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State-Dependent Effects of Loan-to-Value Shocks

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Keywords

Loan-to-Value (LTV) Shocks, Housing Price, Macroeconomic Fluctuations

JEL Classification

E32, E44

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Abstract

This paper presents a Two-Agent New Keynesian (TANK) model with collateral- constrained borrowers and a time-varying shock to loan-to-value (LTV) ratios. A temporary tightening in lending standards in this model leads to a sizable drop in macroeconomic aggregates and significant macroeconomic fluctuations. The analysis shows that effects of shocks to LTV ratios are highly non-linear and state-dependent in the sense that amplification of shocks depends crucially on steady-state LTV ratios. Shocks when LTV ratios are already high lead to effects which are substantially stronger than when the steady-state LTV ratios are comparatively lower. The results in this paper also show that permanent LTV shocks lead to permanent decline in housing prices – a 10 percentage point decline in steady-state LTV ratio from 0.95 results in more than 0.3% decline in housing prices. A novel finding in this paper is that a permanent tightening in lending standards leads to a permanent decline in wages. Additionally, other shocks such as TFP shocks, housing demand shocks and labor supply shocks also show clear state dependence and have highly persistent effects.

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1 Introduction

This paper documents the macroeconomic implications of loan-to-value (LTV) shocks. Using a Two-Agent New Keynesian (TANK) model, this paper presents four results. First, the effects of LTV shocks are highly non-linear and state-dependent. LTV shocks when steady-state LTV ratios are higher have enormously more impact than when initial LTV ratios are lower. Second, a permanent tightening in lending standards leads to a permanent decline in housing price and a long-lasting fall in economic activity. These economic effects are highly persistent and last long after the shock hits the economy. Third, a permanent LTV tightening leads to a permanent reduction in wages. Fourth, other macroeconomic shocks such as TFP shocks, housing demand shocks and labor supply shocks also show state dependence in their effects. To the best of my knowledge, this is the first paper that incorporates time-variation in LTV ratios in a two-agent real model containing only a household and an entrepreneur.

Using a simple RBC model, this paper studies the economic effects of LTV and other macroeconomic shocks. The focus in this paper is on how these various shocks display state-dependence in their effects. In order to do this, I build a parsimonious model consisting of households and entrepreneurs. Households in this model consume, hold housing, supply labor and lend to entrepreneurs who also consume, hire labor and borrow from households. Borrowing of these entrepreneurs is constrained to an exogenous borrowing limit which is subject to a time-varying shock. This borrowing limit restricts entrepreneurs' loans to a fraction of expected value of their assets which consist of physical capital and housing. I then examine the effects of both temporary and permanent LTV shocks in this model and show that their effects are highly state-dependent. I also consider other shocks which are commonly studied in macroeconomic literature such as TFP, labor supply and housing demand shocks and show that these shocks also show state-dependent effects.

To fix ideas, start by considering what happens in the aftermath of an LTV shock. When lending standards are tightened, it reduces the amount of loans that entrepreneurs can borrow and invest in productive capital. Entrepreneurs then respond by delevering and, at higher initial LTV ratios, selling their real estate holdings dramatically which are bought by households. Entrepreneurs then use part of the sale proceeds for increasing their consumption. A permanent tightening of lending standards permanently reduces the amount by which they can take out loans by posting their assets as collateral. Entrepreneurs, in this case, deleverage more dramatically and it has substantially larger impact on macroeconomic variables. This is sum of two effects. Fire sale by entrepreneurs lowers the price of housing which reduces the value of their collaterizable assets and at the same time, it reduces the amount of housing they have that they can post as collateral.

In case of a temporary LTV shock, entrepreneurs deleverage more at higher steady-state LTV ratios than at lower steady-state LTV ratios and changes in their housing stock is starker. For instance, when the steady-state LTV ratio is 0.95, entrepreneurs sell more than 5% of their housing stock and reduce their borrowing by almost 2%. Contrast this with what happens when steady-state LTV ratio is 0.80. In this case, entrepreneurs sell about 2% of their housing stock and reduce their borrowing by about three-quarters of a percent. This indicates that changes in economic variables at higher steady-state LTV ratios is at least twice than what it is at lower steady-state LTV ratios. These changes in key economic variables are the main forces driving the state-dependence in effects of temporary LTV shocks.

When lending standards are tightened permanently, the state-dependence of their effects becomes stronger. Entrepreneurs offload more than 12% of their housing stock, reduce their borrowing by close to 5% and housing price declines by about 0.7% before going up and staying permanently below its prior equilibrium value by slightly more than 0.3%. To put this into context, a 10 percentage point decline in LTV ratio from its steady-state value of 0.95 leads to a permanent decline in housing price by a magnitude which is thrice the change in steady-state LTV ratio. Entrepreneurs use part of their income from selling their housing stock to raise their consumption permanently by 1.5% when steady-state LTV ratio is 0.95. They do so because a pemanent tightening in LTV ratios permanently lowers the amout of loans that housing, as a collaterizable aseet, can bring them. On top of this, fire sale of housing lowers its price further tightening their borrowing constraint. Entrepreneurs, as a result, sell off part of their housing stock and use it to increase their consumption. This reallocation of resources away from housing which is a production intput in this model and into partly increasing entrepreneurial consumption then leads to a persistent decline in borrowing, investment, wages and output. These effects are remarkably state-dependent as entreprenurs engage in bigger fire-sale of housing at higher steady-state LTV ratios leading to larger drop in housing prices and cutting their borrowing more drastically. The movement in these economic variables then have a ripple effects on other macroeconomic variables.

This paper also considers other shocks such as a TFP shock, a housing demand shock and a labor shock and shows that their effects too show clear state-dependence. A negative TFP shock reduces the return entrepreneurs expect on their deployed capital. Entrepreneurs in this case deleverage and sell their housing which reduces its price. This further tightens entrepreneur's borrowing constraint who cut down their investment and consumption. Aggregate consumption and output also fall. These effects are greater at higher steady-state LTV ratios since entrepreneurs deleverage more and sell larger part of their housing stock at higher steady-state LTV ratios. This mechanism lies behind state-dependence in effects of TFP shocks.

Housing demand shocks also display interesting state-dependent effects in this model. In the wake of a negative housing demand shock, entrepreners sell their housing and reduce their borrowing. Housing prices display modest decline and persistently stay below their previous steady-state value even after a decade. Entrepreneurs, at higher steady-state LTV ratios, reallocate a part of their income from selling their housing stock to increase their consumption permanently which shows persistent elevation even after 40 quarters. These effects show state dependence since changes in entrepreneurial housing stock and borrowing, and the resulting housing prices display different behavior at varying steady-state LTV ratios.

I finally consider a negative labor supply shock. Like other shocks discussed before, labor shocks also have state-dependent effects. After a labor shock, employment declines and so do wages, albeit after a short increase at impact. A labor shock reduces the return on entrepreneur's deployed capital who respond to it by reducing their borrowing, selling off part of their housing stock and cutting down their consumption. Housing prices fall due to sell-off by entrepreneurs which further tightens their borrowing constraint. Household consumption also drop because household income falls because of a decline in wages and also because they channel part of their income to buy housing stock being sold off by entrepreneurs. Aggregate consumption, output, investment and capital also decline. All these economic variables display state dependence in their behavior. The underlying mechanism is similar to the case of other shocks though initial impact comes from a different source. Since entrepreneurs delever more and sell off more at higher steady-state LTV ratios, a labor shock has greater impact at higher steady-state LTV ratios.

This paper contributes to the literature at the intersection of macroeconomics and finance. Recently several papers have looked at systematic changes in credit limits over the business cycle and their macroeconomic impact (see, among others, Jensen, Ravn, and Santoro, 2018 and Jensen, Petrella, Ravn, and Santoro, 2020). These papers consider a positive shock to credit limits in a three-agent framework and find that increases in LTV ratios have contributed significantly to business cycle dynamics and increasing skewness observed in US economy. This paper in contrast with these previous works, focuses on building a small two-agent model that can offer a transparent mechanism to understand how exogenous negative shocks to LTV ratios affect the macroeconomic outcomes and how the state-dependence of these shocks propagates to the wider macroeconomy. My focus in this paper in on shining light on how changes in steady-state LTV ratios can affect the macroeconomic dynamics and what their implications are for various economic shocks.

Existing studies have typically abstracted from borrowing costraints on agents when examining impact of LTV shocks. For example, Justiniano, Primiceri, and Tambalotti (2015) look at impact of LTV shocks in a model where agents are never financially constrained and conlcude that these shocks have relatively little macroeconomic impact. This work shows that when financial constraint for agents is taken into account, LTV shocks can have sizable and persistent macroeconomic effects and can cause significant economic fluctuations. This is all the more notable given the simple RBC model in this paper.

I investigate the impact of credit-constrained investment and steady-state changes in LTV ratios by incorporating time-variation in LTV ratios in a Two-Agent New Keynesian (TANK) model. All the investment in this model is made by credit-constrained entrepreneurs. Jensen, Ravn, and Santoro (2018) and Jensen, Petrella, Ravn, and Santoro (2020) study positive changes in credit limits and leverage in a three-agent framework. Kiyotaki, Michaelides, and Nikolov (2011, 2023), Favilukis, Ludvigson, and Van Nieuwerburgh (2017) and Kaplan, Mitman, and Violante (2020) study the effects of changes in credit limits in an overlapping generations framework. Huo and Ríos-Rull (2016) examine the joint role of LTV and interest rate shocks in a heterogeneous agent model. Ferrero (2015) and Boz and Mendoza (2014) examine the effects of changing LTVs in open-economy models. Justiniano, Primiceri, and Tambalotti (2015) and De Veirman (2023) look at time-variation in LTV ratios in a model with sticky prices.

The paper that is closest to the analysis in this work is De Veirman (2023) who embeds the model in Iacoviello (2005) with time-varying LTV shocks and studies their impact. Results in this paper confirm the analysis in De Veirman (2023) but additionally show that state-dependent effects of LTV shocks carry through even to simpler two-agent RBC models featuring only one collateral-constrained borrower (entrepreneurs) and an unconstrained lender (households). This is in stark contrast with the model in Iacoviello (2005) which is a nominal model featuring two collateral-constrained borrowers (impatient households and entrepreneurs) and an unconstrained lender (patient households). Also in terms of results, my paper differs from De Veirman (2023) and shows that permanent LTV tightening has persistent negative effects on macroeconomic aggregates such as aggregate consumption, investment, output and wages. In De Veirman (2023), consumption, investment and output fall in the aftermath of a permanent LTV tightening and return to their steady state after a few quarters. In contrast, I show that after a permanent LTV tightening, these variables persistently stay below their previous steady state and do not return to their equilibrium even after a decade. My results, in this sense, indicate towards highly persistent negative macroeconomic consequences of a permanent LTV tightening. Another additional contribution of this work is to show that other shocks such as TFP shocks, housing shocks and labor shocks too show clear state-dependence[1].

This work is related to Sharma (2023b) who studies the implications of bank-firm lending relationships for changes in steady-state LTV ratios. My focus, on the other hand, is on developing a parsimonious two-agent RBC model that helps understand effects of changes in steady-state LTV ratios on LTV shocks and ignores both bank-firm lending relationships and any role for formal financial sector. Another recent contribution this paper is connected to is Sharma (2023a) who examines effects of collateral shocks in an environment featuring bank-firm lending relationships but does not consider implications of changes in steady-state LTV ratios. Finally, this paper is related to Sharma (2023c) who studies the effects of a shock to the lending standards in a model of bank-firm lending relationships and bank competition on collateral requirements. The current paper focuses on implications of changes in steady-state LTV ratios for LTV and other shocks and abstracts from bank-firm lending relationships, bank competition on collateral requirements or modelling financial sector.

The remainder of this paper is structured as follows. Section 2 presents the model environment in this paper. Section 3 and Section 4 discuss the model solution and results, respectively. Finally, Section 5 concludes.

2 Model

The paper features a discrete-time two-agent New Keynesian (TANK) model. It has (patient) households who consume non-durable consumption goods, supply labor and derive utility from

[1] De Veirman (2023) considers, in addition to LTV shocks, monetary policy shocks and housing demand shocks but does not consider TFP shocks or labor shocks.

holding a durable good (housing). The (impatient) entrepreneurs, in turn, consume non-durable consumption goods, hire labor from households and run firms in the economy. Firms are owned by the entrepreneurs who are subject to a collateral constraint as in Kiyotaki and Moore (1997) which limits their borrowing to a fraction of expected value of their total assets which include physical capital and housing. Because of $\beta^E < \beta^P$, households act as lenders and entrepreneurs act as borrowers in the equilibrium. In what follows, I describe each agent's optimization problem.

2.1 Households

The household's problem bears similarities with those in Iacoviello (2005), Liu, Wang, and Zha (2013) and Justiniano, Primiceri, and Tambalotti (2015). Households have the utility function of the following form:

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \left(\beta^P\right)^t \left\{ \log\left(C_t^P - \gamma^P C_{t-1}^P\right) - \iota_t N_t + \varsigma_t \log H_t^P \right\} \tag{1}$$

where C_t^P, N_t and H_t^P denote consumption, labor and housing respectively of the households, $\beta^P \in (0,1)$ is a discount factor and γ^P measures the degree of habit formation in consumption. The superscript P denotes (patient) households. Household's preference for leisure is subject to an exogenous shock ι_t, the law of motion of which is given by

$$\log \iota_t = (1-\rho_N)\log \iota + \rho_N \log \iota_{t-1} + \sigma_N \epsilon_{N,t} \tag{2}$$

Here, $\epsilon_{N,t}$ is iid innovation which follows a normal distribution with standard deviation σ_N and where $\iota > 0$ and $\rho_N \in (0,1)$. In similar fashion, ς_t is a housing preference shock as in Liu, Wang, and Zha (2013) which follows the following process

$$\log \varsigma_t = (1-\rho_H)\log \varsigma + \rho_H \log \varsigma_{t-1} + \sigma_H \epsilon_{H,t} \tag{3}$$

where $\epsilon_{H,t}$ is iid innovation which follows a normal distribution with standard deviation σ_H and where $\varsigma > 0$ and $\rho \in (0,1)$. The household faces the following budget constraint

$$C_t^P + Q_t^H\left(H_t^P - H_{t-1}^P\right) + R_{t-1}B_{t-1}^P \leq W_t N_t + B_t^P \tag{4}$$

Here, Q_t^H is the price of one unit of housing in terms of consumption goods, W_t is the real wage and R_{t-1} is the gross real interest rate on debt B_{t-1}^P. I assume housing does not depreciate. First order conditions of the households with respect to consumption, debt, housing and labor respectively can be written as

$$\frac{1}{C_t^P - \gamma^P C_{t-1}^P} - \beta^P \mathbb{E}_t \frac{\gamma^P}{C_{t+1}^P - \gamma^P C_t^P} = \lambda_t^P \tag{5}$$

$$\beta^P \mathbb{E}_t \lambda_{t+1}^P = \frac{\lambda_t^P}{R_t} \tag{6}$$

$$\frac{\varsigma_t}{H_t^P} + \beta^P \mathbb{E}_t \left(\lambda_{t+1}^P Q_{t+1}^H\right) = \lambda_t^P Q_t^H \tag{7}$$

$$\iota_t = \lambda_t^P W_t \tag{8}$$

where λ_t^P is the Lagrange multiplier associated with household's budget constraint (4). One can combine household's first-order conditions with respect to consumption (5) and debt (6) to obtain their Euler equation. Equation (7) describes household's Euler equation for housing and links today's housing price to the utility it provides plus the expected capital gain. Equation (8) describes household's consumption-lesiure tradeoff. All the derivations of first order conditions have been relagated to the Appendix A.

2.2 Entrepreneurs

Following Iacoviello (2005) and Liu, Wang, and Zha (2013), the representative entrepreneur maximizes the utility obtained from consuming the non-durable consumption goods

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \left(\beta^E\right)^t \log\left(C_t^E - \gamma^E C_{t-1}^E\right) \tag{9}$$

where β^E and γ^E are as defined before. I assume that entrepreneurs are more impatient than the households, that is, $\beta^E < \beta^P$. Entrepreneurs face a collateral constraint à la Kiyotaki and Moore (1997) that limits the borrowing of each entrepreneur to a fraction of expected value of their assets

$$B_t^E \leq \frac{1}{R_t} \theta_t a_t \tag{10}$$

Here, B_t^E denotes entrepreneur's loan, expected value of entrepreneur's assets is a_t and R_t is the lending rate. Entrepreneur's borrowing is subject to a loan-to-value (LTV) requirement θ_t

which follows the law of motion

$$\log \theta_t = (1 - \rho_\theta) \log \theta + \rho_\theta \log \theta_{t-1} + \sigma_\theta \epsilon_{\theta,t} \tag{11}$$

where $\theta > 0$ is the steady-state LTV ratio, $\epsilon_{\theta,t}$ is iid innovation which follows a normal distribution with standard deviation σ_θ and $\rho_\theta \in (0, 1)$. My goal in this paper is to examine the implications of exogenous changes in credit conditions, including for institutional and regulatory reasons. The LTV shock captures changes in lending that are exogenous from point of view of both lenders and borrowers. Expected value of entrepreneur's assets a_t is given by

$$a_t = \mathbb{E}_t \left(Q^H_{t+1} H^E_t + Q^K_{t+1} K_t \right) \tag{12}$$

In the above equation, Q^K_t denotes the value of installed capital in units of consumption goods, K_t stock of capital and H^E_t stock of housing[2]. Entrepreneurs produce output using a constant returns to scale production function

$$Y_t = A_t \left(N_t\right)^{1-\alpha} \left\{ \left(H^E_{t-1}\right)^\phi \left(K_{t-1}\right)^{1-\phi} \right\}^\alpha \tag{13}$$

where Y_t is output, N_t labor input and $\alpha, \phi \in (0, 1)$ are factor shares. TFP A_t follows the process

$$\log A_t = (1 - \rho_A) \log A + \rho_A \log A_{t-1} + \sigma_A \epsilon_{A,t} \tag{14}$$

with iid innovation $\epsilon_{A,t}$ following a normal process with standard deviation σ_A and where $A > 0$ and $\rho_A \in (0, 1)$. The evolution of capital obeys the following law of motion

$$K_t = (1 - \delta) K_{t-1} + \left[1 - \frac{\Omega}{2} \left(\frac{I_t}{I_{t-1}} - 1 \right)^2 \right] I_t \tag{15}$$

where I_t is firm's investment level, $\delta \in (0, 1)$ the rate of depreciation of capital stock and $\Omega > 0$ is a cost adjustment parameter. The entrepreneur faces the following budget constraint

$$C^E_t + R_{t-1} B^E_{t-1} \leq Y_t - W_t N_t - I_t - Q^H_t \left(H^E_t - H^E_{t-1} \right) + B^E_t \tag{16}$$

[2] Chaney, Sraer, and Thesmar (2012) and Liu, Wang, and Zha (2013) emphasize the importance of real estate as collateral for business loans.

which states that entrepreneur's consumption and debt payment for the previous period should not exceed entrepreneur's output net of wage payment to labor, investment, changes in entrepreneur's housing stock and the debt in the current period. The FOCs of the entrepreneur with respect to consumption, debt, labor, housing, capital and investment, respectively are

$$\lambda_t^E = \frac{1}{C_t^E - \gamma^E C_{t-1}^E} - \beta^E \mathbb{E}_t \frac{\gamma^E}{C_{t+1}^E - \gamma^E C_t^E} \tag{17}$$

$$\lambda_t^E = \beta^E \mathbb{E}_t \lambda_{t+1}^E R_t + \mu_t^E R_t \tag{18}$$

$$W_t = (1-\alpha)\frac{Y_t}{N_t} \tag{19}$$

$$\lambda_t^E Q_t^H = \beta^E \mathbb{E}_t \left[\lambda_{t+1}^E \left(Q_{t+1}^H + \alpha\phi \frac{Y_{t+1}}{H_t^E}\right)\right] + \mu_t^E \theta_t \mathbb{E}_t Q_{t+1}^H \tag{20}$$

$$\kappa_t^E = \alpha(1-\phi)\beta^E \mathbb{E}_t \left(\frac{\lambda_{t+1}^E Y_{t+1}}{K_t}\right) + \beta^E(1-\delta)\mathbb{E}_t \kappa_{t+1}^E + \mu_t^E \theta_t \mathbb{E}_t Q_{t+1}^K \tag{21}$$

$$\lambda_t^E = \kappa_t^E \left[1 - \frac{\Omega}{2}\left(\frac{I_t}{I_{t-1}} - 1\right)^2 - \Omega \frac{I_t}{I_{t-1}}\left(\frac{I_t}{I_{t-1}} - 1\right)\right] + \beta^E \Omega \mathbb{E}_t \left[\kappa_{t+1}^E \left(\frac{I_{t+1}}{I_t}\right)^2 \left(\frac{I_{t+1}}{I_t} - 1\right)\right] \tag{22}$$

where μ_t^E, κ_t^E and λ_t^E are Lagrange multipliers associated with entrepreneur's collateral constraint (10), law of motion of capital (15) and entrepreneur's budget constraint (16). Entrepreneur's first order conditions with respect to consumption (17) and debt (18) may be combined to derive Euler equation for consumption for a collateral-constrained agent. Equation (19) describes entrepreneur's optimal demand for labor. Entrepreneur's Euler equation for land is described by (20) which relates its price today to its expected resale value tomorrow plus the payoff obatained by holding it for a period as given by its marginal productivity and its ability to serve as a collateral. Likewise, (21) is entrepreneur's Euler equation for capital and it links price of capital today to its price tomorrow and the expected payoff from keeping it for a period as given by its marginal productivity and its ability to serve as a collateral. Finally, entrepreneur's Euler equation for the investment is given by (22). Derivation of these first-order conditions have been consigned to Appendix A.

2.3 Aggregation and Market Clearing

Aggregate output in the economy equals consumption of households and entrepreneurs, and entrepreneurial investment

$$C_t^P + C_t^E + I_t = Y_t \tag{23}$$

The total amount of housing in the economy is fixed at H which implies that

$$H_t^P + H_t^E = H \tag{24}$$

Finally, the total net supply of debt in the economy is zero, i.e.

$$B_t^P + B_t^E = 0 \tag{25}$$

3 Model Solution and Prameterization

A period is a quarter in this model. The model is solved by log-linearizing the equilibrium conditions around the steady state and by using perturbation methods. Appendices B, C and D contain the list of equilibrium equations, the list of steady-state conditions and the system of log-linear equations, respectively. The calibration of parameters is standard and as follows (see Table 1). I allow for a relatively significant difference between discount factors of households and entrepreneurs so that steady-state value of Lagrange multiplier on entrepreneur's collateral constraint μ_t^E is different from zero. I therefore set $\beta^P = 0.995$ and $\beta^E = 0.95$. The degree of habit formation in consumption is chosen to be 0.6 which is in line with empirical estimates (Smets and Wouters, 2007). The steady-state value of labor shock ι is set so that households work about 25% of their time in steady state. Following Liu, Wang, and Zha (2013), the value of ς is calibrated to obtain a ratio of residential land to output in steady state around 1.45 at an annual frequency. The labor income share is 0.3 which implies a steady-state capital-output ratio of 1.15, in line with US data (Liu, Wang, and Zha, 2013). The input share of land in production is close to the value estimated in Liu, Wang, and Zha (2013) and Iacoviello (2005). The investment adjustment cost parameter is set to 1.85. The literaure contains estimates which range from 0 (Liu, Wang, and Zha, 2013) to above 26 (Christiano, Motto, and Rostagno, 2010). The capital depreciation rate implies a steady-state ratio of non-residential investment to output a little above 0.13 (Beaudry and Lahiri, 2014).

For calibration of shocks, I follow Smets and Wouters (2007) and set persistence of technology shock to 0.95 and its standard deviation to 0.0014 which is standard in the literature. Smets and Wouters (2007) do not include shocks to housing preferences or labor and I therefore rely on Liu, Wang, and Zha (2013) for their calibration who find these shocks, particularly housing

Table 1: Parameter values

	Value	Description	Source/Target
β^P	0.995	Discount factor, households	Iacoviello (2005)
β^E	0.95	Discount factor, entrepreneurs	Iacoviello (2005)
γ^i, $i = \{P, E\}$	0.6	Habits in consumption, households, entrepreneurs	Smets and Wouters (2007)
ι	3.5	Steady state of labor supply shock	See Text
ς	0.0375	Steady state of housing preference shock	Liu, Wang, and Zha (2013)
α	0.3	Non-labor share of production	See Text
ϕ	0.1	Land share of non-labor input	Iacoviello (2005)
Ω	1.85	Investment adjustment cost parameter	See Text
δ	0.0285	Capital depreciation rate	Beaudry and Lahiri (2014)
ρ_A	0.95	Persistence of technology shock	Smets and Wouters (2007)
ρ_N	0.97	Persistence of labor supply	See Text
ρ_H	0.99	Persistence of housing preference shock	See Text
σ_A	0.0014	Standard deviation of technology shock	Standard
σ_N	0.0014	Standard deviation of labor supply shock	See Text
σ_H	0.014	Standard deviation of housing preference shock	See Text

preference shock, more persistent than technology shock. They also find that standard deviation of housing preference shock is an order of magnitude larger than that of technology and labor shocks. I set these shocks accordingly.

4 Discussion

In this section, I discuss the effects of various shocks and proceed in the following manner. I first consider a temporary LTV shock and show how a one-time temporary shock leads to movement in economic aggregates. I also show that these effects are highly state-dependent. I then consider a permanent LTV shock to show that a permanent change in LTV ratios leads to greater macroeconomic effects, permanent changes in housing prices and they continue to show state-dependence documented before. Finally, I consider a series of other negative shocks – a TFP shock, a housing demand shock and a labor supply shock. I show that all these shocks display clear state-dependence in their effects.

4.1 Effects of a Shock to LTV Ratios

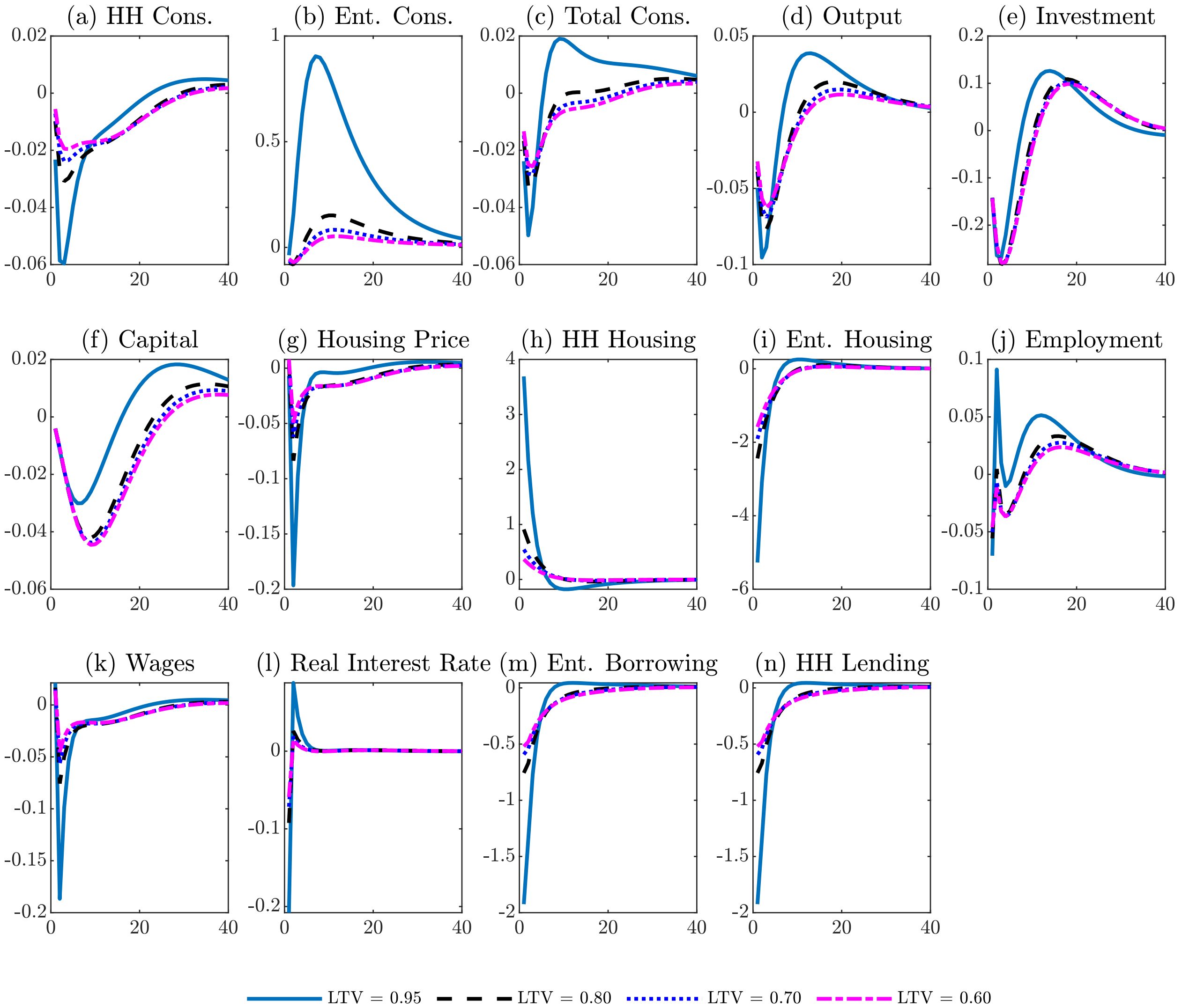
This subsection describes effects of LTV shocks under different steady-state LTV ratios and shows that both temporary and permanent LTV shocks exhibit marked state-dependence. I first discuss impact of a transitory LTV shock before studying effects of a permanent LTV shock. Given the focus in this paper on LTV tightening, I look at negative shocks. For the clarity of exposition and to keep the discussion and presentation manageable, I consider four steady-state LTV ratios – 0.95, 0.80, 0.70 and 0.60[3].

I consider the macroeconomic implications of a temporary negative shock implying a 10 percentage point reduction in steady-state LTV ratios on impact. After a shock, the LTV ratios come back to their steady-state value at the rate $\rho = 0.90$. A temporary LTV shock leads to a tightening of the collateral constraint for entrepreneurs who respond to it by reducing their borrowing and selling their housing (see Figure 1). After a negative LTV shock, entrepreneurs consider their assets less useful in terms of how much they can help them borrow through their use as collateral. Consequently, entrepreneurs sell their housing and increase their consumption using part of proceeds from sale of their housing. This collateral tightening has ripple effects also on investment since entrepreneurs cannot borrow now as much as they were able to before which then also translates into a fall in capital stock, a drop in employment and wages, and a reduction in aggregate output. In virtually every case, higher the steady-state LTV ratio, bigger the drop. This shows that steady-state LTV ratios when the LTV shock hits play an important role in determining the magnitude of the impact. For example, a drop in LTV ratios from 0.95 leads to a fall in entrepreneurial borrowing by almost 2%. This fall in borrowing coincides with a drop in real interest rate by 0.2% percent. Entrepreneurs sell more than 5% of their housing which leads to a drop in housing prices by 0.2%. Entrepreneur's housing stock is bought by households who finance their acquisition through reduction in their consumption. Entrepreneurs then use some of their sale proceeds to increase their consumption which rises by close to 1% and remains elevated for the next 40 quarters. It's noteworthy that difference in increase in peak entrepreneurial consumption when steady-state LTV ratio is 0.95 versus when it is 0.80 is more than 5 times which is not the case when comparing other steady-state LTV ratios such as 0.80, 0.70 and 0.60. This confirms the finding that effects of LTV shocks are highly state-dependent and non-linear. Investment falls on impact and so does capital. This results in a fall in aggregate consumption and output.

Households, on the other hand, respond to a temporary tightening in lending standards by increasing their holdings of housing stock which are being offloaded by entrepreneurs. Remember that housing is in a fixed supply so when the entrepreneurs sell it, it's the households who

[3] De Veirman (2023) computes and shows impulse responses for two steady-state LTV ratios 0.95 and 0.80.

Figure 1: Impact of a Temporary LTV Shock

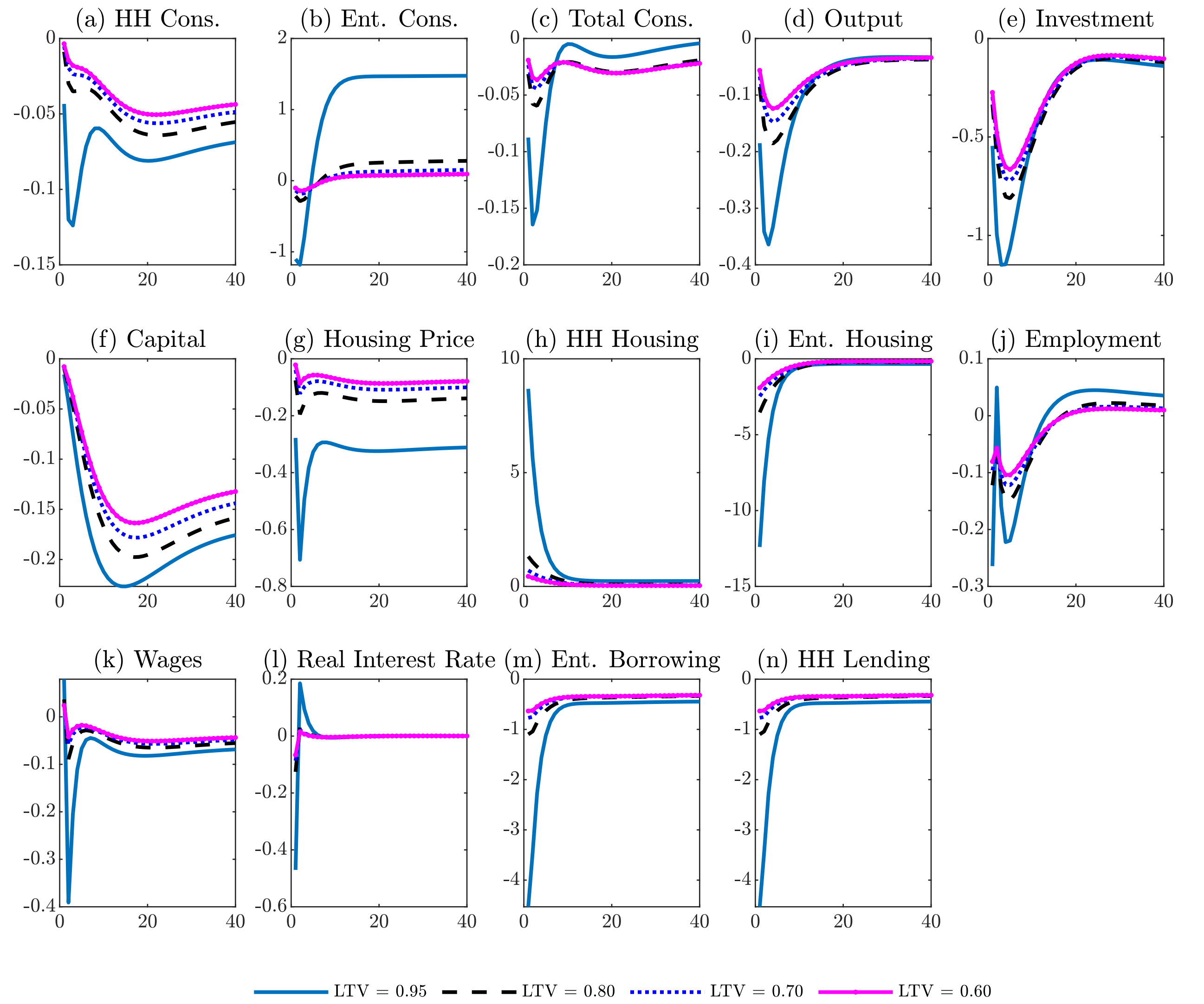


Note: Impact of a temporary shock that leads to 10 percentage point decline in LTV ratio from its steady-state value. Numbers on the horizontal axis are quarters since the shock. Numbers on the vertical axis show percentage deviation from steady state.

buy. Contrary to the entrepreneurs, households respond to the LTV shock by reducing their consumption and using the savings to increase their holdings of housing stock. Lending by households falls since entrepreneurial borrowing plummets and in this model there is one-to-one mapping between the two – households lend and entrepreneurs borrow. There is no financial sector and funds move directly between the two agents. Most of the variables return to their steady state after initial change at the time of the shock which is in line wth transitory nature of the LTV tightening.

I now look at the effects of a quasi-permanent shock to steady-state LTV ratios. The analysis here focuses on impacts of shocks implying a 10 percentage point decline in steady-state LTV ratios on impact and which come back to their steady-state value at a rate governed by $\rho =$

Figure 2: Impact of a Permanent LTV Shock



Note: Impact of a permanent shock that leads to 10 percentage point decline in LTV ratio from its steady-state value. Numbers on the horizontal axis are quarters since the shock. Numbers on the vertical axis show percentage deviation from steady state.

$1 - 10^{-10}$. Figure 2 shows the response of different variables to a permanent tightening in LTV ratios. After the permanent shock hits, entrepreneurs drastically deleverage and sell off their housing. Their existing housing stock now does not allow them to borrow as much as it did before. When the stedy-state LTV is 0.95, entrepreneurial holding of housing stock falls over 13% on impact and house prices, as a result, fall about 0.3% on impact before dropping to over 0.7% and then recovering. They permanently stay 0.3% below their pre-shock level. This is a novel finding and the only papers documenting such an effect, to the best of my knowledge, are De Veirman (2023) and Kiyotaki, Michaelides, and Nikolov (2023). Part of proceeds from house sales is used by entrepreneurs to finance their increased consumption since they no longer invest as much as they did before which leads to a persistent decline in investment, capital stock and output. All these effects are much more pronounced and persistent than in the case of a transitory shock. Interestingly, investment, capital stock and output do not return to their steady-sate value even after 10 years (40 quarters). This points towards highly persistent nature of effects of permanent LTV shocks. This result is in contrast to the finding in De Veirman (2023) who reports that after a permanent tightening, aggregate consumption, investment and output return to their previous equilibirum after a few quarters. Additionally, I find that after a permanent LTV tightening, wages decline permanently which is plausibly due to highly sustained drop in investment and capital stock. This is a novel finding in the literature.

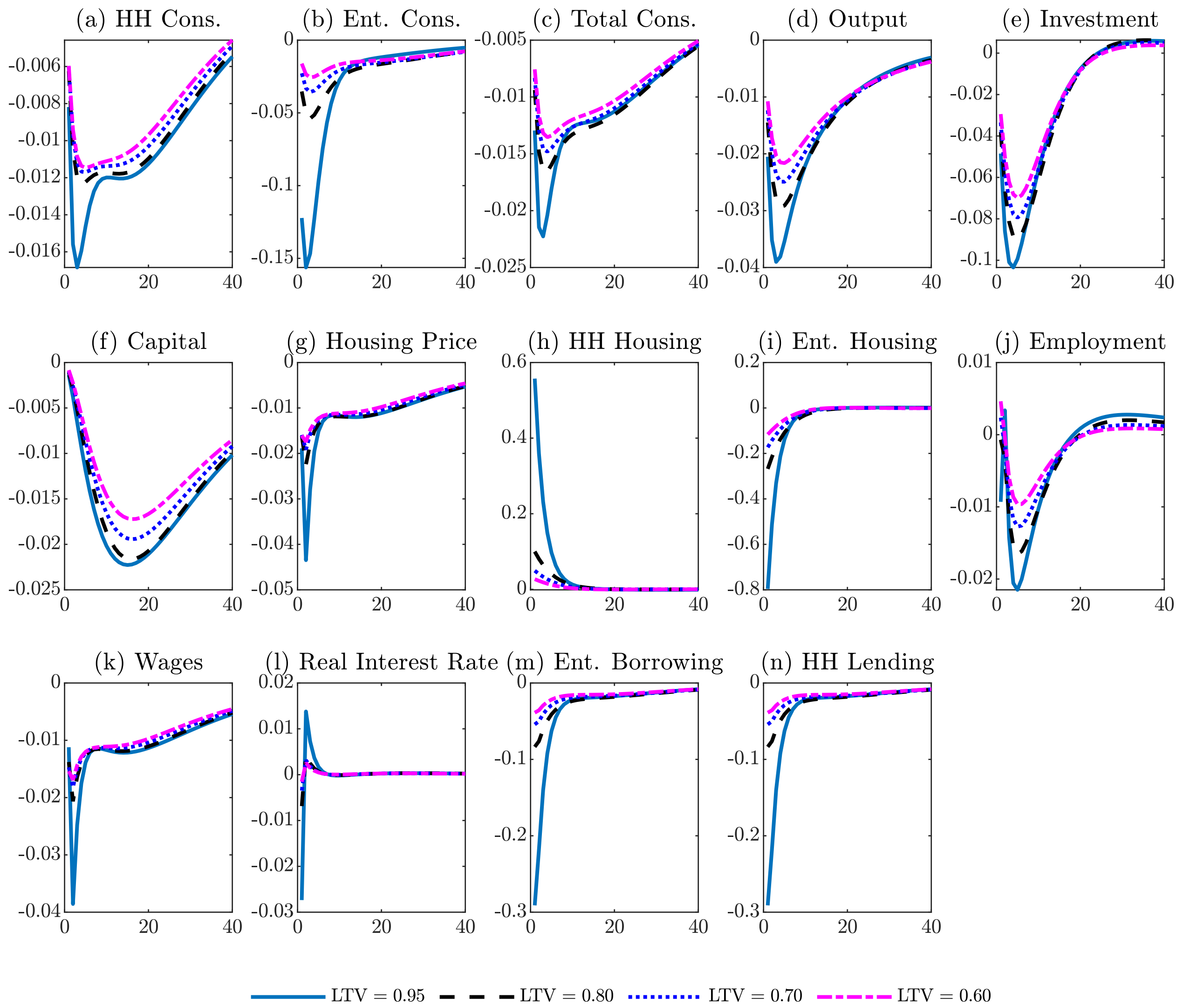
Households, acting like in the case of a transitory LTV shock, respond by reducing their consumption and massively building up new housing positions (buying) by increasing their house holdings by over 8%. There is drastic difference between household's buying of housing stock at different steady-state LTV ratios. When the permanent LTV shock hits and when the steady-state LTV ratio is 0.80, households buy less than 2% of housing stock but they ramp it up to over 8% when the steady-state LTV ratio is 0.95. This reflects a difference of over 400%. Lending by households drops by over 4% which is more than twice the fall in lending by households in the case of a transitory LTV shock. Entrepreneurs' borrowing decline by an equal magnitude as they cut down on their borrowing and investment and increase they consumption. All these effects are much higher than in the case of a transitory shock. Consistent with the pattern observed in the case of transitory shocks, steady-state LTV ratios appear to have a significant state-dependent effects also in the case of a permanent shock. Also notable is the highly persistent nature of permanent LTV shocks on investment, capital stock and employment. A large and persistent drop in investment leads to a fall in stock of capital which then manifests itself in lower employment at impact and persistently lower wages. Permanent shock are magnified a lot when LTV ratio is 0.90 versus when it's 0.80. This confirms the non-linearity in the effects of LTV shocks.

4.2 Effects of Other Shocks

In this subsection, I discuss the implications of changes in steady-state LTV ratios for other shocks in this model. Specifically, I consider a TFP shock, a housing demand shock and a labor supply shock.

Figure 3 shows effects of a negative TFP shock. There are two interesting effects to note here. First, the quantitative effects of a negative TFP shock are muted as compared to an LTV shock. Second, the state-dependence of effects carries through to TFP shocks. TFP shocks seem to have a much higher impact when LTV ratios are higher versus when steady-state LTV ratios are lower. After a negative TFP shock, entrepreneurs sell their housing stock and reduce their borrowing. A negative TFP shock reduces the output and reduces the return entrepreneurs expect on their deployed capital. As a consequence, entrepreneurs delever and reduce their consumption. In the wake of a negative TFP shock, households also reduce their consumption and increase their holding of housing stock which is being sold by entrepreneurs. Their lending declines since entrepreneurs cut down their borrowing. As a result of drop in aggregate lending and borrowing, real interest rate declines at impact before overshooting its steady-state level and then returning to its pre-shock level.

Figure 3: Impact of a Negative TFP Shock



Note: Numbers on the horizontal axis are quarters since the shock. Numbers on the vertical axis show percentage deviation from steady state.

Fall in investment by entrepreneurs leads to a long-lived reduction in the stock of capital and an accompanying decline in wages. In fact, wages remain depressed persistently in the wake of a temporary TFP shock and do not return to their steady-state level even after 40 quarters. Remarkably, housing prices also fall on impact and remain well below their steady-state level even after a decade. The initial fall in housing price is much higher at LTV ratio 0.95 than at LTV ratio 0.80. The fall in housing price in case of steady-state LTV ratio 0.95 is more than 0.04% and it's 0.02% when the steady-state LTV ratio is 0.80. This is a difference of 100%. This highlights the result in this paper that housing price responds significantly not only to LTV shocks but also to temporary TFP shocks and these effects are highly state-dependent.

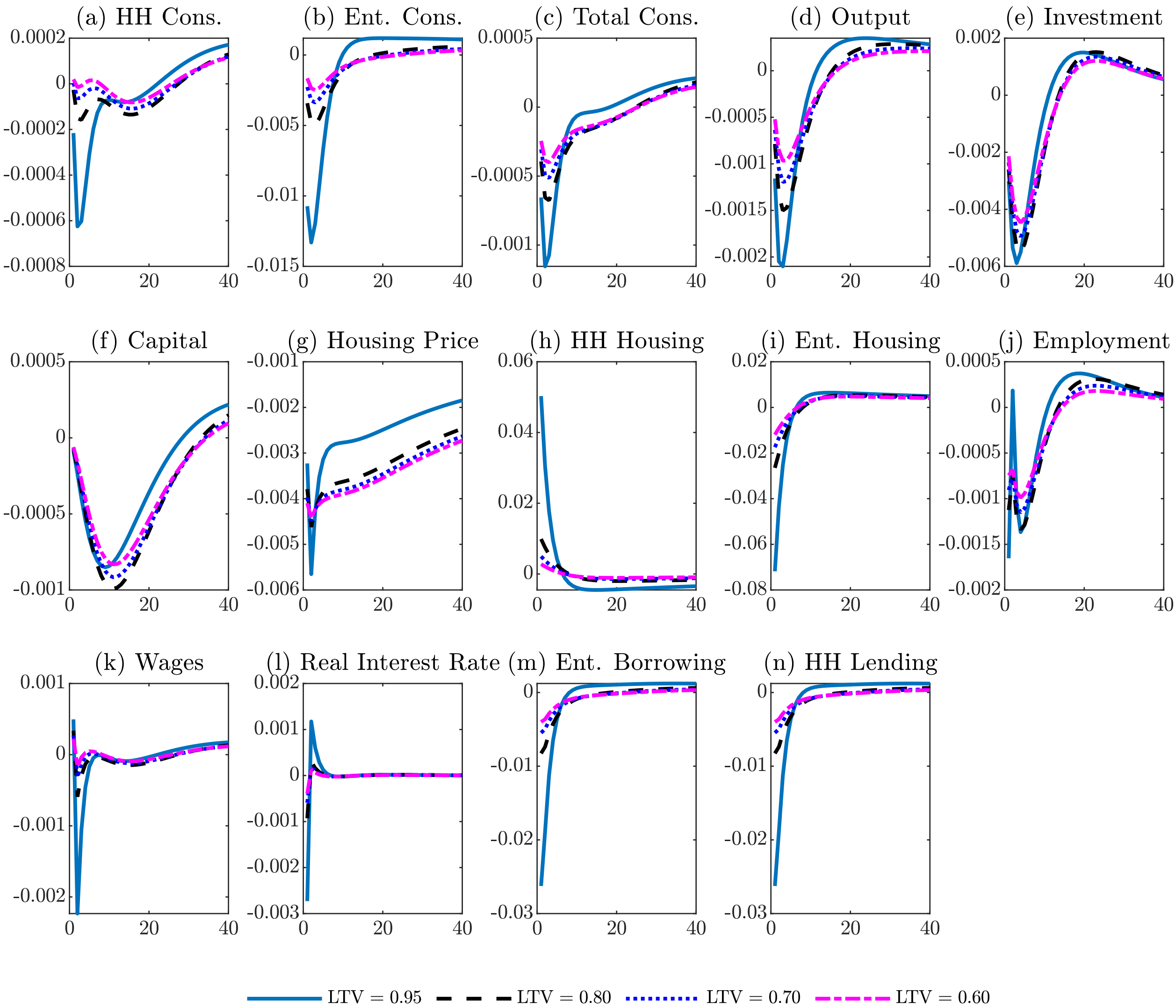
As mentioned before, a negative TFP shock reduces the return entrepreneurs expect on their deployed capital. Entreprneurs then reduce their borrowing, cut down their investment and sell their housing stock. These effects show state-dependence because at higher steady-state LTV ratios, entrepreneurs delever more and sell more of their housing stock. This greater deleveraging and bigger changes in entrepreneur's housing stock then has a ripple effect on other macroeconomic variables.

A housing demand shock in this model has weaker impact than TFP shocks. As Figure 4 shows, housing demand shocks also show clear state-dependent effects. These shocks have a stronger impact when LTV ratios are higher to start with than when they are lower. A negative housing demand shock reduces utility gained by a unit of housing. Note that only households gain direct utility from holding housing while entrepreneurs derive utility from housing indirectly by using it as a production input and as a collaterizable asset. In the wake of a negative housing demand shock, entrepreneurs sell their housing stock and raise their consumption. They raise their consumption permanently when the seady-state LTV ratio is 0.95 in which case, aggregate consumption first falls and then overshoots its previous equilibrium value. This effect is present also at lower steady-state LTV ratios but the magnitude is markedly lower.

Entrepreneurs sell their housing and cut down their investment which reduces the capital stock. They reduce their borrowing at first which leads to a drop in real interest rate. When steady-state LTV ratio is high (in this case, 0.95), entrepreneurial borrowing rises slightly after an initial drop and stays persistently above its previous equilibrium value which results in an initial fall in interest rate before it overshoots its prior steady-state level and then reverts to it.

House prices fall as a result of sell-off by entrepreneurs and remain below their pre-shock level persistently. Reduced housing prices mean that they cannot fetch as much loans as before

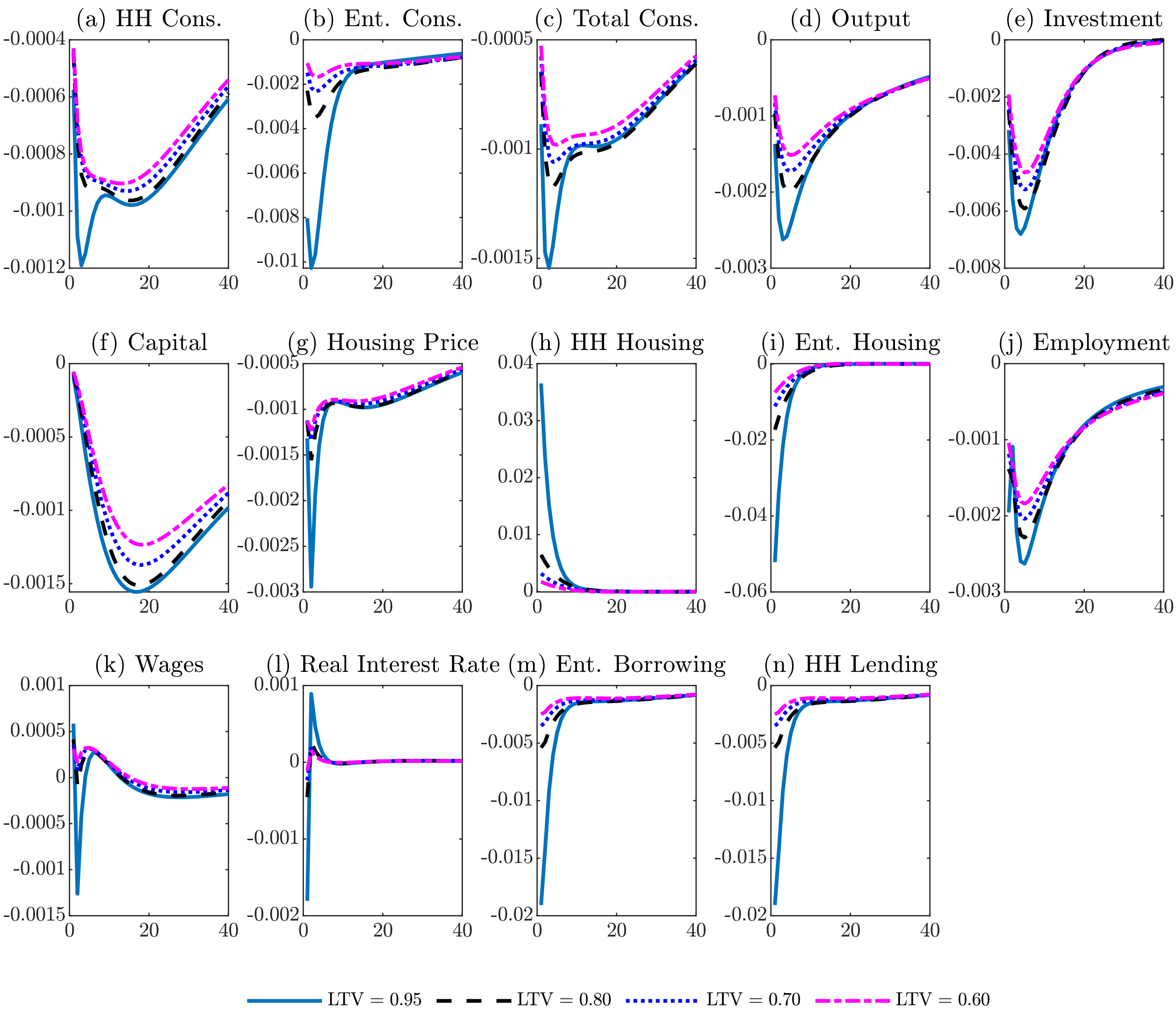
FIGURE 4: IMPACT OF A NEGATIVE HOUSING DEMAND SHOCK



NOTE: Numbers on the horizontal axis are quarters since the shock. Numbers on the vertical axis show percentage deviation from steady state.

and this leads entrepreneurs to reduce their consumption initially. Afterwards, entrepreneurs sell their housing and channel a part of the sale proceeds towards increasing their consumption while funneling the rest of the funds into investment. The logic is as follows. After housing prices decline, they are no longer as valuable as before. Recall that housing serves the dual purpose of being a production input and a collaterizable asset. Entrepreneurs then first sell housing which causes a fall in investment. This results in a persistent reduction in capital stock. Employment, aggregate output and consumption all fall before overshooting their pre-shock levels. Later, entrepreneurs direct part of their sale proceeds from selling houses into investment in capital since it can still produce output. This leads to a boom in investment which overshoots its previous steady-state level after an initial fall and remains elevated for a protracted period. In fact, it does not return to its steady-state value even after 40 quarters. This investment boom is accompanied by an increase in capital stock, employment and wages. This rise in labor and wages lead households to raise their consumption. As a result of rise in investment, capital, labor and wages, aggregate consumption and output overshooot their previous steady-state values after their initial drop and remain elevated for an extended period.

Figure 5: Impact of a Negative Labor Shock



Note: Numbers on the horizontal axis are quarters since the shock. Numbers on the vertical axis show percentage deviation from steady state.

The mechanism driving state-dependence in effects of a negative housing demand shock is related to how entrepreneurs's housing stock changes at various steady-state LTV ratios. At higher steady-state LTV ratios, entrepreneurs respond more forcefully to a negative housing demand shock and sell bigger chunk of their housing stock. Larger sale of housing at higher steady-state LTV ratios results in a larger fall in housing prices which tightens entrepreneur's borrowing constraint. Entrepreneurs then face a higher incentive to sell their housing in a bid to relax their collateral constraint and they sell larger parts of their housing stock causing a greater drop in housing prices. The incentive to sell housing in the face of a negative housing demand shock is smaller at lower steady-state LTV ratios. Consequently, entrepreneurs sell less of their housing stock and it leads to a smaller reduction in housing prices and a smaller effect on other macroeconomic variables.

Figure 5 reports behavior of select economic variables to a negative labor supply shock. Labor shocks have somewhat weaker but persistent effects in this model and they continue to display clear state-dependence. In the aftermath of a labor supply shock, employment, output, agggregate consumption, capital stock and housing prices – all decline persistently. Households reduce their labor supply and cut down their consumption both of which persistently remain below their pre-shock level. Entrepreneurs curtail their investment which reduces the stock of capital in the economy and offload part of their housing stock. This leads to a drop in housing prices and households use this opportunity to increase their buying of housing stock sold off by entrepreneurs. This is accompanied by a modest but sustained drop in aggregate consumption, investment and output. These effects are more pronounced at higher steady-state LTV ratios which shows their state-dependence.

The mechanism behind the state-dependence in effects of labor supply shocks is as follows. After a labor shock, the disutility of working increases and households, as a result, decrease their labor supply. This leads to a fall in wages after a short-lived spike which is plausibly driven by initial shortage of labor. Since labor wages are one of households's income sources (the other being interest income from lending), a fall in it is accompanied by a decline in household's consumption. A decrease in labor supply reduces the return on entrepreneur's deployed capital who consequently scale back their production and cut down their investment. Recall that entrepreneurs combine housing with capital to produce output. This has the implication that when entrepreneurs wind down their investment, they reduce their capital stock and sell their housing. Sale of housing stock leads to a fall in its price which tightens their borrowing constraint and induces them to sell it further. Entrepreneurs reduce their consumption since return on their deployed capital has fallen and they are engaged in winding down their investment and selling their housing which is bought by households. As a result of these events, aggregate investment, capital, consumption and output all decline. This decline is greater at higher steady-state LTV ratios since entrepreneurs engage in greater fire sale of housing at higher LTV ratios in order to relax their borrowing constraint. This illustrates how negative labor shocks show state-dependence and how higher initial LTV ratios amplify labor supply shocks in presence of collateral-constrained investment.

5 Conclusion

This paper provides a simple framework to examine macroeconomic effects of shocks to LTV ratios using a simple two-agent real model. The analysis in this paper shows that effects of LTV shocks are highly state-dependent and non-linear. A shock that leads to the same magnitude of decline in steady-state LTV ratio can have dramatically different effects depending on whether the initial LTV ratios are already high or comparatively lower. The paper also shows that the nature of impact depends upon the nature of the shocks. While transitory shocks lead to interesting macroeconomic fluctuations, permanent shocks lead to permanent decline in housing prices and a sustained reduction in investment, output, capital stock and borrowing. I also show that a permanent LTV tightening leads to a permanent decline in wages which is a novel finding in the literature. In addition, other shocks such as TFP shocks, housing demand shocks and labor supply shocks all show clear state-dependence and have highly persistent effects in the model. These results point towards how the same magnitude of shocks can have dramatically different macroeconomic impact and underscore the importance of taking into account steady-state LTV ratios when studying macroeconomic consequences of various shocks.

For clarity of exposition and in my attempt to keep the analysis as simple as possible, I developed a simple real model in this paper which abstracts from other forces which might have an impact on the effects discussed in this paper. For example, the model is real and features no real or nominal rigidities. Considering them might yield additional insight into the effects of various economic shocks. Additionally, this work completely abstracts from modelling formal financial intermediation. It assumes that households lend and entrepreneurs borrow and banking sector plays no role at all. In reality though, most if not all of this financial transfer happens through banks and other financial intermediaries. Modelling them and their features could have important implications for the results in this paper. Such and other related research efforts are promising and their results can be useful for macroeconomic and macroprudential policy.

References

Beaudry, P. and A. Lahiri (2014): "The Allocation of Aggregate Risk, Secondary Market Trades, and Financial Boom–Bust Cycles," *Journal of Money, Credit and Banking*, 46, 1–42, https://doi.org/10.1111/jmcb.12096.

Boz, E. and E. G. Mendoza (2014): "Financial innovation, the discovery of risk, and the US credit crisis," *Journal of Monetary Economics*, 62, 1–22, https://doi.org/10.1016/j.jmoneco.2013.07.001.

Chaney, T., D. Sraer, and D. Thesmar (2012): "The collateral channel: How real estate shocks affect corporate investment," *American Economic Review*, 102, 2381–2409, https://www.aeaweb.org/articles?id=10.1257/aer.102.6.2381.

Christiano, L. J., R. Motto, and M. Rostagno (2010): "Financial factors in economic fluctuations," https://ssrn.com/abstract=1600166.

De Veirman, E. (2023): "Loan-to-Value Shocks and Macroeconomic Stability," *De Nederlandsche Bank Working Paper*, https://papers.ssrn.com/sol3/papers.cfm?abstract_id=4330441.

Favilukis, J., S. C. Ludvigson, and S. Van Nieuwerburgh (2017): "The macroeconomic effects of housing wealth, housing finance, and limited risk sharing in general equilibrium," *Journal of Political Economy*, 125, 140–223, https://doi.org/10.1086/689606.

Ferrero, A. (2015): "House price booms, current account deficits, and low interest rates," *Journal of Money, Credit and Banking*, 47, 261–293, https://doi.org/10.1111/jmcb.12202.

Huo, Z. and J.-V. Ríos-Rull (2016): "Financial frictions, asset prices, and the great recession," *CEPR Discussion Paper No. DP11544*, https://papers.ssrn.com/sol3/papers.cfm?abstract_id=2847078.

Iacoviello, M. (2005): "House prices, borrowing constraints, and monetary policy in the business cycle," *American economic review*, 95, 739–764, https://www.aeaweb.org/articles?id=10.1257/0002828054201477.

Jensen, H., I. Petrella, S. H. Ravn, and E. Santoro (2020): "Leverage and deepening business-cycle skewness," *American Economic Journal: Macroeconomics*, 12, 245–81, https:

//www.aeaweb.org/articles?id=10.1257/mac.20170319.

Jensen, H., S. H. Ravn, and E. Santoro (2018): "Changing credit limits, changing business cycles," *European Economic Review*, 102, 211–239, https://doi.org/10.1016/j.euroecorev.2017.12.008.

Justiniano, A., G. E. Primiceri, and A. Tambalotti (2015): "Household leveraging and deleveraging," *Review of Economic Dynamics*, 18, 3–20, https://doi.org/10.1016/j.red.2014.10.003.

Kaplan, G., K. Mitman, and G. L. Violante (2020): "The housing boom and bust: Model meets evidence," *Journal of Political Economy*, 128, 3285–3345, https://doi.org/10.1086/708816.

Kiyotaki, N., A. Michaelides, and K. Nikolov (2011): "Winners and losers in housing markets," *Journal of Money, Credit and Banking*, 43, 255–296, https://doi.org/10.1111/j.1538-4616.2011.00374.x.

——— (2023): "Housing, distribution and welfare," *Journal of Money, Credit and Banking, forthcoming*, https://papers.ssrn.com/sol3/papers.cfm?abstract_id=3584351.

Kiyotaki, N. and J. Moore (1997): "Credit cycles," *Journal of political economy*, 105, 211–248, https://doi.org/10.1086/262072.

Liu, Z., P. Wang, and T. Zha (2013): "Land-price dynamics and macroeconomic fluctuations," *Econometrica*, 81, 1147–1184, https://doi.org/10.3982/ECTA8994.

Sharma, V. (2023a): "Collateral shocks, lending relationships and economic dynamics," *CAMA Working Paper 49/2023*, https://cama.crawford.anu.edu.au/sites/default/files/publication/cama_crawford_anu_edu_au/2023-10/49_2023_sharma.pdf.

——— (2023b): "Lending relationships and loan-to-value shocks," *Unpublished Manuscript*, https://sharmavivek.com/assets/lrltv.pdf.

——— (2023c): "Shocks to the lending standards and the macroeconomy," *CAMA Working Paper 55/2023*, https://cama.crawford.anu.edu.au/sites/default/files/publication/cama_crawford_anu_edu_au/2023-11/55_2023_sharma.pdf.

Smets, F. and R. Wouters (2007): "Shocks and frictions in US business cycles: A Bayesian DSGE approach," *American economic review*, 97, 586–606, https://www.aeaweb.org/articles?id=10.1257/aer.97.3.586.

Appendix (For Online Publication)

State-Dependent Effects of Loan-to-Value Shocks

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November 6, 2023

Contents

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A Derivation of FOCs

A.1 Households

The Lagrangian of households is

$$\mathscr{L}_t = \mathbb{E}_t \left\{ \sum_{t=0}^{\infty} \left(\beta^P\right)^t \begin{bmatrix} \log\left(C_t^P - \gamma^P C_{t-1}^P\right) - \iota_t N_t + \varsigma_t \log H_t^P \\ -\lambda_t^P \left[C_t^P + Q_t^H \left(H_t^P - H_{t-1}^P\right) + R_{t-1} B_{t-1}^P - W_t N_t - B_{t-1}^P\right] \end{bmatrix} \right\} \tag{A.1}$$

The problem yields the following first order conditions:

$$\frac{\partial \mathscr{L}}{\partial C_t^P} : \frac{1}{C_t^P - \gamma^P C_{t-1}^P} - \beta^P \mathbb{E}_P \frac{\gamma^P}{C_{t+1}^P - \gamma^P C_t^P} = \lambda_t^P \tag{A.2}$$

$$\frac{\partial \mathscr{L}}{\partial B_t^P} : \beta^P \mathbb{E}_t \lambda_{t+1}^P = \frac{\lambda_t^P}{R_t} \tag{A.3}$$

$$\frac{\partial \mathscr{L}}{\partial H_t^P} : \frac{\varsigma_t}{H_t^P} + \beta^P \mathbb{E}_t \left(\lambda_{t+1}^P Q_{t+1}^H\right) = \lambda_t^P Q_t^H \tag{A.4}$$

$$\frac{\partial \mathscr{L}}{\partial N_t} : \iota_t = \lambda_t^P W_t \tag{A.5}$$

A.2 Entrepreneurs

Entrepreneur's optimization problem can be written as

$$\mathscr{L}_t = \mathbb{E}_t \left\{ \sum_{t=0}^{\infty} \left(\beta^E\right)^t \begin{bmatrix} \log\left(C_t^E - \gamma^E C_{t-1}^E\right) \\ -\lambda_t^E \left[C_t^E + R_{t-1} B_{t-1}^E - Y_t + W_t N_t + I_t + Q_t^H \left(H_t^E - H_{t-1}^E\right) - B_t^E\right] \\ -\mu_t^E \left[R_t B_t^E - \theta_t \mathbb{E}_t \left(Q_{t+1}^H H_t^E + Q_{t+1}^K K_t\right)\right] \\ -\kappa_t^E \left[K_t - (1-\delta) K_{t-1} - \left\{1 - \frac{\Omega}{2}\left(\frac{I_t}{I_{t-1}} - 1\right)^2\right\} I_t\right] \end{bmatrix} \right\} \tag{A.6}$$

where $Y_t = A_t (N_t)^{1-\alpha} \left\{ \left(H_{t-1}^E\right)^{\phi} (K_{t-1})^{1-\phi} \right\}^{\alpha}$ may be inserted for Y_t in the budget constraint. Solving entrepreneur's optmization problem, the first order conditions are:

$$\frac{\partial \mathcal{L}}{\partial C_t^E} : \frac{1}{C_t^E - \gamma^E C_{t-1}^E} - \beta^E \mathbb{E}_t \frac{\gamma^E}{C_{t+1}^E - \gamma^E C_t^E} = \lambda_t^E \tag{A.7}$$

$$\frac{\partial \mathcal{L}}{\partial B_t^E} : \lambda_t^E = \beta^E \mathbb{E}_t \lambda_{t+1}^E R_t + \mu_t^E R_t \tag{A.8}$$

$$\frac{\partial \mathcal{L}}{\partial N_t} : W_t = (1-\alpha) \frac{Y_t}{N_t} \tag{A.9}$$

$$\frac{\partial \mathcal{L}}{\partial H_t^E} : \lambda_t^E Q_t^H = \beta^E \mathbb{E}_t \left\{ \lambda_{t+1}^E \left(Q_{t+1}^H + \alpha\phi \frac{Y_{t+1}}{H_t^E} \right) \right\} + \mu_t^E \theta_t \mathbb{E}_t Q_{t+1}^H \tag{A.10}$$

$$\frac{\partial \mathcal{L}}{\partial K_t} : \kappa_t^E = \alpha (1-\phi) \beta^E \mathbb{E}_t \left(\frac{\lambda_{t+1}^E Y_{t+1}}{K_t} \right) + \beta^E (1-\delta) \mathbb{E}_t \kappa_{t+1}^E + \mu_t^E \theta_t \mathbb{E}_t Q_{t+1}^K \tag{A.11}$$

$$\frac{\partial \mathcal{L}}{\partial I_t} : \lambda_t^E = \kappa_t^E \left\{ 1 - \frac{\Omega}{2} \left(\frac{I_t}{I_{t-1}} - 1 \right)^2 - \Omega \frac{I_t}{I_{t-1}} \left(\frac{I_t}{I_{t-1}} - 1 \right) \right\} + \beta^E \Omega \mathbb{E}_t \left\{ \kappa_{t+1}^E \left(\frac{I_{t+1}}{I_t} \right)^2 \left(\frac{I_{t+1}}{I_t} - 1 \right) \right\} \tag{A.12}$$

B List of Equations

B.1 Households

$$\frac{1}{C_t^P - \gamma^P C_{t-1}^P} - \beta^P \mathbb{E}_t \frac{\gamma^P}{C_{t+1}^P - \gamma^P C_t^P} = \lambda_t^P \tag{B.1}$$

$$\beta^P \mathbb{E}_t \lambda_{t+1}^P = \frac{\lambda_t^P}{R_t} \tag{B.2}$$

$$\frac{\varsigma_t}{H_t^P} + \beta^P \mathbb{E}_t \left(\lambda_{t+1}^P Q_{t+1}^H \right) = \lambda_t^P Q_t^H \tag{B.3}$$

$$\iota_t = \lambda_t^P W_t \tag{B.4}$$

B.2 Entrepreneurs

$$\frac{1}{C_t^E - \gamma^E C_{t-1}^E} - \beta^E \mathbb{E}_t \frac{\gamma^E}{C_{t+1}^E - \gamma^E C_t^E} = \lambda_t^E \tag{B.5}$$

$$\beta^E \mathbb{E}_t \left(\lambda_{t+1}^E R_t \right) + \mu_t^E R_t = \lambda_t^E \tag{B.6}$$

$$W_t = (1-\alpha) \frac{Y_t}{N_t} \tag{B.7}$$

$$\lambda_t^E Q_t^H = \beta^E \mathbb{E}_t \left\{ \lambda_{t+1}^E \left(Q_{t+1}^H + \alpha\phi \frac{Y_{t+1}}{H_t^E} \right) \right\} + \mu_t^E \theta_t \mathbb{E}_t Q_{t+1}^H \tag{B.8}$$

$$\kappa_t^E = \alpha\left(1-\phi\right)\beta^E \mathbb{E}_t\left(\frac{\lambda_{t+1}^E Y_{t+1}}{K_t}\right) + \beta^E\left(1-\delta\right)\mathbb{E}_t\kappa_{t+1}^E + \mu_t^E\theta_t\mathbb{E}_t Q_{t+1}^K \tag{B.9}$$

$$\lambda_t^E = \kappa_t^E\left\{1-\frac{\Omega}{2}\left(\frac{I_t}{I_{t-1}}-1\right)^2 - \Omega\frac{I_t}{I_{t-1}}\left(\frac{I_t}{I_{t-1}}-1\right)\right\} + \beta^E\Omega\mathbb{E}_t\left\{\kappa_{t+1}^E\left(\frac{I_{t+1}}{I_t}\right)^2\left(\frac{I_{t+1}}{I_t}-1\right)\right\} \tag{B.10}$$

$$C_t^E + R_{t-1}B_{t-1}^E = Y_t - W_t N_t - I_t - Q_t\left(H_t^E - H_{t-1}^E\right) + B_t^E \tag{B.11}$$

$$B_t^E = \frac{\theta_t a_t}{R_t} \tag{B.12}$$

$$a_t = \mathbb{E}_t\left(Q_{t+1}^H H_t^E + Q_{t+1}^K K_t\right) \tag{B.13}$$

$$\kappa_t^E = \lambda_t^E Q_t^K \tag{B.14}$$

B.3 Market Clearing and Resource Constraints

$$C_t^P + C_t^E + I_t = Y_t \tag{B.15}$$

$$H_t^P + H_t^E = H \tag{B.16}$$

$$B_t^P + B_t^E = 0 \tag{B.17}$$

$$Y_t = A_t\left(N_t\right)^{1-\alpha}\left\{\left(H_{t-1}^E\right)^{\phi}\left(K_{t-1}\right)^{1-\phi}\right\}^{\alpha} \tag{B.18}$$

$$K_t = \left(1-\delta\right)K_{t-1} + \left\{1-\frac{\Omega}{2}\left(\frac{I_t}{I_{t-1}}-1\right)^2\right\}I_t \tag{B.19}$$

C Steady State Conditions

From household's FOC with respect to consumption (B.1) and labor (B.4), I have

$$\frac{1-\beta^P\gamma^P}{\left(1-\gamma^P\right)C^P} = \lambda^P \tag{C.1}$$

and

$$\iota = \lambda^P W \tag{C.2}$$

respectively. Household's FOC with respect to debt (B.2) yields the steady-state gross interest rate

$$R = \frac{1}{\beta^P} \tag{C.3}$$

which underscores the fact that the time preference of the most patient agent determines the steady-state rate of interest. (B.3) yields

$$\begin{aligned}\frac{\varsigma}{H^P} + \beta^P \lambda^P Q^H &= \lambda^P Q^H \\ \Rightarrow Q^H H^P &= \frac{\varsigma}{\lambda^P \left(1 - \beta^P\right)} \\ \Rightarrow H^P &= \frac{\varsigma}{Q^H \lambda^P \left(1 - \beta^P\right)}\end{aligned} \tag{C.4}$$

I now turn to entrepreneurs. Their consumption FOC (B.5) yields

$$\frac{1 - \beta^E \gamma^E}{\left(1 - \gamma^E\right) C^E} = \lambda^E \tag{C.5}$$

Entrepreneur's FOC with respect to debt (B.6) gives

$$\begin{aligned}\beta^E \lambda^E R^L + \mu^E R &= \lambda^E \\ \Rightarrow \mu^E &= \frac{\lambda^E \left(1 - \beta^E R\right)}{R}\end{aligned} \tag{C.6}$$

The borrowing constraint for entrepreneurs binds only if μ^E is positive. This implies that β^E must be less than R^L. In the baseline calibration, β^E is set to 0.95 whereas the steady state value of R^L is 1.0219 which implies that β^E must be less than 0.9786 which is indeed the case. Entrepreneur's production function is

$$Y = A\left(N\right)^{1-\alpha} \left[\left(H^E\right)^{\phi} \left(K\right)^{1-\phi}\right]^{\alpha} \tag{C.7}$$

From firm's labor choice for househods (B.7),

$$W = \left(1 - \alpha\right) \frac{Y}{N} \tag{C.8}$$

Entrepreneur's FOC with respect to housing (B.8) gives

$$\begin{aligned}\lambda^E Q^H &= \beta^E \lambda^E \left(Q^H + \alpha\phi \frac{Y}{H^E}\right) + \mu^E \theta Q^H \\ \Rightarrow \frac{Q^H H^E}{Y} &= \frac{\beta^E \alpha \phi R}{\left(1 - \beta^E\right) R - \theta \left(1 - \beta^E R\right)}\end{aligned} \tag{C.9}$$

From aggregate law of motion for capital (B.19)

$$K = (1-\delta) K + \left[1 - \frac{\Omega}{2}\left(\frac{I}{I} - 1\right)\right] I$$
$$\Rightarrow I = \delta K \tag{C.10}$$

I have the following steady-state resource constraints

$$Y = C^P + C^E + I \tag{C.11}$$

$$H = H^P + H^E \tag{C.12}$$

$$B^P + B^E = 0 \tag{C.13}$$

Also, I have the following steady-state version of agents' budget constraints (one of them is redundant because of Walras' Law)

$$C^P = WN - (R-1)B^P \tag{C.14}$$

$$C^E = Y - RB^E - WN - I + B^E \tag{C.15}$$

So the steady state is characterized by the vector

$$\left[Y, C^P, C^E, I, H^P, H^E, K, N, W, B^P, B^E, Q^H, Q^K, R, \lambda^P, \lambda^E, \mu^E\right]$$

From entrepreneur's optimal choice of capital (B.9), I have

$$\kappa_t^E = \alpha(1-\alpha)\beta^E \mathbb{E}_t\left(\frac{\lambda_{t+1}^E Y_{t+1}}{K_t}\right) + \beta^E(1-\delta)\mathbb{E}_t\kappa_{t+1}^E + \mu_t^E\theta_t\mathbb{E}_t Q_{t+1}^K$$
$$\Rightarrow \frac{\kappa^E}{\lambda^E}\left(1 - (1-\delta)\beta^E\right) = \alpha(1-\phi)\beta^E\frac{Y}{K} + \frac{\left(1-\beta^E R\right)}{R}\theta Q^K \tag{C.16}$$

Entrepreneur's optimal choice of investment (B.10) yields

$$\lambda_t^E = \kappa_t^E\left[1 - \frac{\Omega}{2}\left(\frac{I_t}{I_t} - 1\right)^2 - \Omega\frac{I_t}{I_t}\left(\frac{I_t}{I_{t-1}} - 1\right)\right] + \beta^E\Omega\mathbb{E}_t\left[\kappa_{t+1}^E\left(\frac{I_{t+1}}{I_t}\right)^2\left(\frac{I_{t+1}}{I_t} - 1\right)\right]$$
$$\Rightarrow \lambda^E = \kappa^E \tag{C.17}$$

Combining this with steady state version of

$$\kappa^E = \lambda^E Q^K \tag{C.18}$$

I obtain $Q^K = 1$ in the steady state. Plugging this into (C.16), I obtain the expression for capital-to-output ratio

$$\begin{aligned}\frac{\kappa^E}{\lambda^E}\left(1-(1-\delta)\beta^E\right) &= \alpha(1-\phi)\beta^E\frac{Y}{K}+\frac{\left(1-\beta^E R\right)}{R}\theta Q^K\\ \Rightarrow \frac{K}{Y} &= \frac{\alpha(1-\phi)R\beta^E}{R\left(1-(1-\delta)\beta^E\right)-\theta\left(1-\beta^E R\right)}\end{aligned} \tag{C.19}$$

Next, combining (B.12) and (B.13)

$$B^E = \frac{\theta}{R}\left(Q^H H^E + Q^K K\right) \tag{C.20}$$

Dividing by Y, the above expression becomes

$$\begin{aligned}\frac{B^E}{Y} &= \frac{\theta}{R}\left(\frac{Q^H H^E}{Y}+\frac{Q^K K}{Y}\right)\\ \Rightarrow \frac{B^E}{Y} &= \alpha\theta\beta^E\left[\frac{\phi}{R\left(1-\beta^E\right)-\theta\left(1-\beta^E R\right)}+\frac{(1-\phi)}{R\left(1-(1-\delta)\beta^E\right)-\theta\left(1-\beta^E R\right)}\right]\end{aligned} \tag{C.21}$$

From entrepreneur's budget constraint (B.11)

$$C^E + RB^E = Y - WN - I + B^E \tag{C.22}$$

Rewriting this in ratio to output

$$\begin{aligned}\frac{C^E}{Y}+\frac{RB^E}{Y} &= 1-\frac{WN}{Y}-\frac{I}{Y}+\frac{B^E}{Y}\\ \Rightarrow \frac{C^E}{Y} &= \alpha-\delta\frac{K}{Y}+(1-R)\frac{B^E}{Y}\end{aligned} \tag{C.23}$$

Steady-state budget constraint of household, in ratio to output, reads

$$\begin{aligned}\frac{C^P}{Y} &= \frac{WN}{Y}+(1-R)\frac{B^P}{Y}\\ &= (1-\alpha)+\frac{(1-R)B^P}{Y}\end{aligned} \tag{C.24}$$

Dividing the above two expressions by each other, I have

$$\begin{aligned}
\frac{\frac{Q^H H^P}{Y}}{\frac{Q^H H^E}{Y}} &= \frac{\frac{\varsigma}{Y\lambda^P\left(1-\beta^P\right)}}{\frac{\beta^E\alpha\phi R}{\left(1-\beta^P\right)R-\theta\left(1-\beta^E R\right)}} \\
\Rightarrow \frac{H^P}{H^E} &= \frac{\varsigma}{Y\frac{1-\beta^P\gamma^P}{\left(1-\gamma^P\right)C^P}\left(1-\beta^P\right)}\frac{\left(1-\beta^E\right)R-\theta\left(1-\beta^E R\right)}{\beta^E\alpha\phi R} \\
\Rightarrow \frac{H^P}{H-H^P} &= \frac{\varsigma\left(1-\gamma^P\right)}{\left(1-\beta^P\right)\left(1-\beta^P\gamma^P\right)}\frac{\left(1-\beta^P\right)R-\theta\left(1-\beta^E R\right)}{\beta^E\alpha\phi R}\frac{C^P}{Y} \qquad \text{(C.25)}
\end{aligned}$$

Steady state version of aggregate resource constraint (B.15) is

$$C^P + C^E + I = Y$$

Dividing by Y

$$\frac{C^P}{Y} = 1 - \frac{C^E}{Y} - \delta\frac{K}{Y} \qquad \text{(C.26)}$$

Combining (C.1), (C.2) and (C.8) gives steady-state equilibrium condition for households

$$\begin{aligned}
\iota &= \lambda^P W \\
\Rightarrow \iota &= \frac{1-\beta^P\gamma^P}{\left(1-\gamma^P\right)C^P}\left(1-\alpha\right)\frac{Y}{N} \\
\Rightarrow N &= \frac{\left(1-\beta^P\gamma^P\right)\left(1-\alpha\right)}{\iota\left(1-\gamma^P\right)}\left(\frac{C^P}{Y}\right)^{-1} \qquad \text{(C.27)}
\end{aligned}$$

From (B.18), steady state output is

$$\begin{aligned}
Y &= A\left(N\right)^{1-\alpha}\left[\left(H^E\right)^\phi\left(K\right)^{1-\phi}\right]^\alpha \\
Y^{1-\alpha} &= A\left(N\right)^{1-\alpha}\left[\left(\frac{H^E}{Y}\right)^\phi\left(\frac{K}{Y}\right)^{1-\phi}\right]^\alpha \\
Y^{1-\alpha} &= A\left(N\right)^{1-\alpha}\left[\left(\frac{H^E}{Y}\right)^\phi\left(\frac{\alpha\left(1-\phi\right)R\beta^E}{R\left(1-\left(1-\delta\right)\beta^E\right)-\theta\left(1-\beta^E R\right)}\right)^{1-\phi}\right]^\alpha \qquad \text{(C.28)}
\end{aligned}$$

From (C.4)

$$Q^H = \frac{\varsigma}{H^P\lambda^P\left(1-\beta^P\right)} \qquad \text{(C.29)}$$

D System of Loglinear Equations

The system of equations log-linearized around their steady state is as below:

D.1 Optimality Conditions of Households

Equations (B.1), (B.2) and (B.4) become

$$\beta^P \gamma^P \mathbb{E}_t \widehat{C}^P_{t+1} - \left(1 + \left(\gamma^P\right)^2 \beta^P\right) \widehat{C}^P_t + \gamma^P \widehat{C}^P_{t-1} = \left(1 - \beta^P \gamma^P\right) \left(1 - \gamma^P\right) \widehat{\lambda}^P \tag{D.1}$$

$$\mathbb{E}_t \widehat{\lambda}^P_{t+1} = \widehat{\lambda}^P_t - \widehat{R}_t \tag{D.2}$$

$$\widehat{\iota}_t = \widehat{\lambda}^P_t + \widehat{W}_t \tag{D.3}$$

Log-linearization of (B.3) yields

$$\left(1 - \beta^P\right) \left(\widehat{\varsigma}_t - \widehat{H}^P_t\right) + \beta^P \mathbb{E}_t \left[\widehat{\lambda}^P_{t+1} + \widehat{Q}^H_{t+1}\right] = \widehat{\lambda}^P_t + \widehat{Q}^H_t \tag{D.4}$$

D.2 Optimality Conditions of Entrepreneurs

From (B.5) and (B.6), I have

$$\beta^E \gamma^E \mathbb{E}_t \widehat{C}^E_{t+1} - \left(1 + \left(\gamma^E\right)^2 \beta^E\right) \widehat{C}^E_t + \gamma^E \widehat{C}^E_{t-1} = \left(1 - \beta^E \gamma^E\right) \left(1 - \gamma^E\right) \widehat{\lambda}^E_t \tag{D.5}$$

and

$$\widehat{\lambda}^E_t = \widehat{R}^L_t + \beta^E R^L \mathbb{E}_t \widehat{\lambda}^E_{t+1} + \left(1 - \beta^E R^L\right) \widehat{\mu}^E_t \tag{D.6}$$

Equation (B.7) yields

$$\widehat{W}_t = \widehat{Y}_t - \widehat{N}_t \tag{D.7}$$

From (B.8), I derive

$$\begin{aligned} \left(\widehat{\lambda}^E_t + \widehat{Q}^H_t\right) = {} & \beta^E \mathbb{E}_t \left(\widehat{\lambda}^E_{t+1} + \widehat{Q}^H_{t+1}\right) + \left(\frac{1}{R} - \beta^E\right) \theta \mathbb{E}_t \left(\widehat{\mu}^E_t + \widehat{\theta}_t + \widehat{Q}^H_{t+1}\right) \\ & + \left[\left(1 - \beta^E\right) - \theta \left(\frac{1}{R} - \beta^E\right)\right] \mathbb{E}_t \left[\widehat{\lambda}^E_{t+1} + \widehat{Y}_{t+1} - \widehat{H}^E_t\right] \end{aligned} \tag{D.8}$$

Equation (B.9) becomes

$$\widehat{Q}_t^K = \left[1 - \beta^E (1-\delta) - \theta \left(\frac{1}{R} - \beta^E\right)\right] \mathbb{E}_t \left[\widehat{\lambda}_{t+1}^E - \widehat{\lambda}_t^E + \widehat{Y}_{t+1} - \widehat{K}_t\right]$$
$$+ \beta^E (1-\delta) \mathbb{E}_t \left(\widehat{Q}_{t+1}^K + \widehat{\lambda}_{t+1}^E - \widehat{\lambda}_t^E\right) + \left(1 - \beta^E R\right) \frac{1}{R} \theta \mathbb{E}_t \left[\widehat{\mu}_t^E - \widehat{\lambda}_t^E + \widehat{\theta}_t + \widehat{Q}_{t+1}^K\right] \quad \text{(D.9)}$$

Equation (B.10) is approximated as

$$\widehat{Q}_t^K = \left(1 + \beta^E\right) \Omega \widehat{I}_t - \beta^E \Omega \mathbb{E}_t \widehat{I}_{t+1} - \Omega \widehat{I}_{t-1} \quad \text{(D.10)}$$

Entrepreneur's budget contraint (B.11) becomes

$$C^E \widehat{C}_t^E + RB^E \left(\widehat{R}_{t-1} + \widehat{B}_{t-1}^E\right) = Y \widehat{Y}_t - WN \left(\widehat{W}_t + \widehat{N}_t\right) - I \widehat{I}_t - Q^H H^E \left(\widehat{H}_t^E - \widehat{H}_{t-1}^E\right) + \widehat{B}_t^E B^E \quad \text{(D.11)}$$

The borrowing constraint (B.12) becomes

$$\widehat{l}_t = \widehat{\theta}_t + \widehat{a}_t - \widehat{R}_t \quad \text{(D.12)}$$

Equation (B.13) which shows entrepreneurs' total assets, becomes

$$\widehat{a}_t = \frac{Q^H H^E}{Q^H H^E + Q^K K} \mathbb{E}_t \left(\widehat{Q}_{t+1}^H + \widehat{H}_t^E\right) + \frac{Q^K K}{Q^H H^E + Q^K K} \mathbb{E}_t \left(\widehat{Q}_{t+1}^K + \widehat{K}_t\right) \quad \text{(D.13)}$$

Linearized version of (B.14) is

$$\widehat{\kappa}_t^E = \widehat{\lambda}_t^E + \widehat{Q}_t^K \quad \text{(D.14)}$$

D.3 Market Clearing and Resource Constraints

Equations (B.15) and (B.16) yield

$$\widehat{Y}_t = \frac{C^P}{Y} \widehat{C}_t^P + \frac{C^E}{Y} \widehat{C}_t^E + \frac{I}{Y} \widehat{I}_t \quad \text{(D.15)}$$

and

$$H^P \widehat{H}_t^P + H^E \widehat{H}_t^E = 0 \quad \text{(D.16)}$$

(B.17) gives

$$\widehat{B}_t^P B^P + \widehat{B}_t^E B^E = 0 \tag{D.17}$$

From (B.18), I have

$$\widehat{Y}_t = \widehat{A}_t + (1-\alpha)\,\widehat{N}_t + \alpha\phi\widehat{H}_{t-1}^E + \alpha\,(1-\phi)\,\widehat{K}_{t-1} \tag{D.18}$$

Equation (B.19) yields

$$\widehat{K}_t = (1-\delta)\,\widehat{K}_{t-1} + \delta\widehat{I}_t \tag{D.19}$$