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Adjusted Net Saving Needs Further Adjusting: Reassessing Human and Resource Factors in Sustainability Measurement

CAMA Working Paper 63/2022
October 2022

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JEL Classification

Q56, E01, Q32, O47, O41

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ISSN 2206-0332

Adjusted Net Saving needs further adjusting: reassessing human and resource factors in sustainability measurement

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13 September 2022

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Acknowledgments: For helpful comments I thank Geir Asheim, Giles Atkinson, Paul Burke, Barbara Fraumeni, Quentin Grafton, Frank Jotzo, Larry Liu, Rob Marks, Martin McCarthy, Warwick McKibbin, Rui Mota, Budy Resosudarmo, David Stern, Simone Valente, Rick van der Ploeg, Matthew Woolf, Rintaro Yamaguchi, and participants at the 2021 Monash Environmental Economics Workshop, the 2022 Australasian Agricultural and Resource Economics Society's conference and the 2022 Sustainable Resource Use and Economic Dynamics conference. I also owe much to inspiration from work by Marty Weitzman, who is sadly missed. This research did not receive any specific grant from funding agencies in the public, commercial, or not-for-profit sectors. All errors and omissions are my own.

1. Introduction

This paper's ultimate purpose is to better inform policy makers about both the overall sustainability of national economies, and the components of sustainability. It does so by highlighting shortcomings in the World Bank's estimates of Adjusted Net Saving, one of its two, regularly computed measures of the sustainability of more than 200 national and regional economies. We will show that their Adjusted Net Saving estimates ideally need further adjusting, because they omit or seriously underestimate "human factors": the benefits of technological change and of human and knowledge capital investment[1] and the cost of population growth; and they omit net growth in natural resource inputs. Yet given our purpose, our analysis has three apparently paradoxical features. We show results only for the global economy, not any national economies; we apparently ignore any actual sustainability problems, by using a theoretical model and global data from 1995-2014 that seem to assume endless, balanced growth, as typically assumed by mainstream growth economists; and notably, we omit any environmental costs like pollution from our further adjustments.

We defend all these features on pragmatic grounds. To include international economic flows, and thus give results for some illustrative national economies, would greatly extend an already long paper, which we view as only a first step towards improved national sustainability estimates. So although our headline result from three separate calibrations of our model is that our further adjustments raise the global estimate of Adjusted Net Saving by 5-11 % of GDP,[2] national estimates could be quite different. In particular, our further adjustments would lower, not raise, the estimates in countries with a combination of low innovation, large depletion instead of net growth of resource stocks, and high population growth; hence the importance of including these adjustments. Our second feature, of assuming always balanced growth, is a rough approximation in the style of Weitzman (1976, 1997, 1999), which greatly simplifies the analysis. It, and the real dollar discount rate of ~7 %/yr that our calibrations calculate was used by the global economy during 1995-2014, effectively ignore any long-term future, as does our third feature, our omission of long-term environmental threats to global sustainability like climate change. We make this omission because, sadly, the global economy almost completely ignored current and future global environmental damage during these years. Our aggregate methodology therefore cannot estimate such damage; and all estimates of global climate damage from extrapolating sub-global data are highly contested anyway,[3] so much so that even the possibility of economic analysis ever resolving this contest can be doubted (Pezzey 2019).

Our estimates raise global Adjusted Net Saving to between 16 and 23 per cent of GDP in 2014, but this should definitely not be seen as a measure of global sustainability, for both empirical and theoretical reasons. The empirical reason is that, as just stressed, we exclude all environmental damages, so our estimates should be seen only as rough, one-sided measures of *economic* sustainability; and on its own our methodology is intended just to inform medium-term, national policies for technology, resources and population, not environmental policies,

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[1] As in most growth economics, we ignore macroeconomic fluctuations, and throughout treat any investment as contributing an identical amount to saving, so total net investment equals total saving.

[2] The World Bank reports some dollar flows as % of gross national income (GNI) and others as % of GDP. GNI and GDP are very similar globally, so throughout we lose no accuracy by using GNI, GDP, gross output and gross production as interchangeable terms. Also, for familiarity we do not use the technically more correct GWP and GWI (for Gross World Product and Income).

[3] As just one example, in 2021 the World Bank used a US$40/t$CO_2$ value of CO_2 emissions in World Development Indicators, while a median expert view in Hänsel et al. (2020) was US$208/t$CO_2$.

especially long-term global ones. The theoretical reason is the one-sidedness just mentioned: as stressed in literature like Asheim (1994) and Pezzey (2004), if current development is significantly unsustainable, then moving to a sustainable path will significantly change prices throughout the economy. Current prices therefore cannot measure overall sustainability, and a positive Adjusted Net Saving is compatible with unsustainability. Our broad message is thus that empirical debates about sustainability should be as full as possible: neither overly pessimistic, by neglecting the power of human innovation and discovery, nor overly optimistic, by omitting the challenge of global population growth.

Our further adjustments bring Adjusted Net Saving per person much closer to the change in wealth per person (with equality in our simplest model). This is actually the World Bank's preferred sustainability measure (World Bank 2021a, p54). Its calculation implicitly includes estimates of the benefits of productivity growth and resource discoveries, and the cost of population growth. So change in per-person wealth gives a more complete picture of economic change, and the Bank's global estimate of it was about 11 percentage points higher than Adjusted Net Saving (ANS) per person in 2014. However, ANS is still useful and important:

> "Measured annually, ANS provides policy makers immediate feedback about the direction of the economy and possible actions they may need to take to ensure long-term growth." (World Bank 2021a, p52)

hence this paper's purpose of further adjusting ANS, so that it becomes a more complete measure. We do so by building on several strands of theoretical and empirical work by academics and institutions since the 1970s, discussed in more detail below. From this literature, we use the analysis in Pezzey (2004, Section 4) as the theoretical basis for a further-adjusted ANS that includes the effects of productivity, population and resources growth. Empirically, the Bank's recent *Wealth of Nations* reports (Lange et al. 2018 and World Bank 2021a) give the methodologies used in their online databases which are our main data sources: World Development Indicators (https://databank.worldbank.org/source/world-development-indicators), which includes ANS; and Wealth Accounts (https://databank.worldbank.org/source/wealth-accounts), which includes wealth per person.

We derive formulae for our further-adjusted, global ANS from a five-fold extension of Stiglitz's (1974) model of optimal growth, which had Cobb-Douglas production from produced capital, a non-renewable resource, and exogenously growing population and total factor productivity. Our extension includes "human" capital, defined as having no spillovers but with some of it produced outside (measured) GDP; "knowledge" capital, defined as having spillovers that result in increasing returns to scale; depreciation of all capitals; and net resource growth, not depletion, thanks to resource discovery and renewal. For stock-flow consistency, we must define human capital as solely the accumulation of net human capital investment, not the much larger, World Bank definition as the present value of lifetime labour income, a value whose growth greatly exceeds any plausible level of net investment. We also assume that all variables grow exponentially, not just asymptotically as in Stiglitz, but always. The global economy's roughly constant growth rate (of per-person consumption) of about 1.6 %/yr during 1995-2014, and other roughly constant growth rates then, approximately justify this assumption.

We then calculate three alternative estimates of our ANS components and totals, by calibrating our model with World Bank and other global data from about 1995 to 2014.[4] Our second and third

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[4] The data in Wealth Accounts were revised in late 2021 to include annual data, and for 2015-18, but world totals were not included. Here we still use the January 2018 estimates, which ended in 2014, hence our estimation of ANS† in 2014, not later.

calibrations entail large increases in estimated net investments in human and knowledge capital, chosen inductively to about halve, or completely eliminate, the Solow residual (the growth in total factor productivity otherwise needed to explain the growth rate); but for all three calibrations, our total ANS is 5-11 % of GDP higher than the World Bank's for 2014, as noted earlier.

The paper's structure is as follows. Section 2 discusses key relevant literature; and then summarises the World Bank's wealth and ANS methodologies, their data sources and main results, and the main differences in our approach. Section 3 defines our model, starting with Stig5, our name for the third, most general model variant. Stig4, the second variant, omits off-GDP investment in human capital; while Stig3, the simplest variant, also omits knowledge capital. The section then computes Stig5's theoretical growth rate, and components of its output and wealth. Section 4 defines our further-adjusted ANS per person, and confirms that in theory, it equals change in wealth per person in Stig3. Section 5 gives the data inputs chosen for calibration, including evidence that the global economy grew nearly exponentially during 1995-2014, and also that two independent estimates of the resource's power in output are inconsistent. It then defines our three calibrations, of Stig3, Stig4 and Stig5 respectively, selected from countless alternatives. Section 6 discusses these calibrations' results for endogenous parameters, and output and wealth components. Section 7 compares the Bank's with our three calibrations of ANS. This establishes our key contribution: to be a fuller measure of economic sustainability, the World Bank's ANS ideally needs major additions and revisions, particularly in countries which strongly depart from global averages, and we view our global results as a useful first step towards this goal. Section 8 concludes.

2. Wealth, saving/investment, and sustainability: key literature, and comparison of World Bank and our methodologies

2.1 Key literature

Although economists' concerns with the sustainability of growth given finite natural resources started with Malthus (1798), modern concerns date from the 1973 oil crisis, which inspired three seminal papers on theoretical sustainability economics (though the s-word was not used then). To simplify, Dasgupta and Heal (1974) showed how optimal growth with capital and a non-renewable resource must eventually be unsustainable, despite unlimited capital-resource substitutability, because of the constant utility discount rate; Stiglitz (1974) showed how high enough growth in total factor productivity (TFP) could avoid this fate; and Solow (1974) showed a different solution, by changing the economy's objective from optimality to intergenerational equity, with Hartwick (1977) providing an investment rule to achieve this.

Separate, seminal work by Weitzman (1976) showed that the change in net national product (NNP) on an optimal path is the (real) interest rate times aggregate net investment; hence that if the interest rate is constant, NNP is the interest on wealth, defined as the present value of all future consumption. This result means that if current net saving – aggregated over all capital stocks, and equated to net investment, following our first footnote – is negative, then current consumption must be unsustainable (because it exceeds NNP), but not the converse: positive net saving does not indicate sustainable consumption. That follows from Weitzman's important, often overlooked caveat (p159) that NNP is not generally attainable as sustainable (i.e. maximum constant) consumption. But standard economic analysis offers no sustainability indicator other than net saving. So a rich stream of papers has interwoven the analysis of optimal growth in economies that use environmental resource inputs, with the analysis of aggregate net saving – usually called genuine saving, with the Adjusted Net Saving label not used by the World Bank until 2003 – as a sustainability indicator. Notable contributions for our purposes are Solow (1986), Hartwick (1990), Pearce and Atkinson (1993), Asheim (1994), Hamilton (1994), Weitzman (1997), Hamilton and Clemens (1999), Asheim and Weitzman (2001), Arrow et al. (2003), Hamilton (2003), Arrow et al. (2004), Pezzey (2004) and Arrow et al. (2012). Together these explored fairly fully how and why to adjust aggregate net saving to include natural resource depletion, knowledge and human capital investment, and the growths of population and TFP, all inputs that Weitzman (1976, p157, p160) presciently noted ought to be included.

The numbers of such papers, and the hundreds of citations that many of them have received, show an enduring and growing economic concern for sustainability, as distinct from optimality, so that recent mainstream reviews of welfare measurement (Fleurbaey 2009, Jorgenson 2018) included discussion of genuine saving or adjusted net saving. Yet Weitzman's caveat remains an inescapable barrier to finding accurate sustainability indicators: it means that if current development is significantly unsustainable, then moving to a sustainable path will significantly change prices throughout the economy, so current prices cannot measure overall sustainability. The other inescapable barrier is valuing pollution, notably global climate damage, which is now widely recognised as a far bigger threat to long-run global sustainability than the finiteness of non-renewable, subsoil resources, the main concern in the 1970s. Significant, pervasive pollution imposes the same per-person cost regardless of the population size, so the economy breaks the constant returns to scale (CRS) assumption made by almost all the above literature, including Pezzey (2004). We duck this barrier here by omitting any climate deductions from our ANS, for reasons given in the introduction, so that as already stated, we view our ANS as a measure of only medium-term, economic sustainability.

2.2 The World Bank's wealth and ANS methodologies, and our departures from them

Appendix A in World Bank (2021a), hereafter WB21, outlines the methodology, based on theory in Hamilton and Liu (2014), which in turn used Hamilton and Hartwick (2005), behind the comprehensive wealth estimates published online in Wealth Accounts (hereafter WA). Unlike World Bank (2006, 2011), which estimated Total wealth as the present value (PV) of sustainable consumption, Lange et al. (2018) and WB21 defined:

Total wealth = Natural capital + Produced Capital + Human capital + Net foreign assets

and calculated each of the four components separately, with Total wealth being the sum of these components. Natural capital comprised mainly energy, minerals, agricultural land, protected areas and forests, valued largely as PVs of rents of these resources. Produced capital comprised physical capital measured by the perpetual inventory method, plus the value of built-up urban land as a mark-up on produced capital. Grounded in the framework of the System of National Accounts (SNA), human capital was estimated as the PV of lifetime incomes of two population groups, aged 15-24 years and 25-65 years, and formed over 60% of total wealth (WB21, p454).

By contrast, in Stig4/5 we define:

Total wealth = Produced Capital + "Human" capital + "Knowledge" Capital
+ PV(labour time income) + PV(natural resource rents)

So that it does not grow just because population is growing, we define "human" capital as solely the accumulation of net investment from output, unlike the much larger definition in WB21 (see Section 3.1 for more detail). "Human" and "knowledge" capital are in speech marks here because our production function effectively defines "human" capital as any capital accumulated by "education" spending that generates no spillovers, and is thus a more restricted concept than in Lucas (1988); and "knowledge" capital as any non-physical form of capital accumulated by "R&D" spending that does generate spillovers. But to avoid clutter, we hereafter mostly drop these qualifying speech marks, including them only when showing and discussing our calibration results. Our Stig3 calibration, which omits knowledge capital, and so has CRS instead of

Stig4/5's increasing returns to scale (IRS), defines net human capital investment as public education expenditure, as reported in World Development Indicators (hereafter WDI), minus depreciation. Our Stig4 and Stig5 calibrations define human and knowledge capital investments as respectively two or three times this level, for inductive reasons explained later. Net foreign assets are zero for the world, so they are absent from Stig3/4/5. (We make tiny adjustments to WA data so they are exactly zero in our calibrations.)

Our PV calculations use an endogenous real dollar discount rate, which we calculate to be ~7 %/yr during 1995-2014. So as seen from 1995, the 20-year horizon to 2015 includes about three-quarters of PV, and the 35-year horizon to 2030 includes over 90% of PV. Hence our assumption of always-exponential growth is consistent with our many other approximations, even though such growth is physically impossible on a finite planet, and there are sound ethical and democratic reasons (e.g. in Hänsel et al. 2020) for not using market-based discount rates to evaluate long-term climate policy. So our always-exponential assumption should in no way be seen as forecasting growth forever, or supporting it as a key objective of much economic policy.

WB21's Appendix A also summarises its ANS methodology, used to calculate results published in WDI, as (hereafter using $_{\text{WB}}$ and † postscripts for conciseness):

$$\text{ANS}_{\text{WB}} = \text{Net investment in produced capital} + \text{(gross) public education expenditure}$$
$$- \text{natural resource depletion} - \text{damage from CO}_2 \text{ and local air pollution.}$$

where, as we will see later, the coverage of natural resources in WDI is narrower than that of natural capital in WA. By contrast, in Stig4/5 we define:

$$\text{ANS}^{\dagger} = \text{Net investment in produced capital} + \text{net investment in human capital}$$
$$+ \text{net investment in knowledge capital} + \text{value of net increase in resource stocks}$$
$$- \text{"population dilution"} + \text{value of TFP growth}$$

Empirically, the biggest differences of $\text{ANS}^{\dagger}$ from ANS_{WB} turn out to be:

- our higher "human factors": the inclusion of the value of TFP growth, following Pezzey (2004, eqn. (52)), and/or estimates of higher net investment in human and knowledge capital; and
- our deduction of "population dilution" := population growth rate × (produced capital + human capital + value of resource stocks), also following Pezzey (2004, eqn. (52)), in order for $\text{ANS}^{\dagger}$ to be a test of the sustainability of per-person consumption when population is growing;

though "following" here is only approximate, because the inclusion of knowledge capital in Stig4/5 breaks the CRS assumption needed for Pezzey's eqn. (52) to hold exactly.

Because our model assumes balanced, exponential growth always, it has constant input shares and a constant wealth-income ratio, as in Kaldor's (1961) stylised facts. Stig3/4/5 therefore cannot address this century's key debate about inequality: the rising share of income going to capital, hence a rising wealth-income ratio, in the very long run, as found by Piketty and Saez (2003) and Piketty and Zucman (2014). However, Table 1 in Section 5.1 will show that the global wealth-income ratio actually fell slightly during 1995-2014, so over this two-decade timescale, our model's omission of inequality is empirically quite valid.

3. Stig5: Stiglitz's theoretical model, extended to include human capital, knowledge capital, capitals depreciation and resource discovery

3.1 Basic definition and caveats

Stig5, our optimal growth model, is formally defined for the aggregate, global economy as:

$$\max_{c(t),R(t)} \int_0^\infty e^{-\rho t} L(t)[c(t)]^{1-\theta} dt, \text{ with } C(t) := L(t)c(t) \qquad [1]$$

$$\text{s.t.} \quad C(t) = F(K,H,N,R,L,t) - \dot{K}(t) - \delta K(t) - \pi\big[\dot{H}(t) + \delta H(t)\big] - \dot{N}(t) - \delta N(t) \qquad [2]$$

$$R(t) = \gamma S(t) - \dot{S}(t) \qquad [3]$$

$$\text{and} \quad F(K,H,N,R,L,t) = A_0 K^\alpha H^\varepsilon N^\nu R^\beta L^{1-\alpha-\varepsilon-\beta} e^{\tau t} \qquad [4]$$

$$\text{with} \quad K(0) = K_0,\ H(0) = H_0,\ N(0) = N_0,\ S(0) = S_0,\ L(t) = L_0 e^{nt}. \qquad [5]$$

where L is population; c is consumption per person and $C := Lc$ is total consumption; $F(.)$ is gross output, also called GNI or GDP here; K is produced capital; H is human capital, where only a fraction π (<1) of gross human capital investment, $\dot{H} + \delta H$, is included in measured GNI, with the remaining $1-\pi$ fraction being "off-GNI"; N is k*N*owledge capital; S is an aggregate resource stock, with R the aggregate resource flow; and $A_0 e^{\tau t}$ is TFP. A_0, K_0, H_0, N_0, S_0, L_0, ρ, θ, δ, π, γ, α, ε, ν, β, τ and n are positive parameters; of these, only the last 11, from ρ to n, will affect Stig5's balanced growth rates. We make the usual neoclassical assumption of perfect competition, including perfect information and foresight. Intuitively (and we will prove in Section 3.2), Stig5 allows a positive, optimal growth rate of per-person consumption, $g(t) := \dot{c}(t)/c(t)$ – hereafter "the economy's growth rate" or just "the growth rate" – to be driven partly or wholly by knowledge and resource growth (ν and γ), not solely by the TFP growth rate (τ) as in Stiglitz's model; and a higher human capital power ε also lowers the model's estimate of τ.

Stig5 thus extends Stiglitz's model in five ways:[5]

- Human capital H is included as a direct productive input in a one-sector model. As noted earlier, [2] defines H to be solely the accumulation of net investment, $\dot{H}$, as required for the sustainability theory in Pezzey (2004) (and Asheim 2004) to apply. Raw labour time, hereafter just "labour time", which grows with population L, is valued as a separate input to the Cobb-Douglas production function in [4], as in Mankiw et al. (1992, equation (8)). The World Bank's SNA-compatible definition of H as the "total present value of the expected future labor income" (WB21, p144) is much larger (though this is far from obvious in WB21's Box 7.1), and incompatible with our theory. The net change $\dot{H}$ would then include population growth and be more than half of GNI, and nH would greatly overstate the population dilution of human capital.[6]
- A fraction $1-\pi$ of gross investment in human capital is excluded from measured GNI.
- All three capitals depreciate at a common rate δ, also as in Mankiw et al. (1992), and in some other major models of human capital and growth like the Rebelo and Uzawa-Lucas models summarised by Barro and Sali-i-Martin (2004, Section 5.2).
- Knowledge capital N is a pure public good input, which has spillovers that cause a basic form of endogenous growth from IRS in production; but as in Dietz and Stern (2015), we do not separately model private agents' knowledge investment decisions.[7]

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[5] The comprehensiveness of Stig5 appears to be new. Perhaps closest to its methodology are theoretical models by Barbier (1999), Valente (2005) and Bretschger and Valente (2011), and calibrated models by Funke and Strulik (2000), Dietz and Stern (2015) and Barbier (2017); but all these have significant differences – both extra and omitted features – compared with Stig5.

[6] A similar remark applies to the sum of health capital and human capital in Arrow et al. (2012).

[7] We also tried two other ways of modelling IRS without using a separate knowledge variable N. One includes knowledge as part of produced capital, as in Dietz and Stern's (2015) equation (1.K), by using a production function $F(.) = A_0 K^{\alpha+\nu} H^\varepsilon R^\beta L^{1-\alpha-\varepsilon-\beta} e^{\tau t}$ instead of [4]. The second includes it as a spillover from human

- The resource stock grows exogenously at a gross rate γ, representing both discovery of subsoil resources and renewal of other natural resources, though for brevity we will call γ just the discovery rate.[8]

Our two simpler models are then:

- Stig4, defined by [1]-[5] but with $\pi = 1$ (no off-GNI investment); and
- Stig3, which has [1]-[5] but $\pi = 1$ and also $v = 0$ (no knowledge capital, hence CRS).

Stig5's theoretical structure and global calibration embody three key aspects of produced (K), human (H) and knowledge (N) capital. The first is that in [2], K, H and N are all directly accumulated by not consuming out of gross output $F(.)$, which we treat as a defining characteristic of "capital". That is why, contrary to dominant current usage as in WB21, we do not call the resource stock "natural capital", except when referring to WA's use of this term.

The second is what Acemoglu (2009, Sec. 10.4) would have called our "neoclassical" treatment of produced capital K and human capital H, which play similar roles in the production and distribution of output in [4] and [2], as in Mankiw et al. (1992), though with endogenous saving rates. Barro and Sala-i-Martin's (2004, p242) comment that this means there is "no meaningful distinction between [such a] model...and a model with a single form of broad capital" is then true in theory. However, keeping K, H, and also N, separate but accumulated from output $F(.)$ allows us to use data points like education spending and R&D spending to suggest separate calibrations of their production powers α, ε and v. We can then divide the PV of measured labour income into the value of accumulated human capital, H, and the PV of the earnings of L, which our model treats as (raw) labour time, in contrast to WB21 as noted in Section 2.2.[9]

The third key aspect of our capital treatment is the IRS in production caused by spillovers from knowledge capital N, in keeping with the endogenous growth literature (e.g. Romer 1994), though without the usual industry or firm-level microeconomic analysis. Our inclusion of N^v in the standard Cobb-Douglas formula avoids the more realistic complexities noted by Kohler et al. (2006) (that knowledge capital is only partly non-rival and partly non-excludable), but still produces a plausible first-order estimate of the empirical importance of knowledge. We will see

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capital, as in $F(.) = A_0K^{\alpha}H^{\varepsilon+v}R^{\beta}L^{1-\alpha-\varepsilon-\beta}e^{\tau t}$. Both yield the same expression for the endogenous interest rate r, so are formally equivalent to Stig4/5.

[8] Hamilton and Atkinson (2013) presented a model of resource discoveries and national income accounting, which includes discovery costs that depend on the discovery rate, cumulative discoveries and cumulative extraction, but was calibrated on subsoil resources only. Their model replaces our $R(t) = \gamma S(t) - \dot{S}(t)$ [3] with $R(t) = D(t) - \dot{S}(t)$, where where the discovery rate $D(t)$ is a choice variable. Subsoil discovery costs are arguably insignificant in the global economy, given Hamilton and Atkinson's rough estimate that costs are 5% of subsoil resource rents, and WDI data (NY.GDP.TOTLRT.ZS minus NY.GDP.FRST.RT.ZS) that subsoil resource rents, though very variable over time, are roughly 2.5% of GDP. So our assumption of costless discoveries is more accurate than several other approximations necessarily made in calibrating Stig3/4/5.

[9] We also considered a richer, two-sector treatment of human capital, based on the Uzawa-Lucas model in Section 5.2.2 of Barro and Sala-i-Martin (2004, p251), but including resource input R^{β} and TFP growth $e^{\tau t}$ as factors of production, with $R(t) = \gamma S(t) - \dot{S}(t)$ as in [3], and replacing $(uH)^{1-\alpha}$ with $(uhL)^{1-\alpha-\beta}$ where h is human capital per person, thus:

$$F(K,R,h,L) = C + \dot{K} + \delta K = A_0K^{\alpha}R^{\beta}(uhL)^{1-\alpha-\beta}e^{\tau t};\ (d/dt)(hL) = B_0(1-u)hL - \delta hL$$

which has a balanced-growth interest rate $r = [B_0(1-\alpha-\beta) + \beta(\gamma+\delta) + \tau]/(1-\alpha) - \delta$ (it can be shown). However, this introduces two extra parameters, u and B_0 which cannot readily be calibrated from GDP data, and does not include knowledge capital; so this model is left to further research.

that IRS requires a downward adjustment of the discount rate used for PV calculations, so that the sum of all capitals and PVs of all factor incomes equals the PV of consumption.

However, making Stig5 theoretically tractable has entailed significant omissions and approximations, and for completeness we list key ones here. First, assuming that all spillovers causing increasing returns come from knowledge capital, while none come from human capital, is semantically quite approximate and against the spirit of Lucas (1988), as noted by Akcigit and Nicholas's (2019) review. Our second and third, perhaps bigger, approximations, are to assume the same depreciation rate for all three capitals in [2], as noted later when choosing the calibration data point for δ; and to assume unit capital-resource substitutability in [4], which is not well-supported empirically (Cohen et al. 2019). Fourth, assuming optimisation by a global planner, rather than by individual firms with explicit modelling of spillovers as in standard endogenous growth models, is not ideal, but in this we follow many other authors, for example Dietz and Stern's (2015, p579) economy-level addition of knowledge spillovers.

Fifth, as discussed in the Introduction, Stig5 ignores climate change, hence we view our ANS as only a measure of medium-term, economic sustainability. Last, our calibrations are mostly, but unavoidably, quite approximate. As will be seen in Section 5, only two parameters in Stig5, the growth rates of population and output, are observable with any accuracy, and methodologies for measuring various capital stocks are inevitably contentious.[10] One advantage our Stig5 can contribute, though, is consistency: in estimating the effects of resource discovery, capitals depreciation, and population growth on other macroeconomic variables; and in boosting WDI data on education and R&D spending to better fit an alternative estimate of β, the resource's power in output [4], which will play a significant role in our calibrations.

3.2 Model solution

We start by defining the growth rates of consumption- and resource-related variables in asymptotic, balanced growth:

$$\lim_{t\to\infty} \dot{C}(t)/C(t) = \lim_{t\to\infty} \dot{F}(t)/F(t) = \lim_{t\to\infty} \dot{K}(t)/K(t) = \lim_{t\to\infty} \dot{H}(t)/H(t) = \lim_{t\to\infty} \dot{N}(t)/N(t) =: g + n \qquad [6]$$

$$\lim_{t\to\infty} \dot{R}(t)/R(t) = \lim_{t\to\infty} \dot{S}(t)/S(t) =: \chi. \qquad [7]$$

Inserting these into $F(.) = A_0 K^{\alpha} H^{\varepsilon} N^{\nu} R^{\beta} L^{1-\alpha-\varepsilon-\beta} e^{\tau t}$ in [4] gives this accounting equation for decomposing the asymptotic (economic) growth rate g into, respectively, the deepenings of produced capital, human capital, knowledge capital (with spillovers) and the resource flow, and TFP growth:

$$g = \alpha g + \varepsilon g + \nu(g+n) + \beta(\chi - n) + \tau \qquad [8]$$

Rearranging [8] gives this reduced-form relationship for the growth rate:

$$g = \frac{\nu n + \beta(\chi - n) + \tau}{1 - \alpha - \varepsilon - \nu}, \qquad [9]$$

and our Appendix A also yields this formula for the asymptotic interest rate in terms of the economic, population and net resource growth rates g, n and χ and the resource discovery rate γ:

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[10] As Ahmed and Bhatti's (2020) survey of TFP measurement put it: "Measurement errors occur because output and employment are directly observable and quantifiable, while capital stock is fundamentally unobservable due to growth accounting issues and several debatable assumptions."

$$\left[\lim_{t\to\infty} F_K(t)\right] - \delta = g + n + \gamma - \chi. \quad [10]$$

As noted earlier, we assume the Stig5 economy is in always-exponential growth from time 0 to ∞, not just asymptotically. Defining $F_0 \coloneqq F(0)$ and $C_0 \coloneqq C(0)$, from [6] and [7] the exact, always-exponential solution from time zero onwards is thus:

$$C(t) \coloneqq L(t)c(t) = C_0 e^{(g+n)t} = L_0 c_0 e^{(g+n)t};\ K(t) = K_0 e^{(g+n)t};\ F(t) = F_0 e^{(g+n)t} \quad [11]$$

$$H(t) = H_0 e^{(g+n)t},\ N(t) = N_0 e^{(g+n)t};\ S(t) = S_0 e^{\chi t},\ R(t) = (\gamma - \chi) S_0 e^{\chi t} \quad [12]$$

This means the interest rate $r := F_K - \delta = \alpha F_0 / K_0 - \delta$, a constant, and (see Appendix B) that the arbitrary parameters K_0, H_0 and N_0 are scaled such that

$$(r+\delta)K_0/\alpha = F_0 = C_0 + (g+n+\delta)(K_0 + \pi H_0 + N_0);\ \pi H_0/\varepsilon = N_0/\nu = K_0/\alpha \quad [13]$$

$$\Rightarrow\ F(t) = F_0 e^{(g+n)t} = C(t)(r+\delta)\ /\ [r + \delta - (\alpha + \varepsilon + \nu)(g + n + \delta)]. \quad [14]$$

Before developing the model further, we reduce algebraic clutter by noting that ρ, the utility discount rate, and θ, the consumption elasticity, are not independent in the model results. Given always-exponential growth, the Ramsey rule from Appendix A is $r = \rho + \theta g$, and ρ and θ in the solution equations can always be arranged into the form $\rho + \theta g$; so to save clutter we replace $\rho + \theta g$ by r from the outset. This is convenient, as ρ and θ are psychological parameters widely seen as the most ethically or empirically contentious ones; and it cuts Stig5's degrees of freedom for estimating growth rates from the 11 noted after [5] to 10. We can then express all growth rates as functions of the 10 parameters α, ε, ν, β, δ, π, n, γ, τ and r.

Of the parameters in [9]-[14], only g, n, χ and π will be calibrated directly from data. The others will be derived, using Stig5's theoretical results, from data points not explicitly in [9]-[14]. Our empirical calibrations will of course fulfil the condition $\gamma > \chi$ needed for resource use $R(t) = (\gamma - \chi)S(t)$ in [3] and [12] to be positive, which via [10] implies $r = \rho + \theta g > g + n$, thus ensuring the welfare integral [1] converges. Our calibrations also fulfil the condition $\nu n + \beta\chi + \tau > \beta n$ needed to make g in [9] fit the positive world growth rate observed during 1995-2014.

3.3 Decompositions of output and wealth

To reduce clutter in the following, we define

$$\mu \coloneqq r + \delta - (\alpha + \varepsilon + \nu)(g + n + \delta). \quad [15]$$

From [14], the decomposition of output at any time t (again see Appendix B) is then:

(Total) Consumption / GNI $= C/F = \mu/(r+\delta)$) [16]
Gross produced capital investment / GNI $= (\dot{K} + \delta K)/F = \alpha\,(g+n+\delta)/(r+\delta) \eqqcolon \hat{\boldsymbol{\alpha}}$)
Measured gross hum. cap. invst. / GNI $= \pi\,(\dot{H} + \delta H)/F = \varepsilon\,(g+n+\delta)/(r+\delta) \eqqcolon \hat{\boldsymbol{\varepsilon}}$)
which we equate to education expenditure / GNI;
Gross knowledge capital investment/GNI $= (\dot{N} + \delta N)/F = \nu\,(g+n+\delta)/(r+\delta) \eqqcolon \hat{\boldsymbol{\nu}}$)
which we equate to R&D expenditure / GNI.

New notations $\hat{\alpha}$, $\hat{\varepsilon}$ and $\hat{\nu}$ have been defined here to further reduce clutter. Their values are exogenous, initially the shares of GNI reported in WDI, as will be described in Section 5.2.

Decomposing wealth is more complex, because the IRS caused by knowledge capital drives the appropriate discount rate for PVs below the (real) interest rate r (which is endogenous but constant, given our always-exponential assumption), say to ηr, $\eta < 1$. We define the appropriate value of η as that which makes the sum of all capitals (K, H including what is generated off-GNI, and N) and the PVs of non-capital factor incomes ($RF_R = \beta F$ and $LF_L = (1–\alpha–\varepsilon–\beta)F$) equal to (total) wealth, $\Theta(t)$, defined as the PV of consumption using the same discount rate:

$$\Theta(t) := \int_t^\infty e^{-\eta r(z-t)} C(z)dz = C(t)/(\eta r - g - n) \quad \text{(from [11])} \qquad [17]$$

$$= K(t) + H(t) + N(t) + \beta F(t)/(\eta r - g - n) + \ (1 - \alpha - \varepsilon - \beta)F(t)/(\eta r - g - n)$$

so from [17] and [16] the decomposition of (total) wealth is:

$$\begin{array}{llcl} \text{Produced capital / wealth} & = K(t)/\Theta(t) & = & \alpha(\eta r - g - n)/\mu \quad) \; [18] \\ \text{Human capital / wealth} & = H(t)/\Theta(t) & = & \varepsilon(\eta r - g - n)/\pi\mu \quad) \\ \text{Knowledge capital / wealth} & = N(t)/\Theta(t) & = & \nu(\eta r - g - n)/\mu \quad) \\ \text{PV (resource revenue, } \beta F\text{) / wealth} & & = & \beta(r + \delta)/\mu =: \beta' \quad) \\ \text{PV (labour time income, } (1–\alpha–\varepsilon–\beta)/F\text{) / wealth} & & = & (1 - \alpha - \varepsilon - \beta)(r + \delta)/\mu \quad) \end{array}$$

where by construction, the sum of these five shares must equal 1. Hence:

$$[(\alpha + \varepsilon/\pi + \nu)(\eta r - g - n) \ + \ (1 - \alpha - \varepsilon)(r + \delta)\ ]/\mu = 1$$

and substituting μ from [15] leads (for details again see Appendix B) to:

$$\eta = [(\alpha + \varepsilon)r + (\varepsilon/\pi - \varepsilon)(g + n) - \delta\nu] \,/\, (\alpha + \varepsilon/\pi + \nu)r. \qquad [19]$$

We define β' as a separate entity in [18] because Section 5.2 gives conflicting empirical estimates of β and β', in that $\mu(r+\delta)$ times estimated β' is about 1.25 times estimated β. We use this conflict to help justify the much higher values of education and R&D spending (multiples of $\hat{\varepsilon}$ and $\hat{\nu}$) and the amount of off-GNI human capital ($1–\pi > 0$) assumed in our third (Stig5) calibration.

Next, the interest rate r is determined by the model as follows. Combining the last equation in [18] with the first in [16], and then using [2] and the others in [16], gives these connections between the resource's power in output, its share of wealth, and consumption's share of output:

$$\beta = \beta'\mu/(r + \delta) = \beta' C(t)/F(t) = \beta'(1 - \hat{\alpha} - \hat{\varepsilon} - \hat{\nu}). \qquad [20]$$

If we then denote

$$F(t)/\Theta(t), \text{ GNI / wealth} =: \Lambda, \qquad [21]$$

inserting this, $C(t)/\Theta(t) = \eta r - g - n$ from [17], and $C(t)/F(t) = \beta/\beta'$ from [20] into $C(t)/\Theta(t) = [C(t)/F(t)][F(t)/\Theta(t)]$ gives $r = (\Lambda\beta/\beta' + g + n)/\eta$. Appendix C then shows that successively inserting η from [19] and using definitions in [16] yields this final result (which simplifies to $r = \Lambda\beta/\beta' + g + n$ in Stig3 where $\nu = 0$ and $\pi = 1$, hence $\eta = 1$) for the interest rate in terms of parameters that we will be able to calibrate:

$$r \ = \ [(\hat{\alpha} + \hat{\varepsilon}/\pi + \hat{\nu})\Lambda\beta/\beta' + (\hat{\alpha} + \hat{\varepsilon} + \hat{\nu})(g + n) + \delta\hat{\nu}] \,/\, (\hat{\alpha} + \hat{\varepsilon}). \qquad [22]$$[11]

4. Definition of ANS†, and its relationship with wealth under constant returns

From (46) in Pezzey (2004), individual, net national product (NNP) is defined as per-person consumption plus per-person net investments (including off-GNI human capital investment,

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[11] This result can be used to show that $(\partial g/\partial\hat{\nu})/(\partial g/\partial\hat{\varepsilon}) = 1 + n/g + (1 - \beta/\beta' + \hat{\nu}\, n/g)(\partial r/\partial\hat{\nu})$, which empirically ≈ $1 + n/g = 1.7$ in all our calibrations. This shows that a counterfactual rise in R&D spending $\hat{\nu}$ would boost the growth rate g much more than the same rise in education spending $\hat{\varepsilon}$. This result is unsurprising, and of limited significance for policy, since it is generated by the double appearance of ν in expression [9] for g, which in turn is generated by Stig5's highly simplifying assumption in [4] that knowledge capital N generates spillovers while human capital H generates none.

since all H contributes to gross output F), corrected for population dilution (the lowering of per-person capital and resource stocks caused by population growth at rate n):

$$\begin{aligned} y(t) &:= c(t) + \mathrm{ANS}(t)/L(t) \\ &= c(t) + \{\dot{K}(t) + \dot{H}(t) + \dot{N}(t) + F_R(t)\dot{S}(t) - n[K(t) + H(t) + F_R(t)S(t)]\}/L(t) \quad [23] \end{aligned}$$

And from Pezzey's (52) and (51), we define (total) Adjusted Net Saving as net investment, corrected for population growth and 'augmented' with the value of TFP growth, $L(t)q^t(t)$:

$$\mathrm{ANS}^\dagger(t) := \dot{K}(t) + \dot{H}(t) + \dot{N}(t) + F_R(t)\dot{S}(t) - n[K(t) + H(t) + F_R(t)S(t)] + L(t)q^t(t),$$

$$\text{where } q^t(t) := \int_t^\infty e^{-(r-n)(z-t)} \frac{\partial y(z)}{\partial z} dz \qquad [24]$$ [12]

Note that $\mathrm{ANS}^\dagger$ includes net knowledge investment, $\dot{N}$, but omits $-nN$, because we assume that as a public good, the per-person value of knowledge N is not diluted by population growth n. This ad hoc treatment of the IRS property of Stig4/5 is one more approximation, because results in Pezzey (2004) assume CRS, among the several we cannot avoid making. We will then compare our $\mathrm{ANS}^\dagger$ estimates with WDI's results for $\mathrm{ANS}_{\mathrm{WB}}$, whose underlying model also breaks CRS because it includes pollution costs.

One feature of [24] that improves the accuracy of our $\mathrm{ANS}^\dagger$ definition is that it includes the benefits of natural resource discoveries and TFP growth – thus breaking conventions in the SNA that these should should not be included in saving (WB21, p53) – and the cost of population dilution. In theory, $\mathrm{ANS}^\dagger$ in Stig3's CRS economy equals the per-person growth rate times total wealth, proved as follows. In Stig3's notation, equations (46)-(49) in Pezzey (2004) mean that the time derivative of augmented, individual NNP, $y^\dagger(t) := y(t) + q^t(t)$, is:

$$\begin{aligned} \dot{y}^\dagger(t) &:= \left(\frac{d}{dt}\right)[y(t) + q^t(t)] = \frac{(r-n)\mathrm{ANS}^\dagger(t)}{L(t)} =: (r-n)ans^\dagger(t) \\ &= (r-n)[y^\dagger(t) - c(t)] \qquad [25] \end{aligned}$$

Integrating this, and using $\int_t^\infty e^{-(r-n)(z-t)} c(z)dz = \Theta(t)/L(t)$ from [17] under CRS shows (see Appendix D) that augmented, individual Adjusted Net Saving, $ans^\dagger$, equals the change in wealth per person, Θ/L:

$$ans^\dagger(t) := \mathrm{ANS}^\dagger(t)/L(t) = [\dot{\Theta}(t) - n\Theta(t)]/L(t) = (d/dt)[\Theta(t)/L(t)]. \qquad [26]$$

This is a population-adjusted equivalent to the utility-based result (3) in Hamilton and Clemens (1999), which is the theoretical basis for statements in WB21 like "In economic theory, investment net of depreciation and depletion equals the change in wealth" (p53). Note that $-n\Theta$ in [26] greatly exceeds the $-n(K+H+F_RS)$ deduction for population dilution in [24], because wealth Θ includes the value of knowledge capital and lifetime labour income. Our treatment of human capital in [2] and [4] means that flows (saving, output and consumption) and stocks (of capitals and resource) can be calibrated consistently, thus minimising the unavoidable departure from equality [26] caused by Stig4/5's IRS property.

In the CRS, Stig3 model, where $\nu = 0$ (no knowledge capital) and $\pi = 1$ (no off-GNI investment), we can convert augmented, individual NNP in [23] (see Appendix E) into this form:

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[12] Strictly speaking, total augmented net investment as defined by (18) and (23) in Pezzey (2004) replaces $L(t)q^t(t)$ in [24] by $Q^t(t) := \int_t^\infty e^{-(r-n)(z-t)} \frac{\partial[L(z)y(z)]}{\partial z} dz$. However, $Lq^t = Q^t$ in our always-exponential economy, so that individual, augmented net investment is as defined in [26], $ans^\dagger := \mathrm{ANS}^\dagger/L$.

$$ans^{\dagger}(t) \coloneqq y^{\dagger}(t) - c(t)$$
$$= \left[\frac{\alpha(g+n)}{r+\delta} + \frac{\varepsilon(g+n)}{r+\delta} + \frac{\beta\chi}{r-g-n} - n\left(\frac{\alpha}{r+\delta} + \frac{\varepsilon}{r+\delta} + \frac{\beta}{r-g-n}\right) + \frac{\tau}{r-g-n}\right]\frac{F(t)}{L(t)}. \qquad [27]$$

Terms here reflect produced and human capital growth (the α and ε terms); net resource growth (the first β term); population dilution (the n term); and the value of time, proportional to TFP growth at rate τ. To get the formula that we later evaluate and contrast with total ANS_{WB}, we multiply [27] by $L(t)/F(t)$. For Stig4/5 we also make a judgmental addition of $v(g+n)/(r+\delta)$ for the contribution of net investment in knowledge capital, but we do not deduct $nv/(r+\delta)$ as a population dilution, because as a pure public good, the value of knowledge is not diluted by population growth. So the final formula used for our calibration results is thus:

$$\frac{\text{ANS}^{\dagger}(t)}{F(t)} = \left[\frac{\alpha(g+n)}{r+\delta} + \frac{\varepsilon(g+n)}{r+\delta} + \frac{\beta\chi}{r-g-n} - n\left(\frac{\alpha}{r+\delta} + \frac{\varepsilon}{r+\delta} + \frac{\beta}{r-g-n}\right) + \frac{\tau}{r-g-n} + \frac{v(g+n)}{r+\delta}\right] \qquad [28]$$

To relate these results to prior literature, note that inserting the always-exponential form of wealth in Stig3, $\Theta(t) = \Theta_0\, exp[(g+n)t]$, into [26] gives:

$$\text{ANS}^{\dagger}(t)/F(t) = ans^{\dagger}(t) = g\Theta(t)/L(t) \ = gc(t)/[(r-g-n)(t)], \qquad [29]$$

where the second equality follows from [17] with $\eta = 1$. Equation [29] (which is only approximate in the IRS case of Stig4/5, when we define $\text{ANS}^{\dagger}$ as [28]) is consistent with Hamilton and Hartwick's (2005) Proposition 1, once their model is extended to include Stig3's extra features like TFP growth and population growth. (For details, again see Appendix E.) Lastly, the Harrod-Domar-Solow formula, that "...in the long run the wealth-income ratio...is equal to the net-of-depreciation saving rate...divided by the income growth rate" (Piketty and Zucman 2014), holds in Stig3. However, Stig3's extra features make the wealth/income ratio much larger than it would be in a classic model: using Stig3's Calibration 1 that we define later, our ratio is 15.5, compared to just 4.5 for the classic model (see Appendix F).

5. Calibrating Stig5 to World Bank and other data for 1995-2014 or 2014

5.1 Evidence for near-exponential growth in 1995-2014, and four calibration data points

This section finally gives the empirical evidence for our key introductory claim that the aggregate world economy was reasonably close to exponential growth during 1995-2014. This evidence gives us three data points used to calibrate constants in Stig5: the growth rate g, the population growth rate n, and the growth rate of resource input, χ. We also show the depreciation rate we use for all capitals, δ.

Our key evidence is the World Bank economic data for 1995-2014 in **Table 1**. As noted earlier, the wealth/GDP and produced capital/GDP ratios changed very little from 1995 to 2014. The GDP growth rate (g+n) fell by only 0.6 percentage points from 1995 to 2014, while the population growth rate fell only about 0.2 points. So as two approximate calibration data points based on Table 1 we will therefore use:

Table 1: Global economic data, 1995-2014

		1995	2000	2005	2010	2014
Ratios:	Total wealth / GDP (yr)	15.1	14.7	14.2	14.4	14.3
	Produced capital / GDP (yr)	3.6	3.5	3.6	3.8	3.8
Yearly growth rates over last 4 or 5 years:	GDP / population	-	2.0%	1.8%	1.4%	1.6%
	Population	-	1.4%	1.3%	1.2%	1.2%
	Wealth	-	2.8%	2.4%	2.9%	2.8%
	Produced capital	-	2.7%	3.8%	3.5%	3.1%
Sources: Wealth, capital: World Bank (2021b); GDP, population: World Bank (2021c).						

the (economy's) **growth rate** in 2014, $\boldsymbol{g}$ **= 1.6 %/yr** (= 2.8 – 1.2 %/yr) [30]

& the **population growth rate** in 2014, $\boldsymbol{n}$ **= 1.2 %/yr**. [31]

Next, as our third calibration data point we will use:

the **depreciation rate of all capitals,** $\boldsymbol{\delta}$ **= 4.2 %/yr** [32]

This is the world average value for produced capital depreciation in Karabarbounis and Neiman (2014, footnote 14). It comes from the Penn World Table, which WB21 often cites. As noted in Section 3.1, assuming human and knowledge capitals depreciate at this same rate is a major simplification, needed to make Stig5 tractable. However, no relevant estimates could be found of economy-wide human capital depreciation, that is, estimates that include retirement and death as well as people's decay of skills while still working. Estimates of knowledge depreciation vary widely. Dietz and Stern (2015, p580) cited Hall et al.'s (2009) estimate of annual depreciation of around 15% of private, firm-level R&D capital, but noted that "a much broader construct of knowledge" is relevant to economy-wide growth modelling.

Natural resources are very diverse, so calibrating a single, aggregate "resource" flow and stock in our global model is also very approximate. **Table 2** gives the best readily available global data: for energy flow and oil and gas reserves, for all material flows by weight, and for an index of food production. The unpredictability of subsoil resource discoveries and climatic impacts of food production means that growth rates, even the 4-5 year averages reported here, are much more volatile than for economic aggregates. Overall, though, they give reasonable support to using, as our fourth definite calibration data point:

Table 2: Global resource data, 1995-2014

	Commodity (measure used)	2000	2005	2010	2014	1995-2014
Yearly growth rates (over last 4 or 5 years, except last column):	Primary energy use (Joules)	1.8%	3.0%	2.1%	1.6%	2.13%
	Oil reserves (barrels)	2.9%	1.2%	3.6%	0.9%	2.20%
	Gas reserves (cubic mtr.)	3.1%	2.5%	2.5%	1.6%	2.48%
						1995-2010
	Material flows (tonnes)	2.4%	4.0%	3.5%	-	3.3%
	Food production (index)	4.5%	4.0%	0.2%	-	2.9%
Sources: Energy/oil/gas: BP (2020); materials: Schandl et al. (2018); food: World Bank (2021d)						

the **resource net growth rate** in 2014, $\boldsymbol{\chi}$ **= 2.0 %/yr.** [33]

(It turned out that choosing χ = 2.2 or even 2.5 %/yr instead changed our calibration results very little.) We will see later that as well as subsoil resources like fossil fuels and minerals, which grow through discovery, our "resource" includes substantial agricultural and forestry land, which yields resource renewal as growing crops, animals and timber.

5.2 Further parameter calibrations needed, and available

Given the 10 independent parameters that we noted in Section 3.2 are needed to determine Stig5's growth rates, the 4 data points selected in [30]-[33] leave 6 degrees of freedom. So we need 6 more data points to completely calibrate Stig5 (or 5 more for Stig4, and 4 for Stig3).

However, we can find 8 calibration data points in WDI and/or WA for possible use in Stig3/4/5, as follows. Obviously these are not all independent; some of them will not be used here, while other calibrations depart greatly from them; and some of them may be mutually inconsistent, as we discuss immediately afterwards. The first two data points in [34], for $\hat{\alpha}$ and $\hat{\varepsilon}$, are for 2014 and come directly from WDI.[13] The third, for R&D spending's share $\hat{\nu}$, is for 2013.[14] The shares of produced capital and natural capital in total wealth are from WA,[15] while the GNI/wealth ratio comes from WDI[16] and WA, all for 2014.

$(\dot{K} + \delta K)/F$, gross produced capital investment / GNI (=: $\hat{\alpha}$) = **26.4%** (WDI)) [34]

$\pi(\dot{H} + \delta H)/F$, gross measured human capital investment / GNI, which we initially equate to:
public education expenditure / GNI (=: $\hat{\varepsilon}$) = **4.1%** (WDI))

$1 - \pi$, fraction of gross human capital investment outside GNI = **0.5** (Kendrick 1976))

$(\dot{N} + \delta N)/F$, gross knowledge capital investment / GNI, which we initially equate to:
R&D expenditure / GNI (=: $\hat{\nu}$) = **2.1%** (WDI))

We will make big changes in the values of $\hat{\varepsilon}$, π and $\hat{\nu}$ used in our selected calibrations of Stig4 and Stig5, but these are better discussed in Section 6 when we give our calibration results.

K/Θ, produced capital's share of wealth = **26.4%** (WA)[17]) [35]

F/Θ, GNI / wealth (=: Λ) = **7.0 %/yr** (WA & WDI))

$\beta F/C$, the PV of βF (resource revenue) as a share of wealth, which we equate to:
natural capital / wealth (=: β') = **9.4%** (WA))

and using assumptions we will discuss shortly, this independent estimate can be made:

β, the resource flow's power in output = **5.2%** (WDI & WA) [36]

Both these resource estimates are biased downwards, because they equate revenue with rents, thus ignoring extraction costs, on which there are no available global data. As discussed below,

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[13] Series Codes are NY.ADJ.NNAT.GN.ZS plus NY.ADJ.DKAP.GN.ZS, and NY.ADJ.AEDU.GN.ZS.

[14] Data for R&D spending's share of GNI (Series Code GB.XPD.RSDV.GD.ZS) were available for 1996-2013 at https://databank.worldbank.org until September 2021. Similar though older shares for major OECD countries were noted by Barro and Sala-i-Martin (2004, p.301).

[15] The Series Codes in WA are respectively NW.PCA.PC, NW.NCA.PC and NW.TOW.PC.

[16] GNI's Series Code in WDI is NY.GNP.MKTP.KD, but this number for F is first multiplied by 1.08, to convert from constant 2010 US\$ to constant 2014 US\$, the numeraire used in WA.

[17] The 26.4% for K/Θ and 9.4% for $\beta F/C$ are calculated from WA after ignoring WA's -0.4% for Net foreign assets as an accounting inconsistency, since there is no foreign entity at the global level.

our three calibrations make different choices of β. In all our calibrations, the largest share of total wealth Θ is taken by the PV of labour time (included in human capital by the World Bank, but not by Stig5), whose share ranges from 60% down to 34% in our calibrations.

We now explain why $(\dot{K} + \delta K)/F$ in [34] and K/Θ in [35] both equal 26.4%, and how they relate to the 7.0 %/yr data point for F/Θ (which is distinct from the endogenous interest rate r derived in [22]). Firstly, WDI's 26.4% for $(\dot{K} + \delta K)/F$ is divided into $\dot{K}/F$ = 10.5% and $\delta K/F$ = 15.9%, and one can then check that (net investment)/depreciation, $\dot{K}/\delta K,$ indeed equals $(g+n)/\delta$, as required by Stig5's always-exponential assumption in [11] that $K(t) = K_0 e^{(g+n)t}$.[18] Secondly, our calibrated values for the gross (produced) investment share of GNI, $\hat{\alpha}$, and the (produced) capital share of wealth, K/Θ, have both been chosen to equal 26.4%, so K/Θ = 26.4% in [35] is not an independent data point. That this is possible follows from the observation that the World Bank's GNI/wealth ratio for 2014 in [35], from WA and WDI, happens to satisfy:

$$F/\Theta \text{ } (=: \Lambda = 7.0\%) = g + n + \delta \text{ from [30]-[32],} \qquad [37]$$

so that, using [6] and [16], the shares of wealth of produced, human and knowledge capitals equal $\hat{\alpha}$, $\hat{\varepsilon}$ and $\hat{\nu}$, the output shares in output of spending on produced capital investment, education and R&D, except when $\pi < 1$:

$$K/\Theta = \hat{\alpha}, \ \ \pi H/\Theta = \hat{\varepsilon} \text{ and } N/\Theta = \hat{\nu}. \qquad [38]$$

There is no theoretical reason why [37] (and hence [38]) should hold in Stig5. It seems likely, though, that the World Bank chose its discount rates and other wealth estimation parameters so they do hold. In any event, [38] will simplify our table of calibration results in Section 6.

We therefore now have 7 independent data points in [34]-[36], still one more than needed, which gives rise to a data inconsistency as follows. Recall [20]:

$$\beta = (C/F)\beta' = (1-\hat{\alpha}-\hat{\varepsilon}-\hat{\nu})\beta'.$$

In our first calibration, consumption's share of output, C/F = 69.5%. Multiplying this by β' = 9.4% from [35] gives β = 6.5%, inconsistent with our independent estimate of the resource flow's power in output, β = 5.2% in [36]. We resolve this in our second and third calibrations by using much higher values for $\hat{\varepsilon}$ and $\hat{\nu}$, the output shares of gross investment in human and knowledge capital. As discussed below in Section 6, we will use twice the data points from [34] in calibrating Stig4, and three times their values in calibrating Stig5, both for inductive reasons, and with broad empirical justification from results in Haveman and Wolfe (1995) and Corrado et al. (2005). In our Stig5 calibration this makes $(1-\hat{\alpha}-\hat{\varepsilon}-\hat{\nu})\beta'$ = 5.2%, consistent with β in [36].

So as promised, we now explain how we estimate the data point [36]. First, we use data for resource rents (WDI's series for Total natural resource rents), because as noted above, data for resource revenues, which are higher than rents as they include extraction costs, are not available; to that extent our values for both β and β' are under-estimates. Next, as **Table 3** shows, resource rents as a share of GDP are highly variable over time, even when aggregated globally, and the heterogeneity and geographic dispersion of natural resources means that any estimate is

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[18] The ratio of 5-year averages for $[(\dot{K}/F)(t)]/[(\delta K/F)(t)]$ reported in WDI is about 0.4-0.6, much wider than the range for GDP growth (g+n) shown in Table 1, but its reported value in 2014 almost exactly equals $(g+n)/\delta$ from [30]-[32].

Table 3: Reported and estimated global resource rent shares of GDP, 1995-2014

	1995	2000	2005	2010	2014	1995-2014 average
(Rents from subsoil assets + forests) / GDP	1.3%	2.1%	3.2%	3.7%	3.1%	
(Subsoil assets + forests) / (Total natural capital)	43.6%	43.9%	51.6%	58.0%	55.6%	
(*Estimated* rents from all natural resources) / GDP	3.1%	4.7%	6.1%	6.4%	5.6%	5.2%

Sources: Series Codes NY.GDP.TOTL.RT.ZS from World Bank (2021c) for rents, (NW.NCA.SSOI.TO + NW.NCA.FORE.TO) / NW.NCA.TO from World Bank (2021b) for values of assets and capital stocks.

inevitably quite approximate.[19] Last, our calibration procedure is indirect, because WDI's Total natural resources rents comprises subsoil assets (fossil fuels and minerals) and forests, but not agricultural land and protected areas, the other two categories of total natural capital reported in WA. So for each year that rents are reported during 1995-2014, we have first divided the GDP share of subsoil plus forest rents reported in WDI, by the share in total natural capital of subsoil plus forest assets reported by WA, to estimate in Table 3's last row the GDP shares of all natural resources rents, above and below ground; and 5.2% is the 1995-2014 average of these shares.

5.3 Defining our calibrations

Here we confirm the three calibrations (combinations of data points), one each of Stig3, Stig4 and Stig5, that we have chosen to report in detail. Our aim is to show that the observed growth rate (g) cannot be explained without including substantial levels of education spending ($\hat{\varepsilon}$), and/or R&D spending ($\hat{\nu}$), and/or off-GNI human capital investment ($1-\pi$), and/or TFP growth rate (τ); and to illustrate, but not decompose in detail, how much boosting the assumed levels of the first three of these human factors can lower the last factor, the TFP growth rate (τ), that is consistent with g.[20] All calibrations use 7 common data points: from [30]-[33] for the growth rates g of per-person consumption, n of population and χ of the resource, and for the common depreciation rate δ; from [34] for the (gross produced capital investment) / GNI ratio $\hat{\alpha}$; and from [35] for the GNI/wealth ratio Λ and the (PV of resource rent) / wealth ratio β'.

So formally, the choices of $\hat{\varepsilon}$, $\hat{\nu}$ and π that define the calibrations are as follows:

Calibration 1, of model Stig3, uses the World Bank's value for (education spending)/GNI, $\hat{\varepsilon} = 4.1\%$ from [34], but sets (R&D spending)/GNI, $\hat{\nu}$, to zero and thus has CRS.

Calibration 2, of model Stig4, uses twice the World Bank's values from [34] for (education spending)/GNI, choosing $\hat{\varepsilon} = 8.1\%$, and for (R&D spending)/GNI, choosing $\hat{\nu} = 4.1\%$, thus giving IRS.

Calibration 3, of model Stig5, triples the World Bank's values in [34] for (education spending)/GNI to $\hat{\varepsilon} = 12.2\%$, and for (R&D spending)/GNI to $\hat{\nu} = 6.1\%$, also giving IRS; and

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[19] Total natural resources rents (Series Code NY.GDP.TOTL.RT.ZS) have more inter-annual variability than suggested by Table 3's first row, with for example a six-fold range between the maximum and minimum during 1970-2020; but the average over this period is 2.7%, very close to the first row's average.

[20] Using [9], [16], [22] and [20], we could derive τ in terms of the calibrated data points g, n, δ, $\hat{\alpha}$, $\hat{\varepsilon}$, $\hat{\nu}$, π, χ and β', but the derivatives $\partial\tau/\partial\hat{\varepsilon}$, $\partial\tau/\partial\hat{\nu}$ and $\partial\tau/\partial\pi$ are too complex to convey any useful intuition; and many more than three calibrations would be needed to give a detailed decomposition of these three effects.

it assumes that only half of gross human capital investment is measured by GNI, i.e. $\pi = 0.5$ from [34], unlike Calibrations 1 and 2 where there is no "off-GNI" investment, i.e. $\pi = 1$.

Our reasons for selecting these calibrations, already mentioned in passing above, are best discussed in detail when showing their results below. We also report there a single result for *Calibration 0*, which zeroes human and knowledge capitals' powers ε and ν and the resource's power β in Stig3. This reduces Stig3 to a Ramsey-Cass-Koopmans model, with produced capital, labour time and TFP as the sole inputs to $F(.)$ in [4]. The TFP growth rate τ is then the sole driver of the growth rate g in [9], with $g = \tau/(1-\alpha)$.

6. Results and discussion of Calibrations 1-3

Table 4 shows the chosen, calibration-based parameters, and resulting estimates derived (endogenous) parameters, and of output, expenditure and wealth shares in Stig3/4/5 using Calibrations 1-3. For context, row (24) shows that the TFP growth rate in Stig3 using Calibration 0 is $\tau = 0.86$ %/yr, which is in the broad range of mainstream estimates (e.g. Barro and Sala-i-Martin 2004, Table 10.1). To avoid spurious accuracy, all numbers are rounded to 2 or 3 significant figures. The italicised, underlined numbers are any endogenously calculated results that do not match the data points chosen in Section 5, which are listed in the penultimate column. Each calibration has at least one mismatched number, because of the inconsistency between β' and β estimates discussed in Section 5.2.

Since results in rows (5)-(7) for the resource discovery rate, the (real) interest rate and the (real, dollar) discount rate are broadly similar for all three calibrations, we mainly discuss the other, bigger differences among the calibrations. Note, though, the importance of the 7-9 %/yr rate of resource discovery, γ (including renewal, since the model's "resource" includes renewable resources), in allowing the resource stock to grow despite growing resource flow; and also the importance of the 7-8 %/yr discount rate, ηr in row (7), noted in Section 2.2 as an approximate justification of Stig3/4/5's always-exponential assumption.

In Calibration 1, TFP growth in row (21) accounts for over 90% of the observed 1.6 %/yr growth rate, which thus gives little or no guidance on what policies might sustain growth. Also the PV of labour time's income is estimated to be about 15 times "human" capital (60.2 % in row (18) versus 4.1% in row (16)), which seems implausible, even given the smaller, investment-based definition of "human" capital used in Stig5. (We temporarily re-insert the speech marks here and in Tables 4 and 5 as a reminder that in our model, "human" capital accumulated by "education" spending, and "knowledge" capital accumulated by "R&D" spending, are not based on any formal definition grounded in the SNA, but are defined as any accumulated skills and knowledge that respectively have no, or positive, spillovers in production [4].) Hence our assuming higher values, out of countless alternatives, of "education" spending, $\hat{\varepsilon}$ and "R&D" spending, $\hat{\nu}$, in Calibration 2 and Calibration 3, in order to lower the estimate of TFP growth needed to explain the growth rate.

In Calibration 2, we set $\hat{\varepsilon}$ to 8.1% and $\hat{\nu}$ to 4.1% of GNI, both twice WDI's estimates of spending shown in [34]. Our broad justification for doubling $\hat{\varepsilon}$ is Haveman and Wolfe's (1995) estimate that about 15% of US (not global) GDP was then spent on raising children; and since then education expenditure, including on early and private education, is likely to have risen considerably. Our broad justification for including a doubled $\hat{\nu}$ is Corrado et al.'s (2005) estimate that "by the end of the [1990s], business fixed investment in intangibles ['knowledge'] was at

Table 4: Results for calibrations of Stig3/4/5 global economy models in 2014

Variable		Symbol and/or formula in Stig3 (v=0, π=1), Stig4 (v>0, π=1) & Stig5 (v>0, π=0.5)	How calculated	Calibr-ation 1 (Stig3)	Calibr-ation 2 (Stig4)	Calibr-ation 3 (Stig5)	Data point	
RATES [/yr]:								
(1)	Population growth	n	Calibrated	1.2%			1.2%	*
(2)	(Economic) growth (of GNI per person)	g	Calibrated	1.6%			1.6%	*
(3)	Capitals depreciation	δ	Calibrated	4.2%			4.2%	&
(4)	Resource net growth	χ	Calibrated	2.0%			2.0%	#
(5)	Resource discovery	$\gamma\ (= r-g-n+\chi)$	Derived	6.8%	7.6%	8.8%		
(6)	Interest	$r(\beta,\beta',v^{\wedge},\delta,g,n,\pi,\Lambda)$	Derived	7.6%	8.4%	9.6%		
(7)	Discount (for PV)	$\eta r := \Lambda\beta/\beta' + g + n$	Derived	7.6%	7.0%	6.6%		
(8)	Total factor productivity (TFP) growth	$\tau\ \ (= (1-\alpha-\varepsilon-v)g - vn - \beta(\chi-n))$	Derived	0.72%	0.35%	0.00%		
(9)	GNI / wealth	$\Lambda := F/\Theta\ (= g+n+\delta)$	Calibrated	7.0%			7.0%	~
POWERS AS INPUTS TO F, GROSS WORLD OUTPUT:								
(10)	Physical capital	$\alpha\ (= \alpha^{\wedge}(r+\delta)/(g+n+\delta))$	Derived	0.447	0.477	0.521		
(11)	"Human" capital	$\varepsilon\ (= \varepsilon^{\wedge}(r+\delta)/(g+n+\delta))$	Derived	0.069	0.147	0.240		
(12)	Labour time	$1-\alpha-\varepsilon-\beta$	Derived	0.419	0.319	0.187		
(13)	Natural resource	$\beta\ (= \beta'(1-\alpha^{\wedge}-\varepsilon^{\wedge}-v^{\wedge}))$	Derived or Calibrated	*0.065*	*0.057*	0.052	0.052	~
(14)	"Knowledge" capital	$v\ (= v^{\wedge}/(r+\delta)/(g+n+\delta))$	Derived	0.000	0.074	0.122		
EXPENDITURE SHARES IN F (GROSS WORLD OUTPUT) & CAPITAL SHARES IN Θ (TOTAL WORLD WEALTH):								
(15)	Gross *K*-investment & Physical capital	$\alpha^{\wedge} := (dK/dt + \delta K) / F = 10.5\%+15.9\% = K/\Theta$	Calibrated	26.4%			26.4%	*
(16)	"Education" spending	$\varepsilon^{\wedge} := \pi(dH/dt+\delta H) / F$	Calibrated or Induced	4.1%	*8.1%*	*12.2%*	4.1%	*
	"Human" capital	$H/\Theta\ (= \varepsilon^{\wedge}/\pi)$				*24.2%*		
(17)	"R&D" spending & "Knowledge" capital	$v^{\wedge} := (dN/dt + \delta N) / F = N/\Theta$	Calibrated or Induced	*0.0%*	*4.1%*	*6.2%*	2.1%	*
(18)	PV of labour time	$1-\alpha^{\wedge}-\varepsilon^{\wedge}/\pi-v^{\wedge}-\beta'$	Derived	*60.2%*	*52.0%*	*33.9%*		
(19)	PV of resource rents	β'	Calibrated	9.4%			9.4%	■
(20)	Consumption	$C/F\ (= 1-\alpha^{\wedge}-\varepsilon^{\wedge}-v^{\wedge})$	Derived	*69.5%*	*61.4%*	*55.2%*		
RESULTS ON GROWTH RATES:								
(21)	Contribution of TFP growth to *g* = 1.6	$\tau/(1-\alpha-\varepsilon-v)$		1.5%	1.1%	0.0%		
(22)	Contribution of knowledge growth to *g*	$vn/(1-\alpha-\varepsilon-v)$		0.0%	0.3%	1.2%		
(23)	Contribn. of resource /psn growth to *g*	$\beta(\chi-n)/(1-\alpha-\varepsilon-v)$		0.1%	0.2%	0.4%		
(24)	Calibration 0 of Stig3 (after Solow 1957): $\varepsilon^{\wedge} = v^{\wedge} = \beta' = 0\ \&\ \pi = 1 \Rightarrow \tau$ = 0.86 %/yr							

Data sources: * = WDI; # = Table 2; & = Karabarbounis & Neiman (2014); ■ = WA ; ~ = WDI & WA

Parameters & results in *underlined italics* do not fit calibrated data values (not all values can be fitted: see text).

least as large as business investment in traditional, tangible ['produced'] capital". Row (8) of Table 4 shows that these higher levels of "education" and "R&D" spending would about halve the TFP growth rate, τ, needed to account for g, from 0.72 to 0.35 %/yr. However, because human capital has no spillovers but knowledge capital does, the extra 4% of GNI allocated to education spending in Calibration 2 lowers τ by only 0.07 %/yr, while the 4% of GNI allocated to R&D spending lowers τ by 0.28 %/yr (a difference not shown on Table 4, but also reflected in the theoretical result $(\partial g/\partial\hat{v})/(\partial g/\partial\hat{\varepsilon}) \approx 1 + n/g$, shown earlier in the footnote to [22]).

So in an inductive approach, we experimented to find a calibration that would completely eliminate the need for any exogenous TFP growth rate τ to explain the observed growth rate g in [30]. One answer is Calibration 3, in which WDI estimates in [34] of "education" and "R&D" spending are tripled, to a combined total of 18.4% of GNI. Just these two changes would make the calculated resource power, $\beta = \beta'(1 - \hat{\alpha} - \hat{\varepsilon} - \hat{v})$ in [20], equal to our independent estimate of 5.2% in [36], while still keeping the share of resource revenue's PV in total wealth at $\beta' = 9.4\%$, which gives some justification for our tripling; and it would lower estimated τ to 0.13 %/yr (not shown in Table 4). However, to make estimated $\tau = 0$, we also need to use the $\pi = 0.5$ data point in [34],[21] based on Kendrick's (1976) estimate that only one-half of gross investment in human capital was included in US GDP, because of unpaid time spent studying, parenting, etcetera. The case for using this old estimate is strengthened by Kendrick's (1994, Table 1A) estimate that total investment was about four times gross private domestic investment recorded in GDP. As seen in rows (16)-(18), "human factors" ("human" plus "knowledge" capital) are then estimated to be worth 30% of total wealth, now much closer to the 34% estimated for the PV of labour time.

7. Comparison of World Bank and our results for Adjusted Net Saving

Here we finally achieve this paper's main aim, of comparing global estimates of the World Bank's ANS_{WB} and our $\text{ANS}^{\dagger}$ for 2014, using results for Calibrations 1-3 in **Table 5**. These show the relative importance of human and knowledge capital, net resource growth, population dilution and TFP growth in ANS estimates. In Table 5 we use the formulae in [28] for the components and sum of $\text{ANS}^{\dagger}/\text{GNI}$, and $g\Theta$ (growth rate times total wealth) divided by F, from L/F times expression [29] for $ans^{\dagger}$ in the Stig3, CRS economy. Because Stig4 and Stig5 include net investment in knowledge capital and thus have IRS, making [29] only approximately true as we noted, $\text{ANS}^{\dagger}/F$ in Calibrations 2 and 3 is noticeably less than, not equal to $g\Theta/F$.

Five differences are obvious from comparing the World Bank's published ANS_{WB} components with our theoretically more consistent $\text{ANS}^{\dagger}$ components in Calibration 1. First, the World Bank includes all public education spending in ANS as human capital investment, whereas Calibration 1 nets out human capital depreciation, hence the difference between their 4.1% and our 1.6%. Second, the Bank follows the SNA, in which "New discoveries of subsoil assets...are only added to the balance sheet, not ANS" (WB21, p53); whereas our Calibrations include our estimate in [33] that the aggregate resource stock and flow grow at an average net rate $\chi = 2.0$ %/yr; hence the 3-4% difference between the -1.6% (i.e. depletion cost) and the growth values shown.

Third, the World Bank deducts 1.6% of GNI for pollution damages from CO_2 and particulate emissions. As noted in the Introduction, climate damages are highly contested, and we omit all

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[21] The effect of π on τ, invisible in [9], occurs via r in [22], hence via α, ε, v in [16], and thus in [9]. And the fact that a zero τ estimate happens for round numbers like triple and half is a happenstance, and countless other calibrations could also result in a zero τ estimate.

Table 5 Global ANS estimates for 2014 by World Bank & by Stig3/4/5 (Calibrations 1-3)

Global ANS components (% of GNI, F)	*Stig3/4/5 formula*	**World Bank (ANS_{WB})*	*Calibr-ation 1 (ANS†)*	*Calibr-ation 2 (ANS†)*	*Calibr-ation 3 (ANS†)*
Net physical capital investment	$\alpha(g+n)/(r+\delta)$	10.5%	10.5%	10.5%	10.5%
Net "human" capital investmt	$\varepsilon(g+n)/(r+\delta)$	-	1.6%	3.2%	9.6%
Gross human capital investment	$\varepsilon(g+n+\delta)/(r+\delta)$	4.1%	-	-	-
Value of net **resource growth**	$\beta\chi/(r-g-n)$	-1.6%	2.7%	2.0%	1.5%
Population dilution of capital and resource stocks (**-ve human factor**)	$-n[(\alpha+\varepsilon)/(r+\delta)$ $+\beta/(r-g-n)]$	-	-6.7%	-7.1%	-7.4%
Value of time (value of growth in total factor productivity, TFP)	$\tau/(r-g-n)$	-	14.8%	6.1%	0.0%
Net "knowledge" capital investment	$v(g+n)/(r+\delta)$	-	0.0%	1.6%	2.5%
Pollution (CO2, particulates)	-	-1.6%	-	-	-
ANS/*F* (sum of above components)	$[(\alpha+\varepsilon)g+v(g+n)]/(r+\delta)$ $+[\beta(\chi-n)+\tau]/(r-g-n)$	11.3%	22.8%	16.5%	16.6%
+ve human factors (human + knowledge capital investment + TFP growth value)	$(\varepsilon+v)(g+n)/(r+\delta)$ $+\tau/(r-g-n)$	4.1%	16.4%	11.0%	12.1%
Growth rate × total wealth	$g\Theta/F$ (= $ANS^\dagger/F$ in CRS)	^22.8%	22.8%	22.8%	22.8%

**2014 data in WDI (World Bank 2021c), where "resource" excludes agricultural land and protected areas*

^Calculated here from World Bank data for g in eq. [30] and F/Θ in eq. [37], but not calculated as such by World Bank

pollution damages in Stig3/4/5. This is not to deny the need for more research on the interaction of climate change and economic growth, though, building on existing work like Fankhauser and Tol (2005) and Dietz and Stern (2015). Climate matters for wealth, too: for example, using a \$208 social cost per tonne of CO_2 from Hänsel et al (2020) would value the anthropogenic, atmospheric CO_2 stock as a substantial cut from WA's estimated global wealth.

Fourth, ANS_{WB} in WDI omits population dilution of capital and resource stocks, and on that account puts global ANS ~7% too high according to Table 5. But fifth, ANS_{WB} also omits TFP growth, so it has no data point to compare with our Calibration 1 estimate that TFP growth (the $\tau/(r-g-n)$ term in [28]) contributes 14.8% to $ANS^\dagger$/GNI.[22] TFP growth minus population dilution explains almost all the 11.5% shortfall of total ANS_{WB}/F below the 22.8% value of $ANS^\dagger/F$ in Calibration 1, a value which from [29] also equals $g\Theta/F$, because Calibration 1 has CRS.

An interesting digression here is that although this equality holds exactly in Calibration 1 by deliberate choice of data points (which however fit reality fairly well as explained in Section 5), Pezzey et al. (2006) found it to be quite untrue in Scotland during 1992-99. Appendix G analyses the numerical differences between the equality in Calibration 1 and the "mismatch" in Pezzey et al., where from their data, $gC/(r-g-n)$ (= $g\Theta$) was roughly 5 times $ANS^\dagger$. The two main causes of this mismatch turn out to be Scotland's higher growth rate in the 1990s (2.5 %/yr compared to 1.6 %/yr from [30]) and a much lower estimated value of time (1.3% of GNI compared to our 14.8%), though we do not investigate here the accuracy or economic causes of these data differences.

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[22] This number is in the same range as the estimates for a modified-DICE model in Table 3 of Pezzey and Burke (2014), though this is somewhat coincidental as their methodology was rather different.

Comparing Calibration 2 to Calibration 1 shows the substantial effect in Stig4 of doubling the estimate of net human capital investment, and including twice WDI's R&D spending share. The calculated value of TFP growth is more than halved; and because of IRS, $\mathrm{ANS}^{\dagger}$ calculated as in [28] is about 6 % of GNI less than, not equal to, the growth rate times wealth ($g\Theta$). In Calibration 3, the inductive but empirically plausible trebling of WDI's estimated human and knowledge capital investments, and having half of human capital investment occurring off-GNI, makes the required TFP growth rate zero as noted earlier, while $\mathrm{ANS}^{\dagger}$ is much the same as in Calibration 2.

A prime purpose of the World Bank's ANS estimates has been to guide country-level policies, especially in developing countries that have low ANS because of low human capital investment and/or high net resource depletion. A key finding from Table 5 for this purpose is thus that human factors – net investment in human and knowledge capital, population dilution and TFP growth – are important components in our global $\mathrm{ANS}^{\dagger}$, but have been ignored or significantly underestimated in $\mathrm{ANS}_{\mathrm{WB}}$. The overall sum of human factors in our estimated global $\mathrm{ANS}^{\dagger}$ is 7-12% of GNI higher than in $\mathrm{ANS}_{\mathrm{WB}}$; but the sum could be negative in countries with low net investments in human and knowledge capital and high rates of population growth. As already stressed, though, our $\mathrm{ANS}^{\dagger}$ tells us little about century-long global sustainability measurement, since climate damage is omitted, and all our $\mathrm{ANS}^{\dagger}$ totals must be positive, to be consistent with the growth rate of about 1.6 %/yr observed during 1995-2014.

8. Conclusions

The world economy cannot grow exponentially forever. It must eventually undergo some mixture of major climate damage and a radical transition to a decarbonised economy, and this alone will make balanced, exponential growth of all its major economic and physical variables impossible in the very long run. Nevertheless, four- or five-year averages of those variables in the World Bank's databases showed that global economic and net resource growths were fairly close to exponential from 1995 to 2014, with a growth rate of per-person consumption of about 1.6 %/yr, and a roughly 2 %/yr growth in inputs and stocks of natural resources, especially fossil fuels. We have used this to justify building a theoretical model of the world economy where growth is always, not just asymptotically, exponential; and making three quite different, approximate calibrations of this model, using the World Bank's and other global data for 1995-2014. The calibrations all derive consumption discount rates of about 7 %/yr, meaning they estimate that markets judged (even though intergenerational equity concerns would reject) the 1995-2014 period to account for about three-quarters of present values seen from 1995; so the empirical inaccuracy of assuming always-exponential growth is quite modest over the medium-term future.

Our model greatly extended Stiglitz's (1974) optimal growth model. We included "human" capital, defined to have no spillovers, and to exclude labour time income so as to achieve stock-flow consistency, unlike the World Bank's much larger definition of human capital. We also included "knowledge" capital, defined to have positive spillovers that cause increasing returns; depreciation of all capitals; discovery (and renewal) of an aggregate natural resource; and some human capital investment that is outside measured GDP. We noted that our Adjusted Net Saving (ANS) per person, which includes both the benefits of total factor productivity growth and resource discovery, and a deduction for the dilution of per-person, productive stocks by population growth, is the best available, albeit one-sided, indicator of whole-economy sustainability, with origins in Weitzman (1976); and that in theory, our ANS per person equals the change in wealth per person, the World Bank's preferred measure of sustainability. This equality also holds exactly in the simplest, constant-returns form of our model; and ANS per person in our more general model is a good deal closer to change in wealth per person than is the World Bank's ANS. So our ANS is a fairly complete measure of future economic prosperity and sustainability, and its components give a more detailed, immediate guide to policy makers than change in wealth.

The key question then addressed by our calibrations is: how much does it matter that the World Bank's version of ANS, omits resource discovery, and also two "human factors", namely the growths of productivity and population, without which ANS is theoretically incomplete? The answer is: a good deal, because our three calibrations all estimate positive "human factors" – the sum of net investment in human capital (e.g. by education spending), net investment in knowledge capital (e.g. by R&D spending) and the value of productivity growth – to be from 7% to 12% of world GDP higher than in the World Bank's ANS; and they estimate the negative human factor of population dilution to be about 7% of world GDP. Also, our estimate that resource discovery was about 7-9 %/yr of existing stocks, so that net resource growth in our ANS is 3-4% of GDP higher than the World Bank's negative value, highlights the importance of net resource growth to recent economic growth. Yet our ANS, which overall is 5-11 % of GDP higher than the World Bank's ANS, omits climate and all other pollution damage, because our methodology cannot estimate such damage, which is hugely contestable anyway. So we are not claiming that the world is more sustainable than the World Bank's ANS suggests: our ANS is only an approximate, one-sided measure of medium-term, economic sustainability, and is neither suitable nor intended as a measure of the very long-term sustainability of well-being.

What our results do mean is that the World Bank's ANS, because of its inherent advantages over change-of-wealth as a practical guide to policy makers, ideally should be further adjusted in the ways suggested here, so as to make its results more informative and accurate. This could be particularly important for individual countries – for example developing countries with high population growth and natural resource dependency, one of the World Bank's main reasons for originally compiling ANS results – where our further adjustments might be negative overall, not strongly positive as in our global calibrations. However, we leave such country-level adjustments to future work, in which both data limitations, and an understandable reluctance to depart from the framework of the System of National Accounts as our further adjustments require, will doubtless be sizeable obstacles.

Our third calibration, chosen inductively but with some empirical justification, completely eliminates any role for total factor productivity growth in explaining global economic growth observed during 1995-2014. To do so this calibration both reclassifies about 18% of GDP, not 4% as in the World Bank's ANS, from consumption to net investment in human and knowledge capitals, and assumes that half of human capital investment occurs outside GDP. As a result, it raises the estimate of human plus knowledge capital to nearly the present value of labour time. Such assumptions rather stretch our one-sector model to its limits, so further research might also try to use a two-sector model of physical and human capital accumulation, where sustained growth is inherently better explained without assuming any productivity growth. Further research should also tackle the vital but vexed issue of pollution damage. Meanwhile, our results, despite their many approximations, help to improve understanding of how far education, R&D and resource policies can make economic growth more sustainable.

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Appendices

Appendix A. Output and net resource growth rates in Stig5

Equating the asymptotic growth rates, denoted $\tilde{\ }$, of the left and right hand sides of [4]:

$\Rightarrow\ \ \tilde{F} = (\alpha+\varepsilon+\nu)\tilde{C} + \beta\tilde{R} + (1-\alpha-\varepsilon-\beta)n + \tau$ [A1]

which, using $\tilde{F} = \tilde{C} =: g+n$ from [6] and $\tilde{R} =: \chi$ from [7]

$\Rightarrow\ \ g = \alpha g + \varepsilon g + \nu(g+n) + \beta(\chi-n) + \tau$ [8]

$\Rightarrow\ \ (1-\alpha-\varepsilon-\nu)g = \nu n + \beta(\chi-n) + \tau$

$\Rightarrow\ \ g = [\nu n+\beta(\chi-n)+\tau]/(1-\alpha-\varepsilon-\nu)$ [9]

To solve the optimisation problem [1]-[5], first rewrite it as:

$$\text{maximise } \int_0^\infty e^{-\rho t}L(t)u[c(t)]dt,$$

$$\begin{aligned}
\text{s.t. } & L(t)c(t) = F(K,H,N,R,L,t) - \dot{K}(t) - \delta K(t) - I^H(t) - I^N(t) \\
& u(c) = c^{1-\theta};\ \ I^H(t) = \pi[\dot{H}(t) + \delta H(t)];\ \ I^N(t) = \dot{N}(t) + \delta N(t) \\
& R(t) = \gamma S(t) - \dot{S}(t);\ \ F(K,H,N,R,L,t) = A_0K^\alpha H^\varepsilon N^\nu R^\beta L^{1-\alpha-\varepsilon-\beta}e^{\tau t} \\
& K(0) = K_0;\ \ H(0) = H_0;\ \ N(0) = N_0;\ \ S(0) = S_0;\ \ L(t) = L_0e^{nt}
\end{aligned}$$

Then form the current-value Hamiltonian

$$J = Lu + \Psi^K\dot{K} + \Psi^H\dot{H} + \Psi^N\dot{N} + \Psi^S\dot{S}$$

$= Lu(C/L) + \Psi^K[F(K,H,N,R,L)-C-\delta K-I^H-I^N] + \Psi^H(I^H/\pi-\delta H) + \Psi^N(I^N-\delta N) + \Psi^S(\gamma S-R)$ [A2]

The first-order conditions with respect to C and K are:

$\partial J/\partial C = 0 = Lu'(c)/L - \Psi^K\ \ \Rightarrow\ \ \dot{\Psi}^K/\Psi^K = \dot{u}'(c)/u'(c) = -\theta\dot{c}/c$ [A3]

$\partial J/\partial K = \rho\Psi^K - \dot{\Psi}^K = \Psi^K(F_K-\delta)\ \ \Rightarrow\ \ \dot{\Psi}^K/\Psi^K = \rho - F_K + \delta$ [A4]

and equating [A3] and [A4] gives the Ramsey rule for the interest rate:

$r := F_K - \delta = \rho + \theta\dot{c}/c$ [A5]

The first-order conditions for human capital H are:

$$\partial J/\partial I^H = 0 = -\Psi^K + \Psi^H/\pi \Rightarrow \Psi^H = \pi\Psi^K$$

$$\partial J/\partial H = \rho\Psi^H - \dot{\Psi}^H = \Psi^KF_H - \Psi^H\delta$$

$\Rightarrow\ \ \dot{\Psi}^H/\Psi^H = \rho + \delta - (\Psi^K/\Psi^H)F_H = \rho + \delta - F_H/\pi = \dot{\Psi}^K/\Psi^K = \rho + \delta - F_K$ from [A4]

Note that assuming the same δ for K, H & N means it now disappears:

$\Rightarrow\ \ F_K = F_H/\pi\ \ \Rightarrow\ \ \alpha F/K = \varepsilon F/\pi H\ \ \Rightarrow\ \ H/K = \varepsilon/\alpha\pi$ [A6]

while the equations for knowledge capital N are:

$$\partial J/\partial I^N = 0 = -\Psi^K + \Psi^N \Rightarrow \Psi^N = \Psi^K$$

$$\partial J/\partial N = \rho\Psi^N - \dot{\Psi}^N = \Psi^KF_N - \Psi^N\delta$$

$\Rightarrow\ \ \dot{\Psi}^N/\Psi^N = \rho+\delta - (\Psi^K/\Psi^N)F_N = \rho+\delta-F_N = \dot{\Psi}^K/\Psi^K = \rho + \delta - F_K$

$\Rightarrow\ \ F_K = F_N\ \ \Rightarrow\ \ \alpha F/K = \nu F/N\ \ \ \ \Rightarrow\ \ N/K = \nu/\alpha$ [A7]

The first-order conditions for resource use are:

$$\partial J/\partial R = 0 = \Psi^K F_R - \Psi^S \;\Rightarrow\; \dot{\Psi}^S/\Psi^S = \dot{\Psi}^K/\Psi^K + \dot{F}_R/F_R = \rho - F_K + \delta + \dot{F}_R/F_R \text{ from [A4]}$$

$$\partial J/\partial S = \rho\Psi^S - \dot{\Psi}^S = \Psi^S\gamma \Rightarrow \dot{\Psi}^S/\Psi^S = \rho - \gamma$$

Equating the right hand sides of these two equations gives the Hotelling rule for the interest rate:

$$r := F_K - \delta = \dot{F}_R/F_R + \gamma \qquad \text{[A8]}$$

Put $F_R = \beta F/R$ with $\tilde{F} = g+n$ and $\tilde{R} = \chi$ into [A8]:

$$\Rightarrow\quad r = g + n - \chi + \gamma \qquad \text{[10]}$$

Appendix B. Output and wealth shares in Stig5

The key approximation of Stig3/4/5 is to assume of an always-exponential solution:

$$C = C_0e^{(g+n)t} = L_0c_0e^{(g+n)t},\; K = K_0e^{(g+n)t},\; F = F_0e^{(g+n)t} \qquad \text{[11]}$$

$$H = H_0e^{(g+n)t},\; N = N_0e^{(g+n)t};\; S = S_0e^{\chi t},\; R = (\gamma-\chi)S_0e^{\chi t} \qquad \text{[12]}$$

Hence from [2], [A6] and [A7]

$$C_0 = F_0 - (g+n+\delta)(K_0+\pi H_0+N_0);\quad \pi H_0 = (\varepsilon/\alpha)K_0;\quad N_0 = (\nu/\alpha)K_0 \qquad \text{[A9]}$$

and from [A5] and [4]:

$$r = F_K - \delta = \alpha F_0/K_0 - \delta,\text{ constant } \;\Rightarrow\; F_0 = (r+\delta)K_0/\alpha \qquad \text{[A10]}$$

and [A9] and [A10]

$$\Rightarrow\quad (r+\delta)K_0/\alpha = F_0 = C_0 + (g+n+\delta)(K_0+\pi H_0+N_0) \qquad \text{[13]}$$

which with [A9]

$$\Rightarrow\quad C_0 = (r+\delta)(K_0/\alpha) - (g+n+\delta)(1+\varepsilon/\alpha+\nu/\alpha)K_0$$

which with [11]

$$\Rightarrow\quad C(t) = [r+\delta - (\alpha+\varepsilon+\nu)(g+n+\delta)]K(t)/\alpha$$

which with [A10] and [11]

$$\Rightarrow\quad F(t) = C(t)(r+\delta)\;/\;[r+\delta - (\alpha+\varepsilon+\nu)(g+n+\delta)] \qquad \text{[14]}$$

Hence, defining

$$\mu := r+\delta - (\alpha+\varepsilon+\nu)(g+n+\delta) \qquad \text{[15]}$$

and using [14], [A10], [A6], [A7] and [11], we have that at all times:

$$\begin{array}{llll}
C/F & = \mu/(r+\delta) & &)\ \text{[16]} \\
(\dot{K}+\delta K)/F & = \alpha(g+n+\delta)/(r+\delta) & =: \hat{\alpha} &) \\
\pi(\dot{H}+\delta H)/F & = \varepsilon(g+n+\delta)/(r+\delta) & =: \hat{\varepsilon} &) \\
(\dot{N}+\delta N)/F & = \nu(g+n+\delta)/(r+\delta) & =: \hat{\nu} &)
\end{array}$$

hence also

$$C/F = (F-\dot{K}-\delta K-\pi\dot{H}-\pi\delta H-\dot{N}-\delta N)/F = 1-\hat{\alpha}-\hat{\varepsilon}-\hat{\nu} \qquad \text{[A11]}$$

Because knowledge capital gives increasing returns to scale, the consumption discount rate that equates two definitions of wealth − as the PV of consumption, and as the sum of all capitals (including knowledge) and PVs of factor incomes − must be ηr, $\eta < 1$, below the interest rate r. We now determine η by equating PV(C) with the sum of capitals and factor PVs.

Using [11], the PV(C) definition of wealth, $\Theta(t)$, is

$$\Theta(t) := \int_t^\infty e^{-\eta r(z-t)}C(z)dz = e^{\eta rt}C_0\int_t^\infty e^{-(\eta r-g-n)z}dz = C_0e^{(g+n)t}/(\eta r-g-n) = C(t)/(\eta r-g-n) \qquad [17]$$

which with the first equation in [16] gives the output/wealth ratio:

$$F(t)/\Theta(t) = (r+\delta)(\eta r-g-n)/\mu =: \Lambda \qquad [21]$$

and then from the other equations in [16]:

$$K(t)/\Theta(t) = \alpha(\eta r-g-n)/\mu \qquad)\ [18]$$
$$H(t)/\Theta(t) = \varepsilon(\eta r-g-n)/\pi\mu \qquad)$$
$$N(t)/\Theta(t) = \nu(\eta r-g-n)/\mu \qquad)$$

Resource income $F_RR = \beta F$ and labour time income $F_LL = (1-\alpha-\beta)F$, so from [17] and [16]:

$$\text{PV(resource income)}/\Theta = \beta F/(\eta r-g-n)\Theta = \beta(r+\delta)/\mu =: \beta' \qquad)$$
$$\text{PV(labour time income)}/\Theta = (1-\alpha-\varepsilon-\beta)(r+\delta)/\mu \qquad)$$

The sum of the shares of capitals and PVs in wealth (which must equal 1) is then

$$K(t)/\Theta(t) + H(t)/\Theta(t) + N(t)/\Theta(t) + \beta(r+\delta)/\mu + (1-\alpha-\varepsilon-\beta)(r+\delta)/\mu$$

So inserting terms from [18], this sum of shares is

$$\alpha(\eta r-g-n)/\mu + \varepsilon(\eta r-g-n)/\pi\mu + \nu(\eta r-g-n)/\mu + \beta(r+\delta)/\mu + (1-\alpha-\varepsilon-\beta)(r+\delta)/\mu$$
$$= [(\alpha+\varepsilon/\pi+\nu)(\eta r-g-n) + (1-\alpha-\varepsilon)(r+\delta)] \,/\, \mu$$
$$= [(\alpha+\varepsilon+\nu)(\eta r-g-n) + (1-\alpha-\varepsilon-\nu)(r+\delta) + (\varepsilon/\pi-\varepsilon)(\eta r-g-n) + \nu(r+\delta)] \,/\, \mu$$
$$= [r+\delta - (\alpha+\varepsilon+\nu)(g+n+\delta) - (\alpha+\varepsilon+\nu)(1-\eta)r + (\varepsilon/\pi-\varepsilon)(\eta r-g-n) + \nu(r+\delta)] \,/\, \mu$$

which, using [15]

$$= [\mu - (\alpha+\varepsilon+\nu)(1-\eta)r + (\varepsilon/\pi-\varepsilon)(\eta r-g-n) + \nu(r+\delta)] \,/\, \mu = 1$$
$$\Rightarrow\ [- (\alpha+\varepsilon+\nu)(1-\eta)r + (\varepsilon/\pi-\varepsilon)(\eta r-g-n) + \nu(r+\delta)] \,/\, \mu = 0$$
$$\Rightarrow\ (\varepsilon/\pi-\varepsilon)(\eta r-g-n) + \nu(r+\delta) = (\alpha+\varepsilon+\nu)(1-\eta)r$$
$$\Rightarrow\ (\alpha+\varepsilon/\pi+\nu)r\eta = (\alpha+\varepsilon)r + (\varepsilon/\pi-\varepsilon)(g+n) - \nu\delta$$
$$\Rightarrow\ \eta = [(\alpha+\varepsilon)r+(\varepsilon/\pi-\varepsilon)(g+n)-\nu\delta] \,/\, (\alpha+\varepsilon/\pi+\nu)r \qquad [19]$$

Appendix C. Interest rate in Stig5

Together [17], [16], [21] and [18]

$$\Rightarrow\ \eta r-g-n = C/\Theta = (C/F)(F/\Theta) = [\mu/(r+\delta)]\Lambda = \Lambda\beta/\beta'$$
$$\Rightarrow\ \eta r = \Lambda\beta/\beta' + g + n = [(\alpha+\varepsilon)r+(\varepsilon/\pi-\varepsilon)(g+n)-\nu\delta] \,/\, (\alpha+\varepsilon/\pi+\nu) \text{ from [19]}$$
$$\Rightarrow\ (\alpha+\varepsilon)r = (\Lambda\beta/\beta'+g+n)(\alpha+\varepsilon/\pi+\nu) - (\varepsilon/\pi-\varepsilon)(g+n) + \nu\delta$$
$$= (\Lambda\beta/\beta')(\alpha+\varepsilon/\pi+\nu) + (\alpha+\varepsilon+\nu)(g+n) + \nu\delta$$
$$\Rightarrow\ r(\alpha+\varepsilon)(g+n+\delta)/(r+\delta) = (\Lambda\beta/\beta')(\alpha+\varepsilon/\pi+\nu)(g+n+\delta)/(r+\delta)$$
$$+ (\alpha+\varepsilon+\nu)(g+n)(g+n+\delta)/(r+\delta) + \delta\nu(g+n+\delta)/(r+\delta)$$

Now use $\alpha(g+n+\delta)/(r+\delta) = \hat{\alpha}$, $\varepsilon(g+n+\delta)/(r+\delta) = \hat{\varepsilon}$ and $\nu(g+n+\delta)/(r+\delta) = \hat{\nu}$ from [16]

$$\Rightarrow\ r(\hat{\alpha}+\hat{\varepsilon}) = (\Lambda\beta/\beta')(\hat{\alpha}+\hat{\varepsilon}/\pi+\hat{\nu}) + (\hat{\alpha}+\hat{\varepsilon}+\hat{\nu})(g+n) + \delta\hat{\nu}$$
$$\Rightarrow\ r = [(\hat{\alpha}+\hat{\varepsilon}/\pi+\hat{\nu})(\Lambda\beta/\beta') + (\hat{\alpha}+\hat{\varepsilon}+\hat{\nu})(g+n) + \delta\hat{\nu}] \,/\, (\hat{\alpha}+\hat{\varepsilon}) \qquad [22]$$

Appendix D. Equivalence of ANS and change in wealth in a general CRS economy

To prove [26], that $ans^{\dagger}(t) = (d/dt)[\Theta(t)/L(t)]$, integrate [25], $\dot{y}^{\dagger}(t) = (r-n)[y^{\dagger}(t)-c(t)]$, to get augmented, individual NNP:

$$y^{\dagger}(t) = c(t) + ans^{\dagger}(t) = (r-n) \int_{t}^{\infty} e^{-(r-n)(z-t)} c(z)dz = (r-n)\Theta(t)/L(t) \tag*{[A12]}$$

where we have used $c(t) = C(t)e^{-nt}$ from [11], hence from [17], wealth per person given CRS is $\Theta(t)/L(t) = \int_{t}^{\infty} e^{-(r-n)(z-t)} c(z)dz$. As Weitzman (1976, p157) foresaw, [A12] is the appropriate extension of his key result (10) on the welfare significance of NNP. Here, augmented, individual NNP, $y^{\dagger}$, is the interest − but at a rate $r-n$, adjusted downwards for population growth − on individual wealth, Θ/L. If we then insert $(r-n)ans^{\dagger}(t)$ from [25] for the numerator and $(r-n)\Theta(t)/L(t)$ from [A12] for the denominator of $\dot{y}^{\dagger}(t)/y^{\dagger}(t)$, and equate this with the growth rate of $(r-n)\Theta(t)/L(t)$, we get

$$\dot{y}^{\dagger}(t)/y^{\dagger}(t) = (r-n)ans^{\dagger}(t) \;/\; [(r-n)\Theta(t)/L(t)] = \dot{\Theta}(t)/\Theta(t) - n$$

$$\Rightarrow \quad ans^{\dagger}(t) = [\dot{\Theta}(t) - n\Theta(t)] \;/\; L(t) = (d/dt)[\Theta(t)/L(t)] \tag*{[26]}$$

Appendix E. Individual NNP and Adjusted Net Saving in Stig3

Rewriting [23] for individual NNP in Stig3, given $N = \nu = 0$ and $\eta = 1$ there, as

$$y \quad = c + \dot{K}/L - nK/L + \dot{H}/L - nH/L + F_R\,(\dot{S}/L - nS/L),$$

and inserting c, $\dot{K}$, K, $\dot{H}$ and H from [16] and $\dot{S}$ and S from [12], gives

$$y \quad = [\mu/(r+\delta) + (\alpha+\varepsilon)g/(r+\delta) + \beta(\chi-n)/(r-g-n)]\; F/L$$

$$= [\mu + (\alpha+\varepsilon)g + (r+\delta)\beta(\chi-n)/(r-g-n)]\; F/L(r+\delta)$$

which, using $\mu + (\alpha+\varepsilon)g = r+\delta - (\alpha+\varepsilon)(n+\delta)$ from [15] with $\nu = 0$

$$= [r+\delta - (\alpha+\varepsilon)(n+\delta) + (r+\delta)\beta(\chi-n)/(r-g-n)]\; F/L(r+\delta)$$

$$= [1 - (\alpha+\varepsilon)\delta/(r+\delta) - (\alpha+\varepsilon)n/(r+\delta) + \beta\chi/(r-g-n) - n\beta/(r-g-n)]\; F/L$$

$$\Rightarrow \quad y(t) = \{1 - \delta(\alpha+\varepsilon)/(r+\delta) + \beta\chi/(r-g-n) - n[(\alpha+\varepsilon)/(r+\delta)+\beta/(r-g-n)]\}\; F(t)/L(t) \tag*{[A13]}$$

thus showing the effects of depreciation at rate δ, the resource's competitive output share β, and stock dilution by population growth n. Next, in Stig3, equation (11) in Pezzey (2003) becomes:

$$(\partial y/\partial t)/y = [(\partial F/\partial t)/F] \;/\; [1 - \delta(K+H)/F + \beta(\gamma S/R-1) - n(K+H+F_R S)/F]$$

which using [4], [16], [12], [10] and [A13]

$$\Rightarrow \quad \partial y/\partial t = \tau y \;/\; \{1 - \delta(\alpha+\varepsilon)/(r+\delta) + \beta[\gamma/(\gamma-\chi)-1] - n[(\alpha+\varepsilon)/r+\delta)+\beta/(\gamma-\chi)]\} = \tau F/L$$

so the individual value of time as defined in [24] is the PV of $\partial y/\partial t$, i.e.

$$q^{t}(t) = [\tau/(r-g-n)]F(t)/L(t) \tag*{[A14]}$$

so using [A13]

$$\Rightarrow q^{t}(t)/y(t) = [\tau/(r-g-n)] \;/\; \{1-\delta(\alpha+\varepsilon)/(r+\delta)+\beta\chi/(r-g-n)-n[(\alpha+\varepsilon)/(r+\delta)+\beta/(r-g-n)]\} \tag*{[A15]}$$

This ratio $q^{t}(t)/y(t)$, here a constant, is Weitzman's (1997) "technological progress premium": the proportional uplift in individual NNP caused by TFP growth as time passes. Weitzman's model had no depreciation δ, resource input β or population growth n, so his premium was not [A15], but just $\tau/(r-g)$ in our notation. In our Calibration 1, the net effect of our three extensions is to raise this premium by a factor of 1.3.

We next compute individual ANS, denoted $ans(t)$, which is individual NNP, $y(t)$ in [A13], minus consumption per person, $c(t) = [\mu/(r+\delta)]F(t)/L(t)$ in [16]; and we finally get augmented, individual ANS, denoted $ans^{\dagger}(t)$, by further adding the individual value of time, $q^t(t)$ in [A14]:

$$\begin{aligned} ans^{\dagger}(t) &= y(t) - c(t) + q^t(t) \\ &= \{1 - \delta\alpha/(r+\delta) - \delta\varepsilon/(r+\delta) + \beta\chi/(r-g-n) - n[\alpha/(r+\delta)+\varepsilon/(r+\delta)+\beta/(r-g-n)] - \mu/(r+\delta) \\ &\qquad + [\tau/(r-g-n)]\}\ F(t)/L(t) \end{aligned}$$

which, using $\mu := r+\delta - (\alpha+\varepsilon)(g+n+\delta)$ from [15] (with $\nu = 0$) gives

$$\begin{aligned} ans^{\dagger}(t) &= [\alpha(g+n)/(r+\delta) + \varepsilon(g+n)/r+\delta) + \beta\chi/(r-g-n) - n[\alpha/(r+\delta) + \varepsilon/(r+\delta) + \beta/(r-g-n) \\ &\qquad + \tau/(r-g-n)]\ F(t)/L(t) \quad [27] \end{aligned}$$

Next, in the always-exponential case of Stig3, substituting $\dot{\Theta}(t) = (g+n)\Theta(t)$ into [26] gives the first result in [29], and substituting $\Theta(t) = C(t)/(r-g-n)$ (from [17] with $\eta = 1$) gives the second result. As for Proposition 1 in Hamilton and Hartwick (2005), when modified for population growth and applied to Stig3, this is $\dot{c}(t) = (r\text{-}n)ans(t) - (d/dt)[ans(t)] = (r\text{-}g\text{-}n)ans(t)$. Substituting $\dot{c}(t) = gc(t) = g(r\text{-}g\text{-}n)\Theta(t)/L(t)$ (from [17] in CRS) and multiplying $ans(\mathrm{t}) = g\Theta(t)/L(t)$ by $L(t)/F(t)$ then gives the first equation in [29].

Appendix F Wealth/income ratio in Stig3

[A12] shows that the ratio between wealth per person, Θ/L, and "income" per person, namely augmented, individual NNP, $y^{\dagger}$, is

$$[\Theta(t)/L(t)]\ /\ y^{\dagger}(t) = 1/(r-n) = 15.5 \text{ in Stig3's Calibration 1.} \qquad [A16]$$

Also from [A12] and using [17], the growth rate of income per person is

$$\dot{y}^{\dagger}(t)/y^{\dagger}(t) = [\dot{\Theta}(t)/\Theta(t)] - \dot{L}(t)/L(t) = g+n - n = g$$

And from [A12], [26] and [17], the ratio of per-person net-of-depreciation saving to income is

$$[y^{\dagger}(t)-c(t)]/y^{\dagger}(t) = ans^{\dagger}(t)\ /\ [(r-n)\Theta(t)/L(t)] = g/(r-n).$$

So from the last two results, the ratio of the net-of-depreciation saving rate to the growth rate of income per person is

$$[g/(r-n)]\ /\ g = 1/(r-n) = \text{(wealth per person) / (income per person) in [A16].}$$

The Harrod-Domar-Solow formula cited by Piketty and Zucman (2014) thus holds true in Stig3 if all quantities in both ratios are individual (to include population growth) and augmented (to include the effect of exogenous technical progress on output). The formula is also true if we exclude all Stig3's extra features, but the ratio is then much smaller. Using [11], the ratio would be

$$\text{net saving rate / growth rate} = [\dot{K}/(C+\dot{K})]/(g+n) = K/(C+\dot{K}) = \text{wealth/income ratio}$$

and in Stig3, using [16] and [15] with $\varepsilon = \nu = 0$,

$$\begin{aligned} K/(C+\dot{K}) &= K\ /\ [\mu K/\alpha + (g+n)K] = \alpha/[\mu+\alpha(g+n)] = \alpha/(r+\delta-\delta\alpha) \\ &= 4.5 \text{ in Table 4's Calibration 1.} \end{aligned}$$

Appendix G Wealth-ANS† equality in Stig3 versus its inequality in Pezzey et al. (2006)

The last two data rows of Table 5 showed how in our Calibration 1 of the world economy, the first formula in [29], $\mathrm{ANS}^\dagger(t)/F(t) = g\Theta(t)/F(t)$, holds exactly true. Table G1 repeats these data, decomposes the result into a product of three terms using the second formula in [29], $gC(t)/[(r\text{-}g\text{-}n)F(t)]$, and then shows all the equivalent data for Scotland during 1992-99 from Pezzey et al. (2006), approximated as closely as possible from their Table 1. (Population growth was close to zero in Scotland then, hence the zero entry for population dilution.) This Table shows that, compared to Calibration 1, the two numerically biggest reasons why Scotland's $\mathrm{ANS}^\dagger/F$ is estimated to be only about a fifth of its $gC/(r\text{-}g\text{-}n)F$ value are a much lower estimated value of time, and a significantly higher growth rate g. We do not investigate the accuracy or economic causes of these data differences, though.

Table G1

Global ANS†/GNI components	*Stig3 Calibration 1 (world, 1995-2014)*	*Pezzey et al. (2006) (Scotland, 1992-1999)*
Net produced capital investment / F	10.5%	6.4%
Net human capital investment / F	1.6%	2.1%
Value of net resource growth / F	2.7%	0.2%
Population dilution / F	−6.7%	0.0%
Value of time / F	14.8%	1,3%
$\mathrm{ANS}^\dagger/F$ (sum of above components)	22.8%	10.1%
$gC/(r-g-n)F$	22.8%	53.5%
g	1.6%	2.5%
$1/(r-g-n)$	20.6	28.4
C/F	69.5%	76.2%