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Loan-to-Value Shocks and Housing in the Production Function

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Keywords

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Loan-to-Value Shocks and Housing in the Production Function

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Abstract

Using a Two-Agent RBC model with time-varying shock to loan-to-value (LTV) ratios, I show that including housing (real estate or land) in the entrepreneurial production function has profound implications for results. In a model in which housing does not play a role as a production input, an LTV tightening has starkly different effects compared to a model in which it is a factor in the production process. In a setup devoid of a role for housing as a production input, differently from the results in the current literature, an LTV tightening leads to a spike in housing price at impact and a lesser fall afterwards. Other macroeconomic variables such as investment and output fall more at lower initial LTV ratios than at higher steady state LTV ratios. The findings of this paper indicate that housing plays an important role in shaping macroeconomic effects of LTV shocks.

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1 Introduction

This paper documents the macroeconomic effects of Loan to Value (LTV) shocks and shows that effects of an LTV tightening depend crucially on whether housing (real estate or land) is an input in entrepreneur's production function. In this paper, I build a simple two-agent RBC model in which households lend to entrepreneurs. These entrepreneurs are collateral-constrained and use housing both as a pledgeable asset to borrow and as a factor in their production process. I show that LTV shocks have very different impact on macroeconomic activity depending upon whether or not housing is a factor in the production process. This is the first paper in the literature that documents this heterogenous impact of LTV shocks.

The focus in this paper on illustrating how macroeconomic dynamics change after an LTV tightening and what role housing plays in it. In order to do so, I build a real model that considers the dual role of housing both as an entrepreneurial asset that can be pledged to obtain loans and as an input in the production process. In contrast with existing literature that has focused on studying state dependence of LTV shocks (see, for example, Sharma, 2023 and De Veirman, 2023), this paper focuses on casting light on how including or excluding housing as a production factor influences the results from the model and shows that findings from the analysis depend critically on this modelling choice. In this work, I explicitly consider the twin role of housing both as an entrepreneurial asset and as an input in production function. I then conduct simulations at various steady state LTV ratios to demonstrate that these differential effects in the aftermath of an LTV shocks persist at all initial LTV ratios considered in this paper.

When housing is an input in the production process, after an LTV tightening, entrepreneurs sell part of their housing stock, reduce their borrowing and use part of the proceeds from sale of housing to increase their consumption. This effect is greater at higher initial LTV ratios which demonstrates state dependence in their effects. However, in an alternative version of the model in which housing is not a factor in the production process, entrepreneurs 'dispose of' or 'destroy' their housing stock after an LTV tightening. This reduces the aggregate supply of housing in the market since it is in fixed supply. As a result, housing prices rise at impact which benefits entrepreneurs since it is a pledgeable asset and it raises their borrowing capacity. It highlights that results from a model that studies the macroeconomic effects of a tightening in lending conditions depend crucially on whether or not housing is a factor in the entrepreneurial production function. In contrast with results in the existing literature (for instance, Sharma, 2023 and De Veirman, 2023), in a model in which housing is not a production factor, an LTV tightening does not lead to a fall in housing price. Rather, it increases at impact. This is a novel finding and is in stark contrast with existing results in the literature that show that housing price declines in the wake of an LTV tightening.

I also show that, at lower steady state LTV ratios, macroeconomic variables fall more when housing is not a production input. In this version of model, investment falls more at lower steady state LTV ratios which then has a spillover effect on other macroeconomic variables such as employment, consumption and output. After initial fall, these variables return to their prior steady state and overshoot it. This recovery is faster than in the version of the model which does feature housing in its production function. Consequently, macroeconomic dynamics in this version of model display greater amplification after inital decline.

This work contributes to the current literature on macroeconomic effects of LTV tightening. A recent paper is De Veirman (2023) who embeds the model in Iacoviello (2005) with time-varying LTV shocks and studies their impact. Another related paper is Sharma (2023) who builds a real model in which LTV ratios are subject to a time-varying shock. These papers focus on state dependence and non-linearity in the effects of LTV shocks and neither of them analyze how the effects of LTV tightening and ensuing macroeconomic dynamics change when housing is not a factor in production input. As I show in this paper, this has profound implications for effects of LTV tightening.

The remainder of this paper is structured as follows. Section 2 presents the model environment in this paper. Section 3 and Section 4 discuss the model solution and results, respectively. Finally, Section 5 concludes.

2 Model

The paper features a discrete-time two-agent RBC model. The setup is identical to Sharma (2023) except that I abstract from all shocks other than a temporary LTV shock. The model has (patient) households who consume non-durable consumption goods, supply labor and derive utility from holding a durable good (housing). The (impatient) entrepreneurs, in turn, consume non-durable consumption goods, hire labor from households and run firms in the economy. Firms are owned by the entrepreneurs who are subject to a collateral constraint which limits their borrowing to a fraction of expected value of their total assets. Because of $\beta^E < \beta^P$, households act as lenders and entrepreneurs act as borrowers in the equilibrium. In what follows, I describe each agent's optimization problem.

2.1 Households

The household's problem bears similarities with those in Iacoviello (2005), Liu, Wang, and Zha (2013) and Justiniano, Primiceri, and Tambalotti (2015). Households have the utility function of the following form:

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \left(\beta^P\right)^t \left\{ \log\left(C_t^P - \gamma^P C_{t-1}^P\right) - \frac{N_t^\eta}{\eta} + \varsigma \log H_t^P \right\} \tag{1}$$

where C_t^P, N_t and H_t^P denote consumption, labor and housing respectively of the households, $\beta^P \in (0,1)$ is a discount factor and γ^P measures the degree of habit formation in consumption. The superscript P denotes (patient) households. The household faces the following budget constraint

$$C_t^P + Q_t^H \left(H_t^P - H_{t-1}^P\right) + R_{t-1} B_{t-1}^P \leq W_t N_t + B_t^P \tag{2}$$

Here, Q_t^H is the price of one unit of housing in terms of consumption goods, W_t is the real wage and R_{t-1} is the gross real interest rate on debt B_{t-1}^P. I assume housing does not depreciate. First order conditions of the households with respect to consumption, debt, housing and labor respectively can be written as

$$\frac{1}{C_t^P - \gamma^P C_{t-1}^P} - \beta^P \mathbb{E}_t \frac{\gamma^P}{C_{t+1}^P - \gamma^P C_t^P} = \lambda_t^P \tag{3}$$

$$\beta^P \mathbb{E}_t \lambda_{t+1}^P = \frac{\lambda_t^P}{R_t} \tag{4}$$

$$\frac{\varsigma}{H_t^P} + \beta^P \mathbb{E}_t \left(\lambda_{t+1}^P Q_{t+1}^H\right) = \lambda_t^P Q_t^H \tag{5}$$

$$N_t^{\eta-1} = \lambda_t^P W_t \tag{6}$$

where λ_t^P is the Lagrange multiplier associated with household's budget constraint (2). One can combine household's first-order conditions with respect to consumption (3) and debt (4) to obtain their Euler equation. Equation (5) describes household's Euler equation for housing and links today's housing price to the utility it provides plus the expected capital gain. Equation (6) describes household's consumption-lesiure tradeoff. All the derivations of first order conditions have been ralegated to the Appendix A.

2.2 Entrepreneurs

Following Iacoviello (2005) and Liu, Wang, and Zha (2013), the representative entrepreneur maximizes the utility obtained from consuming the non-durable consumption goods

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \left(\beta^E\right)^t \log\left(C_t^E - \gamma^E C_{t-1}^E\right) \tag{7}$$

where β^E and γ^E are as defined before. I assume that entrepreneurs are more impatient than the households, that is, $\beta^E < \beta^P$. Entrepreneurs face a collateral constraint à la Kiyotaki and Moore (1997) that limits the borrowing of each entrepreneur to a fraction of expected value of their assets

$$B_t^E \leq \frac{1}{R_t}\theta_t a_t \tag{8}$$

Here, B_t^E denotes entrepreneur's loan, expected value of entrepreneur's assets is a_t and R_t is the lending rate. Entrepreneur's borrowing is subject to a loan-to-value (LTV) requirement θ_t which follows the law of motion

$$\log\theta_t = (1-\rho_\theta)\log\theta + \rho_\theta \log\theta_{t-1} + \sigma_\theta\epsilon_{\theta,t} \tag{9}$$

where $\theta > 0$ is the steady-state LTV ratio, $\epsilon_{\theta,t}$ is iid innovation which follows a normal distribution with standard deviation σ_θ and $\rho_\theta \in (0,1)$. My goal in this paper is to examine the implications of exogenous changes in credit conditions, including for institutional and regulatory reasons. The LTV shock captures changes in lending that are exogenous from point of view of both lenders and borrowers. Changes in maximum LTVs capture not only shifts in macroprudential policy but also government guidelines to agencies that purchase mortgages such as government-sponsored enterprises (GSEs) like Fannie Mae and Freddie Mac about which mortgages can be bought (De Veirman, 2023). These guidelines then influence average LTVs of mortgages originated in the market. Expected value of entrepreneur's assets a_t is given by

$$a_t = \mathbb{E}_t\left(Q_{t+1}^H H_t^E + Q_{t+1}^K K_t\right) \tag{10}$$

In the above equation, Q_t^K denotes the value of installed capital in units of consumption goods, K_t stock of capital and H_t^E stock of housing[1]. Entrepreneurs produce output using a constant returns to scale production function

$$Y_t = A_t (N_t)^{1-\alpha} \left\{ \left(H_{t-1}^E\right)^{\phi} (K_{t-1})^{1-\phi} \right\}^{\alpha} \tag{11}$$

where Y_t is output, N_t labor input and $\alpha, \phi \in (0,1)$ are factor shares. TFP A_t follows the process

$$\log A_t = (1-\rho_A) \log A + \rho_A \log A_{t-1} + \sigma_A \epsilon_{A,t} \tag{12}$$

with iid innovation $\epsilon_{A,t}$ following a normal process with standard deviation σ_A and where $A > 0$ and $\rho_A \in (0,1)$. A number of studies recognize the important role housing (real estate or land) plays as a non-reproducible input in the production process and this provides justification for inluding land (housing) as a factor in the production function (see, among others, Iacoviello, 2005; Iacoviello and Neri, 2010; Lambertini, Mendicino, and Punzi, 2013; Liu, Wang, and Zha, 2013; Iacoviello, 2015; Ravn, 2016; Jensen, Ravn, and Santoro, 2018; Bekiros, Nilavongse, and Uddin, 2020 and Jensen, Petrella, Ravn, and Santoro, 2020). The evolution of capital obeys the following law of motion

$$K_t = (1-\delta) K_{t-1} + \left[1 - \frac{\Omega}{2} \left(\frac{I_t}{I_{t-1}} - 1 \right)^2 \right] I_t \tag{13}$$

where I_t is firm's investment level, $\delta \in (0,1)$ the rate of depreciation of capital stock and $\Omega > 0$ is a cost adjustment parameter. The entrepreneur faces the following budget constraint

$$C_t^E + R_{t-1} B_{t-1}^E \leq Y_t - W_t N_t - I_t - Q_t^H \left(H_t^E - H_{t-1}^E \right) + B_t^E \tag{14}$$

which states that entrepreneur's consumption and debt payment for the previous period should not exceed entrepreneur's output net of wage payment to labor, investment, changes in entrepreneur's housing stock and the debt in the current period. The FOCs of the entrepreneur

[1] Chaney, Sraer, and Thesmar (2012) and Liu, Wang, and Zha (2013) emphasize the importance of real estate as collateral for business loans.

with respect to consumption, debt, labor, housing, capital and investment, respectively are

$$\lambda_t^E = \frac{1}{C_t^E - \gamma^E C_{t-1}^E} - \beta^E \mathbb{E}_t \frac{\gamma^E}{C_{t+1}^E - \gamma^E C_t^E} \tag{15}$$

$$\lambda_t^E = \beta^E \mathbb{E}_t \lambda_{t+1}^E R_t + \mu_t^E R_t \tag{16}$$

$$W_t = (1-\alpha)\frac{Y_t}{N_t} \tag{17}$$

$$\lambda_t^E Q_t^H = \beta^E \mathbb{E}_t \left[\lambda_{t+1}^E \left(Q_{t+1}^H + \alpha\phi \frac{Y_{t+1}}{H_t^E}\right)\right] + \mu_t^E \theta_t \mathbb{E}_t Q_{t+1}^H \tag{18}$$

$$\kappa_t^E = \alpha(1-\phi)\beta^E \mathbb{E}_t \left(\frac{\lambda_{t+1}^E Y_{t+1}}{K_t}\right) + \beta^E (1-\delta)\mathbb{E}_t \kappa_{t+1}^E + \mu_t^E \theta_t \mathbb{E}_t Q_{t+1}^K \tag{19}$$

$$\lambda_t^E = \kappa_t^E \left[1 - \frac{\Omega}{2}\left(\frac{I_t}{I_{t-1}} - 1\right)^2 - \Omega \frac{I_t}{I_{t-1}}\left(\frac{I_t}{I_{t-1}} - 1\right)\right] + \beta^E \Omega \mathbb{E}_t \left[\kappa_{t+1}^E \left(\frac{I_{t+1}}{I_t}\right)^2 \left(\frac{I_{t+1}}{I_t} - 1\right)\right] \tag{20}$$

where μ_t^E, κ_t^E and λ_t^E are Lagrange multipliers associated with entrepreneur's collateral constraint (8), law of motion of capital (13) and entrepreneur's budget constraint (14). Entrepreneur's first order conditions with respect to consumption (15) and debt (16) may be combined to derive Euler equation for consumption for a collateral-constrained agent. Equation (17) describes entrepreneur's optimal demand for labor. Entrepreneur's Euler equation for land is described by (18) which relates its price today to its expected resale value tomorrow plus the payoff obatained by holding it for a period as given by its marginal productivity and its ability to serve as a collateral. Likewise, (19) is entrepreneur's Euler equation for capital and it links price of capital today to its price tomorrow and the expected payoff from keeping it for a period as given by its marginal productivity and its ability to serve as a collateral. Finally, entrepreneur's Euler equation for the investment is given by (20). Derivation of these first-order conditions have been consigned to Appendix A.

2.3 Aggregation and Market Clearing

Aggregate output in the economy equals consumption of households and entrepreneurs, and entrepreneurial investment

$$C_t^P + C_t^E + I_t = Y_t \tag{21}$$

The total amount of housing in the economy is fixed at H which implies that

$$H_t^P + H_t^E = H \tag{22}$$

Finally, the total net supply of debt in the economy is zero, i.e.

$$B_t^P + B_t^E = 0 \tag{23}$$

3 Model Solution and Prameterization

A period is a quarter in this model. The model is solved by log-linearizing the equilibrium conditions around the steady state and by using perturbation methods. Appendices B, C and D contain the list of equilibrium equations, the list of steady-state conditions and the system of log-linear equations, respectively. The calibration of parameters is standard and is summarized in Table 1. I allow for a relatively significant difference between discount factors of households and entrepreneurs so that steady-state value of Lagrange multiplier on entrepreneur's collateral constraint μ_t^E is different from zero. I therefore set $\beta^P = 0.995$ and $\beta^E = 0.95$.

Table 1: Parameter values

	Value	Description	Source/Target
β^P	0.995	Discount factor, households	Iacoviello (2005)
β^E	0.95	Discount factor, entrepreneurs	Iacoviello (2005)
γ^i, $i = \{P, E\}$	0.6	Habits in consumption, households, entrepreneurs	Smets and Wouters (2007)
η	1.01	Frisch elasticity of labor	Iacoviello (2005)
ς	0.1	Weight on housing	Iacoviello (2005)
α	0.3	Non-labor share of production	See Text
ϕ	0.1	Land share of non-labor input	Iacoviello (2005)
Ω	1.85	Investment adjustment cost parameter	See Text
δ	0.0285	Capital depreciation rate	Beaudry and Lahiri (2014)
ρ_A	0.95	Persistence of technology shock	Smets and Wouters (2007)
σ_A	0.0014	Standard deviation of technology shock	Standard

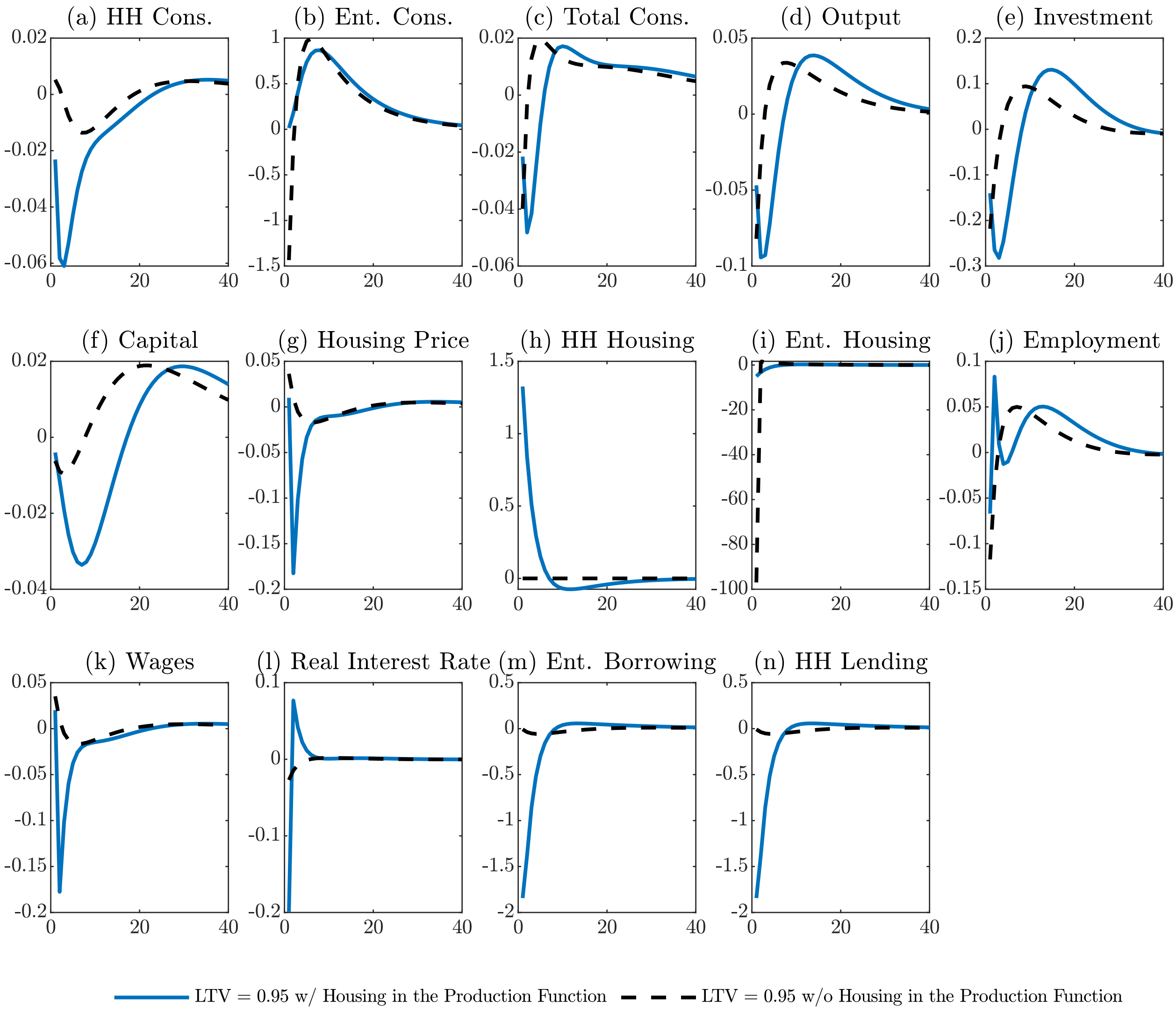
The degree of habit formation in consumption is chosen to be 0.6 which is in line with empirical estimates (Smets and Wouters, 2007). Following Iacoviello (2005), the Frisch elasticity of labor η is set to 1.01 and the wight on housing ς is given a value of 0.1. The labor income share is set to 0.3 (Liu, Wang, and Zha, 2013) and the input share of land in production is close to the value estimated in Liu, Wang, and Zha (2013) and Iacoviello (2005). The investment adjustment cost parameter is set to 1.85 (Ravn, 2016). The literature contains estimates which range from 0 (Liu, Wang, and Zha, 2013) to above 26 (Christiano, Motto, and Rostagno, 2010). The capital depreciation rate implies a steady-state ratio of non-residential investment to output a little above 0.13 (Beaudry and Lahiri, 2014). For calibration of shocks, I follow Smets and Wouters (2007) and set persistence of technology shock to 0.95 and its standard deviation to 0.0014 which is standard in the literature. I disucss the calibration of LTV shocks in the next section where I discuss their impact.

4 Discussion

In this section, I discuss the effects of LTV shocks. In order to keep the presentation clear and discussion easy to follow, I proceed in the following manner. I consider four steady state LTV ratios – 0.95, 0.80, 0.70 and 0.60. I then simulate the model with a temporary LTV shock that causes the respective steady-state ratio to decline by ten percentage point, that is, 0.95 becomes 0.85 after impact. I then separately simulate the model at these four steady-state LTV ratios with and without housing in the production function. The choice to separately simulate the model at these steady-state LTV ratios is to keep the impulse response plots clutter-free and to ease the exposition. I consider a temporary shock which means that the LTV ratios come back to their steady-state value at the rate $\rho = 0.90$.

First consider the case when housing is a factor in production process. A temporary LTV shock leads to a tightening of the collateral constraint for entrepreneurs who respond by reducing their borrowing and selling their housing (see Figure 1) which is bought by households. After a negative LTV shock, entrepreneurs consider their assets less useful in terms of how much those assets can help them borrow through their use as collateral. Consequently, entrepreneurs sell their housing and increase their consumption using part of proceeds from sale of their housing. This reduces housing price and combined with the fact that entrepreneurs have already sold part of their housing stock, it further tightens their borrowing constraint. This collateral tightening has ripple effects on investment since entrepreneurs cannot borrow now as much as they were able to before which then translates into a fall in capital stock, a drop in employment and wages, and a reduction in aggregate output. As Figures 2, 3 and 4 show, in virtually every case, higher the steady-state LTV ratio, bigger the drop. This shows that steady-state LTV ratios when the LTV shock hits play an important role in determining the magnitude of the impact. These state

Figure 1: Impact of a Temporary LTV Shock

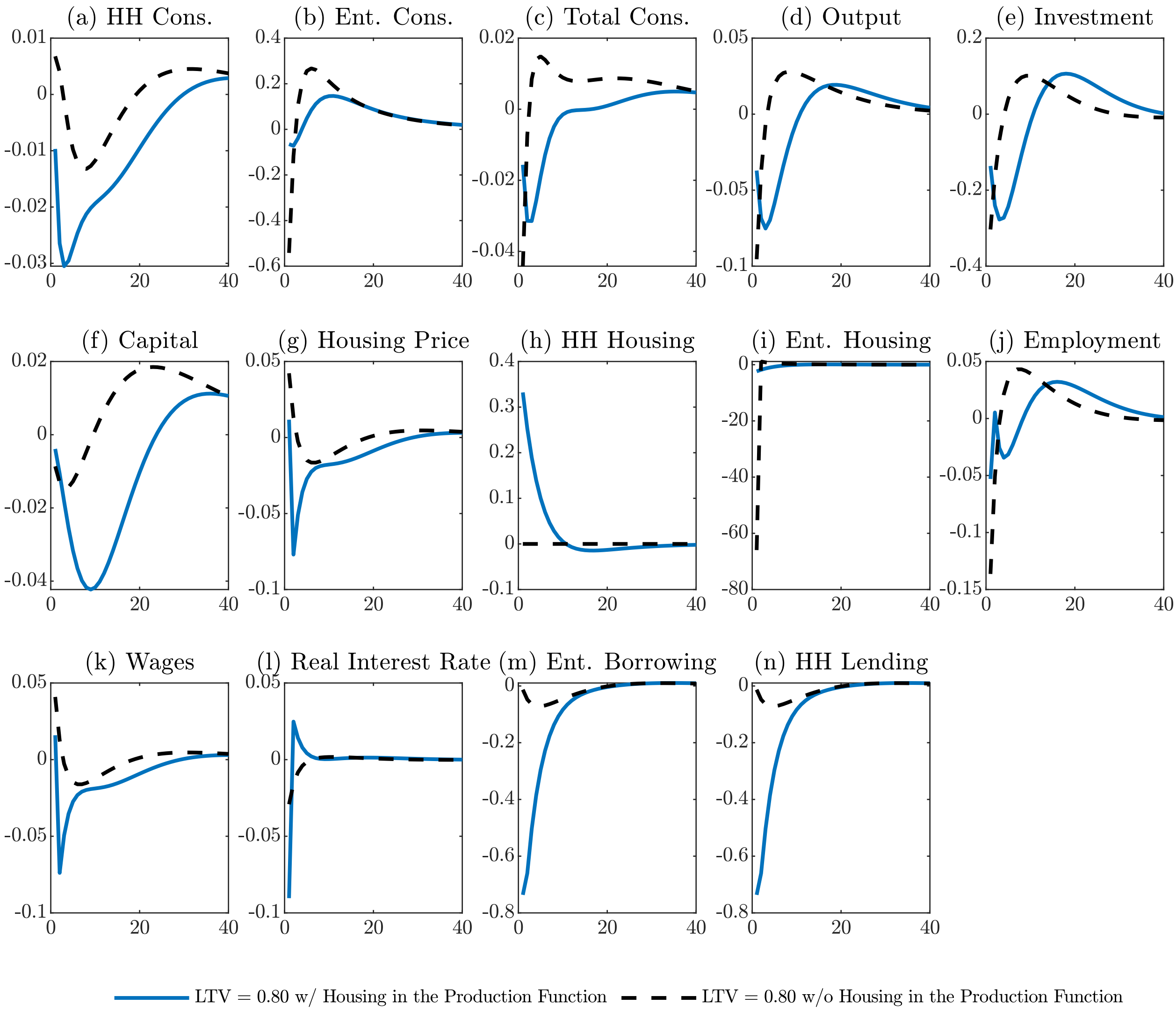


Note: Impact of a temporary shock that leads to 10 percentage point decline in LTV ratio from its steady-state value. Numbers on the horizontal axis are quarters since the shock. Numbers on the vertical axis show percentage deviation from steady state.

dependent effects have been documented in Sharma (2023) and De Veirman (2023).

However, the novelty of this paper is to illustrate the macroeconomic dynamics of a tightening in lending conditions when housing is not an input in the production process. I therefore focus on effects of LTV tightening when housing is not part of production function, that is, I set $\phi = 0$ in the entrepreneur's production function. In response to an LTV shock when initial LTV ratio is 0.95, most of the macroeconomic variables now fall less. Exceptions are employment, and entrepreneurial consumption which falls at impact rather than displaying an increase as in the case of housing being part of production function. Extant literature has documented that after a tightening in lending conditions, housing prices fall (see, e.g., Justiniano, Primiceri, and Tambalotti, 2015; De Veirman, 2023 and Sharma, 2023). The results in this paper provide an

Figure 2: Impact of a Temporary LTV Shock

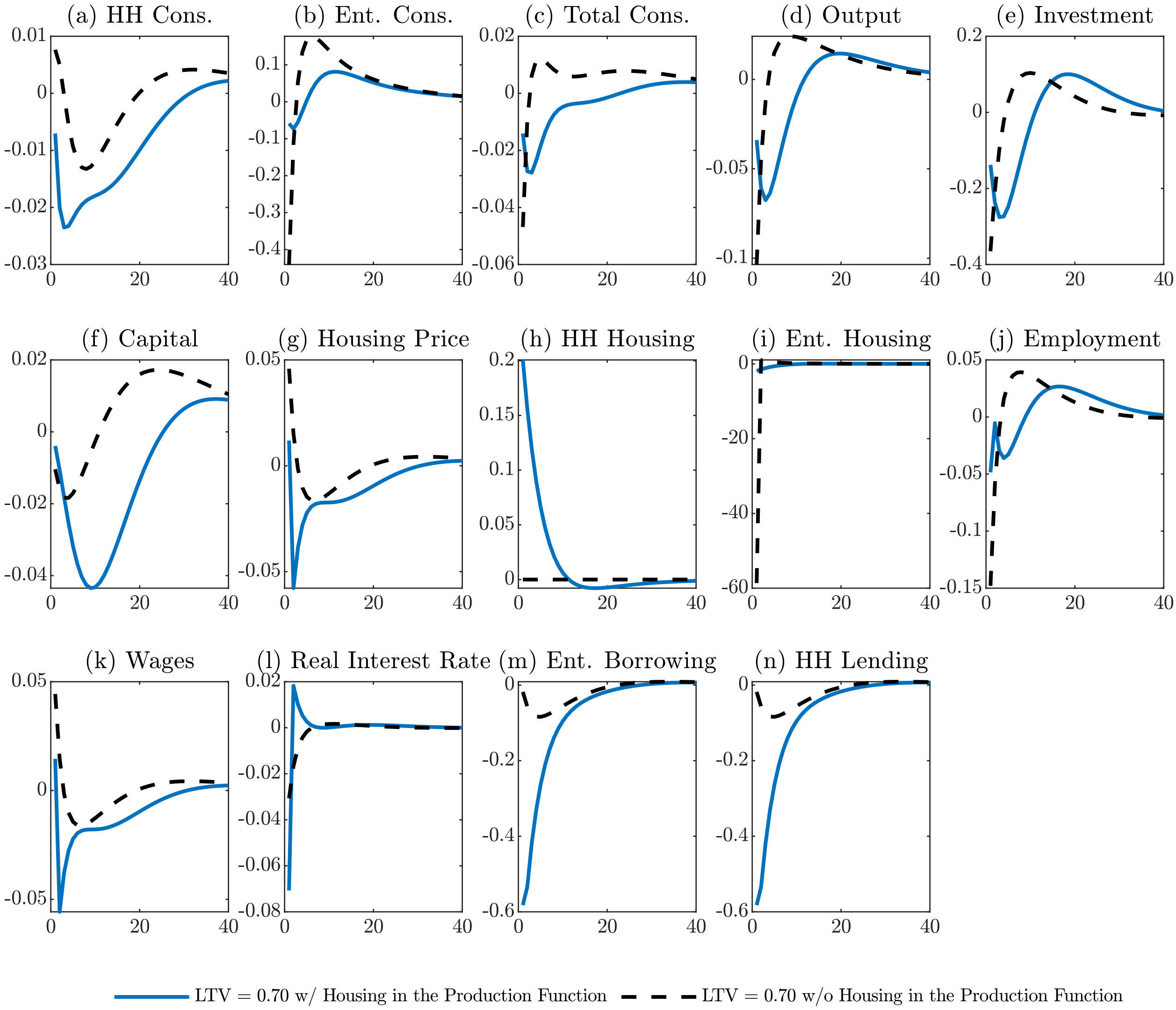


Note: Impact of a temporary shock that leads to 10 percentage point decline in LTV ratio from its steady-state value. Numbers on the horizontal axis are quarters since the shock. Numbers on the vertical axis show percentage deviation from steady state.

alternative novel effect that LTV tightening can have. It shows that in macroeconomic models in which housing does not feature in the entrepreneurial production function, a tightening in lending conditions might have ‘unexpected’ effects on housing prices and other macroeconomic variables.

In the version of the model that ignores role of housing as a production input, when initial LTV ratio is 0.95, entrepreneurs completely ‘dispose of’ their housing stock which reduces the supply of housing in the market. Recall that housing is in fixed supply in the market and as a result, this raises the price of housing at impact before it falls and returns to its prior equilibrium value. This reflects entrepreneur’s desire to obtain economic benefit from their asset holdings since an increase in housing price benefits them by allowing them to borrow more. At

FIGURE 3: IMPACT OF A TEMPORARY LTV SHOCK

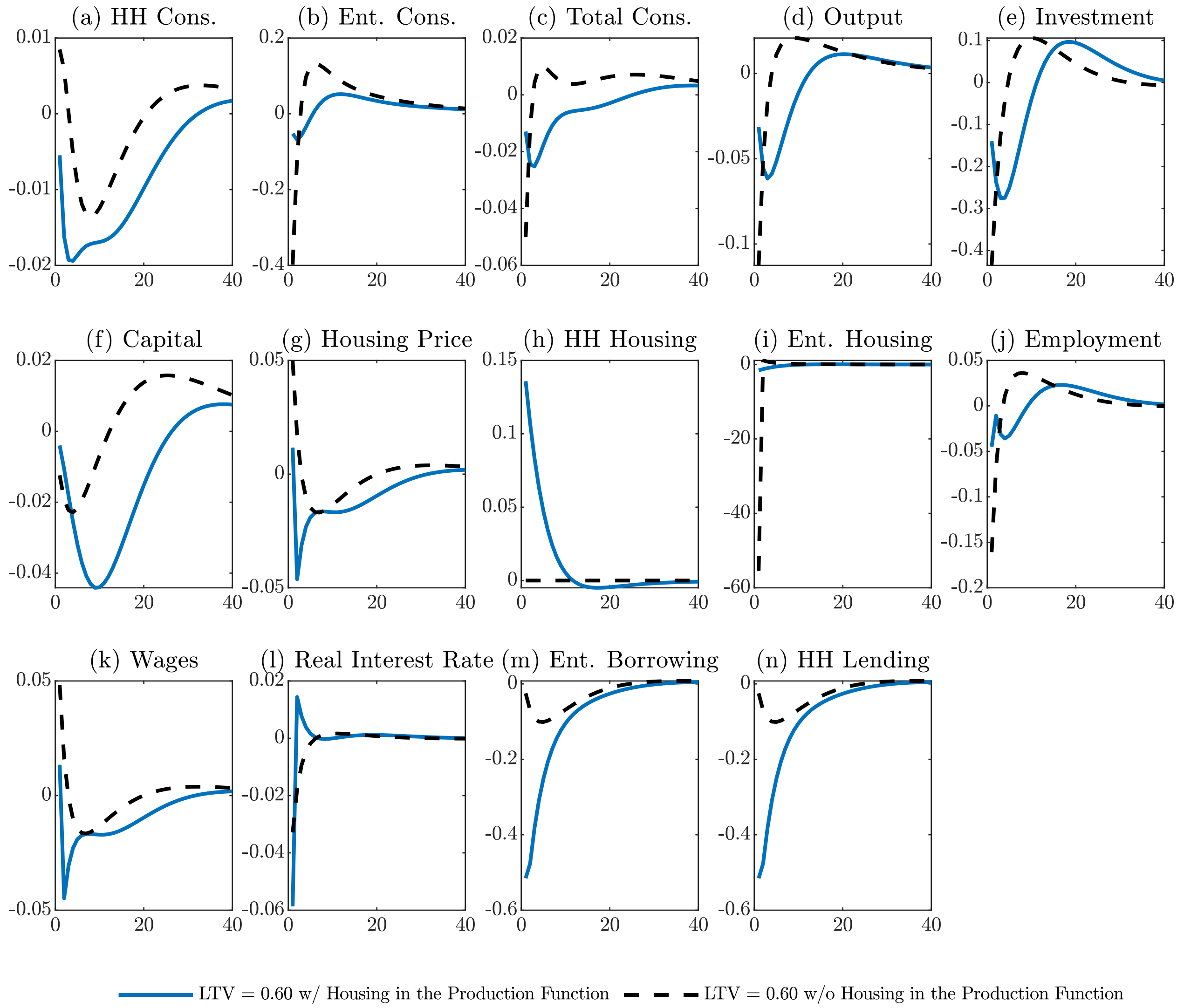


NOTE: Impact of a temporary shock that leads to 10 percentage point decline in LTV ratio from its steady-state value. Numbers on the horizontal axis are quarters since the shock. Numbers on the vertical axis show percentage deviation from steady state.

lower steady state LTV ratios of 0.80, 0.70 and 0.60 (see, Figures 2, 3 and 4), entrepreneurs 'dispose of' a smaller portion of their housing stock which, as in the previous case, reduces the aggregate supply of housing in the market, leading to an increase in its price and benefitting the entrepreneurs by raising their borrowing capacity. This effect, therefore, shows state dependence.

Results from the analysis in this paper show that when housing is not a factor in the production function, macroeconomic activity recovers faster and in general, generates greater economic amplification. Aggregate variables such as total consumption, output, investment and employment fall more before recovering and overshooting their prior steady state. This effect reflects the fact that in a model that excludes housing as a production input, interest rates do not move much after an LTV tightening compared to the alternative model that does include housing

Figure 4: Impact of a Temporary LTV Shock



Note: Impact of a temporary shock that leads to 10 percentage point decline in LTV ratio from its steady-state value. Numbers on the horizontal axis are quarters since the shock. Numbers on the vertical axis show percentage deviation from steady state.

as a production factor and as a consequence, entrepreneurial borrowing and household lending do not respond much. This has spillover impact on larger macroeconomic activity. Initially, investment falls by greater magnitude in the model without housing in the production function before it displays speedy recovery and overshoots its previous steady state. This rapid recovery in investment boosts capital, employment, output and consumption.

Another interesting finding in this paper is that at lower initial LTV ratios, fall in aggregate economic activity is bigger than the case when housing is a factor in the production process. This plausibly reflects the fact that at lower steady state LTV ratios, drop in investment is larger than the case when housing is an input in the production process which then reflects in a bigger fall in employment, aggregate consumption and output. In this sense, these effects display 'reverse state dependence' – economic variables like investment, emplyment, consumption and output fall more and exhibit bigger amplification at lower steady state LTV ratios. This is a novel effect and it contrasts with results in the extant literature that shows that macroeconomic activity shows greater movement in response to an LTV tightening when steady state ratios are high (Sharma, 2023; De Veirman, 2023). This paper, therefore, contributes to the literature by showing that when housing is a not an input in the production process, macroeconomic activity falls more and displays greater amplification at lower initial LTV ratios. This result speaks to macroprudential dimension of LTV tightening when it is implemented to reduce risk of non-payment of loans and associated economic losses. This paper shows that LTV shocks can generate greater macroeconomic volatility at lower steady state LTV ratios and a model gives very different results based on whether housing as a production factor is considered or not.

5 Conclusion

This paper shines a spotlight on important role housing plays as a production input in shaping the macroeconomic effects of LTV tightening. I show that a model that does not feature housing as a production factor, displays very different macroeconomic fluctuations in the aftermath of an LTV shock. In particular, at higher steady state LTV ratios, employment falls more than a model would predict with housing in the production factor while rest of the macroeconomic variables such as consumption and output fall less and return faster to their previous equilibrium. This highlights that macroeconomic actvity recovers faster when housing is not an input in the entrepreneurial production function. At lower intial steady state LTV ratios, most of the aggregate economic variables fall more than they would in a model which features housing as a factor input. I also show that an LTV tightening does not lead to any significant fall in housing price. Instead, housing price rises at impact of the shock. These results have important implications for study of macroeconomic fluctuations in the aftermath of an LTV tightening and indicate towards the important role housing plays in shaping the effects of various macroeconomic shocks.

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Appendix (For Online Publication)

Loan-to-Value Shocks and Housing in the Production Function

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Contents

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A Derivation of FOCs

A.1 Households

The Lagrangian of households is

$$\mathscr{L}_t = \mathbb{E}_t \left\{ \sum_{t=0}^{\infty} \left(\beta^P\right)^t \begin{bmatrix} \log\left(C_t^P - \gamma^P C_{t-1}^P\right) - \frac{N_t^{\eta}}{\eta} + \varsigma \log H_t^P \\ -\lambda_t^P \left[C_t^P + Q_t^H \left(H_t^P - H_{t-1}^P\right) + R_{t-1} B_{t-1}^P - W_t N_t - B_{t-1}^P \right] \end{bmatrix} \right\} \tag{A.1}$$

The problem yields the following first order conditions:

$$\frac{\partial \mathscr{L}}{\partial C_t^P} : \frac{1}{C_t^P - \gamma^P C_{t-1}^P} - \beta^P \mathbb{E}_P \frac{\gamma^P}{C_{t+1}^P - \gamma^P C_t^P} = \lambda_t^P \tag{A.2}$$

$$\frac{\partial \mathscr{L}}{\partial B_t^P} : \beta^P \mathbb{E}_t \lambda_{t+1}^P = \frac{\lambda_t^P}{R_t} \tag{A.3}$$

$$\frac{\partial \mathscr{L}}{\partial H_t^P} : \frac{\varsigma}{H_t^P} + \beta^P \mathbb{E}_t \left(\lambda_{t+1}^P Q_{t+1}^H\right) = \lambda_t^P Q_t^H \tag{A.4}$$

$$\frac{\partial \mathscr{L}}{\partial N_t} : N_t^{\eta-1} = \lambda_t^P W_t \tag{A.5}$$

A.2 Entrepreneurs

Entrepreneur's optimization problem can be written as

$$\mathscr{L}_t = \mathbb{E}_t \left\{ \sum_{t=0}^{\infty} \left(\beta^E\right)^t \begin{bmatrix} \log\left(C_t^E - \gamma^E C_{t-1}^E\right) \\ -\lambda_t^E \left[C_t^E + R_{t-1} B_{t-1}^E - Y_t + W_t N_t + I_t + Q_t^H \left(H_t^E - H_{t-1}^E\right) - B_t^E \right] \\ -\mu_t^E \left[R_t B_t^E - \theta_t \mathbb{E}_t \left(Q_{t+1}^H H_t^E + Q_{t+1}^K K_t\right) \right] \\ -\kappa_t^E \left[K_t - (1-\delta) K_{t-1} - \left\{ 1 - \frac{\Omega}{2} \left(\frac{I_t}{I_{t-1}} - 1 \right)^2 \right\} I_t \right] \end{bmatrix} \right\} \tag{A.6}$$

where $Y_t = A_t (N_t)^{1-\alpha} \left\{ \left(H_{t-1}^E\right)^{\phi} (K_{t-1})^{1-\phi} \right\}^{\alpha}$ may be inserted for Y_t in the budget constraint. Solving entrepreneur's optmization problem, the first order conditions are:

$$\frac{\partial \mathcal{L}}{\partial C_t^E} : \frac{1}{C_t^E - \gamma^E C_{t-1}^E} - \beta^E \mathbb{E}_t \frac{\gamma^E}{C_{t+1}^E - \gamma^E C_t^E} = \lambda_t^E \tag{A.7}$$

$$\frac{\partial \mathcal{L}}{\partial B_t^E} : \lambda_t^E = \beta^E \mathbb{E}_t \lambda_{t+1}^E R_t + \mu_t^E R_t \tag{A.8}$$

$$\frac{\partial \mathcal{L}}{\partial N_t} : W_t = (1-\alpha) \frac{Y_t}{N_t} \tag{A.9}$$

$$\frac{\partial \mathcal{L}}{\partial H_t^E} : \lambda_t^E Q_t^H = \beta^E \mathbb{E}_t \left\{ \lambda_{t+1}^E \left(Q_{t+1}^H + \alpha\phi \frac{Y_{t+1}}{H_t^E} \right) \right\} + \mu_t^E \theta_t \mathbb{E}_t Q_{t+1}^H \tag{A.10}$$

$$\frac{\partial \mathcal{L}}{\partial K_t} : \kappa_t^E = \alpha (1-\phi) \beta^E \mathbb{E}_t \left(\frac{\lambda_{t+1}^E Y_{t+1}}{K_t} \right) + \beta^E (1-\delta) \mathbb{E}_t \kappa_{t+1}^E + \mu_t^E \theta_t \mathbb{E}_t Q_{t+1}^K \tag{A.11}$$

$$\frac{\partial \mathcal{L}}{\partial I_t} : \lambda_t^E = \kappa_t^E \left\{ 1 - \frac{\Omega}{2} \left(\frac{I_t}{I_{t-1}} - 1 \right)^2 - \Omega \frac{I_t}{I_{t-1}} \left(\frac{I_t}{I_{t-1}} - 1 \right) \right\} + \beta^E \Omega \mathbb{E}_t \left\{ \kappa_{t+1}^E \left(\frac{I_{t+1}}{I_t} \right)^2 \left(\frac{I_{t+1}}{I_t} - 1 \right) \right\} \tag{A.12}$$

B List of Equations

B.1 Households

$$\frac{1}{C_t^P - \gamma^P C_{t-1}^P} - \beta^P \mathbb{E}_t \frac{\gamma^P}{C_{t+1}^P - \gamma^P C_t^P} = \lambda_t^P \tag{B.1}$$

$$\beta^P \mathbb{E}_t \lambda_{t+1}^P = \frac{\lambda_t^P}{R_t} \tag{B.2}$$

$$\frac{\varsigma}{H_t^P} + \beta^P \mathbb{E}_t \left(\lambda_{t+1}^P Q_{t+1}^H \right) = \lambda_t^P Q_t^H \tag{B.3}$$

$$N_t^{\eta-1} = \lambda_t^P W_t \tag{B.4}$$

B.2 Entrepreneurs

$$\frac{1}{C_t^E - \gamma^E C_{t-1}^E} - \beta^E \mathbb{E}_t \frac{\gamma^E}{C_{t+1}^E - \gamma^E C_t^E} = \lambda_t^E \tag{B.5}$$

$$\beta^E \mathbb{E}_t \left(\lambda_{t+1}^E R_t \right) + \mu_t^E R_t = \lambda_t^E \tag{B.6}$$

$$W_t = (1-\alpha) \frac{Y_t}{N_t} \tag{B.7}$$

$$\lambda_t^E Q_t^H = \beta^E \mathbb{E}_t \left\{ \lambda_{t+1}^E \left(Q_{t+1}^H + \alpha\phi \frac{Y_{t+1}}{H_t^E} \right) \right\} + \mu_t^E \theta_t \mathbb{E}_t Q_{t+1}^H \tag{B.8}$$

$$\kappa_t^E = \alpha (1-\phi) \beta^E \mathbb{E}_t \left(\frac{\lambda_{t+1}^E Y_{t+1}}{K_t} \right) + \beta^E (1-\delta) \mathbb{E}_t \kappa_{t+1}^E + \mu_t^E \theta_t \mathbb{E}_t Q_{t+1}^K \tag{B.9}$$

$$\lambda_t^E = \kappa_t^E \left\{ 1 - \frac{\Omega}{2} \left(\frac{I_t}{I_{t-1}} - 1 \right)^2 - \Omega \frac{I_t}{I_{t-1}} \left(\frac{I_t}{I_{t-1}} - 1 \right) \right\} + \beta^E \Omega \mathbb{E}_t \left\{ \kappa_{t+1}^E \left(\frac{I_{t+1}}{I_t} \right)^2 \left(\frac{I_{t+1}}{I_t} - 1 \right) \right\} \tag{B.10}$$

$$C_t^E + R_{t-1} B_{t-1}^E = Y_t - W_t N_t - I_t - Q_t \left(H_t^E - H_{t-1}^E \right) + B_t^E \tag{B.11}$$

$$B_t^E = \frac{\theta_t a_t}{R_t} \tag{B.12}$$

$$a_t = \mathbb{E}_t \left(Q_{t+1}^H H_t^E + Q_{t+1}^K K_t \right) \tag{B.13}$$

$$\kappa_t^E = \lambda_t^E Q_t^K \tag{B.14}$$

B.3 Market Clearing and Resource Constraints

$$C_t^P + C_t^E + I_t = Y_t \tag{B.15}$$

$$H_t^P + H_t^E = H \tag{B.16}$$

$$B_t^P + B_t^E = 0 \tag{B.17}$$

$$Y_t = A_t (N_t)^{1-\alpha} \left\{ \left(H_{t-1}^E \right)^\phi (K_{t-1})^{1-\phi} \right\}^\alpha \tag{B.18}$$

$$K_t = (1-\delta) K_{t-1} + \left\{ 1 - \frac{\Omega}{2} \left(\frac{I_t}{I_{t-1}} - 1 \right)^2 \right\} I_t \tag{B.19}$$

C Steady State Conditions

From household's FOC with respect to consumption (B.1) and labor (B.4), I have

$$\frac{1 - \beta^P \gamma^P}{(1-\gamma^P) C^P} = \lambda^P \tag{C.1}$$

and

$$N^{\eta - 1} = \lambda^P W \tag{C.2}$$

respectively. Household's FOC with respect to debt (B.2) yields the steady-state gross interest rate

$$R = \frac{1}{\beta^P} \tag{C.3}$$

which underscores the fact that the time preference of the most patient agent determines the steady-state rate of interest. (B.3) yields

$$\begin{aligned}\frac{\varsigma}{H^P} + \beta^P \lambda^P Q^H &= \lambda^P Q^H \\ \Rightarrow Q^H H^P &= \frac{\varsigma}{\lambda^P\left(1-\beta^P\right)} \\ \Rightarrow H^P &= \frac{\varsigma}{Q^H \lambda^P\left(1-\beta^P\right)}\end{aligned} \tag{C.4}$$

I now turn to entrepreneurs. Their consumption FOC (B.5) yields

$$\frac{1-\beta^E\gamma^E}{\left(1-\gamma^E\right)C^E} = \lambda^E \tag{C.5}$$

Entrepreneur's FOC with respect to debt (B.6) gives

$$\begin{aligned}\beta^E \lambda^E R^L + \mu^E R &= \lambda^E \\ \Rightarrow \mu^E &= \frac{\lambda^E\left(1-\beta^E R\right)}{R}\end{aligned} \tag{C.6}$$

The borrowing constraint for entrepreneurs binds only if μ^E is positive. This implies that β^E must be less than R^L. In the baseline calibration, β^E is set to 0.95 whereas the steady state value of R^L is 1.0219 which implies that β^E must be less than 0.9786 which is indeed the case. Entrepreneur's production function is

$$Y = A\left(N\right)^{1-\alpha}\left[\left(H^E\right)^{\phi}\left(K\right)^{1-\phi}\right]^{\alpha} \tag{C.7}$$

From firm's labor choice for househods (B.7),

$$W = \left(1-\alpha\right)\frac{Y}{N} \tag{C.8}$$

Entrepreneur's FOC with respect to housing (B.8) gives

$$\begin{aligned}\lambda^E Q^H &= \beta^E \lambda^E\left(Q^H + \alpha\phi\frac{Y}{H^E}\right) + \mu^E \theta Q^H \\ \Rightarrow \frac{Q^H H^E}{Y} &= \frac{\beta^E \alpha\phi R}{\left(1-\beta^E\right)R - \theta\left(1-\beta^E R\right)}\end{aligned} \tag{C.9}$$

From aggregate law of motion for capital (B.19)

$$K=(1-\delta)\,K+\left[1-\frac{\Omega}{2}\left(\frac{I}{I}-1\right)\right]I$$
$$\Rightarrow I=\delta K \tag{C.10}$$

I have the following steady-state resource constraints

$$Y=C^{P}+C^{E}+I \tag{C.11}$$

$$H=H^{P}+H^{E} \tag{C.12}$$

$$B^{P}+B^{E}=0 \tag{C.13}$$

Also, I have the following steady-state version of agents' budget constraints (one of them is redundant because of Walras' Law)

$$C^{P}=WN-(R-1)B^{P} \tag{C.14}$$

$$C^{E}=Y-RB^{E}-WN-I+B^{E} \tag{C.15}$$

So the steady state is characterized by the vector

$$\left[Y,C^{P},C^{E},I,H^{P},H^{E},K,N,W,B^{P},B^{E},Q^{H},Q^{K},R,\lambda^{P},\lambda^{E},\mu^{E}\right]$$

From entrepreneur's optimal choice of capital (B.9), I have

$$\kappa_t^{E}=\alpha\,(1-\alpha)\,\beta^{E}\mathbb{E}_t\left(\frac{\lambda_{t+1}^{E}Y_{t+1}}{K_t}\right)+\beta^{E}\,(1-\delta)\,\mathbb{E}_t\kappa_{t+1}^{E}+\mu_t^{E}\theta_t\mathbb{E}_tQ_{t+1}^{K}$$
$$\Rightarrow\frac{\kappa^{E}}{\lambda^{E}}\left(1-(1-\delta)\,\beta^{E}\right)=\alpha\,(1-\phi)\,\beta^{E}\frac{Y}{K}+\frac{\left(1-\beta^{E}R\right)}{R}\theta Q^{K} \tag{C.16}$$

Entrepreneur's optimal choice of investment (B.10) yields

$$\lambda_t^{E}=\kappa_t^{E}\left[1-\frac{\Omega}{2}\left(\frac{I_t}{I_t}-1\right)^{2}-\Omega\frac{I_t}{I_t}\left(\frac{I_t}{I_{t-1}}-1\right)\right]+\beta^{E}\Omega\mathbb{E}_t\left[\kappa_{t+1}^{E}\left(\frac{I_{t+1}}{I_t}\right)^{2}\left(\frac{I_{t+1}}{I_t}-1\right)\right]$$
$$\Rightarrow\lambda^{E}=\kappa^{E} \tag{C.17}$$

Combining this with steady state version of

$$\kappa^E = \lambda^E Q^K \tag{C.18}$$

I obtain $Q^K = 1$ in the steady state. Plugging this into (C.16), I obtain the expression for capital-to-output ratio

$$\begin{aligned} \frac{\kappa^E}{\lambda^E}\left(1-(1-\delta)\beta^E\right) &= \alpha(1-\phi)\beta^E\frac{Y}{K} + \frac{\left(1-\beta^E R\right)}{R}\theta Q^K \\ \Rightarrow \frac{K}{Y} &= \frac{\alpha(1-\phi)R\beta^E}{R\left(1-(1-\delta)\beta^E\right)-\theta\left(1-\beta^E R\right)} \end{aligned} \tag{C.19}$$

Next, combining (B.12) and (B.13)

$$B^E = \frac{\theta}{R}\left(Q^H H^E + Q^K K\right) \tag{C.20}$$

Dividing by Y, the above expression becomes

$$\begin{aligned} \frac{B^E}{Y} &= \frac{\theta}{R}\left(\frac{Q^H H^E}{Y} + \frac{Q^K K}{Y}\right) \\ \Rightarrow \frac{B^E}{Y} &= \alpha\theta\beta^E\left[\frac{\phi}{R\left(1-\beta^E\right)-\theta\left(1-\beta^E R\right)} + \frac{(1-\phi)}{R\left(1-(1-\delta)\beta^E\right)-\theta\left(1-\beta^E R\right)}\right] \end{aligned} \tag{C.21}$$

From entrepreneur's budget constraint (B.11)

$$C^E + RB^E = Y - WN - I + B^E \tag{C.22}$$

Rewriting this in ratio to output

$$\begin{aligned} \frac{C^E}{Y} + \frac{RB^E}{Y} &= 1 - \frac{WN}{Y} - \frac{I}{Y} + \frac{B^E}{Y} \\ \Rightarrow \frac{C^E}{Y} &= \alpha - \delta\frac{K}{Y} + (1-R)\frac{B^E}{Y} \end{aligned} \tag{C.23}$$

Steady-state budget constraint of household, in ratio to output, reads

$$\begin{aligned} \frac{C^P}{Y} &= \frac{WN}{Y} + (1-R)\frac{B^P}{Y} \\ &= (1-\alpha) + \frac{(1-R)B^P}{Y} \end{aligned} \tag{C.24}$$

Dividing the above two expressions by each other, I have

$$\frac{\frac{Q^H H^P}{Y}}{\frac{Q^H H^E}{Y}} = \frac{\frac{\varsigma}{Y\lambda^P\left(1-\beta^P\right)}}{\frac{\beta^E\alpha\phi R}{\left(1-\beta^P\right)R-\theta\left(1-\beta^E R\right)}}$$

$$\Rightarrow \frac{H^P}{H^E} = \frac{\varsigma}{Y\frac{1-\beta^P\gamma^P}{\left(1-\gamma^P\right)C^P}\left(1-\beta^P\right)}\frac{\left(1-\beta^E\right)R-\theta\left(1-\beta^E R\right)}{\beta^E\alpha\phi R}$$

$$\Rightarrow \frac{H^P}{H-H^P} = \frac{\varsigma\left(1-\gamma^P\right)}{\left(1-\beta^P\right)\left(1-\beta^P\gamma^P\right)}\frac{\left(1-\beta^P\right)R-\theta\left(1-\beta^E R\right)}{\beta^E\alpha\phi R}\frac{C^P}{Y} \tag{C.25}$$

Steady state version of aggregate resource constraint (B.15) is

$$C^P + C^E + I = Y$$

Dividing by Y

$$\frac{C^P}{Y} = 1 - \frac{C^E}{Y} - \delta\frac{K}{Y} \tag{C.26}$$

Combining (C.1), (C.2) and (C.8) gives steady-state equilibrium condition for households

$$N^{\eta-1} = \lambda^P W$$

$$\Rightarrow N^{\eta-1} = \frac{1-\beta^P\gamma^P}{\left(1-\gamma^P\right)C^P}\left(1-\alpha\right)\frac{Y}{N}$$

$$\Rightarrow N = \left[\frac{\left(1-\beta^P\gamma^P\right)\left(1-\alpha\right)}{\left(1-\gamma^P\right)}\left(\frac{C^P}{Y}\right)^{-1}\right]^{\frac{1}{\eta}} \tag{C.27}$$

From (B.18), steady state output is

$$Y = A\left(N\right)^{1-\alpha}\left[\left(H^E\right)^{\phi}\left(K\right)^{1-\phi}\right]^{\alpha}$$

$$Y^{1-\alpha} = A\left(N\right)^{1-\alpha}\left[\left(\frac{H^E}{Y}\right)^{\phi}\left(\frac{K}{Y}\right)^{1-\phi}\right]^{\alpha}$$

$$Y^{1-\alpha} = A\left(N\right)^{1-\alpha}\left[\left(\frac{H^E}{Y}\right)^{\phi}\left(\frac{\alpha\left(1-\phi\right)R\beta^E}{R\left(1-\left(1-\delta\right)\beta^E\right)-\theta\left(1-\beta^E R\right)}\right)^{1-\phi}\right]^{\alpha} \tag{C.28}$$

From (C.4)

$$Q^H = \frac{\varsigma}{H^P\lambda^P\left(1-\beta^P\right)} \tag{C.29}$$

D System of Loglinear Equations

The system of equations log-linearized around their steady state is as below:

D.1 Optimality Conditions of Households

Equations (B.1), (B.2) and (B.4) become

$$\beta^P \gamma^P \mathbb{E}_t \widehat{C}^P_{t+1} - \left(1 + \left(\gamma^P\right)^2 \beta^P\right) \widehat{C}^P_t + \gamma^P \widehat{C}^P_{t-1} = \left(1 - \beta^P \gamma^P\right)\left(1 - \gamma^P\right) \widehat{\lambda}^P \tag{D.1}$$

$$\mathbb{E}_t \widehat{\lambda}^P_{t+1} = \widehat{\lambda}^P_t - \widehat{R}_t \tag{D.2}$$

$$(\eta - 1) \widehat{N}_t = \widehat{\lambda}^P_t + \widehat{W}_t \tag{D.3}$$

Log-linearization of (B.3) yields

$$\beta^P \mathbb{E}_t \left[\widehat{\lambda}^P_{t+1} + \widehat{Q}^H_{t+1} + \widehat{H}^P_t\right] = \widehat{\lambda}^P_t + \widehat{Q}^H_t + \widehat{H}^P_t \tag{D.4}$$

D.2 Optimality Conditions of Entrepreneurs

From (B.5) and (B.6), I have

$$\beta^E \gamma^E \mathbb{E}_t \widehat{C}^E_{t+1} - \left(1 + \left(\gamma^E\right)^2 \beta^E\right) \widehat{C}^E_t + \gamma^E \widehat{C}^E_{t-1} = \left(1 - \beta^E \gamma^E\right)\left(1 - \gamma^E\right) \widehat{\lambda}^E_t \tag{D.5}$$

and

$$\widehat{\lambda}^E_t = \widehat{R}^L_t + \beta^E R^L \mathbb{E}_t \widehat{\lambda}^E_{t+1} + \left(1 - \beta^E R^L\right) \widehat{\mu}^E_t \tag{D.6}$$

Equation (B.7) yields

$$\widehat{W}_t = \widehat{Y}_t - \widehat{N}_t \tag{D.7}$$

From (B.8), I derive

$$\begin{aligned}\left(\widehat{\lambda}^E_t + \widehat{Q}^H_t\right) &= \beta^E \mathbb{E}_t \left(\widehat{\lambda}^E_{t+1} + \widehat{Q}^H_{t+1}\right) + \left(\frac{1}{R} - \beta^E\right) \theta \mathbb{E}_t \left(\widehat{\mu}^E_t + \widehat{\theta}_t + \widehat{Q}^H_{t+1}\right) \\ &+ \left[\left(1 - \beta^E\right) - \theta \left(\frac{1}{R} - \beta^E\right)\right] \mathbb{E}_t \left[\widehat{\lambda}^E_{t+1} + \widehat{Y}_{t+1} - \widehat{H}^E_t\right]\end{aligned} \tag{D.8}$$

Equation (B.9) becomes

$$\widehat{Q}_t^K = \left[1 - \beta^E (1-\delta) - \theta \left(\frac{1}{R} - \beta^E\right)\right] \mathbb{E}_t \left[\widehat{\lambda}_{t+1}^E - \widehat{\lambda}_t^E + \widehat{Y}_{t+1} - \widehat{K}_t\right]$$
$$+ \beta^E (1-\delta) \mathbb{E}_t \left(\widehat{Q}_{t+1}^K + \widehat{\lambda}_{t+1}^E - \widehat{\lambda}_t^E\right) + \left(1 - \beta^E R\right) \frac{1}{R} \theta \mathbb{E}_t \left[\widehat{\mu}_t^E - \widehat{\lambda}_t^E + \widehat{\theta}_t + \widehat{Q}_{t+1}^K\right] \quad \text{(D.9)}$$

Equation (B.10) is approximated as

$$\widehat{Q}_t^K = \left(1 + \beta^E\right) \Omega \widehat{I}_t - \beta^E \Omega \mathbb{E}_t \widehat{I}_{t+1} - \Omega \widehat{I}_{t-1} \quad \text{(D.10)}$$

Entrepreneur's budget contraint (B.11) becomes

$$C^E \widehat{C}_t^E + RB^E \left(\widehat{R}_{t-1} + \widehat{B}_{t-1}^E\right) = Y\widehat{Y}_t - WN\left(\widehat{W}_t + \widehat{N}_t\right) - I\widehat{I}_t - Q^H H^E \left(\widehat{H}_t^E - \widehat{H}_{t-1}^E\right) + \widehat{B}_t^E B^E \quad \text{(D.11)}$$

The borrowing constraint (B.12) becomes

$$\widehat{l}_t = \widehat{\theta}_t + \widehat{a}_t - \widehat{R}_t \quad \text{(D.12)}$$

Equation (B.13) which shows entrepreneurs' total assets, becomes

$$\widehat{a}_t = \frac{Q^H H^E}{Q^H H^E + Q^K K} \mathbb{E}_t \left(\widehat{Q}_{t+1}^H + \widehat{H}_t^E\right) + \frac{Q^K K}{Q^H H^E + Q^K K} \mathbb{E}_t \left(\widehat{Q}_{t+1}^K + \widehat{K}_t\right) \quad \text{(D.13)}$$

Linearized version of (B.14) is

$$\widehat{\kappa}_t^E = \widehat{\lambda}_t^E + \widehat{Q}_t^K \quad \text{(D.14)}$$

D.3 Market Clearing and Resource Constraints

Equations (B.15) and (B.16) yield

$$\widehat{Y}_t = \frac{C^P}{Y} \widehat{C}_t^P + \frac{C^E}{Y} \widehat{C}_t^E + \frac{I}{Y} \widehat{I}_t \quad \text{(D.15)}$$

and

$$H^P \widehat{H}_t^P + H^E \widehat{H}_t^E = 0 \quad \text{(D.16)}$$

(B.17) gives

$$\widehat{B}_t^P B^P + \widehat{B}_t^E B^E = 0 \tag{D.17}$$

From (B.18), I have

$$\widehat{Y}_t = \widehat{A}_t + (1-\alpha)\,\widehat{N}_t + \alpha\phi\widehat{H}_{t-1}^E + \alpha\,(1-\phi)\,\widehat{K}_{t-1} \tag{D.18}$$

Equation (B.19) yields

$$\widehat{K}_t = (1-\delta)\,\widehat{K}_{t-1} + \delta\widehat{I}_t \tag{D.19}$$