Crawford School of Public Policy

CAMA

Centre for Applied Macroeconomic Analysis

A comment on Wu and Xia (2016) from a macroeconomic perspective

---

CAMA Working Paper 41/2017
June 2017

Revised version of CAMA Working Paper 48/2015

Leo Krippner
Reserve Bank of New Zealand and
Centre for Applied Macroeconomic Analysis, ANU

Abstract

Counter to the comments in Wu and Xia (2016), I show that the results from macroeconomic models are sensitive to the Shadow Short Rate (SSR) series used. That is, using a standard small macroeconomic vector autorregression model with a range of estimated SSR series obtains counterfactuals for unemployment ranging from 0.4 to 1.8 percentage points, if the Federal Funds Rate rather than the SSR series had been applied in the lower bound period. The counterfactuals for inflation range from -0.2 to -2.2 percentage points. Vetting the various SSR series from several perspectives indicates that some are more preferable than others, but there are reasons to remain cautious on the associated results.

Keywords

shadow rates; lower bound; term structure models; unconventional monetary policy

JEL Classification

E43, G12, G13

Address for correspondence:

(E) cama.admin@anu.edu.au

ISSN 2206-0332

A comment on Wu and Xia (2016) from a macroeconomic perspective

Leo Krippner*

20 June 2017

Abstract

Counter to the comments in Wu and Xia (2016), I show that the results from macroeconomic models are sensitive to the Shadow Short Rate (SSR) series used. That is, using a standard small macroeconomic vector autorregression model with a range of estimated SSR series obtains counterfactuals for unemployment ranging from 0.4 to 1.8 percentage points, if the Federal Funds Rate rather than the SSR series had been applied in the lower bound period. The counterfactuals for inflation range from -0.2 to -2.2 percentage points. Vetting the various SSR series from several perspectives indicates that some are more preferable than others, but there are reasons to remain cautious on the associated results.

JEL classification: E43, G12, G13

Keywords: shadow rates; lower bound; term structure models; unconventional monetary policy.

1 Introduction

In this comment, I show that the results from macroeconomic models that employ estimated benchmark Shadow Short Rate (SSR) series are sensitive to the model specification and data choices used in the SSR estimations. The motivation for this exposition is to counter the comments from Wu and Xia (2016, hereafter WX) that different SSR estimates have little consequence when used in a macroeconomic context. For example, from the WX robustness section (with the theme reiterated often in the paper):

> "Overall, neither changes in the SRTSM [shadow rate term structure model] hence the shadow rate, nor changes in the FAVAR [factor-augmented vector autoregression] alter the key macroeconomic results of this paper, and our results are robust to a wide range of alternatives."

However, these comments are in the context of structural break tests and impulse responses. For many practical quantitative applications of macroeconomic models the shocks also need to be considered. An example is a counterfactual exercise, such as that

*Reserve Bank of New Zealand and Centre for Applied Macroeconomic Analysis. Email: leo.krippner@rbnz.govt.nz. I thank Edda Claus, Iris Claus, Arne Halberstadt, and Glenn Rudebusch for helpful comments.

highlighted in the WX abstract, introduction, and the conclusion. That is, based on the benchmark SSR series, WX estimates that the Federal Reserve's unconventional monetary policy (UMP) accommodation resulted in a 1 percentage point (pp) lower unemployment rate than if the SSR had been constrained at the WX lower bound (LB) of 25 basis points (bps). Different SSR series would obtain different counterfactual results, but WX does not include robustness checks on that aspect.

To gauge the potential sensitivity of counterfactual analysis to alternative SSR series I first, in section 2, present a range of SSR estimates from the WX model using four LB specifications and two datasets. As previously reported in the literature, e.g. Krippner (2015), Bauer and Rudebusch (2017), and Christensen and Rudebusch (2016a), I find that both the levels and profiles of SSR series differ very materially depending on the LB specification and the data used for estimation.

In section 3, I then use the different SSR series to estimate standard small macroeconomic vector autoregression models (VARs) from which I obtain a series of counterfactual results for the unemployment rate and inflation. My estimate using the updated WX benchmark series provides an unemployment counterfactual similar to that of WX noted above, but the estimates using the other SSR series range from 0.4 to 1.8 pp. Those for inflation range from -0.2 to -2.2 pp. The reason for the wide range of results is predominantly that the different levels and profiles of SSR series during the LB period imply materially different series of monetary policy shocks; I confirm the WX result that the impulse responses are not materially different.

Despite the United States exiting the LB period in December 2015, the results from the VAR examples above have important implications. First, if using SSR series as a proxy for near-zero policy rates plus UMP accommodation during the US LB period, researchers should be aware that inferences in hindsight may depend very heavily on the particular SSR series used, which in turn depends on the particular choices made in its estimation. Second, because UMP accommodation remains ongoing in many economies (and could potentially be revisited at some stage in the United States, given the proximity of the policy rate to the LB), the same caveats also apply in foresight. That is, the sensitivity of SSR estimates means that seemingly minor differences in their estimation may result in major differences regarding the overall stance of monetary policy, and hence material ambiguity about its likely influence on the macroeconomy.

One response to the sensitivity of SSR estimates and their associated applications is to avoid using them altogether, necessitating an alternative proxy for UMP accommodation. Alternatively, one could report a series of results based on a range of SSR estimates, perhaps with more weight on the SSR series that accord/s best with the data and the application. To illustrate the latter perspective, I introduce three criteria in section 4 to assess the different SSR series. I find that the SSR series from the shadow/LB model with a simple time-varying LB and the dataset out to 30-years performs better than the other series presented in this comment. Nevertheless, the VAR still shows very significant evidence of a structural break from the start of the LB period, which suggests treating the associated results with caution.

The remainder of the paper follows the outline already noted above, and I offer concluding comments in section 5.

2 Estimates of SSR series

In this section, I outline nine different series of SSR estimates. These are all obtained using the three-factor WX model, but with different LB specifications and datasets as summarized in the first two columns of table 1. I will refer to the associated results in table 1 when I discuss the relative merits for the different SSR series in section 4.

Table 1: Estimation of SSR series and associated results

	SSR estimations				VAR results			UMP
	LB	Data	< LB	Model	π CF	UE CF	SBreak	correl.
Series	bps (1)	set (2)	% (3)	log-L (4)	pp (5)	pp (5)	test (6)	(7)
WX	25 (c)	10-y	73	604.98	-0.77	1.02	409.48	0.19
A	25 (c)	10-y	73	853.00	-0.88	1.18	357.30	0.07
B	17.1 (e)	10-y	40	892.20	-0.61	0.86	394.74	-0.09
C	1.6 (c)	10-y	0	828.15	-0.22	0.40	406.17	-0.46*
D	CMin	10-y	0	880.46	-0.46	0.62	407.90	-0.15
E	25 (c)	30-y	47	-603.18	-2.21	1.80	286.62	0.58*
F	13.8 (e)	30-y	23	-582.56	-1.93	1.65	282.24	0.59*
G	1.6 (c)	30-y	0	-624.13	-1.44	1.29	299.49	0.61*
H	CMin	30-y	0	-570.34	-1.94	1.57	280.05	0.59*
FFR	n/a	n/a	n/a	n/a	n/a	n/a	748.51	0.03

Notes: (1) c = calibrated, e = estimated, CMin is a cumulative minimum LB; (2) 10-y is the updated WX dataset with 3-month to 10-year data, 30-y is the 10-y dataset plus 30-year data; (3) percentage of 3-month data below the given LB specification during the LB period; (4) log-likelihood of estimated shadow/LB model; (5) inflation and unemployment rate counterfactuals using the FFR series instead of the given SSR series in the estimated VAR, as at December 2016; (6) Candelon and Lutkepohl (2001) Chow-type test for a structural break in the estimated VAR at the start of LB period (Dec-2008), all are significant to the 1% level (see footnote 10 in section 4); (7) Kendall correlation, * indicates significance at the 1% level.

Series WX is the benchmark SSR series obtained from Wu's webpage. This series is obtained using a fixed LB calibrated at 25 bp, the WX dataset (updated to December 2016) of 1-month forward rates for horizons of 3 and 6 months, and 1, 2, 5, 7, and 10 years, and the parameters are those from WX (estimated using data up to December 2013). Series A re-estimates the WX model parameters using the dataset up to December 2016, which is the estimation period for all of the remaining models. Series B uses a fixed but estimated LB (17.1 bps), and series C uses a fixed calibrated LB equal to the minimum value of the 3-month data over the sample. Series D uses a time-varying LB, which is simply the cumulative minimum of the 3-month data up to each point in time, analogous to Kortela (2015) and the online implementation of WX for the euro area. Figure 1 illustrates these LB specifications along with the 3-month data to indicate that, on the face of it, each could be justified with respect to the data (or, in the case of the 25 bp LB, with respect to Federal Reserve's interest rate on account balances during the LB period). However, section 4 discusses why some LB specifications will be more preferable.

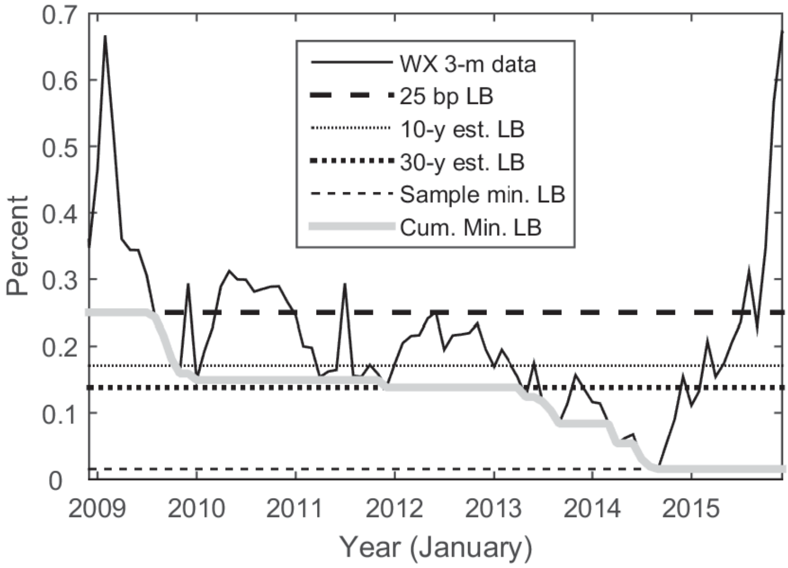

Figure 1: LB specifications and 3-month data (i.e. 1-month rate 3 months forward).

Series E to H use the same LB specifications as series A to D, except I add 1-month forward rates for the 30-year horizon to the WX dataset.[1] This dataset therefore contains horizons that match the full set of benchmark maturities in the US Treasury yield curve. Using the longer-horizon data is consistent with the principle that unconventional monetary policy influences the entire yield curve, and also the composition of the Federal Reserve's asset holdings; i.e. \$0.6 trillion of Treasury securities and \$1.8 trillion of mortgage-backed securities have maturities beyond 10 years (out of total holdings of \$2.5 and \$1.8 trillion in those respective security, within total assets of \$4.5 trillion).[2] Krippner (2015) and Christensen and Rudebusch (2016b) are examples that use data out to 30 years to estimate shadow/LB models.

Figure 2 illustrates the FFR along with the nine alternative SSR series during the LB period. Following WX, all SSR series use the FFR when the target for the FFR was outside the 0 to 25 bp range, and the estimated SSR is used for the LB period, i.e. when the 0 to 25 bp range prevailed from December 2008 to December 2015. Panels 2 and 3 include indicators of major UMP easing and tightening events, using "down" and "up" arrows respectively, and note that QE3 taper period includes eight tapering events.[3] I will refer back to these events in section 4.

[1]I calculate these rates as in WX using the Gürkaynak, Sack, and Wright (2007) dataset.

[2]D'Amico and King (2013) details the Federal Reserve's purchases of Treasury securities and its implications.

[3]The easing events are: (1) 25 November 2008, QE1 announced; (2) 16 December 2008, FFR target set to a range of zero to 25 bps; (3) 18 March 2009, additional security purchases announced; (4) 27 August 2010, QE2 foreshadowed; (5) 3 November 2010 QE2 announced; (6) 9 August 2011, calendar forward guidance announced; 7) 21 September 2011, "Operation Twist"; (8) 25 January 2012, calendar forward guidance extended; (9) 13 September 2012, QE3 announced and calendar forward guidance extended; (10) 12 December 2012, QE3 increased and unemployment-based forward guidance announced. The tightening events are: (1) 22 May 2013, QE3 tapering foreshadowed ("taper tantrum"); (2) 19 June 2013, FOMC members' tightening expectations mentioned; (3) 18 December 2013, QE3 tapering commenced. The subsequent tapering announcements were on 29 January 2014, 19 March 2014, 30 April 2014, 18 June 2014, 30 July 2014, 17 September 2014, and 29 October 2014.

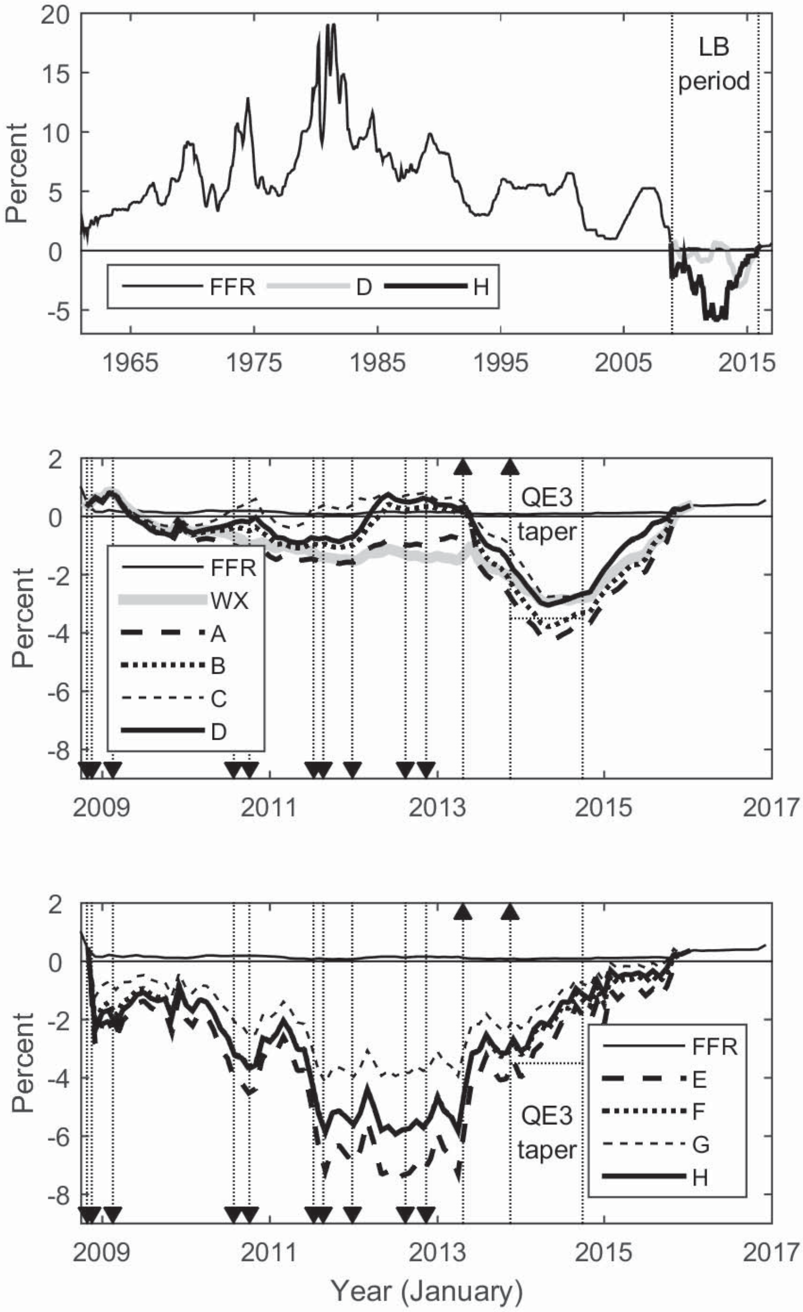

Figure 2: The full-sample FFR with examples of two SSR series for the LB period (panel 1), the SSR series estimated using the 10-year dataset (panel 2), and the estimated SSR series estimated using the 30-year dataset (panel 3). In panels 2 and 3, "down" arrows indicate major UMP easing events and "up" arrows represent major UMP tightening events (see footnote 3 for details).

The variation in magnitudes and profiles of the SSR series during the LB period is readily apparent. Notably, the values of SSR estimates across 2012 and 2013 differ by up to 7 pps, and the lows in the 30-year SSR series are reached around two years before those in the 10-year series. Within the 10-year and 30-year results, it is apparent that even small changes in the LB specification can have a marked effect. Except for the 25 bp LB models, the SSR estimates from the 10-year dataset also contain counterintuitive positive values, up to 0.8 pps, at the beginning and the middle of the LB period.

3 Macroeconomic application

To illustrate the macroeconomic implications of using different SSR series as monetary policy metrics, I estimate a series of standard small monetary VARs and use them for a counterfactual exercise analogous to that in WX.

Each VAR consists of inflation (the annual change in the log level of the personal consumption expenditure less food and energy index), commodity price inflation (the annual change in the log level of the commodity price sub-index of producer price inflation), and the unemployment rate.[4] For the monetary policy variable, I use the SSR series introduced in the previous section. All of the data are monthly. Each VAR is estimated over the sample period from January 1975 to December 2016 and uses six lags. Identification of monetary policy shocks is by Cholesky decomposition with the ordering of the variables as introduced above.

Figure 2 illustrates the IRFs for the VAR with the SSR series H. The IRFs for all of the other estimated VARs are very similar, which concurs with the finding in WX. The similarity is unsurprising because all series use the FFR outside the LB period. Indeed, even the IRFs for the VAR estimated using the FFR over the full sample are also very similar to those in figure 2.

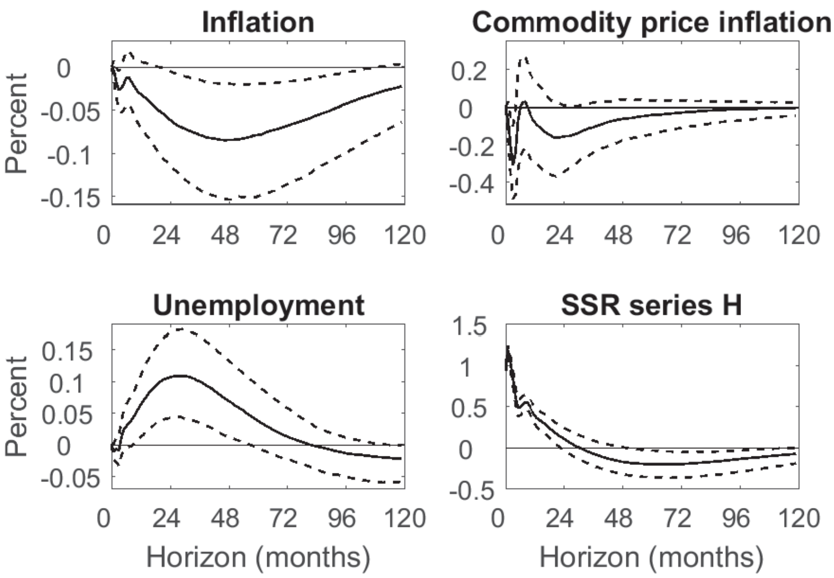

Figure 2: Impulse response functions (IRFs) for a 100 bp tightening shock to the SSR from the VAR estimated with SSR series H. The dashed lines represent 16 and 84 percentiles (analogous to +/- one standard deviation) obtained by bootstrapping.

[4]The results are very similar using the unemployment rate gap, so I have used the unemployment rate to keep all of the macroeconomic data as observables.

My counterfactual exercise for each of the estimated VARs calculates the outcomes for inflation and unemployment if the given SSR series was restricted to equal the FFR during the LB period. In other words, I apply a series of tightening monetary policy shocks, on average, to the SSR series within each VAR. The restriction to the FFR is common to all of the counterfactuals, so the only aspect that differs between each VAR and its associated counterfactual results is the SSR series used.

Figure 3 contains the counterfactual results for the SSR series D and H, as examples to show the differences across the LB period depending on the SSR series used. The first point of note is the large differences by the end of the sample, which mainly reflects the different magnitudes of the two implicit series of shocks applied in the counterfactuals. A more subtle point is that deviations of inflation and unemployment from the actuals begins earlier for the counterfactual using the 30-year dataset than for the 10-year dataset. This difference reflects the different profiles of the SSR series obtained with the 10-year and 30-year datasets.

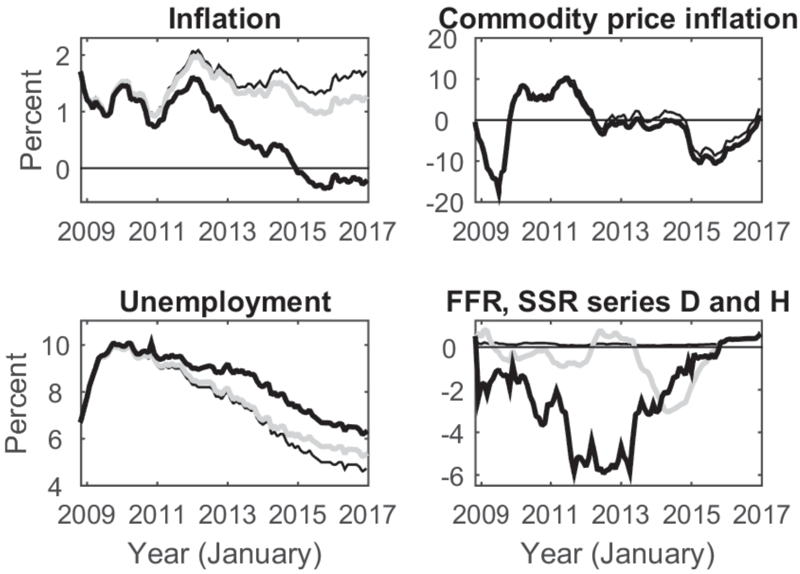

Figure 3: Counterfactuals obtained if the SSR series D and H in subplot 4 (grey and bold lines, respectively) are restricted to equal the FFR (thin line) over the LB period. The thin lines for the other variables are realized values, and the grey and bold lines are the counterfactuals with respect to series D and H, respectively.

Table 1 reports the results of the counterfactual exercise relative to the outcomes without the restrictive shocks. Depending on the SSR series used, inflation ranges from 0.22 to 2.21 pps lower, and unemployment ranges from 0.40 to 1.80 pps higher. Those ranges predominantly reflect the different monetary policy shocks implied by the different SSR series over the unconventional period. More specifically, a counterfactual exercise is dependent on both the shock series and the estimated IRF to a given shock. As mentioned earlier, the estimated IRFs for a monetary policy shock are very similar between the different VARs, but the materially different SSR series provide materially different series of implied monetary policy shocks when compared to the FFR.

To put the results above in the context of WX, I first note that WX provides only point estimates for a counterfactual exercise based on the benchmark WX SSR series, and reports the associated result of a 1 pp lower unemployment rate due to unconventional monetary policies. My estimate using the updated WX SSR series is very similar, at 1.02 pps (albeit for a different sample period). However, the variation in counterfactual results indicates that the WX counterfactual exercise would also be subject to material variability with alternative SSR series, and for the same reasons.

It is worthwhile highlighting that the variation noted above is not simply akin to the typical "different data give different results", which is often found with VAR models. SSR series are not observed data but must be estimated, and section 2 illustrates the substantial variation in SSR series from model and estimation choices. Hence, the substantial variations in the counterfactual results from the VARs presented comes back to the choices made when estimating the SSR series. From the practical perspective of operating monetary policy, if an SSR is used to proxy the degree of UMP accommodation, seemingly immaterial differences in their estimation may result in very large differences in the "data" (i.e. up to 7 pps), which would translate into substantial ambiguity about the potential effect of UMP on macroeconomic outcomes.

I end this section by noting that I have undertaken a range of robustness checks for the VARs, with respect to the number of lags and the sample period.[5] The IRFs and counterfactual results are generally similar to those already presented, as are the structural break results in the following section. The notable exception is that using a sample from January 1961 results in a "price puzzle" for the inflation IRF, as also reported in WX. In that case, the IRF for inflation is positive for the first 36 months before turning negative (both insignificant), resulting in immaterial counterfactual inflation results. I have chosen to present the more intuitive results associated with the 1975 to 2016 sample.

4 Vetting SSR estimates

The results of sections 2 show that estimated SSR series can be sensitive to the choices made in their estimation, and the results of section 3 show that those sensitivities can translate into materially different results in a macroeconomic context during LB periods. Therefore, one should not assume that any SSR series will be suitable as a proxy for the FFR, in the sense that similar quantitative results in all respects would be obtained if another SSR series were used. Related, it is clear that seemingly minor differences in SSR estimations can lead to major differences in subsequent applications.

One response to that sensitivity is to avoid using SSR series estimated from yield curve data altogether, in which case an alternative to the FFR as a monetary policy metric for macroeconomic applications would be required.[6] Another response would be to report the range of results from different SSR series. However, the wide range of counterfactual

[5] Respectively, I have used lag lengths from 2 to 13 months (the Schwatz Bayesian criterion suggests suggest 2, the Akaike information criterion suggests 8, and WX uses 6, 7, 12, and 13), and also a sample from January 1961 to more closely match the sample used for the WX counterfactual and structural break exercises. The start of my dataset is limited by the personal consumption expenditure less food and energy index being available from January 1960, meaning the first datapoint for annual inflation is January 1961.

[6] The Effective Monetary Stimulus proposed in Halberstadt and Krippner (2016) or the financial conditions index in Lombardi and Zhu (2014) are two such alternatives. Or Johannsen and Mertens (2016) provides another method of proxying the FFR during the LB period using a macroeconomic model.

results that I reported in section 3 provides something akin to a qualitative exercise; i.e. UMP provides more accommodation than the near-zero FFR alone (which is generally accepted anyway), but how much and to what effect are very uncertain (hence providing limited practical insight for policy makers).

A potential middle ground between the two extremes above is to more thoroughly vet the different SSR series, and to place more weight on the result/s that accord best with the data. I illustrate this approach by assessing the different SSR series presented in section 2 from the following perspectives:[7]

1. Consistency of the model with the yield curve data. Primarily, I check how often the 3-month data violates the LB specification, which should not occur materially and/or persistently in a LB model. Specifically, the objective of a shadow/LB model is to represent yield curve data subject to a lower bound. If the data falls below the LB of the shadow/LB model, that indicates model mis-specification. I also report the log-likelihood values from the shadow/LB model estimations to indicate the fit to the data within each dataset.[8]

2. Consistency of the SSR series changes with respect to major UMP events. If the SSR series during the LB period reflects UMP accommodation in addition to the near-zero FFR, then the SSR should typically fall (rise) on easing (tightening) UMP announcements, which WX notes for the monthly WX SSR changes in the case studies around the QE1 and "taper tantrum"events. I test this formally using the list of major UMP events from footnote 3, assigning a value of –1 for easings and +1 for tightenings, and calculating the Kendall correlation with changes in the SSR over the month of those events.[9]

3. Consistency of the inter-relationship between the SSR series with macroeconomic data before and after the LB period. For this I use the test for a structural break from Candelon and Lutkepohl (2001) on the VARs that I estimated in section 3.[10]

Table 1 contains the results of these tests. In general, the models using the 10-year datasets produce SSR series that are not positively correlated with unconventional events, and they show much greater evidence of structural breaks in the VARs than all of the 30-year series.[11] Models with a 25 bp or estimated LB also have many instances of the 3-month data violating the given LB (ranging from 23% to 73%), even though the estimated LB models obtain good fits to the data from a statistical perspective. Models

[7] This list is, of course, not exhaustive; other tests could be added, perhaps depending on the context of the application. Other SSR series could also be tested.

[8] The log-likelihood values, or any other goodness of fit measures, are not comparable between the two datasets. The values for the 10-year datasets will always be greater than for the 30-year datasets because the same number of parameters is being used to represent a smaller dataset.

[9] The Kendall correlation is a non-parametric estimate of the association between two quantities based on their concordance. The probabilities are calculated using exact permutation distributions for small samples. See the MatLab function "corr" for further details.

[10] The asymptotic χ^2 1% critical value is 147.4 with 110 degrees of freedom. I have used the bootstrapping method described in Candelon and Lutkepohl (2001), which performs better in small samples, but there is no difference in the significance of the results.

[11] However, the break results are less than for the FFR, which concurs with the results reported in WX and Francis, Jackson, and Owyang (2014).

with the sample minimum LB provide the least best fit to the data and greater evidence of structural breaks, but obviously there are no violations of the LB.

The SSR series H is the best of those I have presented in this comment. In the order of perspectives introduced above: (1) it has no observations of data below the LB, and it has the highest log-likelihood value for the 30-year dataset models, i.e. 570.34; (2) the SSR changes are positively correlated with the series of unconventional policy events, i.e. a highly significant value of 0.59; and (3) the structural break test has the lowest value of 280.05. However, even that value is still well above the 1% critical (see footnote 10), so series H can only be regarded as the "least worse" series from the macroeconomic perspective in this comment.

The estimated effect of UMP using SSR series H is 1.6 pps lower unemployment and 1.9 pps higher inflation, but the still very significant evidence of a structural break from the start of the LB period suggests that these results should be treated with some caution. It might be that even the "best" SSR series does not provide a sufficient proxy for the FFR in the LB period, or it might be that a genuine structural break has occurred. Related to these points, Francis, Jackson, and Owyang (2014) finds that an SSR series similar to series H does not provide a comprehensive gauge of the stance of monetary policy within a VAR estimated only over the LB period, and (Gagnon 2016) notes that the SSR after liftoff (which essentially equates to the effective FFR) will not reflect any ongoing effects of the Federal Reserve's balance sheet. Hence, even if "good" SSR series can be established from tests like those in this comment, it should not be assumed that they will necessarily provide a complete metric for monetary policy over LB periods.

5 Conclusion

I show that the results from macroeconomic models that employ estimated SSR series are sensitive to the model specification choices and data used in the SSR estimations. For example, my counterfactual exercises indicate that the effect of unconventional monetary policy on inflation and the unemployment rate could respectively range from 0.2 to 2.2 pps higher inflation and 0.4 to 1.8 pps lower unemployment rate depending on the SSR series used. The main reason for the variation is that materially different SSR series imply materially different monetary policy shocks. Seemingly minor differences in SSR estimations can therefore lead to major differences in subsequent applications, or when assessing the overall stance of monetary policy.

If researchers decide to use SSR series in empirical work, the results in this comment suggest at least obtaining and reporting a range of results based on alternative SSR series. Suitable vetting of alternative SSR series in the context of the application may be useful for "weeding out" SSR series that accord least with the data and UMP events, and hence more weight can be placed on the remaining result/s.

Appropriately vetted SSR series appear to be useful in the macroeconomic exercise in this comment, although the structural break indicates they may not provide a comprehensive monetary policy metric for LB periods. Until there is consensus on better approaches to representing monetary policy in LB periods, or more definitive evidence to discount SSR series for that purpose, it seems worthwhile for now to retain SSR estimates within the suite of indicators.

References

Bauer, M. and G. Rudebusch (2017). Monetary policy expectations at the zero lower bound. *Journal of Money, Credit and Banking 48(7)*, 1439–1465.

Candelon, B. and H. Lutkepohl (2001). On the reliability of Chow-type tests for parameter constancy in multivariate dynamic models. *Economics Letters 73(2)*, 155–160.

Christensen, J. and G. Rudebusch (2016a). Modeling yields at the zero lower bound: are shadow rates the solution? *Advances in Econometrics 35*, 75–125.

Christensen, J. and G. Rudebusch (2016b). A probability-based stress test of Federal Reserve assets and income. *Journal of Monetary Economics 73*, 26–43.

D'Amico, S. and T. King (2013). Flow and stock effects of large-scale Treasury purchases: evidence on the importance of local supply. *Journal of Financial Economics 108(2)*, 425–448.

Francis, N., L. Jackson, and M. Owyang (2014). How has empirical monetary policy analysis changed after the financial crisis? *Working Paper, Federal Reserve Bank of St. Louis 2014-19A*.

Gagnon, J. (2016). Quantitative easing: an underappreciated success. *Policy Brief, Peterson Institute for International Economics PB16*.

Gürkaynak, R., B. Sack, and J. Wright (2007). The U.S. Treasury yield curve: 1961 to the present. *Journal of Monetary Economics 54(8)*, 2291–2304.

Halberstadt, A. and L. Krippner (2016). The effect of conventional and unconventional euro area monetary policy on macroeconomic variables. *Dicussion Paper, Deutsche Bundesbank 49/2016*.

Johannsen, B. and E. Mertens (2016). A time series model of interest rates with the effective lower bound. *Working Paper, Federal Reserve Board, 2016-033*.

Kortela, T. (2015). A shadow rate model with time-varying lower bound of interest rates. *Discussion Paper, Bank of Finland Research 19/2016*.

Krippner, L. (2015). *Zero Lower Bound Term Structure Modeling: A Practitioner's Guide*. Palgrave-Macmillan.

Lombardi, M. and F. Zhu (2014). A shadow policy rate to calibrate US monetary policy at the zero lower bound. *Working Paper, Bank of International Settlements 452*.

Wu, J. and F. Xia (2016). Measuring the macroeconomic impact of monetary policy at the zero lower bound. *Journal of Money, Credit and Banking 48(2-3)*, 253–291.