The rational expectations equilibria for our economy are composed of a sequence of these stage equilibria. Households and firms have rational expectations about this sequence of stage equilibria and act accordingly.

We select among stage equilibria through history dependence following Cooper (1994). If the economy was in a passive stage equilibrium in $t-1$, firms stay in the passive stage equilibrium in t. Conversely, if the economy was in an active stage equilibrium in $t-1$, firms continue searching with positive effort in t. Sufficiently large shocks to productivity or the discount factor induce firms to adjust search effort, and the economy shifts from one stage equilibrium to the other. Otherwise, the economy stays in the same stage equilibrium as in the previous period.[6] An indicator function, ι_t, with value 0 if the stage equilibrium is passive and 1 if active, keeps track of the stage equilibria. This indicator function is taken as given by all agents.[7]

The number of trading relationships in $t+1$ comprises firms that survive job separation and trading relationship destruction plus newly formed trading relationships:

$$n_{t+1} = (1 - \delta - \widetilde{\delta})n_t + (\phi + (\psi + \sigma_{F,t})(\psi + \sigma_{I,t}))\widetilde{n}_{I,t}. \tag{19}$$

The number of single firms in sector i in $t+1$ includes firms that survive job separation $((1-\delta)\widetilde{n}_{i,t})$, newly created single firms whose vacancies are filled by job-seekers $(\mu_i(\theta_{i,t}) \cdot u_{i,t})$, and firms whose trading relationships exogenously terminate $(\widetilde{\delta} n_{i,t})$, net of the number of single firms that form trading relationships $(\pi_{i,t}\widetilde{n}_{i,t})$:

$$\widetilde{n}_{i,t+1} = (1-\delta)\widetilde{n}_{i,t} + \mu_i(\theta_{i,t})u_{i,t} + \widetilde{\delta} n_{i,t} - \pi_{i,t}\widetilde{n}_{i,t}. \tag{20}$$

Value functions: We can now define the Bellman equations that determine the value, for each sector i, of an unemployed household $(U_{i,t})$, of an employed household in a single firm

[6] The selection based on history dependence is different from Schaal and Taschereau-Dumouchel (2018), who use a global game to select the equilibrium. With a global game, the equilibrium is uniquely and monotonically determined by the economic fundamentals, while, in our model, it is jointly determined by economic fundamentals and past history. As we will see later, our framework allows spells with strong fundamentals while the economy stays in the low-activity equilibrium despite the coexistence of a high-activity equilibrium.

[7] There might exist mixed-strategy Nash equilibria in which firms search with positive variable effort with some probability. We ignore those equilibria because Appendix E shows the mixed strategy is not robust: when one sector changes the probability slightly due to a trembling-hand perturbation, the opposite sector would immediately set the probability to either zero or one. We leave non-Markov strategies, limit cycles, and alternative equilibria selection devices for future investigation.

($\widetilde{W}_{i,t}$) and in a trading relationship ($W_{i,t}$), of a filled job in a single firm ($\widetilde{J}_{i,t}$) and in a trading relationship ($J_{i,t}$), and of a vacant job ($V_{i,t}$). We index all of these value functions by ι_t since they depend on the type of stage equilibrium at t.

The value of an unemployed household in sector i and equilibrium ι is:

$$U_{i,t|\iota_t} = h + \beta\xi_t\mathbb{E}_t\left[\mu_{i,t}\widetilde{W}_{i,t+1} + (1-\mu_{i,t})\,U_{i,t+1} \mid \iota_t\right]. \tag{21}$$

In the current period, the unemployed household receives a payment h. The household finds a job with probability $\mu_{i,t}$ and circulates into employment during the next period, or it fails to find employment with probability $1-\mu_{i,t}$ and remains unemployed. To save space, we ignore the state variables when presenting the equations, but they are described in Appendix J.

The value of a household with a job in a single firm in sector i is:

$$\widetilde{W}_{i,t|\iota_t} = \widetilde{w}_{i,t} + \beta\xi_t\mathbb{E}_t\left\{(1-\delta)\left[\pi_{i,t}W_{i,t+1} + (1-\pi_{i,t})\,\widetilde{W}_{i,t+1}\right] + \delta U_{i,t+1} \mid \iota_t\right\}. \tag{22}$$

The first term on the right-hand side (RHS) is the wage $\widetilde{w}_{i,t}$ (to be determined by Nash bargaining, see Appendix B). In $t+1$, the match that survives job destruction may either form a trading relationship with a firm in the opposite sector with probability $\pi_{i,t}$, gaining the value $W_{i,t+1}$, or otherwise remain a single firm with probability $1-\pi_{i,t}$, with value $\widetilde{W}_{i,t+1}$. With probability δ, the job is destroyed, and the household transitions into unemployment.

The value of a household with a job in a trading relationship in each sector i is:

$$W_{i,t|\iota_t} = w_{i,t} + \beta\xi_t\mathbb{E}_t\left[(1-\delta-\widetilde{\delta})W_{i,t+1} + \widetilde{\delta}\widetilde{W}_{i,t+1} + \delta U_{i,t+1} \mid \iota_t\right]. \tag{23}$$

A worker in this situation receives the wage $w_{i,t}$. In $t+1$, the worker becomes unemployed with probability δ, gaining the value $U_{i,t+1}$. With probability $\widetilde{\delta}$, the trading relationship is terminated, and the value becomes $\widetilde{W}_{i,t+1}$. Otherwise, the match continues, gaining the value $W_{i,t+1}$.

The value of a single firm in sector i is:

$$\widetilde{J}_{i,t|\iota_t} = \max_{\sigma_{i,t}\geq 0}\left\{-\widetilde{w}_{i,t} - c\,(\sigma_{i,t}) + \beta\xi_t\,(1-\delta)\,\mathbb{E}_t\left[\pi_{i,t}J_{i,t+1} + (1-\pi_{i,t})\,\widetilde{J}_{i,t+1} \mid \iota_t\right]\right\}. \tag{24}$$

Single firms have zero revenues but they still need to pay the wage ($\widetilde{w}_{i,t}$) and incur the search costs $c\,(\sigma_{i,t})$. In $t+1$, conditional on surviving job destruction with probability $1-\delta$, the firm forms a trading relationship with probability $\pi_{i,t}$, gaining $J_{i,t+1}$. Otherwise, the firm remains single with value $\widetilde{J}_{i,t+1}$. If the job is destroyed, the firm exits with zero value.

The value of a trading relationship for a sector I firm is:

$$J_{I,t|\iota_t} = z_t p_t - w_{I,t} + \beta\xi_t \mathbb{E}_t \left[(1-\delta-\widetilde{\delta}) J_{I,t+1} + \widetilde{\delta}\widetilde{J}_{I,t+1} \mid \iota_t \right]. \tag{25}$$

The firm's earnings are equal to the revenue from the intermediate good, $z_t p_t$, less the wage $w_{I,t}$. Both p_t and $w_{I,t}$ are determined by Nash bargaining. In $t+1$, with probability $\widetilde{\delta}$, the firm is separated from its partner and becomes a single firm, gaining a value of $\widetilde{J}_{I,t+1}$; with probability δ, the job match is destroyed, and the firm exits the market with zero value. Otherwise the trading relationship continues with value $J_{i,t+1}$.

The value of a trading relationship for a sector F firm is:

$$J_{F,t|\iota_t} = z_t(1-p_t) - w_{F,t} + \beta\xi_t \mathbb{E}_t \left[(1-\delta-\widetilde{\delta}) J_{F,t+1} + \widetilde{\delta}\widetilde{J}_{F,t+1} \mid \iota_t \right]. \tag{26}$$

The profit for the trading relationship in the final-goods sector comprises revenues from selling z_t units of final goods at price 1, net of the costs of purchasing intermediate goods ($z_t p_t$) and paying the wage ($w_{F,t}$). The rest of the equation follows the same interpretation as equation (25).

The value of a vacant job in sector i is:

$$V_{i,t|\iota_t} = -\chi + \beta\xi_t \mathbb{E}_t \left[q\left(\theta_{i,t}\right) \widetilde{J}_{i,t+1} + \left(1 - q\left(\theta_{i,t}\right)\right) \max\left(0, V_{I,t+1}, V_{F,t+1}\right) \mid \iota_t \right]. \tag{27}$$

Equation (27) shows that the value of a vacant job comprises the fixed cost of posting a vacancy χ in t. With probability $q\left(\theta_{i,t|\iota_t}\right)$, the vacancy is filled, and a single firm with value $\widetilde{J}_{i,t+1}$ is created. The last term shows that firms that fail to recruit a worker may choose to be inactive or post a vacancy in either sector in $t+1$.

By free-entry, we have $V_{i,t} = 0$ and the condition that pins down labor market tightness: $\chi = \beta\xi_t \mathbb{E}_t \left[q\left(\theta_{i,t}\right) \widetilde{J}_{i,t+1} \mid \iota_t \right]$. Appendix B describes the Nash bargaining over wages between firms in trading relationships and workers, and prices between the final-goods producer and the intermediate-goods producer within a trading relationship.

Aggregate resource constraint: Finally, the total resources of the economy, equal to $z_t n_t$ (i.e., production per trading relationship times the number of existing trading relationships), are used for aggregate consumption by households, c_t, and to pay for vacancies and inter-firm search:

$$c_t + \sum_{i=I,F} \chi v_{i,t} + \sum_{i=I,F} \widetilde{n}_{i,t} \left(c_0 \widetilde{\sigma}_{i,t}^{0.5} + c_1 \frac{\widetilde{\sigma}_{i,t}^{(1+\nu)/2}}{1+\nu} \right) = z_t n_t. \tag{28}$$

4 Characterizing the equilibrium

The equilibrium definition for our model is standard and we include it in Appendix I. Here, we characterize the optimal search strategy of firms and show how multiple stage equilibria make the effect of shocks persist over time.

Optimal search effort: The value for a firm i of searching with effort $\sigma_{i,t} > 0$ when the search effort in the other sector $\sigma_{j,t}$ given a stage equilibrium ι_t is:

$$\Pi_i\left(\sigma_{i,t} \mid \sigma_{j,t}, \iota_t\right) = -\widetilde{w}_{i,t} - c\left(\sigma_{i,t}\right) + \beta\xi_t\left(1-\delta\right)\mathbb{E}_t\left[\pi_{i,t}(J_{i,t+1} - \widetilde{J}_{i,t+1}) + \widetilde{J}_{i,t+1} \mid \iota_t\right]. \tag{29}$$

The interior solution $\sigma_{i,t} > 0$ (i.e., the best response) satisfies:

$$c_0 + c_1\sigma_{i,t}^{\nu} = \widetilde{\beta} \underbrace{\left(\psi + \sigma_{j,t}\right)}_{\text{Search effort in sector } j} \underbrace{\xi_t}_{\text{discount factor shock}} \underbrace{\mathbb{E}_t\left(J_{i,t+1} - \widetilde{J}_{i,t+1} \mid \iota_t\right)}_{\text{Expected capital gain}} \tag{30}$$

where $\widetilde{\beta} = \beta\left(1-\delta\right)/\tau$ (the wage Nash bargaining implies that the firm bears τ fraction of the search cost). The left-hand side (LHS) of equation (30) is the marginal cost of exerting $\sigma_{i,t}$ to build a trading relationship, while the RHS is the expected benefit of searching for a partner. Since $\mathbb{E}_t(J_{i,t+1} - \widetilde{J}_{i,t+1} \mid \iota_t)$ depends positively on z_t, condition (30) shows how higher ξ_t or z_t (fundamentals) and higher $\sigma_{j,t}$ (search complementarities) lead to higher $\sigma_{i,t}$. Because the optimization problem is non-convex, we also have a corner solution $\sigma_{i,t} = 0$, either because the firms in the other sector search too little or because the discounted expected gains from matching are too small. The next proposition summarizes this argument.

Proposition 1. *The optimal $\sigma_{i,t}$ is equal to:*

$$\sigma_{i,t} = \begin{cases} \left[\frac{\widetilde{\beta}(\psi+\sigma_{j,t})\xi_t\mathbb{E}_t\left(J_{i,t+1}-\widetilde{J}_{i,t+1}\mid\iota_t\right)-c_0}{c_1}\right]^{\frac{1}{\nu}} & \text{if } \widetilde{\beta}\left(\psi+\sigma_{j,t}\right)\xi_t\mathbb{E}_t\left(J_{i,t+1} - \widetilde{J}_{i,t+1} \mid \iota_t\right) > c_0 \\ 0 & \text{otherwise.} \end{cases} \tag{31}$$

This proposition follows directly from equation (30) (see Appendices F and G for additional results and the proof of the existence of two stage equilibria). Sufficiently large shocks to either ξ_t or z_t move the system between interior and corner solutions, generating alternate business cycle phases with robust search effort, a large number of trading relationships, and low unemployment with phases marked by no search effort, few trading relationships, and high unemployment.

Transitional dynamics: To illustrate the deterministic transitional dynamics of the model,

Figure 4 illustrates movements in search effort as a function of ξ_t (a similar figure could be drawn for z_t), for the calibration in Section 5. The dashed line plots the passive stage equilibrium path with low search effort and the solid line the active stage equilibrium path with high search effort. The arrows indicate the direction of the transition dynamics for the endogenous variable to reach the basins of attraction of the system, represented by point $\sigma^p(1)$ for the passive deterministic steady state (DSS) and $\sigma^a(1)$ for the active DSS.[8] The shaded area indicates the range of values of ξ_t that support multiple stage equilibria. The passive stage equilibrium does not exist for sufficiently large values of ξ_t and, conversely, the active stage equilibrium fails to exist for sufficiently small values of ξ_t. In the absence of innovations to ξ_t, the system converges and remains on the original basins of attraction in the passive stage equilibrium, $\sigma^p(1)$, and the active stage equilibrium, $\sigma^a(1)$, depending on the starting stage equilibrium.

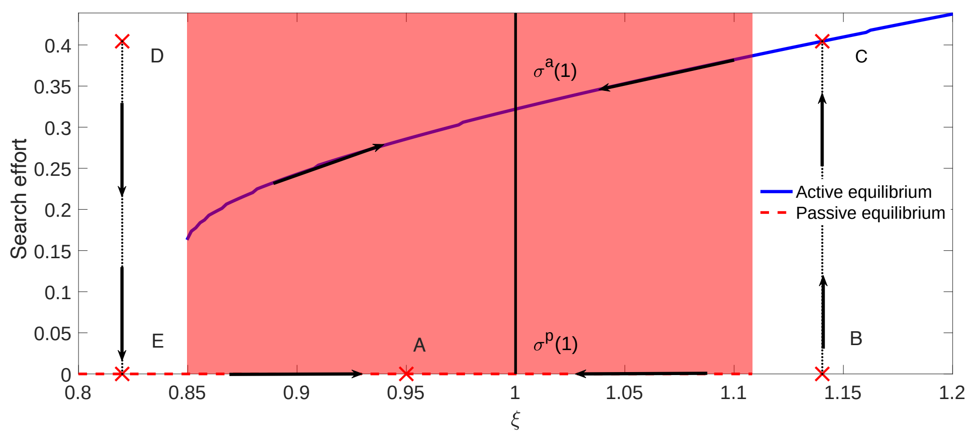


Figure 4: Transitional dynamics for search effort

Temporary shifts to ξ_t, which are sufficiently strong to change search effort, move the system to a new stage equilibrium. For example, if the system starts at point A and a large and positive innovation to ξ_t moves the system to point B, the passive stage equilibrium disappears, and the stage equilibrium of the system becomes active. The economy moves to the new active stage equilibrium at point C, converging to the stationary basin of attraction $\sigma^a(1)$ in the long run. The system remains in the active stage equilibrium until a sufficiently negative innovation to ξ_t returns the system to the passive stage equilibrium. For instance, a large negative innovation to ξ_t, which moves the system from point C to point D, triggers the new passive stage equilibrium at point E, converging to the stationary basin of attraction $\sigma^p(1)$. In comparison, innovations to

[8]The DSS is the steady state to which the economy converges in the absence of shocks. This model has two different DSSs: one with active search and one with passive search. Appendix H shows the solution of the DSSs.

ξ_t that move the system within the shaded area, where both stage equilibria coexist, fail to shift the stage equilibrium because of history dependence.

5 Calibration

We calibrate the model at a monthly frequency for U.S. data over the post-WWII period. Table 5 summarizes the value and the source or target for each parameter.

Table 5: Parameter calibration

Parameter	Value	Source or Target
β	0.996	5% annual risk-free rate
α	0.4	Shimer (2005)
τ	0.4	Hosios condition
χ	1.19	0.45 monthly job-finding rate (Shimer, 2005)
κ	1.25	den Haan et al. (2000)
h	0.3	Thomas and Zanetti (2009)
$\widetilde{\tau}$	0.5	Sectoral symmetry
δ	0.027	5.5% unemployment rate in active DSS
$\widetilde{\delta}$	0.024	3.5 years' duration of trading relationship
ϕ	0.12	22% rate of idleness in recessions
ψ	0.185	Estimation of the matching function in Section 2.2
c_0	0.32	4.45% of revenue spent on search effort
c_1	4.5	12% rate of idleness in booms
ν	2	Ensure concavity of best response function
ρ_ξ	0.6	Livingston Survey
σ_ξ	0.054	Livingston Survey
ρ_z	$0.88^{1/3}$	BLS
σ_z	0.008	BLS

The constant β is set to 0.996 to replicate an average annual interest rate of 5%. In keeping the DMP block of the model standard, we assume a Cobb-Douglas labor market matching function $m(u,v) = u^{1-\alpha}v^{\alpha}$, with $\alpha = 0.4$, the average value in the literature (Petrongolo and Pissarides, 2001). To satisfy the Hosios (1990) condition, we set the wage bargaining power to $\tau = \alpha = 0.4$. We follow den Haan et al. (2000) in selecting the inter-firm matching function that ensures that matching probabilities are between 0 and 1: $H(\widetilde{n}_F, \widetilde{n}_I) = (\widetilde{n}_F \cdot \widetilde{n}_I)/[(\widetilde{n}_F^{\kappa}/2 + \widetilde{n}_I^{\kappa}/2)^{1/\kappa}]$. Also after den Haan et al. (2000), we set $\kappa = 1.25$.

We pick the cost of posting a vacancy $\chi = 1.19$ to match the monthly job-finding rate in the active DSS, $\mu(\theta) = 0.45$, as in Shimer (2005). Then, we select a job-separation rate $\delta = 0.027$ to match an unemployment rate of 5.5% in the active DSS. We set $h = 0.3$ to include the value of leisure and home production and the unemployment benefit, as in Thomas and Zanetti (2009).

Thus, the flow value of unemployment is about 61% of the average wage in the active DSS, which is in the range of replacement rates documented by Hall and Milgrom (2008).

Compared to a standard DMP economy, our model includes seven new parameters: ψ, $\widetilde{\tau}$, $\widetilde{\delta}$, ϕ, c_0, c_1, and ν. We set ψ to 0.185 in accordance with our estimated regression values for equation (8), reported in Table 3. We can rewrite the matching function for firm k in sector i as:

$$\pi_{i,k,t} = \underbrace{\phi + \psi^2}_{\text{constant}} + \underbrace{\psi \sigma_{i,k,t}}_{\text{linear term}} + \underbrace{\sigma_{i,k,t} \varsigma_{j,t}}_{\text{interactive term}} + \underbrace{\psi \varsigma_{j,t}}_{\text{error term}},$$

where $\varsigma_{j,t}$ is the component of $\sigma_{j,t}$ that is orthogonal to $\sigma_{i,k,t}$. Imposing $\pi_{i,k,t} \propto \Delta n^{sup}_{i,k,t}$ yields $\psi = \beta_1/\beta_2 = 1.11/5.99$, where β_1 and β_2 are the coefficients in regression (8).

The bargaining share of the intermediate-goods firm $\widetilde{\tau}$ is set to 0.5, to evenly split the total surplus from matching between firms and make the workers indifferent between working in either sector. The rate of termination of inter-firm matches $\widetilde{\delta}$ is 0.024 to replicate the 3.5 years' average duration of a match documented in Section 2.

Given the previous parameters, c_1 and ϕ pin down the measure of single firms in the active DSS and passive DSS, respectively. The ratio of single firms to employment is the rate of idleness, i.e., the share of time when employed workers are idle due to a lack of activity (Michaillat and Saez, 2015). According to the Institute for Supply Management, the rate of idleness in the U.S. was about 30% for the non-manufacturing sector and 20% for the manufacturing sector during the Great Recession, and 12% for both sectors before this event. Thus, we set $\phi = 0.12$ and $c_1 = 4.5$ to yield a rate of idleness equal to 0.22 and 0.12 in the passive DSS and the active DSS, respectively. We calibrate c_0 to 0.32 to generate a search cost of about 4.45% of output. This value is consistent with the fact that suppliers and customers spend 7.5% and 1.4% of revenues searching for trading partners, respectively, as documented above. Finally, $\nu = 2$ ensures the concavity of the best response function of search effort.

Following Hall (2017), we use the Livingston Survey to calibrate the discount factor shock by obtaining the median 12-months-ahead expected return r_t of the stock market index. We compute the discount factor as $\xi_t = 1/(1 + r_t)$. The monthly AR(1) that fits the series of ξ_t has parameters $\rho_\xi = 0.6$ and $\sigma_\xi = 0.054$. These values, given the rest of the calibration, generate a passive stage equilibrium with 15% probability, consistent with the frequency of recessions in the post-WWII U.S. The quarterly standard deviation (s.d.) of 5% for ξ_t is close to the popular estimate of a quarterly s.d. of 5.7% by Justiniano and Primiceri (2008, Table 1). The persistence of the productivity shock, $\rho_z = 0.88^{1/3}$ matches the observed quarterly autocorrelation of 0.88 and the s.d., $\sigma_z = 0.008$ matches the quarterly s.d. of 0.02, as in Shimer (2005).

Once the model is calibrated, we compute the value functions using value function iteration and exploit the equilibrium conditions of the model to find all variables of interest. See Appendices C and J for details.

6 Quantitative analysis

To study the dynamics of the model, we simulate it for 3,000,000 months and time-average the resulting variables to generate quarterly data. We start the simulation from the active DSS, focusing on the case when only discount factor shocks are present. Appendix K provides a quantitative analysis of the model with productivity shocks. We relegate that case to the appendix because productivity shocks of plausible magnitude cannot move the system between stage equilibria, unless those shocks are permanent.

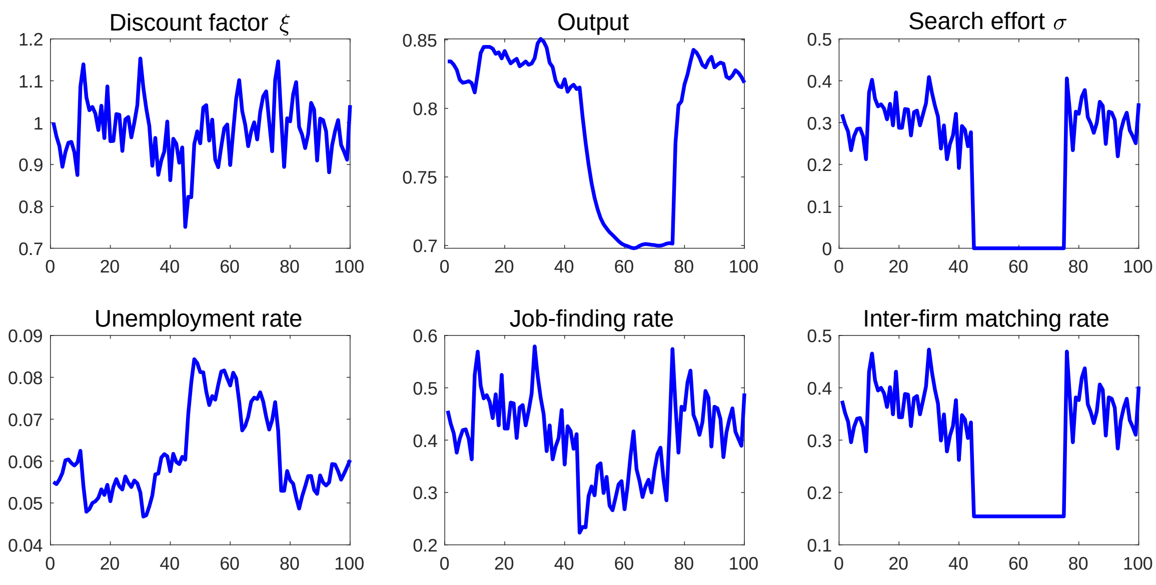


Figure 5: Simulated variables for the first 100 periods with shocks to ξ_t

Figure 5 reports the responses of key variables to shocks to ξ_t (top left panel) for the first 100 periods. The economy begins at a positive search effort with high output, low unemployment, and a high job-finding rate. Then, in period 45, a large negative shock to ξ_t pushes the economy to a prolonged drop in output (top center panel) as trading relationships terminate faster than they are replaced due to low effort (top right panel). Low effort generates a high unemployment rate and low job-finding and inter-firm matching rates (bottom panels). While the mean-reversion of ξ_t increases job-finding and decreases unemployment, the recovery is mild, since the economy stays in the passive stage equilibrium until a large positive discount factor shock that makes households more patient shifts the economy back to the active stage equilibrium in period 74. In such a way, our model endogenizes, through varying change effort, the idea of a regime-switching process in the evolution of output postulated by Hamilton (1989).

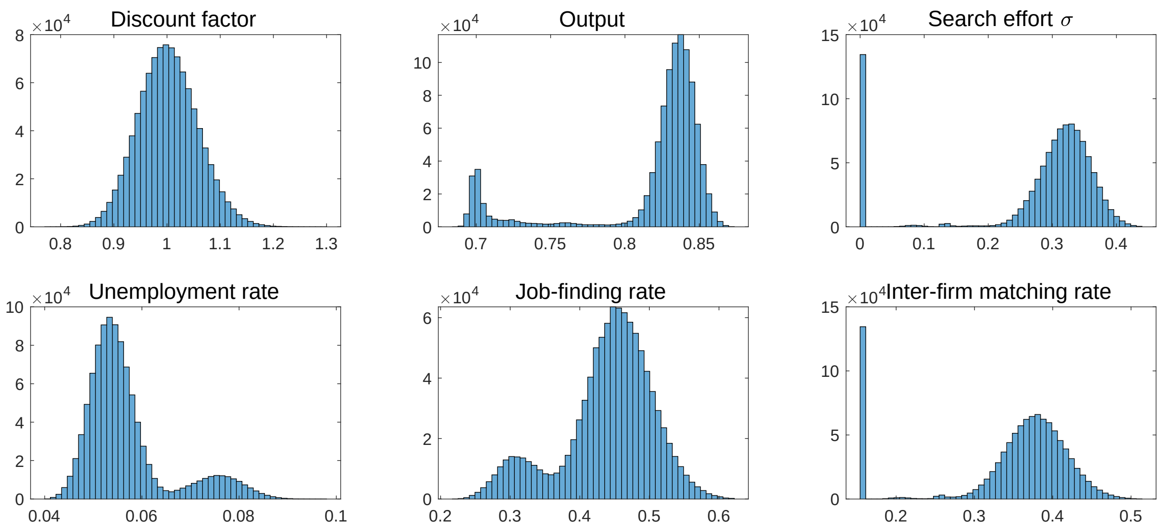


Figure 6: Ergodic distribution with shocks to ξ_t

Figure 6 plots the ergodic distribution of selected variables implied by the entire simulation. Endogenous switches between passive and active stage equilibria generate a distinctive bimodal distribution of aggregate variables resembling those documented in Adrian et al. (2019) or the ones from models with increasing returns to scale to search à la Diamond (1982), even if the discount factor shock has a unimodal distribution.[9] Consistent with U.S. data regarding recessions, our model predicts that the economy spends about 85% of the time in the active stage equilibrium and 15% in the passive stage equilibrium. In the former, the unemployment rate fluctuates around 5.5%. In the latter, unemployment fluctuates around 7.5%. The job-finding rate moves around 45% in the active stage equilibrium and 30% in the passive stage equilibrium.

Figure 7 compares the empirical distribution of real GDP per capita and the unemployment rate (continuous line) with the ergodic distribution in the model (discontinuous line).[10] Both

[9]See Pizzinelli et al. (2020) and Schaal and Taschereau-Dumouchel (2018) for additional evidence on skewness and bimodality in macroeconomic variables.

[10]Real GDP per capita is quarterly from 1960 to 2018 and is linearly detrended in logs. The unemployment

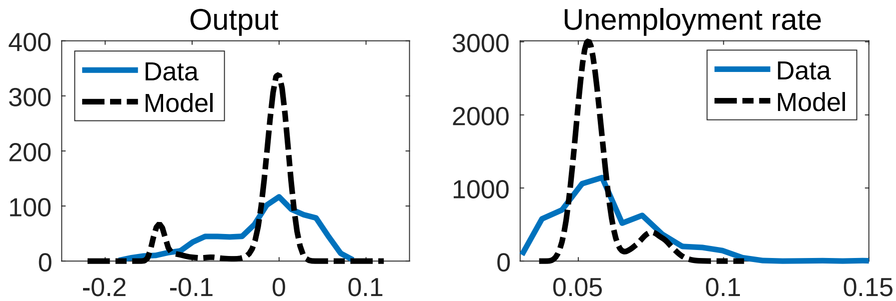


Figure 7: Distribution of unemployment rate and output in the data

the data and the model show skewness and bimodality.[11] The fair similarity between the data and the model offers support for the model, in particular if we recall that while we are using shocks *only* to the discount factor in our model, a combination of different shocks drives the dynamics of the real data. We will revisit this issue in more detail in Section 8.

Table 6: Second moments

		u	v	v/u	lp	ξ
(a) Quarterly U.S. data, 1951-2016						
Autocorrelation coefficient		0.95	0.95	0.95	0.90	–
Standard deviation		0.20	0.21	0.40	0.02	–
	u	1	-0.92	-0.98	-0.25	–
Correlation matrix	v		1	0.98	0.29	–
	v/u			1	0.27	–
	lp				1	–
(b) Benchmark model						
Autocorrelation coefficient		0.76	0.44	0.63	0.91	0.35
Standard deviation		0.12	0.27	0.36	0.04	0.05
	u	1	-0.67	-0.84	-0.85	-0.47
Correlation matrix	v		1	0.97	0.49	0.81
	v/u			1	0.65	0.76
	lp				1	0.11
	ξ					1

Note: Following Shimer (2005), all variables are reported in logs as deviations from an HP trend with $\lambda = 10^5$.

Panel (a) of Table 6 reports second moments of observed business cycle statistics following the structure in Shimer (2005, Table 1). Panel (b) reports second moments of the benchmark model with two DSSs. Appendix L reports the simulation of the model without search complementarities. The results are nearly identical when the filtering is done with a linear trend, as we used in Figure 7.

rate is monthly from 1960 to 2018.

[11]The Hartigan dip test for unimodality (Hartigan and Hartigan, 1985) rejects unimodality in the data for real GDP per capita and the unemployment rate with 11% and 6% significance levels, respectively.

Several lessons arise from Table 6. First, our model generates a robust internal propagation: the autocorrelation coefficients of the aggregate variables are significantly larger than in the model without complementarities and much closer to the observed ones. Complementarities in search effort plus history dependence amplify and prolong the effect of shocks. Second, our model generates empirically plausible s.d.'s for the selected variables that are much larger than those in the model without complementarities. This property of the model comes from the amplification of shocks described above. Third, our model produces endogenous movements in labor productivity ("lp" in the table) because of the time-varying fraction of the trading relationships over the total number of firms, $n_{i,t}/\left(\widetilde{n}_{i,t}+n_{i,t}\right)$. The business cycle statistics for labor productivity generated by our benchmark model are close to those in the data.

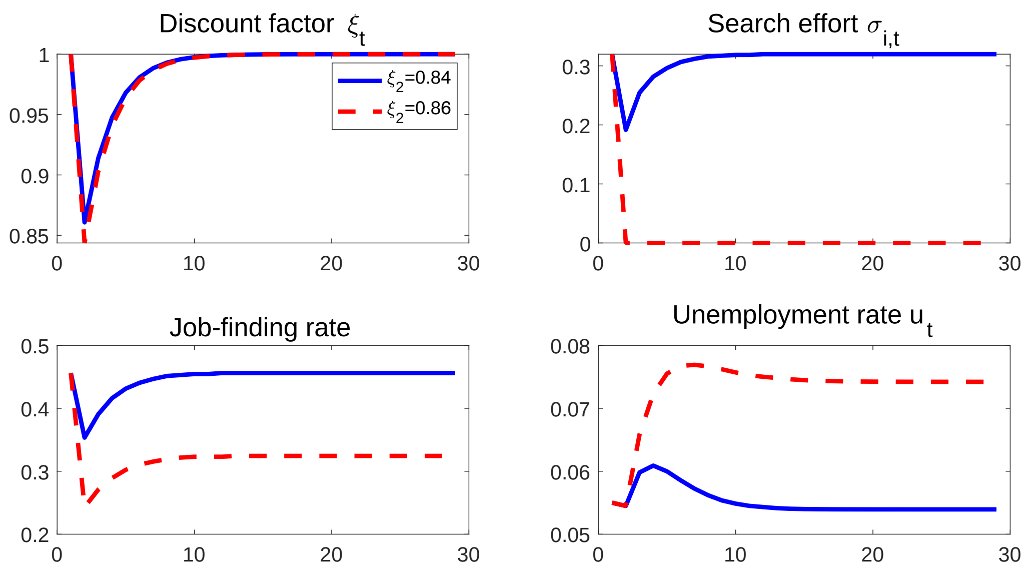


Figure 8: GIRFs to a negative discount factor shock

Figure 8 shows generalized impulse response functions (GIRF) of selected variables to a 16% (solid line) and 14% (dashed line) shock to ξ_t, respectively (since the model is not linear, we use the adjective "generalized"). In $t=1$, the economy starts from the active DSS. In $t=2$, a one-period innovation to the discount factor hits the economy (recall, however, that households and firms have rational expectations that this innovation can arrive with some probability). When the contractionary shock to ξ_t is 14%, the firm's search effort declines in response to the fall in the stream of benefits in forming a trading relationship, generating a mild decline in labor market tightness and a rise in the unemployment rate. This shock is too small to move the system to the passive stage equilibria and the variables converge to the original DSS. However, when the fall in ξ_t is sufficiently large, the system moves to the equilibrium with zero search effort, low output, and high unemployment. While the shock is only a bit larger (16% vs. 14%), its effects are quite different: search complementarities induce large nonlinearities in the model.

7 Evidence on the shocks to the discount factor

The mechanism in our model builds on two legs: the shocks to the discount factor and history dependence. We will not discuss the latter. As argued in the introduction, history dependence is an intuitive selection device that has shown empirical success in experiments (Van Huyck et al., 1990, 1991). We focus, instead, on the shocks to the discount factor.

Discount factor shocks have been documented, among many others, by Justiniano and Primiceri (2008), Fernández-Villaverde et al. (2015), Cochrane (2011), Hall (2016, 2017), and Ikeda et al. (2020). These authors have argued that, beyond shocks to preferences, discount factor shocks can also represent demographic shifts, movements in financial frictions, fluctuations in risk tolerance, and changes in fiscal and monetary policy that we abstract from in the model.

To relate measures of the discount factor to changes in aggregate output, unemployment, and inter-firm matching, we use the s.d. of the discount factor as the ratio of the current market price of a future cash receipt to the expected value of the payment (our households are risk-neutral and, hence, we do not need to adjust for risk).

There are three popular measures of the discount factor. In measure 1, we proxy the discount rate r_t as we did in the calibration using the measure of expected returns from the Livingston Survey. In measure 2, we follow Hall (2017) and construct the series for the market discount rate for dividends payable from one year (12 months) to two years (24 months) as: $\xi_t = p_t/(\mathbb{E}_t \sum_{\tau=13}^{24} d_{t+\tau})$, where p_t is the market price in month t of the claim of future dividends inferred from option prices and the stock price, and d_t is the dividend paid in month t. The data on p_t are from Binsbergen et al. (2012). Finally, in measure 3, we proxy the discount factor using the price-dividend ratio (p/d) of the stock market, as described in Cochrane (2011).

All three measures agree that i) there was a sizable decline in the discount factor during the Great Recession (as our theory requires) and ii) the series display high variance (reflecting the large sensitivity of the discount factor over the business cycle, also required by our theory). The low correlation across the three measures is not surprising, since each of these series reflects discounting from different financial players and assets (Hall, 2017). See Appendix M for details.

Table 7: Correlation between discount rates and aggregate variables

Correlation coefficient	(a) Unemployment rate	(b) GDP	(c) Intermediate input	(d) Match creation
Livingston Survey	-0.55	0.53	0.42	0.16
S&P dividend strip p/d ratio	-0.33	0.50	0.21	0.32
P/d ratio	-0.75	0.80	0.53	0.79

Note: Discount rates and unemployment: monthly data from January 1996 to May 2009. GDP: quarterly data from 1996Q1 to 2009Q1. Intermediate input: annual data from 1997 to 2009. Rate of match creation: annual data from 1996 to 2009. Series are HP filtered.

Table 7 shows that the three measures of the discount factor are negatively correlated with unemployment (column (a)), positively correlated with GDP and input of intermediate goods (columns (b) and (c)), and positively correlated with match creation (column (d)), measured from Compustat Customer Segment data. These patterns corroborate the link between shocks to the discount factor and movements in unemployment, production, and inter-firm matching highlighted by our model.

8 The volatility of shocks and aggregate performance

Our model links nonlinearly the volatility of shocks with aggregate outcomes. This feature has two sharp implications. First, when volatility is high, the distribution of output is bimodal, as the economy often switches between stage equilibria. However, when volatility is low, the distribution of output is unimodal. This is quite a unique prediction that distinguishes our model from most other business cycle models. Second, when volatility is low, large shocks have particularly persistent effects. Once a large shock pushes the economy into a new stage equilibrium, the economy will remain in it for a very long time because the probability of another large shock that will switch equilibria is low.[12] Again, this is a distinctive property of our model. We show now that both implications of our model hold in the data.

High volatility and bimodality: Our model predicts that the bimodality of the distribution of output will be particularly salient in periods of high volatility. To show that this is also the case

[12]See Appendix N for an illustration of how the duration of each stage equilibrium is inversely related to the volatility of the shocks. Appendix O reports the histograms of the model's endogenous variables for alternative levels of the volatility of the shocks.

in the data, we estimate the one-quarter-ahead conditional distribution of output with the non-parametric approach proposed by Adrian et al. (2019). Denote $y_t = (ln(GDP_{t+1}), ln(pd_{t+1}))$ and $x_t = (ln(GDP_t), ln(pd_t))$, where $ln(GDP_t)$ and $ln(pd_t)$ are the quarterly real GDP per capita and price-dividend ratio in logs as deviations from an HP trend, respectively.[13] The log price-dividend ratio is a proxy for the discount factor, the driving shock in our model.

We compute the joint distribution function of y conditional on x as:

$$\widehat{p}(y \mid x) = \frac{\frac{1}{T-1}\sum_{t=2}^{T} K_y(y - y_t) K_x(x - x_t)}{\frac{1}{T-1}\sum_{t=2}^{T} K_x(x - x_t)},$$

where $K_y(\cdot)$ and $K_x(\cdot)$ are independent kernels for y and x, respectively, defined as:

$$K_y(y - y_t) = \frac{1}{\omega_{1,y}}\varphi\left(\frac{y_1 - y_{1t}}{\omega_{1,y}}\right) + \frac{1}{\omega_{2,y}}\varphi\left(\frac{y_2 - y_{2,t}}{\omega_{2,y}}\right)$$
$$K_x(x - x_t) = \frac{1}{\omega_{1,x}}\varphi\left(\frac{x_1 - x_{1t}}{\omega_{1,x}}\right) + \frac{1}{\omega_{2,x}}\varphi\left(\frac{x_2 - x_{2,t}}{\omega_{2,x}}\right),$$

where $\omega_{i,y}$ and $\omega_{i,x}$ are the bandwidths for the ith variable of y and x, respectively, and $\varphi(\cdot)$ is the normal pdf. We set the bandwiths to be proportional to the in-sample unconditional standard deviation: $\omega_{1,y} = \omega_{1,x} = c \cdot \sigma(ln(GDP_t))$, $\omega_{2,y} = \omega_{2,x} = c \cdot \sigma(ln(pd_t))$, where c is calibrated to 0.3 as in Adrian et al. (2019).

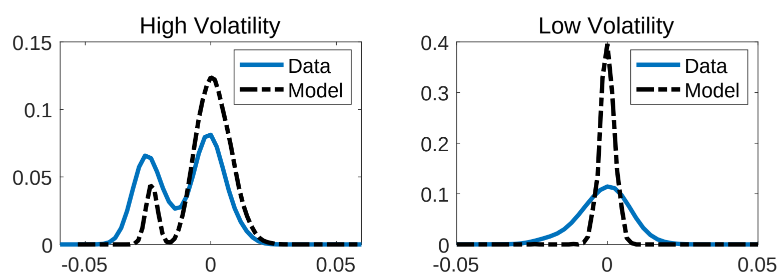


Figure 9: Marginal conditional distribution of output

The solid curves in Figure 9 plot the estimated conditional marginal distribution of output in 2008.Q4 (left panel) and 2017.Q4 (right panel). Bimodality is pronounced in 2008.Q4, when volatility was high ($\sigma(ln(pd_t)) = 0.075$ vs. a sample mean of 0.052).[14] In contrast, there was no bimodality in 2017.Q4, when volatility was low ($\sigma(ln(pd_t)) = 0.031$). This result is general across the sample 1960.Q1-2021.Q1. The correlation between the Hartigan dip statistic for each

[13] We obtain the monthly p-d ratio from Robert Shiller's website: http://www.econ.yale.edu/ shiller/data.htm, then convert it to quarterly using time-averaging.

[14] We compute $\sigma(ln(pd_t))$ as the standard deviation of the monthly p-d ratio in a 12-month window around each quarter.

quarter's marginal conditional distribution of output and the volatility of the price-dividend ratio is -0.13, which is statistically significant at the 5% level.[15]

To replicate the same exercise using our model, we simulate two sets of 1,000 economies. In the first set, we set $\sigma_\xi = 0.081$, 50% higher than in our benchmark calibration in Table 5. Each economy runs for one quarter (three months) and starts from the active DSS with the initial discount factor two standard deviations below its mean. In the second set, we simulate another 1,000 economies for one quarter when $\sigma_\xi = 0.027$, 50% lower than our benchmark calibration. To make the two sets perfectly comparable, we construct the discount factors in the second simulation as our random draws in the first simulation multiplied by 1/3.

The dashed curves in Figure 9 plot the marginal conditional distribution of output for each of these two simulated samples. The marginal conditional distribution of output implied by the model in the high volatility case (the left panel) is bimodal and fairly close to the solid curve in the same panel. The right panel shows that the conditional distribution is unimodal when volatility is low, similar to the data.

The Great Moderation and the persistence of business cycles: Our model predicts that a lower volatility of fundamentals is associated with more prolonged stage equilibrium spells. This prediction is consistent with the U.S. data.

In Figure 10, the upper panel plots the U.S. employment rate (blue curve) and its trend (orange curve) estimated from an HP filter with $\lambda = 1600$ from 1996 to 2017. The light-orange bars indicate labor market downturns. Inspired by the NBER's methodology, we define a labor market downturn as starting when the employment rate falls below the trend for two quarters and ending when the employment rate rises above the trend for two quarters. As noted by many researchers (Jaimovich and Siu, 2012 and references therein), the figure shows how the three labor market downturns after 1984 were longer than the previous ones. Precisely after 1984, the U.S. economy experienced a substantial reduction in aggregate volatility, which Fernández-Villaverde et al. (2015) attribute, in part, to a lower volatility of shocks to fundamentals. To illustrate this point, the bottom panel in Figure 10 plots the cyclical component of real GDP per capita, with a grey area to indicate the Great Moderation after 1984.

Our model suggests an intrinsic connection between the Great Moderation and the increasing

[15] Since the dip statistic is an increasing function of the probability of unimodality, a negative correlation means unimodality is much more likely to be rejected when volatility is high.

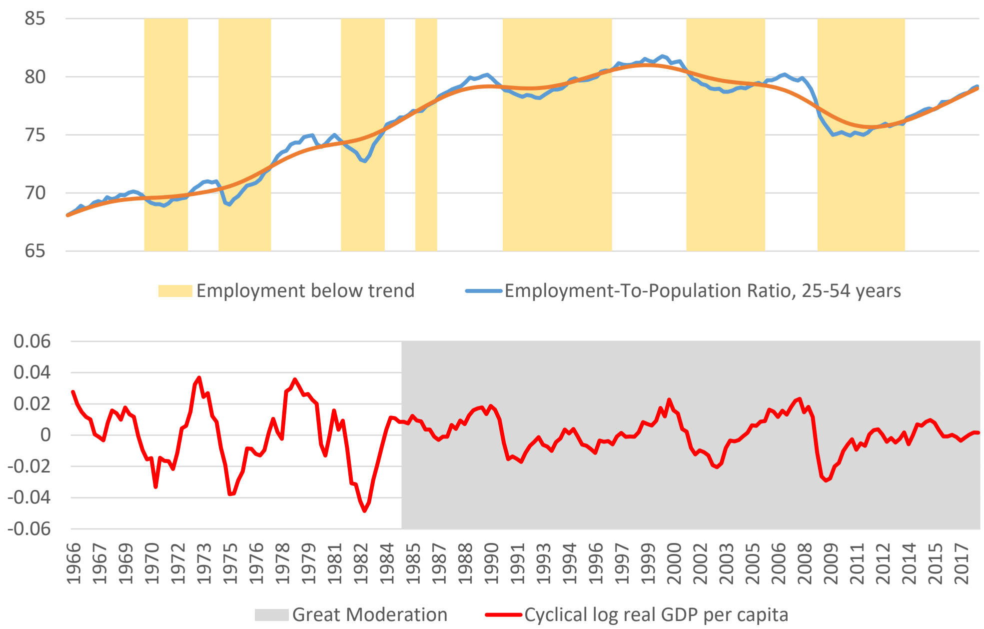


Figure 10: The Great Moderation and labor market downturns

persistence in labor market downturns, like the one that followed the financial crisis of 2008. While the Great Moderation improves macroeconomic stability and reduces the occurrences of recessions, it makes these recessions and the associated labor market downturns more durable.

9 The role of fiscal policy

In our model, government spending that stimulates search effort may permanently move the system from a passive to an active stage equilibrium, inducing a large fiscal multiplier. To explore this idea, we embed government spending in our economy. We focus on government spending (government consumption expenditures and gross investment) while ignoring transfers because, in our model, output is not demand-determined.

9.1 Government spending as a set of final-goods producers

The government owns single final-goods firms, $\widetilde{n}^G_{F,t}$, that operate together with private firms. The only difference between government-owned and private firms is that the former do not use labor.[16] The formation of private firms remains endogenous, as described by equation (20).

[16] It would be easy to modify the model to force government-owned firms to hire workers to operate (and, thus, be fully symmetric to private firms). We prefer our assumption of jobless government firms because it allows

Government spending is equal to the output produced by government-owned firms in trading relationships (i.e., the government "buys" the output of its firms and the government-owned firms use those resources to pay the private intermediate firm and in production) plus the single government-owned firms' search cost. This spending is financed through lump-sum taxes.

We model higher government spending as an exogenous increase in the number of single firms in the final-goods sector. These additional firms can be interpreted as new public projects such as building a new school. Thus, the law of motion for government single final-goods firms is $\widetilde{n}_{F,t+1}^{G} = (1 - \delta - \pi_F)\, \widetilde{n}_{F,t}^{G} + \epsilon_t^G$, where ϵ_t^G are the new government-owned single firms created in t.[17] Like private firms, government-owned firms must form a trading relationship with firms in the intermediate-goods sector to manufacture goods (for example, a public school requires CFRPs produced by private firms). Trading relationships with government-owned firms follow $n_{F,t+1}^{G} = \left(1 - \delta - \widetilde{\delta}\right) n_{F,t}^{G} + \pi_F \widetilde{n}_{F,t}^{G}$. A government-owned firm exits the market when its trading relationship is terminated (either because the relationship itself fails with probability $\widetilde{\delta}$ or the job in the private firm disappears with probability δ).

The inflow ϵ_t^G changes the matching probabilities $\pi_{I,t} = [\phi + (\psi + \sigma_I)(\psi + \sigma_F)]\, H\left(1, \widetilde{\theta}_t\right)$, in the inter-firm matching market and $\pi_F = [\phi + (\psi + \sigma_I)(\psi + \sigma_F)]\, H\left(\frac{1}{\widetilde{\theta}_t}, 1\right)$, where $\widetilde{\theta}_t = (\widetilde{n}_{F,t} + \widetilde{n}_{F,t}^{G})/\widetilde{n}_{I,t}$ is the new inter-firm matching market tightness ratio.

Since H is increasing in both arguments, $\epsilon_t^G > 0$ increases the matching probability for intermediate-goods firms (more potential partners) and decreases the matching probability for final-goods firms (stiffer competition for partners). These changes in matching probabilities, in turn, move search effort and, potentially, the stage equilibrium of the economy.

Government spending is equal to $g_t = z_t n_{F,t}^{G} + \widetilde{n}_{F,t}^{G}\left(c_0 \widetilde{\sigma}_{F,t}^{0.5} + c_1 \frac{\widetilde{\sigma}_{F,t}^{(1+\nu)/2}}{1+\nu}\right)$. Gross aggregate output comprises government and private production (as per standard national accounting conventions): $y_t = z_t(n_{F,t}^{G} + n_{F,t})$, and it is used for private consumption, government spending, and search costs. The aggregate resource constraint is $y_t = c_t + g_t + \sum_{i=I,F} \chi v_i + \sum_{i=I,F} \widetilde{n}_{i,t}\left(c_0 \widetilde{\sigma}_{i,t}^{0.5} + c_1 \frac{\widetilde{\sigma}_{i,t}^{(1+\nu)/2}}{1+\nu}\right)$.

for a rapid increase in government spending. Job matching requires time, and it would mean that government spending would only phase in slowly. Unfortunately, this slow phase-in would make our quantitative results less comparable with existing findings in the literature.

[17] We assume that government spending shocks hit once per year. With probability 1/12, ϵ_t^G is drawn from the uniform distribution with the support $[0, \widetilde{n}_{F,t}/2]$. Otherwise, $\epsilon_t^G = 1$. This specification ensures a non-negative measure of government firms and that the inter-firm matching market tightness ratio does not explode.

9.2 Shocks to government spending and equilibria switches

We assume that the economy is in the passive stage equilibrium (i.e., $\sigma_I = \sigma_F = 0$) before the arrival of a positive government spending shock, ϵ_t^G. Upon the realization of the shock, the passive stage equilibrium continues to exist if and only if:

$$\widetilde{\beta}\xi_t\psi H\left(1,\widetilde{\theta}_t\right)\mathbb{E}_t\left(J_{I,t+1}-\widetilde{J}_{I,t+1}\mid\iota=0\right)<c_0, \tag{32}$$

and

$$\widetilde{\beta}\xi_t\psi H\left(\widetilde{\theta}_t^{-1},1\right)\mathbb{E}_t\left(J_{F,t+1}-\widetilde{J}_{F,t+1}\mid\iota=0\right)<c_0. \tag{33}$$

where $\widetilde{\beta}=\beta\left(1-\delta\right)/\tau$. Equation (32) shows that the passive stage equilibrium disappears if the increase in government-owned single firms tightens the inter-firm matching market enough.

Proposition 2. *Starting from the passive stage equilibrium, the size of government spending needed to move the system to the active stage equilibrium is:*

$$\frac{\widetilde{n}_{F,t}^G}{\widetilde{n}_{I,t}}>\Psi\left[\frac{c_0}{\widetilde{\beta}\xi\psi\mathbb{E}_t\left(J_{I,t+1}-\widetilde{J}_{I,t+1}\mid\iota=0\right)}\right]-\frac{\widetilde{n}_{F,t}}{\widetilde{n}_{I,t}}, \tag{34}$$

with $\Psi'>0$.[18]

Equation (34) determines that the magnitude of the policy intervention that moves the economy to an active stage equilibrium is proportional to the cost-benefit ratio of forming a trading relationship, and it decreases with the measure of private firms in the final-goods sector relative to intermediate-goods firms. A large quantity of private final-goods firms improves the trading relationship prospects for intermediate-goods firms, decreasing the magnitude of government spending needed to move to the active stage equilibrium.

9.3 The fiscal multiplier

We measure now the response of the economy to expansionary fiscal policy shocks and the size of the fiscal multiplier. Once we introduce government spending, we have 12 state variables. Due to this large number of state variables, we implement a dimensionality reduction algorithm inspired

[18] Denote $h\left(\widetilde{\theta}\right)=H\left(1,\widetilde{\theta}\right)$. Ψ is the inverse function of $h\left(\cdot\right)$. As $h\left(\cdot\right)$ is strictly increasing in $\widetilde{\theta}$ by assumption, Ψ is also a strictly increasing function. In our calibration: $h\left(\theta\right)=2^{\frac{1}{\kappa}}\left(1+\widetilde{\theta}_t^{-\kappa}\right)^{-1/\kappa}$, $\Psi\left(x\right)=\left(2x^{-\kappa}-1\right)^{-1/\kappa}$.

by Krusell and Smith (1998) that is of interest in itself and applicable to similar problems. See Appendix J.2 for computational details.

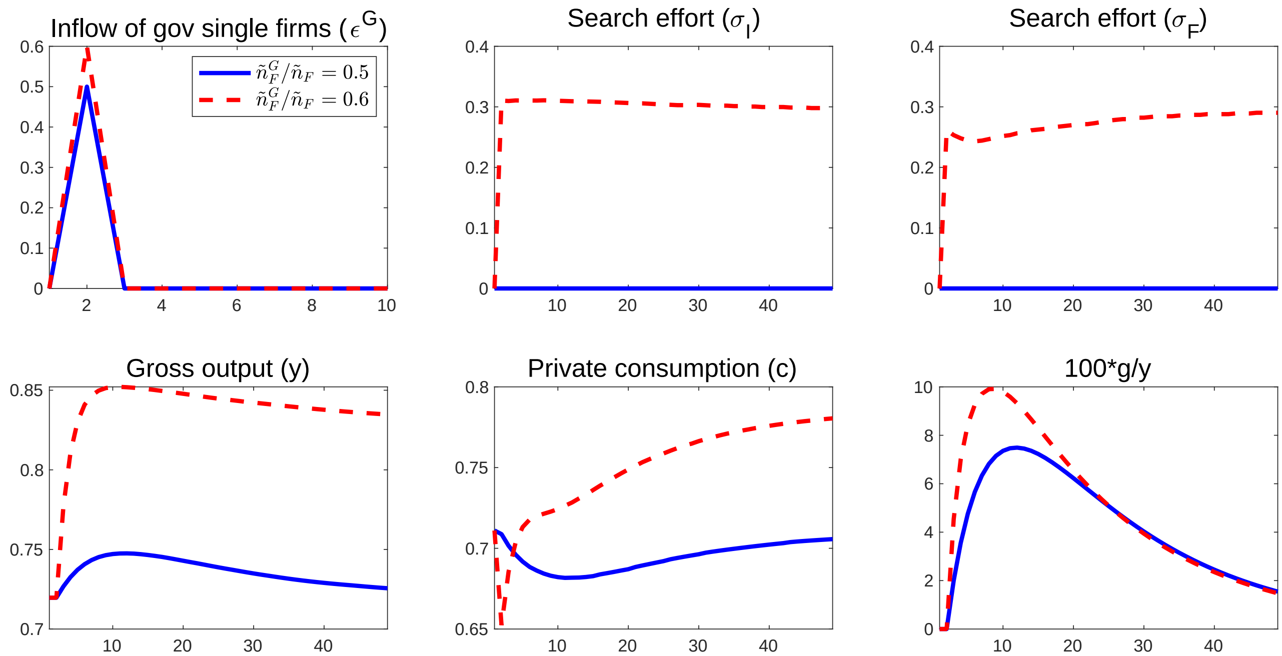
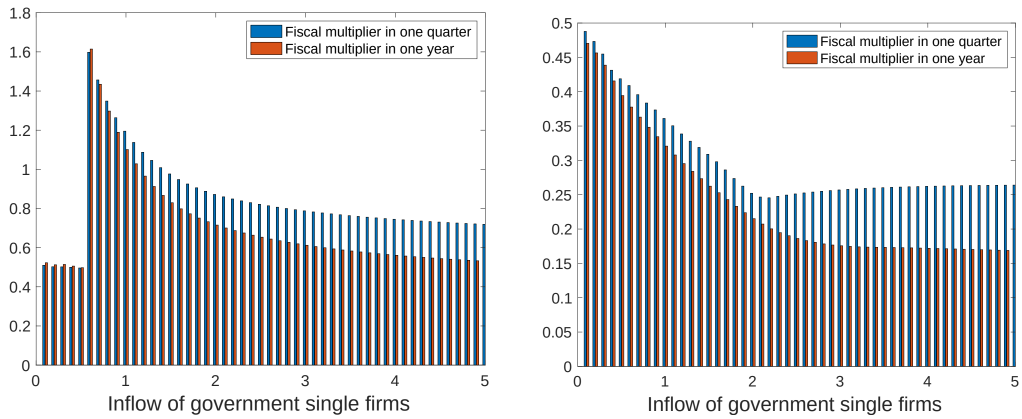


Figure 11: GIRFs to positive government spending shock

Figure 11 shows the GIRFs to the same 50% (dotted line) and 60% (solid line) shocks to the relative size of the final-goods sector that we just described when the economy starts at the passive DSS (Appendix P shows the responses for the system that starts from the active DSS). Since the 60% fiscal expansion satisfies Proposition 2, it produces a significant and persistent increase in output and a fall in unemployment. Nevertheless, this fiscal expansion crowds out private consumption upon impact. This reaction is due to two mechanisms. First, a rise in government-owned firms reduces, in the short run, the formation of trading relationships that produce goods for private consumption. Second, the shift of equilibrium triggers an increase in the cost associated with vacancy posting and the formation of trading relationships, which further reduces private consumption. The first mechanism still exists in the 50% fiscal expansion, inducing a small drop in private consumption.

We calculate the fiscal multiplier for our economy, defined as the ratio of the cumulative change in output over one quarter and one year, generated by the one-period change in government spending triggered by the inflow of government-owned single firms in the final-goods sector (we could compute the fiscal multiplier at other horizons if desired). Panel (a) in Figure 12 shows the fiscal multiplier as a function of the inflow of government-owned single firms when the economy is in the passive stage equilibrium at the start of the fiscal expansion. Panel (b) replicates the exercise for the case when the economy is in the active stage equilibrium.



(a) Passive stage equilibrium **(b)** Active stage equilibrium

Figure 12: Fiscal multiplier

In the passive stage equilibrium, a sufficiently large fiscal expansion generates a multiplier larger than one since it triggers a jump in search effort. The fiscal multiplier peaks at the threshold where we shift from the passive to the active stage equilibrium. In our calibration, the peak quarterly fiscal multiplier, 1.59, is at a 55% increase in the number of government-owned firms, which is equivalent to a 5.4% increase in government spending relative to output in the first quarter (since the increase in government spending is persistent, the overall size of the fiscal intervention is larger than the impact change of 5.4%). Any larger stimulus reduces the fiscal multiplier because the crowding out of private consumption outweighs the increase in output from the fiscal expansion. Similarly, a fiscal expansion below the threshold generates a less than unitary fiscal multiplier since it creates a large crowding-out effect and no equilibrium switch.

Panel (b) in Figure 12 shows that the fiscal multiplier is substantially lower in the active stage equilibrium. The increased costs of forming trading relationships for private firms in the final-goods sector reduce private output, and we have a less than unitary fiscal multiplier for any size of the fiscal stimulus. The multiplier declines with the size of government spending for a crowding-out effect across a wide range of time horizons.

Our results in Figure 12 agree with the recent empirical literature that has documented the acute state dependence of fiscal multipliers. See, for example, Auerbach and Gorodnichenko (2012), Owyang et al. (2013), and Ghassibe and Zanetti (2020). Our model accounts for such state dependence of fiscal multipliers.

10 Conclusion

This paper has documented five novel facts about the role of search effort in forming trading relationships by combining a variety of micro and macro datasets. These five facts can be parsimoniously interpreted as suggesting the existence of search complementarities in the formation of trading relationships. We have built a dynamic general equilibrium model, disciplined with our new firm-level evidence on search effort, to account for those five novel facts and explore its quantitative implications.

The analysis opens exciting avenues for additional research. Empirically, the role of agent and spatial heterogeneity in search effort and trading relationships formation deserves further exploration. Quantitatively, a direct extension would be to embed strategic complementarities in richer models of the business cycle, such as those including money, nominal rigidities, and financial frictions. We will pursue some of those ideas in future work.

References

Acemoglu, D., Carvalho, V. M., Ozdaglar, A., and Tahbaz-Salehi, A. (2012). The network origins of aggregate fluctuations. *Econometrica*, 80(5):1977–2016.

Adrian, T., Boyarchenko, N., and Giannone, D. (2019). Multimodality in macro-financial dynamics. Staff Report 903, Federal Reserve Bank of New York.

Auerbach, A. J. and Gorodnichenko, Y. (2012). Measuring the output responses to fiscal policy. *American Economic Journal: Economic Policy*, 4(2):1–27.

Beaudry, P., Galizia, D., and Portier, F. (2016). Is the macroeconomy locally unstable and why should we care? In *NBER Macroeconomics Annual 2016, Volume 31*, NBER Chapters, pages 479–530. National Bureau of Economic Research, Inc.

Beaudry, P., Galizia, D., and Portier, F. (2018). Reconciling Hayek's and Keynes' views of recessions. *Review of Economic Studies*, 85(1):119–156.

Beaudry, P., Galizia, D., and Portier, F. (2020). Putting the cycle back into business cycle analysis. *American Economic Review*, 110(1):1–47.

Binsbergen, J. v., Brandt, M., and Koijen, R. (2012). On the timing and pricing of dividends. *American Economic Review*, 102(4):1596–1618.

Bulow, J. I., Geanakoplos, J. D., and Klemperer, P. D. (1985). Multimarket oligopoly: Strategic substitutes and complements. *Journal of Political Economy*, 93(3):488–511.

Burdett, K. and Mortensen, D. (1980). Search, layoffs, and labor market equilibrium. *Journal of Political Economy*, 88(4):652–72.

Chatterjee, S., Cooper, R., and Ravikumar, B. (1993). Strategic complementarity in business formation: Aggregate fluctuations and sunspot equilibria. *Review of Economic Studies*, 60(4):795–811.

Cochrane, J. H. (2011). Discount rates. *Journal of Finance*, 66(4):1047–1108.

Cooper, R. (1994). Equilibrium selection in imperfectly competitive economies with multiple equilibria. *Economic Journal*, 104(426):1106–1122.

Cooper, R. and John, A. (1988). Coordinating coordination failures in Keynesian models. *Quarterly Journal of Economics*, 103(3):441–463.

den Haan, W. J., Ramey, G., and Watson, J. (2000). Job destruction and propagation of shocks. *American Economic Review*, 90(3):482–498.

Diamond, P. (1982). Aggregate demand management in search equilibrium. *Journal of Political Economy*, 90(5):881–894.

Diamond, P. and Fudenberg, D. (1989). Rational expectations business cycles in search equilibrium. *Journal of Political Economy*, 97(3):606–619.

Eeckhout, J. and Lindenlaub, I. (2018). Unemployment cycles. Mimeo, UCL.

Fernández-Villaverde, J., Guerrón, P., and Rubio-Ramírez, J. F. (2015). Estimating dynamic equilibrium models with stochastic volatility. *Journal of Econometrics*, 185(1):216–229.

Ghassibe, M. and Zanetti, F. (2020). State dependence of fiscal multipliers: the source of fluctuations matters. Mimeo, University of Oxford.

Hall, R. E. (2014). What the cyclical response of advertising reveals about markups and other macroeconomic wedges. Mimeo, Stanford University.

Hall, R. E. (2016). *Macroeconomics of Persistent Slumps*, volume 2 of *Handbook of Macroeconomics*, pages 2131–2181. Elsevier.

Hall, R. E. (2017). High discounts and high unemployment. *American Economic Review*, 107(2):305–330.

Hall, R. E. and Milgrom, P. (2008). The limited influence of unemployment on the wage bargain. *American Economic Review*, 98(4):1653–1674.

Hamilton, J. D. (1989). A new approach to the economic analysis of nonstationary time series and the business cycle. *Econometrica*, 57(2):357–384.

Hartigan, J. A. and Hartigan, P. M. (1985). The dip test of unimodality. *The Annals of Statistics*, 13(1):70–84.

Heizer, J., Render, B., and Munson, C. (2016). *Operations Management: Sustainability and Supply Chain Management*. Pearson Education, 12th edition.

Hosios, A. J. (1990). On the efficiency of matching and related models of search and unemployment. *Review of Economic Studies*, 57(2):279–298.

Howitt, P. (1985). Transaction costs in the theory of unemployment. *American Economic Review*, 75(1):88–100.

Ikeda, D., Li, S., Mavroeidis, S., and Zanetti, F. (2020). Testing the effectiveness of unconventional monetary policy in Japan and the United States. Papers 2012.15158, arXiv.org.

Jaimovich, N. and Siu, H. E. (2012). The trend is the cycle: Job polarization and jobless recoveries. Working Paper 18334, National Bureau of Economic Research.

Jones, C. I. (2013). Misallocation, economic growth, and input-output economics. In Acemoglu, D., Arellano, M., and Dekel, E., editors, *Advances in Economics and Econometrics: Tenth World Congress*, volume 2, pages 419–456. Cambridge University Press.

Justiniano, A. and Primiceri, G. E. (2008). The time-varying volatility of macroeconomic fluctuations. *American Economic Review*, 98(3):604–641.

Kaplan, G. and Menzio, G. (2016). Shopping externalities and self-fulfilling unemployment fluctuations. *Journal of Political Economy*, 124(3):771–825.

Krusell, P. and Smith, A. A. (1998). Income and wealth heterogeneity in the macroeconomy. *Journal of Political Economy*, 106(5):867–896.

Liu, P., Theodoridis, K., Mumtaz, H., and Zanetti, F. (2019). Changing macroeconomic dynamics at the zero lower bound. *Journal of Business & Economic Statistics*, 37(3):391–404.

Michaillat, P. and Saez, E. (2015). Aggregate demand, idle time, and unemployment. *Quarterly Journal of Economics*, 130(2):507–569.

Owyang, M. T., Ramey, V. A., and Zubairy, S. (2013). Are government spending multipliers greater during periods of slack? Evidence from twentieth-century historical data. *American Economic Review*, 103(3):129–134.

Petrongolo, B. and Pissarides, C. A. (2001). Looking into the black box: A survey of the matching function. *Journal of Economic Literature*, 39(2):390–431.

Pizzinelli, C., Theodoridis, K., and Zanetti, F. (2020). State Dependence in Labor Market Fluctuations. *International Economic Review*, 16(3):1027–1072.

Schaal, E. and Taschereau-Dumouchel, M. (2018). Coordinating Business Cycles. Mimeo.

Shimer, R. (2005). The cyclical behavior of equilibrium unemployment and vacancies. *American Economic Review*, 95(1):25–49.

Sterk, V. (2016). The Dark Corners of the Labor Market. Discussion Papers 1603, Centre for Macroeconomics (CFM).

Stevenson, W. J. (2018). *Operations Management*. McGraw-Hill/Irwin, 13th edition.

Thomas, C. and Zanetti, F. (2009). Labor market reform and price stability: An application to the Euro Area. *Journal of Monetary Economics*, 56(6):885–899.

Van Huyck, J. B., Battalio, R. C., and Beil, R. O. (1990). Tacit coordination games, strategic uncertainty, and coordination failure. *American Economic Review*, 80(1):234–248.

Van Huyck, J. B., Battalio, R. C., and Beil, R. O. (1991). Strategic uncertainty, equilibrium selection, and coordination failure in average opinion games. *Quarterly Journal of Economics*, 106(3):885–910.

Weitzman, M. (1982). Increasing returns and the foundations of unemployment theory. *Economic Journal*, 92(368):787–804.

Appendix

We include a series of appendices with further details of the model and the quantitative exercises.

A Use signaling costs to measure search efforts

As an alternative exercise, we approximate the search effort by the signaling costs that make firms more visible to potential trading partners. Following Hall (2014), we measure an industry i's signaling cost as the value of its intermediate input from the four industries of publishing, motion picture/sound recording, broadcasting/telecommunications, and data processing/internet publishing/other information services, obtained from the BEA input-output tables. We use first log differences to remove the industry-specific trend in the measured signaling cost. The difference between this measurement and our measurement in the main paper is that the former gauges the search effort outsourced from the other industries, while the latter focuses on the search effort exerted within the industry.

More precisely, we measure search costs as the intermediate input from the industries above by deriving a measure of the signaling cost for industry i's connected industries by weighting signaling costs by the value of input-output intermediate goods traded with industry i. Then, we estimate

$$\Delta\sigma_{i,t}^{sig} = \omega\Delta\sigma_{con(i),t}^{sig} + \nu_i + \gamma_t + \epsilon_{i,t},$$

where $\Delta\sigma_{i,t}^{sig}$ is the measure of changes in the signaling cost of industry i; $\Delta\sigma_{con(i),t}^{sig}$ is the change in signaling cost of industry i's connected industries, which include both industry i's customer and supplier industries; and υ_i and γ_t are the industry and the year fixed effects, respectively. Column 1 in Table 8 shows that signaling costs are positively correlated between connected industries, which –as in the main text– supports the existence of search complementarities.

To ensure that results are not driven by common shocks, we also implement the two-stage regression approach described in the main text. Table 8 in column 2 shows that changes in signaling costs are positively correlated between connected industries, supporting the existence of search complementarities.

Table 8: Search efforts are positively correlated between connected industries

	(5)	(6)
	One-stage	Two-stage
$\sigma^{sig}_{con(i),t}$	0.48***	0.40***
	(0.06)	(0.06)
Time FE	Yes	Yes
Industry FE	Yes	Yes
R^2	0.12	0.10
Observations	1,239	1,239

Note: Data are yearly from 1998 to 2017. Standard errors are in parentheses. *** denotes significance at the 1% level.

B Wages and prices

During each period t, wages are pinned down by Nash bargaining between firms in trading relationships and workers:

$$\max_{w_{i,t}} (W_{i,t} - U_{i,t})^{1-\tau} J^{\tau}_{i,t} \tag{35}$$

and between single firms and workers:

$$\max_{\widetilde{w}_{i,t}} (\widetilde{W}_{i,t} - U_{i,t})^{1-\tau} \widetilde{J}^{\tau}_{i,t}, \tag{36}$$

where the parameter $\tau \in [0,1]$ is the firm's bargaining power.

The price for goods manufactured in the intermediate-goods sector is determined by Nash bargaining between the final-goods producer and the intermediate-goods producer within the trading relationship:

$$\max_{p_t} (J_{F,t} - \widetilde{J}_{F,t})^{1-\widetilde{\tau}} (J_{I,t} - \widetilde{J}_{I,t})^{\widetilde{\tau}}, \tag{37}$$

where the parameter $\widetilde{\tau} \in [0,1]$ is the intermediate-goods producer's bargaining power.

C Total surplus

The total surplus of a labor market match at time t in a trading relationship in either sector $i \in \{I, F\}$ of the economy is $TS_{i,t} = W_{i,t} - U_{i,t} + J_{i,t}$. Analogously, the total surplus of a filled job in a single firm is $\widetilde{TS}_{i,t} = \widetilde{W}_{i,t} - U_{i,t} + \widetilde{J}_{i,t}$.

Given the bargaining weight, τ, common across sectors, Nash bargaining for wages implies:

$$J_{i,t} = \tau TS_{i,t}, \tag{38}$$

$$W_{i,t} - U_{i,t} = (1-\tau) TS_{i,t}, \tag{39}$$

$$\widetilde{J}_{i,t} = \tau \widetilde{TS}_{i,t}, \tag{40}$$

$$\widetilde{W}_{i,t} - U_{i,t} = (1-\tau) \widetilde{TS}_{i,t}. \tag{41}$$

The free-entry condition of the labor market is:

$$\chi = \beta \xi_t \tau H\left(\widetilde{\theta}_t, 1\right) \mathbb{E}_t \left(\widetilde{TS}_{I,t+1}\right) = \beta \xi_t \tau H\left(1, 1/\widetilde{\theta}_t\right) \mathbb{E}_t \left(\widetilde{TS}_{F,t+1}\right). \tag{42}$$

The total surplus of establishing a trading relationship is the sum of the capital gain from matching for the firms in the intermediate-goods sector, $J_{I,t} - \widetilde{J}_{I,t}$, and final-goods sector, $J_{F,t} - \widetilde{J}_{F,t}$ is $TSJV_t = J_{I,t} - \widetilde{J}_{I,t} + J_{F,t} - \widetilde{J}_{F,t}$. The price for intermediate goods, p_t, is set according to the Nash bargaining rules $J_{I,t} - \widetilde{J}_{I,t} = \widetilde{\tau} TSJV_t$ and $J_{F,t} - \widetilde{J}_{F,t} = (1-\widetilde{\tau}) TSJV_t$, where $\widetilde{\tau}$ is the intermediate-goods producer's bargaining power.

We derive now the total surplus of a filled job in a trading relationship, $TS_{i,t}$. Using the equations for $W_{I,t}$, $J_{I,t}$, and $U_{I,t}$ in the definition of $TS_{I,t}$, we get:

$$W_{I,t} + J_{I,t} - U_{I,t} = z_t p_t - h + \beta \xi_t \mathbb{E}_t \left[\begin{array}{c} \left(1-\delta-\tilde{\delta}\right)\left(W_{I,t+1} + J_{I,t+1} - U_{I,t+1}\right) \\ +\tilde{\delta}\left(\widetilde{W}_{I,t+1} + \widetilde{J}_{I,t+1} - U_{I,t+1}\right) - \mu_{I,t}\left(\widetilde{W}_{I,t+1} - U_{I,t+1}\right) \end{array} \right], \tag{43}$$

or, equivalently,

$$TS_{I,t} = z_t p_t - h + \beta \xi_t \mathbb{E}_t \left[\left(1-\delta-\tilde{\delta}\right) TS_{I,t} + \left(\tilde{\delta} - \mu_{I,t}(1-\tau)\right) \widetilde{TS}_{I,t}\right], \tag{44}$$

where, in the interest of space, we omit the variable ι_t.

Analogously, the total surplus of a filled job in a trading relationship for the firm in the final-goods sector F is:

$$TS_{F,t} = z_t (1-p_t) - h + \beta \xi_t \mathbb{E}_t \left[\left(1-\delta-\tilde{\delta}\right) TS_{F,t} + \left(\tilde{\delta} - \mu_{F,t}(1-\tau)\right) \widetilde{TS}_{F,t}\right]. \tag{45}$$

Next, we derive the total surplus of a filled job in a single firm, $\widetilde{TS}_{i,t}$. The equations for $\widetilde{W}_{I,t}$, $\widetilde{J}_{I,t}$, and $U_{I,t}$ yield:

$$\widetilde{J}_{I,t}+\widetilde{W}_{I,t}-U_{I,t}=-h-c\left(\sigma^*_{I,t}\right)+ \beta\xi_t\mathbb{E}_t\begin{bmatrix}(1-\delta)\left(1-\pi^*_{I,t}\right)\left(\widetilde{J}_{I,t+1}+\widetilde{W}_{I,t+1}-U_{I,t+1}\right)+\\(1-\delta)\,\pi^*_{I,t}\left(W_{I,t+1}+J_{I,t+1}-U_{I,t+1}\right)-\mu_{I,t}\left(\widetilde{W}_{I,t+1}-U_{I,t+1}\right)\end{bmatrix}, \quad (46)$$

where $\sigma^*_{I,t}$ is the search effort that maximizes $\widetilde{J}_{I,t}$ and $\pi^*_{I,t}$ is the matching probability induced by $\sigma^*_{I,t}$. By using the definition of $\widetilde{TS}_{i,t}$ above, we re-arrange the previous equation as:

$$\widetilde{TS}_{I,t}=-h-c\left(\sigma^*_{I,t}\right) + \beta\xi_t\mathbb{E}_t\left[(1-\delta)\,\pi_{I,t}TS_{I,t+1}+\left((1-\delta)\left(1-\pi_{I,t}\right)-(1-\tau)\,\mu_{I,t}\right)\widetilde{TS}_{I,t+1}\right]. \quad (47)$$

Nash bargaining for wages, as shown by equation (40), indicates that firm and worker choose search effort to maximize their joint surplus $\widetilde{TS}_{I,t}$. Specifically, since $\sigma^*_{I,t}$ maximizes $\widetilde{J}_{I,t}$, it also maximizes $\widetilde{TS}_{I,t}$. Thus, equation (47) becomes:

$$\widetilde{TS}_{I,t}=\max_{\sigma_{I,t}\geq 0}\Big\{-h-c\left(\sigma_{I,t}\right) + \beta\xi_t\mathbb{E}_t\left[(1-\delta)\,\pi_{I,t}TS_{I,t+1}+\left((1-\delta)\left(1-\pi_{I,t}\right)-(1-\tau)\,\mu_{I,t}\right)\widetilde{TS}_{I,t+1}\right]\Big\}, \quad (48)$$

and where $\pi_{I,t}$ is an increasing function of $\sigma_{I,t}$.

We denote the gain for total surplus from forming a trading relationship as $\Delta TS_{i,t} = TS_{i,t}-\widetilde{TS}_{i,t}$, and rewrite equation (48) as:

$$\widetilde{TS}_{I,t}=\max_{\sigma_{I,t}\geq 0}\Big\{-h-c\left(\sigma_{I,t}\right) + \beta\xi_t\mathbb{E}_t\left[(1-\delta)\,\pi_{I,t}\Delta TS_{I,t+1}+\left((1-\delta)-(1-\tau)\,\mu_{I,t}\right)\widetilde{TS}_{I,t+1}\right]\Big\}. \quad (49)$$

Similarly, we write the total surplus for single firms in the final-goods sector as:

$$\widetilde{TS}_{F,t}=\max_{\sigma_{F,t}\geq 0}\Big\{-h-c\left(\sigma_{F,t}\right) + \beta\xi_t\mathbb{E}_t\left[(1-\delta)\,\pi_{F,t}TS_{F,t+1}+\left((1-\delta)\left(1-\pi_{F,t}\right)-(1-\tau)\,\mu_{F,t}\right)\widetilde{TS}_{F,t+1}\right]\Big\}, \quad (50)$$

or, equivalently,

$$\widetilde{TS}_{F,t} = \max_{\sigma_{F,t}\geq 0} \Big\{ -h - c\left(\sigma_{F,t}\right) \\ + \beta\xi_t\mathbb{E}_t\left[(1-\delta)\,\pi_{F,t}\Delta TS_{F,t+1} + ((1-\delta)-(1-\tau)\,\mu_{F,t})\,\widetilde{TS}_{F,t+1}\right]\Big\}. \tag{51}$$

Finally, we derive the total surplus of a trading relationship, $TSJV_{i,t}$. The Nash bargaining for the intermediate goods price and wage yields $\Delta TS_{I,t} = \frac{\widetilde{\tau}}{\tau}TSJV_t$ and $\Delta TS_{F,t} = \left(\frac{1-\widetilde{\tau}}{\tau}\right)TSJV_t$. Using equations (44) and (48) in the definition of $\Delta TS_{i,t}$ produces:

$$\Delta TS_{I,t} = \min_{\sigma_{I,t}} \Big\{ z_t p_t + c\left(\sigma_{I,t}\right) + \beta\left[\left(1-\delta-\tilde{\delta}\right) - (1-\delta)\,\pi_{I,t}\right]\xi_t\mathbb{E}_t\left(\Delta TS_{I,t+1}\right)\Big\}, \tag{52}$$

or after using the Nash bargaining condition $\Delta TS_{I,t} = \frac{\tau}{\widetilde{\tau}}TSJV_t$:

$$TSJV_t = \min_{\sigma_{I,t}} \Big\{ \frac{\tau}{\widetilde{\tau}}\left[z_t p_t + c\left(\sigma_{I,t}\right)\right] + \beta\left[\left(1-\delta-\tilde{\delta}\right) - (1-\delta)\,\pi_{I,t}\right]\xi_t\mathbb{E}_t\left(TSJV_{t+1}\right)\Big\}. \tag{53}$$

Analogously, the total surplus of a trading relationship from sector F's optimization problem is:

$$TSJV_t = \min_{\sigma_{F,t}} \Big\{ \frac{\tau}{1-\widetilde{\tau}}\left[z_t\left(1-p_t\right) + c\left(\sigma_{F,t}\right)\right] \\ + \beta\left[\left(1-\delta-\tilde{\delta}\right) - (1-\delta)\,\pi_{F,t}\right]\xi_t\mathbb{E}_t\left(TSJV_{t+1}\right)\Big\}. \tag{54}$$

Combining equation (53)$\times\widetilde{\tau}$+equation (54)$\times\left(1-\widetilde{\tau}\right)$, p_t cancels out and we find:

$$TSJV_t = \tau\cdot z_t + \beta\left(1-\delta-\tilde{\delta}\right)\xi_t\mathbb{E}_t\left(TSJV_{t+1}\right) \\ + \min_{\sigma_{I,t}}\Big\{\tau\cdot c\left(\sigma_{I,t}\right) - \beta\left(1-\delta\right)\pi_{I,t}\xi_t\mathbb{E}_t\left(\widetilde{\tau}\cdot TSJV_{t+1}\right)\Big\} \\ + \min_{\sigma_{F,t}}\Big\{\tau\cdot c\left(\sigma_{F,t}\right) - \beta\left(1-\delta\right)\pi_{F,t}\xi_t\mathbb{E}_t\left[\left(1-\widetilde{\tau}\right)\cdot TSJV_{t+1}\right]\Big\}. \tag{55}$$

The first-order conditions for $\{\sigma_{I,t}, \sigma_{F,t}\}$ in equation (55) are:

$$\beta\left(1-\delta\right)\left(\psi+\sigma_{F,t}\right)H\left(\widetilde{\theta}_t, 1\right)\widetilde{\tau}\xi_t\mathbb{E}_t\left(TSJV_{t+1}\right) = \tau\left[c_0 + c_1\left(\sigma_{I,t}\right)^{\nu}\right], \tag{56}$$

$$\beta\left(1-\delta\right)\left(\psi+\sigma_{I,t}\right)H\left(1, \widetilde{\theta}_t^{-1}\right)\left(1-\widetilde{\tau}\right)\xi_t\mathbb{E}_t\left(TSJV_{t+1}\right) = \tau\left[c_0 + c_1\left(\sigma_{F,t}\right)^{\nu}\right]. \tag{57}$$

The active stage equilibrium exists if and only if there exists a pair $\left(\sigma_{I,t}, \sigma_{F,t}\right) > 0$ that jointly solves equations (56) and (57). In the symmetric equilibrium for which $\widetilde{\tau} = 1/2$ and $\widetilde{\theta}_t = 1$, equations (56) and (57) become:

$$\widetilde{\beta}\left(\psi + \sigma_{F,t}\right)\xi_t \mathbb{E}_t\left(J_{I,t+1} - \widetilde{J}_{I,t+1}\right) = c_0 + c_1\left(\sigma_{I,t}\right)^{\nu}, \tag{58}$$

$$\widetilde{\beta}\left(\psi + \sigma_{I,t}\right)\xi_t \mathbb{E}_t\left(J_{F,t+1} - \widetilde{J}_{F,t+1}\right) = c_0 + c_1\left(\sigma_{F,t}\right)^{\nu}, \tag{59}$$

where $\widetilde{\beta} = \beta\left(1-\delta\right)/\tau$. Equivalently, we can express the first-order conditions as:

$$\beta\left(1-\delta\right)\left(\psi + \sigma_{F,t}\right)\xi_t \mathbb{E}_t\left(\Delta TS_{I,t+1}\right) = c_0 + c_1\left(\sigma_{I,t}\right)^{\nu} \tag{60}$$

$$\beta\left(1-\delta\right)\left(\psi + \sigma_{I,t}\right)\xi_t \mathbb{E}_t\left(\Delta TS_{F,t+1}\right) = c_0 + c_1\left(\sigma_{F,t}\right)^{\nu}. \tag{61}$$

D The deterministic steady states of the model

We study now the existence of the deterministic steady states (DSSs) of the model that appear when we shut down the shocks ξ_t and z_t by making them constant and equal to their mean values (both equal to 1). The model encompasses two types of DSSs: a passive DSS with zero search effort ($\sigma_I = \sigma_F = 0$) and active DSSs with positive search effort ($\sigma_I > 0$, $\sigma_F > 0$).

Proposition 3. *The level of output is strictly lower and the unemployment rate is strictly higher in a passive DSS than in an active DSS.*

Proof. We consider the case of symmetric sectors, so we drop the sector subscripts. We first show that the labor market tightness ratio is strictly lower in the passive DSS, i.e., $\theta^{pas} < \theta^{act}$, or, equivalently $\widetilde{TS}^{pas} < \widetilde{TS}^{act}$, as implied by the free-entry condition of the labor market.

We start with

$$\widetilde{TS}^{act} = -h + \left[c_0\sigma^{act} + c_1\frac{\left(\sigma^{act}\right)^{\nu+1}}{1+\nu}\right] + \beta\left\{\left(1-\delta\right)\pi^{act}\cdot\Delta TS^{act} + \left[\left(1-\delta\right) - \mu^{act}\left(1-\tau\right)\right]\widetilde{TS}^{act}\right\}, \tag{62}$$

and

$$\mu^{act} = \left(\frac{\beta\tau\widetilde{TS}^{act}}{\chi}\right)^{\frac{\alpha}{1-\alpha}}. \tag{63}$$

The values for $\widetilde{TS}^{act}$ and θ^{act} solve equations (62) and (63). We rewrite equation (63) as

$$\chi = \beta\tau q^{act}\widetilde{TS}^{act} = \beta\tau q^{pas}\widetilde{TS}^{pas}. \tag{64}$$

Given the Cobb-Douglas matching function for the labor market, equation (64) is equivalent to:

$$\theta^{act} = \left(\frac{\beta\tau\widetilde{TS}^{act}}{\chi}\right)^{\frac{1}{1-\alpha}}. \tag{65}$$

Applying equation (64) to equation (62), delivers:

$$\begin{aligned}\left(\frac{1-\tau}{\tau}\right)\chi\theta^{act} = \Big\{ -h - \left[c_0\sigma^{act} + c_1\frac{\left(\sigma^{act}\right)^{\nu+1}}{1+\nu}\right] \\ + \beta\left(1-\delta\right)\pi^{act}\cdot\Delta TS^{act} - \left[1-\beta\left(1-\delta\right)\right]\widetilde{TS}^{act}\Big\},\end{aligned} \tag{66}$$

where we used $\mu^{act} = \theta^{act}q^{act}$. In equation (64), θ is strictly increasing in $\widetilde{TS}$. In equation (66), θ is linear and strictly decreasing in the total surplus for a single firm, $\widetilde{TS}$. Since σ^{act} and ΔTS^{act} are solved below in equations (94) and (95), they are treated as constant terms here.

Hence, values for θ^{act} and $\widetilde{TS}^{act}$ solve:

$$\begin{aligned}\left(\frac{1-\tau}{\tau}\right)\chi\theta = \Big\{ -h - \left[c_0\sigma^{act} + c_1\frac{\left(\sigma^{act}\right)^{\nu+1}}{1+\nu}\right] \\ + \beta\left(1-\delta\right)\pi^{act}\cdot\Delta TS^{act} - \left[1-\beta\left(1-\delta\right)\right]\widetilde{TS}\Big\}\end{aligned} \tag{67}$$

$$\theta = \left(\frac{\beta\tau\widetilde{TS}}{\chi}\right)^{\frac{1}{1-\alpha}}. \tag{68}$$

Similarly, values for $\widetilde{TS}^{pas}$ and θ^{pas} solve:

$$\left(\frac{1-\tau}{\tau}\right)\chi\theta = \left[-h + \beta\left(1-\delta\right)\pi^{pas}\cdot\Delta TS^{pas}\right] - \left[1-\beta\left(1-\delta\right)\right]\widetilde{TS}, \tag{69}$$

$$\theta = \left(\frac{\beta\tau\widetilde{TS}}{\chi}\right)^{\frac{1}{1-\alpha}}. \tag{70}$$

In equations (67) and (69), θ is linear and strictly decreasing in $\widetilde{TS}$. In equations (68) and (70), θ is strictly increasing in $\widetilde{TS}$.

For $\theta^{act} > \theta^{pas}$, it must be that the intercept term in equation (66) is greater than the intercept term in equation (69), which occurs if:

$$-h - \left[c_0 \sigma^{act} + c_1 \frac{\left(\sigma^{act}\right)^{\nu+1}}{1+\nu} \right] + \beta \left(1-\delta\right) \pi^{act} \cdot \Delta TS^{act} > -h + \beta \left(1-\delta\right) \pi^{pas} \cdot \Delta TS^{pas}. \quad (71)$$

To simplify notation, denote $W\left(\sigma\right) = -h + \frac{W_1(\sigma)}{W_2(\sigma)}$, where

$$\begin{aligned} W_1\left(\sigma\right) &= \left[\beta\left(1-\delta-\tilde{\delta}\right) - 1\right]\left[c_0\sigma + c_1\frac{\sigma^{\nu+1}}{1+\nu}\right] + \beta\left(1-\delta\right)\left[\phi + \left(\psi+\sigma\right)^2\right], \\ W_2\left(\sigma\right) &= 1 - \beta\left\{\left(1-\delta-\tilde{\delta}\right) - \left(1-\delta\right)\left[\phi + \left(\psi+\sigma\right)^2\right]\right\}. \end{aligned}$$

It can be shown that equation (71) is equivalent to $W\left(\sigma^{act}\right) > W\left(0\right)$. We verify that, for $\sigma \in \left(0, \sqrt{1-\phi} - \psi\right)$, $\frac{dW_1}{d\sigma}/W_1 > \frac{dW_2}{d\sigma}/W_2$, which implies $dW/d\sigma > 0$. Consequently, equation (71) holds, and $\theta^{act} > \theta^{pas}$.

Since the job-finding rate is strictly increasing in labor market tightness, $\mu^{act} > \mu^{pas}$. Since $u = \delta/\left(\delta+\mu\right)$ in the DSS, $u^{act} < u^{pas}$ holds.

Finally, we show that $y^{act} > y^{pas}$. Since $y = n$ and $n = 1 - \tilde{n} - u$, $y^{act} > y^{pas}$ is equivalent to showing that $\tilde{n}^{act} + u^{act} < \tilde{n}^{pas} + u^{pas}$.

In the DSS, it holds that:

$$\tilde{n} + u = \frac{\tilde{\delta} + \left(\pi + \mu + \delta\right)\frac{\delta}{\delta+\mu}}{\delta + \pi + \tilde{\delta}}. \quad (72)$$

The RHS of equation (72) is strictly decreasing in both μ and π. Given that $\mu^{pas} < \mu^{act}$ and $\pi^{pas} < \pi^{act}$, it holds that $\tilde{n}^{act} + u^{act} < \tilde{n}^{pas} + u^{pas}$, or, equivalently, $y^{act} > y^{pas}$. □

Intuitively, zero search effort in the passive DSS implies few trading relationships and low production. A small probability of forming a trading relationship reduces the value of a single firm and generates a fall in posted vacancies and an increase in unemployment.

The next two propositions establish conditions for the existence of the different DSSs.

Proposition 4. *The passive DSS exists if and only if*

$$\frac{\beta(1-\delta)\tau\psi}{2 - 2\beta\left[\left(1-\delta-\tilde{\delta}\right) - \left(1-\delta\right)\left(\phi+\psi^2\right)\right]} < c_0. \quad (73)$$

Proof. This proposition holds if it is optimal for firms in one sector to search with zero effort when firms in the opposite sector search with zero effort. In such a case, the Nash equilibrium with zero search effort exists in the passive DSS.

The firm's maximization problem in the passive DSS is:

$$\widetilde{TS}^{pas} = \max_{\sigma \geq 0} -h - \left(c_0\sigma + c_1\frac{\sigma^{\nu+1}}{1+\nu}\right) + \beta \begin{Bmatrix} (1-\delta)\left[\phi + \psi\left(\psi+\sigma\right)\right] \cdot \Delta TS^{pas} \\ + \left[(1-\delta) - \mu^{pas}\left(1-\tau\right)\right] \widetilde{TS}^{pas} \end{Bmatrix}. \tag{74}$$

The total surplus of a single firm $\widetilde{TS}^{pas}$ is strictly concave in σ, for $\sigma > 0$. Hence, the corner solution $\sigma = 0$ is optimal if and only if the first-order derivative is non-positive at $\sigma = 0$:

$$c_0 + c_1 0^\nu \geq \beta\left(1-\delta\right)\psi \Delta TS^{pas}, \tag{75}$$

where ΔTS^{pas} is given by equation (92), or, equivalently:

$$c_0 > \frac{\beta\tau\left(1-\delta\right)\psi}{2 - 2\beta\left[\left(1-\delta-\tilde{\delta}\right) - \left(1-\delta\right)\left(\phi+\psi^2\right)\right]}, \tag{76}$$

where we assume $z^{ss} = 1$, $\xi^{ss} = 1$, and $\widetilde{\tau} = 0.5$. □

Proposition 4 states that the passive DSS exists for any sufficiently large value of c_0—that is, when the benefit from an additional unit of search effort is lower than the cost associated with it. In such a case, $\sigma_I = \sigma_F = 0$. The critical cost for the existence of the passive DSS is c_0. In comparison, c_1 does not appear in Proposition 4.

Proposition 5. *An active DSS exists if and only if there exists* $\sigma \in \left(0, \sqrt{1-\phi} - \psi\right)$ *that solves*

$$\frac{1 + \left(c_0\sigma + c_1\frac{\sigma^{1+\nu}}{1+\nu}\right)}{2 - 2\beta\left[\left(1-\delta-\tilde{\delta}\right) - \left(1-\delta\right)\left(\phi + \left(\sigma+\psi\right)^2\right)\right]} = \frac{c_0 + c_1\sigma^\nu}{\beta(1-\delta)\tau\left(\psi+\sigma\right)}. \tag{77}$$

Proof. This proposition holds if there exist $\sigma \in \left(0, \sqrt{1-\phi} - \psi\right)$ (to guarantee that the matching probability is bounded by one) and $\Delta TS \in \mathbb{R}$ that solve equations (95) and (94).

By substituting equation (95) into equation (94), we get:

$$\frac{1+\left(c_0\sigma + c_1\frac{\sigma^{\nu+1}}{1+\nu}\right)}{2-2\beta\left[\left(1-\delta-\tilde{\delta}\right)-(1-\delta)\left[\phi+(\sigma+\psi)^2\right]\right]} = \frac{c_0+c_1\sigma^{\nu}}{\beta\tau\left(1-\delta\right)\left(\psi+\sigma\right)}, \tag{78}$$

where we assume $\widetilde{\tau}=1/2$, $\xi^{ss}=1$, $z^{ss}=1$. □

The LHS of equation (77) captures the marginal gain of searching with positive effort in the active stage equilibrium. The RHS reflects the marginal cost of searching. In the active DSS, both quantities must be equal. Proposition 5 defines the parameter values that guarantee the existence of the active DSS. The restriction $\sigma \in \left(0, \sqrt{1-\phi}-\psi\right)$ ensures that the matching probability $\phi + (\psi+\sigma_I)(\psi+\sigma_F)$ is within (0,1).

Proposition 6. *The active and passive DSSs coexist if and only if equations (73) and (77) hold simultaneously.*

Equations (73) and (77) can hold simultaneously, since they depend on different parameter combinations. The passive DSS characterized by equation (73) is uniquely pinned down when $\sigma_I = \sigma_F = 0$. In comparison, the system allows for multiple active DSSs, since equation (77) can hold for different symmetric $(\sigma_{I,t}, \sigma_{F,t}) > 0$. When the best response function is strictly concave (i.e., $\nu > 1$), the system admits, at most, two DSSs (if $\nu < 1$, we would only have one active and unstable equilibrium). The argument is formalized next.

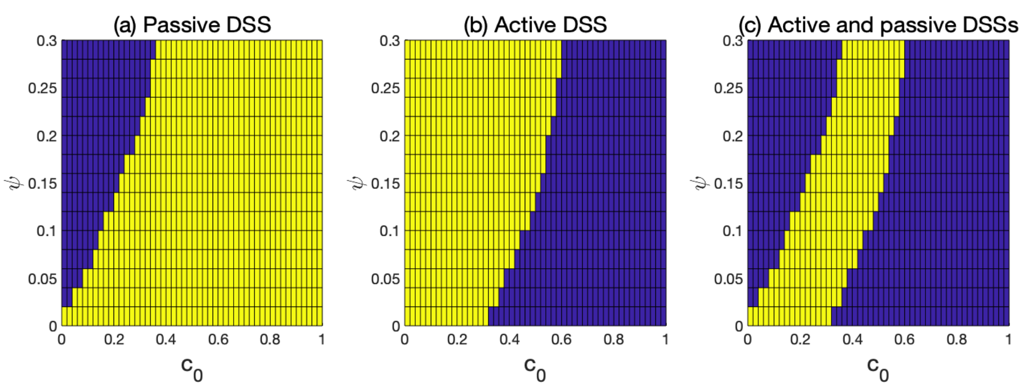


Figure 13: Existence of DSSs

Figure 13 illustrates, for a range of values of c_0 (x-axes) and ψ (y-axes), the conditions for the existence of a passive DSS, an active DSS, and the coexistence of DSSs stated in the main text when the model is calibrated as in Section 5. The yellow-shaded area shows the values that guarantee the existence of such a DSS, while the blue area shows the non-existence region. Panel (a) shows that conditional on the value of ψ, the passive DSS exists when the value of c_0 is sufficiently large. Panel (b) demonstrates that the active DSS exists for sufficiently low values of c_0. Panel (c) shows that two DSSs exist when c_0 is in the medium range.

Lemma 1. *The system has a unique passive DSS and at most two active DSSs.*

Appendix F establishes the stability of the DSSs and it shows that a slight deviation of a subset of firms from their best response will fail to cause the system to deviate from the initial DSS permanently.

E Mixed-strategy Nash equilibria

This appendix discusses the role of mixed-strategy Nash equilibria in our model. We first establish the condition for the existence of a mixed-strategy Nash equilibrium in the DSS (the case with stochastic shocks is similar, but more cumbersome to derive). Then, we argue that such a mixed-strategy Nash equilibrium exists and is unique for the calibration in Section 5. However, this mixed-strategy Nash equilibrium is unstable: a small deviation from the mixed-strategy makes the system converge to the pure-strategy Nash equilibrium.

In a mixed-strategy setting, firms randomize their search effort by choosing $\sigma = 0$ with probability q and choosing $\sigma = \hat{\sigma}$ with probability $(1-q)$. We numerically verify that the solution to equation (60) is unique in the range of $0 < \hat{\sigma} < \sqrt{1-\phi} - \psi$. So firms cannot randomize their search effort by choosing between multiple positive efforts. The random choice is independent across firms. Due to the law of large numbers, the average search effort in both sectors is $\overline{\sigma} = q \cdot 0 + (1-q)\,\hat{\sigma}$. For a single firm, the inter-firm matching probability is given by $\pi(\sigma) = \phi + \psi(\psi+\sigma)(\psi+\overline{\sigma})$. In the mixed-strategy Nash equilibrium, the inter-firm matching probability takes two values: $\pi(0) = \phi + \psi(\psi+\overline{\sigma})$ and $\pi(\hat{\sigma}) = \phi + (\psi+\hat{\sigma})(\psi+\overline{\sigma})$.

A mixed-strategy Nash equilibrium consists of a tuple $\{q, \hat{\sigma}\}$ with $\hat{\sigma} \in \left(0, \sqrt{1-\phi} - \psi\right)$ and $q \in (0,1)$. The tuple $\{q, \hat{\sigma}\}$ implies that single firms are indifferent between choosing $\sigma = 0$ and $\sigma = \hat{\sigma}$, i.e., $\widetilde{TS}(0) = \widetilde{TS}(\hat{\sigma})$. Since $\Delta TS(0) = TS - \widetilde{TS}(0)$ and $\Delta TS(\hat{\sigma}) = TS - \widetilde{TS}(\hat{\sigma})$, it holds that $\Delta TS(0) = \Delta TS(\hat{\sigma})$. We denote $\Delta TS(0) = \Delta TS(\hat{\sigma}) = \Delta TS$.

According to equation (52):

$$\Delta TS = z^{ss} p^{ss} + \beta\left[\left(1-\delta-\tilde{\delta}\right) - (1-\delta)\,\pi(0)\right]\Delta TS, \tag{79}$$

where $c(0) = 0$. From equation (49), the single firm's total surplus with zero search effort is:

$$\widetilde{TS}(0) = -h + \beta\left[(1-\delta)\pi(0)\Delta TS + ((1-\delta) - (1-\tau)\theta^{\alpha})\widetilde{TS}(0)\right]. \tag{80}$$

Analogously, the single firm's total surplus by choosing $\hat{\sigma}$ search effort satisfies:

$$\widetilde{TS}(\hat{\sigma}) = -h - c(\hat{\sigma}) + \beta\left[(1-\delta)\pi(\hat{\sigma})\Delta TS + ((1-\delta) - (1-\tau)\theta^{\alpha})\widetilde{TS}(\hat{\sigma})\right]. \tag{81}$$

Since $\widetilde{TS}(0) = \widetilde{TS}(\hat{\sigma})$ in the mixed-strategy Nash equilibrium, combining equations (80) and (81) delivers:

$$c(\hat{\sigma}) = \beta(1-\delta)(\pi(\hat{\sigma}) - \pi(0))\Delta TS. \tag{82}$$

Finally, according to the first-order condition for $\{\sigma_{I,t}, \sigma_{F,t}\}$ in equation (60):

$$\beta(1-\delta)(\psi + \overline{\sigma})\Delta TS = c_0 + c_1\hat{\sigma}^{\nu}. \tag{83}$$

In sum, we have the three equations (79), (82), and (83) and three unknowns (i.e., $\hat{\sigma}$, q, ΔTS). The mixed-strategy Nash equilibrium exists if the system of equations has a solution for the three unknowns. Using the calibration in Section 5, the mixed-strategy Nash equilibrium is $q = 0.3425$, $\hat{\sigma} = 0.0164$, and $\Delta TS = 2.7417$. The average search effort $\overline{\sigma}$ is $(1-q)\times\hat{\sigma} = 0.0107$.

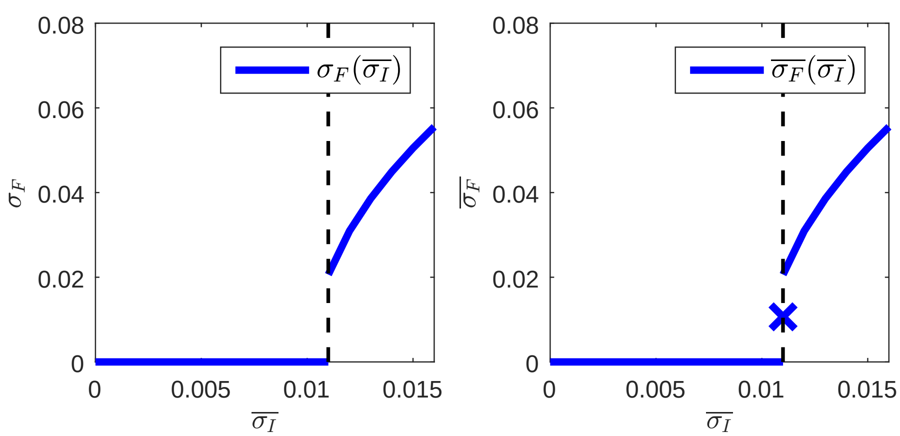


Figure 14: Best response in the mixed-strategy Nash equilibrium

The left panel in Figure 14 displays the firm's optimal search effort in sector F as a function of $\overline{\sigma}_I$. The firm chooses a positive search effort if $\overline{\sigma}_I > 0.0107$ (i.e., for values to the right of the vertical dashed line). The firm chooses a zero search effort if $\overline{\sigma}_I < 0.0107$ (i.e., for values to the left of the vertical dashed line). The firm is indifferent between choosing $\sigma = 0.0164$ and $\sigma = 0$ if $\overline{\sigma}_I = 0.0107$ (i.e., if sector I uses the mixed-strategy $q_I = 0.3425$, $\hat{\sigma}_I = 0.0164$).

The right panel in Figure 14 plots $\overline{\sigma}_F$ as a function of $\overline{\sigma}_I$. The firm chooses a positive search effort if $\overline{\sigma}_I > 0.0107$ (i.e., for values to the right of the vertical dashed line). The firm would choose a zero search effort if $\overline{\sigma}_I < 0.0107$ (i.e., for values to the left of the vertical dashed line). If $\overline{\sigma}_I < 0.0107$ (i.e., if sector I uses the mixed-strategy $q_I = 0.3425$, $\hat{\sigma}_I = 0.0164$), a fraction 0.3425 of firms chooses $\sigma = 0$, while the rest of the firms choose $\sigma = 0.0164$, which implies $\overline{\sigma}_F = 0.0107$ (i.e., the cross marker).

Figure 14 shows that the mixed-strategy Nash equilibrium is unstable: a decrease in $\overline{\sigma}_I$ induces all firms in sector F to search with zero effort and the system converges to the pure-strategy Nash equilibrium with zero search effort (i.e., passive stage equilibrium). Similarly, an increase in $\overline{\sigma}_I$ induces all firms in sector F to search with positive effort; hence, the system converges to the pure-strategy Nash equilibrium with positive search effort (i.e., active stage equilibrium).

F Stability of DSSs

The next proposition establishes the stability of the DSSs. This stability guarantees that a slight deviation of a subset of firms from their best response will fail to cause the system to deviate from the initial DSS permanently.

Proposition 7. *Suppose the active and passive DSSs coexist. The passive DSS is stable. When two active DSSs coexist, one DSS is stable and the other DSS is unstable. When only one active DSS exists, it is unstable.*

Proof. We first show that the Nash equilibrium in the passive DSS is stable. To do so, we demonstrate that there exists an $\epsilon > 0$, such that when a firm in sector j deviates from the passive DSS by searching with a small and positive effort bounded by ϵ, it remains optimal for the firm in the opposite sector i to search with zero effort:

$$c_0 + c_1 0^{\nu} > \beta\left(1-\delta\right)\left(\psi+\sigma_j\right)\mathbb{E}\left(\Delta TS_i\right), \tag{84}$$

where $\sigma_j \in (0, \epsilon)$. The RHS of equation (84) is a function of σ_j, which is continuous at $\sigma_j = 0$ (note that $\mathbb{E}\left(\Delta TS_i\right)$ is a continuous function of σ_j). Given the existence of the passive DSS, we know that $c_0 + c_1 0^\nu > \beta(1-\delta)\psi\Delta TS^{pas}$. Because of continuity, there exists $\epsilon > 0$, so that equation (84) holds when $\sigma_j < \epsilon$.

Next, we show that one Nash equilibrium in the active DSS is stable when two active DSSs exist. The best response function of sector i implied by equations (60) and (61) in the active DSS is:

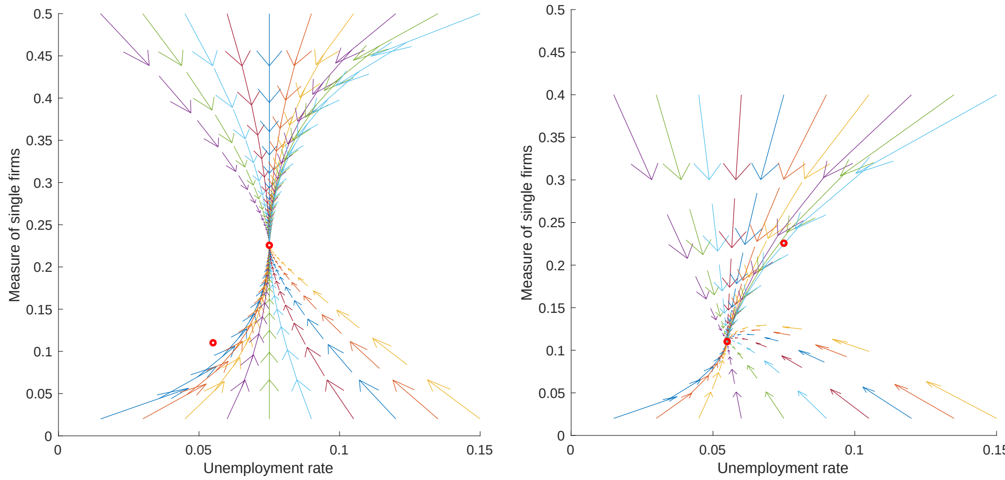
$$\sigma_i = \begin{cases} \left[\frac{\beta(1-\delta)(\psi+\sigma_j)\Delta TS^{act}-c_0}{c_1}\right]^{\frac{1}{\nu}} & \text{if } \beta(1-\delta)(\psi+\sigma_j)\Delta TS^{act} \geq c_0 \\ 0 & \text{if } \beta(1-\delta)(\psi+\sigma_j)\Delta TS^{act} < c_0, \end{cases} \tag{85}$$

which is strictly increasing and concave in σ_j since $c_1 > 0$ and $\nu > 1$. When two active DSSs exist, the best response curve (85) intersects with the 45-degree line at $\sigma_F = \sigma_I = \sigma^*$ and $\sigma_F = \sigma_I = \sigma^{**}$ with $0 < \sigma^* < \sigma^{**} < \sqrt{1-\phi} - \psi$. Due to strict concavity, we have $\frac{d\sigma_i}{d\sigma_j}|_{\sigma_i=\sigma_j=\sigma^*} > 1$ and $\frac{d\sigma_i}{d\sigma_j}|_{\sigma_i=\sigma_j=\sigma^{**}} < 1$. Therefore, the active Nash equilibrium at $\sigma_F = \sigma_I = \sigma^*$ is unstable, while the one at $\sigma_F = \sigma_I = \sigma^{**}$ is stable.

Finally, consider the case when the passive DSS and one active DSS exist, where $\sigma_F = \sigma_I = \sigma^*$ and $0 < \sigma^* < \sqrt{1-\phi} - \psi$. Since the passive DSS exists, the inequality $c_0 > \beta(1-\delta)\psi\Delta TS^{pas}$ holds. In addition, we have that $\Delta TS^{act} < \Delta TS^{pas}$, which results from equations (92) and (93). We also have that $c_0 > \beta(1-\delta)\psi\Delta TS^{act}$. So $\sigma_i(\sigma_j) = 0$ in the active DSS for $\sigma_j \in [0, \hat{\sigma}]$ with $\hat{\sigma} = \frac{c_0}{\beta(1-\delta)\Delta TS^{act}} - \psi$. Since $\sigma_F = \sigma_I = \sigma^*$ is the only intersection between $\sigma_i(\sigma_j)$ and the 45-degree line in the range $\sigma_j \in [\hat{\sigma}, \sigma^*]$ with $\sigma_i(\hat{\sigma}) = 0$, we must have $\frac{d\sigma_i}{d\sigma_j}|_{\sigma_i=\sigma_j=\sigma^*} \geq 1$. When the derivative is equal to one, the best response curves are tangent to the 45-degree line; when the derivative is greater than one, the best response curve may have two intersections with the 45-degree line, in which case we have $0 < \sigma^* < \sqrt{1-\phi} - \psi < \sigma^{**}$ which ensures that only one intersection (σ^*) is the active stage equilibrium. Since the derivative is greater than or equal to one, the active stage Nash equilibrium at $\sigma_F = \sigma_I = \sigma^*$ is unstable. □

For the remainder of the analysis, we mainly focus on stable DSSs. Also, we can study the transition path from an arbitrary point in the state space of the system to the DSS. The endogenous state variables of the system are the unemployment rates $(u_{I,t}, u_{F,t})$, the measure of single firms $(\tilde{n}_{I,t}, \tilde{n}_{F,t})$, the measure of firms in trading relationships $(n_{I,t}, n_{F,t})$, and the current equilibrium (ι_t). Knowledge of $\tilde{n}_{i,t}$ and $u_{i,t}$ gives us $n_{i,t} = 1 - \tilde{n}_{i,t} - u_{i,t}$.

Figure 15 shows the transition path of the system to the DSS for different initial values of the unemployment rate (x-axes) and the measure of single firms (y-axes) and the calibration in Section 5. Since we consider the case of a symmetric economy, the analysis is representative



(a) Initial passive stage equilibrium **(b)** Initial active stage equilibrium

Figure 15: Transition path to the DSS

of each sector. Panel (a) shows the transition path to the DSS when the system starts from a passive stage equilibrium (with each red dot representing a DSS of the system). Given history dependence, the system remains in the passive stage equilibrium and converges to the passive DSS indicated by the higher red circle, where the unemployment rate is 7.5% and the measure of single firms is 22%. Analogously, panel (b) shows that the system converges to the active and stable DSS, when it starts from an active stage equilibrium. In the active DSS (the lower red dot), the unemployment rate is 5.5%, and the measure of single firms is 12%.

G Existence of two stage equilibria

The following propositions characterize the conditions for the existence of passive and active stage equilibria and their coexistence.

Proposition 8. *The passive stage equilibrium exists if and only if*

$$\frac{\partial \Pi_i\left(0|0,\iota_t=0\right)}{\partial \sigma_{i,t}} \leq 0 \quad \text{for } i = I, F \tag{86}$$

or equivalently

$$c_0 > \widetilde{\beta}\psi\xi_t \mathbb{E}_t(J_{i,t+1} - \widetilde{J}_{i,t+1} \mid \iota_t = 0). \tag{87}$$

Proposition 8 states that the passive stage equilibrium exists when the marginal benefit from increasing search effort is negative.Thus, the existence of the passive stage equilibrium requires either a low ξ_t or a small z_{t+1} (and, hence, a low $\mathbb{E}_t(J_{i,t+1} - \widetilde{J}_{i,t+1} \mid \iota_t = 0)$).

Proposition 9. *The active stage equilibrium exists if and only if there exists a pair of positive search efforts ($\{\sigma_{I,t}\sigma_{F,t}\} > 0$) that satisfies:*

$$\frac{\partial \Pi_i \left(\sigma_{i,t} \mid \sigma_{j,t}, \iota_t = 1\right)}{\partial \sigma_{I,t}} = 0 \quad \text{for } i = \{I, F\} \tag{88}$$

or, equivalently,

$$c_0 + c_1 \sigma_{i,t}^{\nu} = \widetilde{\beta} \left(\psi + \sigma_{j,t}\right) \xi_t \mathbb{E}_t(J_{i,t+1} - \widetilde{J}_{i,t+1} \mid \iota_t = 1), \tag{89}$$

with $(\sigma_{I,t}, \sigma_{F,t}) > 0$ and the second derivatives of Π_i are negative.

Proposition 9 states that an active stage equilibrium exists when the optimal response of the firm is to choose $\sigma_{i,t} > 0$ that satisfies equation (89). The next proposition states the condition for the coexistence of the two stage equilibria.

Proposition 10. *The active and passive stage equilibria coexist if and only if Propositions 8 and 9 hold simultaneously.*

H Solving for the DSSs

To solve for the DSSs, we evaluate the equilibrium conditions of the model when the variables are constant over time and the exogenous shocks take their average value. The model entails a passive and an active (stable) DSS. We disregard the active and unstable DSS in our analysis. We denote the variables referring to the passive and active DSS with superscript "pas" and "act," respectively.

Using equation (55), the total surplus of a trading relationship in the passive DSS is:

$$\begin{aligned} TSJV^{pas} &= \tau \cdot z^{ss} + 2\tau \cdot c\left(0\right) \\ &\quad + \beta \xi^{ss} \left[\left(1 - \delta - \tilde{\delta}\right) - \widetilde{\tau} \left(1 - \delta\right) \pi_I^{pas} - \left(1 - \widetilde{\tau}\right) \left(1 - \delta\right) \pi_F^{pas} \right] TSJV^{pas} \end{aligned} \tag{90}$$

where $c\left(0\right) = 0$, $\pi_I^{pas} = \left(\phi + \psi^2\right) H \left(1, \widetilde{\theta}^{pas}\right)$, and,

$$\pi_F^{pas} = \left(\phi + \psi^2\right) H \left(1, 1/\widetilde{\theta}^{pas}\right). \tag{91}$$

As in our baseline calibration, we set $\widetilde{\tau} = 0.5$ and assume a symmetric equilibrium so that $\widetilde{\theta}^{pas} = 1$. Applying these conditions in equation (90) yields:

$$TSJV^{pas} = \frac{\tau \cdot z^{ss}}{1 - \beta \xi^{ss} \left[\left(1 - \delta - \tilde{\delta} \right) - (1 - \delta) \left(\phi + \psi^2 \right) \right]}. \tag{92}$$

The gain of total surplus from forming a trading relationship in the passive DSS is determined by $\Delta TS_I^{pas} = \frac{\widetilde{\tau}}{\tau} TSJV^{pas}$ and $\Delta TS_F^{pas} = \left(\frac{1-\widetilde{\tau}}{\tau} \right) TSJV^{pas}$, which are useful in deriving the total surplus of a filled job in the passive DSS.

Analogously, the total surplus of a trading relationship in the active DSS is:

$$TSJV^{act} = \frac{\tau \cdot \left[z^{ss} + \left(c_0 \sigma_I^{act} + c_1 \frac{\left(\sigma_I^{act} \right)^{\nu+1}}{1+\nu} \right) + \left(c_0 \sigma_F^{act} + c_1 \frac{\left(\sigma_F^{act} \right)^{\nu+1}}{1+\nu} \right) \right]}{1 - \beta \xi^{ss} \left[\left(1 - \delta - \tilde{\delta} \right) - \widetilde{\tau} (1 - \delta) \pi_I^{act} - (1 - \widetilde{\tau}) (1 - \delta) \pi_F^{act} \right]}, \tag{93}$$

where:

$$\pi_I^{act} = \left[\phi + \left(\sigma_F^{act} + \psi \right) \left(\sigma_I^{act} + \psi \right) \right] H \left(1, \widetilde{\theta}^{act} \right),$$

and

$$\pi_F^{act} = \left[\phi + \left(\sigma_F^{act} + \psi \right) \left(\sigma_I^{act} + \psi \right) \right] H \left(1, 1/\widetilde{\theta}^{act} \right).$$

By imposing the symmetry conditions $\widetilde{\tau} = 1/2$, $\widetilde{\theta}^{act} = 1$ and $\sigma_F^{act} = \sigma_I^{act} = \sigma^{act}$, equation (93) becomes:

$$TSJV^{act} = \frac{\tau \cdot \left[z^{ss} + 2 \left(c_0 \sigma^{act} + c_1 \frac{\left(\sigma^{act} \right)^{\nu+1}}{1+\nu} \right) \right]}{1 - \beta \xi^{ss} \left[\left(1 - \delta - \tilde{\delta} \right) - (1 - \delta) \left[\phi + \left(\sigma^{act} + \psi \right)^2 \right] \right]}. \tag{94}$$

In the active DSS, the first-order condition for $\{\sigma_{I,t}, \sigma_{F,t}\}$ described by equations (56) and (57) is:

$$\frac{\beta (1 - \delta) \left(\psi + \sigma^{act} \right) \xi^{ss} TSJV^{act}}{2} = \tau \left[c_0 + c_1 \left(\sigma^{act} \right)^{\nu} \right]. \tag{95}$$

Equations (94) and (95) can be used to solve numerically for σ^{act} and $TSJV^{act}$.

The gain of total surplus from forming a trading relationship in the active DSS is determined by $\Delta TS_I^{act} = \frac{\widetilde{\tau}}{\tau} TSJV^{act}$ and $\Delta TS_F^{act} = \left(\frac{1-\widetilde{\tau}}{\tau} \right) TSJV^{act}$. Next, we derive the total surplus of a filled job in a single firm and the job-finding rate in the DSS. Using equation (47), the total surplus of a filled job for a single firm in sector I in the passive DSS is:

$$\widetilde{TS}_I^{pas} = -h + \beta \left\{ (1-\delta)\,\pi_I^{pas} \cdot \Delta TS_I^{pas} + \left[(1-\delta) - \mu_I^{pas}\,(1-\tau)\right] \widetilde{TS}_I^{pas} \right\}, \tag{96}$$

where ΔTS_I^{pas} and π_I^{pas} were solved analytically as in equations (91) and (92). Using the matching function and free-entry condition in the labor market, the job-finding rate in the passive DSS is:

$$\mu_I^{pas} = \left(\frac{\beta\tau\widetilde{TS}_I^{pas}}{\chi} \right)^{\frac{\alpha}{1-\alpha}}. \tag{97}$$

Equations (96) and (97) are solved numerically for $\widetilde{TS}_I^{pas}$ and μ_I^{pas}.

Applying the same approach, we solve for $\widetilde{TS}_F^{pas}$ and μ_F^{pas}. Analogously, the total surplus of a filled job in a single firm and the job-finding rate in the active DSS solves:

$$\begin{aligned} \widetilde{TS}_i^{act} = & -h + \left[c_0\sigma_i^{act} + c_1 \frac{\left(\sigma_i^{act}\right)^{\nu+1}}{1+\nu} \right] \\ & + \beta \left\{ (1-\delta)\,\pi_i^{act} \cdot \Delta TS_i^{act} + \left[(1-\delta) - \mu_i^{act}\,(1-\tau)\right] \widetilde{TS}_i^{act} \right\}, i \in F, I \end{aligned} \tag{98}$$

and

$$\mu_i^{act} = \left(\frac{\beta\tau\widetilde{TS}_i^{act}}{\chi} \right)^{\frac{\alpha}{1-\alpha}}, i \in F, I. \tag{99}$$

The total surplus of a filled job in a trading relationship in the DSS is $TS_i^l = \widetilde{TS}_i^l + \Delta TS_i^l$, $i \in \{I, F\}, l \in \{act, pas\}$. The firm's asset value in the DSS is $J_i^l = \tau TS_i^l$, $\widetilde{J}_i^l = \tau \widetilde{TS}_i^l$, $i \in \{I, F\}, l \in \{act, pas\}$. Finally, we can derive the DSS value for the remaining variables. Substituting the job-finding rate into the matching function of the labor market, we get $\theta^{pas} = (\mu^{pas})^{\frac{1}{\alpha}}$ and $\theta^{act} = (\mu^{act})^{\frac{1}{\alpha}}$.

The value for the unemployment rate, the measure of single firms, and the measure of trading relationships in the passive and active DSS are:

$$\begin{aligned} u^{pas} &= \frac{\delta}{\delta + \mu^{pas}} \\ u^{act} &= \frac{\delta}{\delta + \mu^{act}} \\ \tilde{n}^{pas} &= \frac{\tilde{\delta} + \left(\mu^{pas} - \tilde{\delta}\right) u^{pas}}{\delta + \pi^{pas} + \tilde{\delta}} \end{aligned}$$

$$\tilde{n}^{act} = \frac{\tilde{\delta} + \left(\mu^{act} - \tilde{\delta}\right) u^{act}}{\delta + \pi^{act} + \tilde{\delta}}$$

$$n^{pas} = 1 - u^{pas} - \tilde{n}^{pas}$$

$$n^{act} = 1 - u^{act} - \tilde{n}^{act}.$$

The value for total final output in the passive and active DSSs is $y^{pas} = z^{ss} n^{pas}$ and $y^{act} = z^{ss} n^{act}$.

I Equilibrium

A recursive, symmetric equilibrium of type ι_t for our economy is a collection of Bellman equations $U_{i,t}$, $\widetilde{W}_{i,t}$, $W_{i,t}$, $\widetilde{J}_{i,t}$, $J_{i,t}$, and $V_{i,t}$, a variable search effort $\sigma_{i,t}$, and sequences for unemployment u_t, single firms $\widetilde{n}_{i,t}$, trading relationships n_t, the price of the intermediate good p_t, and wages $\widetilde{w}_{i,t}$ and $w_{i,t}$, all for $i \in \{I, F\}$, such that:

1. $U_{i,t}$, $\widetilde{W}_{i,t}$, $W_{i,t}$, $\widetilde{J}_{i,t}$, $J_{i,t}$, and $V_{i,t}$ satisfy equations (21)-(27).
2. The free-entry condition $V_{i,t} = 0$ holds.
3. $\sigma_{i,t}$ maximizes the asset value of the single firm $\widetilde{J}_{i,t}$.
4. The sequences of unemployment u_t, single firms $\widetilde{n}_{i,t}$, and trading relationships n_t follow the laws of motion in equations (13), (20), and (19), respectively.
5. The intermediate-goods price p_t and the wage for single and trading relationships, $\widetilde{w}_{i,t}$ and $w_{i,t}$, respectively, are determined by the Nash bargaining equations (35)-(37).
6. The type of equilibrium ι_t is consistent with $\sigma_{i,t}$.
7. ξ_t and z_t follow their stochastic processes.
8. The aggregate resource constraint (28) is satisfied.

J Model solution

In this appendix, we outline the algorithm to solve the model numerically.

J.1 Solution without government spending

We first discuss the solution to the benchmark case without government spending. The vector of state variables is $S_t = (z_t, \xi_t, \iota_{t-1}, u_t, n_t, \widetilde{n}_t)$, where we omit the sector subscripts. At the beginning of period t, S_t is taken as given. The states z_t and ξ_t are exogenous, and the states ι_{t-1}, u_t, n_t, and $\widetilde{n}_t$ are endogenous and predetermined. To derive the solution of the system, we require the value functions $TSJV(S_t)$, and $\widetilde{TS}(S_t)$; two policy functions $\sigma(S_t)$, and $\theta(S_t)$; and the transition rule of $\iota_t = \iota(\iota_{t-1}, S_t)$. The transition rule for the other endogenous states (u_t, n_t and $\widetilde{n}_t$) is directly given by the model once the other functions have been found.

Because of sectoral symmetry, $\widetilde{\theta}_t = \widetilde{n}_{F,t}/\widetilde{n}_{I,t} = 1$. As we show below in equation (100), a fixed $\widetilde{\theta}$ implies that the value functions, policy functions, and the transition rule for ι_t depend on $(z_t, \xi_t, \iota_{t-1})$ only.

Step 1: Solve for $TSJV$, σ, and ι. Equation (55) can be rewritten as:

$$TSJV(z_t, \xi_t, \iota_{t-1}) = \min_{\sigma_t \geq 0} \tau \cdot [z_t + 2c(\sigma_t)] + \beta \left\{ \left(1 - \delta - \tilde{\delta}\right) - (1-\delta)[\phi + (\psi + \sigma_t)(\psi + \overline{\sigma_t})] \right\} * \xi_t \mathbb{E}_t [TSJV(z_{t+1}, \xi_{t+1}, \iota_t)], \tag{100}$$

where $\overline{\sigma_t}$ is the search effort in the opposite sector, taken as given by the firms. In the symmetric equilibrium, $\sigma_t = \overline{\sigma_t}$.

The equilibrium type ι_t is determined by the best response functions implied by equation (100) and the history dependence of equilibrium selection. Specifically, if $\iota_{t-1} = 0$ (passive stage equilibrium in $t-1$), we first verify whether the passive stage equilibrium continues to exist in period t by checking whether:

$$\arg\min_{\sigma_t \geq 0} 2c(\sigma_t) - \widetilde{\beta}[\phi + (\psi + \sigma_t)\psi] \xi_t \mathbb{E}_t [TSJV(z_{t+1}, \xi_{t+1}, \iota_t = 0)] = 0 \tag{101}$$

holds. If it does, the passive stage equilibrium exists and persists ($\iota_t = \iota_{t-1} = 0$). Otherwise, the passive stage equilibrium does not exist and we switch to the active stage equilibrium ($\iota_t = 1$).

Analogously, if $\iota_{t-1} = 1$ (active stage equilibrium in $t-1$), we verify whether the active stage equilibrium continues to exist in period t by checking whether:

$$\arg\min_{\sigma_t \geq 0} 2c(\sigma_t) - \widetilde{\beta}[\phi + (\psi + \sigma_t)(\psi + \sigma^*)] \xi_t \mathbb{E}_t [TSJV(z_{t+1}, \xi_{t+1}, \iota_t = 1)] > 0 \tag{102}$$

holds. If it does, the active stage equilibrium exists and persists ($\iota_t = \iota_{t-1} = 1$). Otherwise, the active stage equilibrium does not exist and we switch to the passive stage equilibrium ($\iota_t = 0$).

We use value function iteration methods to solve for the value function $TSJV$, the policy function σ, and the transition rule of ι using equation (100) and conditions (101) and (102).

Step 2: Solve for $\widetilde{TS}$ and θ. Equation (47) can be rewritten as:

$$\widetilde{TS}\left(z_t, \xi_t, \iota_{t-1}\right) = -h - c\left(\sigma_t\right) + \beta \xi_t \mathbb{E}_t \left[\begin{array}{c} \left(1-\delta\right) \pi_t \Delta TS\left(z_{t+1}, \xi_{t+1}, \iota_t\right) + \\ \left(\left(1-\delta\right) - \left(1-\tau\right)\theta^{\alpha}\left(z_t, \xi_t, \iota_{t-1}\right)\right) \widetilde{TS}\left(z_{t+1}, \xi_{t+1}, \iota_t\right) \end{array} \right], \quad (103)$$

where we used $\Delta TS_{t+1} = TS_{t+1} - \widetilde{TS}_{t+1}$ and $\mu_t = \theta_t^{\alpha}$.

The free-entry condition of the labor market (equation 42) can be rewritten as:

$$\chi = \beta \xi_t \tau \theta^{\alpha-1}\left(z_t, \xi_t, \iota_{t-1}\right) \mathbb{E}_t \left[\widetilde{TS}\left(z_{t+1}, \xi_{t+1}, \iota_t\right)\right]. \quad (104)$$

With $\Delta TS_t = \tilde{\tau} TSJV_t/\tau$, σ_t, and ι_t being solved in step 1, we find the value function $\widetilde{TS}$ and the policy function θ with equations (103) and (104) by using value function iteration.

J.2 Solution with government spending

We consider now the case with government spending. This case is challenging to solve since, in general, it implies sectoral asymmetry. The model's vector of state variables is: $S_t = \left(z_t, \xi_t, \epsilon_t^G, \iota_{t-1}, u_t^F, u_t^I, n_t^F, n_t^I, n_t^G, \widetilde{n}_t^F, \widetilde{n}_t^I, \widetilde{n}_t^G\right)$. States z_t, ξ_t, and ϵ_t^G are exogenous, and states $\iota_{t-1}, u_t^F, u_t^I, n_t^F, n_t^I, n_t^G, \widetilde{n}_t^F, \widetilde{n}_t^I$, and $\widetilde{n}_t^G$ are endogenous. To derive the solution of the system, we need the solution for the value functions $TSJV\left(S_t\right)$, $\widetilde{TS}_F\left(S_t\right)$, and $\widetilde{TS}_I\left(S_t\right)$ (the other value functions can be derived from these three value functions), the four policy functions $\sigma_I\left(S_t\right)$, $\sigma_F\left(S_t\right)$, $\theta_I\left(S_t\right)$, and $\theta_F\left(S_t\right)$, and the transition rule of $\iota_t = \iota\left(\iota_{t-1}, S_t\right)$. The transition rule of the other endogenous states is directly given once the other functions have been found.

In the asymmetric case, the value functions, the policy functions, and the transition rule of ι_t depend on the entire vector of states S_t rather than a subset of S_t as in Appendix J.1. The reason is that the measure of single firms $\left(\widetilde{n}_t^F, \widetilde{n}_t^I, \widetilde{n}_t^G\right)$ determines the inter-firm market tightness ratio $\widetilde{\theta}_t$, which affects firms' value and policy. In addition, the transition rule of $\left(\widetilde{n}_t^F, \widetilde{n}_t^I, \widetilde{n}_t^G\right)$ depends on the $\left(u_t^F, u_t^I, n_t^F, n_t^I, n_t^G\right)$.

Given the high dimension of the state space, we simplify the model solution with a forecast rule for $\tilde{\theta}$ that only depends on a small number of state variables. This approach is inspired by similar ideas in Krusell and Smith (1998). Intuitively, firms do not need to know $\left(u_t^F, u_t^I, n_t^F, n_t^I, n_t^G, \widetilde{n}_t^F, \widetilde{n}_t^I, \widetilde{n}_t^G\right)$ to make decisions if the forecast rule is accurate, which greatly reduces the dimension of the state space when solving the value and policy functions.

We choose the forecast rule:

$$\log\left(\widetilde{\theta}_{t+1}\right) = \left(a_{\tilde{\theta}} + a_{\tilde{\theta},\iota}\iota_{t-1}\right)\log\left(\widetilde{\theta}_t\right) + \left(a_z + a_{z,\iota}\iota_{t-1}\right)\log\left(z_t\right) + \left(a_\xi + a_{\xi,\iota}\iota_{t-1}\right)\log\left(\xi_t\right) + \left(a_G + a_{G,\iota}\iota_{t-1}\right)\epsilon_t^G, \tag{105}$$

where $A = \left(a_{\tilde{\theta}}, a_{\tilde{\theta},\iota}, a_z, a_{z,\iota}, a_\xi, a_{\xi,\iota}, a_G, a_{G,\iota}\right)$ is the vector of coefficients to be determined.

To do so, we proceed as follows:

Step 1: Initialize the algorithm. We initialize the forecast rule with some initial guess:

$$A^{(0)} = \left(a_{\tilde{\theta}}^{(0)}, a_{\tilde{\theta},\iota}^{(0)}, a_z^{(0)}, a_{z,\iota}^{(0)}, a_\xi^{(0)}, a_{\xi,\iota}^{(0)}, a_G^{(0)}, a_{G,\iota}^{(0)}\right). \tag{106}$$

Step 2: Solve for $TSJV$, σ_F, σ_I, and ι. Equation (55) can be rewritten as:

$$\begin{aligned} TSJV\left(z_t, \xi_t, \epsilon_t^G, \iota_{t-1}, \widetilde{\theta}_t\right) &= \tau \cdot z_t + \beta\left(1 - \delta - \tilde{\delta}\right)\xi_t \mathbb{E}_t\left[TSJV\left(z_{t+1}, \xi_{t+1}, \epsilon_{t+1}^G, \iota_t, \widetilde{\theta}_{t+1}\right)\right] \\ &+ \min_{\sigma_{I,t}} \tau \cdot c\left(\sigma_{I,t}\right) - \beta\left(1-\delta\right)\pi_{I,t}\xi_t\mathbb{E}_t\left[\widetilde{\tau} TSJV\left(z_{t+1}, \xi_{t+1}, \epsilon_{t+1}^G, \iota_t, \widetilde{\theta}_{t+1}\right)\right] \\ &+ \min_{\sigma_{F,t}} \tau \cdot c\left(\sigma_{F,t}\right) - \beta\left(1-\delta\right)\pi_{F,t}\xi_t\mathbb{E}_t\left[\left(1-\widetilde{\tau}\right) TSJV\left(z_{t+1}, \xi_{t+1}, \epsilon_{t+1}^G, \iota_t, \widetilde{\theta}_{t+1}\right)\right], \end{aligned} \tag{107}$$

where $\pi_{I,t} = \left[\phi + \left(\psi + \sigma_{F,t}\right)\left(\psi + \sigma_{I,t}\right)\right] H\left(\widetilde{\theta}_t, 1\right)$, $\pi_{F,t} = \left[\phi + \left(\psi + \sigma_{F,t}\right)\left(\psi + \sigma_{I,t}\right)\right] H\left(1, 1/\widetilde{\theta}_t\right)$,

$$\log\left(\widetilde{\theta}_{t+1}\right) = \left(a_{\tilde{\theta}}^{(q)} + a_{\tilde{\theta},\iota}^{(q)}\iota_{t-1}\right)\log\left(\widetilde{\theta}_t\right) + \left(a_z^{(q)} + a_{z,\iota}^{(q)}\iota_{t-1}\right)\log\left(z_t\right) + \left(a_\xi^{(q)} + a_{\xi,\iota}^{(q)}\iota_{t-1}\right)\log\left(\xi_t\right) + \left(a_G^{(q)} + a_{G,\iota}^{(q)}\iota_{t-1}\right)\epsilon_t^G,$$

and $A^{(q)} = \left(a_{\tilde{\theta}}^{(q)}, a_{\tilde{\theta},\iota}^{(q)}, a_z^{(q)}, a_{z,\iota}^{(q)}, a_\xi^{(q)}, a_{\xi,\iota}^{(q)}, a_G^{(q)}, a_{G,\iota}^{(q)}\right)$ is the vector of coefficients of the forecast rule in the q-th iteration.

The equilibrium type ι_t is determined by the best response functions implied by equation (100) and the history dependence of equilibrium selection. If $\iota_{t-1} = 0$ (passive stage equilibrium in period $t-1$), we verify whether the passive stage equilibrium still exists in the current period t, i.e., $\iota_t = 0$, by checking whether:

$$\arg\min_{\sigma_{I,t}\geq 0} c\left(\sigma_{I,t}\right) - \widetilde{\beta}\left[\phi + \left(\psi + \sigma_{I,t}\right)\psi\right]\xi_t \mathbb{E}_t\left[\widetilde{\tau} TSJV\left(z_{t+1}, \xi_{t+1}, \epsilon^G_{t+1}, \iota_t = 0, \widetilde{\theta}_{t+1}\right)\right] = 0 \tag{108}$$

$$\arg\min_{\sigma_{F,t}\geq 0} c\left(\sigma_{F,t}\right) - \widetilde{\beta}\left[\phi + \left(\psi + \sigma_{F,t}\right)\psi\right]\xi_t \mathbb{E}_t\left[\left(1-\widetilde{\tau}\right) TSJV\left(z_{t+1}, \xi_{t+1}, \epsilon^G_{t+1}, \iota_t = 0, \widetilde{\theta}_{t+1}\right)\right] = 0 \tag{109}$$

hold. If these conditions hold, $\iota_t = \iota_{t-1} = 0$. Otherwise, $\iota_t = 1$.

Analogously, if $\iota_{t-1} = 1$ (active stage equilibrium in $t-1$), we verify whether the active stage equilibrium still exists in the current period, i.e., $\iota_t = 1$, by checking whether:

$$\arg\min_{\sigma_{I,t}\geq 0} c\left(\sigma_{I,t}\right) - \widetilde{\beta}\left[\phi + \left(\psi + \sigma_{I,t}\right)\left(\psi + \sigma_{F,t}\right)\right]\xi_t \mathbb{E}_t\left[\widetilde{\tau} TSJV\left(z_{t+1}, \xi_{t+1}, \epsilon^G_{t+1}, \iota_t = 1, \widetilde{\theta}_{t+1}\right)\right] > 0 \tag{110}$$

and

$$\arg\min_{\sigma_{F,t}\geq 0} c\left(\sigma_{F,t}\right) - \widetilde{\beta}\left[\phi + \left(\psi + \sigma_{I,t}\right)\left(\psi + \sigma_{F,t}\right)\right]\xi_t \mathbb{E}_t\left[\left(1-\widetilde{\tau}\right) TSJV\left(z_{t+1}, \xi_{t+1}, \epsilon^G_{t+1}, \iota_t = 1, \widetilde{\theta}_{t+1}\right)\right] > 0 \tag{111}$$

hold. If these conditions hold, $\iota_t = \iota_{t-1} = 1$. Otherwise, $\iota_t = 0$.

Given the forecast rule with $A^{(q)}$, we can solve for the value function $TSJV$, the policy function σ, and the transition rule ι with equation (107) and conditions (108)-(111) using value function iteration.

Step 3: Solve for $\widetilde{TS}$ and θ. Equation (47) can be rewritten, for $i \in \{I, F\}$, as:

$$\widetilde{TS}\left(z_t, \xi_t, \epsilon^G_t, \iota_{t-1}, \widetilde{\theta}_t\right) = -h - c\left(\sigma_{i,t}\right) + \beta \xi_t \mathbb{E}_t \begin{bmatrix} \left(1-\delta\right)\pi_{i,t}\Delta TS\left(z_{t+1}, \xi_{t+1}, \epsilon^G_{t+1}, \iota_t, \widetilde{\theta}_{t+1}\right) + \\ \left(\left(1-\delta\right) - \left(1-\tau\right)\theta^{\alpha}_i\left(z_t, \xi_t, \epsilon^G_t, \iota_{t-1}, \widetilde{\theta}_t\right)\right) \\ *\widetilde{TS}\left(z_{t+1}, \xi_{t+1}, \epsilon^G_{t+1}, \iota_t, \widetilde{\theta}_{t+1}\right) \end{bmatrix}, \tag{112}$$

where we have used the fact that $\Delta TS_{i,t+1} = TS_{i,t+1} - \widetilde{TS}_{i,t+1}$ and $\mu_{i,t} = \theta^{\alpha}_{i,t}$.

We also have, for $i \in \{I, F\}$, the free-entry condition implied by equation (42):

$$\chi = \beta \xi_t \tau \theta_{i,t}^{\alpha-1} \left(z_t, \xi_t, \epsilon_t^G, \iota_{t-1}, \widetilde{\theta}_t \right) \mathbb{E}_t \left[\widetilde{TS} \left(z_{t+1}, \xi_{t+1}, \epsilon_{t+1}^G, \iota_t, \widetilde{\theta}_{t+1} \right) \right]. \tag{113}$$

With $\Delta TS_{i,t}$, $\sigma_{I,t}$, $\sigma_{F,t}$ and ι_t being solved in step 2 (in particular, $\Delta TS_t = \widetilde{\tau} TSJV_t / \tau$), we can solve for the value function $\widetilde{TS}_{i,t}$ and the policy function $\theta_{i,t}$ approximately with equations (112) and (113) using value function iteration.

Step 4: Simulate the model. We simulate the model for 10,000 periods (disregarding the first 2,000 as a burn-in) with random draws of $\left\{ z_t, \xi_t, \epsilon_t^G \right\}$. Then, we compute the realized equilibrium inter-firm market tightness ratio $\widetilde{\theta}_t$.

Step 5: Update the forecast rule. Based on the simulated data, we update the coefficient of the forecast rule $A^{(q)}$ with $A^{(q+1)}$ using ordinary least squares. If $A^{(q)}$ and $A^{(q+1)}$ are sufficiently close to each other, we stop the iteration. Otherwise, we return to step 2. The converged forecasting rule explains the fluctuations of $\widetilde{\theta}_t$ well, with an R^2 of 0.91.

K Simulations based on shocks to productivity

In this appendix, we complete our discussion of the effects of technology shocks in the model. Figure 16 plots the ergodic distribution of selected variables for the case where we only have AR(1) shocks to technology, z_t (for transparency, we eliminate the discount factor shocks). As outlined in the paper, persistent exogenous disturbances to the technological process fail to move the system to a different equilibrium, the equilibrium is always active, and the ergodic distributions of the variables of interest are unimodal.

In Figure 17, we plot the ergodic distribution of selected variables for the case where we have shocks both to technology, z_t and to the discount factor, ξ_t. We recover bimodality, but this feature is induced by the shocks to ξ_t and their ability to switch equilibria. The main effect of the shocks to productivity is to spread out the ergodic distribution in Figure 6 in the main text (only shocks to ξ_t) around its two modes.

Figure 18 shows the GIRFs to a range of persistent negative productivity shocks when the economy starts from the active DSS. Negative productivity shocks are unable to generate a shift in equilibrium even when their magnitude gets very large. In each case, the costly search effort

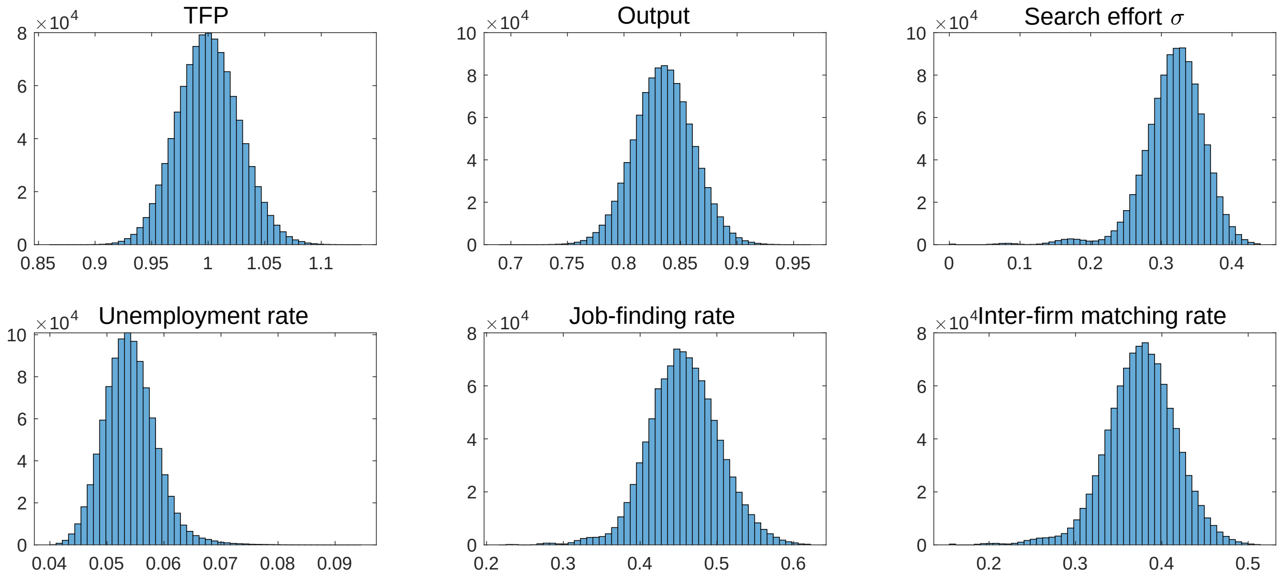


Figure 16: Ergodic distribution with AR(1) shocks to z_t

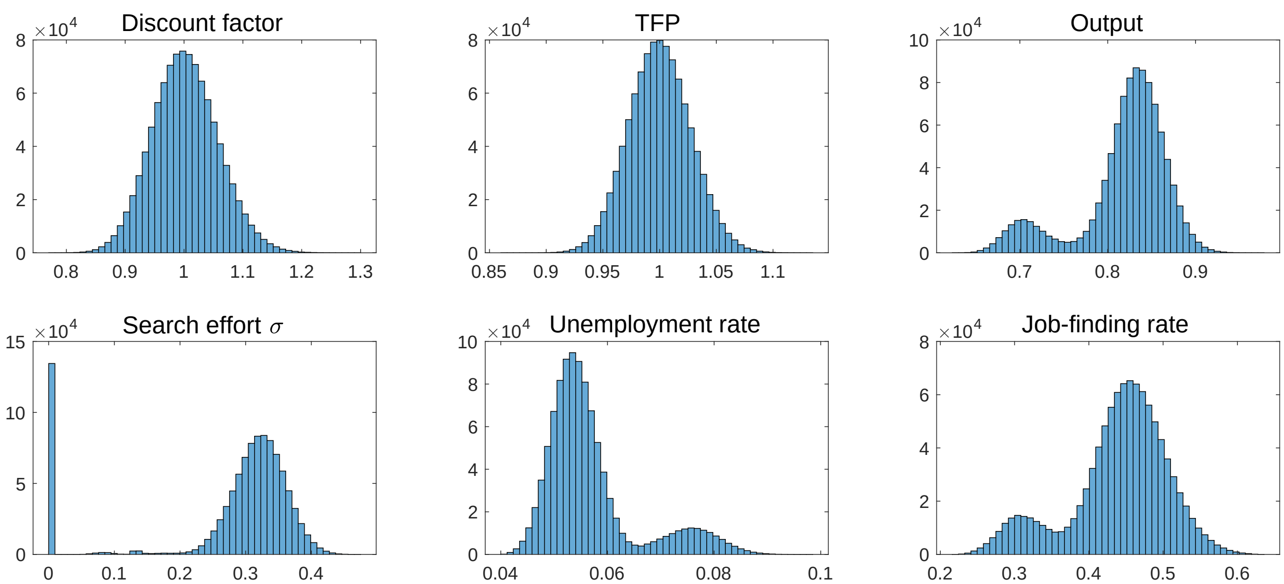


Figure 17: Ergodic distribution with shocks to ξ_t and shocks to z_t

falls after the productivity shock, and then gradually recovers. The effect of a productivity shock on the labor market tightness ratio and the unemployment rate is also transitory. The mechanism is that the gain of matching with a partner $(TS - \widetilde{TS})$ in the active stage equilibrium is inelastic with the change in productivity. This result is similar to the one in Shimer (2005), who points out that the gain of matching with a worker, $\widetilde{TS}$ and TS, is inelastic with the change in productivity in a canonical DMP model. Since $\widetilde{TS}$ and TS move in the same direction in reaction to productivity shocks, the response of $TS - \widetilde{TS}$ to productivity shocks is even weaker.

As a result, the existence condition for the active stage equilibrium in equation (89) keeps holding: if we start at the active DSS, firms find it desirable to search actively for a partner even when productivity is low.

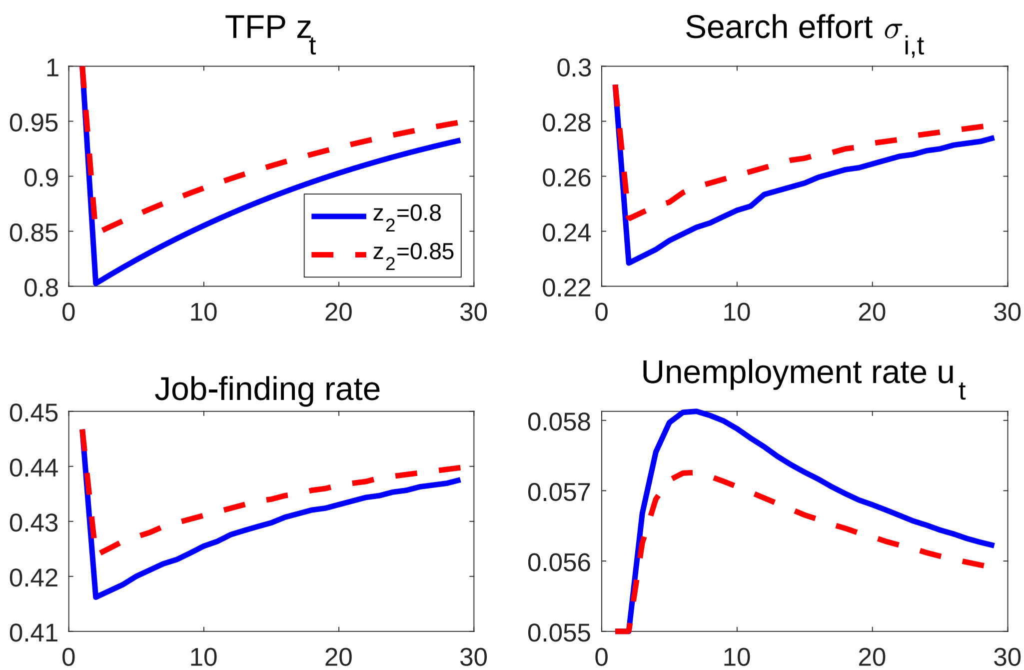


Figure 18: GIRFs to a negative productivity shock

We also experiment with permanent changes in productivity. In $t = 1$, the economy starts from the active DSS with positive search effort, and in $t = 2$ a permanent fall in productivity hits the economy. This permanent shock may shift the equilibrium of the system by affecting the expected gain of match $\mathbb{E}_t(J_{i,t+1} - \widetilde{J}_{i,t+1})$. For example, in an economy in the active stage equilibrium, a sufficiently large fall in z_t decreases the expected gain from trading relationship formation and moves the system to the passive stage equilibrium.

We assess the magnitude of the fall in z_t needed to move the system from the active to the passive stage equilibrium. Figure 19 shows the GIRFs to a 30% (solid line) and 40% (dashed line) permanent decline in productivity (z_t). The first shock is unable to move the system to the active stage equilibrium because the expected gain from inter-firm matching is relatively inelastic to permanent changes in productivity. Productivity shocks induce $\widetilde{J}_{i,t+1}$ and $J_{i,t+1}$ to comove, leading to a weak response of $\mathbb{E}_t(J_{i,t+1} - \widetilde{J}_{i,t+1})$ to the shock. This finding is consistent with Shimer (2005). In comparison, a sufficiently large productivity shock of 40% pushes the economy to the passive stage equilibrium. This analysis suggests that a permanent productivity shock is unlikely to move the system between equilibria unless the shock is massive.

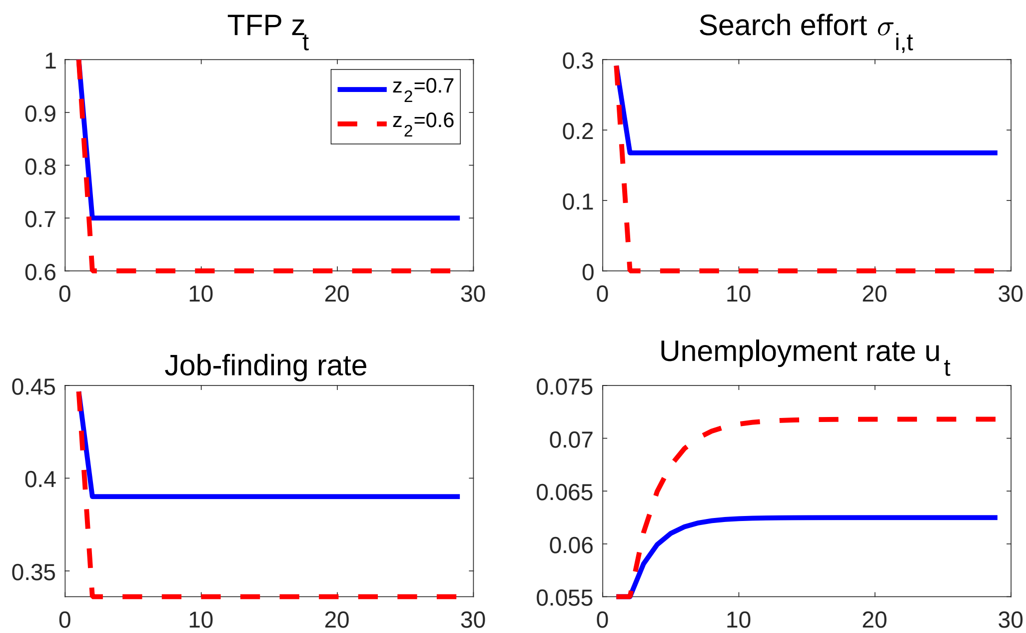


Figure 19: GIRFs to a negative permanent productivity shock

L Second moments of the model without search complementarities

Table 9 reports the simulations of the model without search complementarities.

Table 9: Second moments, model without search complementarities

		u	v	v/u	lp	ξ
Autocorrelation coefficient		0.52	0.13	0.35	1	0.35
Standard deviation		0.03	0.07	0.09	0	0.05
	u	1	-0.51	-0.74	0	-0.74
Correlation matrix	v		1	0.96	0	0.96
	v/u			1	0	1.00
	lp				1	0
	ξ					1

Note: Following Shimer (2005), all variables are reported in logs as deviations from an HP trend with $\lambda = 10^5$.

M Measures of the discount factor

Figure 20 plots the three measures of the discount factor (discount factor from dividend strip, the price-to-dividend ratio, and the Livingston Survey) for the period between January 1996 and May 2009. All three measures agree that i) there was a sizable decline in the discount factor

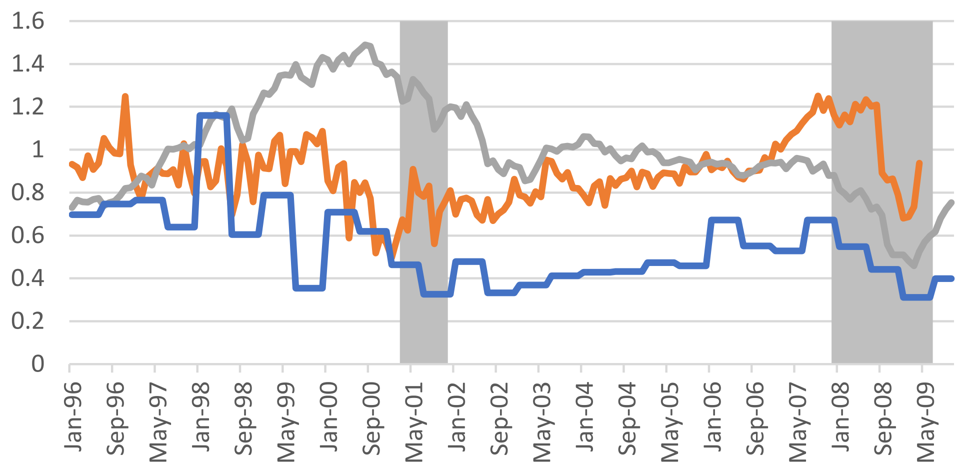


Figure 20: Alternative measures of the discount factor

Note: Alternative measures of the discount factor from dividend strip (orange line), the price-to-dividend ratio (gray line), and the Livingston Survey (blue line).

during the Great Recession (as our theory requires) and ii) the series display high variance (reflecting the large sensitivity of the discount factor over the business cycle, also required by our theory). The low correlation across the three measures is not surprising, since each of these series reflects discounting from different financial players and assets (see Hall, 2017).

N Volatility and duration of stage equilibria

To gain intuition, we derive an analytical characterization of the effect of volatility on the likelihood and duration of each stage equilibrium by simplifying the model in Section 3. First, we assume that firms produce their output without labor. Thus, we can drop the whole DMP module of the model and set a constant measure of size 1 of firms in each sector. Second, we assume that $\widetilde{\delta} = 1$, i.e., all trading relationships terminate after one period. Also, trading relationships start producing in the same period in which firms match. Hence, the firm's problem is equivalent to a sequence of stage maximization problems and we do not need to specify a discount factor. To ease the algebra, we also set $\rho_z = 0$, and as in the calibration in Section 5, $\widetilde{\tau} = 0.5$ and $\nu = 2$.

Under these simplifications, each firm optimally chooses the level of its search effort, $\sigma_{i,t}$, given the search effort of the firms in the opposite sector, $\sigma_{-i,t}$, and productivity, z_t, by maximizing:

$$J_{i,t}(\sigma_{i,t}, \sigma_{-i,t}, z_t) = (\phi + (\psi + \sigma_{i,t})(\psi + \sigma_{-i,t}))\frac{z_t}{2} - c_0\sigma_{i,t} - c_1\frac{\sigma_{i,t}^3}{3}.$$

The first term of the RHS is the inter-firm matching probability defined in equation (18) multiplied by half the expected production, $\pi_{i,t} z_t$ (recall the equal split of output between the firms given $\widetilde{\tau} = 0.5$) minus the cost of searching.

The interior solution $\sigma_{i,t} > 0$ satisfies:

$$c_0 + c_1 \sigma_{i,t}^2 = (\psi + \sigma_{-i,t}) \frac{z_t}{2}. \tag{114}$$

Otherwise, $\sigma_{i,t} = 0$. Hence, as in the benchmark model, the simplified model entails passive and active stage equilibria. The passive stage equilibrium with zero search effort exists if and only if $c_0 > \psi \frac{z_t}{2}$. Thus, a productivity threshold $\bar{z} = \frac{2c_0}{\psi}$ determines whether the passive stage equilibrium exists.

Lemma 2. *The passive stage equilibrium exists if and only if* $z_t < \bar{z}$.

Recall that we assumed that $\psi > 0$. If $\psi = 0$, a passive stage equilibrium always exists regardless of the value of z_t.

In an active stage equilibrium, firms in each sector optimally choose a positive search effort that comes from finding the fixed point of the product of equation (114) for each sector:

$$\sigma_{F,t} = \sigma_{I,t} = \frac{z_t + \sqrt{z_t^2 + 8\psi z_t - 16 c_0 c_1}}{4c_1}. \tag{115}$$

This optimal search effort is increasing in z_t.[19] From equation (115), the threshold for the active stage equilibrium is $\underline{z} = 4\left(\sqrt{\psi^2 c_1^2 + c_1 c_0} - \psi c_1\right)$, and we get the following lemma.[20]

Lemma 3. *An active stage equilibrium exists if and only if* $z_t \geq \underline{z}$.

Proposition 11 merges lemmas 2 and 3.

Proposition 11. *The economy retains multiple equilibria if* $z_t \in (\underline{z}, \bar{z})$. *The passive stage equilibrium is the unique equilibrium if* $z_t \leq \underline{z}$. *The active stage equilibrium is the unique equilibrium if* $z_t \geq \bar{z}$.

Proposition 11 establishes that if economic fundamentals are sufficiently weak or strong, the stage equilibrium is unique, either passive or active; otherwise, we have two stage equilibria.

[19] There is a second fixed point, $\sigma_{i,t} = \frac{z_t - \sqrt{z_t^2 + 8\psi z_t - 16 c_0 c_1}}{4c_1}$. However, this solution is locally unstable.

[20] To prevent the marginal search cost from converging to zero when $\sigma_{i,t}$ is zero, the term c_0 must be positive. If $c_0 = 0$, it yields $\underline{z} = 0$. In such an instance, the active stage equilibrium exists for any positive value of z_t.

Sufficiently large shocks to z_t move the system between the two alternative stage equilibria. Proposition 11 is empirically relevant because we can calibrate ψ to a small number so that $\bar{z}$ is low and c_1 to a large number so that $\underline{z}$ is high. In that way, the model will allow multiple stage equilibria for a wide range of productivity $\underline{z} < 1 < \bar{z}$.

Since we have set $\rho_z = 0$, $\log(z_t) \sim \mathcal{N}(0, \sigma_z^2)$. Using the distribution for z_t and the thresholds $\underline{z}$ and $\overline{z}$, we derive the transition matrix between equilibria:

	Active	Passive
Active	$1 - \Phi[\log(\underline{z})/\sigma_z]$	$\Phi[\log(\underline{z})/\sigma_z]$
Passive	$1 - \Phi[\log(\bar{z})/\sigma_z]$	$\Phi[\log(\bar{z})/\sigma_z]$

where $\Phi(\cdot)$ is the cdf of the standard normal distribution. The next proposition establishes that aggregate volatility plays a critical role in the selection and duration of each stage equilibrium.

Proposition 12. *The expected duration of a passive stage equilibrium spell is* $\frac{1}{\Phi[\log(\underline{z})/\sigma_z]}$*, and the expected duration of an active stage equilibrium spell is* $\frac{1}{1-\Phi[\log(\bar{z})/\sigma_z]}$*. The duration of each equilibrium is inversely related to the volatility of* z_t*.*

Proposition 12 shows that a reduction in volatility induces the system to remain for a prolonged spell in one stage equilibrium, with a decreased probability for the system to move to the alternative stage equilibrium. However, if a large change in fundamentals triggers a change in the stage equilibrium, the economy would stay there for a long time.

Next, we use our benchmark model to gauge the changes in the volatility of shocks. Table 10 reports business cycle statistics for a low (column (a)) and a high (column (b)) variance of shocks to the discount factor (σ_ξ). As before, we simulate the model for 3,000,000 months and time average to obtain quarterly data. The first and second rows report the number of periods and the average duration of the passive stage equilibrium, respectively, and the third row reports the transition matrix between equilibria. We calibrate high and low volatility by following Justiniano and Primiceri (2008), who estimate that the volatility of the discount factor is equal to 0.07 before 1984 and 0.04 after that date.

The passive stage equilibrium materializes with a probability of around 7% in the low-volatility economy, in contrast with a 26% probability in the high-volatility economy. Despite the lower chance of moving to a passive stage equilibrium, the low-volatility economy stays longer on average in a passive stage equilibrium, 11.8 quarters, than the high-volatility economy,

Table 10: Variance of shocks and duration of equilibria

	(a) $Std(\xi) = 0.04$		(b) $Std(\xi) = 0.07$	
Fraction of periods in passive stage equilibrium	0.07		0.26	
Average number of quarters in passive stage equilibrium	11.8		4.5	
Transition matrix				
	Active	Passive	Active	Passive
Active	0.99	0.01	0.94	0.06
Passive	0.08	0.92	0.20	0.80

4.5 quarters. Low volatility induces less frequent but long-lasting periods of low output and high unemployment.

The last two rows in Table 10 report the transition matrix between equilibria. The low-volatility economy transitions between equilibria infrequently. The probability of moving from active stage equilibrium to passive stage equilibrium is equal to 1%, and the probability of a reverse move from passive stage equilibrium to active stage equilibrium is equal to 8%. The rotation among equilibria gets much higher in the high-volatility economy, as the probability of moving from an active to a passive stage equilibrium is 6%, and the probability of a reverse move is 20%.

These dynamics are consistent with the large and persistent low employment-to-population ratio in the aftermath of the financial crisis of 2007-2009. The financial crisis was preceded by a long spell of stable economic conditions during the Great Moderation that started in the mid-1980s, which the model identifies as a prerequisite for the persistence in the low employment-to-population ratio.

O Volatility of shocks

The panels in the left column of Figure 21 plot the ergodic distribution of endogenous variables with shocks to ξ in the case of high volatility ($\sigma_\xi^{high} = 1.5\sigma_\xi$). The panels in the middle column of Figure 21 repeat the same exercise, but in the case of low volatility ($\sigma_\xi^{high} = 0.75\sigma_\xi$). In both cases, we see the bimodal distributions that we discussed in the main text and the long left tail of output when the volatility of ξ_t is high. Finally, the panels in the right column of Figure 21 display the results when the volatility is ultra low ($\sigma_\xi^{ultralow} = 0.5\sigma_\xi$), in which case the bimodality disappears.

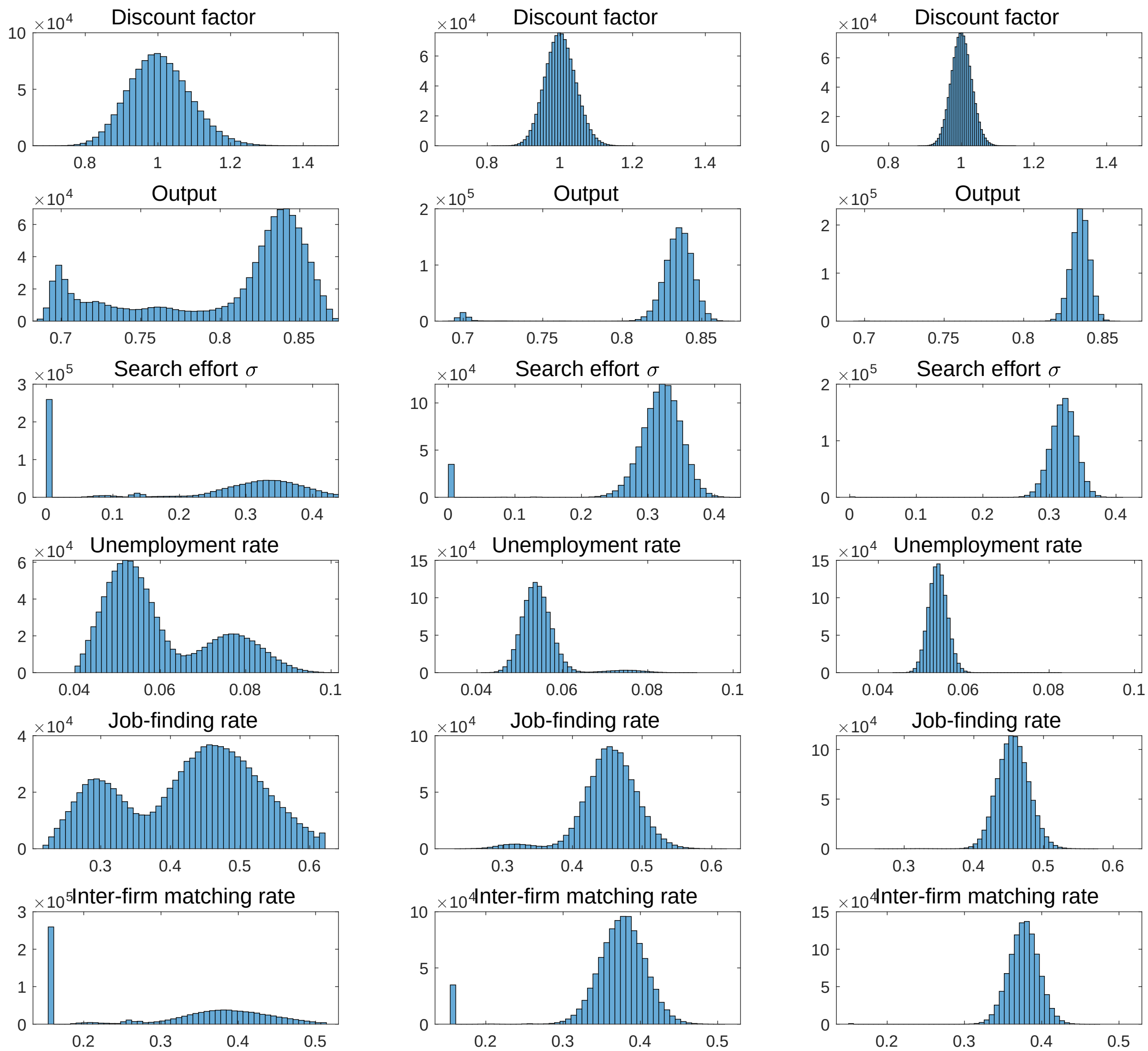


Figure 21: Ergodic distribution with shocks to ξ with different volatilities

Note: The left, middle, and right columns display the ergodic distributions for high, low, and ultra low volatility cases, respectively.

P GIRFs to government spending shock in the active stage equilibrium

This appendix studies the effect of government spending shocks when the economy starts from the active stage equilibrium. Figure 22 shows the response in the level of selected variables to a 50% (the solid line) and a 60% (the dashed line) government spending shock. Since the economy is already in the active stage equilibrium, the effects of the fiscal expansion are limited and transitory.

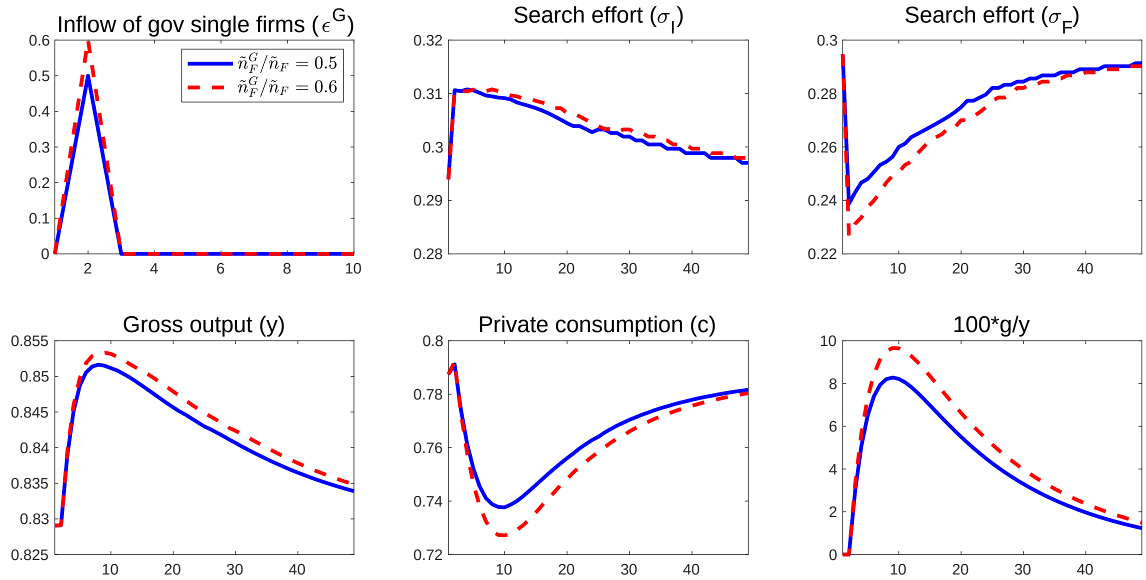


Figure 22: GIRFs to positive government spending shock in the active stage equilibrium

Q Separation rate of inter-firm matches

This appendix shows that our main results are robust to different calibrations for the inter-firm matches' separation rate, $\widetilde{\delta}$. Panels (a)-(c) of Figure 23 display the histogram of output for low $\widetilde{\delta}$ (50% lower than our benchmark calibration), medium $\widetilde{\delta}$ (our benchmark calibration), and high $\widetilde{\delta}$ (50% higher than our benchmark calibration), respectively. The three panels show a bimodal distribution of output, while a higher $\widetilde{\delta}$ entails a higher probability of the passive equilibrium and lower modes of the output distribution.

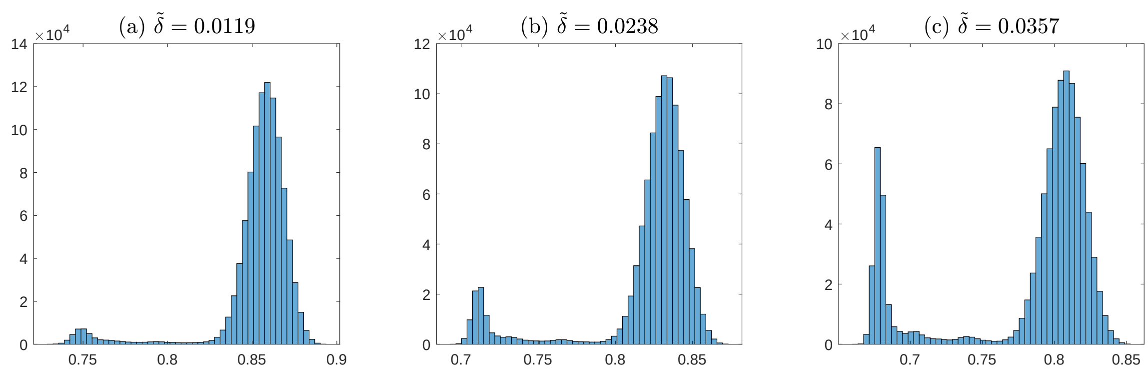


Figure 23: Histogram of output for different $\widetilde{\delta}$