Australian National University

Crawford School of Public Policy

CAMA

Centre for Applied Macroeconomic Analysis

Collateral Shocks, Lending Relationships and Economic Dynamics

---

CAMA Working Paper 49/2023
October 2023

Vivek Sharma
University of Melbourne
CASMEF
Centre for Applied Macroeconomic Analysis, ANU

Abstract

What are the effects of changing bank lending conditions in a model in which borrowers have endogenously-persistent credit relationships with lenders? This paper answers this question in a simple Two-Agent New Keynesian (TANK) setup. Fluctuations in collateral requirements, termed collateral shocks in this paper, result in a rise in spread, a drop in bank credit and amplification of macroeconomic volatility. These effects are amplified by presence of lending relationships and are greater at higher persistence and volatility of the collateral shocks. The results in this paper underscore that credit relationships matter when collateral shocks hit the economy and a model that assumes away the existence of these lending relationships, risks underestimating their effects.

Keywords

Collateral Shocks, Lending Relationships, Economic Activity

JEL Classification

E32, E44

Address for correspondence:

(E) cama.admin@anu.edu.au

ISSN 2206-0332

Collateral Shocks, Lending Relationships and Economic Dynamics

Vivek Sharma*

University of Melbourne, CAMA, CASMEF

September 20, 2023

Abstract

What are the effects of changing bank lending conditions in a model in which borrowers have endogenously-persistent credit relationships with lenders? This paper answers this question in a simple Two-Agent New Keynesian (TANK) setup. Fluctuations in collateral requirements, termed collateral shocks in this paper, result in a rise in spread, a drop in bank credit and amplification of macroeconomic volatility. These effects are amplified by presence of lending relationships and are greater at higher persistence and volatility of the collateral shocks. The results in this paper underscore that credit relationships matter when collateral shocks hit the economy and a model that assumes away the existence of these lending relationships, risks underestimating their effects.

Keywords: Collateral Shocks, Lending Relationships, Economic Activity

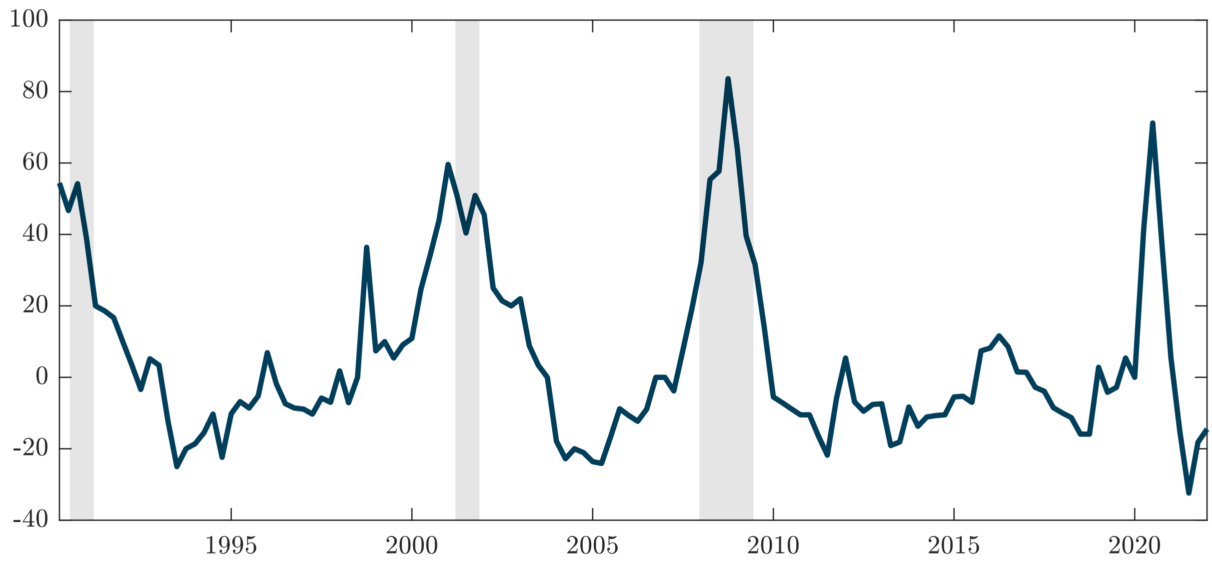
JEL Classification: E32, E44

*sharma.v2@unimelb.edu.au, https://sharmavivek.com/.

1 Introduction

This paper examines the impact of changing bank lending conditions on macroeconomic activity. Recently, Becard and Gauthier (2022) have shown that changing bank lending conditions, or collateral shocks, are a significant driver of US business cycle. Using a New Keynesian model featuring both household and firm borrowing, Becard and Gauthier (2022) show that collateral shock is able to reproduce the comovement of consumption, output, investment and employment. There is no work, however, that examines the effects of collateral shocks in an environment which features lending relationships between lenders and borrowers. The model in Becard and Gauthier (2022) includes a banking sector as in Jakab and Kumhof (2015) which accepts deposits and makes loans. This modelling of banking sector, however, abstracts from lending relationships between borrowers and lenders. Existence of lending relationships between banks and firms have been documented across various economies (Ongena and Smith, 2000; Kosekova, Maddaloni, Papoutsi, and Schivardi, 2023). This paper takes this aspect of banking sector seriously and builds a framework that allows for endogenously-persistent lending relationships between banks and their borrowers.

Figure 1: Net Percentage of Banks Tightening Standards for Business Loans



Note: Business loans refer to commercial and industry loans to large and middle-market firms. Data from Senior Loan Officer Opinion Survey on Bank Lending Practices administered by the Federal Reserve. Shaded areas refer to NBER recession dates.

As Figure 1 shows, bank lending standards evolve systematically over the business cycle. Becard and Gauthier (2022) call the exogenous fluctuation in collateral requirements imposed by banks on their borrowers collateral shocks. Collateral shocks in this paper is a "stand-in" for a broad variety of developments in the financial sector. It could, for example, be used to describe scenarios where banks lose confidence in value of assets posted as collateral and consequently require higher haircuts. It can, for instance, also describe situations where, for any number of reasons, banks discount assets posted as collateral more heavily. In this paper, I build a model that takes into account this systematic variation in credit standards and introduces it into a framework that features credit relationships. I model lending relationships[1] in this paper using deep habits in banking which have been successfully used to model countercyclical spreads in banking and study borower "hold up" effect (see, among others, Aliaga-Díaz and Olivero, 2010, Ravn, 2016, Airaudo and Olivero, 2019 and Shapiro and Olivero, 2020). To the best of my knowledge, this is the first paper that studies implications of lending relationships for collateral shocks.

To keep the model simple, in this paper I focus on lending relationships between firms and banks and abstract from household lending[2]. The analysis in this paper shows that lending relationships generate significant amplification of collateral shocks. I further find that higher volatility and persistence of collateral shocks amplifies their effects. It's notable that I obtain these results in a parsimonious two-agent RBC model devoid of any real or nominal rigidities typically found in the New Keynesian literature. The key mechanism behind these effects is jump in spread between bank lending and deposit rates. To understand the mechanism behind these results, consider what happens when the economy features no credit relationships between lenders and borrowers and a collateral shock materializes. In this case, the spread does not move at all since bank lending and deposit rates fall by the same magnitude. Bank credit, however, still falls because a collateral shock reduces the value of assets used as collateral to take bank loans which then leads to a drop in investment, output and aggregate consumption. In the other case when credit relationships are present, in the wake of a collateral shock, banks dramatically increase their spread since the value of assets posted as collateral with them is less now. This spurt in spread results in a greater fall in bank credit and a much higher impact of this fall in financial intermediation on wider macroeconomy. Investment, output and aggregate consumption, in this case, drop more than in the case of no bank-firm lending relationships. After an initial rise, the bank spread falls rapidly and overshoots its previous steady-state level

[1]I will use the terms 'lending relationships' and 'credit relationships' synonymously in this paper.

[2]This does not take away from the message in this paper. As shown by Becard and Gauthier (2022), including household debt will likely only strengthen the results in this paper. Incorporating household debt in this model is straightforward and I leave it as a future research exercise.

before returning to its previous equilibrium value. This reflects the fact that after initial surge in spread and fall in bank credit, spread falls rapidly as banks scramble to "lock-in" customers which then results in spread falling below its previous steady-state value. This causes an increase in bank credit, investment, output and aggregate consumption. This mechanism lies behind macroeconomic amplification in this paper. These findings echo financial accelerator effects of Bernanke, Gertler, and Gilchrist (1999) and show how presence of borrower-lender relationships can act as a "financial accelerator". These results highlight the important role of bank-firm lending relationships in shaping macroeconomic dynamics in the aftermath of a collateral shock. It also underscores the fact that a macroeconomic model that assumes away presence of borrower-lender credit relationships, might miss true economic dynamics and might underestimate the effects of collateral shocks.

This paper contributes to the literature on the intersection of macroeconomics and banking. As mentioned before, Becard and Gauthier (2022) build a New Keynesian model with both household and entrepreneurial borrowing and show that collateral shocks drive significant part of macroeconomic fluctuations in the US economy. Differently from them, my paper turns its attention on presence of bank-firm lending relationships and builds a parsimonious RBC model. It then asks what the implications are of a collateral shock in this model. The model in this paper is similar to Sharma (2023a), however, the focus in this paper is on collateral shocks while the focus in Sharma (2023a) is on understanding implications of changes in steady-state loan-to-value (LTV) ratios on effects of LTV shocks. This work is also related to Sharma (2023b) who studies state-dependence in effects of LTV shocks in a simple two-agent RBC model. My work, on the other hand, focuses on implications of lending relationships for collateral shocks.

The rest of this paper is structured as follows. Section 2 presents the model and Section 3 discusses model solution and parameterization. Section 4 presents the results and Section 5 concludes.

2 Model

The paper features a Two-Agent New Keynesian (TANK) model and bears resemblance to the setup in Iacoviello (2005), Liu, Wang, and Zha (2013) and Justiniano, Primiceri, and Tambalotti (2015). It has (patient) households who consume, supply labor and make deposits with a bank. Households are the ultimate owners of the banks and receive their profits. The (impatient) entrepreneurs, in turn, consume non-durable consumption good and run firms in the economy. They are subject to a collateral constraint which limits their borrowing to a fraction of expected value of their assets which include productive capital and land. The entrepreneurs borrow from banks and develop endogenously-persistent credit relationships with them. Lending relationships in this paper are modelled by using the deep habits framework developed first by Ravn, Schmitt-Grohé, and Uribe (2006) and used later in studying banking sector by Aliaga-Díaz and Olivero (2010), Ravn (2016), Airaudo and Olivero (2019) and Shapiro and Olivero (2020), among others. These banks raise deposits from households which is their only source of funding and lend them to entrepreneurs who combine them with productive capital to produce output. In what follows, I describe each agent's optimization problem.

2.1 Households

Households have the utility function of the following form:

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \left(\beta^P\right)^t \left\{ \log\left(C_{i,t}^P - \gamma^P C_{i,t-1}^P\right) - \frac{N_{i,t}^{\eta}}{\eta} + \varsigma \log H_{i,t}^P \right\} \tag{1}$$

where $C_{i,t}^P$, $N_{i,t}$ and $H_{i,t}^P$ denote consumption, labor and housing respectively of the households, $\beta^P \in (0,1)$ is a discount factor, γ^P measures the degree of habit formation in consumption, η is Frisch elasticity of labor supply and ς is a weight on housing. The superscript P denotes (patient) households. The household faces the following budget constraint

$$C_{i,t}^P + Q_t^H \left(H_{i,t}^P - H_{i,t-1}^P\right) + \int_0^1 D_{ik,t}\mathrm{d}k \leq W_t N_{i,t} + \int_0^1 \Pi_{ik,t}\mathrm{d}k + R_{t-1}^D \int_0^1 D_{ik,t-1}\mathrm{d}k \tag{2}$$

where Q_t^H is the price of one unit of housing in terms of consumption goods, W_t is the real wage and R_{t-1}^D is the gross risk-free interest rate on the stock of deposits $D_{ik,t-1}$ of household i in bank k at the end of period $t-1$. I assume housing does not depreciate. Profits obtained by household i from bank k are denoted by $\Pi_{ik,t}$. After imposing symetric equilibrium, FOCs of the households can be written as

$$\frac{1}{C_t^P - \gamma^P C_{t-1}^P} - \beta^P \mathbb{E}_t \frac{\gamma^P}{C_{t+1}^P - \gamma^P C_t^P} = \lambda_t^P \tag{3}$$

$$\beta^P \mathbb{E}_t \lambda_{t+1}^P = \frac{\lambda_t^P}{R_t^D} \tag{4}$$

$$\frac{\varsigma}{H_t^P} + \beta^P \mathbb{E}_t \left(\lambda_{t+1}^P Q_{t+1}^H\right) = \lambda_t^P Q_t^H \tag{5}$$

$$N_t^{\eta-1} = \lambda_t^P W_t \tag{6}$$

where λ_t^P is the Lagrange multiplier associated with household's budget constraint (2). One can combine household's first-order conditions with respect to consumption (3) and bank deposits (4) to obtain their Euler equation. Equation (5) describes household's Euler equation for housing and links today's housing price to the utility it provides plus the expected capital gain. Equation (6) describes household's consumption-lesiure tradeoff. First order conditions of the problem are derived in the Appendix A.1.

2.2 Entrepreneurs

Following Iacoviello (2005) and Liu, Wang, and Zha (2013), entrepreneur j maximizes the utility obtained from consuming the non-durable consumption goods

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \left(\beta^E\right)^t \log\left(C_{j,t}^E - \gamma^E C_{j,t-1}^E\right) \tag{7}$$

where β^E and γ^E are as defined before. I assume that entrepreneurs are more impatient than the (patient) households, that is, $\beta^E < \beta^P$. Entrepreneurs face a collateral constraint as in Kiyotaki and Moore (1997) that limits the borrowing of each entrepreneur to a fraction of their assets

$$l_{jk,t} \leq \frac{1}{R_{k,t}^L} \theta_t a_{j,t} \tag{8}$$

Here, $l_{jk,t}$ denotes entrepreneur j's loan from bank k, expected value of entrepreneur's assets is $a_{j,t}$ and $R_{k,t}^L$ is the bank-specific lending rate. At the beginnig of period t, entrepreneurs are hit by a shock θ_t that converts the value of their assets $a_{j,t}$ to $\theta_t a_{j,t}$. The collateral shock is assumed to satisfy

$$\log\theta_t = (1-\rho_\theta)\log\theta + \rho_\theta \log\theta_{t-1} + \sigma_\theta \epsilon_{\theta,t} \tag{9}$$

where θ is the steady-state collateral shock and $\epsilon_{\theta,t}$ are shocks with mean one. Expetced valued of entrepreneur's assets $a_{j,t}$ is given by

$$a_{j,t} = \mathbb{E}_t \left(Q^H_{t+1} H^E_{j,t} + Q^K_{t+1} K_{j,t} \right) \tag{10}$$

In the above equation, Q^K_t denotes the value of installed capital in units of consumption goods, $K_{j,t}$ is stock of capital and $H^E_{j,t}$ stock of housing.

Entrepreneurs have deep habits in banking relationships and and I let $x_{j,t}$ denote entrepreneur j's effective/habit-adjusted borrowing. Given the continuum of banks in the economy who compete under monopolistic competition, this can be written as

$$x_{j,t} = \left[\int_0^1 \left(l_{jk,t} - \gamma^L s_{k,t-1} \right)^{\frac{\xi-1}{\xi}} \mathrm{d}k \right]^{\frac{\xi}{\xi-1}} \tag{11}$$

where stock of habits $s_{k,t-1}$ evolves according to

$$s_{k,t-1} = \rho_s s_{k,t-2} + (1 - \rho_s)\, l_{k,t-1} \tag{12}$$

Here, $\gamma^L \in (0,1)$ denotes the degree of habit formation in demand for loans and $\rho_s \in (0,1)$ measures the persistence of these habits. The parameter ξ denotes the elasticity of substitution between loans from different banks and is thus a measure of the market power of each individual bank.

Given his total need for financing $x_{j,t}$, each entrepreneur chooses $l_{jk,t}$ to solve the following problem

$$\min_{l_{jk,t}} \int_0^1 R^L_{k,t} l_{jk,t} \mathrm{d}k \tag{13}$$

subject to collateral constraint (8) and his effective borrowing (11). The first order condition associated with this problem gives entrepreneur j's optimal demand for loans from bank k

$$l_{jk,t} = \left(\frac{R^L_{k,t}}{R^L_t} \right)^{-\xi} x_t + \gamma^L s_{k,t-1} \tag{14}$$

where $R^L_t \equiv \left[\int_0^1 (R^L_{k,t})^{1-\xi} \mathrm{d}k \right]^{\frac{1}{1-\xi}}$ is the aggregate lending rate. Production function of each entrepreneur is

$$Y_{j,t} = A_t \left(N_{j,t} \right)^{1-\alpha} \left\{ \left(H^E_{j,t-1} \right)^{\phi} \left(K_{j,t-1} \right)^{1-\phi} \right\}^{\alpha} \tag{15}$$

where $Y_{j,t}$ is output, $N_{i,t}$ is labor input and $\alpha, \phi \in (0,1)$ are factor shares. TFP A_t follows the process

$$\log A_t = (1-\rho_A)\log A + \rho_A \log A_{t-1} + \sigma_A \epsilon_{A,t} \tag{16}$$

with iid innovation $\epsilon_{A,t}$ following a normal process with standard deviation σ_A where $A > 0$ and $\rho_A \in (0,1)$. The evolution of capital obeys the following law of motion

$$K_{j,t} = (1-\delta)\, K_{j,t-1} + \left[1 - \frac{\Omega}{2}\left(\frac{I_{j,t}}{I_{j,t-1}} - 1\right)^2\right] I_{j,t} \tag{17}$$

where $I_{j,t}$ is firm j's investment level, $\delta \in (0,1)$ the rate of depreciation of capital stock and $\Omega > 0$ is a cost adjustment parameter. The entrepreneur faces the following budget constraint

$$C^E_{j,t} + \int_0^1 R^L_{k,t-1} l_{jk,t-1} \mathrm{d}k \leq Y_{j,t} - W_t N_{j,t} - I_{j,t} - Q^H_t \left(H^E_{j,t} - H^E_{j,t-1}\right) + x_{j,t} \tag{18}$$

After imposing symmetric equilibrium, the FOCs of the entrepreneurs are

$$\lambda^E_t = \frac{1}{C^E_t - \gamma^E C^E_{t-1}} - \beta^E \mathbb{E}_t \frac{\gamma^E}{C^E_{t+1} - \gamma^E C^E_t} \tag{19}$$

$$\lambda^E_t = \beta^E \mathbb{E}_t \lambda^E_{t+1} R^L_t + \mu^E_t R^L_t \tag{20}$$

$$W_t = (1-\alpha)\frac{Y_t}{N_t} \tag{21}$$

$$\lambda^E_t Q^H_t = \beta^E \mathbb{E}_t \left[\lambda^E_{t+1}\left(Q^H_{t+1} + \alpha\phi \frac{Y_{t+1}}{H^E_t}\right)\right] + \mu^E_t \theta_t \mathbb{E}_t Q^H_{t+1} \tag{22}$$

$$\kappa^E_t = \alpha(1-\phi)\,\beta^E \mathbb{E}_t \left(\frac{\lambda^E_{t+1} Y_{t+1}}{K_t}\right) + \beta^E (1-\delta)\, \mathbb{E}_t \kappa^E_{t+1} + \mu^E_t \theta_t \mathbb{E}_t Q^K_{t+1} \tag{23}$$

$$\lambda^E_t = \kappa^E_t \left[1 - \frac{\Omega}{2}\left(\frac{I_t}{I_{t-1}} - 1\right)^2 - \Omega \frac{I_t}{I_{t-1}}\left(\frac{I_t}{I_{t-1}} - 1\right)\right] + \beta^E \Omega \mathbb{E}_t \left[\kappa^E_{t+1}\left(\frac{I_{t+1}}{I_t}\right)^2\left(\frac{I_{t+1}}{I_t} - 1\right)\right] \tag{24}$$

where μ^E_t, κ_t and λ^E_t are Lagrange multipliers associated with entrepreneur's collateral constraint (8), law of motion of capital (17) and entrepreneur's budget constraint (18). Entrepreneur's first order conditions with respect to consumption (19) and loans (20) may be combined to derive his Euler equation for consumption. Equation (21) describes entrepreneur's optimal demand for labor. Entrepreneur's Euler equation for land is described by (22) which relates its price today to its expected resale value tomorrow plus the payoff obatained by holding it for a period as given by its marginal productivity and its ability to serve as a collateral. Likewise, (23) is entrepreneur's Euler equation for capital and it links price of capital today to its price tomorrow and the expected payoff from keeping it for a period as given by its marginal productivity and its ability to serve as a collateral. Finally, entrepreneur's Euler equation for the investment is given by (24). All the derivations of first order conditions have been relegated to Appendix A.2.

2.3 Banking Sector

Banks in this model accept deposits from households and make loans to entrepreneurs. Banks take the interest rate on deposits R_t^D as given. Each individual bank k chooses its lending rate $R_{k,t}^L$ and its total amount of lending $L_{k,t}$.

$$\Pi_{k,t} = R_{k,t-1}^L L_{k,t-1} + \int_0^1 D_{ik,t} \mathrm{d}i - L_{k,t} - R_{t-1}^D \int_0^1 D_{ik,t-1} \mathrm{d}i \tag{25}$$

The balance sheet of bank k is

$$L_{k,t} = \int_0^1 D_{ik,t} \mathrm{d}i \tag{26}$$

where $L_{k,t}$ denotes total loans made by bank k to all entrepreneurs, that is, $L_{k,t} \equiv \int_0^1 l_{jk,t} \mathrm{d}j$. Each bank takes the demand for its loans as given

$$L_{k,t} = \int_0^1 l_{jk,t} \mathrm{d}j = \int_0^1 \left[\left(\frac{R_{k,t}^L}{R_t^L} \right)^{-\xi} x_t + \gamma^L s_{k,t-1} \right] \mathrm{d}j \tag{27}$$

Each bank chooses $L_{k,t}$ and $R_{k,t}^L$ to maximize its profits subject to (26) and (27). Considering a symmetric equilibrium in which all banks optimally choose the same lending rate, the FOCs for banks' optimization problem are:

$$\varrho_t^E = \mathbb{E}_t q_{t,t+1} \left[\left(R_t^L - R_t^D \right) + \gamma^L \left(1 - \rho_s \right) \mathbb{E}_t \varrho_{t+1}^E \right] \tag{28}$$

and

$$\xi \varrho_t^E x_t \frac{1}{R_t^L} = \mathbb{E}_t q_{t,t+1} L_{k,t} \tag{29}$$

where ϱ_t^E is the Lagrange multiplier on demand for bank's loans (27) and can be interpreted as shadow value to the bank of lending an extra dollar. Banks are owned by households and consequently their stochastic discount factor is given by $q_{t,t+1} = \beta^P \mathbb{E}_t \frac{\lambda_{t+1}^P}{\lambda_t^P}$. The optimality condition (28) states that shadow value of lending an extra dollar is given by repayment minus cost of borrowing that extra dollar from the households. The second term on the right-hand side reflects the fact that if a given bank lends an extra dollar in this period, the borrower of that dollar will develop will develop a habit for loans from that bank and as a result, will borrow more from it also in the susbesequent period. The size of this effect depends on degree γ^L and duration ρ_s of deep habits. In absence of deep habits, the latter term disappears. Equation (29) equates the profit gain from a marginal increase in bank's lending rate to the marginal cost. Bank's marginal cost is on the left-hand side and indicates a loss in its market share as it increases its lending rate. The marginal benefit of a higher lending rate appears on the right-hand side and shows the discounted gain made by repayment of loans made at higher lending rates. All the derivations are contained in Appendix A.3.

2.4 Aggregation and Market Clearing

Aggregate resource constraint of the economy is

$$C_t^P + C_t^E + I_t = Y_t \tag{30}$$

The clearing condition for the housing market is

$$H_t^P + H_t^E = H \tag{31}$$

where H is the total fixed supply of housing.

3 Model Solution and Prameterization

A period in the model refers to a quarter. Appendices B, C and D contain the list of equilibrium equations, the list of steady-state conditions and the system of log-linear equations, respectively. The calibration of parameters is rather standard and is summarized in Table 1. I allow for a relatively significant difference between discount factors of households and entrepreneurs so that steady-state value of μ_t^E is different from zero. The degree of habit formation in consumption is chosen to be 0.6 which is in line with empirical estimates (Smets and Wouters, 2007). The Frisch elasticity of labor supply η is chosen to be 1.01 and the value of weight on housing ς is set to 0.1 (Iacoviello, 2005).

The labor income share is 0.3 which implies a steady-state capital-output ratio of 1.15, in line with US data (Liu, Wang, and Zha, 2013). The input share of land in production is close to the value estimated in Liu, Wang, and Zha (2013) and Iacoviello (2005). The investment adjustment cost parameter is given a value of 1.85 (Ravn, 2016). The literaure contains estimates which range from 0 (Liu, Wang, and Zha, 2013) to above 26 (Christiano, Motto, and Rostagno, 2010). The capital depreciation rate implies a steady-state ratio of non-residential investment to output slightly above 0.13 as in Beaudry and Lahiri (2014).

For parameters in the banking sector, I rely on Aliaga-Díaz and Olivero (2010). I set the deep habit parameter in lending γ^L to 0.72 (Aliaga-Díaz and Olivero, 2010; Ravn, 2016). I set the autocorrelation parameter in stock of habits in lending ρ_s to 0.85 used by both Ravn, Schmitt-Grohé, and Uribe (2006) and Aliaga-Díaz and Olivero (2010). For elasticity of substitution between different loan varieties ξ, I pick the value of 190 used in Aliaga-Díaz and Olivero (2010) while Melina and Villa (2018) use a value of 427.

Table 1: Parameter values

	Value	Description	Source/Target
β^P	0.995	Discount factor, households	Iacoviello (2005)
β^E	0.95	Discount factor, entrepreneurs	Iacoviello (2005)
γ^i, $i = \{P, E\}$	0.6	Habits in consumption, households, entrepreneurs	Smets and Wouters (2007)
η	1.01	Frisch elasticity of labor	Iacoviello (2005)
ς	0.1	Weight on utility from housing	Iacoviello (2005)
α	0.3	Non-labor share of production	See Text
ϕ	0.1	Land share of non-labor input	See Text
Ω	1.85	Investment adjustment cost parameter	Ravn (2016)
δ	0.0285	Capital depreciation rate	See Text
γ^L	0.72	Deep habit formation	Aliaga-Díaz and Olivero (2010)
ρ_s	0.85	Persistence of stock of deep habits	Aliaga-Díaz and Olivero (2010)
ξ	190	Elasticity of substitution between banks	Aliaga-Díaz and Olivero (2010)
ρ_A	0.95	Persistence of technology shock	Smets and Wouters (2007)
θ	1	Steady-state value of collateral shock	Normalization
ρ_θ	0.90	Persistence of collateral shock	See Text
σ_A	0.0014	Standard deviation of technology shock	Standard
σ_θ	0.011	Standard deviation of collateral shock	See Text

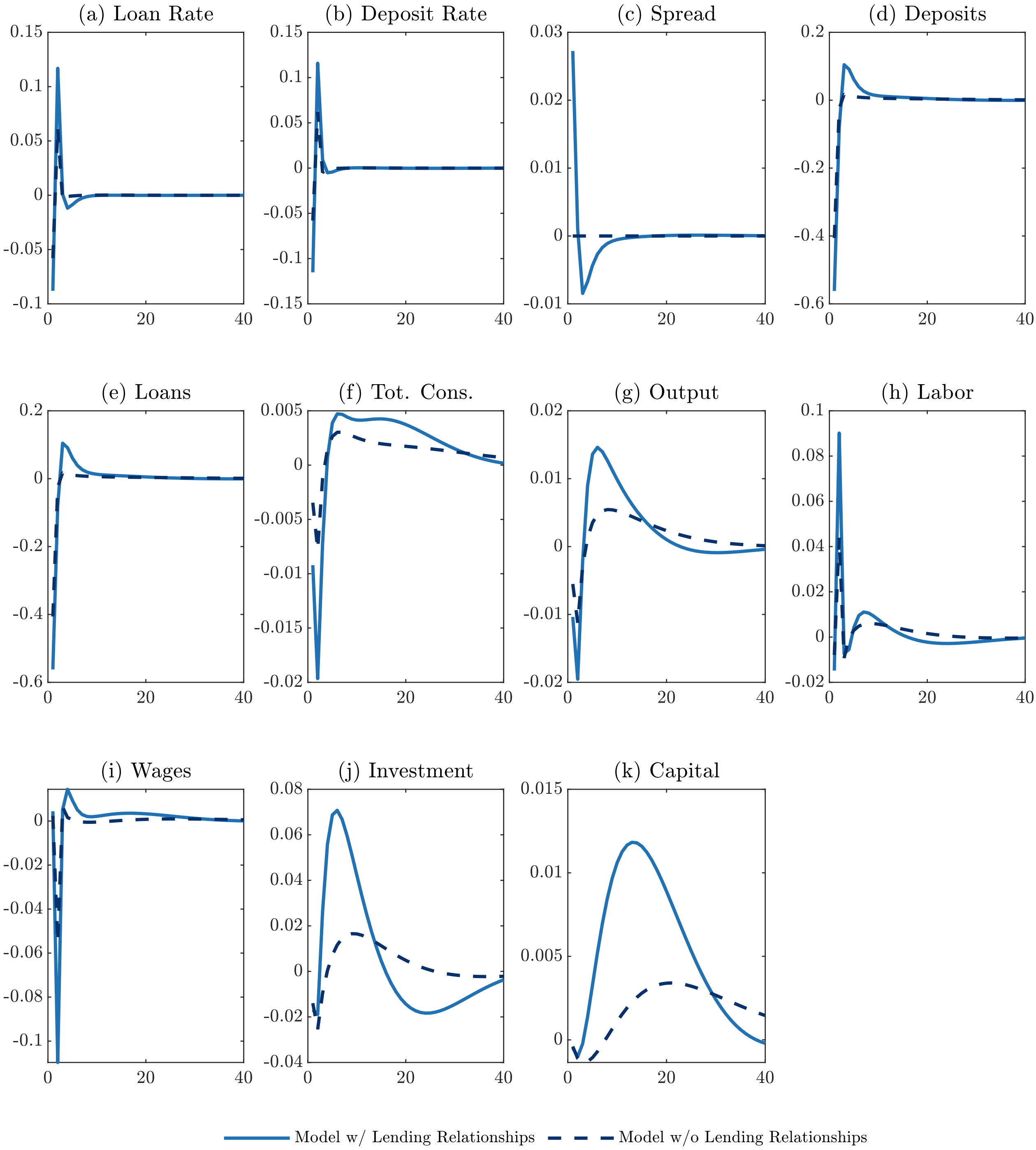
Following Smets and Wouters (2007), I set the persistence of TFP shock to 0.95 and its standard deviation to 0.0014 which is standard in the literature. I normalize the steady-state value of collateral shock θ to 1. For autocorrelation parameter of collateral shock ρ_θ, I set a value of 0.90 and for standard deviation of collateral shock σ_θ, I set a value of 0.011. I call these values baseline. Later, I run simulations with 50% higher volatility, that is, 0.0156 and a higher persistence of 0.95. These experiments show how the effects of collateral shocks change at higher volatilites and increased persistence. I also run another simulation with alternative calibration of collateral shocks in which I follow Becard and Gauthier (2022) and set 0.0215 for volatility of collateral shock σ_θ and 0.9729 for autocorrelation parameter ρ_θ.

4 Discussion

I discuss the effects of a collateral shock in this section. Figure 2 shows the effects of a collateral shock on key macroeconomic variables. After a collateral shock, when credit relationships are present, the spread between lending and deposit rates rises and loans fall by close to 0.6% percent. Investment falls on impact before overshooting its previous level which leads to a boom in capital stock. Consumption and output fall before they rise and exceed their previous level. Because of uptick in investment after an initial fall, labor falls on impact before quickly rising and then returning to its previous level. This leads to a short-lived fall in wages before it returns to its previous level. These effects are significantly higher in presence of credit relationships versus the case when lending relationships are absent. This suggests that collateral shocks amplify macroeconomic fluctuations and these effects are much greater when bank-firm lending relationships are considered.

The mechanism driving these results is as follows. In the aftermath of a collateral shock, spread between lending and deposit rates rises which does not move at all in absence of credit relationships. This surge in spread leads to a much greater fall in loans obtained by entrepreneurs which leads to a drop in investment and output at impact. After an intial rise in spread, it falls quickly below its previous steady-state level as banks scramble to "lock-in" more customers which then leads to an increase in lending, higher investment and larger output. These results highlight how presence of lending relationships amplify these macroeconomic fluctuations which are relatively muted in their absence. Note that these effects are present also in the case of no credit relationships between borrowers and lenders. In that case, after a collateral shock, spread does not move but bank credit still falls since collateral shocks reduce the amount of loans borrowers can obtain from the bank. This then leads to a fall in investment, output and consumption. These effects are, however, comparatively smaller than the case when there are lending relationships between banks and firms. Presence of credit relationships, in this case,

Figure 2: Impact of a Collateral Shock

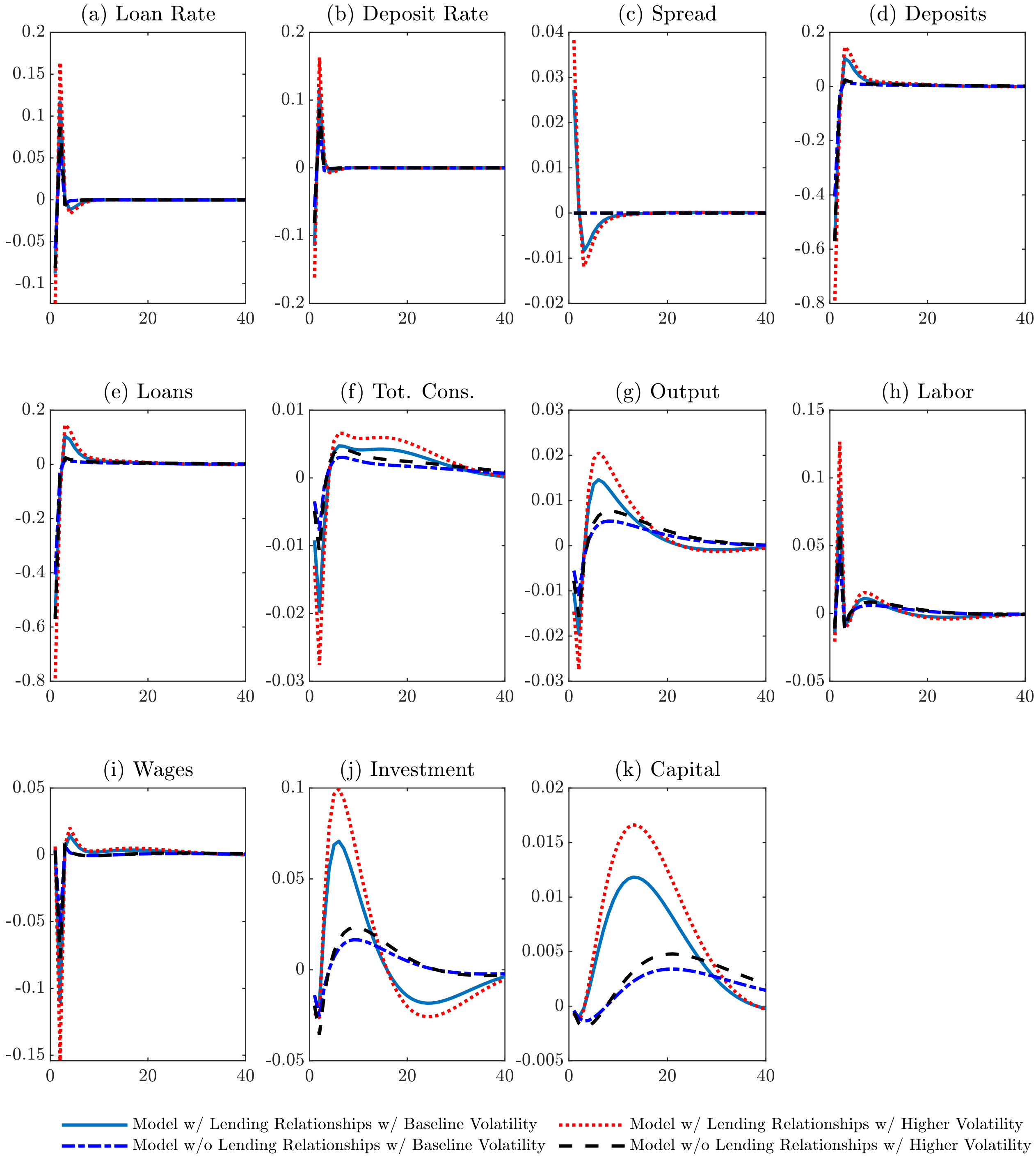


Note: Numbers on the horizontal axis are quarters since the shock. Numbers on the vertical axis show percentage deviation from steady state.

works as a "financial accelerator" (Bernanke, Gertler, and Gilchrist, 1999) which amplifes the movements in spread, bank credit and rest of the macroeconomy.

In order to see how how larger volatility of collateral shock effects the economy, I run an experiment in which I raise the baseline volatility by 50%. Figure 3 displays the results of a collateral shock at higher volatility. I plot the impulse responses for both cases – with and without lending relationships. It's clear that higher volatility of collateral shock increases its

Figure 3: Impact of a Collateral Shock at Different Levels of Volatilities

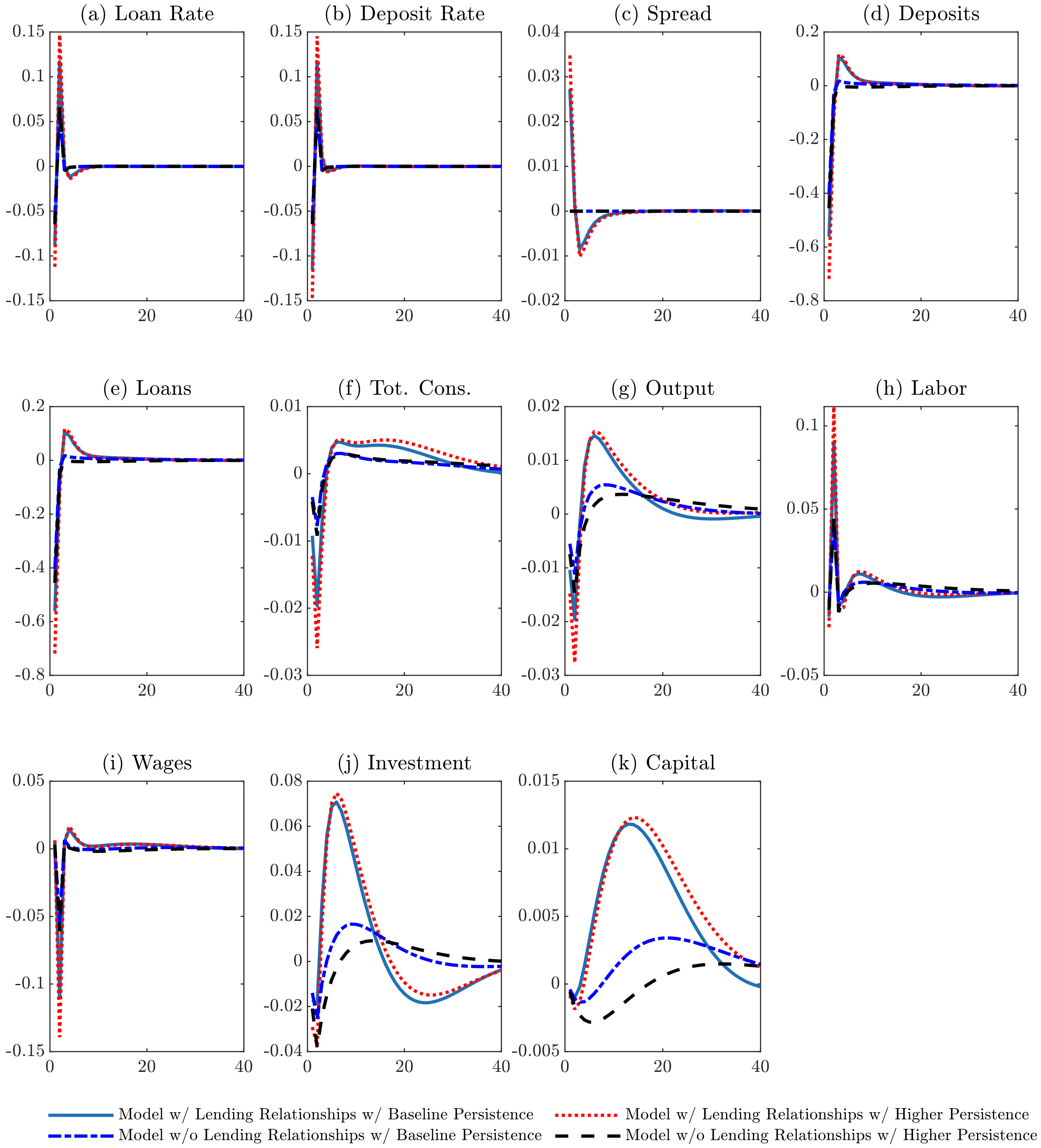


Note: Numbers on the horizontal axis are quarters since the shock. Numbers on the vertical axis show percentage deviation from steady state.

macroeconomic effects with larger effects on impact and more amplification. These effects, consistent with the observation in Figure 2, are much larger in presence of bank-firm lending relationships compared to the case when these credit relationships do not exist.

I follow this exercise with another experiment in which I increase the persistence of collateral shock from 0.90 to 0.95. Figure 4 shows the impulse responses of a collateral shock at higher persistence. Like the previous case, I plot the impulse responses for both cases – when lending

Figure 4: Impact of a Collateral Shock at Different Levels of Persistence

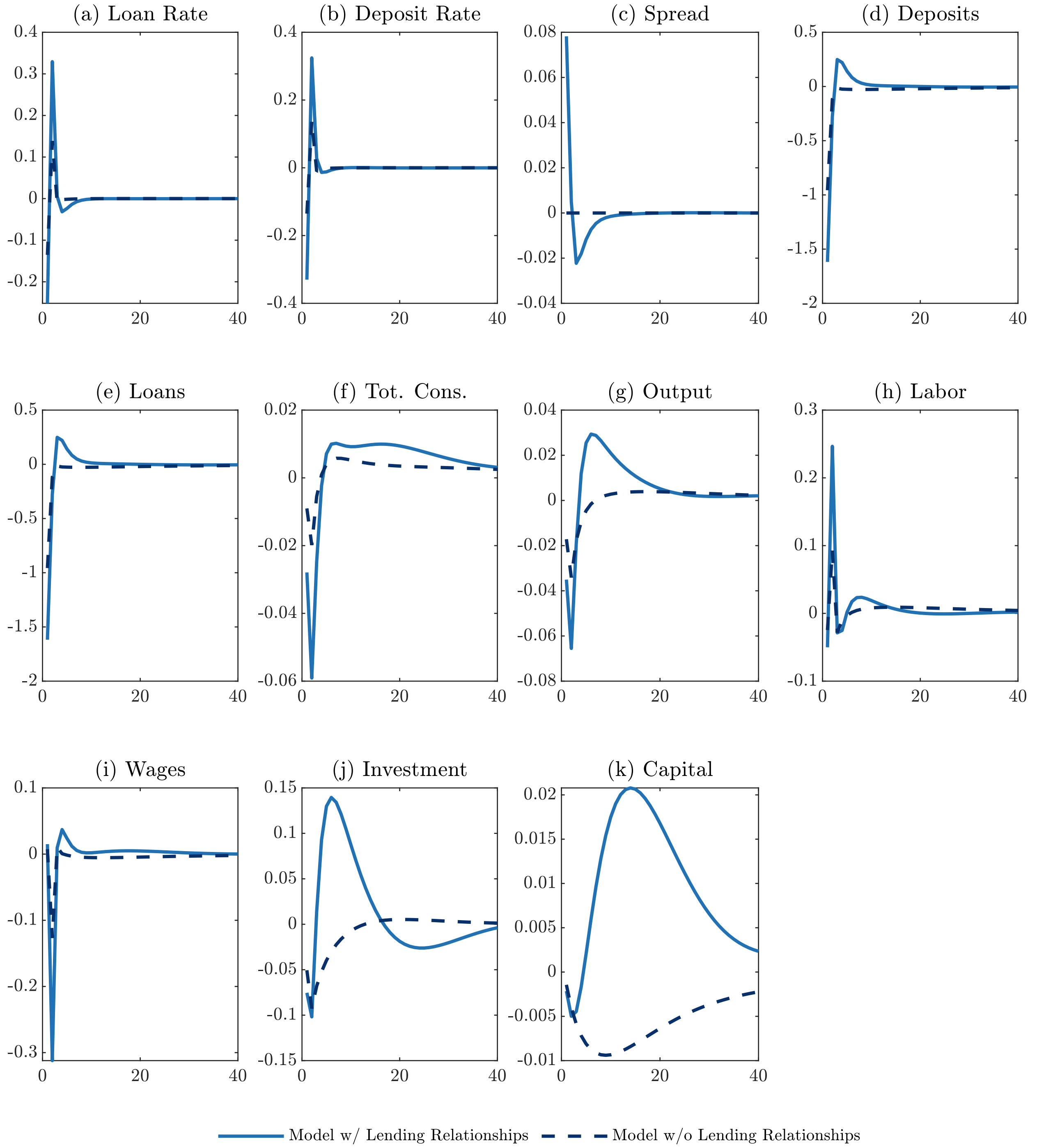


Note: Numbers on the horizontal axis are quarters since the shock. Numbers on the vertical axis show percentage deviation from steady state.

relationships are present and when they are not. At increased persistence, collateral shocks have greater impact on macroeconomic variables and these effects are more noticeable in presence of lending relationships.

I then conduct a final experiment. In this exercise, I calibrate the volatility and persistence of volatility shock following Becard and Gauthier (2022). It's notable that Becard and Gauthier (2022) estimate a much higher numerical values for these parameters and in this sense, the

Figure 5: Impact of a Collateral Shock: Alternative Calibration (Becard and Gauthier, 2022)



Note: Numbers on the horizontal axis are quarters since the shock. Numbers on the vertical axis show percentage deviation from steady state.

responses of macroeconomic variables in Figures 2, 3 and 4 can be seen as lower bound of effects of a collateral shock. Figure 5 plots the impulse responses of key macroeconomic variables to a collateral shock using the calibration values of shock parameters in Becard and Gauthier (2022). It's apparent that a collateral shock has much larger impact in this case. Loans fall by more than 1.5% in case of lending relationships which is almost thrice the baseline case where it declines by close to 0.6%. This suggests that collateral shocks have significant impact on economic dynamics and these effects are much bigger in case when bank-firm lending relationships are factored into analysis.

5 Conclusion

This paper develops a parsimonious model in which firms borrow from banks and develop endogenously-persistent lending relationships with them. A collateral shock in this model, defined as changing lending conditions, leads to significant rise in spread, fall in bank credit and heightened macroeconomic volatility. These fluctuations are amplifed by presence of lending relationships. The key factor driving these effects is movement in spread between bank lending and deposit rates which rises significantly after a collateral shock before it falls below its previous stady-state level. This amplifies movements in macroeconomic variables. Further, the effects of collateral shocks are increasing in their volatility and persistence. The central result from this paper is that collateral shocks can cause large macroeconomic fluctuations which are amplified by presence of borrower-lender relationships. A model that fails to take into account the presence of these credit relationships may end up underesimating the macroeconomic effects and volatility in the aftermath of a collateral shock.

References

Airaudo, M. and M. P. Olivero (2019): "Optimal monetary policy with countercyclical credit spreads," *Journal of Money, Credit and Banking*, 51, 787–829.

Aliaga-Díaz, R. and M. P. Olivero (2010): "Macroeconomic implications of "deep habits" in banking," *Journal of Money, Credit and Banking*, 42, 1495–1521, https://doi.org/10.1111/j.1538-4616.2010.00351.x.

Beaudry, P. and A. Lahiri (2014): "The Allocation of Aggregate Risk, Secondary Market Trades, and Financial Boom–Bust Cycles," *Journal of Money, Credit and Banking*, 46, 1–42, https://doi.org/10.1111/jmcb.12096.

Becard, Y. and D. Gauthier (2022): "Collateral shocks," *American Economic Journal: Macroeconomics*, 14, 83–103, https://www.aeaweb.org/articles?id=10.1257/mac.20190223.

Bernanke, B. S., M. Gertler, and S. Gilchrist (1999): "The financial accelerator in a quantitative business cycle framework," *Handbook of macroeconomics*, 1, 1341–1393, https://doi.org/10.1016/S1574-0048(99)10034-X.

Christiano, L. J., R. Motto, and M. Rostagno (2010): "Financial factors in economic fluctuations," https://ssrn.com/abstract=1600166.

Iacoviello, M. (2005): "House prices, borrowing constraints, and monetary policy in the business cycle," *American economic review*, 95, 739–764, https://www.aeaweb.org/articles?id=10.1257/0002828054201477.

Jakab, Z. and M. Kumhof (2015): "Banks are not intermediaries of loanable funds–and why this matters," *Bank of England working paper*.

Justiniano, A., G. E. Primiceri, and A. Tambalotti (2015): "Household leveraging and deleveraging," *Review of Economic Dynamics*, 18, 3–20, https://doi.org/10.1016/j.red.2014.10.003.

Kiyotaki, N. and J. Moore (1997): "Credit cycles," *Journal of political economy*, 105, 211–248, https://doi.org/10.1086/262072.

Kosekova, K., A. Maddaloni, M. Papoutsi, and F. Schivardi (2023): "Firm-bank relationships: a cross-country comparison," *ECB Working Paper*.

Liu, Z., P. Wang, and T. Zha (2013): "Land-price dynamics and macroeconomic fluctuations," *Econometrica*, 81, 1147–1184, https://doi.org/10.3982/ECTA8994.

Melina, G. and S. Villa (2018): "Leaning against windy bank lending," *Economic Inquiry*, 56, 460–482, https://doi.org/10.1111/ecin.12491.

Ongena, S. and D. C. Smith (2000): "What determines the number of bank relationships? Cross-country evidence," *Journal of Financial intermediation*, 9, 26–56, https://doi.org/10.1006/jfin.1999.0273.

Ravn, M., S. Schmitt-Grohé, and M. Uribe (2006): "Deep habits," *The Review of Economic Studies*, 73, 195–218, https://doi.org/10.1111/j.1467-937X.2006.00374.x.

Ravn, S. H. (2016): "Endogenous credit standards and aggregate fluctuations," *Journal of Economic Dynamics and Control*, 69, 89–111, https://doi.org/10.1016/j.jedc.2016.05.011.

Shapiro, A. F. and M. P. Olivero (2020): "Lending relationships and labor market dynamics," *European Economic Review*, 127, 103475.

Sharma, V. (2023a): "Lending relationships and loan-to-value shocks," *Unpublished Manuscript*, https://sharmavivek.com/assets/lrltv.pdf.

——— (2023b): "State-dependent effects of loan-to-value shocks," *Unpublished Manuscript*, https://sharmavivek.com/assets/sdltv.pdf.

Smets, F. and R. Wouters (2007): "Shocks and frictions in US business cycles: A Bayesian DSGE approach," *American economic review*, 97, 586–606, https://www.aeaweb.org/articles?id=10.1257/aer.97.3.586.

Appendix (For Online Publication)

Collateral Shocks, Lending Relationships and Economic Dynamics

Vivek Sharma[3]

University of Melbourne, CAMA, CASMEF

September 20, 2023

Contents

[3] sharma.v2@unimelb.edu.au, https://sharmavivek.com/.

A Derivation of FOCs

A.1 Households

The Lagrangian of patient households is

$$\mathscr{L}_t = \mathbb{E}_t \left\{ \sum_{t=0}^{\infty} \left(\beta^P\right)^t \left[\begin{array}{c} \log\left(C_{i,t}^P - \gamma^P C_{i,t-1}^P\right) - \frac{N_{i,t}^{\eta}}{\eta} + \varsigma \log H_{i,t}^P \\ -\lambda_{i,t}^P \left[\begin{array}{c} C_{i,t}^P + Q_t^H \left(H_{i,t}^P - H_{i,t-1}^P\right) + \int_0^1 D_{ik,t} \mathrm{d}k \\ -W_t N_{i,t} - \int_0^1 \Pi_{ik,t} \mathrm{d}k - R_{t-1}^D \int_0^1 D_{ik,t-1} \mathrm{d}k \end{array} \right] \end{array} \right] \right\} \tag{A.1}$$

The problem yields the following first order conditions (here, I ignore all the i's denoting individual patient households):

$$\frac{\partial \mathscr{L}}{\partial C_t^P} : \frac{1}{C_t^P - \gamma^P C_{t-1}^P} - \beta^P \mathbb{E}_P \frac{\gamma^P}{C_{t+1}^P - \gamma^P C_t^P} = \lambda_t^P \tag{A.2}$$

$$\frac{\partial \mathscr{L}}{\partial D_t} : \beta^P \mathbb{E}_t \lambda_{t+1}^P = \frac{\lambda_t^P}{R_t^D} \tag{A.3}$$

$$\frac{\partial \mathscr{L}}{\partial H_t^P} : \frac{\varsigma}{H_t^P} + \beta^P \mathbb{E}_t \left(\lambda_{t+1}^P Q_{t+1}^H\right) = \lambda_t^P Q_t^H \tag{A.4}$$

$$\frac{\partial \mathscr{L}}{\partial N_t^P} : N_t^{\eta-1} = \lambda_t^P W_t \tag{A.5}$$

A.2 Entrepreneurs

Entrepreneur's optimization problem features two parts. The first part consists of choosing how much to borrow from each individual bank, $l_{jk,t}$ to minimize his total interest rate expenditure. This problem can be framed as

$$\min_{l_{jk,t}} \left[\int_0^1 R_{k,t}^L l_{jk,t} \mathrm{d}k\right] - \chi_t^E \left[x_{j,t} - \left(\int_0^1 \left(l_{jk,t} - \gamma^L s_{k,t-1}\right)^{\frac{\xi-1}{\xi}} \mathrm{d}k \right)^{\frac{\xi}{\xi-1}} \right] \tag{A.6}$$

The first order condition for this problem is

$$R_{k,t}^L = -\frac{\xi}{\xi-1} \chi_t^E \left(\int_0^1 \left(l_{jk,t} - \gamma^L s_{k,t-1}\right)^{\frac{\xi-1}{\xi}} \mathrm{d}k \right)^{\frac{1}{\xi-1}} \frac{\xi-1}{\xi} \left(l_{jk,t} - \gamma^L s_{k,t-1}\right)^{-\frac{1}{\xi}} \tag{A.7}$$

This can be rewritten as

$$
\begin{aligned}
R^L_{k,t} &= -\chi^E_t \left(\int_0^1 \left(l_{jk,t} - \gamma^L s_{k,t-1}\right)^{\frac{\xi-1}{\xi}} \mathrm{d}k \right)^{\frac{1}{\xi-1}} \left(l_{jk,t} - \gamma^L s_{k,t-1}\right)^{-\frac{1}{\xi}} \\
R^L_{k,t} \left(l_{jk,t} - \gamma^L s_{k,t-1}\right) &= -\chi^E_t \left(\int_0^1 \left(l_{jk,t} - \gamma^L s_{k,t-1}\right)^{\frac{\xi-1}{\xi}} \mathrm{d}k \right)^{\frac{1}{\xi-1}} \left(l_{jk,t} - \gamma^L s_{k,t-1}\right)^{\frac{\xi-1}{\xi}} \\
\int_0^1 R^L_{k,t} \left(l_{jk,t} - \gamma^L s_{k,t-1}\right) \mathrm{d}k &= -\chi^E_t \left(\int_0^1 \left(l_{jk,t} - \gamma^L s_{k,t-1}\right)^{\frac{\xi-1}{\xi}} \mathrm{d}k \right)^{\frac{1}{\xi-1}} \int_0^1 \left(l_{jk,t} - \gamma^L s_{k,t-1}\right)^{\frac{\xi-1}{\xi}} \mathrm{d}k \\
\int_0^1 R^L_{k,t} \left(l_{jk,t} - \gamma^L s_{k,t-1}\right) \mathrm{d}k &= -\chi^E_t \left(\int_0^1 \left(l_{jk,t} - \gamma^L s_{k,t-1}\right)^{\frac{\xi-1}{\xi}} \mathrm{d}k \right)^{\frac{\xi}{\xi-1}} \qquad \text{(A.8)}
\end{aligned}
$$

Now, using $\left(\int_0^1 \left(l_{jk,t} - \gamma^L s_{k,t-1}\right)^{\frac{\xi-1}{\xi}} \mathrm{d}k \right)^{\frac{\xi}{\xi-1}} = x_{j,t}$, the previous equation can be written as

$$
x_{j,t} = -\frac{1}{\chi^E_t} \left[\int_0^1 R^L_{k,t} \left(l_{jk,t} - \gamma^L s_{k,t-1}\right) \mathrm{d}k \right] \qquad \ddagger
$$

Define the aggregate lending rate as $R^L_t \equiv \left[\int_0^1 \left(R^L_{k,t}\right)^{1-\xi} \right]^{\frac{1}{1-\xi}}$ and note that at the optimum, the following condition must hold

$$
R^L_t x_{j,t} = \int_0^1 R^L_{k,t} \left(l^E_{jk,t} - \gamma^L s^E_{k,t-1}\right) \mathrm{d}k
$$

Now, ‡ can be rewritten as

$$
\begin{aligned}
x_{j,t} &= -\frac{1}{\chi^E_t} \left[R^L_t x_{j,t} \right] \\
-\chi^E_t &= R^L_t
\end{aligned}
$$

Inserting this in first order condition (A.8)

$$
\begin{aligned}
R_{k,t}^{L} &= -\frac{\xi}{\xi-1}\chi_t^E\left(\int_0^1\left(l_{jk,t}-\gamma^L s_{k,t-1}\right)^{\frac{\xi-1}{\xi}}\mathrm{d}k\right)^{\frac{1}{\xi-1}}\frac{\xi-1}{\xi}\left(l_{j,t}-\gamma^L s_{k,t-1}\right)^{-\frac{1}{\xi}}\\
R_{k,t}^{L} &= R_t^L\left(\int_0^1\left(l_{jk,t}-\gamma^L s_{k,t-1}\right)\mathrm{d}k\right)^{\frac{1}{\xi-1}}\left(l_{jk,t}-\gamma^L s_{k,t-1}\right)^{-\frac{1}{\xi}}\\
R_{k,t}^{L} &= R_t^L\left(x_t\right)^{\frac{1}{\xi}}\left(l_{jk,t}-\gamma^L s_{k,t-1}\right)^{-\frac{1}{\xi}}\\
\left(l_{jk,t}-\gamma^L s_{k,t-1}\right)^{\frac{1}{\xi}} &= \left(x_t\right)^{\frac{1}{\xi}}\frac{R_t^L}{R_{k,t}^L}\\
l_{jk,t} &= \left(\frac{R_t^L}{R_{k,t}^L}\right)^{\xi}x_t+\gamma^L s_{k,t-1}\\
l_{jk,t} &= \left(\frac{R_{k,t}^L}{R_t^L}\right)^{-\xi}x_t+\gamma^L s_{k,t-1}
\end{aligned}
$$

The second part of entrepreneur's optimization problem can be written as

$$
\mathscr{L}_t=\mathbb{E}_t\left\{\sum_{t=0}^{\infty}\left(\beta^E\right)^t\left[\begin{array}{c}\log\left(C_{j,t}^E-\gamma^E C_{j,t-1}^E\right)\\ -\lambda_{j,t}^E\left[\begin{array}{c}C_{j,t}^E+R_{k,t-1}^L\int_0^1 l_{jk,t-1}\mathrm{d}k-Y_{j,t}+W_tN_{j,t}+I_{j,t}\\ +Q_t^H\left(H_{j,t}^E-H_{j,t-1}^E\right)-x_{j,t}\end{array}\right]\\ -\mu_{j,t}^E\left[R_{k,t}^L\int_0^1 l_{jk,t}\mathrm{d}k-\int_0^1\theta_t\mathrm{d}k\mathbb{E}_t\left(Q_{t+1}^H H_{j,t}^E+Q_{t+1}^K K_{j,t}\right)\right]\\ -\kappa_{j,t}^E\left[K_{j,t}-\left(1-\delta\right)K_{j,t-1}-\left\{1-\frac{\Omega}{2}\left(\frac{I_{j,t}}{I_{j,t-1}}-1\right)^2\right\}I_{j,t}\right]\\ -\epsilon_{j,t}^E\left[x_{j,t}-\left\{\int_0^1\left(l_{jk,t}-\gamma^L s_{k,t-1}\right)^{\frac{\xi-1}{\xi}}\mathrm{d}k\right\}^{\frac{\xi}{\xi-1}}\right]\end{array}\right]\right\} \tag{A.9}
$$

where $Y_{j,t}=A_t\left(N_{j,t}\right)^{1-\alpha}\left\{\left(H_{j,t-1}^E\right)^{\phi}\left(K_{j,t-1}\right)^{1-\phi}\right\}^{\alpha}$ may be inserted for $Y_{j,t}$ in the budget constraint. Solving entrepreneur's optmization problem, the first order conditions are (I ignore all

j's here):

$$\frac{\partial \mathscr{L}}{\partial C_t^E} : \frac{1}{C_t^E - \gamma^E C_{t-1}^E} - \beta^E \mathbb{E}_t \frac{\gamma^E}{C_{t+1}^E - \gamma^E C_t^E} = \lambda_t^E \tag{A.10}$$

$$\frac{\partial \mathscr{L}}{\partial x_t} : \lambda_t^E = \epsilon_t^E \tag{A.11}$$

$$\frac{\partial \mathscr{L}}{\partial l_t} : \epsilon_t^E = \beta^E \mathbb{E}_t \lambda_{t+1}^E R_t^L + \mu_t^E R_t^L \tag{A.12}$$

$$\frac{\partial \mathscr{L}}{\partial N_t} : W_t = (1-\alpha) \frac{Y_t}{N_t} \tag{A.13}$$

$$\frac{\partial \mathscr{L}}{\partial H_t^E} : \lambda_t^E Q_t^H = \beta^E \mathbb{E}_t \left\{ \lambda_{t+1}^E \left(Q_{t+1}^H + \alpha\phi \frac{Y_{t+1}}{H_t^E} \right) \right\} + \mu_t^E \theta_t \mathbb{E}_t Q_{t+1}^H \tag{A.14}$$

$$\frac{\partial \mathscr{L}}{\partial K_t} : \kappa_t^E = \alpha (1-\phi) \beta^E \mathbb{E}_t \left(\frac{\lambda_{t+1}^E Y_{t+1}}{K_t} \right) + \beta^E (1-\delta) \mathbb{E}_t \kappa_{t+1}^E + \mu_t^E \theta_t \mathbb{E}_t Q_{t+1}^K \tag{A.15}$$

$$\frac{\partial \mathscr{L}}{\partial I_t} : \lambda_t^E = \kappa_t^E \left\{ 1 - \frac{\Omega}{2} \left(\frac{I_t}{I_{t-1}} - 1 \right)^2 - \Omega \frac{I_t}{I_{t-1}} \left(\frac{I_t}{I_{t-1}} - 1 \right) \right\} + \beta^E \Omega \mathbb{E}_t \left\{ \kappa_{t+1}^E \left(\frac{I_{t+1}}{I_t} \right)^2 \left(\frac{I_{t+1}}{I_t} - 1 \right) \right\} \tag{A.16}$$

Using $\lambda_t^E = \epsilon_t^E$ from (A.11), (A.12) becomes

$$\beta^E \mathbb{E}_t \left(\lambda_{t+1}^E R_t^L \right) + \mu_t^E R_t^L = \lambda_t^E \tag{A.17}$$

A.3 Banks

The problem of banks is to choose their lending rate and the total amount of lending. The bank considers deep habits in loan demand. The bank solves the following problem

$$\max_{L_{k,t}, R_{k,t}^L} \Pi_t = R_{k,t-1}^L L_{k,t-1} - R_{t-1}^D L_{k,t-1} + \varrho_t^E \left(\int_0^1 \left[\left(\frac{R_t^L}{R_{k,t}^L} \right)^{\xi} x_t + \gamma^L s_{k,t-1} \right] \mathrm{d}j - L_{k,t} \right)$$

The first order condition for $L_{k,t}$ is

$$\mathbb{E}_t q_{t,t+1} R_{k,t}^L - \mathbb{E}_t q_{t,t+1} R_t^D + \gamma^L (1-\rho_s) \mathbb{E}_t \Big(q_{t,t+1} \varrho_{t+1}^E - \varrho_t^E \Big) = 0$$

Rearranging terms

$$\varrho_t^E = \mathbb{E}_t q_{t,t+1} \Big[\left(R_{k,t}^L - R_t^D \right) + \gamma^L (1-\rho_s) \mathbb{E}_t \varrho_{t+1}^E \Big] \tag{A.18}$$

The first order condition for $R^L_{k,t}$ is

$$\mathbb{E}_t q_{t,t+1} L^E_{k,t} + \xi \varrho^E_t \left(\frac{R^L_t}{R^L_{k,t}}\right)^{\xi-1} x_t \left(\frac{-R^L_t}{\left(R^L_{k,t}\right)^2}\right) = 0$$

Moving terms around

$$\mathbb{E}_t q_{t,t+1} L^E_{k,t} = \xi \varrho^E_t x_t \left(\frac{R^L_t}{R^L_{k,t}}\right)^{\xi-1} \left(\frac{R^L_t}{\left(R^L_{k,t}\right)^2}\right) \tag{A.19}$$

In a symmetric equilibrium all banks have the same lending rate $R^L_{k,t} = R^L_t, \forall k$ and consequently lend the same amount $L_{k,t} = L_t, \forall k$. Bank's first order condition in this case can be rewritten as

$$\varrho^E_t = \mathbb{E}_t q_{t,t+1} \Big[\left(R^L_t - R^D_t\right) + \gamma^L \left(1 - \rho_s\right) \mathbb{E}_t \varrho^E_{t+1} \Big] \tag{A.20}$$

$$\frac{\xi \varrho^E_t x_t}{R^L_t} = \mathbb{E}_t q_{t,t+1} L_t \tag{A.21}$$

where I have imposed $L_t = l_t$ in a symmetric equilibrium.

B List of Equations

B.1 Households

$$\frac{1}{C^P_t - \gamma^P C^P_{t-1}} - \beta^P \mathbb{E}_t \frac{\gamma^P}{C^P_{t+1} - \gamma^P C^P_t} = \lambda^P_t \tag{B.1}$$

$$\beta^P \mathbb{E}_t \lambda^P_{t+1} = \frac{\lambda^P_t}{R^D_t} \tag{B.2}$$

$$\frac{\varsigma}{H^P_t} + \beta^P \mathbb{E}_t \left(\lambda^P_{t+1} Q^H_{t+1}\right) = \lambda^P_t Q^H_t \tag{B.3}$$

$$N^{\eta-1}_t = \lambda^P_t W_t \tag{B.4}$$

B.2 Entrepreneurs

$$\frac{1}{C^E_t - \gamma^E C^E_{t-1}} - \beta^E \mathbb{E}_t \frac{\gamma^E}{C^E_{t+1} - \gamma^E C^E_t} = \lambda^E_t \tag{B.5}$$

$$\beta^E \mathbb{E}_t \left(\lambda^E_{t+1} R^L_t\right) + \mu^E_t R^L_t = \lambda^E_t \tag{B.6}$$

$$W_t = (1-\alpha)\frac{Y_t}{N_t} \tag{B.7}$$

$$\lambda_t^E Q_t^H = \beta^E \mathbb{E}_t \left\{ \lambda_{t+1}^E \left(Q_{t+1}^H + \alpha\phi \frac{Y_{t+1}}{H_t^E} \right) \right\} + \mu_t^E \theta_t \mathbb{E}_t Q_{t+1}^H \tag{B.8}$$

$$\kappa_t^E = \alpha(1-\phi)\beta^E \mathbb{E}_t \left(\frac{\lambda_{t+1}^E Y_{t+1}}{K_t} \right) + \beta^E (1-\delta)\mathbb{E}_t \kappa_{t+1}^E + \mu_t^E \theta_t \mathbb{E}_t Q_{t+1}^K \tag{B.9}$$

$$\lambda_t^E = \kappa_t^E \left\{ 1 - \frac{\Omega}{2}\left(\frac{I_t}{I_{t-1}} - 1 \right)^2 - \Omega \frac{I_t}{I_{t-1}} \left(\frac{I_t}{I_{t-1}} - 1 \right) \right\} + \beta^E \Omega \mathbb{E}_t \left\{ \kappa_{t+1}^E \left(\frac{I_{t+1}}{I_t} \right)^2 \left(\frac{I_{t+1}}{I_t} - 1 \right) \right\} \tag{B.10}$$

$$s_t = \rho_s s_{t-1} + (1-\rho_s) l_t \tag{B.11}$$

$$x_t = \left(l_t - \gamma^L s_{t-1} \right) \tag{B.12}$$

$$L_t = l_t \tag{B.13}$$

$$C_t^E + R_{t-1}^L l_{t-1} = Y_t - W_t N_t - I_t - Q_t \left(H_t^E - H_{t-1}^E \right) + x_t \tag{B.14}$$

$$l_t = \frac{\theta_t a_t}{R_t^L} \tag{B.15}$$

$$a_t = \mathbb{E}_t \left(Q_{t+1}^H H_t^E + Q_{t+1}^K K_t \right) \tag{B.16}$$

$$\kappa_t^E = \lambda_t^E Q_t^K \tag{B.17}$$

B.3 Banks

$$\varrho_t^E = \mathbb{E}_t q_{t,t+1} \Big[\left(R_t^L - R_t^D \right) + \gamma^L (1-\rho_s) \mathbb{E}_t \varrho_{t+1}^E \Big] \tag{B.18}$$

$$\xi \varrho_t^E x_t \frac{1}{R_t^L} = \mathbb{E}_t q_{t,t+1} L_t \tag{B.19}$$

$$\Pi_t = R_{t-1}^L L_{t-1} + D_t - L_t - R_{t-1}^D D_{t-1} \tag{B.20}$$

$$L_t = D_t \tag{B.21}$$

$$q_{t,t+1} = \beta^P \mathbb{E}_t \frac{\lambda_{t+1}^P}{\lambda_t^P} \tag{B.22}$$

B.4 Market Clearing and Resource Constraints

$$C_t^P + C_t^E + I_t = Y_t \tag{B.23}$$

$$H_t^P + H_t^E = H \tag{B.24}$$

$$Y_t = A_t (N_t)^{1-\alpha} \left\{ \left(H^E_{t-1}\right)^{\phi} (K_{t-1})^{1-\phi} \right\}^{\alpha} \tag{B.25}$$

$$K_t = (1-\delta) K_{t-1} + \left\{ 1 - \frac{\Omega}{2} \left(\frac{I_t}{I_{t-1}} - 1 \right)^2 \right\} I_t \tag{B.26}$$

C Steady State Conditions

All $i's$, $j's$ and $k's$ denoting individual household, entrepreneur and bank respectively are ignored. From household's FOC with respect to consumption (B.1) and labor (B.4), I have

$$\frac{1-\beta^P\gamma^P}{\left(1-\gamma^P\right)C^P} = \lambda^P \tag{C.1}$$

and

$$N^{\eta-1} = \lambda^P W \tag{C.2}$$

respectively. Household's FOC with respect to deposit (B.2) yields the steady-state gross interest rate

$$R^D = \frac{1}{\beta^P} \tag{C.3}$$

underscoring that the time preference of most patient individual determines the steady-state rate of interest. From (B.3), I obtain

$$\begin{aligned} \frac{\varsigma}{H^P} + \beta^P\lambda^P Q^H &= \lambda^P Q^H \\ \Rightarrow Q^H H^P &= \frac{\varsigma}{\lambda^P\left(1-\beta^P\right)} \\ \Rightarrow H^P &= \frac{\varsigma}{Q^H\lambda^P\left(1-\beta^P\right)} \end{aligned} \tag{C.4}$$

I next turn to entrepreneurs. Their consumption FOC (B.5) yields

$$\frac{1-\beta^E\gamma^E}{\left(1-\gamma^E\right)C^E} = \lambda^E \tag{C.5}$$

Entrepreneur's FOC with respect to loans (B.6) gives

$$\begin{aligned} \beta^E\lambda^E R^L + \mu^E R^L &= \lambda^E \\ \Rightarrow \mu^E &= \frac{\lambda^E\left(1-\beta^E R^L\right)}{R^L} \end{aligned} \tag{C.6}$$

The borrowing constraint for entrepreneurs binds only if μ^E is positive. This implies that β^E must be less than R^L. In the baseline calibration, β^E is set to 0.95 whereas the steady state value of R^L is 1.0219 which implies that β^E must be less than 0.9786 which is indeed the case.

The production function is

$$Y = A\left(N\right)^{1-\alpha}\left[\left(H^E\right)^{\phi}\left(K\right)^{1-\phi}\right]^{\alpha} \tag{C.7}$$

From firm's labor choice for households (B.7),

$$W = \left(1-\alpha\right)\frac{Y}{N} \tag{C.8}$$

From entrepreneur's FOC with respect to housing (B.8), I have

$$\begin{aligned} \lambda^E Q^H &= \beta^E \lambda^E \left(Q^H + \alpha\phi\frac{Y}{H^E}\right) + \mu^E \theta Q^H \\ \Rightarrow \frac{Q^H H^E}{Y} &= \frac{\beta^E \alpha\phi R^L}{\left(1-\beta^E\right)R^L - \theta\left(1-\beta^E R^L\right)} \end{aligned} \tag{C.9}$$

From aggregate law of motion for capital (B.26)

$$\begin{aligned} K &= \left(1-\delta\right)K + \left[1 - \frac{\Omega}{2}\left(\frac{I}{I}-1\right)\right]I \\ &\Rightarrow I = \delta K \end{aligned} \tag{C.10}$$

I have the following steady-state resource constraints

$$Y = C^P + C^E + I \tag{C.11}$$

$$H = H^P + H^E \tag{C.12}$$

$$L = D \tag{C.13}$$

Also, I have the following steady-state version of agents' budget constraints (one of them is redundant because of Walras' Law)

$$C^P = WN - (R^D - 1)D + \Pi \tag{C.14}$$

$$C^E = Y - R^L l - WN - I - x \tag{C.15}$$

So the steady state is characterized by the vector

$$\left[Y, C^P, C^E, I, H^P, H^E, K, N, L, D, Q^H, Q^K, R^D, R^L, W, \lambda^P, \lambda^E, \mu^E\right]$$

From entrepreneur's optimal choice of capital (B.9), I have

$$\begin{aligned}
\kappa_t^E &= \alpha(1-\alpha)\beta^E \mathbb{E}_t\left(\frac{\lambda_{t+1}^E Y_{t+1}}{K_t}\right) + \beta^E(1-\delta)\mathbb{E}_t\kappa_{t+1}^E + \mu_t^E\theta_t\mathbb{E}_t Q_{t+1}^K \\
\Rightarrow \frac{\kappa^E}{\lambda^E}&\left(1-(1-\delta)\beta^E\right) = \alpha(1-\phi)\beta^E\frac{Y}{K} + \frac{\left(1-\beta^E R^L\right)}{R^L}\theta Q^K
\end{aligned} \tag{C.16}$$

Entrepreneur's optimal choice of investment (B.10) yields

$$\begin{aligned}
\lambda_t^E(j) = \kappa_t^E(j)&\left[1 - \frac{\Omega}{2}\left(\frac{I_t(j)}{I_t(j-1)} - 1\right)^2 - \Omega\frac{I_t(j)}{I_t(j-1)}\left(\frac{I_t(j)}{I_{t-1}(j)} - 1\right)\right] \\
&+ \beta^E\Omega\mathbb{E}_t\left[\kappa_{t+1}^E(j)\left(\frac{I_{t+1}(j)}{I_t(j)}\right)^2\left(\frac{I_{t+1}(j)}{I_t(j)} - 1\right)\right] \\
\Rightarrow \lambda^E = \kappa^E&
\end{aligned} \tag{C.17}$$

Combining this with steady state version of

$$\kappa^E = \lambda^E Q^K \tag{C.18}$$

I obtain $Q^K = 1$ in the steady state. Plugging this into (C.16), I obtain the expression for capital-to-output ratio

$$\begin{aligned}
\frac{\kappa^E}{\lambda^E}\left(1-(1-\delta)\beta^E\right) &= \alpha(1-\phi)\beta^E\frac{Y}{K} + \frac{\left(1-\beta^E R^L\right)}{R^L}\theta Q^K \\
\Rightarrow \frac{K}{Y} &= \frac{\alpha(1-\phi)R^L\beta^E}{R^L\left(1-(1-\delta)\beta^E\right) - \theta\left(1-\beta^E R^L\right)}
\end{aligned} \tag{C.19}$$

Next, combining (B.15) and (B.16) yields

$$l = \frac{\theta}{R^L}\left[Q^H H^E + Q^K K\right] \tag{C.20}$$

Dividing by Y, the above expression becomes

$$\frac{l}{Y} = \frac{\theta}{R^L}\left[\frac{Q^H H^E}{Y} + \frac{Q^K K}{Y}\right]$$

Plugging in the values of $\frac{Q^H H^E}{Y}$ and $\frac{K}{Y}$ and using that $Q^K = 1$, I have

$$\frac{l}{Y} = \alpha\theta\beta^E\left[\frac{\phi}{R^L\left(1-\beta^E\right)-\theta\left(1-\beta^E R^L\right)} + \frac{(1-\phi)}{R^L\left(1-(1-\delta)\beta^E\right)-\theta\left(1-\beta^E R^L\right)}\right] \tag{C.21}$$

From entrepreneur's budget constraint (B.14)

$$C^E + R^L l = Y - WN - I + x \tag{C.22}$$

Rewriting this in ratios to output

$$\begin{aligned} \frac{C^E}{Y} + \frac{R^L l}{Y} &= 1 - \frac{WN}{Y} - \frac{I}{Y} + \frac{x}{Y} \\ \Rightarrow \frac{C^E}{Y} &= \alpha - \delta\frac{K}{Y} + \left(1-\gamma^L - R^L\right)\frac{l}{Y} \end{aligned} \tag{C.23}$$

Dividng (C.4) by Y and then dividing it again by (C.9) gives

$$\begin{aligned} \frac{\frac{Q^H H^P}{Y}}{\frac{Q^H H^E}{Y}} &= \frac{\frac{\varsigma}{Y\lambda^P\left(1-\beta^P\right)}}{\frac{\beta^E\alpha\phi R^L}{\left(1-\beta^P\right)R^L - \theta\left(1-\beta^E R^L\right)}} \\ \Rightarrow \frac{H^P}{H^E} &= \frac{\varsigma}{Y\frac{1-\beta^P\gamma^P}{\left(1-\gamma^P\right)C^P}\left(1-\beta^P\right)}\frac{\left(1-\beta^E\right)R^L - \theta\left(1-\beta^E R^L\right)}{\beta^E\alpha\phi R^L} \\ \Rightarrow \frac{H^P}{H-H^P} &= \frac{\varsigma\left(1-\gamma^P\right)}{\left(1-\beta^P\right)\left(1-\beta^P\gamma^P\right)}\frac{\left(1-\beta^P\right)R^L - \theta\left(1-\beta^E R^L\right)}{\beta^E\alpha\phi R^L}\frac{C^P}{Y} \end{aligned} \tag{C.24}$$

From entrepreneur's stock of habits for loans (B.11)

$$\begin{aligned} s_t &= \rho_s s_{t-1} + \left(1-\rho_s\right)l_t \\ s &= l \end{aligned} \tag{C.25}$$

Entrepreneur's effective demand for loans (B.12) gives

$$x_t = \left(l_t - \gamma^L s_{t-1}\right)$$
$$\Rightarrow x = \left(1 - \gamma^L\right) l \tag{C.26}$$

Total loans of entrepreneurs (B.13)

$$L = l \tag{C.27}$$

From bank's balance sheet condition (B.21), total deposits must equal total loans

$$D = L \tag{C.28}$$

Steady state version of stochastic discount factor (B.22) reads

$$q = \beta^P \tag{C.29}$$

The steady-state version of bank's first order condition (B.18) with respect to loans reads

$$\varrho_t = \beta^P \left[R^L - R^D + \gamma^L \left(1 - \rho_s\right) \varrho^E\right]$$

which can be simplified to yield

$$\varrho^E = \beta^P \frac{R^L - R^D}{1 - \beta^P \gamma^L \left(1 - \rho_s\right)} \tag{C.30}$$

The steady-state version of bank's second first order condition with respect to lending rate (B.19) writes

$$\xi \varrho^E x \frac{1}{R^L} = \beta^P L$$

Steady-state version of aggregate resource constraint (B.23) is

$$C^P + C^E + I = Y$$
$$\Rightarrow \frac{C^P}{Y} = 1 - \frac{C^E}{Y} - \delta \frac{K}{Y} \tag{C.31}$$

Combining (C.1), (C.2) and (C.8) gives steady-state equilibrium condition for households

$$\begin{aligned} N^{\eta-1} &= \lambda^P W \\ \Rightarrow N^{\eta-1} &= \frac{1-\beta^P\gamma^P}{(1-\gamma^P)\,C^P}(1-\alpha)\frac{Y}{N} \\ \Rightarrow N &= \left[\frac{\left(1-\beta^P\gamma^P\right)(1-\alpha)}{\iota\left(1-\gamma^P\right)}\left(\frac{C^P}{Y}\right)^{-1}\right]^{\frac{1}{\eta}} \end{aligned} \tag{C.32}$$

From (B.25), steady state output is

$$\begin{aligned} Y &= A\,(N)^{1-\alpha}\left[\left(H^E\right)^{\phi}(K)^{1-\phi}\right]^{\alpha} \\ Y^{1-\alpha} &= A\,(N)^{1-\alpha}\left[\left(\frac{H^E}{Y}\right)^{\phi}\left(\frac{K}{Y}\right)^{1-\phi}\right]^{\alpha} \\ Y^{1-\alpha} &= A\,(N)^{1-\alpha}\left[\left(\frac{H^E}{Y}\right)^{\phi}\left(\frac{\alpha\,(1-\phi)\,R^L\beta^E}{R^L\left(1-(1-\delta)\,\beta^E\right)-\theta\left(1-\beta^E R^L\right)}\right)^{1-\phi}\right]^{\alpha} \end{aligned} \tag{C.33}$$

From (C.4)

$$Q^H = \frac{\varsigma}{H^P\lambda^P\left(1-\beta^P\right)} \tag{C.34}$$

D System of Loglinear Equations

The system of equations log-linearized around their steady state is as below:

D.1 Optimality Conditions of Households

(A.2), (A.3) and (A.5) become

$$\beta^P\gamma^P\mathbb{E}_t\widehat{C}^P_{t+1}-\left(1+\left(\gamma^P\right)^2\beta^P\right)\widehat{C}^P_t+\gamma^P\widehat{C}^P_{t-1}=\left(1-\beta^P\gamma^P\right)\left(1-\gamma^P\right)\widehat{\lambda}^P \tag{D.1}$$

$$\mathbb{E}_t\widehat{\lambda}^P_{t+1}=\widehat{\lambda}^P_t-\widehat{R}^D_t \tag{D.2}$$

$$(\eta-1)\,\widehat{N}_t=\widehat{\lambda}^P_t+\widehat{W}_t \tag{D.3}$$

Log-linearization of (A.4) gives

$$\beta^P\mathbb{E}_t\left[\widehat{\lambda}^P_{t+1}+\widehat{Q}^H_{t+1}+\widehat{H}^P_t\right]=\widehat{\lambda}^P_t+\widehat{Q}^H_t+\widehat{H}^P_t \tag{D.4}$$

D.2 Optimality Conditions of Entrepreneurs

From (B.5) and (B.6), I have

$$\beta^E\gamma^E\mathbb{E}_t\widehat{C}^E_{t+1} - \left(1+\left(\gamma^E\right)^2\beta^E\right)\widehat{C}^E_t + \gamma^E\widehat{C}^E_{t-1} = \left(1-\beta^E\gamma^E\right)\left(1-\gamma^E\right)\widehat{\lambda}^E_t \tag{D.5}$$

(B.7) yields

$$\widehat{W}_t = \widehat{Y}_t - \widehat{N}_t \tag{D.6}$$

From (B.8), I derive

$$\begin{aligned}\left(\widehat{\lambda}^E_t + \widehat{Q}^H_t\right) &= \beta^E\mathbb{E}_t\big(\widehat{\lambda}^E_{t+1} + \widehat{Q}^H_{t+1}\big) + \left(\frac{1}{R^L} - \beta^E\right)\theta\mathbb{E}_t\big(\widehat{\mu}^E_t + \widehat{\theta}_t + \widehat{Q}^H_{t+1}\big)\\ &+ \left[\left(1-\beta^E\right) - \theta\left(\frac{1}{R^L} - \beta^E\right)\right]\mathbb{E}_t\big[\widehat{\lambda}^E_{t+1} + \widehat{Y}_{t+1} - \widehat{H}^E_t\big]\end{aligned} \tag{D.7}$$

(B.9) becomes

$$\begin{aligned}\widehat{Q}^K_t &= \big[1-\beta^E(1-\delta) - \theta\big(\frac{1}{R^L} - \beta^E\big)\big]\mathbb{E}_t\big[\widehat{\lambda}^E_{t+1} - \lambda^E_t + \widehat{Y}_{t+1} - K_t\big]\\ &+ \beta^E(1-\delta)\mathbb{E}_t\big(\widehat{Q}^K_{t+1} + \widehat{\lambda}^E_{t+1} - \widehat{\lambda}^E_t\big) + \left(1-\beta^E R^L\right)\frac{1}{R^L}\theta\mathbb{E}_t\left[\widehat{\mu}^E_t - \widehat{\lambda}^E_t + \widehat{\theta}_t + \widehat{Q}^K_{t+1}\right]\end{aligned} \tag{D.8}$$

(B.10) is approximated as

$$\widehat{Q}^K_t = \left(1+\beta^E\right)\Omega\widehat{I}_t - \beta^E\Omega\mathbb{E}_t\widehat{I}_{t+1} - \Omega\widehat{I}_{t-1} \tag{D.9}$$

From (B.11) and (B.13), I get

$$\widehat{s}_t = \rho_s\widehat{s}_{t-1} + (1-\rho_s)\,\widehat{l}_t \tag{D.10}$$

and

$$\widehat{x}_t = \frac{\widehat{l}_t}{1-\gamma^L} - \frac{\gamma^L\widehat{s}_{t-1}}{1-\gamma^L} \tag{D.11}$$

(B.14) becomes

$$C^E\widehat{C}^E_t + R^L l\left(\widehat{R}^L_{t-1} + \widehat{l}_{t-1}\right) = Y\widehat{Y}_t - WN\left(\widehat{W}_t + \widehat{N}_t\right) - I\widehat{I}_t - Q^H H^E\left(\widehat{H}^E_t - \widehat{H}^E_{t-1}\right) + x\widehat{x}_t \tag{D.12}$$

(B.15) gives

$$\widehat{l}_t = \widehat{\theta}_t + \widehat{a}_t - \widehat{R}_t^L \tag{D.13}$$

(B.16) yields

$$\widehat{a}_t = \frac{Q^H H^E}{Q^H H^E + Q^K K} \mathbb{E}_t \left(\widehat{Q}_{t+1}^H + \widehat{H}_t^E \right) + \frac{Q^K K}{Q^H H^E + Q^K K} \mathbb{E}_t \left(\widehat{Q}_{t+1}^K + \widehat{K}_t \right) \tag{D.14}$$

D.3 Optimality Conditions of Banks

From (B.18), I obtain

$$\frac{\varrho^E}{\beta^P} \widehat{\varrho}_t^E - \varrho^E \gamma^L (1 - \rho_s) \mathbb{E}_t \widehat{\varrho}_{t+1}^E = \left[R^L - R^D + \varrho^E \gamma^L (1 - \rho_s) \right] \mathbb{E}_t \widehat{q}_{t,t+1} + R^L \widehat{R}_t^L - R^D \widehat{R}_t^D \tag{D.15}$$

Log-linearization of (B.19) yields

$$\xi \varrho^E x \left(\widehat{\varrho}_t^E + \widehat{x}_t \right) = \beta^P R^L L \left(\widehat{R}_t^L + \widehat{L}_t + \mathbb{E}_t \widehat{q}_{t,t+1} \right) \tag{D.16}$$

D.4 Market Clearing and Resource Constraints

From (B.13), I obtain

$$\widehat{L}_t = \widehat{l}_t \tag{D.17}$$

(B.23) and (B.24) yield

$$\widehat{Y}_t = \frac{C^P}{Y} \widehat{C}_t^P + \frac{C^E}{Y} \widehat{C}_t^E + \frac{I}{Y} \widehat{I}_t \tag{D.18}$$

and

$$H^P \widehat{H}_t^P + H^E \widehat{H}_t^E = 0 \tag{D.19}$$

From (B.21), I get

$$\widehat{L}_t = \widehat{D}_t \tag{D.20}$$

From (B.25), I have

$$\widehat{Y}_t = \widehat{A}_t + (1 - \alpha) \widehat{N}_t + \alpha \phi \widehat{H}_{t-1}^E + \alpha (1 - \phi) \widehat{K}_{t-1} \tag{D.21}$$

(B.26) gives

$$\widehat{K}_t = (1 - \delta) \widehat{K}_{t-1} + \delta \widehat{I}_t \tag{D.22}$$

Linearized versions of (B.17) and (B.22) are

$$\widehat{\kappa}_t^E = \widehat{\lambda}_t^E + \widehat{Q}_t^K \tag{D.23}$$

$$\widehat{q}_{t,t+1} = \widehat{\lambda}_{t+1}^P - \widehat{\lambda}_t^P \tag{D.24}$$