Crawford School of Public Policy

CAMA

Centre for Applied Macroeconomic Analysis

Endogenous Business Cycles with Small and Large Firms

CAMA Working Paper 20/2025
April 2025

Qazi Haque
University of Adelaide
Centre for Applied Macroeconomic Analysis, ANU

Oscar Pavlov
University of Tasmania
Centre for Applied Macroeconomic Analysis, ANU

Mark Weder
Aarhus University
Centre for Applied Macroeconomic Analysis, ANU

Abstract

Recent decades have seen a rise in the market power of large firms. We propose a theory in which their technology involves the ability to produce multiple products. Large firms interact with smaller competitors and market share reallocations via product creation generate heterogeneous markup dynamics across the firm types. Higher market shares of large firms increase the parameter space for macroeconomic indeterminacy. Bayesian estimation of the general equilibrium model suggests the importance of the endogenous amplification of the product creation channel and animal spirits play a non-trivial role in driving U.S. business cycles.

Keywords

indeterminacy, business cycles, multi-product firms, animal spirits, Bayesian estimation

JEL Classification

E32

Address for correspondence:

(E) cama.admin@anu.edu.au

ISSN 2206-0332

The Centre for Applied Macroeconomic Analysis in the Crawford School of Public Policy has been established to build strong links between professional macroeconomists. It provides a forum for quality macroeconomic research and discussion of policy issues between academia, government and the private sector.

The Crawford School of Public Policy is the Australian National University's public policy school, serving and influencing Australia, Asia and the Pacific through advanced policy research, graduate and executive education, and policy impact.

Endogenous Business Cycles with Small and Large Firms*

Qazi Haque
The University of Adelaide

Oscar Pavlov†
University of Tasmania

Mark Weder
Aarhus University

March 30, 2025

Abstract

Recent decades have seen a rise in the market power of large firms. We propose a theory in which their technology involves the ability to produce multiple products. Large firms interact with smaller competitors and market share reallocations via product creation generate heterogeneous markup dynamics across the firm types. Higher market shares of large firms increase the parameter space for macroeconomic indeterminacy. Bayesian estimation of the general equilibrium model suggests the importance of the endogenous amplification of the product creation channel and animal spirits play a non-trivial role in driving U.S. business cycles.

Keywords: Indeterminacy, business cycles, multi-product firms, animal spirits, Bayesian estimation.

JEL Classification: E32.

*This paper was formerly distributed and presented under the title "Superstar Firms and Aggregate Fluctuations". The authors are indebted to the Editor, an Associate Editor and two anonymous referees for excellent comments and exchange of ideas. Also thanked are seminar and conference participants at Aarhus, Adelaide, ASSET Lisbon, Bank of Japan, Barcelona, Cardiff, EcoSta, EDMM Adelaide, Keio, Kyoto, Le Mans, MMF Manchester, Sydney, T2M Amsterdam and WAMS Kuala Lumpur. Weder acknowledges generous support from Danmarks Nationalbank and the Australian Research Council under the grant DP200101963.

†Corresponding author. Tasmanian School of Business and Economics, University of Tasmania and CAMA, Hobart TAS 7001, Australia. E-mail address: oscar.pavlov@utas.edu.au.

1 Introduction

Firms are not identical. Many markets are polarized and populated by a few relatively big firms mixed with a greater number of smaller competitors with less market power (De Loecker et al., 2020).[1] What is it that drives this heterogeneity and, in particular, what makes large firms special?[2] Bernard et al. (2022) and Hottman et al. (2016) quantify the contributions of various factors to firm heterogeneity and find that product scope and branding explain most of the variations in firm sales. In this paper, we study the implications of product scope adjustment for macroeconomic dynamics in an economy with big and small firms. In particular, we show how the constellation of firm size and market power create equilibrium indeterminacy that opens up the possibility of endogenous business cycles in which beliefs of the economic agents can become self-fulfilling.[3] Through the lens of the model, we evaluate the importance of these animal spirits for observed fluctuations in macroeconomic aggregates.

The mechanism that brings about firm heterogeneity parallels Neary (2010), who puts forward the idea that the technology of big firms

> "involves the ability to produce a large number of products. In that case, the small number of superstar firms are multi-product firms, while the remaining insiders which constitute the competitive fringe are single-product firms. This configuration is consistent with the empirical evidence [...]." [Neary, 2010, p. 15].

Indeed, Bernard et al. (2010) report that large firms produce multiple products: 40 percent of U.S. manufacturing firms are multi-product producers and account for almost 90 percent of total output. Within the firms in her dataset, Guo (2023) finds that firms with large product scopes are large in size. Broda and Weinstein (2010) document that almost all product creation and destruction occurs within firms, implying that the adjustments to the intensive margin are a more important driver of product creation than the entry and exit of firms. Cao et al. (2022) and Kehrig and Vincent (2025) provide related findings.

Motivated by these observations, this paper aspires to improve our understanding of the interactions between large and small firms and the implications for equilibrium indeterminacy and business cycles driven by animal spirits. To this end, we develop a tractable framework in which some, but not all, firms possess the ability to produce multiple goods. These firms endogenously choose their product scope owing to consumers' preferences for variety, which in turn leads to their market shares and market power being higher relative to that of ordinary mono-product firms. Our design of multi-product firms

[1] See also Autor el al. (2020) and Edmond et al. (2023).

[2] In this paper we use the terms big, large and multi-product firms interchangeably.

[3] See Farmer (2016) for the evolution of endogenous business cycles stemming from indeterminacy.

is based on the love of variety effect akin to Minniti and Turino (2013) and Pavlov and Weder (2017). However, unlike their economies, we blend large firms together with smaller mono-product competitors and the resulting interactions drive the key dynamics in our model.[4]

Our theory predicts that big firms set higher markups and prices and grab a larger market share. It is thereby consistent with De Loecker et al.'s (2020) findings that the rise of aggregate markups is primarily driven by large firms with higher market shares, while markups of small firms remained mostly unchanged. Our theory also explains the coexistence of large and small firms and how the interactions between them can drive systematic differences in the cyclicality of their market power. First and foremost, markups of large firms can be procyclical which follows from their market shares increasing with product creation at the hands of the love of variety arising from a Dixit and Stiglitz (1977) demand system. On the flip side, the markups of small firms are countercyclical as they lose market share by way of large firms' product scope changes. Such heterogeneous markup dynamics are consistent with recent findings by Burstein et al. (2025) using French administrative firm-level data. Second, a higher market share of big firms is compatible with a greater gap between the markups of large and small firms which is in line with De Loecker et al. (2020).

The interaction between large and small firms in our framework is complementary to Burstein et al. (2025) who propose an economy populated by oligopolistic producers in which exogenous firm-level shocks determine a firm's size. Firms behave strategically and markups depend on market shares. Thus, large realizations of firm-level shocks lead to higher markups. The fundamental difference in the current paper is that a subset of firms is large and set higher markups, both because of their ability to produce multiple products. Moreover, consistent with empirical evidence given by Guo (2023) and Broda and Weinstein (2010), the product scope of multi-product firms is procyclical. If these firms expand their product scopes they capture larger market shares, while mono-product firms lose out. Hence, it is the endogenous product scope adjustment, rather than exogenous firm-level shocks, that explains the heterogeneous markup dynamics across firms. Moreover, in an external validation, we find that model-estimated net product creation and net business formation are both consistent with empirical evidence, all without employing micro data to identify these variables.

The artificial economy gives rise to business cycles kindled by animal spirits. This echoes Benhabib and Farmer (1994) and Wen (1998) in which increasing returns in production can lead to local indeterminacy and sunspot equilibria. Similarly, Galí (1994), Jaimovich (2007) and Benhabib and Wang (2013) develop frameworks in which coun-

[4] Helpman and Niswonger (2022) propose a similar framework. Kurozumi and Van Zandweghe (2022) formulate a model in which large firms (superstars) differ from ordinary ones due to higher total factor productivity.

tercyclical markups lead to indeterminacy. Instead, the mechanism in our setup arises from the endogenous efficiency wedge that flows from the product creation within large firms as well as their interactions with small firms. This channel shares aspects of the efficiency channel in Benhabib and Farmer (1994). However, markups fluctuations – both at the aggregate and across firm levels – do not have to be countercyclical as they would be in the absence of firm heterogeneity. We find this functioning of indeterminacy constitutes an important contribution for the following reason. Empirical support for countercyclical markups remains contested with widespread disagreement concerning the cyclical behavior of aggregate markups (e.g. Bils et al., 2018, or Nekarda and Ramey, 2020). This disagreement about empirical markups puts doubt on the plausibility of models that are built on aggregate markups that move countercyclically, which includes the existing indeterminacy literature like the studies mentioned at the beginning of the paragraph. Our paper addresses this tension by laying out an artificial economy that is prone to indeterminacy irrespective of the aggregate markup cyclicality. Moreover, the model can additionally replicate markup facts at disaggregated level. Burstein et al. (2025), using administrative firm-level data, find markups of large (small) firms to be procyclical (countercyclical). Our model can replicate this heterogeneity while remaining agonistic about aggregate markup cyclicality.

Finally, given the model is susceptible to sunspot fluctuations, a question that arises is how important are animal spirits in driving business cycles? Naturally, this is a quantitative question and an answer requires estimating a version of the model with both fundamental shocks as well as animal spirits.[5] Our full-information Bayesian estimation points to the importance of the endogenous amplification mechanism of product creation within large firms. In terms of U.S. business cycles, shocks to technology and the marginal efficiency of investment explain the bulk of the fluctuations, similar to the findings of medium-scale models (Smets and Wouters, 2007, or Justiniano et al., 2011). While the relative contributions of supply and demand disturbances to U.S. aggregate output fluctuations are roughly the same, we find that a small but non-trivial portion of these fluctuations, in particular for aggregate investment, is driven by realized animal spirits, i.e., non-fundamental swings between euphoria and pessimism.

This paper comes in five parts. It continues by presenting the baseline model from which we have stripped off various bells and whistles that we insert into the full model when estimating it. The approach allows us to highlight the main mechanisms that drive our results. Section 3 discusses the local dynamics by presenting the parametric zones for indeterminacy. Section 4 presents the Bayesian estimation of the full model. We end the paper by listing our conclusions.

[5] Lubik and Schorfheide (2004), Dai et al. (2020), Pintus et al. (2022) and Hirose et al. (2023), among others, perform versions of Bayesian estimations of models with indeterminacy as in the present paper.

2 Model

The economy is populated by two groups of firms. One group consists of smaller mono-product firms. We will coin them ordinary or small firms. The other group of firms produces multiple products and, consequently, these firms have more market power. We will call them big or large firms. Both groups of firms produce differentiated goods and adjust their markups according to fluctuations in their market shares. The firms' goods are bought by a perfectly competitive sector that welds the varieties together into the final good that is used for household consumption or added to the capital stock. People rent out labor and capital services. Firms and households are price takers on factor markets. Time evolves in discrete steps and we suppress the time index in the following static equations for notational ease.

2.1 Final goods

Final output Y is a combination of products produced by N ordinary firms and M multi-product firms. M and N are constant for now so that we can pinpoint the role of time-varying product scopes as opposed to firm dynamics of entry and exit that we will introduce later. Similar to Shimomura and Thisse (2012), final output is

$$Y = \left(\sum_{i=1}^{N} x(i)^{\frac{\sigma-1}{\sigma}} + \sum_{j=1}^{M} Y(j)^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}} \tag{1}$$

in which $\sigma > 1$ stands for the constant elasticity of substitution and $x(i)$ is the amount produced by mono-product firm i. Since large firms are multi-product firms, $Y(j)$ is a composite good as in Feenstra and Ma (2009) and Minniti and Turino (2013)

$$Y(j) = \left(\int_0^{S(j)} x(j,s)^{\frac{\sigma-1}{\sigma}} ds \right)^{\frac{\sigma}{\sigma-1}} \tag{2}$$

in which $S(j)$ stands for the product scope and $x(j,s)$ denotes the amount of variety s produced by firm j.[6] The symmetry of the elasticity of substitution σ across these CES bundlers allows to concentrate on the key effects that arise from the market structure. The CES aggregators imply a *love of variety* $\nu = 1/(\sigma - 1)$. We relax the tight connection between ν and σ, and examine the implications of non-symmetric elasticities in Section 3. The variety effect in (2) provides the benefit of product creation for the large firm. If it were zero, they would not have any incentive to become multi-product firms. The profit maximization problem yields two demand functions for the firms' goods

$$x(i) = \left(\frac{p(i)}{P} \right)^{-\sigma} Y$$

[6] The formulation echoes Brander and Eaton's (1984) market segmentation in which the inner nest corresponds to a bundle of a single firm's varieties.

$$x(j,s) = \left(\frac{p(j,s)}{P}\right)^{-\sigma} Y$$

and the aggregate price index

$$P = \left(\sum_{i=1}^{N} p(i)^{1-\sigma} + \sum_{j=1}^{M} P(j)^{1-\sigma}\right)^{\frac{1}{1-\sigma}}$$

with

$$P(j) = \left(\int_0^{S(j)} p(j,s)^{1-\sigma} ds\right)^{\frac{1}{1-\sigma}}.$$

2.2 Intermediate good firms

Varieties supplied by large firms are produced using labor $h(j,s)$ and capital services $\kappa(j,s) \equiv Uk(j,s)$. The variable U stands for the utilization rate set by the owners of physical capital and it is the same for every unit of capital k rented. Firms hire the two services on perfectly competitive factor markets at the wage rate w and the rental rate of capital services r. Multi-product firm j maximizes profits

$$\pi(j) = \int_0^{S(j)} \left[p(j,s)x(j,s) - wh(j,s) - r\kappa(j,s)\right] ds$$

subject to the production technology

$$\int_0^{S(j)} x(j,s)ds = \int_0^{S(j)} [\kappa(j,s)^{\alpha} h(j,s)^{1-\alpha} - \phi_s]ds - \phi_m, \qquad 0 < \alpha < 1.$$

The variety-level fixed cost ϕ_s restricts the amount of varieties the firm produces. While we do not model the sorting into the two firm groups endogenously, these costs provide a reason why some firms remain of the mono-product kind: they face prohibitively high fixed costs to produce multiple varieties. The firm-level fixed costs ϕ_m provide economies of scope and help pinning down steady state profits. Ordinary firm i only produces a single variety and its production technology is

$$x(i) = \kappa(i)^{\alpha} h(i)^{1-\alpha} - \phi_n$$

in which the fixed cost ϕ_n is calibrated so that it has zero profits at the steady state. Given that both groupings of firms hire on the same factor markets, the first-order conditions are

$$w = (1-\alpha)\Lambda\kappa(j,s)^{\alpha} h(j,s)^{-\alpha} = (1-\alpha)\Lambda\kappa(i)^{\alpha} h(i)^{-\alpha} \tag{3}$$

$$r = \alpha\Lambda\kappa(j,s)^{\alpha-1} h(j,s)^{1-\alpha} = \alpha\Lambda\kappa(i)^{\alpha-1} h(i)^{1-\alpha} \tag{4}$$

in which

$$\Lambda \equiv \alpha^{-\alpha}(1-\alpha)^{\alpha-1} r^{\alpha} w^{1-\alpha} \tag{5}$$

are the marginal costs that are the same for both firm types.

The number of firms in the economy is endogenously determined and thereupon we cannot simply presume that the Dixit and Stiglitz (1977) approximation of constant markups is satisfied. We adopt Yang and Heijdra's (1993) suggestion that all firms understand the effect of their pricing on the aggregate price index P. The formulation then renders markups time-varying.[7] As each variety is produced with the same technology, firm j charges the same price for all of its varieties, i.e., $p(j,s) = p(j,k) = p(j)$. Then, markups are

$$\mu(j) \equiv \frac{p(j)}{\Lambda} = \frac{\sigma\left(1-\left(\frac{P(j)}{P}\right)^{1-\sigma}\right)}{\sigma\left(1-\left(\frac{P(j)}{P}\right)^{1-\sigma}\right)-1}$$

and

$$\mu(i) \equiv \frac{p(i)}{\Lambda} = \frac{\sigma\left(1-\left(\frac{p(i)}{P}\right)^{1-\sigma}\right)}{\sigma\left(1-\left(\frac{p(i)}{P}\right)^{1-\sigma}\right)-1}$$

in which

$$\left(\frac{P(j)}{P}\right)^{1-\sigma} = \frac{P(j)Y(j)}{PY} \equiv \epsilon(j) \quad \text{and} \quad \left(\frac{p(i)}{P}\right)^{1-\sigma} = \frac{p(i)x(i)}{PY} \equiv \epsilon(i).$$

The markups are hence positively related to the firms' market shares ϵ. Gamber (2023) finds such a positive relationship between markups and market shares for large U.S. firms. Ordinary firms, being by comparison small and more numerous, display markup elasticities with respect to their market shares that are relatively lower than the large firms' and thus also closer to the Dixit and Stiglitz case. For a multi-product firm the market share is increasing in the number of varieties $S(j)$ which is endogenously determined via maximizing profits

$$\pi(j) = \left(\frac{p(j)-\Lambda}{p(j)}\right) PY\epsilon(j) - \Lambda[S(j)\phi_s + \phi_m]$$

with respect to $S(j)$. Each big firm takes into account the effect of its product scope on its own prices, prices of all other firms, and the aggregate price index. The first-order condition, $\partial\pi(j)/\partial S(j) = 0$, implies

$$\Lambda\phi_s = \sigma PY \left(\frac{p(j)-\Lambda}{p(j)}\right)^2 \frac{\partial\epsilon(j)}{\partial S(j)} + Y\epsilon(j)\left(\frac{p(j)-\Lambda}{p(j)}\right)\frac{\partial P}{\partial S(j)} \tag{6}$$

where $\partial\epsilon(j)/\partial S(j) > 0$ and $\partial P/\partial S(j) < 0$ (see the Appendix for details). The term on the left-hand side in (6) represents the direct cost of expanding the product scope. The

[7] In Jaimovich (2007) and Burstein et al. (2025) firms are also aware that their actions affect aggregates. In their economies, this concerns sectoral price indices. Alternatively, we could have assumed Kimball (1995)-type aggregation for similar markup patterns to ours.

first term on the right-hand side represents the gain to market share from the love of variety in the CES bundler (2).[8] The second term indicates that profits drop in response to the higher product scope that reduces the aggregate price index. Put differently, the variety effect is necessary for multi-product firms to exist as, otherwise, all firms would be of the mono-product type.

2.3 Symmetric equilibrium

In the symmetric equilibrium each large firm produces the same number of varieties $S(j) = S$, charges the same price $p(j) = p_m$, and has the same market share $\epsilon(j) = \epsilon_m$. Similarly, for the ordinary firms $p(i) = p_n$ and $\epsilon(i) = \epsilon_n$ hold. The markups arrange to

$$\mu_m = \frac{\sigma(1-\epsilon_m)}{\sigma(1-\epsilon_m)-1} > \mu_n = \frac{\sigma(1-\epsilon_n)}{\sigma(1-\epsilon_n)-1} > \frac{\sigma}{\sigma-1} \tag{7}$$

and

$$\epsilon_m = Sp_m^{1-\sigma} > \epsilon_n = p_n^{1-\sigma}$$

with the final good set as the numeraire $P = 1$. Multi-product firms have larger market shares and markups than ordinary firms owing to the variety effect and the resulting product-scope nature. Since both ordinary and large firms hire labor and capital services from the same factor markets and both have constant returns to scale production functions (abstracting from fixed costs), (3) and (4) imply

$$\frac{w}{r} = \frac{1-\alpha}{\alpha}\frac{UK_m}{H_m} = \frac{1-\alpha}{\alpha}\frac{UK_n}{H_n}$$

in which $K_m = MSk_m$, $K_n = Nk_n$, $H_m = MSh_m$ and $H_n = Nh_n$. Therefore, all firms choose identical capital-labor intensities and factor markets are in equilibrium, that is, $K = K_m + K_n$ and $H = H_m + H_n$. From (5) and (7), we can see that multi-product firms charge a higher price than their ordinary counterparts:

$$p_m = \mu_m \alpha^{-\alpha}(1-\alpha)^{\alpha-1} r^{\alpha} w^{1-\alpha} > p_n.$$

Large firms set higher prices because their products are valued highly by their customers due to the love of variety. This large firms' pricing finds support in Hottman et al. (2016) and Mongey and Waugh (2025). Relatedly, Kehrig and Vincent (2021) report that low labor share establishments, which correspond to large firms in our model due to their higher markups, also charge higher prices.[9] Summing production and demand functions

[8] While a higher product scope increases the demand for the firm's output, the introduction of new varieties cannibalizes the sales of existing ones. Hottman et al. (2016) provide quantitative evidence of significant cannibalization effects for multi-product firms.

[9] Foster et al. (2008) report that firms' revenue productivity is less dispersed than physical productivity, and this could imply multi-product firms setting lower prices. This productivity effect is absent in our model's technologies.

of ordinary firms

$$\sum_{i=0}^{N} x(i) = \sum_{i=0}^{N} \left(\frac{p(i)}{P}\right)^{-\sigma} Y = \sum_{i=0}^{N} \left(\kappa(i)^{\alpha} h(i)^{1-\alpha} - \phi_n\right)$$

and then applying symmetry yields

$$Y = \frac{p_n}{N\epsilon_n}\left(U^{\alpha} K_n^{\alpha} H_n^{1-\alpha} - N\phi_n\right).$$

Similarly, multi-product firms' output is

$$\sum_{j=1}^{M} \int_0^{S(j)} x(j,s)ds = \sum_{j=1}^{M} \int_0^{S(j)} \left(\frac{p(j,s)}{P}\right)^{-\sigma} Y ds = \sum_{j=1}^{M} \left(\int_0^{S(j)} \left[\kappa(j,s)^{\alpha} h(j,s)^{1-\alpha} - \phi_s\right] ds - \phi_m\right)$$

and

$$Y = \frac{p_m}{M\epsilon_m}(U^{\alpha} K_m^{\alpha} H_m^{1-\alpha} - MS\phi_s - M\phi_m).$$

Lastly, the first-order condition (6) can be rearranged to define the product scope

$$S = f(\mu_m, \mu_n, N, M, \sigma)\frac{Y}{\phi_s p_m}.$$

It is strongly procyclical and the derivation of the function f can be found in the Appendix.

2.4 Households

Households are characterized by a representative agent who chooses sequences of consumption C_t and hours worked H_t to maximize discounted lifetime utility

$$\sum_{t=0}^{\infty} \beta^t \left(\ln C_t - \upsilon \frac{H_t^{1+\chi}}{1+\chi}\right) \qquad 0 < \beta < 1, \upsilon > 0, \chi \geq 0$$

in which β is the discount rate and υ denotes the disutility of working. The agent owns all firms and receives their profits Π_t. The period-budget is constrained by

$$w_t H_t + r_t U_t K_t + \Pi_t \geq I_t + C_t$$

in which I_t is investment that adds to the capital stock

$$K_{t+1} = (1-\delta_t)K_t + I_t$$

and the depreciation rate varies according to

$$\delta_t = \frac{1}{\theta} U_t^{\theta} \qquad \theta > 1.$$

The first-order conditions from the agent's maximization problem comprise of the labor supply

$$\upsilon H_t^{\chi} C_t = w_t$$

the Euler equation

$$\frac{1}{C_t} = \frac{1}{C_{t+1}}\beta\left(r_{t+1}U_{t+1} + 1 - \delta_t\right)$$

and the optimal rate of capital utilization

$$r_t = U_t^{\theta-1}.$$

The steady state versions of these equations then pin down $\theta = \left(1/\beta - 1 + \delta\right)/\delta$.

3 Dynamics and steady state

Let us now discuss the existence of the mixed market structure with two types of firms as well as local dynamic properties of the model. The equilibrium conditions are log-linearized around the steady state and the dynamical system is arranged to

$$\begin{bmatrix} \widehat{K}_{t+1} \\ \widehat{C}_{t+1} \end{bmatrix} = \mathbf{J} \begin{bmatrix} \widehat{K}_t \\ \widehat{C}_t \end{bmatrix}.$$

Hatted variables denote percentage deviations from their steady state values and $\mathbf{J}$ is the 2×2 Jacobian matrix of partial derivatives. Consumption C_t is a non-predetermined variable and capital K_t is predetermined. Indeterminacy, and the potential presence of animal spirits, requires both roots of $\mathbf{J}$ to be inside the unit circle.

From (7), for given calibrations of markups and σ, the market shares in the steady state are

$$\epsilon_m = 1 - \frac{\mu_m}{\mu_m - 1}\frac{1}{\sigma} > \epsilon_n = 1 - \frac{\mu_n}{\mu_n - 1}\frac{1}{\sigma}.$$

Since these shares sum to unity, $M\epsilon_m + N\epsilon_n = 1$, we can then calibrate the market share of multi-product firms $M\epsilon_m$ to pin down the number of firms in the steady state. It is then straightforward to show that for each calibration of μ_n, the lower bound on σ is $\sigma^{\min} \equiv \mu_n/(\mu_n - 1)$. As σ approaches the lower bound, the number of ordinary firms approaches infinity and their markups become constant at $\sigma/(\sigma-1)$ as in the monopolistic competition framework. In other words, the Yang and Heijdra (1993) aggregate price index effect disappears for the ordinary firms and their markups are fixed as in Dixit and Stiglitz (1977). This case is also where the love of variety $\nu = 1/(\sigma-1)$ hits its maximum. For the upper bound, $\sigma^{\max}$ cannot be greater than either

$$\frac{\mu_n}{\mu_n - 1}\left(1 + \frac{N\epsilon_n}{M\epsilon_m}\right)$$

or

$$\frac{\mu_m}{\mu_m - 1}\left(1 + \frac{M\epsilon_m}{N\epsilon_n}\right)$$

to guarantee $M \geq 1$ and $N \geq 1$.

Figure 1 visualizes the feasible parameter space for the existence of both kind of firms. Standard parameters are calibrated at a quarterly frequency to $\alpha = 0.3$, $\beta = 0.99$, and

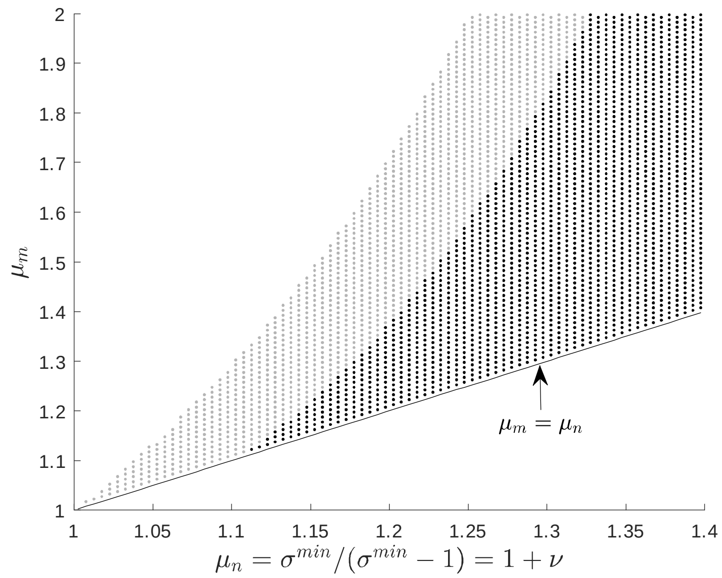

Figure 1: Indeterminacy (dark shaded zone) and determinacy (light shaded zone) without entry and exit.

$\delta = 0.025$. We also set $\chi = 0$ for labor being indivisible as in Hansen (1985) and Rogerson (1988).[10] We initially set the market share of multi-product firms at 60 percent.

The feasible markup combinations in Figure 1 remain unaltered for different calibrations of σ. If $\sigma = \sigma^{\min}$, then the ordinary firms are monopolistic competitors, i.e. $N \to \infty$ and they have constant markups at $\mu_n = \sigma^{\min}/\left(\sigma^{\min} - 1\right) = 1 + \nu$. If $\sigma^{\min} < \sigma \leq \sigma^{\max}$, however, ordinary firms' markups would no longer be constant but will become countercyclical. This markup motion is linked to the product creation of large firms that negatively affects ordinary firms' market shares. Therefore, in contrast to Minniti and Turino (2013) and others, markups can be variable even without entry and exit. Along the graph's lower boundary, the 45 degree line where $\mu_m = \mu_n$, the markups of both sets of firms would be identical. It is also the configuration along which $M \to \infty$ and large firms would produce only a single product and effectively are monopolistic competitors. Off the 45 degree line, however, they produce multiple products, their markups are procyclical and always higher than ordinary firms' markups. The reason is that product creation, both dynamically and in steady state, steals market share from ordinary firms, which raises big firms' markups. Finally, at the upper boundary, the number of big firms approaches one and the product scope becomes large.

[10] We assume indivisible labor for easier comparison to previous studies in the indeterminacy literature, for example Benhabib and Farmer (1994), Farmer and Guo (1994), Wen (1998), Jaimovich (2007), Benhabib and Wang (2013), and Pintus et al. (2022).

3.1 Indeterminacy mechanism and firm dynamics

Figure 1 also splits the feasible area into indeterminacy and determinacy zones. The darker zone denotes indeterminacy whereas in the lighter shaded area and on the 45 degree line the economy's equilibrium is unique. A necessary condition for indeterminacy is the presence of a certain level of market power, precisely $\mu_m > \mu_n = 1.104$.[11] In other words, without multi-product firms, the economy would always be determinate. Indeterminacy arises from product creation and the associated variety effect since there is no entry or exit of firms. The indeterminacy result is best understood by means of the usual equilibrium wage-hours locus (Farmer and Guo, 1994). Product creation within large firms makes this locus upwardly sloping by the presence of love of variety in the CES aggregator (2). The composite good from each large firm can be created more efficiently the greater the product scope and variations in product scope generate an endogenous efficiency wedge. Then, if economic sentiments shift into optimistic gear, the labor supply curve shifts up along the upwardly sloping wage-hours locus, thereby validating the animal spirits. Product scope adjustments together with firm heterogeneity thus provide a novel mechanism for indeterminacy and markup dynamics by way of market share reallocations even without entry and exit of firms. As large firms' markups are procyclical, for a given μ_n, raising (steady state) μ_m increases the markup elasticity that can push the economy into its determinacy region in Figure 1. This happens because the contractionary effect of the procyclical markup overcomes the efficiency gain from product creation. However, the outcome disappears once we consider multi-product firms' dynamics in interaction with the entry and exit of ordinary firms.

Next, we allow the number of ordinary firms N_t to vary over time and adjust per free entry that forces their profits to zero. That is, each period firm i's profit is

$$\pi_t(i) = \left(\frac{p_t(i) - \Lambda_t}{p_t(i)}\right) P_t Y_t \epsilon_t(i) - \Lambda_t \phi_n = 0$$

which in symmetric equilibrium boils down to

$$p_{n,t} = (\mu_{n,t} - 1)\frac{\epsilon_{n,t} Y_t}{\phi_n}$$

to determine the number of ordinary firms.[12] Assuming that entry and exit of big firms is relatively insignificant at business cycle frequencies, we continue with a constant M. Figure 2 displays how entry and exit affects the indeterminacy region. The necessary and sufficient condition for indeterminacy is $\mu_m \geq \mu_n = 1.104$. Indeterminacy not only remains but now exists for a greater range of parameters due to the interaction between entry of ordinary firms and the product scope decisions of big firms. Entry pushes the

[11] This corresponds to the increasing returns necessary for indeterminacy in Wen (1998).

[12] The entry decision is static to keep the model tractable. Indeterminacy remains when we introduce dynamic entry as in Bilbiie et al. (2012). You can find this model version in the Appendix.

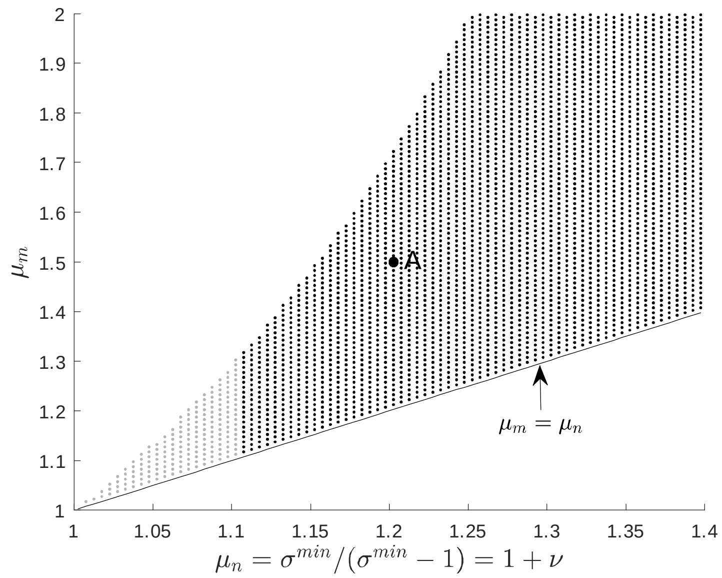

Figure 2: Indeterminacy (dark shaded zone) and determinacy (light shaded zone) with entry and exit.

market shares of both firm types downwards. However, large firms are able to defend their market shares by increasing their product scopes. Since higher product scopes and a larger number of ordinary firms both increase efficiency at the hand of the variety effect, the upwardly sloping wage-hours locus becomes steeper.

What is the effect of an increase in the market share of multi-product firms? In Figure 3, $M\epsilon_m$ is now increased to 75 percent from the previous calibration of 60 percent. The indeterminacy zone increases further. If we compare points A and B in Figures 2 and 3, thus keeping the markups constant, the higher market share of multi-product firms supports a higher M while each individual firm has the same product scope. The higher market share also allows for a greater gap between the markups of big and small firms, which is consistent with the rise and diverging markups reported in De Loecker et al. (2020).

As emphasized earlier, the love of variety governs the gain to product creation and is the central amplification mechanism for equilibrium indeterminacy. Similar to Benassy (1996), we now separate the variety effect ν from the elasticity of substitution σ. Isolating the variety effect allows us to directly set the firms' benefit of product creation without changing the steady state number of firms or their market power. Specifically, the CES bundlers are now

$$Y_t = \left(N_t^{\frac{\nu(\sigma-1)-1}{\sigma}} \sum_{i=1}^{N_t} x_t(i)^{\frac{\sigma-1}{\sigma}} + M^{\frac{\nu(\sigma-1)-1}{\sigma}} \sum_{j=1}^{M} Y_t(j)^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}} \tag{8}$$

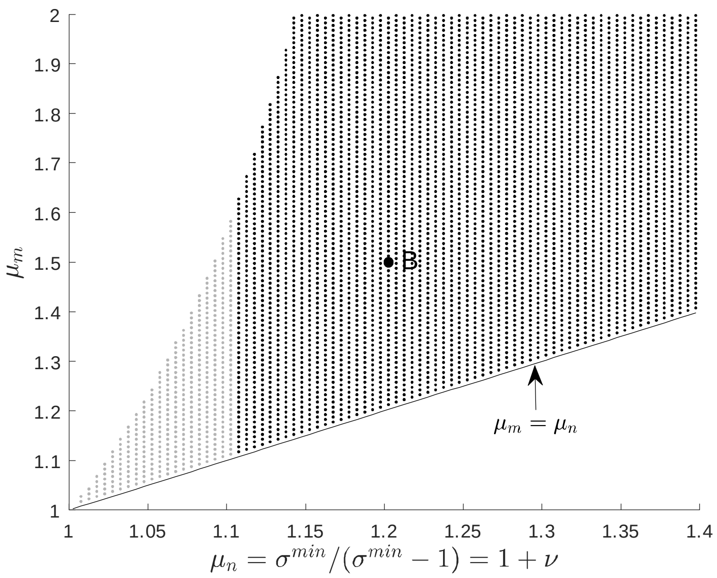

Figure 3: Indeterminacy (dark shaded zone) and determinacy (light shaded zone) with higher market shares of multi-product firms.

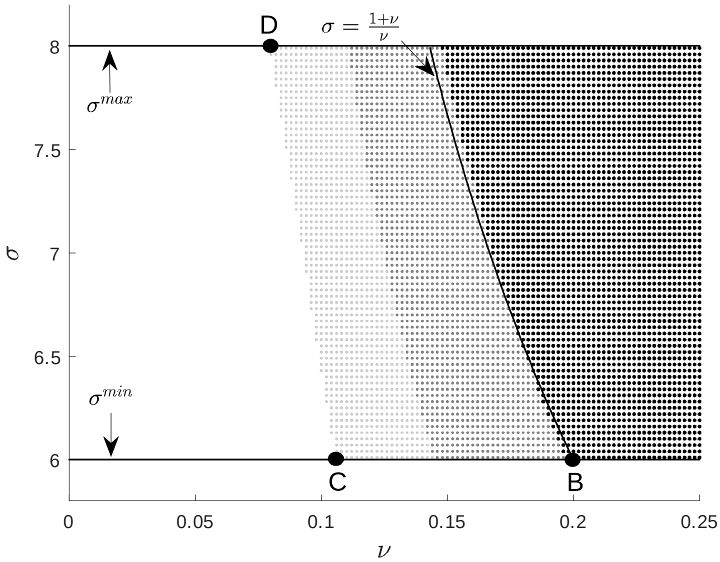

Figure 4: Indeterminacy (shaded zones) and determinacy (unshaded zone) with separated variety effect ν. From left to right: determinacy with all markups countercyclical, indeterminacy with all markups countercyclical, indeterminacy with only large firms' markups procyclical, indeterminacy with both large firms' and the average markup procyclical.

and

$$Y_t(j) = \left(S_t(j)^{\frac{\nu(\sigma-1)-1}{\sigma}} \int_0^{S_t(j)} x_t(j,s)^{\frac{\sigma-1}{\sigma}} ds \right)^{\frac{\sigma}{\sigma-1}} \tag{9}$$

in which $\nu > 0$ denotes the love of variety. Setting $\nu = 1/(\sigma - 1)$ brings back the CES aggregators from Section 2. Figure 4 plots parameter zones by varying σ and ν for given steady state markups, $\mu_m = 1.5$ and $\mu_n = 1.2$. Beginning from the left, low levels of ν imply determinacy and in this unshaded area all markups are countercyclical. The shaded zone involves indeterminacy and it is partitioned into three areas in which, beginning from the left, all firms' markups are countercyclical, large firms' markups are procyclical and lastly, both large firms' and the economy's average markup are procyclical. Here, we compute the average markup μ_t as an employment weighted average as suggested by Edmond et al. (2023)

$$\mu_t = \sum_{i=0}^{N_t} \mu_t(i) \frac{h_t(i)}{H_t} + \sum_{j=0}^{M} \mu_t(j) \frac{\int_0^{S_t(j)} h_t(j,s) ds}{H_t}. \tag{10}$$

Importantly, the cyclicality of markups discussed here is invariant to the type of economic disturbance hitting the economy. As you can see in Figure 4, indeterminacy requires a certain amount of love for variety as explained earlier. For orientation, at $\nu = 1/(\sigma^{\min} - 1)$ the model is in the same point B as in Figure 3. Beginning from B is a line connecting the combinations at which $\nu = 1/(\sigma - 1)$, i.e., the formulation in (1) and (2), which was used to construct Figures 1-3. Along the graph's lower boundary $\sigma^{\min}$, ordinary firms are monopolistic competitors with constant markups. Still, for $\nu \geq 0.104$, that is to the right of C, the model displays indeterminacy through variety effects only and consequently, the Yang and Heijdra (1993) markup formulation is not necessary for the result. For $\sigma^{\min} < \sigma \leq \sigma^{\max}$ ordinary firms' markups are no longer constant but countercyclical. This explains why the boundary between determinacy and indeterminacy is not vertical, i.e., point D is to the left of point C: if ordinary firms' markups are sufficiently countercyclical, indeterminacy can arise at $\nu = 0.08$ (at $\sigma^{\max}$) instead of $\nu = 0.104$ (at $\sigma^{\min}$).

Lastly, through the lens of our model, the finding reported by Burstein et al. (2025), namely that large firms' markups are procyclical, can not arise in the determinacy region of the model. However, markups of large firms can become countercyclical for a smaller love of variety (lightest shaded zone in Figure 4). This is because a lower variety effect implies a weaker gain to product creation and an ability to steal market share from ordinary firms. The entry of ordinary firms then reduces market shares of both firm types.

Next, we relax the symmetry of elasticities of substitutions across the CES bundlers that now become

$$Y_t = \left(N_t^{\frac{\nu(\sigma-1)-1}{\sigma}} \sum_{i=1}^{N_t} x_t(i)^{\frac{\sigma-1}{\sigma}} + M^{\frac{\nu(\sigma-1)-1}{\sigma}} \sum_{j=1}^{M} Y_t(j)^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}}$$

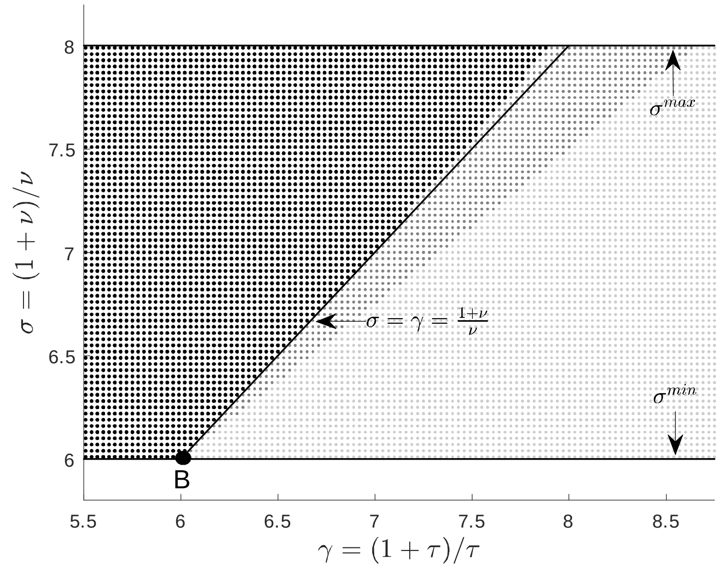

Figure 5: Indeterminacy under different elasticities of substitution. From left to right: both large firms' and average markups are procyclical (dark shaded zone), only large firms' markups are procyclical, all markups are countercyclical (lightest shaded zone).

and

$$Y_t(j) = \left(S_t(j)^{\frac{\tau(\gamma-1)-1}{\gamma}} \int_0^{S(j)} x(j,s)^{\frac{\gamma-1}{\gamma}} ds \right)^{\frac{\gamma}{\gamma-1}}.$$

How does a differing degree of substitutability influence the dynamics of market power, pricing and aggregate dynamics? Figure 5 maps dynamic regions in the $\gamma-\sigma$-space, where $\nu = 1/(\sigma-1)$ and $\tau = 1/(\gamma-1)$.[13] For orientation, we added point B and, furthermore, the 45 degree line exactly corresponds to Figure 4's $\sigma = (1+\nu)/\nu$ condition.[14] If $\sigma < \gamma$, the elasticity of substitution between varieties within multi-product firms is higher than the elasticity of substitution between different firms. This implies that consumers are generally more willing to substitute between products offered by the same firm than between those offered by different firms. Thus, as one moves to the right of the 45 degree line, $\tau < \nu$ and the gain from product creation within multi-producers relative to that coming from entry of ordinary firms falls. Eventually, the product scope expansions are insufficient to raise large firms' market shares and their markups become countercyclical.

[13] In our model's symmetric equilibrium, the only role of γ is to determine the love of variety τ, while σ still affects markup dynamics independently of ν.

[14] In Figure 5, the entire area is indeterminate due to the high value of the variety effect implied by $\sigma^{\min}$ and $\sigma^{\max}$.

4 Estimation

So far, we have shown that large firms' endogenous product scope decisions and their interaction with small firms can lead to macroeconomic instability. This situation opens the possibility of animal spirits driving business cycles and we examine their importance in combination with various fundamental shocks next. In doing so, we extend our model by exogenous growth, fundamental aggregate supply and demand shocks, as well as external consumption habits. We continue with a separable love of variety and endogenous entry and exit of ordinary firms as described in the previous section. Lastly, we return to the model with CES bundlers (8) and (9), in which the variety effects are symmetric, as the data cannot simultaneously identify both variety effects.

4.1 Bells and whistles

We add a mix of aggregate supply and demand disturbances to the model. The first such fundamental shock takes the form of labor augmenting technological progress A_t and it affects all firms equally. Aggregate output is now

$$Y_t = \frac{p_{m,t}}{M\epsilon_{m,t}}[(U_tK_{m,t})^\alpha(A_tH_{m,t})^{1-\alpha} - \phi_{s,t}MS_t - \phi_{m,t}M] = \frac{p_{n,t}}{N_t\epsilon_{n,t}}[(U_tK_{n,t})^\alpha(A_tH_{n,t})^{1-\alpha} - N_t\phi_{n,t}]$$

in which all three fixed costs grow at the average rate of technological progress. Technological progress is non-stationary and follows the process

$$\ln A_t = \ln A_{t-1} + \ln a_t$$

with

$$\ln a_t = (1-\psi_A)\ln a + \psi_A \ln a_{t-1} + \varepsilon_t^A$$

in which $0 \leq \psi_A < 1$ governs the persistence of the shock, $\ln a$ is the average growth rate and ε_t^A is an i.i.d. disturbance with variance σ_A^2. Next, shifts of marginal efficiency of investment z_t affect the transformation of investment to physical capital as in Greenwood et al. (1988)

$$K_{t+1} = (1-\delta_t)K_t + z_tI_t.$$

The technological shifter follows the exogenous process

$$\ln z_t = (1-\psi_z)\ln z + \psi_z \ln z_{t-1} + \varepsilon_t^z.$$

As laid out by Justiniano et al. (2011), the shock can be a proxy for capturing disturbances in financial markets. Intuitively, a positive shock to z_t represents a boom in financial markets that reduces borrowing costs for firms, leading to a rise in investment.

The first fundamental demand disturbance is a taste shock Δ_t that increases the marginal utility of consumption as in Christiano (1988). Lifetime utility then becomes

$$E_0 \sum_{t=0}^{\infty} \beta^t \left(\Delta_t \ln(C_t - bC_{t-1}) - \upsilon \frac{H_t^{1+\chi}}{1+\chi}\right)$$

in which E_0 denotes the expectations operator and the parameter $0 \leq b < 1$ determines the degree of external consumption habits. The taste shock follows the process

$$\ln \Delta_t = (1 - \psi_\Delta) \ln \Delta + \psi_\Delta \ln \Delta_{t-1} + \varepsilon_t^\Delta.$$

Besides the interpretation of purely changing tastes, the shock could also be interpreted as affecting the economy's labor wedge, i.e., the gap between the marginal rate of substitution between consumption and leisure and the marginal product of labor. Hence, it can be interpreted as a stand-in for other shocks that affect this wedge. The second demand shock is to government expenditures G_t financed by lump sum taxes. Consequently, the economy's resource constraint becomes $Y_t = C_t + I_t + G_t$. Government spending follows a stochastic trend

$$A_t^g = (A_{t-1}^g)^{\psi_{ag}} (A_{t-1})^{1-\psi_{ag}}$$

in which ψ_{ag} governs the smoothness of the trend relative to the trend in output. Then, detrended government spending is $g_t \equiv G_t / A_t^g$ and follows

$$\ln g_t = (1 - \psi_g) \ln g + \psi_g \ln g_{t-1} + \varepsilon_t^g.$$

The non-fundamental animal spirits shock is modelled as an expectation error to output that is unrelated to any fundamental changes in the economy.[15] Under indeterminacy, the economy's response to fundamentals is not uniquely determined, and we model the behavior of output as

$$\widehat{Y}_t = E_{t-1}\widehat{Y}_t + \Omega_A \varepsilon_t^A + \Omega_z \varepsilon_t^z + \Omega_\Delta \varepsilon_t^\Delta + \Omega_g \varepsilon_t^g + \varepsilon_t^s$$

in which the parameters Ω_A, Ω_z, Ω_Δ and Ω_g determine the effects of technology, investment, taste and government shocks on output. The term ε_t^s is i.i.d., independent of fundamentals, comes with variance σ_s^2 and it can be thought of as profit-seeking businessmen exercising their animal spirits.[16]

4.2 Bayesian estimation

The model is estimated by full-information Bayesian methods using U.S. data with the observables made up of quarterly real per capita growth rates of output, consumption, investment, government spending and the logarithm of per capita hours worked. Justiniano

[15] Farmer et al. (2015) show that when the variance-covariance matrix of shocks remains unrestricted, the specific choice of the forward-looking variable for the expectation error is irrelevant. However, fundamental shocks are assumed to be uncorrelated, as is standard in the literature, meaning that the variance-covariance matrix is not unrestricted. Notwithstanding, we confirm our results are robust to the choice of the expectation error.

[16] In what follows, our empirical findings are not sensitive to the choice of modelling sunspot equilibria. We have also estimated the model using Bianchi and Nicolò's (2021) approach of solving and estimating the linear rational expectations models under indeterminacy. The results remain robust and are reported in the Appendix.

et al. (2011) use credit spread data to identify investment shocks. Similarly, we adopt the spread between BAA corporate bonds and the market yield on 30 year Treasury securities to identify disturbances to the marginal efficiency of investment as in[17]

$$\text{spread}_t = \varkappa \widehat{z}_t \qquad \varkappa < 0. \tag{11}$$

We focus on the 1990:I-2019:IV period to coincide with the rise of market power in levels and in dispersion reported in De Loecker et al. (2020) and also to abstract from the COVID-19 pandemic as our small-scale model is not designed to deal with its complexities. The Appendix sets out the sources and construction of the data.

We follow Bilbiie et al. (2012) and deflate Y_t, C_t, I_t, and G_t in the model by a data-consistent price index to obtain variables that are better comparable to observed data which does not take into account the welfare improvements of product variety at quarterly frequency. For example, data-consistent output is

$$Y_t^d \equiv \frac{P_t Y_t}{p_t} \equiv \frac{P_t Y_t}{p_{n,t}} N_t \epsilon_{t,n} + \frac{P_t Y_t}{p_{m,t}} M \epsilon_{t,m}$$

which removes the welfare gains that originate from entry and product scope adjustments. In line with this procedure, we set the shocks to government expenditures and animal spirits to directly affect the data-consistent variables G_t^d and Y_t^d, respectively. Accordingly, the measurement equations are

$$\begin{bmatrix} 100\ln(Y_t/Y_{t-1}) \\ 100\ln(C_t/C_{t-1}) \\ 100\ln(I_t/I_{t-1}) \\ 100\ln(G_t/G_{t-1}) \\ 100\,(\ln H_t/H) \\ spread_t \end{bmatrix} = \begin{bmatrix} \widehat{Y^d}_t - \widehat{Y^d}_{t-1} + \widehat{a}_t \\ \widehat{C^d}_t - \widehat{C^d}_{t-1} + \widehat{a}_t \\ \widehat{I^d}_t - \widehat{I^d}_{t-1} + \widehat{a}_t \\ \widehat{G^d}_t - \widehat{G^d}_{t-1} + \widehat{a^g}_t - \widehat{a^g}_{t-1} + \widehat{a}_t \\ \widehat{H}_t \\ \varkappa \widehat{z}_t \end{bmatrix} + \begin{bmatrix} \overline{a} \\ \overline{a} \\ \overline{a} \\ \overline{a} \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} \varepsilon_t^{m.e.} \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

in which $\overline{a} = 100(a-1)$, $a_t^g = A_t^g/A_t = (a_{t-1}^g)^{\psi_{ag}} a_t^{-1}$, $\varepsilon_t^{m.e.}$ is a measurement error restricted to account for not more than ten percent of output growth and H stands for the average hours worked over the sample period.

We have six shocks in the model: four fundamental shocks, a sunspot shock and a measurement error. In what follows, we only show the estimation results under indeterminacy based on the fact that the implications for markup cyclicality in our artificial economy matches the empirical evidence from Burstein et al. (2025), that large (small) firms' markups are procyclical (countercyclical), only under indeterminacy.

[17] We have also considered a measurement error in (11) as in Justiniano et al. (2011), but it only explains one percent of the spread. To be in line with Pagan and Robinson (2022) in avoiding excess shocks, we chose to exclude the measurement error.

4.2.1 Calibration and priors

We calibrate a subset of the model parameters. To begin with, we set the quarterly growth rate of labor augmenting technological progress to 0.34 percent to be consistent with the growth rate of per capita real GDP over the sample period and set the share of government expenditures G/Y to 0.19. The steady state profit share is calibrated at 14 percent, which is half-way between the estimates of Barkai (2020) and Hasenzagl and Pérez (2023) for the post-1990 period. Reflecting on the markup heterogeneity reported in De Loecker et al. (2020), we set the markup of large firms to $\mu_m = 1.5$ and that of smaller firms, the bottom 50 percentile, to $\mu_n = 1.2$. Bernard et al. (2010) report that 39 percent of U.S. manufacturing firms produced multiple products (five digit SIC categories) and accounted for 87 percent of manufacturing's output. For the aggregate economy, we calibrate the total market share of multi-product firms at a slightly lower value. Our choice targets the share of fixed costs in output as well as the average markup. Concretely, we set the market share of large firms at 75 percent, i.e., $M\epsilon_m = 0.75$. Such share aligns with Kehrig and Vincent's (2025), who report that multi-plant firms manufacture 78 percent of value added. Jointly, the calibration implies an average markup of 1.41, which is near the De Loecker et al. (2020) average, and the share of fixed costs in output works out to be about 20 percent. This value agrees with Abraham et al. (2024), Bridgman and Herrendorf (2024), and Nekarda and Ramey (2020). Together with setting $\alpha = 0.3$, the aggregate labor share turns out to be 60 percent, which is similar to Elsby et al. (2013). Furthermore, the labor share of large firms is 57 percent, while that of small firms is 70 percent, which is qualitatively consistent with Autor et al. (2020) who find that big firms have relatively smaller labor shares. Lastly, we set the elasticity σ to seven, which helps matching the relative volatility of net product creation and net business formation in the data, as will be discussed in Section 4.4. The values of the other standard parameters remain the same as in Section 3.[18]

The remaining parameters are estimated.[19] These include the love of variety ν, external habits b, the coefficient mapping the credit spread to investment shocks $\varkappa$, and parameters that govern the stochastic processes: ψ_A, ψ_z, ψ_Δ, ψ_g, ψ_{ag}, σ_s, σ_A, σ_z, σ_Δ, σ_g, Ω_A, Ω_z, Ω_Δ, Ω_g, and $\sigma^{m.e.}$. Table 1 presents the prior and posterior distributions. We employ a normal distribution, truncated at zero, for the variety effect ν. Since this parameter is central to our amplification mechanism which generates indeterminacy, we set the prior to give a prior probability of (in)determinacy of roughly 50 percent. A wide

[18] Following the indeterminacy literature, we continue to assume indivisible labour and set $\chi = 0$ in our estimation. However, we have also estimated the model with $\chi > 0$ but find that the indeterminate model with indivisible labor fits the data better in terms of marginal data density. This is primarily due to the indeterminate model being better at matching the volatility and cyclicality of hours worked, as we show in the Appendix.

[19] All estimations are done using Dynare (https://www.dynare.org). The posterior distributions are based on 500,000 draws from two separate chains with a 25-30% acceptance rate for each chain.

Table 1: Prior and posterior distributions

Prior					Posterior	
Name	Range	Density	Mean	Std. Dev.	Mean	90% Interval
ν	R^+	Normal	0.11	0.05	0.23	[0.20 0.27]
b	[0,1)	Beta	0.5	0.1	0.43	[0.34,0.51]
ψ_A	[0,1)	Beta	0.5	0.2	0.00	[0.00,0.01]
ψ_z	[0,1)	Beta	0.5	0.2	0.81	[0.75,0.88]
ψ_Δ	[0,1)	Beta	0.5	0.2	0.97	[0.96,0.99]
ψ_g	[0,1)	Beta	0.5	0.2	0.99	[0.99,0.99]
ψ_{ag}	[0,1)	Beta	0.5	0.2	0.71	[0.52,0.90]
σ_s	R^+	Inverse Gamma	0.1	Inf	0.22	[0.19,0.24]
σ_A	R^+	Inverse Gamma	0.1	Inf	0.62	[0.55,0.68]
σ_z	R^+	Inverse Gamma	0.1	Inf	0.06	[0.04,0.09]
σ_Δ	R^+	Inverse Gamma	0.1	Inf	0.79	[0.67,0.91]
σ_g	R^+	Inverse Gamma	0.1	Inf	0.79	[0.71,0.88]
$\sigma^{m.e.}$	$[0, 0.18]$	Uniform	0.09	0.05	0.18	[0.18,0.18]
Ω_A	[-3,3]	Uniform	0	1.73	-0.44	[-0.54,-0.35]
Ω_z	[-3,3]	Uniform	0	1.73	1.74	[0.87,2.65]
Ω_Δ	[-3,3]	Uniform	0	1.73	0.39	[0.29,0.48]
Ω_g	[-3,3]	Uniform	0	1.73	0.05	[-0.01,0.11]
$\varkappa$	[-20,0]	Uniform	-10	5.77	-5.32	[-7.55,-3.00]

The table presents the prior and posterior distributions for model parameters and shocks under indeterminacy. Standard deviations are in percent terms.

uniform distribution is employed for the expectation error parameters Ω_A, Ω_z, Ω_Δ, Ω_g and the credit spread coefficient $\varkappa$. The shock processes follow the standard inverse gamma distribution.

4.2.2 Estimation results

Table 1 reports the estimation results. The love of variety ν, which is a key parameter in the model, turns out to be non-trivial with a posterior mean of 0.23. It indicates the presence of a strong amplification mechanism via product creation in explaining the macroeconomic time series. The table further reports a close-to-zero persistence of the permanent technology shock and, consistent with the real business cycle model, a positive shock causes a fall in detrended output at impact. The investment shock is moderately persistent and as expected, raises output. Finally, both demand shocks are highly persistent and also cause an increase in output. The table also shows the estimated shock volatilities including a non-negligible estimate for the sunspot shock.

Table 2 displays the second moments of the observables, both for actual data and its model counterparts computed at the posterior mean. Our admittedly small-scale model captures the behavior of U.S. macroeconomic variables reasonably well. The model somewhat overpredicts the volatilities of output and consumption and underpredicts hours worked. One outlier is investment for which the model strongly overpredicts its vari-

Table 2: Business cycle dynamics

	Data			Model		
x	σ_x	$\rho(x, \ln(Y_t/Y_{t-1}))$	ACF	σ_x	$\rho(x, \ln(Y_t/Y_{t-1}))$	ACF
$\ln(Y_t/Y_{t-1})$	0.58	1	0.29	0.71	1	0.42
$\ln(C_t/C_{t-1})$	0.47	0.67	0.38	0.59	0.56	0.42
$\ln(I_t/I_{t-1})$	1.66	0.79	0.62	3.01	0.78	0.56
$\ln(G_t/G_{t-1})$	0.77	0.25	0.24	0.83	0.08	0.06
$\ln(H_t/H)$	6.16	0.20	0.99	4.88	0.10	0.99
spread$_t$	0.60	-0.58	0.85	0.59	-0.24	0.81

Business cycle statistics for the artificial economy are calculated at the posterior mean. σ_x denotes the standard deviation of variable x, $\rho(x, \ln(Y_t/Y_{t-1}))$ is the correlation of variable x and output growth, and ACF is the first order autocorrelation coefficient.

Table 3: Unconditional variance decomposition (in percent)

	$\ln\left(\frac{Y_t}{Y_{t-1}}\right)$	$\ln\left(\frac{C_t}{C_{t-1}}\right)$	$\ln\left(\frac{I_t}{I_{t-1}}\right)$	$\ln\left(\frac{G_t}{G_{t-1}}\right)$	$\ln\left(\frac{H_t}{H}\right)$	$spread_t$
ε_t^s	10.59	0.49	19.71	0.00	2.05	0.00
ε_t^A	30.99	38.86	19.01	9.23	11.43	0.00
ε_t^z	17.35	0.74	30.21	0.00	17.88	100
ε_t^Δ	32.50	59.80	24.00	0.00	56.06	0.00
ε_t^g	2.16	0.11	7.07	90.77	12.59	0.00
$\varepsilon_t^{m.e.}$	6.42	0.00	0.00	0.00	0.00	0.00

Variance decompositions are performed at the posterior mean.

ance.[20] Correlations with output are well replicated. As a result of its rich internal propagation mechanism, the artificial economy captures data's autocorrelation functions remarkably even without the myriad of real frictions often employed in medium-scale models to generate such persistence.

Table 3 displays the forecast error variance decompositions which reveal the relative contribution of each of the six shocks to the macroeconomic aggregates. Supply shocks to technology and investment explain about half of U.S. business cycle fluctuations. The latter shocks account for a large fraction of investment growth but, overall, investment shocks' importance shrinks when compared to Justiniano et al. (2011). On the demand side, preference shocks are the most dominant, explaining more than half of consumption and hours worked, while government expenditure shocks are a negligible source of business cycles. The effect of animal spirits on the business cycle is non-trivial: they drive a modest fraction of output and a sizeable portion of investment. While the result suggests that actual cycles are at least partially of endogenous nature, the modest effect of animal spirits contrasts with Dai et al. (2020) and Pavlov and Weder (2017). We see two main reasons for this finding. First, these papers' estimations were conducted over a much longer and more volatile period, i.e., from 1955 onwards, while the current

[20] We ran an alternative estimation using endogenous priors as in Christiano et al. (2011) that matches investment data better with the key results remaining robust.

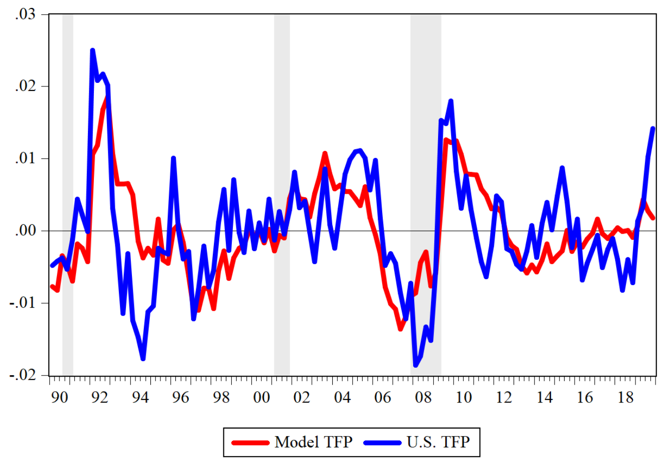

Figure 6: Total factor productivity. Percentage deviations from HP-trend.

paper is concerned with more recent trends. Second, their theoretical frameworks and, thus propagation mechanisms, primarily rely on countercyclical markups which tend to assuage co-movement problems and make sunspots more important. Overall, the relative contributions of supply and demand disturbances to movements in output growth are comparable at around 50 percent each.[21]

4.3 External validation for technology and sunspot shocks

We identify shocks by estimating them in a system and it is thus fair to ask if the estimated shocks are meaningfully labeled. As we estimate the model without employing data on total factor productivity (TFP) or animal spirits, we now externally validate estimated shocks by comparing them to their empirical counterparts.

For TFP, we consult Fernald's (2014) series in its utilization adjusted form. To make the data comparable, we convert both series into level indices that we then Hodrick-Prescott filter to take out low frequency movements. Figure 6 reports that the estimated series resemblances the empirical data, with a positive correlation of 0.63. This result supports our interpretation of the shocks.[22]

We also compare the estimated animal spirits with the University of Michigan's sentiment index. We construct a level-index from the smoothed estimates of animal spirits shocks parallel to what we have done for TFP.[23] Figure 7 indicates a positive correlation of the two series at 0.42. To us, this pattern signals that the estimated shocks can be meaningfully coined animal spirits, thus describing people's extrinsic expectations and

[21] We have estimated the model with different calibrations of σ and market shares and the contribution of animal spirits remains in the region of slightly above ten percent.

[22] Furthermore, TFP from the indeterminacy model and that from the determinate model version with two technology shocks are virtually identical.

[23] The series are normalized to make them comparable. The Appendix features a parallel figure using a Business Tendency Index.

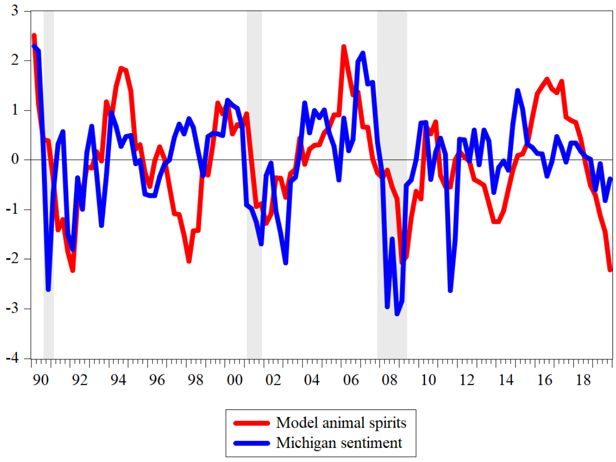

Figure 7: Sentiments and animal spirits. Normalized deviations from HP-trend.

how these expectations alternate between euphoric and pessimistic states. This being said, the Michigan sentiment is a composite of extrinsic and intrinsic parts and a less than perfect correlation is expected. Furthermore, the estimated series appears to lead the Michigan index at upper business cycle turning points and both indices begin to fall right before each of the three NBER recessions.

4.4 The cyclical patterns of product scope and business formation

Our model economy, and in particular its indeterminacy mechanism, is based on the variety effect associated with variations of the product scope and business formation. This last section will inspect if the cyclicality and relative volatility of product scope and firm dynamics are, despite the model's small scale nature and simplicity, reasonable and in line with micro evidence.

Axarloglou (2003), Broda and Weinstein (2010), Lee and Mukoyama (2018) and Guo (2023) all have put forward evidence of strong procyclicalities of product scope. Broda and Weinstein's is a key paper here and, most importantly for us, it provides an empirical series for product creation constructed from the *ACNielsen Homescan* database.[24] They define net product creation as the value of new products net of the value of disappearing products, reported as a fraction of total value. This is then plotted next to consumption sales growth over the period 2000:I-2003:III, documenting a strong positive co-movement. We use real GDP as a measure of aggregate economic activity and define a corresponding measure of net product creation NPC_t as

$$NPC_t \equiv \frac{p_{m,s} M x_{m,s}(S_t - S_s) + p_{n,s} x_{n,s}(N_t - N_s)}{Y_s} \tag{12}$$

[24] We thank Broda and Weinstein for providing their net product creation data.

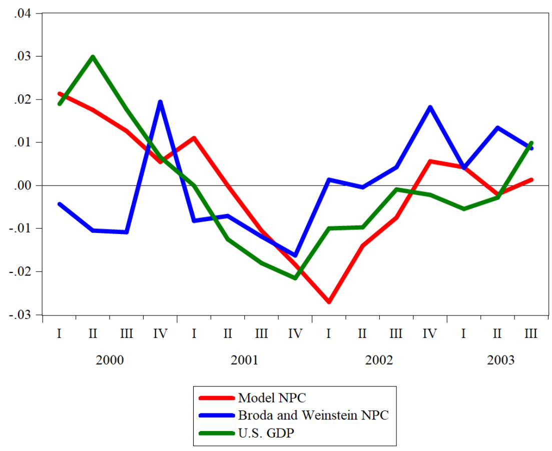

Figure 8: Net product creation and real GDP (year-on-year growth rates).

in which x_m (p_m) and x_n (p_n) are the outputs (prices) per variety of our two types of firms. Using the smoothed variables from our estimation, we construct year-on-year growth rates, i.e., we set $s = t - 4$ in (12) to be comparable with Broda and Weinstein's data.[25]

Figure 8 presents our artificial net product creation alongside Broda and Weinstein's, as well as year-on-year U.S. GDP growth rates to address cyclicality. All series are demeaned making them comparable. Except for the first few quarters, the model's net product creation is positively correlated with Broda and Weinstein's measure. In fact, the correlation over 2000:IV-2003:III is 0.41 and the relative standard deviation of the data compared to the model is well matched at 1.02.[26] Figure 8 also shows a strong positive correlation of 0.63 between the model's product creation and real GDP growth, including a marked slowdown during the 2001 recession.[27] The corresponding correlation for Broda and Weinstein's series is 0.76.

Next, we construct a series for net business formation for the model and U.S. data. In the model, net business formation NBF_t is simply the change in the number of mono-product firms (since multi-product firms are constant in number) over the total number of firms

$$NBF_t \equiv \frac{N_t - N_s}{M + N_s}.$$

Then, we use *Bureau of Labor Statistics*' (BLS) openings and closings of U.S. establishments to construct a data equivalent. Fortunately, the data allows covering a longer

[25] In the numerator, we value new products in period s quantities and prices. The results remain quantitatively indistinguishable if we were to value these based on period t.

[26] The first three observations in Broda and Weinstein's (2010) *ACNielsen Homescan*-based product creation and consumption sales growth do not resemble the pattern in the aggregate data for economic activity, such as GDP, industrial production or aggregate retail sales. Since we use GDP data in our estimation, we decided to drop the first three quarters when computing these statistics, as the correlation of their net creation data with aggregate output would otherwise be close to zero.

[27] The correlation is 0.70 over our full estimation sample.

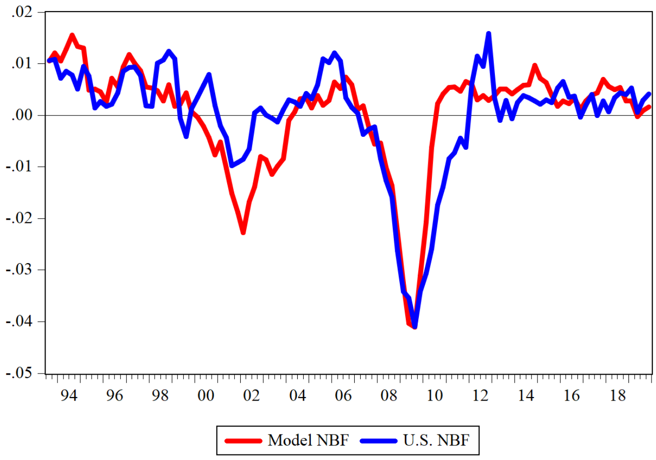

Figure 9: Model and data net business formation (year-on-year growth rates).

period beginning in 1993. Figure 9 presents year-on-year growth rates for both series. The correlation between the two series is 0.80 and the standard deviations are virtually identical.

Finally, net product creation in the model turns out to be more volatile than net business formation. The relative volatility $\sigma(NPC_t)/\sigma(NBF_t)$ is 2.28. The corresponding number in the data is 2.09 when comparing Broda and Weinstein's net product creation measure to the net business formation series from BLS. The relatively higher volatility of net product creation suggests that the intra-firm extensive margin to the overall fluctuations of product creation is more important than the contribution stemming from the entry of new firms, which is in line with Minniti and Turino (2013). In fact, the last result gives additional support for calibrating the elasticity of substitution σ at 7: if σ were calibrated at $\sigma^{\min}$ or $\sigma^{\max}$, the relative standard deviation would be 1 or 4.00. We make sense of this finding, driven mainly by variations of $\sigma(NBF_t)$, as follows: suppose σ is at its upper limit $\sigma^{\max}$, then the number of ordinary firms reaches a minimum and, as a form of low business dynamism, the volatility of firm entry and exit falls. Consequently, the relative standard deviation of net product creation hits an upper bound.

In sum, our takeaway from the two exercises is that the model-generated net product creation and net business formation are both reasonable and consistent with micro evidence. This result provides additional support for the key mechanisms of our theory, all without employing micro data to identify these variables.

5 Concluding remarks

The rise of market power in the last decades is primarily driven by the upper portions of the firm size distribution. This paper proposes a general equilibrium theory of large firms in which their technology involves the ability to produce multiple products. This ability allows them to set larger markups and grab a larger market share than ordinary mono-product firms. Consistent with recent findings, endogenous product creation and the resulting market share reallocations generate heterogeneous firm and markup dynamics. At the aggregate level, the co-presence of large multi-product and small mono-product firms has implications for macroeconomic instability. Higher market shares of large firms expand the parametric space for equilibrium indeterminacy and, thus, endogenous business cycles. We assess the relative importance of animal spirits and canonical fundamental disturbances in driving aggregate fluctuations. A full-information Bayesian estimation of the general equilibrium model reveals the importance of endogenous amplification of the product creation channel. Several external validations confirm that model-estimated technology shocks, sentiments, net product creation and net business formation are consistent with empirical evidence. Through the lens of our theory, animal spirits play a non-trivial role in driving U.S. business cycles, in particular for fluctuations of aggregate investment.

References

[1] Abraham, F., Y. Bormans, J. Konings and W. Roeger (2024): "Price-Cost Margins, Fixed Costs and Excess Profits", *Economic Journal* **134**, 2655–2684.

[2] Autor, D., D. Dorn, L. Katz, C. Patterson and J. Van Reenen (2020): "The Fall of the Labor Share and the Rise of Superstar Firms", *Quarterly Journal of Economics* **135**, 645–709.

[3] Axarloglou, K. (2003): "The Cyclicality of New Product Introductions", *Journal of Business* **76**, 29–48.

[4] Barkai, S. (2020): "Declining Labor and Capital Shares", *Journal of Finance* **75**, 2421–2463.

[5] Benassy, J.–P. (1996): "Taste for Variety and Optimum Production Patterns in Monopolistic Competition", *Economics Letters* **52**, 41–47.

[6] Benhabib, J. and R. Farmer (1994): "Indeterminacy and Increasing Returns", *Journal of Economic Theory* **63**, 19–41.

[7] Benhabib, J. and P. Wang (2013): "Financial Constraints, Endogenous Markups, and Self-fulfilling Equilibria", *Journal of Monetary Economics* **60**, 789–805.

[8] Bernard, A., S. Redding and P. Schott (2010): "Multi-product Firms and Product Switching", *American Economic Review* **100**, 70–97.

[9] Bernard, A., E. Dhyne, G. Magerman and K. Manova: (2022): "The Origins of Firm Heterogeneity: A Production Network Approach", *Journal of Political Economy* ***130,*** 1765–1804.

[10] Bianchi, F. and G. Nicolò (2021): "A Generalized Approach to Indeterminacy in Linear Rational Expectations Models", *Quantitative Economics* **12**, 843–868.

[11] Bilbiie, F., F. Ghironi and M. Melitz (2012): "Endogenous Entry, Product Variety, and Business Cycles", *Journal of Political Economy* **120**, 304–345.

[12] Bils, M., P. Klenow and B. Malin (2018): "Resurrecting the Role of the Product Market Wedge in Recessions", *American Economic Review* **108**, 1118–1146

[13] Brander, J. and J. Eaton (1984): "Product Line Rivalry", *American Economic Review* **74**, 323–334.

[14] Bridgman, B. and B. Herrendorf (2024): "Labor Share, Markups, and Input-Output Linkages – Evidence from the U.S. National Accounts", Bureau of Economic Analysis, mimeo.

[15] Broda, C. and D. Weinstein (2010): "Product Creation and Destruction: Evidence and Price Implications", *American Economic Review* **100**, 691–732.

[16] Burstein, A., V. Carvalho and B. Grassi (2025): "Bottom-up Markup Fluctuations", *Quarterly Journal of Economics*, forthcoming.

[17] Cao, D., H. Hyatt, T. Mukoyama and E. Sager (2022): "Firm Growth through New Establishments", Georgetown University, mimeo.

[18] Christiano, L. (1988): "Why Does Inventory Investment Fluctuate So Much?", *Journal of Monetary Economics* **21**, 247–280.

[19] Christiano, L., M. Trabandt and K. Walentin (2011): "Introducing Financial Frictions and Unemployment into a Small Open Economy Model", *Journal of Economic Dynamics and Control* **35**, 1999-2041.

[20] Dai, W., M. Weder, and B. Zhang (2020): "Animal Spirits, Financial Markets, and Aggregate Instability", *Journal of Money, Credit and Banking* **52**, 2053–2083.

[21] De Loecker J., J. Eeckhout and G. Unger (2020): "The Rise of Market Power and the Macroeconomic Implications", *Quarterly Journal of Economics* **135**, 561–644.

[22] Dixit, A. and J. Stiglitz (1977): "Monopolistic Competition and Optimum Product Diversity", *American Economic Review* **67**, 297–308.

[23] Edmond, C., V. Midrigan and D. Xu (2023): "How Costly Are Markups?", *Journal of Political Economy* **131**, 1619–1675.

[24] Elsby, M., B. Hobijn and A. Şahin (2013): "The Decline of the U.S. Labor Share", *Brookings Papers on Economic Activity*, 1–52.

[25] Farmer, R. (2016): "The Evolution of Endogenous Business Cycles", *Macroeconomic Dynamics* **20**, 554–557.

[26] Farmer, R. and J.-T. Guo (1994): "Real Business Cycles and the Animal Spirits Hypothesis", *Journal of Economic Theory* **63**, 42–72.

[27] Farmer, R., V. Khramov and G. Nicolò (2015): "Solving and Estimating Indeterminate DSGE Models", *Journal of Economic Dynamics and Control* **54**, 17–36.

[28] Feenstra, R. and H. Ma (2009): "Optimal Choice of Product Scope for Multiproduct Firms Under Monopolistic Competition". In: Helpman, E., D., Marin., Verdier, T. (Eds.), The Organization of Firms in a Global Economy. Harvard University Press, Cambridge, pp. 173–199.

[29] Fernald, J. (2014): "A Quarterly, Utilization-Adjusted Series on Total Factor Productivity", Federal Reserve Bank of San Francisco, mimeo.

[30] Foster, L., J. Haltiwanger and C. Syverson (2008): "Reallocation, Firm Turnover, and Efficiency: Selection on Productivity or Profitability?", *American Economic Review* **98**, 394–425.

[31] Galí, J. (1994): "Monopolistic Competition, Business Cycles, and the Composition of Aggregate Demand", *Journal of Economic Theory* **63**, 73–96.

[32] Gamber, W. (2023): "The Importance of Entry in the Business Cycle: What Are the Roles of Markups, Adjustment Costs, and Heterogeneity?", *Journal of Political Economy: Macroeconomics* **1**, 551–585.

[33] Greenwood, J., Z. Hercowitz and G. Huffman (1988): "Investment, Capacity Utilization, and the Real Business Cycle", *American Economic Review* **78**, 402–417.

[34] Guo, D. (2023): "Heterogenous Oligopolistic Multiproduct Firms and the Business Cycle: Evidence and Theory", Xiamen University, mimeo.

[35] Hansen, G. (1985): "Indivisible Labor and the Business Cycle", *Journal of Monetary Economics* **16**, 309–327.

[36] Hasenzagl, T. and L. Pérez (2023): "The Micro-Aggregated Profit Share", University of Minnesota, mimeo.

[37] Helpman, E. and B. Niswonger (2022): "Dynamics of Markups, Concentration and Product Span", *American Economic Journal: Macroeconomics* **14**, 42–81.

[38] Hirose, Y., T. Kurozumi and W. Van Zandweghe (2023): "Inflation Gap Persistence, Indeterminacy, and Monetary Policy", *Review of Economic Dynamics* **51**, 867–887.

[39] Hottman, C., S. Redding and D. Weinstein (2016): "Quantifying the Sources of Firm Heterogeneity ", *Quarterly Journal of Economics* **131**, 1291–1364.

[40] Jaimovich, N. (2007): "Firm Dynamics and Markup Variations: Implications for Sunspot Equilibria and Endogenous Economic Fluctuations", *Journal of Economic Theory* **137**, 300–325.

[41] Justiniano A., G. Primiceri and A. Tambalotti (2011): "Investment Shocks and the Relative Price of Investment", *Review of Economic Dynamics* **14**, 102–121.

[42] Kehrig, M. and N. Vincent (2021): "The Micro-Level Anatomy of the Labor Share Decline", *Quarterly Journal of Economics* **136**, 1031–1087.

[43] Kehrig, M. and N. Vincent (2025): "Good Dispersion, Bad Dispersion", *Review of Economic Studies*, forthcoming.

[44] Kimball, M. S. (1995): "The Quantitative Analytics of the Basic Neomonetarist Model", *Journal of Money, Credit and Banking* **27**, 1241-1277.

[45] Kurozumi, T. and W. Van Zandweghe (2022): "Macroeconomic Changes With Declining Trend Inflation: Complementarity With the Superstar Firm Hypothesis", *European Economic Review* **141**, 103998.

[46] Lee, Y. and T. Mukoyama (2018): "A Model of Entry, Exit, and Plant-level Dynamics over the Business Cycle", *Journal of Economic Dynamics and Control* **96**, 1–25.

[47] Lubik, T. and F. Schorfheide (2004): "Testing for Indeterminacy: An Application to U.S. Monetary Policy", *American Economic Review* **94**, 190–217.

[48] Minniti, A. and F. Turino (2013): "Multi-product Firms and Business Cycle Dynamics", *European Economic Review* **57**, 75–97.

[49] Mongey, S. and M. Waugh (2025): "Pricing Inequality", Federal Reserve Bank of Minneapolis, mimeo.

[50] Neary, P. (2010): "Two and a Half Theories of Trade", *The World Economy* **33**, 1–19.

[51] Nekarda, C. and V. Ramey (2020): "The Cyclical Behavior of the Price-Cost Markup", *Journal of Money, Credit and Banking* **52**, 319–353.

[52] Pagan, A. and T. Robinson (2022): "Excess Shocks Can Limit the Economic Interpretation", *European Economic Review* **145**, 104120.

[53] Pavlov, O. and M. Weder (2017): "Product Scope and Endogenous Fluctuations", *Review of Economic Dynamics* **24**, 175–191.

[54] Pintus, P., Y. Wen and X. Xing (2022): "The Inverted Leading Indicator Property and Redistribution Effect of the Interest Rate", *European Economic Review* **148**, 104219.

[55] Rogerson, R. (1988): "Indivisible Labor, Lotteries and Equilibrium", *Journal of Monetary Economics* **21**, 3–16.

[56] Shimomura, K and J. Thisse (2012): "Competition Among the Big and the Small", *RAND Journal of Economics* **43**, 329–347.

[57] Smets, F and R. Wouters (2007): "Shocks and Frictions in US Business Cycles: A Bayesian DSGE Approach", *American Economic Review* **97**, 586–606.

[58] Wen, Y. (1998): "Capacity Utilization under Increasing Returns to Scale", *Journal of Economic Theory* **81**, 7–36.

[59] Yang, X. and B. Heijdra (1993): "Monopolistic Competition and Optimum Product Diversity: Comment", *American Economic Review* **83**, 295–301.

A Online appendix

This Appendix is not for publication and it contains:

A.1 Data sources and construction

A.2 Derivation of the product scope

A.3 Indeterminacy with dynamic entry and exit of firms

A.4 Further external validation

A.5 Estimation results using Bianchi and Nicolò's (2021) method

A.6 Estimation results with less elastic labor supply

A.1 Data sources and construction

This Appendix details the source and construction of the U.S. data used in Section 4. All data is quarterly and for the period 1990:I-2019:IV.

1. Gross Domestic Product. Seasonally adjusted at annual rates, billions of chained (2012) dollars. Source: Bureau of Economic Analysis, NIPA Table 1.1.6.

2. Gross Domestic Product. Seasonally adjusted at annual rates, billions of dollars. Source: Bureau of Economic Analysis, NIPA Table 1.1.5.

3. Personal Consumption Expenditures, Nondurable Goods. Seasonally adjusted at annual rates, billions of dollars. Source: Bureau of Economic Analysis, NIPA Table 1.1.5.

4. Personal Consumption Expenditures, Services. Seasonally adjusted at annual rates, billions of dollars. Source: Bureau of Economic Analysis, NIPA Table 1.1.5.

5. Personal Consumption Expenditures, Durable Goods. Seasonally adjusted at annual rates, billions of dollars. Source: Bureau of Economic Analysis, NIPA Table 1.1.5.

6. Gross Private Domestic Investment, Fixed Investment, Residential. Seasonally adjusted at annual rates, billions of dollars. Source: Bureau of Economic Analysis, NIPA Table 1.1.5.

7. Gross Private Domestic Investment, Fixed Investment, Nonresidential. Seasonally adjusted at annual rates, billions of dollars. Source: Bureau of Economic Analysis, NIPA Table 1.1.5.

8. Government consumption expenditures and gross investment. Seasonally adjusted at annual rates, billions of dollars. Source: Bureau of Economic Analysis, NIPA Table 1.1.5.

9. Nonfarm Business Hours. Index 2012=100, seasonally adjusted. Source: Bureau of Labor Statistics, Series Id: PRS85006033.

10. Civilian Noninstitutional Population. 16 years and over, thousands. Source: Bureau of Labor Statistics, Series Id: LNU00000000Q.

11. GDP Deflator = (2)/(1).

12. Real Per Capita Output, Y_t = (1)/(10).

13. Real Per Capita Consumption, C_t = [(3) + (4)]/(11)/(10).

14. Real Per Capita Investment, I_t = [(5) + (6) + (7)]/(11)/(10).

15. Real Per Capita Government Expenditures, G_t = (8)/(11)/(10).

16. Per Capita Hours Worked, H_t = (9)/(10).

17. Moody's Seasoned Baa Corporate Bond Yield [DBAA], retrieved from FRED, Federal Reserve Bank of St. Louis; https://fred.stlouisfed.org/series/DBAA.

18. Market Yield on U.S. Treasury Securities at 30-Year Constant Maturity, Quoted on an Investment Basis [DGS30], retrieved from FRED, Federal Reserve Bank of St. Louis; https://fred.stlouisfed.org/series/DGS30.

19. Credit spread, (17) − (18).

20. Deviation from average credit spread, $spread_t$ = (19)− average of (19).

21. University of Michigan. Consumer Sentiment [UMCSENT], retrieved from FRED, Federal Reserve Bank of St. Louis; https://fred.stlouisfed.org/series/UMCSENT.

22. Total Factor Productivity. "A Quarterly Utilization-Adjusted Series on Total Factor Productivity", retrieved from https://www.johnfernald.net/TFP.

23. Organization for Economic Co-operation and Development, Business Tendency Surveys (Manufacturing): Confidence Indicators: Composite Indicators: OECD Indicator for United States [BSCICP03USM665S], retrieved from FRED, Federal Reserve Bank of St. Louis; https://fred.stlouisfed.org/series/BSCICP03USM665S.

24. Number of establishments for net business formation is constructed from Total Private Openings and Closings, Number of Establishments, Levels and Rate. Source: Bureau of Labor Statistics, Series Id: BDS0000000000000000120003LQ5, BDS0000000000000000120006LQ5, BDS0000000000000000120003RQ5, BDS0000000000000000120006RQ5.

A.2 Derivation of the product scope

This Appendix derives the firm's optimal product scope. Firm j maximizes profits with respect to $S(j)$ and takes into account the effect of its product scope on its own prices, prices of other firms, and the aggregate price index. First we rewrite profits as

$$\pi(j) = \left(\frac{p(j) - \Lambda}{p(j)}\right) PY\epsilon(j) - \Lambda[S(j)\phi_s + \phi_m]$$

and obtain the first-order condition

$$\frac{\partial \pi(j)}{\partial S(j)} = \sigma PY \left(\frac{p(j) - \Lambda}{p(j)}\right)^2 \frac{\partial \epsilon(j)}{\partial S(j)} + Y\epsilon(j)\left(\frac{p(j) - \Lambda}{p(j)}\right)\frac{\partial P}{\partial S(j)} - \Lambda\phi_s = 0.$$

Then

$$\frac{\partial \epsilon(j)}{\partial S(j)} = \frac{\epsilon(j)}{S(j)} - (\sigma - 1)\epsilon(j)\left[\frac{1}{p(j)}\frac{\partial p(j)}{\partial S(j)} - \frac{1}{P}\frac{\partial P}{\partial S(j)}\right]$$

and for other multi-product firms

$$\frac{\partial \epsilon(k)}{\partial S(j)} = -(\sigma - 1)\epsilon(k)\left[\frac{1}{p(k)}\frac{\partial p(k)}{\partial S(j)} - \frac{1}{P}\frac{\partial P}{\partial S(j)}\right]$$

and ordinary firms

$$\frac{\partial \epsilon(i)}{\partial S(j)} = -(\sigma - 1)\epsilon(i)\left[\frac{1}{p(i)}\frac{\partial p(i)}{\partial S(j)} - \frac{1}{P}\frac{\partial P}{\partial S(j)}\right].$$

Next, rewrite the aggregate price index as

$$P = \left(\sum_{i=1}^{N} p(i)^{1-\sigma} di + \sum_{k=1}^{M} S(k)p(k)^{1-\sigma}\right)^{\frac{1}{1-\sigma}}.$$

From here, we use symmetry to simplify. After some algebra, $\partial P/\partial S(j)$ can be expressed as

$$\frac{\partial P}{\partial S(j)} = N\frac{\epsilon_n}{p_n}\frac{\partial p(i)}{\partial S(j)} + \frac{\epsilon_m}{p_m}\left((M-1)\frac{\partial p(k)}{\partial S(j)} + \frac{\partial p(j)}{\partial S(j)}\right) + \frac{1}{1-\sigma}\frac{\epsilon_m}{S}$$

where

$$\frac{\partial p(i)}{\partial S(j)} = -\sigma(\sigma-1)(\mu_n - 1)(1 - 1/\mu_n)\epsilon_n\left[\frac{\partial p(i)}{\partial S(j)} - p_n\frac{\partial P}{\partial S(j)}\right]$$

$$\frac{\partial p(k)}{\partial S(j)} = -\sigma(\sigma-1)(\mu_m - 1)(1 - 1/\mu_m)\epsilon_m\left[\frac{\partial p(k)}{\partial S(j)} - p_m\frac{\partial P}{\partial S(j)}\right]$$

$$\frac{\partial p(j)}{\partial S(j)} = \sigma(\mu_m - 1)(1 - 1/\mu_m)\left(p_m\frac{\epsilon_m}{S} - (\sigma-1)\epsilon_m\left[\frac{\partial p(j)}{\partial S(j)} - p_m\frac{\partial P}{\partial S(j)}\right]\right).$$

Putting all these together, it can then be shown that $\frac{\partial P}{\partial S(j)} < 0$, $\frac{\partial p(k)}{\partial S(j)} < 0$, $\frac{\partial p(i)}{\partial S(j)} < 0$, $\frac{\partial p(j)}{\partial S(j)} > 0$, $\frac{\partial \epsilon(k)}{\partial S(j)} < 0$, $\frac{\partial \epsilon(i)}{\partial S(j)} < 0$, and $\frac{\partial \epsilon(j)}{\partial S(j)} > 0$. Finally, $\frac{\partial \epsilon(j)}{\partial S(j)}$, $\frac{\partial P}{\partial S(j)}$, and $\frac{\partial p(j)}{\partial S(j)}$ can be substituted in the first-order condition $\frac{\partial \pi(j)}{\partial S(j)} = 0$ to find the product scope

$$S = f(\mu_m, \mu_n, N, M, \sigma)\frac{Y}{\phi_s p_m}$$

where

$$f = \frac{\frac{\epsilon_m(\mu_m-1)^2\sigma}{\mu_m} - \frac{\epsilon_m^2(\mu_m-1)\left(1+\frac{(\mu_m-1)\sigma(\sigma-1)}{\mu_m\left(1+\frac{\epsilon_m(\mu_m-1)^2\sigma(\sigma-1)}{\mu_m}\right)}\right)}{(\sigma-1)\left[1+\epsilon_m M\left(\frac{\mu_m}{\mu_m+\epsilon_m(\mu_m-1)^2\sigma(\sigma-1)}-1\right)+\epsilon_n N\left(\frac{\mu_n}{\mu_n+\epsilon_n(\mu_n-1)^2\sigma(\sigma-1)}-1\right)\right]}}{\left(1+\frac{\epsilon_m(\mu_m-1)^2\sigma(\sigma-1)}{\mu_m}\right)}$$

and $\epsilon_m = 1 - \frac{\mu_m}{\mu_m-1}\frac{1}{\sigma}$ and $\epsilon_n = 1 - \frac{\mu_n}{\mu_n-1}\frac{1}{\sigma}$.

A.3 Indeterminacy with dynamic entry and exit of small firms

This Appendix presents the version of the model where the entry of ordinary firms is dynamic as in Bilbiie et al. (2012) and shows that indeterminacy remains. A prospective entrant i computes its expected value

$$v_t(i) = E_t \sum_{l=1}^{\infty} Q_{t,l} \pi_{n,t+l}(i)$$

where $Q_{t,l}$ is the stochastic discount factor and $\pi_{n,t}(i)$ denotes profits of ordinary firms. There is a time-to-build lag in that period t entrants begin operating in period $t+1$ and the number of firms evolves according to

$$N_t = (1-\delta_n)(N_{t-1} + N_{E,t-1})$$

where δ_n is the exogenous exit probability and $N_{E,t}$ is the number of entrants. Entry occurs until the expected value, $v_t(i)$, is equal to the sunk cost of entry. To enter, f_E amount of labor needs to be hired and since labor is paid the real wage w_t, this sunk cost is equal to

$$v_t(i) = w_t f_E.$$

The production function for new firms is thus

$$N_{E,t} = \frac{H_{E,t}}{f_E}$$

where $H_{E,t}$ is the amount of labor hired for the production of new firms. In a symmetric equilibrium, a representative household enters period t with mutual fund share holdings x_t and has the budget constraint

$$C_t + I_t + v_t(N_t + N_{E,t})x_{t+1} = (\pi_{n,t} + v_t)N_t x_t + w_t H_t + r_t U_t K_t + M\pi_{m,t}$$

where $\pi_{m,t}$ are profits from a constant number of multi-product firms and $H_t = H_{E,t} + H_{n,t} + H_{m,t}$. The Euler equation for share holding is then

$$v_t = E_t \beta(1-\delta_n)\frac{C_t}{C_{t+1}}(\pi_{n,t+1} + v_{t+1}).$$

Imposing the equilibrium condition $x_{t+1} = x_t = 1$ for all t gives

$$C_t + I_t + v_t N_{E,t} = \pi_{n,t} N_t + w_t H_t + r_t U_t K_t + M\pi_{m,t} \equiv Y_t$$

where Y_t is GDP consisting of consumption, investment in capital, and investment in new firms. Total investment is then

$$X_t \equiv I_t + v_t N_{E,t}$$

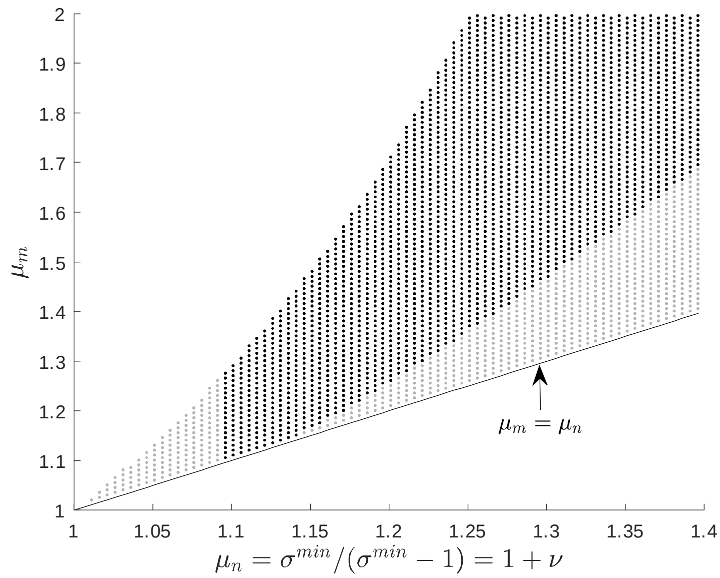

Figure A1: Indeterminacy (dark shaded zone), determinacy (left light shaded zone), source (right light shaded zone) with dynamic entry of ordinary firms.

and the CES aggregator is now

$$Y_{g,t} \equiv C_t + I_t = \left(\sum_{i=1}^{N_t} x_t(i)^{\frac{\sigma-1}{\sigma}} + \sum_{j=1}^{M} Y_t(j)^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}}.$$

Small firms no longer have firm-level fixed costs and the symmetric equilibrium goods production is then

$$Y_{g,t} = \frac{p_{n,t} U_t^{\alpha} K_{n,t}^{\alpha} H_{n,t}^{1-\alpha}}{N_t \epsilon_{n,t}} = \frac{p_{m,t} U_t^{\alpha} K_{m,t}^{\alpha} H_{m,t}^{1-\alpha} - p_{m,t} M S_t \phi_s}{M \epsilon_{m,t}}.$$

We calibrate the model as in Section 3 and additionally set $\delta_n = 0.025$ as in Bilbiie et al. (2012). Analogous to Figure 2, Figure A1 plots the feasible parameter zones where multi-product firms can exist. The indeterminacy region largely remains but the lighter zone on the right side indicates an unstable equilibrium (a source) where markups are not sufficiently different. The lighter zone on the left representing determinacy remains.

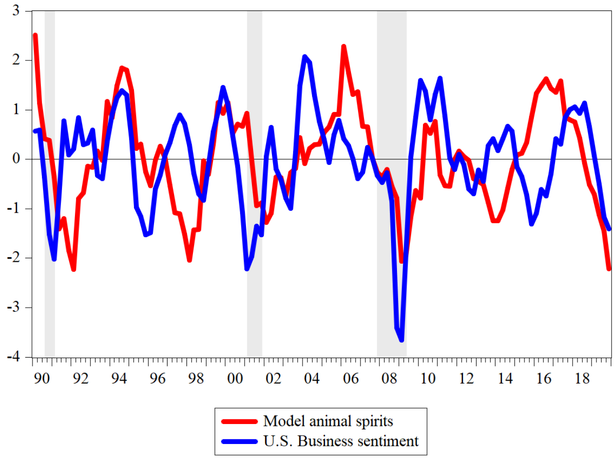

Figure A2: Figure A2: Business Tendency Surveys and animal spirits. Normalized deviations from HP-trend.

A.4 Further external validation

Figure A2 repeats the external validation of the expectational shocks comparing them to the Business Tendency Surveys (Manufacturing). The plotted data has been constructed as in the main part of the paper. The two series continue displaying positive co-movements.

A.5 Estimation results using Bianchi and Nicolò (2021) method

Bianchi and Nicolò (2021) develop a new method to solve and estimate linear rational expectations (LRE) models under indeterminacy. Their characterization of indeterminate equilibria is equivalent to Lubik and Schorfheide (2004) and Farmer et al. (2015). We closely follow Bianchi and Nicolò (2021) and in the following briefly sketch their methodology while referring the readers to their paper for detailed exposition. Following Bianchi and Nicolò (2021), we append the following autoregressive process to the original LRE model

$$\omega_t = \varphi^* \omega_{t-1} + \varepsilon_t^s - \eta_t$$

in which ε_t^s is the animal spirit shock as before and η_t can be any element of the forecast error vector. As in our baseline analysis, we include the forecast error associated with (data-consistent) output $\eta_t = \widehat{Y}_t^d - E_{t-1}\widehat{Y}_t^d$. The main insight of the Bianchi and Nicolò (2021) approach consists of choosing this auxiliary process in a way that delivers the 'correct' solution. When the original model is indeterminate, the auxiliary process must be explosive so that the augmented representation satisfies the Blanchard-Kahn condition, although it does not for the original model. Accordingly, we set φ^* such that its absolute value is outside the unit circle. As before, we estimate the standard deviation of the animal spirit shock, σ_s. In addition, the animal spirit shock is potentially related to the structural shocks of the model and we capture this association by estimating the correlation between the non-fundamental and fundamental shocks using a uniform prior distribution over the interval $[-1, 1]$. The resulting model is estimated using Bayesian techniques as in the baseline analysis.

Table A1 reports the parameter estimates. The parameter estimates turn out to be quite similar to the baseline results, except for the standard deviation of the animal spirit shock which now turns out to be higher than before and the correlations of the animal spirit shock with the fundamental shocks which appear only in the Bianchi-Nicolò method.[28] Nevertheless, as Table A2 shows, the forecast error variance decompositions are virtually indistinguishable from our baseline results.

[28] The higher standard deviation of the animal spirit shock is driven by the alternative way of introducing non-fundamental shocks in the estimation under the Bianchi-Nicolò method.

Table A1: Prior and posterior distributions (Bianchi-Nicolò method)

Prior					Posterior	
Name	Range	Density	Mean	Std. Dev.	Mean	90% Interval
ν	R^+	Normal	0.11	0.05	0.24	[0.20,0.27]
b	[0,1)	Beta	0.5	0.1	0.43	[0.35,0.52]
ψ_A	[0,1)	Beta	0.5	0.2	0.00	[0.00,0.01]
ψ_z	[0,1)	Beta	0.5	0.2	0.82	[0.75,0.89]
ψ_Δ	[0,1)	Beta	0.5	0.2	0.97	[0.96,0.99]
ψ_g	[0,1)	Beta	0.5	0.2	0.99	[0.99,0.99]
ψ_{ag}	[0,1)	Beta	0.5	0.2	0.70	[0.52,0.89]
σ_s	R^+	Inverse Gamma	0.1	Inf	0.47	[0.43,0.52]
σ_A	R^+	Inverse Gamma	0.1	Inf	0.61	[0.55,0.68]
σ_z	R^+	Inverse Gamma	0.1	Inf	0.07	[0.04,0.10]
σ_Δ	R^+	Inverse Gamma	0.1	Inf	0.79	[0.67,0.91]
σ_g	R^+	Inverse Gamma	0.1	Inf	0.80	[0.71,0.88]
$\sigma^{m.e.}$	$[0, 0.18]$	Uniform	0.09	0.05	0.18	[0.18,0.18]
$\rho_{s,A}$	[-1,1]	Uniform	0	0.58	-0.56	[-0.67,-0.46]
$\rho_{s,z}$	[-1,1]	Uniform	0	0.58	0.23	[0.14,0.31]
$\rho_{s,\Delta}$	[-1,1]	Uniform	0	0.58	0.63	[0.51,0.74]
$\rho_{s,g}$	[-1,1]	Uniform	0	0.58	0.07	[-0.02,0.17]
$\varkappa$	[-20,0]	Uniform	-10	5.77	-5.03	[-7.32,-2.66]

Table A2: Unconditional variance decomposition (Bianchi-Nicolò method)

	$\ln\left(\frac{Y_t}{Y_{t-1}}\right)$	$\ln\left(\frac{C_t}{C_{t-1}}\right)$	$\ln\left(\frac{I_t}{I_{t-1}}\right)$	$\ln\left(\frac{G_t}{G_{t-1}}\right)$	$\ln\left(\frac{H_t}{H}\right)$	$spread_t$
ε_t^s	11.64	0.56	20.98	0.00	2.27	0.00
ε_t^A	30.73	38.44	18.48	9.33	11.03	0.00
ε_t^z	18.27	0.82	30.76	0.00	19.29	100
ε_t^Δ	30.88	60.06	22.72	0.00	54.87	0.00
ε_t^g	2.12	0.12	7.06	90.67	12.55	0.00
$\varepsilon_t^{m.e.}$	6.35	0.00	0.00	0.00	0.00	0.00

Table A3: Log marginal densities (modified harmonic mean)

Indeterminacy, $\chi = 0$	Determinacy, $\chi = 0$	Determinacy, $\chi = 1$	Determinacy, $\chi = 5$
-770.62	-897.94	-868.20	-886.26

Table A4: Business cycle dynamics for the indeterminacy and determinacy economies

	Data		Indeterminacy $\chi = 0$		Determinacy $\chi = 1$	
x	σ_x	$\rho(x, \ln(Y_t/Y_{t-1}))$	σ_x	$\rho(x, \ln(Y_t/Y_{t-1}))$	σ_x	$\rho(x, \ln(Y_t/Y_{t-1}))$
$\ln(Y_t/Y_{t-1})$	0.58	1	0.71	1	0.95	1
$\ln(C_t/C_{t-1})$	0.47	0.67	0.59	0.56	0.65	0.71
$\ln(I_t/I_{t-1})$	1.66	0.79	3.01	0.78	3.40	0.90
$\ln(G_t/G_{t-1})$	0.77	0.25	0.83	0.08	0.99	0.29
$\ln(H_t/H)$	6.16	0.20	4.88	0.10	11.86	0.03
spread_t	0.60	-0.58	0.59	-0.24	0.67	-0.01

A.6 Estimation results with less elastic labor supply

This Appendix compares the fit of the indeterminate economy estimated in Section 4 against determinacy versions of the model with a less elastic labor supply, i.e., $\chi > 0$. Since the animal spirits shock is no longer available under determinacy, and in order to keep the number of shocks equal to the observables, we add a temporary technology innovation A_t^T that affects all firms equally and with persistence ψ_T and variance σ_T. For example, the output of an ordinary firm is now

$$x_t(i) = A_t^T \kappa_t(i)^\alpha \left[A_t h_t(i)\right]^{1-\alpha} - \phi_{n,t}$$

where

$$\ln A_t^T = (1 - \psi_T) \ln A^T + \psi_T \ln A_{t-1}^T + \varepsilon_t^T.$$

You will see from Table A3's log-data densities, for the determinacy model versions, an elasticity of labor supply that implies $\chi > 0$ improves the fit of the estimated model. However, U.S. data continues favoring the indeterminate model. The main reason rests on the indeterminate model better matching the volatility and cyclicality of hours worked. This is shown in Table A4, which compares the second moments between the data, the indeterminate model with indivisible labor, and the best-fitting determinate model with $\chi = 1$. Specifically, hours worked turns out to be twice as volatile in the determinate model compared to the data, while the indeterminate model fits the data relatively well. In addition, hours worked are mildly procyclical in both the data and the indeterminate model, but are acyclical in the determinate model.