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### Abstract

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E22; E23; O33; O47

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# Unmeasured investment and the puzzling lost decades of the Japanese economy

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## 1. Introduction

The basic neoclassical growth model accounts well for the Japanese economy prior to the 1990s, provided that variations in population growth, depreciation rates, total factor productivity (TFP) and government purchases are incorporated<sup>1</sup>. The behaviour of the Japanese economy during the 1990s and 2000s, however, is often significantly inconsistent with this model when compared with not only the depression in labour hours but also most aggregate series that business cycle theorists study. To be specific, the model predicts a steady mid to late 1990s and early 2000s economy, when in fact it was depressed. For example, over the period of 1995 to 2007, Japanese nominal GDP fell from \$5.33 to \$4.36 trillion while the nominal wages at current USD fell around 10% according to statistics from World Bank<sup>2</sup>. Accordingly, the 1990s and 2000s are called the lost decades or the lost 20 years of Japan. Existing literature argues that the decline in TFP is the main cause of the lost decades in Japan (Hayashi and Prescott, 2002; Fukao et al., 2006; Griffin and Odaki, 2009), which is inconsistent with the counterfactual predictions generated by the neoclassical growth model when TFP shocks are incorporated.

Following McGrattan and Prescott (2010) and McGrattan and Prescott (2014), this study extends the base model by introducing intangible investment and non-neutral technology change with respect to the production of intangible investment goods and finds that, in the light of the new theory proposed by McGrattan and Prescott (2010) and McGrattan and Prescott (2014), the lost decades in Japan are much less puzzling. Most intangible investment is excluded from gross domestic output (GDP) because to measure it is difficult. Examples of intangible investment include research and development (R&D), advertising, organization capital, staff training, etc.

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<sup>1</sup> See, for example, Figure 3 in Kobayashi and Inaba (2006).

<sup>2</sup> The World Bank data (<http://data.worldbank.org/country/japan>).

These investments<sup>3</sup> are treated as expenditure and do not appear in the national account. However, these investments are made for realizing future profits and are reflected in the valuation of a company when the company goes public or is sold (Hulten and Hao, 2008; Eisfeldt and Papanikolaou, 2013; Asker et al., 2014). The importance of intangible investment in economic activities has been widely confirmed in the literature (Atkeson and Kehoe, 2005; Corrado et al., 2009; Fukao et al., 2009; van Ark et al., 2009; Marrano et al., 2009; Awano et al., 2010; Corrado and Hulten, 2010; Tronconi and Marzetti, 2011; Arato and Yamada, 2012; Borgo et al., 2013; Corrado et al., 2013; Haskel and Wallis, 2013; Miyagawa and Hisa, 2013; Eisfeldt and Papanikolaou, 2013; Gourio and Rudanko, 2014a; Eisfeldt A L and Papanikolaou, 2014; Gourio and Rudanko, 2014b; Clausen and Hirth, 2016; Chun and Nadiri, 2016), and missing this critical element might cause problems for macroeconomic theories.

There is both macroeconomic and microeconomic evidence suggesting that the growth of unmeasured investment was extremely low during the lost decades. According to Fukao et al. (2009) and the intangible investment data they used, the growth rate of real intangible investment was low and sometimes negative during the lost decades. From 1985 to 1992, real intangible investment in the Japanese economy grew by 48% while it only grew respectively by 14% and 9.6% from 1992 to 1999 and from 1999 to 2006 in real terms<sup>4</sup>. If we look at the industrial level data, it is clear that the intangible investment is significantly low compared with previous periods. Taking Japan's semiconductor industry as an example, from 1985 to 1992, its intangible investment quadrupled, while decreased by 5% between 1999 and 2006<sup>5</sup>. Moreover, during the Asian Financial Crisis, Japanese output and working hours fell significantly while labour

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<sup>3</sup> Not include software. Software is capitalized and appears in the national account.

<sup>4</sup> Data from JIP database 2011 (<http://www.rieti.go.jp/en/database/JIP2011/>).

<sup>5</sup> Data from JIP database 2011 (<http://www.rieti.go.jp/en/database/JIP2011/>).

productivity rose or fell much less than the output and working hours<sup>6</sup>, which is inconsistent with the predictions of current macro theories that assume business cycles are, at least partially, driven by shocks of total factor productivity. McGrattan and Prescott (2014) argue that the current business cycle theory is likely to miss the unmeasured intangible investment based on similar phenomena that took place in US during the downturn of 2008-2009.

The fact that measured change in labour productivity is significantly different from the measured output and hours change is consistent with a theory that distinguishes economic income and measured income, which need not move together and did not move together during most of the time between 1990s and 2000s. To uncover what actually happened during Japan's lost decades, I incorporate the intangible investment into the basic neoclassical growth model following McGrattan and Prescott (2010) and McGrattan and Prescott (2014). There are two activities in the economy: the production of final goods and services, and the production of intangible investment goods<sup>7</sup>. Following McGrattan and Prescott (2010) and McGrattan and Prescott (2014), I assume hours allocated to these two activities are measured accurately, and reported income is underestimated by the amount of intangible investments. Given the inaccurate nature of the intangible investment measurement, I use the extended model to determine the path for the intangible investment following McGrattan and Prescott (2010) and show why including the missing intangible investment is important for understanding the lost decades of Japan.

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<sup>6</sup> Data from Total Economy Database (<https://www.conference-board.org/data/economydatabase/>). In terms of labour productivity per hour, it rose both in 1998 and 1999. In terms of labour productivity per person, it fell slightly in 1998 and rose in 1999.

<sup>7</sup> The intangible investment in this study is different from that in Corrado et al. (2009) and the literature based on Corrado et al. (2009). The intangible investment in this study is derived from macroeconomic theory while that of Corrado et al. (2009) is derived from the available data.

I allow the rates of technological change to differ across both the sector producing final goods and services and the sector producing intangible investment goods following McGrattan and Prescott (2010) and McGrattan and Prescott (2014)<sup>8</sup>. To generate working hours consistent with the reality, one could have modified the basic growth model by introducing large and variable shocks to preferences for leisure or labour market frictions, which is a common practice in business cycle research (McGrattan and Prescott, 2010). The advantage of the new theory proposed by McGrattan and Prescott (2010) and McGrattan and Prescott (2014) is that it can avoid introducing large changes of preferences for leisure or labour market frictions that often cannot be justified by any observations on tax rates, which makes this theory better satisfy the input justification criterion. That is, the exogenous input of this theory is more consistent with micro and macro evidence.

Another requirement for a successful theory is to satisfy the prediction criterion. That is, a theory must not produce counterfactual predictions, at least. A stronger requirement is to make correct predictions for data that were not used to set parameters and exogenous input. Therefore, I follow McGrattan and Prescott (2009) by using only TFP shocks and government wedge shocks because if I use all exogenous inputs, then I will obtain a perfect match between data and theory no matter what theory I use.

This study is the first to apply this new theory to an economy other than the US and therefore provides important evidence for the applicability of this new theory. To confirm the robustness of the extension proposed by McGrattan and Prescott (2010), I further apply the extension to an alternate neoclassical growth model with tangible investment adjustment costs. This study also provides a better explanation for the lost decades of Japan compared with previous literature. For

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<sup>8</sup> For the rationale of this modelling choice, please see McGrattan and Prescott (2010) and McGrattan and Prescott (2014).

example, Hayashi and Prescott (2002) treat the working hours in Japan as exogenous in some years during the prediction period and therefore have not fully explained the depression in working hours during the lost decades; Kobayashi and Inaba (2006) argue that labour market frictions play an important role in the lost decades of Japan but the increased frictions cannot be justified by any observations on tax rates. By applying the method proposed by McGrattan and Prescott (2010) and McGrattan and Prescott (2014) to the Japanese economy between 1995<sup>9</sup> and 2006<sup>10</sup>, I find that the prediction results of the new theory are much more consistent with the actual data of Japan compared with those of the base model, which indicates that this new theory is also applicable to Japan. My findings suggest that the standard productivity measures greatly underestimate the actual fall in labour productivity during most of the time in Japanese lost decades.

This paper is organized as follows. The following section demonstrates the prediction results from the basic neoclassical theory. Section 3 provides the evidence of decreased intangible investment during the research period. Section 4 shows the extended theory and its predictions. Section 5 draws the conclusion.

## **2. Predictions of the basic theory without intangible investment**

My starting point is the basic neoclassical growth model used in the study of business cycles. The basic model used in this study is a simplified version of that used in McGrattan and Prescott (2010) by eliminating most of the tax rates except the labour income tax<sup>11</sup>. Therefore, it is closer to the

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<sup>9</sup> The effective labour tax rate is an important element of the new theory proposed by McGrattan and Prescott (2010) and McGrattan and Prescott (2014). However, some national account data needed in the calculation of the effective labour tax rate is unavailable for Japan before 1994. Therefore, I decide to choose 1995 as the initial year.

<sup>10</sup> During the Global Financial Crisis, both the base model and the model extended with intangible investment and non-neutral technology do not work well due to dramatic financial frictions, though the extended model works better. However, the prediction results of both models are consistent, which indicates that the movement of measured output and unmeasured investment is consistent in Japan during the GFC. Therefore, I choose 2006 as the terminal year.

<sup>11</sup> Other tax rates or frictions except the labour income tax rate generally remain stable over the research period in Japan and therefore are already embodied in either the initial investment wedge that is normalized to 1 or the initial

model used in the business cycle accounting literature (Kobayashi and Inaba, 2006; Chari et al., 2007; Kersting, 2008). In the basic model, I treat TFP, labour income tax rate, population and the public consumption exogenously, which is consistent with McGrattan and Prescott (2010) and McGrattan and Prescott (2014).

In a standard one-sector neoclassical growth model, given the initial capital stock  $k_0$ , a representative household chooses consumption  $c$ , investment  $x$  and working hours  $h$  to maximize

$$E_0 \left[ \sum_{t=0}^{\infty} \beta^t U(c_t, h_t) N_t \right]$$

subject to

$$c_t + x_t = (1 - t_{wt})w_t h_t + r_t k_t + T_t$$

$$k_{t+1} = [(1 - \delta)k_t + x_t]/(1 + n)$$

The lowercase variables are written in per capita terms and  $N_t = N_0(1 + n)^t$  is the population in time  $t$ .  $r_t$  is the capital rent while  $w_t$  is the labour wage rate. Households discount their utility at the discount rate  $\beta$ , and the capital depreciation rate is  $\delta$ .  $t_{wt}$  is the labour income tax, which is the main component of labour wedge according to the theory of business cycle accounting.

The aggregate production function is labour augmented and in the form of Cobb-Douglas, which is as follows:

$$Y_t = (A_t H_t)^{1-ak} K_t^{ak}$$

Capital letters denote aggregate variables.  $A_t$  is TFP that varies over time,  $H_t$  is the total working hours and  $K_t$  is mainly the tangible capital stock<sup>12</sup>.  $K_t$  is calculated by applying the perpetual

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parameter settings. Therefore, it is reasonable to use this simplified version.

<sup>12</sup> It includes software, however.

inventory method to the investment in the national account. Firms rent capital and employ labour.  $ak$  is the capital income share in the production. If profits are maximized, then both the rental rate of capital and the wage rate of labour are respectively equal to the marginal product of each. The clear condition of the goods and service market is  $N_t(c_t + x_t + g_t) = Y_t$ .  $g_t$  is the government purchase or the government wedge.

Following McGrattan and Prescott (2010), I first calibrate the model based on the data of the initial year, and then compute the model's equilibrium path with households having perfect foresight of future changes in labour income tax rates, TFP, public consumption and populations. In appendix A, I discuss the data sources of the variables and the parameterization of the model. The parameters used to compute the equilibrium path of this model are summarized in Table A1. The effective labour income tax rate, the public consumption, and the TFP are reported in Table A2. The process of computing the equilibrium path is described in detail in Appendix A. The tax rate changes I consider in this study is the effective labour income tax rate  $t_{wt}$ , which is constructed using the method proposed by Mendoza (1994). The method proposed by Mendoza (1994) is also used in Prescott (2004) and McGrattan and Prescott (2010). The data for constructing effective labour income tax rate is obtained from OECD national account and revenue statistics<sup>13</sup>.

The utility function used in this study is standard in the business cycle literature, as follows:

$$U(c, l) = \log c + \psi \log(1 - h)$$

$\psi$  is the leisure preference parameter. Assume a technology progress rate  $\gamma$ , and the technical progress rate is derived from the average growth rate of GDP per working age person between

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<sup>13</sup> OECD statistics (<https://stats.oecd.org>)

1995 and 2006, which is standard in the business cycle literature. Therefore, we have the first order conditions as follows:

$$\frac{\psi \hat{c}_t}{1 - h_t} = (1 - t_{wt}) \frac{(1 - ak) \hat{y}_t}{h_t}$$

$$\mu_t = \hat{\beta} E_t \mu_{t+1} [(1 - \delta) + \frac{ak \hat{y}_{t+1}}{\hat{k}_{t+1}}]$$

where  $\hat{\beta} = \beta / (1 + \gamma)$ ,  $\mu = 1 / \hat{c}$ ,  $E_t$  denotes expectation. The hat on a variable indicates that it has been detrended by  $(1 + \gamma)^t$ . For example,  $\hat{c}_t = c_t / (1 + \gamma)^t$ .

To close the model, I add the resource constraint and the motion of capital:

$$\hat{c}_t + \hat{x}_t + \hat{g}_t = \hat{y}_t$$

$$\hat{y}_t = (A_t \hat{h}_t)^{1-ak} \hat{k}_t^{ak}$$

$$\hat{k}_{t+1} = [(1 - \delta) \hat{k}_t + \hat{x}_t] / [(1 + n)(1 + \gamma)]$$

The initial capital stock in 1995 is derived from the Penn World Table 8.1<sup>14</sup>. Following McGrattan and Prescott (2010), I choose the depreciation rate  $\delta$  based on the capital stock and the investment of Japan in 1995. Then I choose the utility parameter  $\psi$  so that the model's consumption share, investment share and factor inputs share are consistent with Japan level in 1995 (See Appendix A and B for details). Moreover, the perfect foresight of the household is assumed following McGrattan and Prescott (2010). Then, I incorporate the technology changes and government purchase changes into the model above and obtain the prediction results as follows:

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<sup>14</sup> For the detailed introduction of Penn World Table 8.1, please see Feenstra et al. (2015).

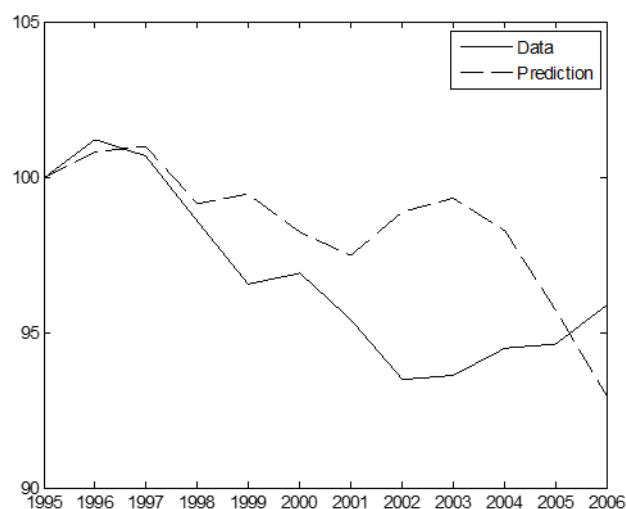


Figure 1. Japan and basic model per capita hours worked  
(Annual, 1995=100, 1995-2006)

In Figure 1, I plot the model's predicted per capita working hours and the actual per capita working hours, indexed so that 1995 equals 100. The difference between the two series is noticeable. Actual per capita hours were depressed during the research period while the predicted per capita hours remained steady and sometimes even boomed between 1995 to 2006.

In Figure 2, I plot the model's predicted output along with the real GDP of Japan. Both series are adjusted according to the population growth and a technological progress rate of  $1.015^t$ . Although the depression in output was not as large as the depression in hours, the model predicts that the economy should have boomed.

In Figure 3, I plot the model's predicted tangible investment along with the actual tangible investment of Japan. Obviously, significant deviations from the actual data is observed. The same phenomenon happens in consumption, which is displayed in Figure 4.

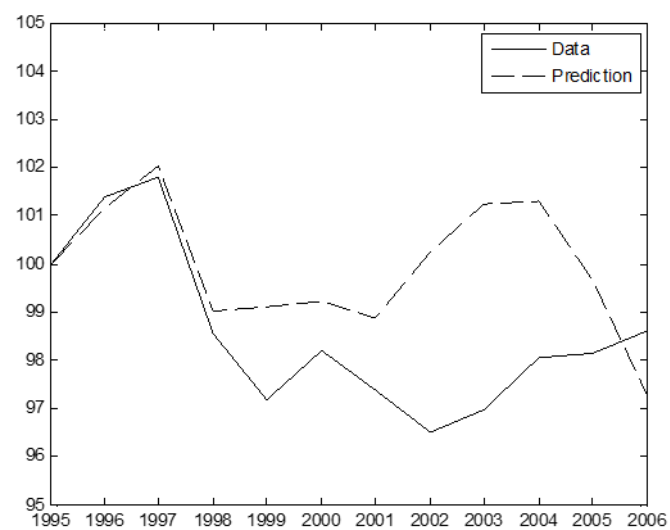


Figure 2. Japan and the basic model real GDP per capita (Detrended by  $1.015^t$ , annual, 1995=100, 1995-2006)

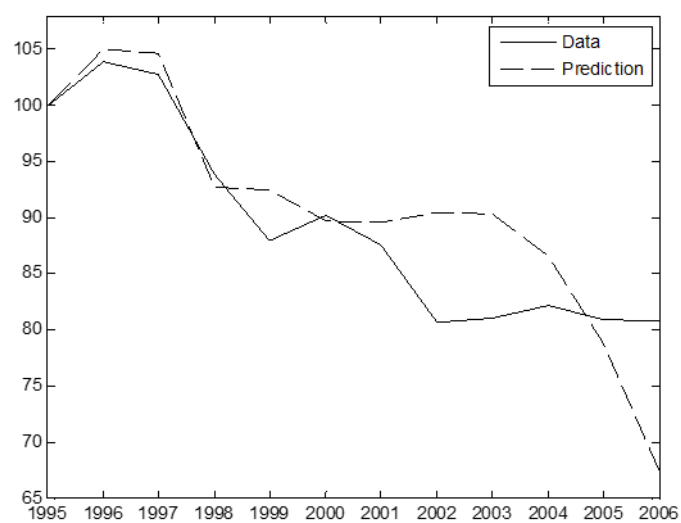


Figure 3. Japan and the basic model tangible investment per capita (Detrended by  $1.015^t$ , annual, 1995=100, 1995-2006)

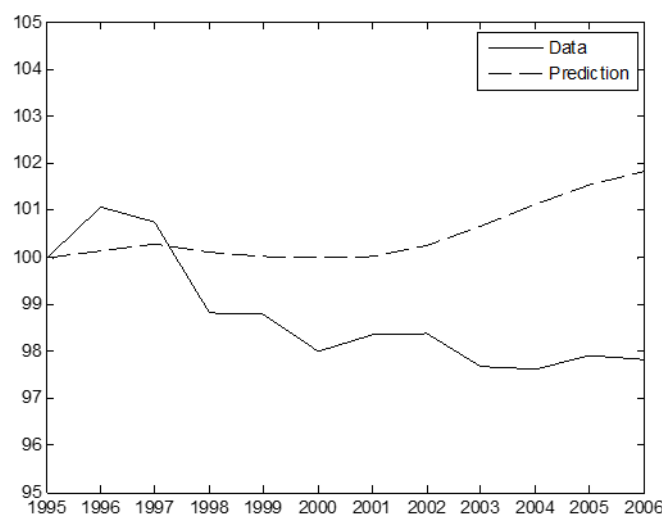


Figure 4. Japan and the basic model consumption per capita  
(Detrended by  $1.015^t$ , annual, 1995=100, 1995-2006)

### 3. Evidence of decreased intangible investment

I present evidence that suggests that unmeasured intangible investment was low during most of the time between 1995 and 2006. If all incomes were included in national accounts, we would expect both the growth rate of labour productivity per hour and the growth rate of output to be low during a depression. Since intangible investments are expensed in the national account, the measurement of labour productivity is overstated to a significant extent when these investments are low. An examination of Japan's national accounts reveals that the growth rate of labour productivity was high compared with the growth rate of output and working hours in many years between 1995 and 2006 according to Figure 5, suggesting that the growth rate of unmeasured intangible investment is often lower than that of output and working hours.

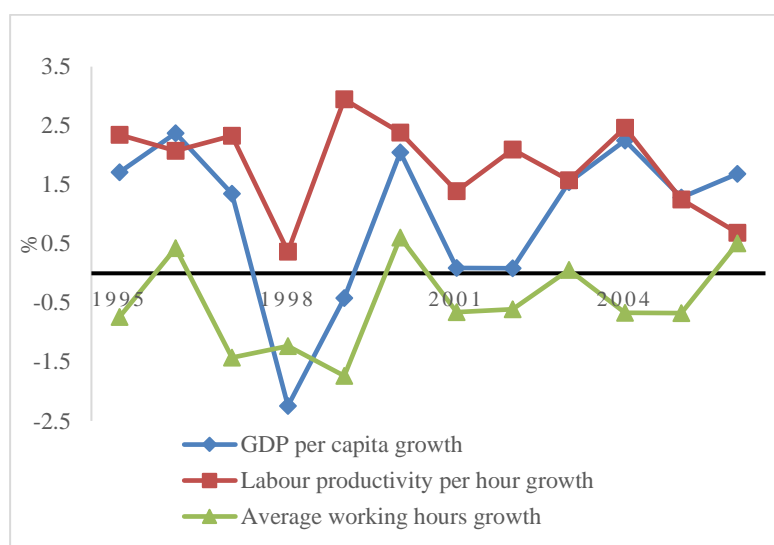


Figure 5. The growth of GDP per capita, labour productivity per hour and average working hours in Japan (Annual)

Source: Author's own construction; data from Total Economy Database (<https://www.conference-board.org/data/economydatabase/>).

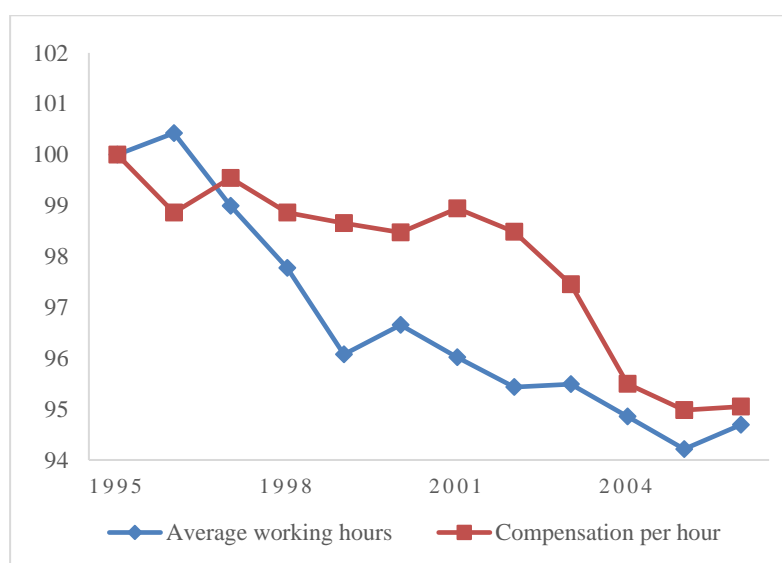


Figure 6. The average working hours and compensation per hour in Japan (Compensation per hour detrended by  $1.015^t$ , annual, 1995=100, 1995-2006)

Source: Author's own construction; data from Total Economy Database (<https://www.conference-board.org/data/economydatabase/>) and OECD national account statistics.

In Figure 6, I plot the average working hours and compensation per hour in Japan. The average working hours have been adjusted according to the population growth rate, and the compensation per hour is adjusted according to the price level provided by the Penn World Table 8.1 and technology growth rate. It is evident that the movements of average working hours and compensation per hours are inconsistent during the research period. In some years, the

compensation per hour grew while the working hours per labour declined, which may indicate that the unmeasured investment was low in many years.

#### 4. Predictions of the extended theory with intangible investment

According to McGrattan and Prescott (2010) and McGrattan and Prescott (2014), the technology used in producing final goods and services should be different from that used in producing intangible investment. Therefore, the extended theory should include not only intangible investment but also non-neutral technology. The household problem remains the same as that in section 2 except that there is an additional investment choice. I examine the extended model's predictions and show that these predictions are in conformity with Japanese observations between 1995 and 2006.

#### Extensions

The aggregate production comprises two aggregation production relations:

$$y_t = (A_t^1 h_t^1)^{1-ak-ai} (k_{Tt}^1)^{ak} (k_{It})^{ai}$$

$$x_{It} = (A_t^2 h_t^2)^{1-ak-ai} (k_{Tt}^2)^{ak} (k_{It})^{ai}$$

Firms produce final goods and services using intangible capital  $k_I$ , tangible capital  $k_T^1$  and labour  $h^1$ . Firms produce new intangible capital  $x_I$ , such as new brands, new products R&D, staff training, etc., using intangible capital  $k_I$ , tangible capital  $k_T^2$  and labour  $h^2$ . It is difficult to acquire the parameters of the intangible investment production because of a lack of data. For simplicity, I assume the income shares of the three production factors are the same between the two activities following McGrattan and Prescott (2010). According to McGrattan and Prescott (2010), varying the parameters of the production function of intangible investment does not change the implications of the new theory.

Following McGrattan and Prescott (2010) and McGrattan and Prescott (2014),  $k_I$  is an input to both sectors. It is not split between them as is the case for tangible capital and labour because it can be used in different productions simultaneously. For example, a brand name is used both to sell final goods and services and to develop new brands; new knowledge from R&D are used by both producers and researchers. Given initial capital  $(k_{It}, k_{Tt})$ , the maximization problem of household is

$$E_0[\sum_{t=0}^{\infty} \beta^t U(c_t, h_t) N_t],$$

subject to

$$c_t + x_{Tt} + q_t x_{It} = (1 - t_{wt}) w_t h_t + r_{Tt} k_t + r_{It} k_{It} + T_t.$$

$$k_{T,t+1} = [(1 - \delta_T) k_{T,t} + x_{Tt}] / (1 + n)$$

$$k_{I,t+1} = [(1 - \delta_I) k_{I,t} + x_{It}] / (1 + n)$$

As before, all lowercase variables are in per capita units and there is a growth of population at rate  $n$ . The relative price of intangible investment is  $q_t$ . The rental rate of intangible capital and tangible capital are respectively denoted by  $r_{It}$  and  $r_{Tt}$ . The transfer payment is denoted by  $T_t$  and is exogenous in household's decision problem. Labour income is  $w_t h_t$ . The first order conditions are as follows:

$$\frac{\psi \hat{c}_t}{1 - h_t} = (1 - t_{wt}) \frac{(1 - ak - ai) \hat{y}_t}{h_t^1} \quad (1)$$

$$\mu_t = \hat{\beta} E_t \mu_{t+1} [(1 - \delta_T) + \frac{ak \hat{y}_{t+1}}{\hat{k}_{t+1}}] \quad (2)$$

$$q_t \mu_t = \hat{\beta} E_t \mu_{t+1} [q_{t+1} (1 - \delta_T) + \frac{ai(\hat{y}_{t+1} + q_{t+1} x_{It+1})}{\hat{k}_{t+1}}] \quad (3)$$

$$\frac{\hat{y}_t}{h_t^1} = \frac{q_t \hat{x}_{It}}{h_t^2} \quad (4)$$

$$\frac{\hat{y}_t}{k_t^1} = \frac{q_t \hat{x}_{It}}{k_t^2} \quad (5)$$

The hat on a variable indicates that it has been detrended by  $(1 + \gamma)^t$ . To close the model, I again add the resource constraint and the capital motion:

$$\hat{c}_t + \hat{x}_t + \hat{g}_t = \hat{y}_t \quad (6)$$

$$\hat{y}_t = (A_t^1 h_t^1)^{1-ak-ai} (\hat{k}_{Tt}^1)^{ak} (\hat{k}_{It})^{ai} \quad (7)$$

$$\hat{x}_{It} = (A_t^2 h_t^2)^{1-ak-ai} (\hat{k}_{Tt}^2)^{ak} (\hat{k}_{It})^{ai} \quad (8)$$

$$\hat{k}_{T,t+1} = [(1 - \delta_T) \hat{k}_{T,t} + \hat{x}_{Tt}] / [(1 + n)(1 + \gamma)] \quad (9)$$

$$\hat{k}_{I,t+1} = [(1 - \delta_I) \hat{k}_{I,t} + \hat{x}_{It}] / [(1 + n)(1 + \gamma)] \quad (10)$$

### Explaining the seemingly high wages

I showed earlier that there is a large deviation between predictions of the basic growth model and Japanese data. The model predicts that after tax real wage should remain steady between 1995 and 2006, leading to steady per capita hours and output. With the extended model, the measurement of the real wage is different and is consistent with the behaviour of output and hours.

The basic model measures the real wage as

$$\bar{w}_t = (1 - ak) \frac{y_t}{(h_t^1 + h_t^2)}$$

where  $ak$  is the capital share,  $y$  is the measured value added, and  $h_t^1 + h_t^2$  is the total working hours. The problem with the measurement of labour productivity on the right side of equation is that some hours are used to produce intangible investment. The hours used to produce  $y$  are  $h_t^1$  and, therefore, the real wage measurement should be

$$w_t = (1 - ak - ai) \frac{y_t}{h_t^1}$$

where  $ai$  is the intangible capital share.  $\frac{y_t}{h_t^1}$  is the labour productivity in producing final goods and services. The labour hours  $h_t^2$  is used to produce intangible investment and is not part of the labour input in producing  $y$ . If the relative size of  $h_t^2$  to  $h_t^1 + h_t^2$  decreases, then  $\bar{w}_t/w_t$  increases and the percentage overstatement of true wages becomes more significant.

Moreover, the technology used in producing intangible investment and final goods and services should be different due to the different nature of them, and therefore should be influenced by different productivity shocks. This would imply a decrease in  $A_t^2/A_t^1$ . My hypothesis is that  $A_t^2/A_t^1$  did decrease significantly, which led to a decrease in the relative hours allocated to the intangible investment production, namely  $h_t^2/(h_t^1 + h_t^2)$ .

### Identifying total factor productivities

The scale of the inputs and outputs of both production functions has to be determined in order to identify the total factor productivities. This requires splitting the hours and tangible capital between two production activities as well as determining the magnitude of intangible investment and capital.

To identify how much labour is allocated to the two production activities, I use the fact that the after tax real wage rate equals the marginal rate of substitution between leisure and consumption, following McGrattan and Prescott (2010). That is, using equation (1) in the first order conditions of the extended theory. Then, we have

$$h_t^1 = (1 - t_{wt}) \frac{(1 - ak - ai)\hat{y}_t}{\psi \hat{c}_t} (1 - h_t)$$

Please note that observations on consumption  $\hat{c}_t$ , total hours  $h_t$ , final goods and services  $\hat{y}_t$  and the labour tax rate  $t_{wt}$  are available. Hours used in producing intangible investment is determined residually, which is  $h_t^2 = h_t - h_t^1$ .

The marginal products of labour in the two activities should be equal, which is the equation (4) in the first order conditions of the extended theory. Therefore, we have

$$q_t \hat{x}_{It} = \frac{h_t^1}{h_t^2} \hat{y}_t \quad (11)$$

which is the measurement of intangible investment. As per McGrattan and Prescott (2010), the derivation of intangible investment relies heavily on theory and observations on consumption, total working hours, final goods and services as well as labour tax rate. This method has an advantage over direct measurement when some or all of the intangible investment is not or cannot be measured (accurately) due to data availability issues.

The allocation of tangible capital across the two activities is determined in a similar way to the allocation of labour hours. To be specific, the marginal products of tangible capital in the two activities should also be equal, which is equation (5) in the first conditions of the extended theory.

Re-arrange this equation, we have

$$\hat{k}_t^1 = \frac{\hat{y}_t \hat{k}_t}{(q_t \hat{x}_{It} + \hat{y}_t)}$$

Again, tangible capital allocated to the production of intangible investment is determined residually as

$$\hat{k}_t^2 = \hat{k}_t - \hat{k}_t^1$$

If there is a sequence of the price  $q_t$  of the intangible investment, the already-computed sequence of outputs  $q_t \hat{x}_{It}$  can be used to infer the sequence of the intangible investment, and with a given initial value for intangible capital stock  $\hat{k}_{I,1995}$ , I can use equation (10) to determine the sequence of intangible stocks. To achieve the above, I calculate the sequence of the intangible investment price  $q_t$  based on the intertemporal condition of intangible investment, which is equation (3) in the first order conditions of the extended theory:

$$q_t \mu_t = \hat{\beta} E_t \mu_{t+1} [q_{t+1} (1 - \delta_T) + \frac{ai(\hat{y}_{t+1} + q_{t+1} x_{It+1})}{\hat{k}_{It+1}}]$$

$q_{t+1} x_{It+1}$  is derived from equation (11), and  $\hat{k}_{It+1}$  is derived from equation (10), which is the motion of intangible capital. Since we have the observations on output and consumption,  $q_{t+1}$  can be obtained given  $q_t$ . I normalize  $q_{1995}=1$  following McGrattan and Prescott (2009) and then the sequence of  $q$  is obtained. Figure 7 demonstrates the ratio of intangible investment  $q_t \hat{x}_{It}$  derived from the extended theory to the measured output, which is consistent with the evidence discussed in section 3.

Finally, I obtain the varying TFP and government wedge according to equation (6), (9) and (10), and incorporate them along with the effective labour tax rate into the extended model to derive the prediction results in the following section.

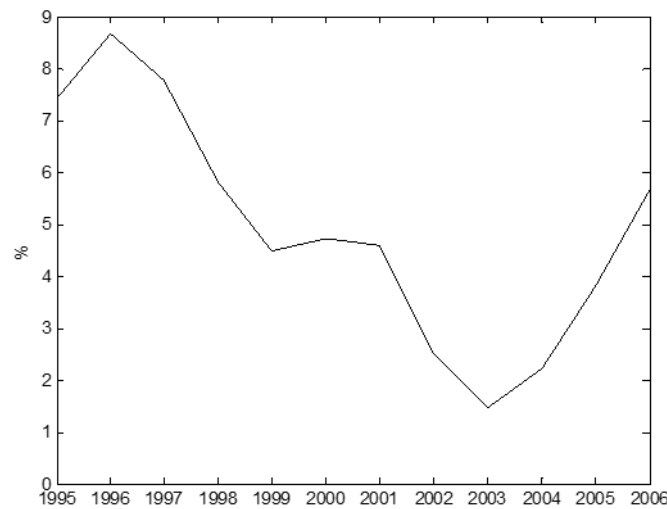


Figure 7. Extended model intangible investment measured output ratio  
(Annual, 1995-2006)

Source: Author's own calculation

### Model predictions

Treating the TFP sequence and the government wedge sequence as the exogenous input, I calibrate the model based on the data in 1995, compute the equilibrium of all variables and compare them with actual Japanese data. All of the parameters used in computing the equilibrium path are described in Appendix A and summarized in Table A1. Moreover, a sensitivity analysis is conducted in Appendix B.

In Figure 8, I display the results for per capita total working hours. Unlike the comparative results from the basic model (Figure 1), the predictions here are much more consistent with the actual data. The extended model predicts a slight fall in hours used in producing final goods and investment during most of the time between 1995 and 2006. However, because the fall in hours used in producing intangible investment is much more than those used in producing final goods and investment, the model predicts a significant depression in per capita total working hours. In Figure 9, unsurprisingly, the modelled per capita output and the actual per capita output are close. In Figure 10, the predicted per capita tangible investment is almost the same as the actual

data. In Figure 11, the predicted per capita consumption also generally captures the trend of the actual data.

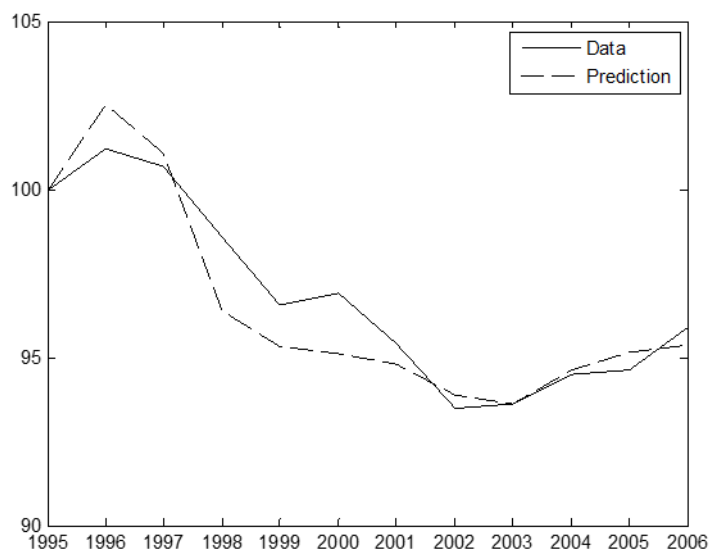


Figure 8. Extended model per capita total hours worked in Japan  
(Annual, 1995=100, 1995-2006)

Source: Author's own calculation



Figure 9. Extended model per capita real GDP in Japan  
(Annual, series detrended by  $1.015^t$ , 1995=100, 1995-2006)

Source: Author's own calculation

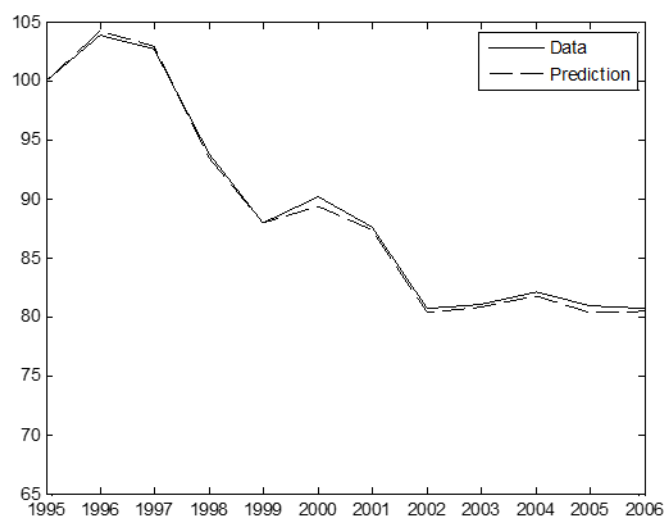


Figure 10. Extended model per capita tangible investment in Japan  
(Annual, series detrended by  $1.015^t$ , 1995=100, 1995-2006)

Source: Author's own calculation

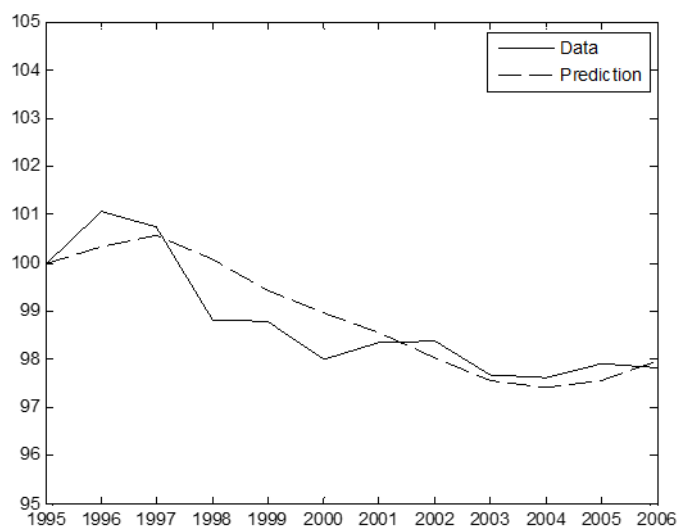


Figure 11. Extended model per capita consumption in Japan  
(Annual, series detrended by  $1.015^t$ , 1995=100, 1995-2006)

Source: Author's own calculation

What does all this mean for Japanese labour productivity? If some output is unmeasured relative to input, then the change in productivity is biased when the unmeasured output does not move together with the measured output. The extended model's predictions for macro variables with or without intangible investment demonstrate how distorted the standard data and basic model are for assessing the lost score of Japan.

In Figure 12, I compare two series of predictions derived from the extended model. One is the model's predictions for output per hour without intangible investment included. The other is the predictions for output per hour with intangible investment included. Obviously, with intangible investment incorporated, the puzzling labour productivity growth during the Asian Financial Crisis is no longer puzzling: labour productivity actually declined. Moreover, it is clear that the deviation of measured labour productivity and the actual labour productivity is significant. During the lost decades, the labour productivity of Japan was actually depressed, rather than booming.

In Figure 13, I compare the extended model's two measurements of total investment: one without intangible investment and the other with intangible investment. Again, the two series of predictions are quite different, which indicates that the measured investment dramatically underestimates the actual decline in investment.

In summary, the results above show that standard accounting measurements and predictions of the standard model without intangible investment do not accurately reflect what was going on in Japan between 1995 and 2006. Therefore, the extended model is needed when conducting aggregate analyses.

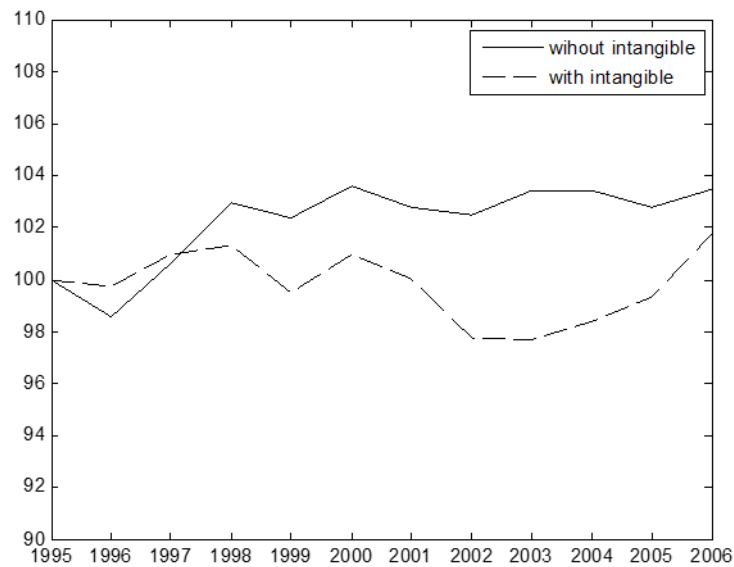


Figure 12. Extended model labour productivity in Japan  
(Annual, series detrended by  $1.015^t$ , 1995=100, 1995-2006)

Source: Author's own calculation

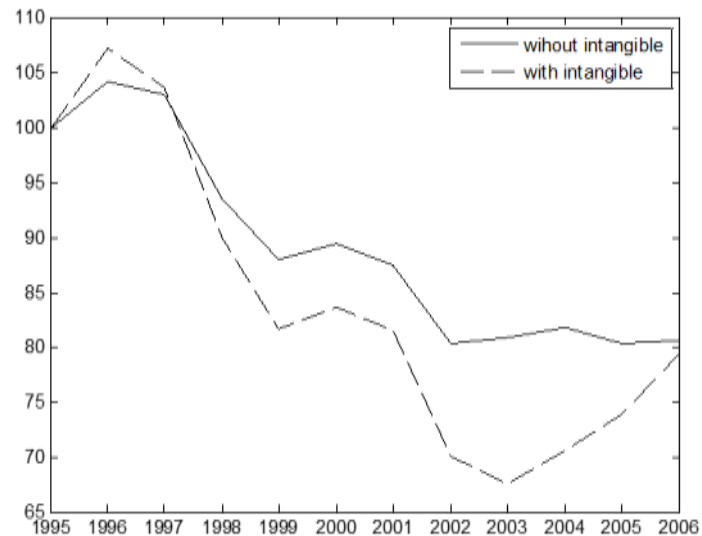


Figure 13. Extended model per capita investment in Japan  
(Annual, series detrended by  $1.015^t$ , 1995=100, 1995-2006)

Source: Author's own calculation

## 5. Results with alternative model settings

In this section I test the robustness of the extension developed by McGrattan and Prescott (2010) with an alternative model with tangible investment adjustment costs. Christiano and Davis (2006) indicate that introducing tangible investment adjustment costs can affect the prediction results of

neoclassical growth models. In this section, I modify the capital accumulation equations both in the basic model and the extended model to incorporate quadratic tangible investment adjustment costs.

In both the basic model and the extended model, the law of the motion of tangible capital turns into

$$\hat{k}_{t+1} = \left[ (1 - \delta)\hat{k}_t + \hat{x}_t - \Phi\left(\frac{\hat{x}_t}{\hat{k}_t}\right)\hat{k}_t \right] / (1 + n)$$

where

$$\Phi\left(\frac{\hat{x}_t}{\hat{k}_t}\right) = \frac{\phi}{2} \left(\frac{\hat{x}_t}{\hat{k}_t} - \lambda_T\right)^2$$

The constant  $\lambda_T$  of tangible capital is set at  $\lambda_T = na - (1 - \delta)$  so that the adjustment cost is equal to zero at steady state. The parameter  $\phi$  is calibrated to match the marginal Tobin's Q to one:

$$\frac{d\log q_T}{d\log\left(\frac{\hat{x}}{\hat{k}}\right)} = 1$$

where  $q_T$  is the effective price of tangible investment relative to consumption:

$$q_T = \frac{1}{1 - \Phi'}$$

This leads to  $\phi = \frac{\hat{k}}{\hat{x}}$ .

The simulation results are similar to the original models: the basic alternative model fails to generate satisfying predictions and the extended alternative model improves the predictions significantly. Therefore, the extension proposed by McGrattan and Prescott (2010) is robust. The value of the additional parameter  $\lambda_T$  as well as the simulation results are demonstrated in Appendix C.

## 6. Conclusion

The basic neoclassical growth model accounts well for the Japanese economy prior to the 1990s, provided that variations in population growth, depreciation rates, total factor productivity (TFP) and government purchase are incorporated. The behaviour of the Japanese economy during the 1990s and 2000s, however, is often significantly inconsistent with the model predictions, which is also inconsistent with the argument in the literature that the decline in TFP is the main cause of the lost decades in Japan (Hayashi and Prescott, 2002; Fukao et al., 2006; Griffin and Odaki, 2009).

Following McGrattan and Prescott (2010) and McGrattan and Prescott (2014), I find that the unmeasured intangible investment as well as non-neutral technological change in intangible investment production led to the puzzling behaviour of the Japanese economy between 1995 and 2006. This change resulted in a depression in intangible investment, which is not reflected in the measured output. After applying the new theory, the puzzling lost decades in Japan becomes less puzzling.

This study is the first to apply this new theory to a country other than the US and finds that this new theory works well in Japan, even using a simpler version. Significant improvements in predictions are seen compared with the standard neoclassical model. The results remain robust when tangible investment adjustment costs are added. It also provides important evidence of the applicability of this new theory to other economies, and strengthens the argument of McGrattan and Prescott (2010) that the new theory with intangible investment should be used in aggregate analyses.

## **Appendix A Data and parameters**

The main sources of data in this study are the Penn World Table 8.1, OECD revenue and national accounts statistics, and the World Bank DataBank. The variables from the Penn World Table 8.1 that this study has used include total labour hours, real GDP, consumption share, investment share, government and net export share, labour compensation share; the variables from OECD revenue and national accounts statistics are used to calculate the effective labour income tax based on the method proposed by Mendoza (1994); the working age population (age 15-64) data is obtained from the World Bank DataBank. The calibration process used in this study follow McGrattan and Prescott (2009) and are demonstrated in Table A1. The exogenous inputs for simulation of the standard model and the extended model are respectively demonstrated in Table A2 and Table A3.

Table A1 Model parameters

Parameter	Expression	Value
<b>Common parameters</b>		
Growth in population	$n$	-0.003
Growth in technology	$\gamma$	0.015
Discount factor	$\beta$	0.98
<b>Standard model, no intangible investment</b>		
Utility parameter	$\psi$	4.44
Depreciation rate	$\delta$	0.07
Capital share	$ak$	0.35
<b>Extended model, with intangible investment</b>		
Utility parameters	$\psi$	3.43
Tangible depreciation rate	$\delta_T$	0.07
Intangible depreciation rate <sup>15</sup>	$\delta_I$	0
Tangible capital share	$ak$	0.3276
Intangible capital share	$ai$	0.2064

Growth in population is derived from the working age population data; growth in technology is obtained from the average growth rate in value added per labour; the discount factor follows McGrattan and Prescott (2010).

### Parameters calculation and exogenous inputs for the standard model

Calibration is based on level data in 1995 instead of the average data and the following equations:

$$\delta = \frac{\hat{x}}{\hat{k}} + 1 - (1 + n)(1 + \gamma)$$

$$r = \frac{[1 - \beta(1 - \delta)]}{\beta}$$

$$ak = \frac{r\hat{k}}{\hat{y}}$$

$$\psi = \frac{(1 - t_{w,1995})(1 - ak)(1 - h)\hat{y}}{\hat{c}h}$$

$$z = \left(\frac{\hat{k}}{\hat{y}}\right)^{ak/(ak-1)} \frac{\hat{y}}{h}$$

<sup>15</sup> The depreciation rate of intangible capital is following McGrattan and Prescott (2010). However, I will conduct sensitivity analysis in Appendix B to show that the choice of the depreciation rate does not affect the results much.

z is the initial technology level; other notations are the same as those in the text. The exogenous inputs include the TFP, effective labour income tax and the government wedge, which are listed in Table A2.

Table A2 Exogenous inputs for the standard model

Year	Labour income tax	Government wedge	Technology parameter
	$t_w$	$\hat{g}$	A
1995	0.226005	0.148440931	3.19515
1996	0.226332	0.145587261	3.225424
1997	0.230106	0.154571707	3.255914
1998	0.225499	0.157813769	3.161069
1999	0.224183	0.160119394	3.167338
2000	0.236472	0.168758844	3.222646
2001	0.240673	0.165521832	3.24456
2002	0.236517	0.175462343	3.281042
2003	0.236543	0.183147381	3.325961
2004	0.243221	0.191610003	3.376623
2005	0.251445	0.19400152	3.396088
2006	0.257525	0.19977311	3.39723

### Parameters calculation and exogenous inputs for the extended model

Again, calibration is based on level data in 1995 and the following equations:

$$r_T = \frac{[1 - \beta(1 - \delta_T)]}{\beta}$$

$$r_I = \frac{q[1 - \beta(1 - \delta_I)]}{\beta}$$

$$\hat{k}_I = \frac{\hat{y} - r_T \hat{k}_T - 1995 \text{ labour compensation}}{r_I - q[(1 + \gamma)(1 + n) - 1 + \delta_I]}$$

$$\hat{x}_I = ((1 + \gamma)(1 + n) - 1 + \delta_I) \hat{k}_I$$

$$ak = \frac{r_T \hat{k}_T}{\hat{y} + q \hat{x}_I}$$

$$ai = \frac{r_I \hat{k}_I}{\hat{y} + q \hat{x}_I}$$

$$z_1 = \left( \frac{\hat{y}}{h_1^{1-ak-ai} \hat{k}_T^{ak} \hat{k}_I^{ai}} \right)^{\frac{1}{1-ak-ai}}$$

$$z_2 = \left( \frac{\hat{x}_I}{h_2^{1-ak-ai} \hat{k}_T^{ak} \hat{k}_I^{ai}} \right)^{\frac{1}{1-ak-ai}}$$

$z_1$  and  $z_2$  are respectively the initial production technology of final goods and services and intangible investment. The exogenous inputs include TFP for final goods and services and intangible investment, effective labour income tax and the government wedge.

Table A3 Exogenous inputs for the extended model

Year	Labour income tax	Government wedge	Technology parameter	
	$t_w$	$\hat{g}$	$A_1$	$A_2$
1995	0.226005	0.148440931	1.331588	0.421171
1996	0.226332	0.145587261	1.381845	0.457688
1997	0.230106	0.154571707	1.376854	0.437985
1998	0.225499	0.157813769	1.269788	0.369422
1999	0.224183	0.160119394	1.237311	0.320089
2000	0.236472	0.168758844	1.276741	0.343663
2001	0.240673	0.165521832	1.280091	0.336908
2002	0.236517	0.175462343	1.247973	0.249418
2003	0.236543	0.183147381	1.254313	0.200235
2004	0.243221	0.191610003	1.30684	0.251584
2005	0.251445	0.19400152	1.357024	0.328368
2006	0.257525	0.19977311	1.409351	0.408921

## Appendix B Varying the depreciation rate of intangible capital

The depreciation rate of intangible capital is chosen to be 0 following McGrattan and Prescott (2010). In the following I will vary the depreciation to show that the results remain robust given different depreciation rate of intangible capital.

$$\delta_I = 0.1,$$

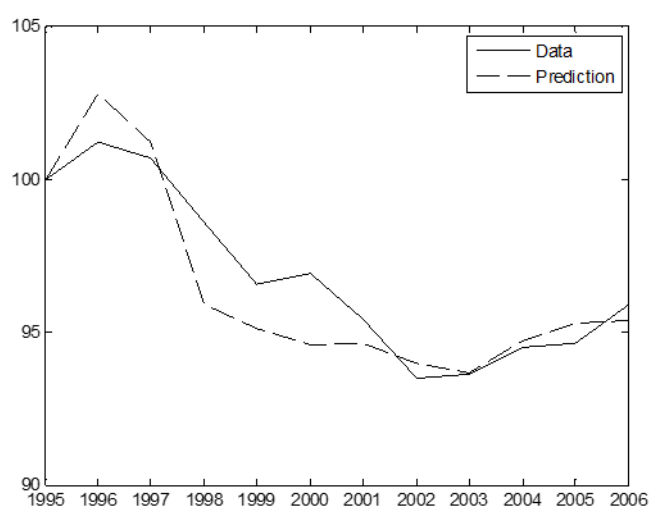


Figure B1. Extended model per capita total hours worked in Japan with  $\delta_I = 0.1$   
(Annual, 1995=100, 1995-2006)

Source: Author's own calculation

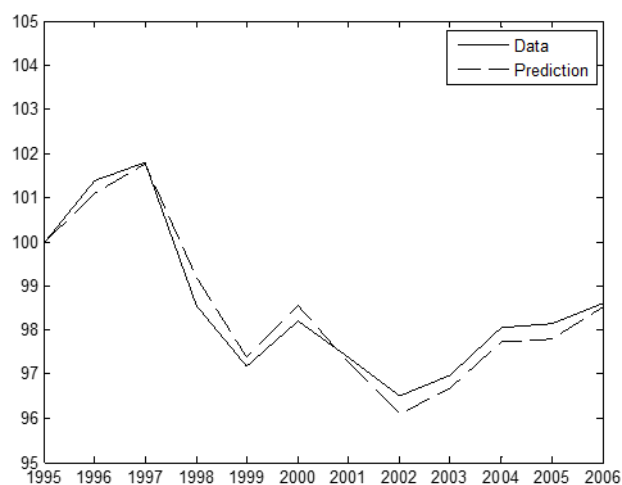


Figure B2. Extended model per capita real GDP in Japan with  $\delta_I = 0.1$   
(Annual, series detrended by  $1.015^t$ , 1995=100, 1995-2006)

Source: Author's own calculation

$\delta_I = 0.2$ ,

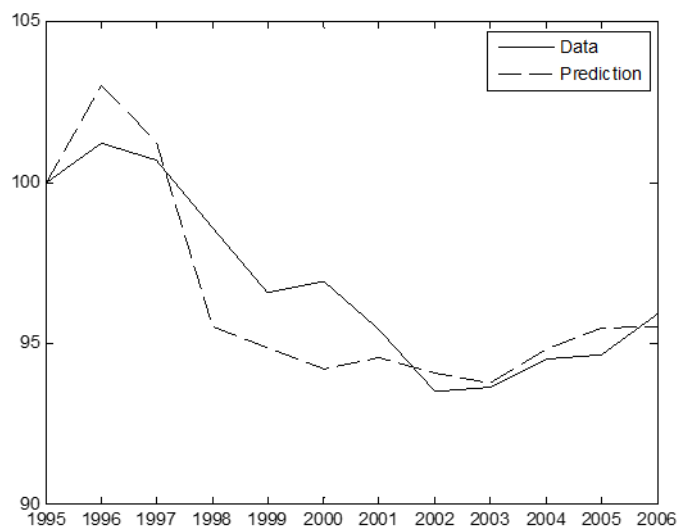


Figure B3. Extended model per capita total hours worked in Japan with  $\delta_I = 0.2$   
(Annual, 1995=100, 1995-2006)

Source: Author's own calculation

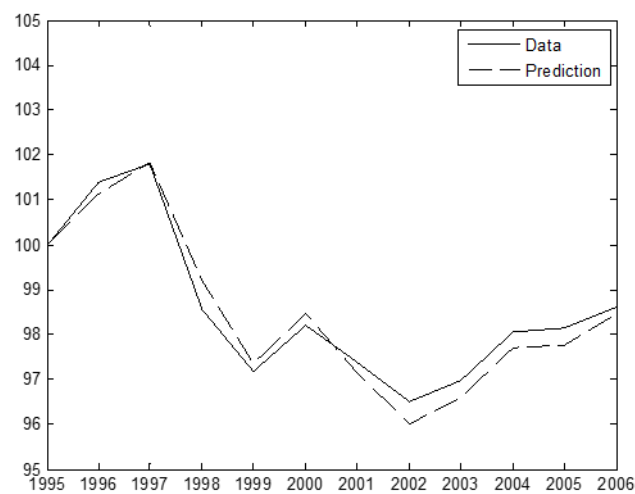


Figure B4. Extended model per capita real GDP in Japan with  $\delta_I = 0.2$   
(Annual, series detrended by  $1.015^t$ , 1995=100, 1995-2006)

Source: Author's own calculation

## Appendix C Simulation results of models with tangible capital adjustment costs

According to the calibration method of  $\lambda_T$  in section 5, the value of  $\lambda_T$  is 0.0821. Simulation results of the alternative models are demonstrated as follows:

### The basic alternative model

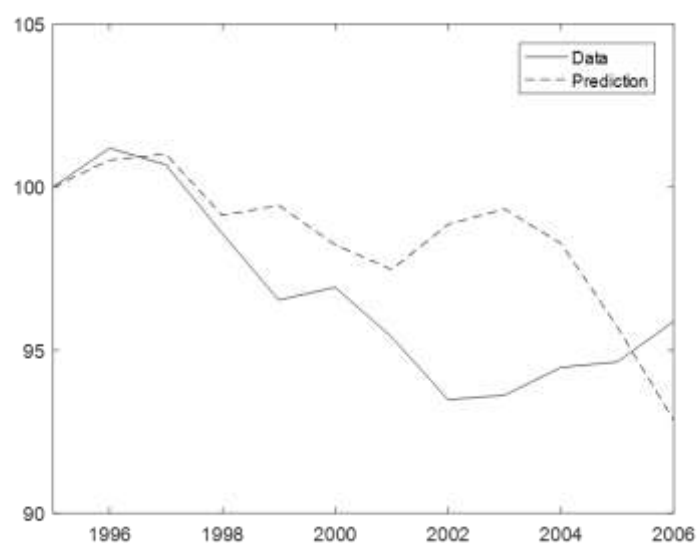


Figure C1. Basic alternative model per capita total hours worked in Japan

(Annual, 1995=100, 1995-2006)

Source: Author's own calculation

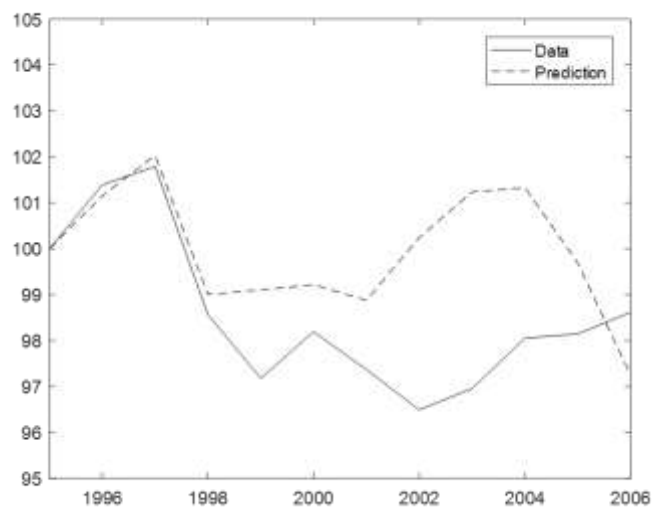


Figure C2. Basic alternative model per capita real GDP in Japan  
(Annual, 1995=100, 1995-2006)

Source: Author's own calculation

### The extended alternative models

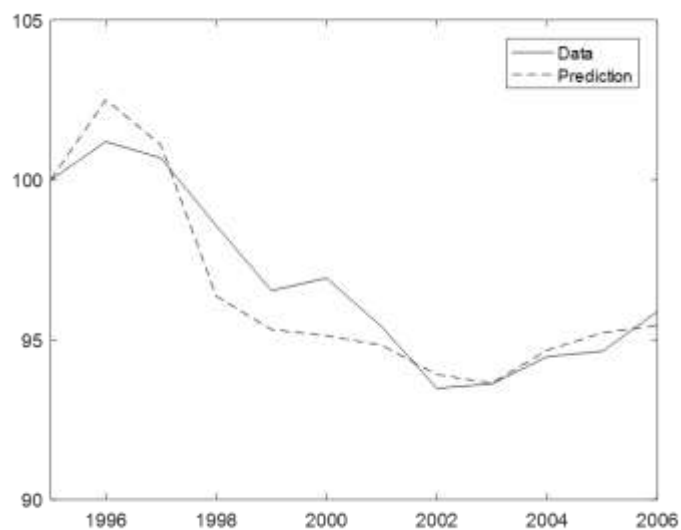


Figure C3. Extended alternative model per capita total hours worked in Japan  
(Annual, 1995=100, 1995-2006)

Source: Author's own calculation

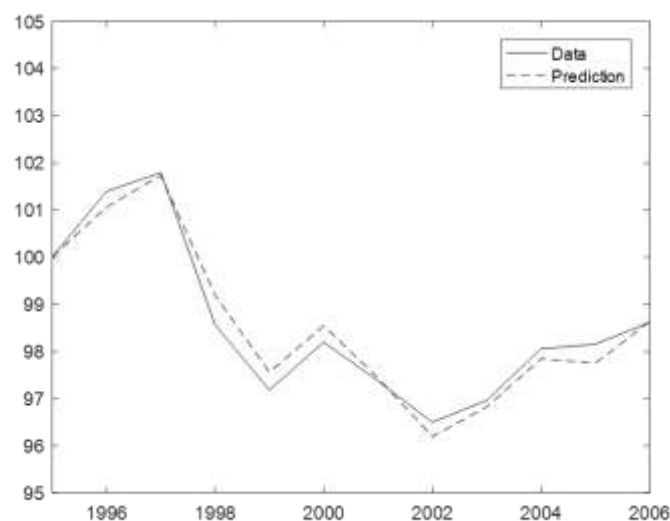


Figure C4. Extended alternative model per capita real GDP in Japan  
(Annual, 1995=100, 1995-2006)

Source: Author's own calculation

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