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A Note on Imperfect Credibility

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Keywords

imperfect credibility, monetary policy, sustainable policy

JEL Classification

E52, E58, E61

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A Note on Imperfect Credibility*

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Abstract

We explore how outcomes of optimal monetary policy with *loose commitment* (Schaumburg and Tambalotti, 2007; Debortoli and Nunes, 2010) can be observationally equivalent, or interpretable as outcomes of deeper optimal policy under *sustainable plans* (Chari and Kehoe, 1990). Both interpretations of "imperfect credibility" in optimal monetary policy design are attempts to rationalize outcomes that lie in between the conventional extremes of optimal policy under commitment and under discretion. In a standard monetary-policy framework, when we match impulse responses of inflation and the output gap to large enough markup shocks, we find that a small probability ($1 - \alpha = 0.05$) of replanning in the quasi/loose commitment world corresponds to $N = 18$ in the N-period punishment optimal sustainable monetary policy, in terms of observable outcomes. For plausible cases of loose-commitment model economies (with α between 0.77 and 1) we can find an observationally equivalent sustainable-plan economy indexed by some N.

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1 Background and Problem Statement

In this note, a monetary-policy plan is an infinite sequence of history/state-contingent policy selections of inflation and output gap outcomes. As in the mainstream literature on optimal monetary policy, we define *imperfect credibility* generically as the imperfect ability of a monetary-policy plan to influence the private sector's belief about the continuation of its policy plan into the indefinite future. However, in the literature, this notion of imperfect credibility has taken on two alternative, and (structurally) very different, modelling interpretations. The first approaches the notion using a game-theoretic (or equivalently a contract-design) modelling device: This is the *limited commitment* approach to time-consistent policy design, and can

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be couched in terms of a *sustainable plans* problem (see, e.g., Chari and Kehoe, 1990; Kurozumi, 2008; Sunakawa, 2015). The second approach to imperfect credibility takes on the interpretation of a policy planner reoptimizing on its previously given policy plans (with exogenous probability): This is also known as *stochastic replanning* (Roberds, 1987), *quasi commitment* (Schaumburg and Tambalotti, 2007), or, *loose commitment* (Debortoli and Nunes, 2010; Debortoli et al., 2014).[1]

When agents are forward-looking, a policy maker can enhance social welfare by committing to future policy. However, once agents have fixed their expectations on such a plan, the policy maker is tempted to renege on their previous commitments. This *time inconsistency* problem has been the central concern of optimal dynamic policy design since the seminal studies by Kydland and Prescott (1977) and Barro and Gordon (1983). In the conventional monetary-policy literature, commitment to and discretion in policy plans are two polar cases. It is unrealistic to suppose that policy makers will always uphold previous commitments. The situation where policy is always discretionary, even though there are gains from commitment, is also troubling in a normative sense. Hence the interest in the literature on these two alternative views of *imperfect credibility* in policy plans, where policy reality is often thought to lie somewhere in between commitment and discretion.

Under quasi/loose commitment, a policy maker will renege on its commitment with a constant probability, say, $1-\alpha$. When $\alpha = 1$ (or when $\alpha = 0$), such a policy collapses to the traditional commitment (or discretion) policy regime. Thus, it becomes possible to analyze "a continuum of monetary policy rules characterized by differing degrees of credibility" (see Schaumburg and Tambalotti, 2007). Debortoli and Lakdawala (2016) estimate a prototypical dynamic stochastic general equilibrium model *à la* Smets and Wouters (2007) and find that the probability with which the US Federal Reserve reneges on its previous commitment is around 19%.

In the quasi/loose commitment approach to imperfect credibility, the key friction to commitment, α, is a free parameter. Thus, analyses using quasi/loose commitment are subject to the Lucas (1976) critique. Although computationally and quantitatively appealing, what could possibly be a theoretical justification for such a quasi/loose commitment? It has been conjectured that the sustainable plans approach to imperfect policy credibility may provide a deeper foundation for the more reduced-form quasi/loose commitment story. For example, Schaumburg and Tambalotti (2007), who utilized the quasi/loose commitment approach, noted that:

> We also view our approach to credibility as an alternative to the one built on the game theoretic apparatus of Abreu et al. (1986, 1990). ... [D]ifferently from that literature, we do not explore the set of competitive equilibria that can be sustained by punishing governments that renege on their promises. In fact, we bypass this issue completely by assuming that policy makers have access to a commitment technology that guarantees (some of) their promises. In our model, credibility is not the attribute of a particular policy plan, the plan which policy makers optimally choose not to deviate from when behaving sequentially.

[1] See, for example, Debortoli and Nunes (2010) for fiscal policy, Debortoli and Nunes (2013) for the optimal level of debt, and, Bodenstein et al. (2012) for the optimal monetary policy under the zero lower bound.

> Rather it is a quality of the central bank as perceived by the public. ... [I]n this sense, quasi-commitment assumes a given level of policy credibility, rather than explaining its origin. ... [S]o we propose quasi-commitment as a modeling device to explore decision-making procedures intermediate between discretion and commitment, along a dimension that can be usefully interpreted as credibility.

In reference to their usage of the quasi/loose commitment framework, Bodenstein et al. (2012) acknowledged that while their approach is (computationally) more tractable for larger models, “a reputation framework would be required to capture the effects of policies on building and losing credibility.” Interestingly, they further conjectured that “a reputation model enhanced with random coordination failures among private agents may share similarities” with their loose commitment approach, “despite the obvious differences in assumptions.”

In this note, we make the quantitative connection between these two ideas. Specifically, we take up and explore the claims that the two approaches to imperfect policy credibility—quasi/loose commitment and sustainable plans—are “alternatives” (Schaumburg and Tambalotti, 2007) and “may share similarities” (Bodenstein et al., 2012), from a dynamic outcome or behavioral perspective. In particular, using the standard analytical framework for optimal monetary policy (Woodford, 2003), we compare impulse responses between quasi/loose commitment and the *optimal sustainable monetary policy* (Kurozumi, 2008; Sunakawa, 2015). As a consequence, we open up a question on which framework, if they are similar observationally, can be properly identified by an econometrician *vis-à-vis* observed data?

In the *sustainable plans* approach (Kurozumi, 2008; Sunakawa, 2015), a monetary policy plan (viz. central bank) is either sustainable (credible) or not, whereas, in the language of quasi/loose commitment, a policy plan may be credible with some probability. To connect these two ideas, we consider tighter notions of the sustainability constraint (interpreted as simple penal codes), where there is some finite duration N of the punishment phase if a policy maker were to deviate from its original plan (see also, Loisel, 2008). The standard assumption for the threat of “forever reversion to a discretion equilibrium” (Kurozumi, 2008; Sunakawa, 2015) would be the limiting case of $N \nearrow \infty$. We aim to understand how α, the degree of quasi/loose commitment, can or cannot be mapped back, and behaviorally equivalent to, the severity of the punishment threat, N.

We provide the following insights: First, even though quasi/loose commitment is based on an *ad-hoc* assumption to rationalize a policy equilibrium between commitment and discretion, our study shows that it can be a reasonably good approximation of a sustainable policy equilibrium behavior. In a standard calibration of the monetary-policy framework, when we match impulse responses of inflation and the output gap to large enough markup shocks, we have that $\alpha = 0.95$ in the quasi/loose commitment world corresponds to $N = 18$ in the N-period punishment optimal sustainable monetary policy; and both equilibria’s outcomes are very close to the standard commitment regime.

2 Model

We employ the standard framework for optimal monetary policy by Woodford (2003). The state variable is u_t, a cost-push shock realized at the beginning of date $t \in \mathbb{N}$. Let $\mathbb{E}_t \equiv \mathbb{E}\left\{\cdot | h^t\right\}$ denote the expectation operator conditional on information at date t, as summarized by some t-history h^t. Denote $\pi_t \equiv \pi_{\tilde{\sigma}}\left(h^t\right)$ and $x_t \equiv x_{\tilde{\sigma}}\left(h^t\right)$, respectively, as the inflation-rate and the output-gap outcomes induced by some policy plan $\tilde{\sigma} = \left\{h^t \mapsto \tilde{\sigma}_t\left(h^t\right) : t \in \mathbb{N}\right\}$, given observed history of relevant states $h^t := (x_{-1}, u_0, ..., x_{t-1}, u_t)$. The cost-push (markup) shock u_t is a distortionary shock which induces a time-varying wedge between the model's competitive equilibrium and efficient allocations. It is assumed to follow an AR(1) process:[2]

$$u_t = \rho u_{t-1} + e_t; \qquad e_t \sim N\left(0, \sigma_e^2\right), |\rho| < 1.$$

Given policy plan $\tilde{\sigma}$, the total expected value to the planner (and society) at the initial state is $W_t \equiv W_{\tilde{\sigma}}\left(h^t\right)$.[3] The central bank's objective function is given by

$$W_t = -\mathbb{E}_t\left\{\sum_{\tau=0}^{\infty}\beta^\tau L\left(\pi_{t+\tau}, x_{t+\tau}\right)\right\}; \qquad L\left(\pi_t, x_t\right) = \left(\pi_t^2 + \lambda x_t^2\right), \tag{1}$$

where $\lambda = \kappa/\varepsilon$. For any fixed policy plan $\tilde{\sigma}$, the competitive equilibrium is sufficiently characterized by the Phillips curve constraint:

$$\pi_t = \beta\mathbb{E}_t\pi_{t+1} + \kappa x_t + u_t; \qquad \kappa := \frac{(1-\theta)(1-\beta\theta)(\sigma+\eta)}{\theta(1+\eta\varepsilon)}, \tag{2}$$

where the composite parameter κ is a function on underlying microeconomic taste and technology parameters: β, σ, η, ε and θ, respectively, denote the subjective discount factor, the inverse of the intertemporal elasticity of substitution, the inverse of Frisch elasticity, the elasticity of substitution among differentiated products, and the Calvo (1983) parameter, where $1-\theta$ is the probability of re-optimization of prices.

Timing of events and actions. The following timeline holds for all policy regimes we consider below. This follows the setting in Kurozumi (2008). At the beginning of each date $t \in \mathbb{N}$: (T1) The shock u_t is realized. (T2) The central bank chooses a policy plan $\tilde{\sigma}'$ (or continues with a previously promised plan $\tilde{\sigma}$). (T3) Simultaneously with (T2), measure-zero and homogeneous agents form rational expectations $\mathbb{E}_t\pi_{t+s}$, for all $s \geq 0$, consistent with the central bank's plan. (T4) Current outcomes (π_t, x_t) are realized consistent with competitive equilibrium condition (2).

[2]When computing the optimal sustainable monetary policy, the AR(1) process is approximated by a finite-state Markov chain following Tauchen (1986).

[3]Woodford (2003) demonstrates how the flow criterion function L can be derived from a second-order accurate approximations of the representative household preference function and the competitive equilibrium conditions.

2.1 Standard policy regimes

Commitment equilibrium. In this regime, the policy maker chooses a once-and-for-all plan $\tilde{\sigma} = \{x_t, \pi_t\}_{t\in\mathbb{N}}$ at date $t = 0$ to maximize the objective function (1) subject to the functional equation (2). The optimal plan satisfies the condition

$$\pi_t = -\frac{1}{\varepsilon}\left(x_t - x_{t-1}\right), \tag{3}$$

which is interpretable as an optimal targeting rule (under commitment). Given a process for u_t, the Phillips curve functional equation (2), together with the targeting rule in equation (3), characterize the equilibrium under monetary policy *commitment*. Denote the value of an optimal commitment plan to the policy maker as $V^C\left(u_t, x_{t-1}\right)$, which additionally depends on an auxiliary state variable x_{t-1} since the commitment plan ties the policy maker's hands to its past promise.

Discretion equilibrium. In this policy regime, the policy maker chooses (π_t, x_t) each period to minimize the per-period loss function $L\left(\pi_t, x_t\right)$ in (1), subject to the constraint (2) and taking expectations $\mathbb{E}_t\pi_{t+1}$ as fixed. The optimal targeting rule under policy discretion can be derived as

$$\pi_t = -\frac{1}{\varepsilon}x_t. \tag{4}$$

Given a process for u_t, equation (2) together with the targeting rule in equation (4) characterize the (Markov perfect) equilibrium under monetary policy *discretion*. The total expected value to the policy maker (and society) at any state u_t can be calculated analytically as

$$V^D\left(u_t\right) = -\frac{1+\varepsilon\kappa}{\left(1-\beta\rho+\varepsilon\kappa\right)^2\left(1-\beta\rho^2\right)}\left(u_t^2 + \frac{\beta\sigma_e^2}{1-\beta}\right). \tag{5}$$

3 Imperfect credibility in monetary policy: Two stories

Now, we describe the two popular ways that have been taken to define environments with imperfect credibility in monetary-policy plans. Each has a completely different interpretation of the notion of imperfect credibility.

3.1 Quasi/Loose Commitment

In this environment, we have a reduced-form notion of limited (or "loose") commitment indexed by a parameter, $\alpha \in [0,1]$. The parameter $1-\alpha$ measures the (common-knowledge) probability that the policy maker will re-optimize over its initial policy plan (i.e., renege on its original optimal plan). As shown in Schaumburg and Tambalotti (2007) and Bodenstein et al. (2012), under quasi/loose commitment, the welfare maximization problem by the monetary authority is:

$$Q\left(u_t, x_{t-1}\right) = \max\left\{-\left(\pi_t^2 + \frac{\kappa}{\varepsilon}x_t^2\right) + \beta\mathbb{E}_t\left[\alpha Q\left(u_{t+1}, x_t\right) + (1-\alpha)\, Q^R\left(u_{t+1}, 0\right)\right]\right\}, \tag{6}$$

subject to:

$$\pi_t = \beta\mathbb{E}_t\left[(1-\alpha)\,\pi_{t+1} + \alpha\pi_{t+1}^R\right] + \kappa x_t + u_t. \tag{7}$$

The function Q delivers the planner's valuation of the optimal plan implemented at the initial state, whereas, Q^R, is supported by a new optimal plan when the planner reneges on the original plan and ignores the past.[4] The expected duration of the initial optimal plan is $\lceil\frac{1}{1-\alpha}\rceil$ periods. (The notation $\lceil\cdot\rceil$ denotes the ceiling function.)

3.2 Sustainable Plans Policy Regime

Consider now the *sustainable plans equilibrium* (see Chari and Kehoe, 1990; Kurozumi, 2008; Sunakawa, 2015). Suppose that a planner deviates from a given plan that induces total expected welfare $W\left(\cdot, u_t\right)$ at current state u_t. The worst credible equilibrium is a continuation to the (Markov-perfect) discretion equilibrium *forever*, which delivers the total expected payoff $V^D\left(u_t\right)$.[5] In this setting, the constrained-efficient planner maximizes the objective (1) subject to the Phillips curve constraint (2), and, the sustainability constraint:

$$V^S\left(u_t, x_{t-1}\right) \geq V^D\left(u_t\right), \tag{8}$$

for all date t and given u_t, where $V^S\left(u_t, x_{t-1}\right)$ is the value of the optimal sustainable plan at the given initial state. Condition (8) encodes the requirement that social welfare under the optimal sustainable monetary policy to be at least as high as that under a deviation to the discretion equilibrium, $V^D\left(u_t\right)$.

So that we can compare between these two stories of imperfect credibility (in Section 4), we will consider tighter versions of the sustainability constraint (8). That is, instead of a threat of reversion to forever discretion, consider the case that in an event of deviation from an initial plan, the continuation to a discretion equilibrium will only last for N periods, for sure, and this is public information.[6] In this setting, the RHS of the sustainability constraint (8) will be modified by a nonstationary value of discretion, $V_0^D\left(u_t\right)$, where the nonstationarity arises from the finite-duration of punishment. We describe how this problem is characterized in our online

[4]From that date onward, if the event that the planner reneges on its past plan is realized, then one just resets the function Q^R as Q.

[5]The *forever* discretion equilibrium is the worst sustainable equilibrium in this model: This was proven by Kurozumi (2008).

[6]In equivalent game-theoretic settings, one could directly interpret this as a Nash reversion (a.k.a. trigger) punishment strategy with an exogenously specified penal code where it takes N periods to exit the punishment phase. Loisel (2008), in the context of the same monetary-policy model, also interprets N as the time taken for a deviating central bank to regain its "reputation". Loisel (2008) showed that even if we endogenized N, in equilibrium agents will not behave strategically with respect to N, and the outcome is equivalent to one where we set N exogenously. In the limit where $N \nearrow +\infty$, we approach the threat of forever-discretion-equilibrium version of the sustainability constraint in (8).

appendix.

4 Comparison

We use a standard calibration of the NK model to discipline the comparison exercise. This is summarized in Table 1.

Table 1: Parameter Values

Parameters	Values	Explanation
β	.99	Subjective discount factor
σ	1	Inverse of intertemporal elasticity of substitution
η	1	Inverse of Frisch elasticity
ε	6	Elasticity of substitution among differentiated products
θ	.75	Calvo parameter
ρ	0	Shock persistence
σ_e^2	0.154^2	Variance of shock

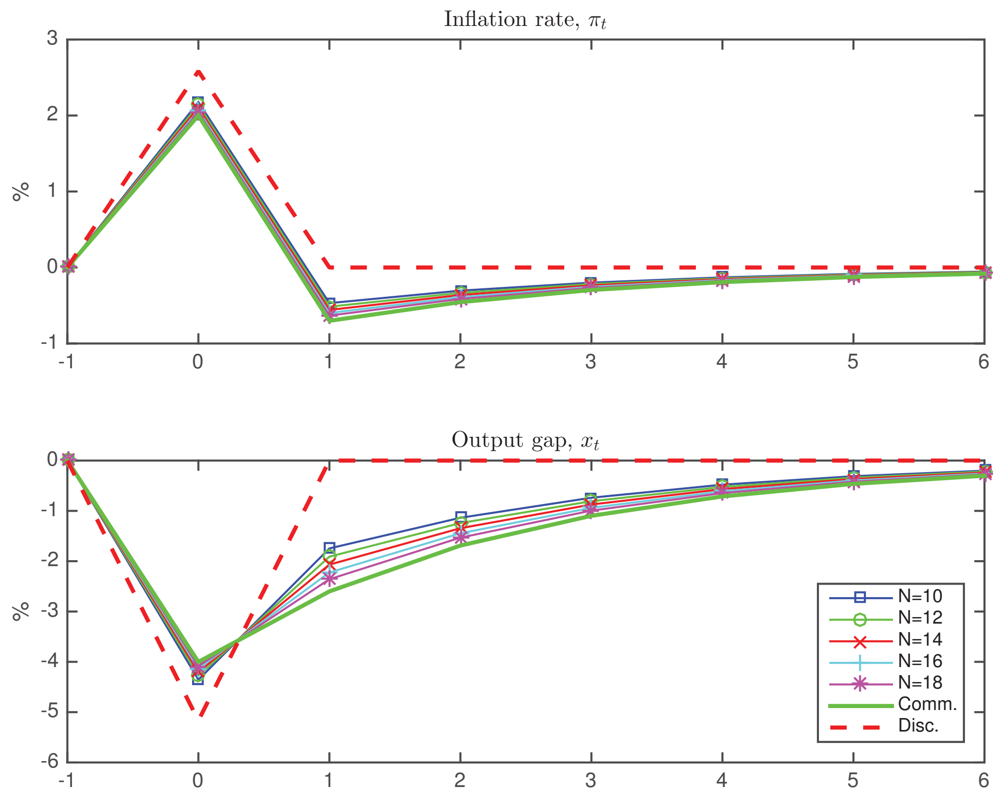

Figure 1: Sustainable plans (various N) vs. Commitment vs. Discretion equilibria.

Consider impulse responses of (annualized) inflation and the output gap to a positive markup shock (of $5 \times \sigma_e$ percent) induced by the policy plan under an assumed commitment regime versus under a discretion regime. These, regimes respectively, are given by the solid green line and the dashed red line in both panels of Figure 1. Given a positive markup shock , we observe the well-known insight that the response of both variables under the commitment regime is less aggressive and is more gradual than that under the discretion equilibrium.[7] The former reflects the endogenous history dependence in policy implementation.

Next consider the dynamics under sustainable plans equilibria for various lengths of the punishment phase, N. Numerically, there are no equilibria if N is too small. Intuitively, if N is too small, the sustainability constraint will bind almost surely, since the outside option value will be relatively higher than that under any conjectured sustainable plan. This makes the numerical solution not well defined. In the figure, we plot the cases where equilibria do exist, beginning from an economy with $N = 10$. What is interesting to observe is that for N large enough, the responses of inflation and output gap become close to those under the assumed commitment equilibrium. The intuition is as follows: For N sufficiently large, the value from deviating from a given sustainable plan becomes lower—i.e., deviating from commitment is not so attractive to the policy maker—since the threat of the punishment phase is prolonged. In other words, the sustainability constraint is almost surely slack when N is large enough.

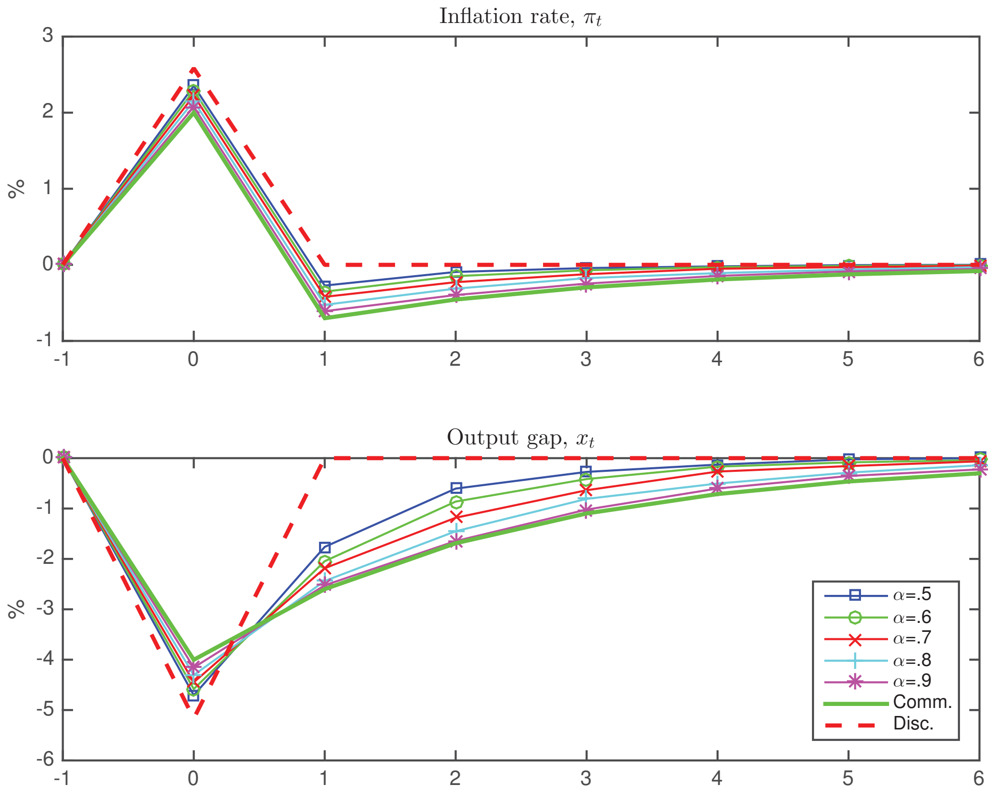

Figure 2: Loose Commitment (various α) vs. Commitment vs. Discretion equilibria.

Now consider the regime of loose commitment in Figure 2. As we increase (lower) the

[7] We consider this large markup shock solely for the purpose of comparing impulse responses in all our comparisons below. In the sustainable-plans equilibrium, for the sustainability constraint to bind given just a one-time markup shock, one needs to have an unexpected and sufficiently noisy one-off markup shock. This, however is not necessary if we focus on simulations with repeated shocks over time. Note also that the unexpected and large markup shock allows for the (non-linear) sustainable-plans equilibrium impulse responses to be distinguished from those under commitment. However, the size of the standard deviation of the markup shock does not matter for the impulse responses under the loose-commitment solution. (The latter is still a solution to a linear-quadratic dynamic program so the impulse response dynamics will be independent of the markup noise statistic.) Thus, the impulse responses in the loose-commitment equilibria will still vary with α, even if we consider a small one-time markup shock.

parameter α toward unity (zero), the loose-commitment equilibrium outcomes for inflation and output gap approach that of the standard commitment (discretion) regime's. In fact, if we have either $\alpha = 1$ or $\alpha = 0$, the loose commitment solution is exactly that of the respective commitment and discretion regime's outcomes.

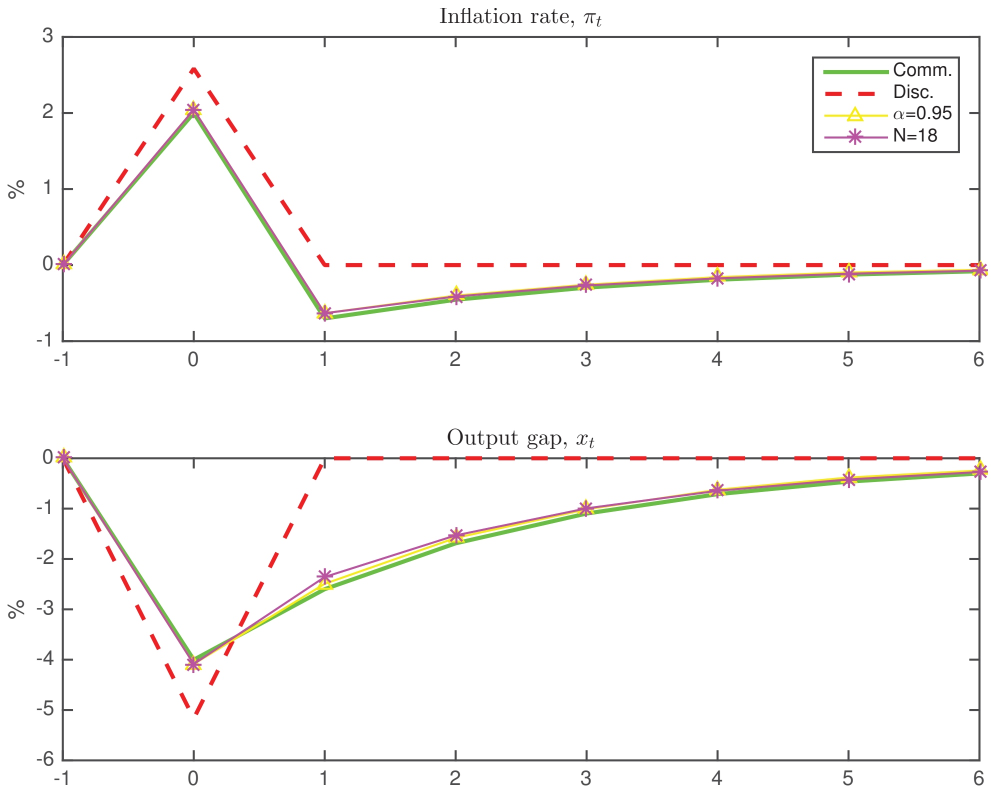

Figure 3: Loose Commitment ($\alpha = 0.95$) vs. Commitment vs. Sustainable Plan ($N = 18$).

In our computation, we find that $N = 18$ for the sustainable plans economy suffices to replicate the dynamics of inflation and output gap of the hypothetical commitment equilibrium. Moreover, $N = 18$ appears to be observationally equivalent to the loose commitment equilibrium with $\alpha \approx 0.95$. (It suffices for us to match these equilibria's outcomes in terms of their impulse-response-function statistics.[8]) This is shown in Figure 3. Also, we can consider the set of loose-commitment regimes, indexed by α, that most closely match with a corresponding N-period-punishent sustainable-plan equilibrium, in terms of their impulse response statistics. The result of this exercise is reported in Figure 4.

Given the lowest feasible setting of $N = 10$ (in terms of existence of a sustainable-plan equilibrium), we have a corresponding $\alpha \approx 0.77$ that yields a loose-commitment equilibrium dynamic which matches the $N = 10$ sustainable-plan equilibrium. Interestingly, in a Bayesian estimation of a more complicated version of this model, Debortoli and Lakdawala (2016) found

[8]The value $\alpha \approx 0.95$ was obtained by searching across different values of $\alpha \in [0, 1]$, solving each corresponding loose commitment equilibrium, and obtaining each loose-commitment equilibrium's impulse response functions (for inflation and output gap) as a function of α. Denote these impulse response functions at the peak (i.e., Period 0) as $IRF_0(\alpha)$. For a fixed N in the optimal sustainable plan economy, we also have its corresponding impulse response functions $IRF_0(N)$. Then $\alpha \approx 0.95$ minimizes the simple average of the ℓ_1-norm, $\sum_{j\in\{\pi,x\}} \left| IRF_{0,j}(\alpha) - IRF_{0,j}(N) \right|$, where $N = 18$.

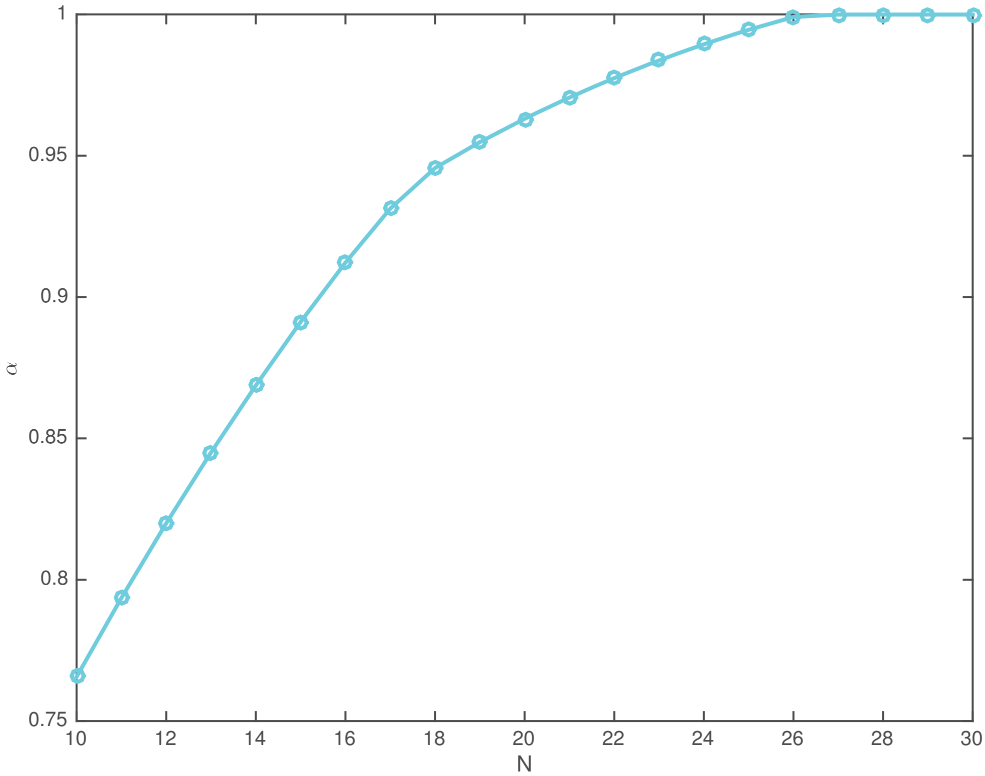

Figure 4: Loose Commitment (α) vs. Sustainable Plan (N): Matching impulse response dynamics.

that the posterior mean (mode) of their estimate of the equivalent of α is 0.82 (0.81).[9] While we cannot make a direct claim to the empirical validity of our result here, we can still argue that, informally, the set of observational equivalence between the two notions of imperfect credibility, as indexed by the (N, α) locus in Figure 4, involves quantitatively plausible values of α when shocks are large.

5 Conclusion

In a well-known monetary policy framework, we showed that when large enough shocks hit the economy two competing notions of *imperfect credibility* in monetary policy design—*loose commitment* versus *sustainable plans*—have similar dynamic outcomes in terms of inflation and output gap. These are two key variables that central bankers pay attention to in practice. Moreover, the instance in which they are similar is also very close to the outcomes induced by the framework's assumed commitment regime.

However, this leaves open a question related to an inverse problem: Is an observer or statistician, faced with data consistent with either model of sustainable plans or loose commitment policy, able to identify one or the other regime given the empirical distribution of the data? This is potentially a big problem for policy modelling that is expected to be empirically plausi-

[9]Their fifth (ninety-fifth) percentile of the posterior distribution of α is 0.77 (0.85), given a uniform prior with mean 0.50.

ble too. We do not have the answer to this question yet.

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SUPPLEMENTARY (ONLINE) APPENDIX

A Sustainable plan with N-period punishment

We first characterize the optimal sustainable plan as a modified recursive Euler operator in section A.1, assuming that the value functions V^S and V_0^D are already known. Then, we explain how the value function $u \mapsto V_0^D(u)$ is determined simultaneously with the value function $(u, x_{-1}) \mapsto V^S(u, x_{-1})$ through a successive function approximation algorithm utilizing the recursion from section A.1 combined with a backward induction step described in section A.2.

A.1 Characterizing the optimal sustainable plan

The relevant natural state variables are $s := (u, x_{-1})$. Define a record-keeping function $z := \Psi_{t-1}/(\Psi_{t-1} + \psi_t)$, where $\Psi_t = \sum_{s=0}^{t} \psi_t$ and $\psi_t \equiv \psi(u_t, x_{t-1})$ is a Lagrange multiplier or gradient function on the sustainability constraints. This sufficiently encodes history dependence in the constrained-efficient optimal sustainable plan, and in a recursive way.[10] A recursive characterization of the optimal sustainable policy plan is a list of policy functions $s \mapsto (\pi, x, z)(s)$, and, value functions $s \mapsto (V^S, V_0^D)(s)$, such that:

$$
\begin{aligned}
V^S(s) &= -[\pi(s)]^2 - \lambda[x(s)]^2 + \beta \sum_{u'} p(u'|u) V^S[u', x(s)]; \\
\pi(s) &= -\frac{\lambda}{\kappa}[x(s) - z(s)x_{-1}]; \\
\pi(s) &= \kappa x(s) + u + \beta \sum_{u'} p(u'|u)\pi[u', x(s)]; \\
V^S(s) &\geq V_0^D(u);
\end{aligned}
$$

A.2 Determining V_0^D

Below, we denote a nonstationary value function at punishment stage n as $V_n^D(\cdot)$, where $n \in \{0, 1, ..., N\}$. (Stage N is where the policy maker has just existed the punishment phase.) The timing of events is as follows:

- Suppose the policy maker deviates from the current policy plan at date t.
- Private agents observe it and punish the policymaker for $t, ..., t + N - 1$.

[10]Intuitively, if $z(s) = 1$ almost everywhere, and the sustainability constraints are never binding, then we have the traditional commitment regime.

- The policymaker is allowed to commit to a new plan in $t+N$; i.e., $x_{t+N-1} = 0$ in Period $t+N$.

Note that each function V_n^D does not depend on a current date t per se, but on the stage $n \in \{0, 1, ..., N-1\}$ *relative to any date t* where the policy deviation occured. Thus V_0^D, means the total expected value of reversion to an N-period punishment phase, at the beginning of that phase ($n = 0$). Once we have this function, we have the outside option value for the constrained-efficient planner in the description of an N-period punishment sustainable plan.

A.2.1 Backward induction

By construction, the policymaker is allowed to commit to a new plan N periods after an initial deviation from a given plan. Thus we can set $V_N^D(u) \equiv V_N^D(u, x_{-1}) = V^S(u, 0)$ and $\pi_N^D(u) \equiv \pi_N^D(u, x_{-1}) = \pi(u, 0)$. The following backward induction is used to obtain $V_0^D(u)$.

1. In period $t+N-1$, at the ultimate punishment stage $n = N-1$, solve

$$
\begin{aligned}
V = \max_{\pi, x} &\left\{ -\pi^2 - \lambda x^2 + \beta \sum_{u'} p(u'|u) V_N^D(u') \right\}, \\
\text{s.t.} \quad \pi &= \kappa x + u + \beta \sum_{u'} p(u'|u) \pi_N^D(u').
\end{aligned}
$$

   for $\pi = \pi_{N-1}^D(u)$ and $V = V_{N-1}^D(u)$, for every u, where p is the Markov matrix for the stochastic process of u.

2. Repeat this for $n = N-2, ..., 0$ to obtain $V_{N-2}^D(u), ..., V_0^D(u)$.

A.2.2 Iterative procedure

$V_0^D(u)$ depends on $V^S(u, x_{-1})$, which in turn, depends on $V_0^D(u)$. We will denote $V^{S,(i)}$ as a *candidate guess of a (sustainable-equilibrium) value function after exiting an N-period punishment phase*. Note that in a sustainable equilibrium, it would be that $\lim_{i \to \infty} V^{S,(i)} = V^S$. The idea is that, if we have found V^S, then we would also know V_0^D.

To find the fixed point in terms of the functions V^S, we iterate on the following steps:

1. Set an initial guess for the value function $V^{S,(0)}$ (after the final punishment stage) as the value under a commitment equilibrium:

$$V^{S,(0)}(u, x_{-1}) \leftarrow V^C(u, x_{-1});$$

   and, get its corresponding policy as that under a commitment equilibrium:

$$\pi^{(0)}(u, x_{-1}) \leftarrow \pi^C(u, x_{-1}),$$

   for all (u, x_{-1}). This a good, but arbitrary, initial guess.

2. For each iteration $i \geq 0$, set

$$V_N^{D,(i)}(u) \leftarrow V^{S,(i)}(u, 0)$$

and set policy

$$\pi_N^{D,(i)}(u) \leftarrow \pi^{(i)}(u, 0)$$

for all u.

3. Given function $V_N^{D,(i)}$, solve by backward induction (see section A.2.1), for $V_0^{D,(i)}$ (the candidate approximant for the equilibrium V_0^D).

4. Given function $V_0^{D,(i)}$, solve for a candidate pair of sustainable equilibrium value and policy functions using the recursions defined in section A.1. We get updated guesses: $V^{S,(i+1)}(u, x_{-1})$ and $\pi^{(i+1)}(u, x_{-1})$ for all (u, x_{-1}).

5. Repeat Steps 2-4 until the sequence of function approximants converge: $V^{S,(i)} \rightarrow V^S$ and $\pi^{(i)} \rightarrow \pi$.