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Keywords

global r-star, Beveridge-Nelson decomposition, block exogeneity, foreign shocks

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How important is global r-star for open economies?*†

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1 Introduction

The neutral interest rate, henceforth r-star or r^*, is a key variable in setting monetary policy. Conceptually, it measures the level of the short term real interest rate where monetary policy would be neither contractionary nor expansionary, although other definitions of r-star are often considered and measured (Reis, 2025). While r-star is unobserved and known to be estimated with a large degree of imprecision, the consensus narrative is that it has fallen in the last two decades to very low, perhaps even negative levels in the aftermath of the global financial crisis. While there is some debate and it is unclear if r-star has increased more recently since the pandemic, it is generally accepted that it is still at very low levels in historical terms (see, e.g., Lunsford and West, 2019; Gourinchas, Rey, and Sauzet, 2022). Because the consensus view is that r-star has fallen not just for the United States, but more broadly internationally, if one accepts that risk-adjusted returns should equalise internationally under perfect capital mobility, a natural conclusion that one would draw is that r-stars in open economies have an important global component (see, e.g., Gourinchas et al., 2022; Clarida, 2019).

Our main contribution is to quantify the role of global r-star for a set of advanced open economies; namely Australia, Canada, the Euro Area, New Zealand, Norway, Sweden, and the United Kingdom. Apart from the Euro Area, which is still subject to exogenous global forces, all of these economies can be plausibly described as *small* open economies and have been modelled as such in prior work. We nonetheless add that the question we are posing is somewhat different from most of the extant literature on global r-star. To be precise, while there has been considerable research on measuring the level of global r-star and quantifying why it may have fallen due to factors such as safe assets, demographics, and productivity (e.g., Rachel and Summers, 2019; Cesa-Bianchi, Harrison, and Sajedi, 2022; Ferreira and Shousha, 2023), our focus is instead on asking how much global forces play a role in the determination of domestic r-stars from the perspective of a relatively small open economy, although also see Zhang, Martínez-García, Wynne, and Grossman (2021) for a country-by-country analysis of r-star for a similar set of small open economies.[1] While determining the drivers of global r-star is an interesting issue in its own right, we argue that a consideration of what drives domestic r-stars may also be important for two reasons. First, for domestic monetary policy, it is the domestic r-star that is important for a central bank to gauge the stance of their policy. Second, given that a small open economy takes

[1] Zhang et al. (2021) identify the role of foreign shocks using a structural model estimated with domestic variables, the effective exchange rate, and terms of trade and compare the fit of a closed-economy version of the model to an open-economy version. They find the open-economy model fits better for the six economies they consider of Australia, Canada, South Korea, Sweden, Switzerland, and the United Kingdom, with the estimated r-stars appearing to co-move for the different economies, similar to what is found in other studies that consider country-by-country estimates such as Clarida (2019) and Han and Ma (2024). However, they do not directly measure the importance of global shocks for the country-level r^*'s.

foreign conditions as given, the foreign determinants of domestic r-star are clearly beyond the control of domestic policymakers. A natural question is whether there is any room for local factors to influence the domestic r-star. While the domestic monetary authority has to take the level of domestic r-star as given for short to medium run stabilisation policy, factors such as domestic government debt and productivity differentials relative to the rest of the world are outcomes that could be partly due to domestic economic policies, begging the question of whether, and if so how much, local factors determine domestic r-star.

We document three key results. First, r-star for the United States can be treated as global r-star from perspective of the seven open economies we consider. In particular, we find that the shocks that drive the U.S. r-star are close to being the only foreign shocks that drive the domestic r-stars. This is a useful result because the extant literature usually takes a more complicated approach of estimating global r-star by postulating a factor structure of interest rates across economies (e.g., Del Negro, Giannone, Giannoni, and Tambalotti, 2019). While this is a reasonable way to measure global r-star, our objective is more about quantifying the role of global r-star for open economies than just measuring global r-star itself. Recognising that shocks driving the U.S.'s r-star are also a function of the shocks that drive global r-star, it suffices for our approach to only consider U.S. interest rates (as opposed to a large cross-section of interest rates) for capturing the foreign block of our model of foreign and domestic interest rates. Our results suggest that this is a reasonable approximation. Second, we find local shocks matter for domestic r-stars, and their share is non-trivial and often substantial. This result suggests at least part of the determination of r-star for the seven open economies may be due to domestic economic policies. The fact that local shocks matter for r-star also provides an interpretation for why the literature has often found evidence against real interest rates cointegrating one-for-one across economies, contrary to what would be expected under perfect capital mobility. Third, we find that, despite domestic shocks mattering, the general decline of domestic r-stars since the mid 2000s (and especially post global financial crisis) has been largely due to global shocks. While there is some degree of heterogeneity across the economies we consider, the lowering of global r-star since the global financial crisis can account for somewhere between a large to overwhelming amount of the overall decline in the domestic r-stars. For some economies such as Canada and the Euro Area, the counterfactual r-star would have changed very little relative to 2007 in the absence of the shocks to global r-star.

From a modelling standpoint, we build on Morley, Tran, and Wong (2024) (MTW hereafter). MTW show how to estimate r-star by applying a multivariate Beveridge and Nelson (1981) using a standard Bayesian VAR, with a correction for any apparent model misspecification by applying a secondary univariate Beveridge-Nelson decomposition to the preliminary estimates. However, we depart from MTW in two important ways to directly tackle our question of interest. First, we extend the Bayesian VAR model into an open economy setting. Building on existing small open economy models that distinguish between foreign and domestic blocks (e.g., Zha, 1999; Justiniano and Preston, 2010; Kamber and Wong, 2020), the two block structure enables us to estimate domestic r-star for the small open economy while accounting for the foreign block. While this approach allows us to *estimate* the level of domestic r-star incorporating foreign data, it does not in itself capture the role of foreign shocks in driving domestic r^*. Thus, our second departure from MTW is to account for whether movements in domestic r^* are due to identified foreign or domestic shocks. The two block structure quantifies the role of foreign shocks driving r^* by drawing on a thriving small open economy literature which explicitly identifies the role of foreign shocks in determining domestic variables (e.g., Zha, 1999; Justiniano and Preston, 2010). While the identification of foreign shocks naturally stems from the SVAR literature, our approach builds more specifically from Kamber and Wong (2020), who show that one can link the identified structural shocks to changes in the Beveridge-Nelson trend and cycle. We therefore adopt the Kamber and Wong (2020) approach, but allowing for the MTW correction.

We see our paper as contributing directly to two threads of what is now a fairly broad literature on r-star. First, we contribute directly to the measurement of r-star. A key reference paper in this body of work is Holston, Laubach, and Williams (2017), who apply the well-known Laubach and Williams (2003) model developed for the U.S. economy to Canada, the Euro Area, and the United Kingdom, and draw the broad conclusion that r-star seems to be falling internationally.[2] Because the Laubach and Williams (2003) model was developed for the U.S. as a closed economy, the Holston et al. (2017) extension to the non-U.S. economies does not allow for an explicit role of global r-star, apart from recognising that the estimates for other economies appear to be falling similarly to those for the U.S. Using a different methodology, Kiley (2020) discusses how not allowing for a global dimension may imply an over-estimation of U.S. r-star. Del Negro et al. (2019), Ferreira and Shousha (2023), and Cesa-Bianchi et al. (2022) all model open economy features to allow for global influences on r-star. From a measurement perspective, our approach is very much tied to MTW. Nonetheless, since MTW builds on the Beveridge-Nelson decomposition, it is also related to unobserved components models such as considered by Laubach and Williams (2003), Del Negro et al. (2019), and, more recently, Schmitt-Grohé and Uribe (forthcoming), since the Beveridge-Nelson decomposition is directly linked to unobserved components models given the Beveridge-Nelson trend being the long-horizon forecast and r-star assumed to be a random walk for the unobserved components models (see Morley, Nelson, and Zivot, 2003). Second, we contribute to understanding the role global factors play in the fall of domestic r-star

[2] While widely applied, the Laubach and Williams (2003) model and its estimation are not immune to criticisms (see Berger and Kempa, 2019; Buncic, Pagan, and Robinson, 2023).

for open economies. Del Negro et al. (2019), Kiley (2020), Hamilton, Harris, Hatzius, and West (2016), and Ferreira and Shousha (2023) model how global factors matter for r-star, although their focus varies from quantifying why global r-star fell to what this implies for the U.S. economy. By contrast, while we consider how much global factors matter for r-star, we do not directly consider why domestic r-stars fell beyond whether shocks are global or domestic in origin. We note that viewing factors such as demography, productivity, or safe assets makes it difficult to disentangle whether these declines in r^* are global or domestic in origin given almost all advanced economies are undergoing similar demographic transitions, and factors such as productivity or risk premiums can be endogenous in practice, making it challenging to separate cause and effect.[3] However, to the extent that the U.S. r-star is a good proxy for global r-star, the findings in MTW for a multivariate Beveridge-Nelson decomposition using a wide range of variables, but producing very similar estimates to those we find for the U.S., support the idea that changes in global r-star are driven by a mix of U.S.-related productivity, demographic, and safe-asset supply and demand factors, as might be expected under a high degree of integration across international asset markets (see Del Negro et al., 2019).

The rest of this paper is structured as follows. Section 2 presents a simple theoretical framework to motivate our empirical approach. Section 3 presents the methods used to estimate r-star using the Beveridge-Nelson decomposition together with MTW correction in an open economy context that links r-star to identified domestic and foreign shocks. Section 4 discusses the data and estimation. Section 5 reports our empirical results. Section 6 considers policy implications and offers some concluding remarks.

2 Theoretical framework

We first describe a theoretical framework to guide our empirical strategy. This framework is a simplified version of the theory presented in Del Negro et al. (2019).

Suppose we have investors based in two economies. While the framework is sufficiently general to consider any arbitrary pair of economies, we specifically consider a large vs. small open economy. For relatability, we label the large economy "the U.S." and the small open economy "Canada". These investors trade one period bonds denominated in U.S. (US\$) and Canadian dollars (C\$). Let $i_t^{US\$}$ be the nominal yield on the bond. Let M_{t+1}^{US} be the marginal rate of substitution between consumption today and tomorrow for the U.S. investor (i.e., the U.S. stochastic discount factor), and $P_t^{US\$}$ be the price of consumption

[3]We note that it is the sort of argument that led to approaches such as Rachel and Summers (2019) and Cesa-Bianchi et al. (2022) that just treat a set of advanced economies as one large closed economy.

in U.S. dollars. The, the pricing equation will be

$$\mathbb{E}_t\left[M_{t+1}^{U.S.}(1+\chi_t^{US\$})(1+i_t^{US\$})\frac{P_t^{US\$}}{P_{t+1}^{US\$}}\right]=1. \tag{1}$$

The $\chi_t^{US\$}$ term reflects preferences for U.S. bonds. One interpretation includes the "convenience yield" notion in Del Negro et al. (2019), but for our purpose, we consider any premium that investors are willing to place on U.S. assets. These could include preferences for the U.S. dollar given its central role in the global financial system as a safe haven asset or a particular faith that the U.S. government will repay its debt.

The analogous equation for how U.S. investors price the Canadian bond is as follows:

$$\mathbb{E}_t\left[M_{t+1}^{US}(1+\chi_t^{C\$})(1+i_t^{C\$})\frac{S_{t+1}}{S_t}\frac{P_t^{US}}{P_{t+1}^{US}}\right]=1, \tag{2}$$

where $i_t^{C\$}$ is the yield on the Canadian bond, and S_t is the spot nominal exchange rate, defined as $\frac{US\$}{C\$}$. Taking a first order log-linear approximation of equations (1) and (2), we can write the long-run components as follows (see Del Negro et al., 2019):[4]

$$r_{U.S.,t}^* \approx \quad \bar{r}_t^{US\$} = \quad \bar{m}_t^{US} - \bar{\chi}_t^{US\$}, \tag{3}$$

$$r_{Can,t}^* \approx \quad \bar{r}_t^{C\$} = \quad \bar{m}_t^{US} - \bar{\chi}_t^{C\$} - \Delta\bar{q}_t, \tag{4}$$

where $\bar{x}_t$ represents the Beveridge and Nelson (1981) (BN herafter) trend of a variable x_t, with $\bar{x}_t = \lim_{j\to\infty}\mathbb{E}_t x_{t+j} - \delta t$ and δ representing any deterministic drift in x_t. $\Delta\bar{q}_t$ is the trend of the change in the natural log of the real exchange rate. We define $r_t^i \equiv ln(1+i_t^i) - \mathbb{E}_t\pi_{t+1}^i$, where $\pi_t^i = ln(P_t^i/P_{t-1}^i)$ is the inflation rate, and $m_t^i = -\mathbb{E}_t lnM_{t+1}^i$ is the negative of the expected growth of marginal utility. We define r^* to be the trend of the (simple, not continuously compounded) real interest rate, thus equating $e^{\bar{r}}-1$ with r^*. Del Negro et al. (2019) highlight that, at the very minimum, the term $\bar{m}_t^{US}$ in equations (3) and (4) creates a common factor structure for interest rates across all economies, motivating them to extract a common global factor for r^*.

We can use equations (3) and (4) to make three particular points to help guide our empirical strategy:

Beveridge-Nelson trend: First, any empirical strategy that attempts to extract r^* using a BN trend is consistent with the framework that we have set up. In this regard, while Del Negro et al. (2019) and Laubach and Williams (2003) use multivariate unobserved

[4] In principle, higher order moments may matter, but we keep with much of the empirical literature (e.g., Del Negro et al., 2019; Lunsford and West, 2019) by considering just the first order terms. Del Negro et al. (2019) also allow an additional term that reflects the preferences of domestic investors, which we abstract from given that it does not play any role in our exposition.

components models to estimate r^*, it is known that multivariate unobserved components models have an VARIMA representations and that the BN trend for the alternate representation is equivalent to the Kalman-filtered trend for the unobserved components model. Indeed, Del Negro et al. (2019) explicitly refer to the variables in equations (3) and (4) as "Beveridge and Nelson" trends. Our empirical strategy directly makes use of extracting the BN trend of real interest rates from Bayesian VARs, as motivated by MTW, but it should be clear that the underlying concept that our empirical strategy appeals to has a lot in common with using unobserved components models to estimate r^*.

A two-block model with global and domestic shocks Second, while Del Negro et al. (2019) choose to work with the factor structure representation across the r^*'s, we can also view equations (3) and (4) as a two-block model of the U.S. and Canada r^*'s, where the permanent shocks to real interest rates are the shocks to the terms on the right hand side (i.e., $\bar{m}_t^{US}$, $\bar{\chi}_t^i$ and $\bar{\Delta} q_t$). In VAR or DSGE settings, these permanent shocks can originate from the U.S., Canada, or even a third economy, but the exposition should make clear that common permanent shocks will be the basis of any comovement in r^* across the two economies and, by extension, globally. Consider, for example, typical small open economy models such as Zha (1999) and Justiniano and Preston (2010), which model the U.S. as the large economy and the other country as the small open economy. This is a typical identification strategy in the structural VAR literature that relies only on a block exogeneity assumption with the small open economy too small to affect the world. Within this setting of block exogeneity, the U.S. shocks are global shocks. Everything that determines U.S. r^* in this setting is a function of foreign shocks and the small open economy's r^* is driven by both foreign and domestic shocks. From equations (3) and (4), such a structure will imply that only foreign and, by extension, U.S. shocks determine $\bar{m}_t^{U.S.}$ and $\bar{\chi}_t^{U.S.}$. Canadian r^* will be determined by global shocks through $\bar{m}_t^{U.S.}$, but possibly also through $\bar{\chi}_t^{C\$}$ and $\Delta \bar{q}_t$. The latter terms will also capture local shocks if these shocks are indeed important in determining Canadian r^*. Furthermore, equations (3) and (4) also clarify how cointegration of real interest rates could occur. In the above example, U.S. and Canadian real interest rates will cointegrate $[1 - 1]$ if and only if there are no permanent shocks to their respective risk premia, $\chi^{US\$}$ and $\chi^{C\$}$, or the drift in the real exchange rate, $\mathbb{E}[\Delta q_t]$. Suppose for sake of argument that $\bar{\chi}^{US\$} = \Delta \bar{q}_t = 0$. Whether real interest rates between the two economies cointegrate or not depends on $\bar{\chi}_t^{C\$}$. If shocks to the risk premium of the Canadian bond were not permanent, but there is some constant spread that investors demand to hold Canadian assets in the long run, real interest rates will cointegrate $[1 - 1]$ with a constant difference of $\bar{\chi}^{C\$}$. Suppose instead that shocks to the Canadian risk premium were permanent. We can now see that r^*_{Can} would be a sum of two random walks. Real interest rates in both economies would not cointegrate because $\bar{\chi}_t^{C\$}$ would act as a wedge stopping the two interest rates from cointegrating. However, the two interest rates would cointegrate *conditionally* when conditioning only on shocks to $\bar{m}_t^{U.S.}$. Specifically, the effects of a shock to $\bar{m}_t^{U.S.}$ on $r^*_{U.S.}$ and r^*_{Can} will be one-for-one.

Interpreting drivers of r^* Third, given the structure implied by equations (3) and (4) would naturally emerge from most general equilibrium models, explanations for changes in r^* using such models are embedded within our framework, albeit via a different lens of interpretation. The economy-specific risk premiums are sufficiently general to accommodate interpretations related to safe assets or convenience yields. We can also show that the stochastic discount factor m_t^i is sufficiently flexible to accommodate many commonly postulated drivers of r^*. For example, in a representative agent framework with standard CRRA preferences, the stochastic discount factor can be written as follows:

$$M_{t+1} = \beta \left(\frac{C_{t+1}}{C_t} \right)^{-\sigma}, \tag{5}$$

where β is the discount factor of households, $1/\sigma$ is the intertemporal elasticity of substitution, and the $\frac{C_{t+1}}{C_t}$ term in parentheses is the growth rate of per capita consumption. Therefore, in terms of the Beveridge-Nelson trend, at least with this functional form for utility, the $\bar{m}_t$ term would be the following linear function of trend consumption growth:

$$\bar{m}_t = -\ln \beta + \sigma \Delta \bar{c}_t. \tag{6}$$

This is an interpretation that is closest to Laubach and Williams (2003), who link trend growth for real activity directly with r^*. The point is that the $\bar{m}_t$ term accommodates common interpretations of what drives r^* such as productivity and demographics that could affect the stochastic discount factor. Nonetheless, where our approach differs from much of the extant work is that we do not directly ascertain these individual drivers, but by using block exogeneity, we seek to understand whether these drivers originate globally or locally. For example, we can rewrite equation (4) in terms of the Canadian investor,

$$\bar{r}_t^{C\$} = \bar{m}_t^{Can} - \bar{\chi}_t^{C\$}, \tag{7}$$

which combined with equation (4) implies

$$\bar{m}_t^{Can} = \bar{m}_t^{US} - \Delta \bar{q}_t. \tag{8}$$

If the drift in $\bar{m}_t^i$ were driven by a technological leader such as the U.S., this would be a foreign shock from the perspective of Canada through the $\bar{m}_t^{US}$ term in equation (4). Furthermore, if Canadian trend growth were just the U.S. trend growth, then there would be no drift in the exchange rate (i.e., $\Delta \bar{q}_t = 0$) and the $\bar{m}_t^{Can}$ component would be fully determined by foreign shocks. However, if Canada experiences long-run productivity growth differentials relative to the U.S., this would be reflected as a domestic shock component in $\bar{m}_t^{Can}$ with domestic shocks also reflected in the drift in the real exchange rate, where equation (8) is effectively a version of the well known Balassa-Samuelson effect.

3 Empirical methods

The discussion in Section 2 motivates a two-block empirical model, where we will use the Beveridge and Nelson (1981) (BN) decomposition to estimate r^* for both the U.S. and the domestic open economy. We will also use the structure from our theoretical framework to identify permanent shocks since equations (7) and (8) make it clear that it is permanent shocks one needs to identify in order to distinguish between global and domestic shocks to r^*. In presenting the details of our empirical strategy, we first discuss how to obtain an estimate of r^* using a multivariate Beveridge-Nelson decomposition, with a correction for possible model misspecification. We then present our two-block open economy model before discussing shock identification. We also provide an interpretation of our empirical strategy as it relates to our theoretical framework. Throughout, we take r^* to be the trend of the short-term risk-free real interest rate, since this is the quantity for which r^* is often considered in policy settings and should not be subject to the various premia that can influence longer-term or riskier bonds.

3.1 Estimating r^* using the BN Decomposition

We first outline the concepts of how we estimate r^*, building on MTW. We employ the BN decomposition, taking r^* as the trend of the short-term real interest rate used in our analysis. The BN decomposition has proven a useful approach to separate trend from cycle in a wide variety of settings (e.g., Evans and Reichlin, 1994; Morley and Piger, 2012; Kamber, Morley, and Wong, 2018). Our approach is to apply the BN decomposition to a multivariate time series model (see Evans and Reichlin, 1994; Morley and Wong, 2020). Equating the stochastic trend component of a short-term real interest rate with its BN trend is implicit in a lot of the literature on r^*. In particular, given that the Kalman-filtered estimated trend from an unobserved components model and the BN trend for an equivalent reduced-form time series model are the same, as shown by Morley, Nelson, and Zivot (2003), methods such as Laubach and Williams (2003) and Del Negro et al. (2019) based on unobserved components models are consistent with the notion that r^* is the BN trend of the real interest rate.[5]

[5] Schmitt-Grohé and Uribe (forthcoming) consider an unobserved components model that allows correlation between trend and cycle movements, thus making it even more similar to the BN decomposition.

Letting r_t be the short term real interest rate at time t, define r_t^* as the long-horizon conditional forecast of the *level* of real interest rate at time t given the full relevant conditioning information Ω_t:

$$r_t^* \equiv \lim_{j\to\infty} \mathbb{E}_t \left[r_{t+j} \mid \Omega_t \right]. \tag{9}$$

Let $\boldsymbol{X}_t$ be a vector of variables with Δr_t, the first difference of the short term real interest rate be its k^{th} element. We can write the law of motion of the state equation in $\boldsymbol{X}_t$ in state-space or companion form:

$$\boldsymbol{X}_t = \boldsymbol{B}\boldsymbol{X}_{t-1} + \boldsymbol{H}\boldsymbol{e}_t, \tag{10}$$

where $\boldsymbol{B}$ is a companion matrix whose eigenvalues are all within the unit circle, $\boldsymbol{e}_t$ is a vector of stationary, and possibly serially uncorrelated, forecast errors with covariance matrix $\boldsymbol{\Sigma}$, and $\boldsymbol{H}$ is a matrix which maps the forecast errors to the companion form. Following Morley (2002) and defining $\boldsymbol{\iota}_k$ as a selector row vector with 1 as its k^{th} element and zero otherwise, an estimate of r_t^* conditional on the model implied by equation (10), which we denote as $\hat{r}_t^*$, is

$$\hat{r}_t^* = \lim_{j\to\infty} \mathbb{E}_t \left[r_{t+j} \mid \hat{\Omega}_t \right] = r_t + \boldsymbol{\iota}_k \boldsymbol{B}(\mathbf{I} - \boldsymbol{B})^{-\mathbf{1}}\mathbf{X_t}. \tag{11}$$

where $\hat{\Omega}_t \equiv \{\boldsymbol{B}, \boldsymbol{X}_t, \boldsymbol{X}_{t-1}, \boldsymbol{X}_{t-2}, \ldots\}$. We make the 'hat' notation explicit in equation (11) because the $\hat{r}_t^*$ estimate is based on potentially a smaller information set, $\hat{\Omega}_t$ such that it does not necessarily coincide with r_t^*, which can only be perfectly recovered with a potentially larger information set, Ω_t.

As MTW show, a smaller information set may manifest as model misspecification, such as the possibility of there being error due to indirectly measuring real interest rates using a proxy for inflation expectations. According to the BN decomposition, the estimate of r^* by applying equation (11) should be a random walk, where the first difference of $\hat{r}^*$ should form a martingale difference sequence. MTW show that possible misspecification of the the model in equation (10) will lead to serial correlation in the first difference of $\hat{r}^*$ which could take the form of a complicated ARMA structure.[6] In this case, this serial correlation can be modelled to correct the preliminary estimate $\hat{r}^*$. Let $\theta(L)$ be a possibly infinite-order lag polynomial where $\theta(L) = \sum_{j=0}^{q} \theta_j L^q$, with $\theta_0 = 1$. The first difference

[6] We note that the exposition so far allows for $\boldsymbol{e}_t$ to be possibly serially correlated. Ideally, an analyst would include a sufficient set of variables in equation (10) such that there is no evident residual serial correlation in $\boldsymbol{e}_t$. Nonetheless, as MTW show, $\boldsymbol{e}_t$ can generally appear to be serially uncorrelated, but due to aggregation of small amounts of serial correlation in individual forecast errors, $\Delta\hat{r}_t^*$ can exhibit a higher degree of serial correlation.

of the preliminary estimate can be written as[7]

$$\Delta \hat{r}^*_t = \theta(L)\eta_t. \tag{12}$$

Applying a univariate BN decomposition conditional on the process implied by equation (12), a corrected estimate of r^*, which we denote as $\tilde{r}^*$, is the univariate BN trend of $\hat{r}^*$ such that

$$\tilde{r}^*_t = \tilde{r}^*_{t-1} + \theta(1)\eta_t \tag{13}$$

and, through the law of iterated expectations, $\tilde{r}^*$ is consistent with

$$\tilde{r}^*_t = \lim_{j\to\infty} \mathbb{E}_t \left[\hat{r}^*_{t+j} \mid \hat{\Omega}_t, \theta(L) \right]. \tag{14}$$

This corrected estimate of r^* is effectively based on a larger information set relative to what is implied under equation (11) in that it is consistent with estimating r^* using the BN decomposition with the information set $\hat{\Omega}_t$ and $\theta(L)$. We also note that even if $\Delta\hat{r}^*$ is not a martingale difference sequence, and hence the need to apply the correction, $\Delta\tilde{r}^*$ will be serially uncorrelated due to the consideration of enough moving average terms to remove any serial correlation. Finally, no *a priori* assumption about whether the correction results in a smoother or more volatile estimate of r^* is being made, although one could impose a smoothing prior by setting a prior mean that $\theta(1) < 1$. In particular, the correction means that fluctuations in $\tilde{r}^*_t$ are re-scaled forecast errors for $\hat{r}^*_t$, as implied by equation (13). Then, if $\theta(1) < 1$ and considering an invertible representation, which we impose in estimation, $|\theta(1) - 1| < 1$ and the variance of $\theta(1)\eta_t$ in equation (13) will necessarily be less than the variance of $\theta(L)\eta_t$ in equation (12).

3.2 A two-block model

Our empirical specification builds on MTW by blending in features of empirical small-open economy models such as Zha (1999), Justiniano and Preston (2010), and Kamber and Wong (2020). The core ingredients of such models are to model separate foreign and domestic blocks, where the domestic block, being the small open economy, is too small to affect the foreign block. The latter will be a key identification assumption which dichotomises all of the shocks in the model to either being foreign or domestic in origin, with only foreign shocks affecting variables in the foreign block, while both foreign and domestic shocks can affect variables in the domestic block.

Let $\boldsymbol{Y}_t$ be a vector of variables, containing vectors of variables from the foreign and

[7] We write equation (12) as a possibly infinite-order MA process because all that is needed to apply the correction for serial correlation is the implied MA form. This form is sufficiently flexible because even if $\Delta\hat{r}_t$ can be represented by an ARMA, this would imply $\rho(L)\Delta\hat{r}^*_t = \vartheta(L)\eta_t$, but through the Wold theorem, $\theta(L) = \rho(L)^{-1}\vartheta(L)$.

domestic block respectively which we label $\boldsymbol{Y}_t^F$ and $\boldsymbol{Y}_t^D$. A two-block open economy VAR model is given as follows:

$$\boldsymbol{Y}_t = \boldsymbol{\Phi}_1 \boldsymbol{Y}_{t-1} + \ldots + \boldsymbol{\Phi}_p \boldsymbol{Y}_{t-p} + \boldsymbol{e_t}, \tag{15}$$

$$\begin{bmatrix} \boldsymbol{Y}_t^F \\ \boldsymbol{Y}_t^D \end{bmatrix} = \begin{bmatrix} \boldsymbol{\Phi}_{11}^1 & \boldsymbol{0} \\ \boldsymbol{\Phi}_{21}^1 & \boldsymbol{\Phi}_{22}^1 \end{bmatrix} \begin{bmatrix} \boldsymbol{Y}_{t-1}^F \\ \boldsymbol{Y}_{t-1}^D \end{bmatrix} + \ldots + \begin{bmatrix} \boldsymbol{\Phi}_{11}^p & \boldsymbol{0} \\ \boldsymbol{\Phi}_{21}^p & \boldsymbol{\Phi}_{22}^p \end{bmatrix} \begin{bmatrix} \boldsymbol{Y}_{t-1}^F \\ \boldsymbol{Y}_{t-1}^D \end{bmatrix} + \begin{bmatrix} \boldsymbol{A}_{11} & \boldsymbol{0} \\ \boldsymbol{A}_{21} & \boldsymbol{A}_{22} \end{bmatrix} \begin{bmatrix} \boldsymbol{\epsilon}_t^F \\ \boldsymbol{\epsilon}_t^D \end{bmatrix}, \tag{16}$$

$$\text{where } \boldsymbol{A} \begin{bmatrix} \boldsymbol{\epsilon}_t^F \\ \boldsymbol{\epsilon}_t^D \end{bmatrix} = \boldsymbol{e}_t, \boldsymbol{A} = \begin{bmatrix} \boldsymbol{A}_{11} & \boldsymbol{0} \\ \boldsymbol{A}_{21} & \boldsymbol{A}_{22} \end{bmatrix}, \boldsymbol{A}\boldsymbol{A}' = \mathbb{E}\boldsymbol{e}_t\boldsymbol{e}_t' = \boldsymbol{\Sigma}, \quad \begin{bmatrix} \boldsymbol{\epsilon}_t^F \\ \boldsymbol{\epsilon}_t^D \end{bmatrix} \sim (\boldsymbol{0}, \boldsymbol{I}). \tag{17}$$

Here, $\boldsymbol{e}_t$ is a vector of forecast errors from all the equations, which is analogous to $\boldsymbol{e}_t$ in equation (10) when the model is cast in state-space form. The sub-vectors $\boldsymbol{\epsilon}_t^F$ and $\boldsymbol{\epsilon}_t^D$ reflect underlying foreign and domestic shocks, respectively, which are all normalised to have unit variance and to be orthogonal to one another. Note that, in equation (16), we are setting $\boldsymbol{\Phi}_{1,2}^j = \boldsymbol{0}$, $j = \{1, \ldots p, \}$, where the blocks of zeros are imposing the small-open economy block exogeneity restriction that the domestic economy is too small to affect the large foreign economy.

We briefly note that, unlike block exogeneity strategies that directly impose the restriction $\boldsymbol{A}_{12} = \boldsymbol{0}$ in estimation, we will use an alternative approach that *recovers* structural shocks for which $\boldsymbol{A}_{12} = \boldsymbol{0}$. We will clarify this issue further when we discuss the identification of shocks, but for now, it suffices to note that the identification strategy will impose $\boldsymbol{A}_{12} = \boldsymbol{0}$. Even so, we are deliberate in explicitly expressing $\boldsymbol{A}_{12} = \boldsymbol{0}$ in equation (16) because only such a structure in which domestic shocks do not affect the foreign economy is consistent with standard empirical small open economy models. In other words, despite not directly imposing such a restriction in estimation, we still require $\boldsymbol{A}_{12} = \boldsymbol{0}$ in order to interpret equation (16) as a standard empirical small open economy model like in Zha (1999), Justiniano and Preston (2010), Kamber and Wong (2020), etc.

MTW showed in the U.S. context that, because the term spread consists of almost all of the useful information for identifying the transitory variation in interest rates, an appropriate and parsimonious approach to estimating r^* would be to use a bivariate model with a short and long term real interest rate, where we allow for a common stochastic trend between the short and long term interest rate. The one common trend effectively imposes a restriction that implies either a bivariate vector error correction model between the short and long term interest rate or a bivariate vector autoregression with the first difference of one of the interest rates and the term spread (see King, Plosser, Stock, and Watson, 1991). We adopt the latter approach here as it makes it more straightforward to estimate and later impose the required restrictions to identify the structural shocks in the model. Thus, the following is our baseline specification of the variables:

$$\boldsymbol{Y}_t^F = \begin{bmatrix} \Delta r_{U.S.,t}^L \\ r_{U.S.,t}^L - r_{U.S.,t}^S - \mu_{U.S} \end{bmatrix}, \boldsymbol{Y}_t^D = \begin{bmatrix} \Delta r_{Dom,t}^L \\ r_{Dom,t}^L - r_{Dom,t}^S - \mu_{Dom} \end{bmatrix}, \tag{18}$$

where r denotes a real interest rate, the superscripts S or L refer to short- and long-term real interest rates, respectively, and the subscripts $U.S.$ and Dom denote the U.S. and domestic small open economy, respectively. The μ's are the mean or steady-state term spreads between two interest rates.[8]

To summarise, we have a two-block model consisting of a foreign block with U.S. variables and a domestic block with variables for a small open economy. The small open economy is too small to affect the foreign block, which implies block exogeneity and will be used to identify foreign and domestic shocks. Each block is individually a bivariate VAR, or equivalently a bivariate VECM with the term spread being the cointegrating error. The model is also set up in a manner where the foreign block can be viewed as the bivariate version of the MTW model estimated using U.S. data and indeed produces similar estimates to MTW.

Under the implied model structure, one can use the Beveridge-Nelson decomposition to obtain the preliminary estimate of r^* for both the U.S. and the small open economy, consistent with equation (11). If either or both of these preliminary estimates contains serial correlation, possibly due to model misspecification, we can apply the ARMA correction proposed by MTW to these preliminary estimates as individual steps, following the logic of equations (12) to (14). We can thus obtain estimates of r^* for both the U.S. and domestic economy through the same model.

3.3 Shock identification

Estimation of the model in equation (15) will recover the VAR coefficients (i.e the $\boldsymbol{\Phi}$'s) and the forecast errors with their associated covariance matrix (i.e., $\boldsymbol{e}_t$ and $\boldsymbol{\Sigma}$). These provide enough information to apply the BN decomposition and the associated correction following MTW to provide estimates of r^* for both the U.S. and the domestic small open economy. However, in order to interpret movements in r^* as arising from global or local forces, we need to identify and recover the domestic and foreign shocks as in equation (16) (i.e., $\boldsymbol{\epsilon}_t^F$ and $\boldsymbol{\epsilon}_t^D$).

[8] It should make little difference whether one chooses the use the first difference of the long or short rate in the model in a large sample. However, we opt to use the long rate given the zero lower bound at some points of the sample would imply that the only variation for the short rate is coming from changes in inflation. If there are shifts in the natural rate during the period of the zero lower bound, these will be better reflected by the long rate, hence our choice to work with the long rate. An alternative approach would be to work with a "shadow" short rate, as in Han and Ma (2024), although these typically move with the long rate, so should produce similar results the approach we take.

For each block, we identify one permanent and one transitory shock. I.e.,

$$\boldsymbol{\epsilon}_t^F = \begin{bmatrix} \epsilon_{P,t}^F \\ \epsilon_{T,t}^F \end{bmatrix}, \boldsymbol{\epsilon}_t^D = \begin{bmatrix} \epsilon_{P,t}^D \\ \epsilon_{T,t}^D \end{bmatrix}, \tag{19}$$

where the P or T subscripts refer to the shock being permanent or transitory, respectively. Note that permanent refers to whether the shock has long-run effects on the real interest rate only in its the specific block (i.e., it says nothing about whether the shock is permanent or transitory for the interest rate in the other block). For example, the foreign permanent shock is permanent in terms of the U.S. real interest rate and the domestic permanent shock is permanent in terms of the domestic real interest rate.

By considering the permanent and transitory dichotomy, we can follow King and Morley (2007) to identify the shocks driving the BN trends by imposing long-run restrictions, where transitory shocks as those with no long-run effect on the *level* of the real interest rate in its specific block, as in the long-run identification approach considered by Blanchard and Quah (1989), King et al. (1991). To implement the restrictions needed to identify the shocks, define the lag polynomial $\boldsymbol{\Phi}(L) = \boldsymbol{I} - \sum_{j=1}^{p} \boldsymbol{\Phi}_j L^j$ and working off equation (15), we can write the model in its VAR and vector MA forms as

$$\boldsymbol{\Phi}(L)\boldsymbol{Y}_t = \boldsymbol{e}_t, \tag{20}$$

$$\boldsymbol{Y}_t = \boldsymbol{\Psi}(L)\boldsymbol{e}_t, \tag{21}$$

where $\boldsymbol{\Phi}(L)^{-1} = \boldsymbol{\Psi}(L) = \sum_{j=0}^{\infty} \boldsymbol{\Psi}_0 L^j$. Given we have rewritten the variables as a vector MA process in terms of the forecast errors in equation (21), the long-run multiplier to a unit innovation in any of the forecast errors is contained in $\boldsymbol{\Psi}(1)$. In order to describe the process in terms of the foreign and domestic shocks $\left[\boldsymbol{\epsilon}_t^F, \boldsymbol{\epsilon}_t^D\right]'$, we need to solve for $\boldsymbol{A}$ in equation (16) that is consistent with our structural model. Letting $\boldsymbol{C}(1)$ describe the long-run multipliers, or the cumulative effects, on the variables of the foreign and domestic shocks, $\boldsymbol{C}(1)$ needs to have a structure where $\boldsymbol{C}(1)\boldsymbol{C}(1)' = \boldsymbol{\Psi}(1)\boldsymbol{\Sigma}\boldsymbol{\Psi}(1)'$ for the both the auto and cross covariance structure of the variables to be consistent. As it turns out, the structural model that we have in mind can be obtained by solving for $\boldsymbol{C}(1)$ as a Cholesky decomposition of $\boldsymbol{\Psi}(1)\boldsymbol{\Sigma}\boldsymbol{\Psi}(1)'$, and subsequently solving for $\boldsymbol{A} = \boldsymbol{\Psi}(1)^{-1}\boldsymbol{C}(1)$. It is worthwhile recalling that, despite not directly imposing $\boldsymbol{A}_{12} = \boldsymbol{0}$ in equation (16), as is standard in small open economy models, $\boldsymbol{A}_{12} = \boldsymbol{0}$ follows from our identification scheme because $\boldsymbol{C}(1)$ is lower triangular.[9]

We now show how the estimated r^* from the Beveridge-Nelson decomposition is related

[9]This small open economy structure follows because the top right 2×2 blocks of the $\boldsymbol{\Phi}$'s, $\boldsymbol{\Psi}(1)$, and $\boldsymbol{C}(1)$ are a block of zeros, with the $\boldsymbol{\Phi}$'s being due to block exogeneity, $\boldsymbol{\Psi}(1)$ inheriting the block of zeros as an inverse of the matrix $\boldsymbol{\Phi}(1)$ with the same structure, and $\boldsymbol{C}(1)$ having the same block of zeros due to the identification scheme. Therefore, standard matrix multiplication will return the product of two such matrices, $\boldsymbol{A}$, as also inheriting the same block of zeros.

to the identified shocks. As a starting point, we first show how the forecast errors in the model pin down $\hat{r}^*$, which is the preliminary estimate of r^* from the model above. By applying the multivariate BN decomposition and taking the estimated trend in the previous period as given, let the first difference of the long rate be in the k^{th} position in the vector of variables. The preliminary estimate from the model $\hat{r}^*_t$ is[10]

$$\hat{r}^*_t = \hat{r}^*_{t-1} + \boldsymbol{\iota}_k \boldsymbol{\Psi}(1)\boldsymbol{e}_t, \tag{22}$$

$$\hat{r}^*_t = \hat{r}^*_{t-1} + \boldsymbol{\iota}_k \boldsymbol{C}(1) \begin{bmatrix} \boldsymbol{\epsilon}^F_t \\ \boldsymbol{\epsilon}^D_t \end{bmatrix}, \tag{23}$$

where

$$\boldsymbol{\Psi}(1) = \begin{bmatrix} \psi(1)_{1,1} & \psi(1)_{2,1} & 0 & 0 \\ \psi(1)_{2,1} & \psi(1)_{2,2} & 0 & 0 \\ \psi(1)_{3,1} & \psi(1)_{3,2} & \psi(1)_{3,3} & \psi(1)_{3,4} \\ \psi(1)_{4,1} & \psi(1)_{4,2} & \psi(1)_{4,3} & \psi(1)_{4,4} \end{bmatrix}, \boldsymbol{C}(1) = \begin{bmatrix} c(1)_{1,1} & 0 & 0 & 0 \\ c(1)_{2,1} & c(1)_{2,2} & 0 & 0 \\ c(1)_{3,1} & c(1)_{3,2} & c(1)_{3,3} & 0 \\ c(1)_{4,1} & c(1)_{4,2} & c(1)_{4,3} & c(1)_{4,4} \end{bmatrix}.$$

Expanding on equations (22) and (23),

$$\hat{r}^*_t = \hat{r}^*_{t-1} + \psi(1)_{k,1}e_{1,t} + \psi(1)_{k,2}e_{2,t} + \psi(1)_{k,3}e_{3,t} + \psi(1)_{k,4}e_{4,t}, \tag{24}$$
$$\hat{r}^*_t = \hat{r}^*_{t-1} + c(1)_{k,1}\epsilon^F_{P,t} + c(1)_{k,2}\epsilon^F_{T,t} + c(1)_{k,3}\epsilon^D_{P,t} + c(1)_{k,4}\epsilon^D_{T,t}, \tag{25}$$

where the estimate of r^* from the model can be written in terms of its forecast errors or the identified foreign and domestic shocks. From equations (16) and (18), given the U.S. is in the 1[st] position in the vector of variables and expanding on the relevant elements of $\boldsymbol{\Psi}(1)$ and $\boldsymbol{C}(1)$, the preliminary estimate for the U.S., which we label $\hat{r}^*_{U.S.,t}$, is

$$\hat{r}^*_{U.S.,t} = \hat{r}^*_{U.S.,t-1} + \psi(1)_{1,1}e_{1,t} + \psi(1)_{1,2}e_{2,t}, \tag{26}$$
$$\hat{r}^*_{U.S.,t} = \hat{r}^*_{U.S.,t-1} + c(1)_{1,1}\epsilon^F_{P,t}. \tag{27}$$

Equations (26) and (27) clarify how block exogeneity and the restrictions yield the estimate of r^* for the U.S. The estimated U.S. r^* is a function of the two forecast errors in the foreign block, but only a function of the one identified foreign permanent shock. Therefore, while the BN decomposition only relies on the forecasts errors of the reduced-form model, the identification scheme effectively apportions the variation in the BN trend

[10]Note that we are defining r^* as the trend of the short term interest rate, but the difference of the long rate appears in equation (18). However, since the two share the same stochastic trend, their trend levels are identical up to a constant in the long run, which is $\mu_{U.S.}$ and μ_{Dom} for the foreign and domestic blocks, respectively. We can obtain r^* by applying the approach in Morley (2002) to calculate the trend of the long-rate and subsequently subtracting either $\mu_{U.S.}$ and μ_{Dom} to obtain the r^* for the U.S. and the domestic small open economy, respectively.

to the number of identified permanent shocks for the trend in question. Since only one permanent shock drives the U.S. r^* in our model, *all* of the permanent variation that arises from the two forecast errors is apportioned to the one permanent shock.

We can repeat this exercise for the domestic small open economy. The preliminary estimate for the domestic economy, which we label $\hat{r}^*_{Dom,t}$ becomes

$$\hat{r}^*_{Dom,t} = \hat{r}^*_{Dom,t-1} + \psi(1)_{3,1}e_{1,t} + \psi(1)_{3,2}e_{2,t} + \psi(1)_{3,3}e_{3,t} + \psi(1)_{3,4}e_{4,t}, \quad (28)$$

$$\hat{r}^*_{Dom,t} = \hat{r}^*_{Dom,t-1} + c(1)_{3,1}\epsilon^F_{P,t} + c(1)_{3,2}\epsilon^F_{T,t} + c(1)_{3,3}\epsilon^D_{P,t}. \quad (29)$$

The domestic r^* is a function of all of the forecast errors in the model. Therefore, variation in *all* of the forecast errors can change the estimate of r^*. Even so, the domestic r^* is only a function of the first three identified structural shocks. Since the domestic block has only one permanent shock, there is only one domestic shock that can determine domestic r^*. Meanwhile, the domestic r^* is a function of all of the foreign shocks. That is, the domestic r^* is a function of both the shock that drives U.S. r^*, but also possibly the foreign shock that only has transitory effects on U.S. interest rates.

Finally, note that equations (27) and (29) apply to the preliminary estimate. From equations (12) and (13), because $\theta(1)$ of a one-unit innovation to $\Delta\hat{r}_t$ is permanent, let $\theta(1)_j$, $j \in \{U.S., dom\}$ be the parameters used to apply the univariate correction for the U.S. and domestic economies, respectively. Applying the correction is simply

$$\tilde{r}^*_{U.S.,t} = \tilde{r}^*_{U.S.,t-1} + \theta(1)_{U.S.}c(1)_{1,1}\epsilon^F_{P,t}, \quad (30)$$

$$\tilde{r}^*_{Dom,t} = \tilde{r}^*_{Dom,t-1} + \theta(1)_{dom}\left[c(1)_{3,1}\epsilon^F_{P,t} + c(1)_{3,2}\epsilon^F_{T,t} + c(1)_{3,3}\epsilon^D_{P,t}\right]. \quad (31)$$

3.4 Interpretation

We now discuss how to interpret the results from the empirical model described in Sections 3.2 and 3.3. As perfect capital mobility underlies many of the no arbitrage conditions prevailing in our discussion in Section 2, we first consider how to interpret the results from the empirical model described in Sections 3.2 and 3.3 under such conditions. As discussed in Sections 3.2, we expect r^* to be identical globally if there are no domestic shocks and so global r^* would equate to r^* in for every economy. In such circumstances, this would imply two sharp predictions for our empirical model. First, implied by equations (23), (30), and (31), *only* ϵ^P_t would matter for real interest rates in the long run. This is analogous to saying that r^* in *any* economy must be driven by the same set of shocks since this is the only set of shocks that drive r^* globally across the world. From equation (30), this is already by construction for the U.S. r^*, but for domestic r^*, this would correspond to restrictions that both $c(1)_{3,2} = c(1)_{3,3} = 0$. That is, for the domestic economy, no other foreign shocks should matter in the long run apart from the shock that characterises the U.S. r^* and domestic shocks should also not matter in the long run.[11] Second, this also implies real interest rates will co-integrate [1 -1] for every pair of economies in the world since global r^* equates r^* in every economy, real interest rates globally will converge to the same level in the long run. From equation (30) and (31), this must imply $\theta(1)_{U.S.}c(1)_{1,1} = \theta(1)_{dom}c(1)_{3,1}$, which is identical to saying domestic r^* changes one-for-one with the U.S. r^*.

Although likely too idealized in practice, the risk neutral and perfect capital mobility setting is a useful starting point to interpret our empirical results. In particular, we can think of any of $c(1)_{3,2} = 0$, $c(1)_{3,3} = 0$, or $\theta(1)_{U.S.}c(1)_{1,1} = \theta(1)_{dom}c(1)_{3,1}$ not holding as arising from departures from some assumption in the risk neutral and perfect capital mobility setting. As discussed by Del Negro et al. (2019) and in our simplified exposition in Section 2, we expect a factor structure for r^* globally where each economy, i, loads on a global r^* and has an idiosyncratic component which is orthogonal across i. This is a useful characterization because any of $c(1)_{3,2} = 0$, $c(1)_{3,3} = 0$, or $\theta(1)_{U.S.}c(1)_{1,1} = \theta(1)_{dom}c(1)_{3,1}$ not holding could be directly attributed to an economy-specific idiosyncratic component in r^*. As we discussed in Section 2, these idiosyncratic components reflect various wedges between the i^{th} economy's r^* and the global r^*.

We consider the domestic and U.S. blocks in turn. First, for the domestic small open economy, if domestic shocks matter for domestic r^*, this implies that $c(1)_{3,3} \neq 0$ and that there is a permanent shock arising domestically that can drive a wedge between the global and domestic r^*'s. While our empirical strategy does not attempt to fine-tune the interpretation further, this component could, as per the discussion in Section 2, relate to risk premiums attached to the domestic small open economy or could arise due to fundamentals such as an increase in relative domestic productivity or large changes in the domestic government's debt position. Second, we can consider a possible role for a U.S. idiosyncratic component since it is the only economy in the foreign block. Because ϵ_t^P is the only shock that drives the U.S. r^*, if the U.S. had a non-trivial idiosyncratic component, the U.S. r^* would be a combination of the global r^* and the U.S. idiosyncratic component (see equation (3)). The presence of a significant idiosyncratic component for the U.S. would then lead the U.S. and domestic real interest rates not to adjust one-for-one with each other such that $\theta(1)_{U.S.}c(1)_{1,1} = \theta(1)_{dom}c(1)_{3,1}$ would no longer hold as the U.S. idiosyncratic component of r^* would drive a further wedge between the domestic and global r^*'s. Specifically, the open economy response to U.S. r^* shocks would be a linear combination of a one-for-one response to global r^* and a zero response to the U.S. idiosyncratic component, with weights related to the relative magnitudes of the true shocks to global r^* and the U.S. idiosyncratic component. The existence of an additional

[11] We can trivially rule out $\theta(1) = 0$ since this implies a unit MA root so that there would be no permanent shocks to real interest rates and we would not need the BN decomposition to estimate a constant r^*.

U.S. idiosyncratic shock would also mean our shock identification that assumes just two orthogonal shocks for the U.S. real interest rate would end up recovering mixtures of the three true shocks due to non-invertibility for the vector MA form such that the recovered U.S. transitory shock from the VAR could appear significant with $c(1)_{3,2} = 0$ no longer holding as the "mongrel" shock (Canova and Ferroni, 2022) could end up being correlated with and effectively trying to control for movements in the U.S. idiosyncratic component when estimating the $c(1)_{3,1}$ effect of the recovered U.S. permanent shock on the domestic r^*.

This reasoning about possible distortions in estimating the effects of global shocks if there is an idiosyncratic component in the r^* for the foreign economy informs our choice of using the U.S. for the foreign block, as it is the economy with the likely most trivial idiosyncratic component, thus giving rise to our interpretation that we are identifying the role of global r^* for the open economies under consideration. Given the U.S.'s central role in the global financial markets and its comparatively open capital markets, a U.S. specific idiosyncratic component is likely to be comparatively small. However, the extent to which the U.S. r^* provides a good characterisation of global r^* from the small open economy's perspective is ultimately an empirical question that can be answered within our framework. Specifically, because $\theta(1)_{U.S.}c(1)_{1,1} = \theta(1)_{dom}c(1)_{3,1}$ and $c(1)_{3,2} = 0$ are both testable implications for our model, testing these implications allows us to see if there is any implicit role of a non-trivial U.S. idiosyncratic component.

We acknowledge that it is an open question whether using the U.S. for the foreign block is sufficient to capture all of the relevant foreign shocks for the open economies under consideration. As is known, to recover shocks from a structural VAR, we require all the relevant modelled information to span the underlying shocks. We therefore do not require including the whole global economy in the foreign block, but we do require the variables in the model to span the global shocks. This, fortunately, is also a testable implication for our purposes. Forni and Gambetti (2014) show that for a VAR to that includes sufficient information to span the shocks, no other variable should Granger cause the variables in the system. The need to span the relevant shocks provides a further reason why the U.S. should be in the foreign block. Given the U.S.'s central role in the global economy, and with relatively free financial markets, as long as shocks in the rest of the world are reflected in U.S. interest rates, then the specification of the foreign block using just U.S. interest rates would be justified. To the extent that there are foreign shocks that are relevant and are not shocks to the U.S. r^*, as long as U.S. interest rates (and especially so the term spread) contain the relevant information to reflect these shocks, we can safely omit other variables that also reflect the shocks. This can be verified using Forni and Gambetti (2014)'s procedure. We therefore test whether variables such as exchange rates or a global activity indicator Granger cause the variables in our four-variable system. We relegate the results of these tests to Section C of the appendix, but briefly note here that our system appears informationally sufficient, and that our results are robust to even allowing for departures from our baseline system to consider more variables.

4 Data and estimation

We estimate our two-block model for domestic economies of Australia, Canada, the Euro Area, New Zealand, Norway, Sweden and the United Kingdom, with the United States always serving as the foreign economy. The coverage of economies largely aligns with Holston et al. (2017), Del Negro et al. (2019), and Ferreira and Shousha (2023), noting that the latter two estimated the model for various European countries separately, whereas we have chosen to estimate the Euro Area as a whole, similar to Holston et al. (2017). The data is from the OECD, which we sourced via FRED, where we have considered a short-term and a long-term nominal interest rate for each economy. We obtain the ex-ante real short (long) term interest rate by subtracting from the nominal interest rate the average of the last four (twenty) quarters of year-on-year inflation for the short (long) rate. Section A of the appendix provides more details of the data. The sample coverage for all economies ends in 2024Q2. The start dates are as follows: U.S. 1964Q4, Australia 1986Q1, Canada 1964Q4, Euro Area 2002Q4, New Zealand 1992Q1, Norway 1995Q1, Sweden 1987Q1, and U.K. 1986Q1. The start date of the sample is often determined by the availability of both a short and long term interest rate, where the short rate is a 3 month interbank interest rate and the long rate is the interest rate on a 10 year government bond. We note that we have sometimes chosen to start the sample at a later date if we are aware of institutional features where the underlying security was not freely traded. When interest rates are regulated, the shocks we would obtain for the interest rate may no longer identify r^*.[12]

Given that we use quarterly data, we specify the standard four lags for the VAR in equation (15) and follow MTW by considering an MA(8) model when conducting the correction of preliminary estimates. We estimate the model using Bayesian methods. Specifically, the two block small open economy model specified by equations (15) and (16) can be estimated using a standard Gibbs sampler with Normal-inverse Wishart priors (see Kamber and Wong, 2020). We thus specify a standard Minnesota type prior on the BVAR coefficients. For the correction, MTW show how to implement a Metropolis-within-Gibbs step to apply the correction. While most of the estimation follows MTW, we implement two slightly non-standard features to the estimation of a BVAR that are tailored to the exercise at hand.

First, we only fit a mean for the spread, but not the change of the interest rate. That is, we estimate μ_{US} and μ_{Dom} in equation (18). The imposition of a zero mean

[12]One example is both Australia and New Zealand, which only saw deregulation of their interest rates and financial markets in the 1980s and 1990s, respectively (e.g., see Orden and Fisher, 1993, for details).

for the interest rate equation recognises that r^* should be a random walk without drift, and was also imposed in MTW. By contrast, a constant or a non-zero mean for Δr_t^i would imply a deterministic drift in r^*, which is at odds with the broader literature that typically models r^* as not having a deterministic drift (e.g., Del Negro et al., 2019). While what we describe so far can be achieved trivially via selectively specifying a constant in some equations in the BVAR and not others, we opt for a slightly different approach. Specifically, we adopt the "steady-state" priors developed by Villani (2009) to estimate μ_{US} and μ_{Dom}. Operationally, our only difference relative to Villani (2009) is that his exposition specifies a mean for every variable in the BVAR, whereas we only estimate a mean for the spread equations, owing to our motivation to not induce a deterministic drift in r^*. This step requires a slight modification relative to Villani (2009) and the accommodation of this feature leads to drawing both μ_{US} and μ_{Dom} in a separate block of the Gibbs sampler. Our motivation to use a "steady-state" prior on the spread, though, deserves some elaboration. Because analysts often do not have strong views *a priori* about the values of the intercepts in a BVAR, the standard approach is to specify an extremely diffuse prior. This is ill-advised in our setting for reasons similar to those argued by Villani (2009). Due to persistence in many macroeconomic time series, the mean is typically not estimated precisely. Because the means are a function of the intercepts in a VAR, a diffuse prior on the intercepts leads to this uncertainty directly feeding into the estimates of the means. However, the unconditional mean is often an object that an analyst has more of an *a priori* view on, motivating Villani (2009) to develop the "steady-state" priors as a mechanism to build this information into estimation of the model. We note that it is also the imprecision in estimating the mean (and by extension the drifts in levels when needed) that also led to some of our previous work to concentrate the mean out of the likelihood by simply using a sample average (e.g., Morley and Wong, 2020; Berger, Morley, and Wong, 2023; Morley, Rodríguez-Palenzuela, Sun, and Wong, 2023). This represents one extreme where we are not treating the mean (or drift) as a random variable in our Bayesian estimation. By contrast, the proposed approach here of incorporating some prior information when estimating the mean spread allows us to have a full Bayesian treatment when estimating our model.

The second non-standard feature in our approach is that we ensure estimates for the foreign block are *always* identical no matter which small open economy we consider in a pair of economies. From equations (15) and (16), we can see that the foreign block is always identical. However, in practice, because the U.S. sample is almost always longer than for the open economy under consideration due to data availability, constraining the foreign block to have the same sample span as the domestic economy in question would lead to different parameters estimated for the foreign block for small open economies with different sample availabilities. This would complicate discussion and comparison of the role of foreign shocks across the seven economies considered in our analysis. Another way of viewing our argument is that, given equations (15) and (16), U.S. r^* should be identical across all seven pairs of economies. Our solution, then, is to estimate missing observations for the domestic block that retain the same correlation structure with the foreign block as in the shorter sample, but so that the sample size for domestic block with the augmented missing observations aligns with that of the foreign block. It should be clear that the in-filled observations have no effect on the estimates for the foreign block given the block exogeneity restriction. The in-filled observations are only used to estimate the parameters in the domestic block, which one can then use for the BN decomposition of the observed (and discarding the in-filled) data to obtain r^* for the small open economy. The intuition for how we estimate missing observations is as follows. Because the overlapping sample between the domestic and foreign block effectively ties down the covariance matrix of the residuals (i.e., $\mathbf{\Sigma}$), given the foreign block does not require the domestic block due to block exogeneity, we can condition on the structure of $\mathbf{\Sigma}$ to in-fill the residuals for the shorter-sample domestic block conditional on the residuals in the foreign block. When conditioning on the in-filled residuals in the foreign block, one can easily reconstruct the missing observations for the domestic block. Once the missing observations are drawn as a step in the Gibbs sampler, we can then draw from the conditional posterior for a standard BVAR, conditioning on both the missing observations and data. Importantly, the in-filled residuals will retain the same covariance structure as the in-sample residuals such that the in-filled residuals will not distort the estimation of $\mathbf{\Sigma}$.

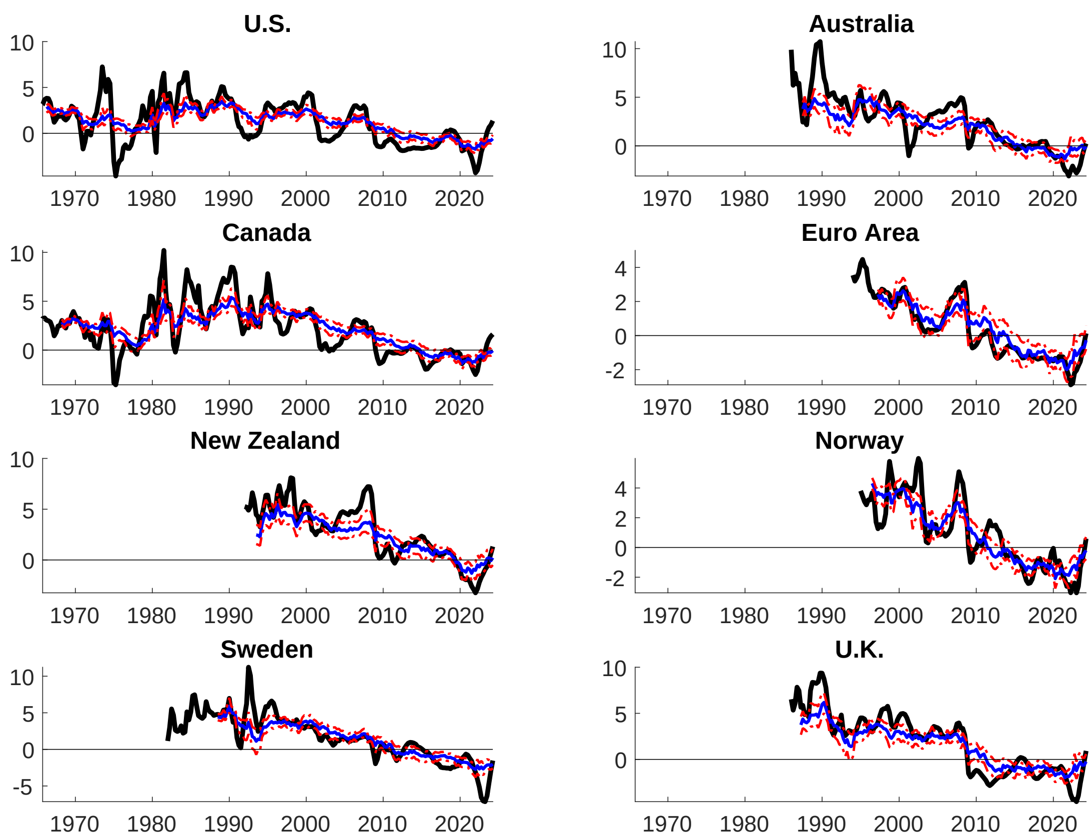
These two slightly non-standard features in our estimation thus add two additional steps to the standard BVAR estimation where one draws the BVAR coefficients conditional on the covariance matrix and vice versa. In particular, we draw μ_{US} and μ_{Dom} as a step in the Gibbs sampler and we also draw the missing observations for the domestic block as another step in the Gibbs sampler, resulting in four instead of the standard two steps in the sampler. For the missing observations, we place a flat prior on the in-filled residuals. For the mean spread, we set a prior of $N(1, 0.3^2)$, embedding our prior belief that the steady-state term spread is 1% with a standard deviation of 0.3. Conditional on the BVAR, we obtain preliminary $\hat{r}^*$ estimates for both the U.S. and the domestic small open economy. Following MTW, we then apply the correction as a Metropolis-within-Gibbs steps, with the correction applied separately for the U.S. and domestic economy. We set a Minnesota-like shrinkage on the MA(8) parameters, shrinking longer lags more tightly around zero. We also restrict the MA parameters to an invertible representation and mix in an additional prior on the cumulative sum of the MA coefficients, $\theta(1)$, as $N(0.7, 0.3^2)$.[13] This additional prior embeds our belief there is some smoothing in the correction step and is informed by the smoothing for the U.S. found in MTW with the

[13]Because the individual parameters have a prior that shrinks around zero, the overall implicit prior mean on the sum of the MA coefficients will be in between 0.7 and 1, while the standard error will also be a bit larger than 0.3.

posterior mean of $\theta(1)$ being around 0.5 when using a flat prior. We highlight that this prior is not strongly informative and we obtain similar results using a flat prior that only restricts the MA parameters to the invertibility region in the correction step. Nonetheless, the prior on the MA parameters helps with regularising the MA parameters, which can sometimes lead to better mixing of the MCMC chain as a short sample period can result in imprecise estimates stemming from known issues with estimating MA models such as multi-modality. It also provides a way, in principle, to impose a stronger smoothing prior on r^*, if desired.

5 Empirical results

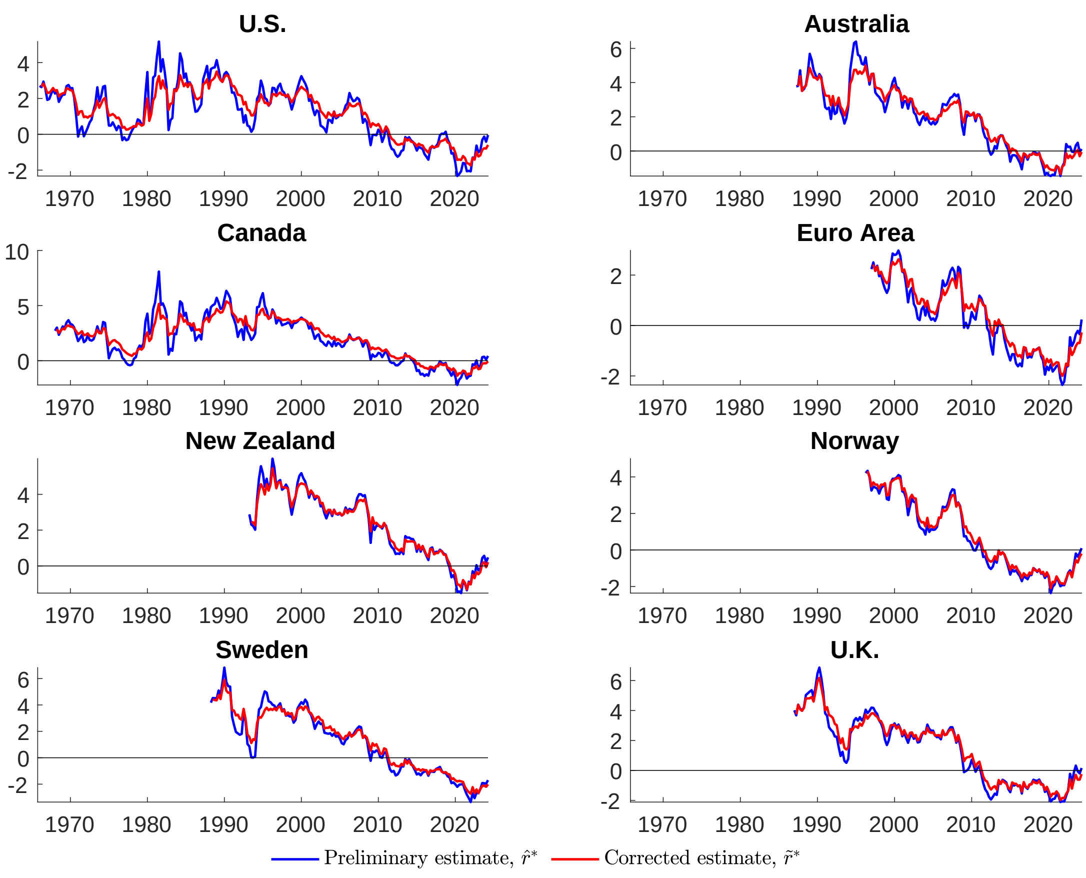
Figure 1: Corrected $\tilde{r}^*$ estimates



Notes: Posterior median estimates with 90% credible interval. Also presented is the ex-ante short term real interest rate.

We begin by presenting the r^* estimates. Figure 1 presents the corrected $\tilde{r}^*$ estimates. Recall that because we have modelled the foreign, or U.S., block to be identical across all seven models (with the same variables and sample span), there is only one set of estimates for the U.S. The U.S. r^* estimates are fairly standard relative to the broader literature, falling since around 1980s, with a slightly uptick since the COVID-19 pandemic. Not surprisingly given our approach, the U.S. estimates are very similar to those in MTW, suggesting that all the same productivity, demographic, and safe asset supply and demand drivers are responsible for movements in the U.S. r^* as found in MTW. For the open economies, the 90% credible sets suggests a fair amount of heterogeneity in terms of estimation uncertainty, with some economies (e.g., Euro Area, U.K.) featuring more estimation uncertainty, while others (e.g., Canada, Sweden) appear to be more precisely estimated. To the extent that alternative estimates of r^* contain a non-trivial degree of estimation uncertainty, the estimation uncertainty implied by the width of our credible sets are no wider than what one would expect from the extant literature, and possibly even tighter in some cases.

Figure 2: Comparison of preliminary and corrected estimates of r^*



Notes: The posterior median estimates of the corrected measure is presented alongside the posterior median preliminary estimate (i.e., $\hat{r}^*$).

Figure 2 compares the posterior median of the preliminary estimate (i.e., $\hat{r}^*$ from equation (11)) to the corrected estimate also presented in Figure 1. We present just the posterior median estimates to preserve the readability of the figure. There is clearly some heterogeneity in the extent to which the correction matters for a given economy.

For some economies like the U.S., Australia, and Canada, the corrected estimate is a lot smoother than the preliminary estimate. For others, such as Norway and the U.K., while the correction does smooth the preliminary estimates, the differences are not as large.

As we have discussed and reiterate here, if there is no serial correlation in the first differences of the preliminary estimates, the correction would have little effect. We also reiterate that the correction could lead to more volatile or smoother corrected estimates of r^*, depending on the nature of misspecification and residual serial correlation for the preliminary estimate. As is clear in Figure 2, the correction, to the extent it matters, generally leads to a smoothing of the preliminary estimate, though the extent to which it does so differs across the different economies. This smoothing is consistent with the fact that, when we obtain posterior distribution of $\theta(1)$, the mass of the distribution is less than one. We therefore conclude that the correction procedure is generally helpful in obtaining smoother estimates of r^*, but it an empirical issue and not mechanically imposed whether it does so.

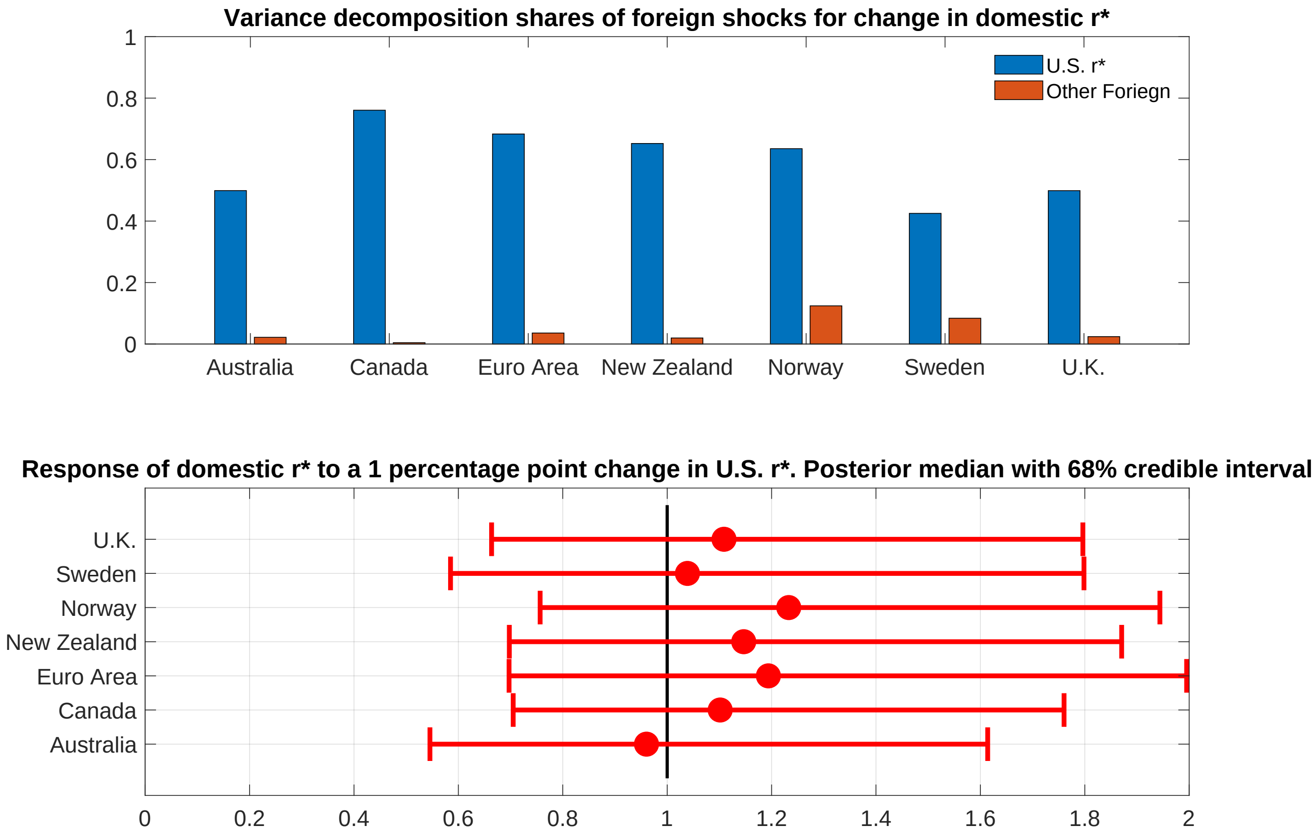
5.1 How important is global r^*?

Having obtained what we believe are plausible estimates of r^*, we now move to the central question of the paper, which is how important global r^* is for the various open economies under consideration. We first investigate the extent that one can just use the U.S. r^* as a proxy for global r^* when studying the role of foreign determinants of domestic r^*. Given that the model dichotomises all fluctuations in the change of r^* into domestic and foreign shocks, a variance decomposition is a natural starting point for such analysis. From equation (31), we can rearrange and apply the variance operator to obtain

$$var(\Delta\tilde{r}^*_{Dom,t}) = \theta(1)^2_{dom}\left\{c(1)^2_{3,1}var(\epsilon^F_{P,t}) + c(1)^2_{3,2}var(\epsilon^F_{T,t}) + c(1)^2_{3,3}var(\epsilon^D_{P,t})\right\}. \qquad (32)$$

Because the variance of the underlying structural shocks are normalized to unity, we can thus use the other terms to decompose the total variation of the change in the estimated r^*. The top panel of Figure 3 presents a variance decomposition for the change in domestic r^*, where we consider, on average, the proportion of variation attributed to the two foreign shocks. As is clear from equation (32), the change in domestic r^* can only be driven by either of the two foreign shocks and the permanent domestic shock, with any remainder not presented in Figure 3 implied to be due to the domestic permanent shock since the shares for all of the shocks must sum up to 100%. Because U.S. r^* is only driven by the first foreign shock, we label this as a "U.S. r^* shock". For all of the open economies, it is clear that the U.S. r^* shock drives almost all the variation attributed to foreign shocks, leaving almost no role for the other foreign shock. Indeed, 90% credible sets (not reported for readability) for the share attributed to the other foreign shocks always reach essentially to 0, suggesting that the only important foreign shock for the domestic r^*, are the same

Figure 3: The importance of global r^*



Notes: The top panel presents the share of the variance of the change in the estimated r^* attributed to the foreign shocks. The bottom panel presents the posterior median and 68% credible set of the estimated percent change in domestic r^* in response to a 1 percentage point change in U.S. r^*. The horizontal line marks out the point where domestic r^* changes one-for-one with U.S. r^*.

shocks that drive U.S. r^*. Therefore, from a small open economy perspective, shocks to global r^* are essentially equivalent to shocks to U.S. r^*. It also tell us, from equation (31) and (32), that it is likely $c(1)_{3,2} = 0$.[14] As discussed in the previous section, this result is consistent with there being no idiosyncratic component in the U.S. r^*.

We also consider whether changes in the U.S. r^* transmit one-for-one to the domestic r^*'s, as one would expect under perfect capital mobility and standard no arbitrage conditions in the long run. Because we know that the permanent foreign shock is the only shock that can change U.S. r^*, from equations (30) and (31), U.S. and domestic r^* can only change one-for-one if $\theta(1)_{U.S.}c(1)_{1,1} = \theta(1)_{dom}c(1)_{3,1}$. The bottom panel presents the posterior median estimate and the associated bounds of the 68% credible set of the ratio of these two quantities, $\frac{\theta(1)_{dom}c(1)_{3,1}}{\theta(1)_{U.S.}c(1)_{1,1}}$. If this ratio is 1, then domestic r^* changes one-for-one with

[14] This is also confirmed by the 90% credible sets for $c(1)_{3,2}$ always containing zero.

the U.S. In general, while this ratio is sometimes estimated imprecisely, the posterior median estimates are close to 1, with 1 always well within the 68% credible interval. The evidence thus supports the idea that domestic r^* changes one-for-one with U.S. r^*. Again, as discussed in the previous section, this result is consistent with there being no idiosyncratic component in the U.S. r^*.

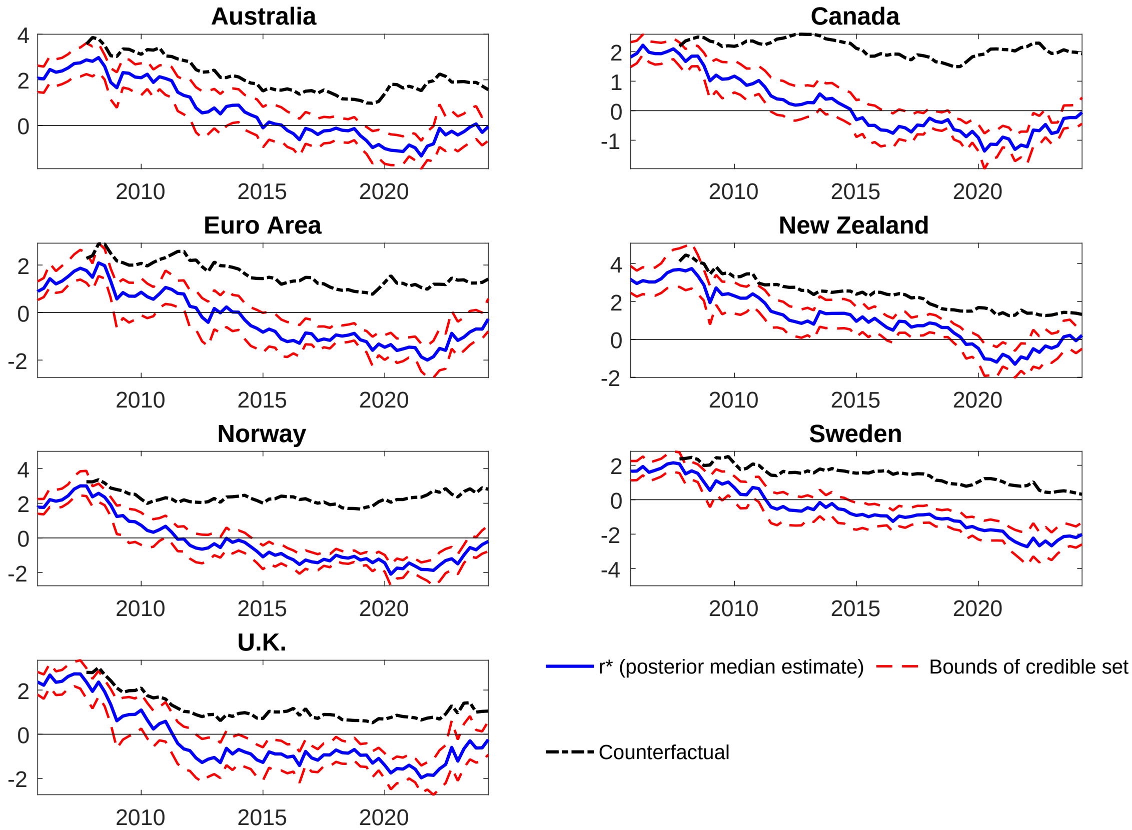
The first key conclusion to draw from from Figure 3 then is that, from the perspective of the seven open economies at least, the U.S. r^* provides a good proxy for global r^*. In particular, we find almost no role for other foreign shocks, and so the foreign shocks that determine domestic r^* are almost identical to the ones that drive U.S. r^*. Moreover, the fact that our results also suggest that domestic r^* moves one for one with U.S. r^* means that any U.S. specific-idiosyncratic component that drives U.S. r^* is evidently negligible from the perspective of the seven open economies. Our analysis therefore suggests that one does not necessarily require a large panel of interest rates to understand the implications of global r^* on domestic r^*. Tracking U.S. r^* appears to be sufficient. This is likely driven by the U.S.'s central role in the global financial system. Our results thus nuances the dilemma inherent in the work by Cesa-Bianchi et al. (2022) and Rachel and Summers (2019), where their solution to measuring global r^* is to treat a set of economies as being one large closed economy. Our analysis also suggests that, given the U.S.'s central role in the global financial system and capital mobility where factor prices have to equalise internationally, whatever drives global r^* likely drives the overwhelming share of what drives U.S. r^*. To be clear, global drivers like high safe asset demand due to a global savings glut can help drive the U.S. r^*, as found, for example, in MTW. So an interpretation of U.S. r^* as global r^* is possible to reconcile with U.S. r^* being driven by global forces.

Second, despite an important role of global shocks for domestic r^*'s, we find that domestic shocks also matter for these open economies. Foreign shocks drive anywhere between 35% to 70% of the variation in the change in r^*, but this share is not 100%. Considering our sample consists of advanced economies for which capital is relatively mobile, it also suggests that capital mobility alone does not guarantee that all of the variation in r^* is driven purely by international factors. Instead, our results suggest that domestic factors matter, which in turn implies that domestic economic policies could potentially mitigate or offset the effects of international factors.

Third, because we find that the only foreign shocks which matter for the open economies are captured by U.S. r^* shocks and that domestic r^* moves one for one with U.S. r^*, it suggests that, in the absence of domestic shocks, real interest rates would cointegrate [1 -1] between any pair of economies. The broader empirical literature has often found mixed results on whether real interest rates cointegrate internationally (e.g., Mishkin, 1984; Cumby and Mishkin, 1986; Chinn and Frankel, 1995). The mixed evidence is somewhat puzzling considering capital mobility and standard no-arbitrage conditions in the long run would imply that real returns should equalise internationally. Our results provide a slightly different perspective of these results. We find that, conditional on global shocks, the theory is supported, and indeed, returns should equalise internationally. That is, this cointegration relationship does hold conditionally, but not unconditionally when also taking into account local shocks. Because we find that domestic shocks have a non-trivial role in driving variation in domestic r^*, this causes domestic r^* to contain an additional permanent component, which drives permanent deviations between domestic and U.S. r^*, and so leads to the mixed results when one tries to uncover the long-run relationship between interest rates amongst the various economies under consideration.

5.2 Did global shocks drive declines in r^*?

Figure 4: Estimates of actual and counterfactual levels of r^*



Notes: Posterior medians of the estimated r^* for various economies are presented alongside with the estimated posterior median counterfactual levels when identified global shocks are set to zero from 2007Q1 to the end of the sample. Presented alongside are the associated 90% credible sets for the r^* estimates.

Given a large role of global r^* for these open economies and one-for-one relationship, an immediate question of interest is the whether global forces drove the decline in r^* across the various open economies. While there is debate about when the decline started, the evidence in Holston et al. (2017) at least suggests that this decline has been ongoing since at least early 2000s.[15] The variance decompositions reported in Figure (3) quantify the *average* variation in r^* due to the various shocks, but they do not directly address whether the levels of r^* are low due to global or domestic shocks. This is a subtle, but important, distinction because, while shocks can be positive or negative, and these permanent shock drive the *change* in r^*, the *level* of r^* is an accumulation of the historical realisations of permanent shocks. Nonetheless, because our approach decomposes the change of r^* into identified foreign and domestic shocks that are orthogonal to one another, we can run simple counterfactuals based on historical decompositions relative to some initial level under the assumption that either foreign or domestic shocks are set to zero after the starting point. More precisely, let $\tilde{r}^*_{t-\tau}$ be the estimate of r^* τ period ago, which we are taking at the initial level. Recursively substituting into equation (31) yields

$$\tilde{r}^*_{Dom,t} = \tilde{r}^*_{Dom,t-\tau} + \sum_{j=0}^{\tau-1} \theta(1)_{dom} \boldsymbol{\iota}_3 \boldsymbol{C}(1) \begin{bmatrix} \boldsymbol{\epsilon}^F_{t-j} \\ \boldsymbol{\epsilon}^D_{t-j} \end{bmatrix}. \tag{33}$$

The counterfactual level of r^* can then be obtained by setting a particular counterfactual sequence of shocks. We use this approach to ask how much of the level of the fall in r^* since 2007Q4 can be attributed to global shocks by setting these shocks to zero and tracing out the counterfactual path r^* would have taken in their absence. Our choice of 2007Q4 is largely driven by the fact that r^* internationally has fallen since the global financial crisis. 2007Q4, as the point just before the financial crisis is thus a useful starting point since this is close to the peak level of r^* for many of the economies in recent history.

Figure 4 presents counterfactual levels of r^* if global shocks had not occurred since 2007Q4, while Table 1 presents a decomposition of the decline, where the first column presents the total change in estimated r^* from 2007Q4 to 2024Q2, the second column the part of the decline attributed to global shocks, and the third expressing this proportion attributed to global shocks as a proportion of the total change.[16] Unsurprisingly, r^*'s

[15]It is more unclear whether r^* rose temporarily in the 1980s and 1990s before falling or that this has been a steady decline since the 1970s. For example, Del Negro et al. (2019) show that r^* rose internationally in the 1970s before starting a decline at some point in the 1980s, a pattern that is similar to what we find. While Holston et al. (2017) find a general steady decline in r^* in the U.S. since even the 1960s, their results for the Euro Area, U.K., and Canada suggests there were bouts in the 1980s and 1990s where r^* recovered to higher levels. By contrast, the general decline since the 2000s seems a common feature across many different studies. We also note that broad comparisons are also challenging because of different modelling assumptions and the data also differs to the extent that Del Negro et al. (2019) use annual data, while Holston et al. (2017) use quarterly data.

[16]Note that the foreign shocks are identical across all models because the foreign block is identical. We also note that the foreign transitory shock are also set to zero in the counterfactual, but just turning off the U.S. r^* shock produces essentially identical results given we show in the historical decomposition of Figure 3 that the foreign transitory shock has almost no effect on domestic r^*'s and that the foreign share of r^* is essentially the shocks driving global r^*.

Table 1: Decomposition of declines in r^* from 2007Q4 to 2024Q2

	Total Change	Change due to global shocks	Proportion accounted for by global shocks
Australia	-347	-162	47
Canada	-251	-199	79
Euro Area	-291	-168	58
New Zealand	-452	-110	24
Norway	-405	-301	74
Sweden	-483	-233	48
U.K.	-359	-129	36

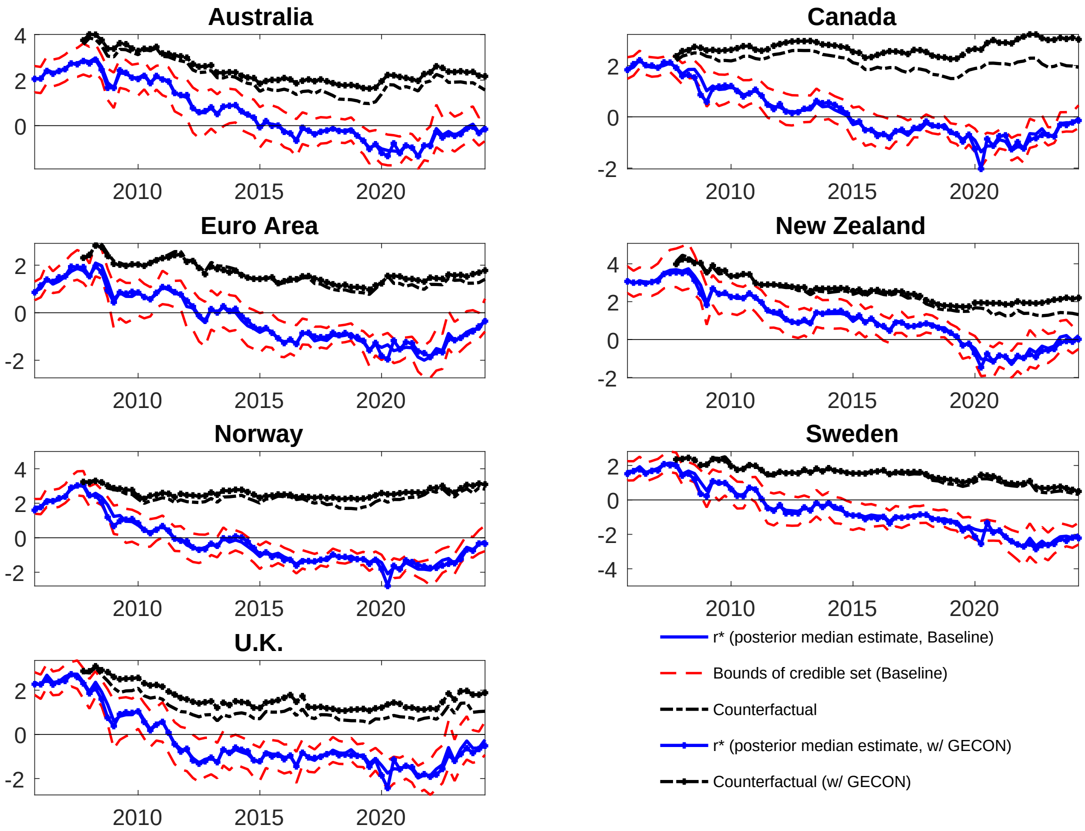
Notes: The first column presents the total change in basis points for the posterior median estimate of r^* from 2007Q4 to 2024Q2. The second column shows the posterior median estimate of the decline attributed to foreign shocks in basis points. The third column presents the proportion in percent of the decline attributed to global shocks.

across all seven economies fell since 2007Q4. These declines range from about 250 basis points for Canada to as much as nearly 500 basis points for New Zealand and Sweden. While there is heterogeneity across the seven open economies, the global influences on the fall in r^* is apparent. In particular, for all of the economies, the counterfactual level at the end of the sample is above the upper bound of the 90% credible interval of the r^* estimates. In other words, global shocks played a significant role in the fall of r^* for each open economy since 2007Q4. At the same time, how important the global shocks were for domestic r^*'s differs. For Canada, the Euro Area and Norway, much of, to almost all of, the decline can be attributed to global shocks. For the rest, while domestic shocks can in part explain some of the decline in r^* since 2007Q4, global shocks still account for as much as nearly half of the decline in r^*. Even for New Zealand and the U.K., the latter experiencing Brexit during the period in question, while domestic shocks account for a large share of the decline, over a quarter to a third of their decline in r^* can still be accounted for by global shocks. The results in MTW based on a larger model for U.S. r^* over a similar time period speak to these global shocks being related to both lower U.S. productivity growth and increased safe asset demand.

5.3 Allowing for other foreign shocks

As discussed, a key assumption in our analysis is that the U.S. interest rates in the foreign block are sufficient to capture the effects of global shocks for the respective domestic r^*'s. We now consider possible departures from this setting, specifically allowing for the possibility that there may be other foreign shocks not captured by U.S. interest rates. Consider economies such as New Zealand, Sweden, or Norway. The U.S. interest rates may not be sufficient to capture all sources of global shocks since sources of foreign shocks may originate from key trading partners like Australia for New Zealand or the Euro Area

Figure 5: Estimates of actual and counterfactual levels of r^* given more information



Notes: Posterior medians of the estimated r^* for various economies are presented alongside with the estimated posterior median counterfactual estimate where there are no foreign shocks from 2007Q1 to the end of the sample from the baseline specification and including the global economic activity indicator. Presented alongside are the associated 90% credible set of the r^* estimate from the baseline.

for Norway and Sweden.

As shown by our informational sufficiency tests in Section C of the appendix, there is little evidence against our baseline four variable system being informationally sufficient, although we note there is some mixed evidence when we consider the global economic activity indicator by Baumeister et al. (2022). Because we do not find evidence that the global economic activity indicator Granger causes the U.S. block, but mixed evidence that it may Granger cause selected domestic blocks, this suggest that the global economic activity indicator may be capturing non-U.S. foreign shocks for some of the open economies. We thus re-estimated all our models including the global economic activity indicator in the foreign block. Figure 5 presents the estimated r^*'s and counterfactuals when we consider the global economic activity indicator in the foreign block relative to our baseline results. We mark out the estimates obtained from the system where we consider the global economic activity indicator. As is clear, the estimated r^* is largely unchanged when we include the global economic activity indicator. The counterfactual r^*'s do change slightly. In particular, it appears that if we allow for non-U.S. foreign shocks, the counterfactual r^*'s would be slightly higher in some economies relative to our benchmark. We also show in Section D in the appendix that our results for the variance decompositions are largely unchanged when we consider the global economic activity indicator. Therefore, we conclude that, even if one obtains mixed results for informational sufficiency, our results are largely unchanged when we allow for other sources of non-U.S. foreign shocks.

6 Conclusion

In this paper, we have studied how important global shocks are for determining r-star for a set of seven open economies: Australia, Canada, Euro Area, New Zealand, Norway, Sweden, and the United Kingdom. We build on Morley, Tran, and Wong (2024), by using the Beveridge-Nelson decomposition based on a standard small-open-economy Bayesian VAR. Our method should add to the toolkit of various methods policy institutions and analysts use to estimate r-star for open economies.

Given our method makes use of a Bayesian VAR, we bring to bear standard tools from the VAR literature to disentangle the global versus local determinants of r-star. We document three main findings. First, we find that from the perspective of the seven open economies under consideration, U.S. r-star is a good proxy for global r-star. Second, we find that local shocks are important for domestic r-stars, leaving open the potential that domestic economic policy can complement or offset the global forces that also drive r-star. Third, when we consider declines in r-star since 2007Q4, we find that of the roughly 200 to 400 basis points declines, global shocks can often account for more than half to all of the fall. Therefore, while local shocks can matter, the long-term decline in r-star appears to be due to global forces such as a lower US productivity growth and an increase in safe asset demand.

References

Banbura, Marta, Domenico Giannone, and Lucrezia Reichlin (2010): "Large Bayesian vector auto regressions," *Journal of Applied Econometrics*, 25 (1), 71–92.

Baumeister, Christiane, Dimitris Korobilis, and Thomas K. Lee (2022): "Energy Markets and Global Economic Conditions," *The Review of Economics and Statistics*, 104 (4), 828–844.

Berger, Tino and Bernd Kempa (2019): "Testing for time variation in the natural rate of interest," *Journal of Applied Econometrics*, 34 (5), 836–842.

Berger, Tino, James Morley, and Benjamin Wong (2023): "Nowcasting the output gap," *Journal of Econometrics*, 232 (1), 18–34.

Beveridge, Stephen and Charles R Nelson (1981): "A new approach to decomposition of economic time series into permanent and transitory components with particular attention to measurement of the 'business cycle'," *Journal of Monetary Economics*, 7 (2), 151–174.

Blanchard, Olivier Jean and Danny Quah (1989): "The Dynamic Effects of Aggregate Demand and Supply Disturbances," *American Economic Review*, 79 (4), 655–673.

Buncic, Daniel, Adrian Pagan, and Tim Robinson (2023): "Recovering Stars in Macroeconomics," CAMA Working Papers 2023-43, Centre for Applied Macroeconomic Analysis, Crawford School of Public Policy, The Australian National University.

Canova, Fabio and Filippo Ferroni (2022): "Mind the Gap! Stylized Dynamic Facts and Structural Models," *American Economic Journal: Macroeconomics*, 14 (4), 104–35.

Carriero, Andrea, Todd E. Clark, and Massimiliano Marcellino (2015): "Bayesian VARs: Specification Choices and Forecast Accuracy," *Journal of Applied Econometrics*, 30 (1), 46–73.

Cesa-Bianchi, Ambrogio, Richard Harrison, and Rana Sajedi (2022): "Decomposing the drivers of Global R," Bank of England working papers 990, Bank of England.

Chinn, Menzie D and Jeffrey A Frankel (1995): "Who drives real interest rates around the Pacific Rim: the USA or Japan?" *Journal of International Money and Finance*, 14 (6), 801–821.

Clarida, Richard (2019): "The global factor in neutral policy rates: Some implications for exchange rates, monetary policy, and policy coordination," *International Finance*, 22 (1), 2–19.

Cumby, Robert E. and Frederic S. Mishkin (1986): "The international linkage of real interest rates: The European-US connection," *Journal of International Money and Finance*, 5 (1), 5–23.

Del Negro, Marco, Domenico Giannone, Marc P. Giannoni, and Andrea Tambalotti (2019): "Global trends in interest rates," *Journal of International Economics*, 118 (C), 248–262.

Evans, George and Lucrezia Reichlin (1994): "Information, forecasts, and measurement of the business cycle," *Journal of Monetary Economics*, 33 (2), 233–254.

Ferreira, Thiago R.T. and Samer Shousha (2023): "Determinants of global neutral interest rates," *Journal of International Economics*, 145 (C).

Forni, Mario and Luca Gambetti (2014): "Sufficient information in structural VARs," *Journal of Monetary Economics*, 66 (C), 124–136.

Gelper, Sarah and Christophe Croux (2007): "Multivariate out-of-sample tests for Granger causality," *Computational Statistics & Data Analysis*, 51 (7), 3319–3329.

Gourinchas, Pierre-Olivier, Hélène Rey, and Maxime Sauzet (2022): "Global Real Rates: A Secular Approach," CEPR Discussion Papers 16941, C.E.P.R. Discussion Papers.

Hamilton, James D., Ethan S. Harris, Jan Hatzius, and Kenneth D. West (2016): "The Equilibrium Real Funds Rate: Past, Present, and Future," *IMF Economic Review*, 64 (4), 660–707.

Han, Yang and Jun Ma (2024): "Estimating the Interest Rate Trend in a Shadow Rate Term Structure Model," Working paper.

HOLSTON, KATHRYN, THOMAS LAUBACH, AND JOHN C. WILLIAMS (2017): "Measuring the natural rate of interest: International trends and determinants," *Journal of International Economics*, 108 (S1), 59–75.

JUSTINIANO, ALEJANDRO AND BRUCE PRESTON (2010): "Can structural small open-economy models account for the influence of foreign disturbances?" *Journal of International Economics*, 81 (1), 61–74.

KADIYALA, K. RAO AND SUNE KARLSSON (1997): "Numerical methods for estimation and inference in Bayesian VAR models," *Journal of Applied Econometrics*, 12 (2), 99–132.

KAMBER, GÜNEŞ, JAMES MORLEY, AND BENJAMIN WONG (2018): "Intuitive and reliable estimates of the output gap from a Beveridge-Nelson filter," *Review of Economics and Statistics*, 100, 550–566.

KAMBER, GÜNEŞ AND BENJAMIN WONG (2020): "Global factors and trend inflation," *Journal of International Economics*, 122, 103265.

KILEY, MICHAEL T. (2020): "The Global Equilibrium Real Interest Rate: Concepts, Estimates, and Challenges," *Annual Review of Financial Economics*, 12 (1), 305–326.

KING, ROBERT G., CHARLES I. PLOSSER, JAMES H. STOCK, AND MARK W. WATSON (1991): "Stochastic Trends and Economic Fluctuations," *American Economic Review*, 81 (4), 819–840.

KING, THOMAS B. AND JAMES MORLEY (2007): "In Search of the Natural Rate of Unemployment," *Journal of Monetary Economics*, 54, 550–564.

LAUBACH, THOMAS AND JOHN C. WILLIAMS (2003): "Measuring the Natural Rate of Interest," *The Review of Economics and Statistics*, 85 (4), 1063–1070.

LUNSFORD, KURT G. AND KENNETH D. WEST (2019): "Some Evidence on Secular Drivers of US Safe Real Rates," *American Economic Journal: Macroeconomics*, 11 (4), 113–139.

MISHKIN, FREDERIC S (1984): "Are Real Interest Rates Equal across Countries? An Empirical Investigation of International Parity Conditions," *Journal of Finance*, 39 (5), 1345–1357.

MORLEY, JAMES AND JEREMY PIGER (2012): "The Asymmetric Business Cycle," *The Review of Economics and Statistics*, 94 (1), 208–221.

MORLEY, JAMES, DIEGO RODRÍGUEZ-PALENZUELA, YIQIAO SUN, AND BENJAMIN WONG (2023): "Estimating the euro area output gap using multivariate information and addressing the COVID-19 pandemic," *European Economic Review*, 153 (C).

MORLEY, JAMES, TRUNG DUC TRAN, AND BENJAMIN WONG (2024): "A Simple Correction for Misspecification in Trend-Cycle Decompositions with an Application to Estimating r*," *Journal of Business & Economic Statistics*, 42 (2), 665–680.

MORLEY, JAMES AND BENJAMIN WONG (2020): "Estimating and accounting for the output gap with large Bayesian vector autoregressions," *Journal of Applied Econometrics*, 36 (1), 1–18.

MORLEY, JAMES C (2002): "A state-space approach to calculating the Beveridge-Nelson decomposition," *Economics Letters*, 75 (1), 123–127.

Morley, James C, Charles R Nelson, and Eric Zivot (2003): "Why Are the Beveridge-Nelson and Unobserved-Components Decompositions of GDP So Different?" *The Review of Economics and Statistics*, 85 (2), 235–243.

Orden, David and Lance A Fisher (1993): "Financial Deregulation and the Dynamics of Money, Prices, and Output in New Zealand and Australia," *Journal of Money, Credit and Banking*, 25 (2), 273–292.

Rachel, Lukasz and Lawrence H. Summers (2019): "On Secular Stagnation in the Industrialized World," *Brookings Papers on Economic Activity*, 50 (1 (Spring), 1–76.

Reis, Ricard (2025): "The Four R-stars: From Interest Rates to Inflation and Back," Working paper.

Schmitt-Grohé, Stephanie and Martín Uribe (forthcoming): "The Macroeconomic Consequences of Natural Rate Shocks," *Review of Economics and Statistics.*

Villani, Mattias (2009): "Steady-state priors for vector autoregressions," *Journal of Applied Econometrics*, 24 (4), 630–650.

Zha, Tao (1999): "Block recursion and structural vector autoregressions," *Journal of Econometrics*, 90 (2), 291–316.

Zhang, Ren, Enrique Martínez-García, Mark A. Wynne, and Valerie Grossman (2021): "Ties that bind: Estimating the natural rate of interest for small open economies," *Journal of International Money and Finance*, 113 (C), 102315.

A Data

The data were all sourced from FRED. Additional data sources which were used for the informational sufficiency tests in Section C are also reported below.

	FRED mnemonic	Underlying source
Interest rates		
Interest Rates: 3-Month or 90-Day Rates and Yields: Interbank Rates: Total for United States	IR3TIB01USM156N	OECD
Interest Rates: 3-Month or 90-Day Rates and Yields: Interbank Rates: Total for Australia	IR3TIB01AUM156N	OECD
Interest Rates: 3-Month or 90-Day Rates and Yields: Interbank Rates: Total for Canada	IR3TIB01CAM156N	OECD
Interest Rates: 3-Month or 90-Day Rates and Yields: Interbank Rates: Total for Euro Area (19 Countries)	IR3TIB01EZM156N	OECD
Interest Rates: 3-Month or 90-Day Rates and Yields: Interbank Rates: Total for New Zealand	IR3TIB01NZM156N	OECD
Interest Rates: 3-Month or 90-Day Rates and Yields: Interbank Rates: Total for Norway	IR3TIB01NOM156N	OECD
Interest Rates: 3-Month or 90-Day Rates and Yields: Interbank Rates: Total for Sweden	IR3TIB01SEM156N	OECD
Interest Rates: 3-Month or 90-Day Rates and Yields: Interbank Rates: Total for United Kingdom	IR3TIB01GBM156N	OECD
Interest Rates: Long-Term Government Bond Yields: 10-Year: Main (Including Benchmark) for United States	IRLTLT01USM156N	OECD
Interest Rates: Long-Term Government Bond Yields: 10-Year: Main (Including Benchmark) for Australia	IRLTLT01AUM156N	OECD
Interest Rates: Long-Term Government Bond Yields: 10-Year: Main (Including Benchmark) for Canada	IRLTLT01CAM156N	OECD

Interest Rates: Long-Term Government Bond Yields: 10-Year: Main (Including Benchmark) for Euro Area (19 Countries)	IRLTLT01EZM156N	OECD
Interest Rates: Long-Term Government Bond Yields: 10-Year: Main (Including Benchmark) for New Zealand	IRLTLT01NZM156N	OECD
Interest Rates: Long-Term Government Bond Yields: 10-Year: Main (Including Benchmark) for Norway	IRLTLT01NOM156N	OECD
Interest Rates: Long-Term Government Bond Yields: 10-Year: Main (Including Benchmark) for South Africa	IRLTLT01ZAM156N	OECD
Interest Rates: Long-Term Government Bond Yields: 10-Year: Main (Including Benchmark) for Sweden	IRLTLT01SEM156N	OECD
Interest Rates: Long-Term Government Bond Yields: 10-Year: Main (Including Benchmark) for United Kingdom	IRLTLT01GBM156N	OECD
Consumer price indices		
Consumer Price Indices (CPIs, HICPs), COICOP 1999: Consumer Price Index: All Items Non-Food Non-Energy for United States	USACPICORQINMEI	OECD
Consumer Price Indices (CPIs, HICPs), COICOP 1999: Consumer Price Index: All Items Non-Food Non-Energy for Australia	AUSCPICORQINMEI	OECD
Consumer Price Indices (CPIs, HICPs), COICOP 1999: Consumer Price Index: All Items Non-Food Non-Energy for Canada	CANCPICORQINMEI	OECD
Consumer Price Index: Harmonised Prices: All Items: Total for the Euro Area (19 Countries)	EA19CPHPTT01IXEBM	OECD
Consumer Price Indices (CPIs, HICPs), COICOP 1999: Consumer Price Index: All Items Non-Food Non-Energy for New Zealand	CPGRLE01NZQ659N	OECD
Consumer Price Indices (CPIs, HICPs), COICOP 1999: Consumer Price Index: Total for Norway	NORCPIALLMINMEI	OECD

Consumer Price Indices (CPIs, HICPs), COICOP 1999: Consumer Price Index: Total for Sweden	SWECPIALLMINMEI	OECD
Consumer Price Indices (CPIs, HICPs), COICOP 1999: Consumer Price Index: Total for United Kingdom	GBRCPIALLMINMEI	OECD
Real exchange rates		
Real Narrow Effective Exchange Rate for United States	RNUSBIS	BIS
Real Narrow Effective Exchange Rate for Australia	RNAUBIS	BIS
Real Narrow Effective Exchange Rate for Canada	RNCABIS	BIS
Real Narrow Effective Exchange Rate for Euro Area	RNXMBIS	BIS
Real Narrow Effective Exchange Rate for New Zealand	RNNZBIS	BIS
Real Narrow Effective Exchange Rate for Norway	RNNOBIS	BIS
Real Narrow Effective Exchange Rate for Sweden	RNSEBIS	BIS
Real Narrow Effective Exchange Rate for United Kingdom	RNGBBIS	BIS
Activity indicator		
Global economic activity indicator		Christiane Baumeister's website

B Estimation procedure

We first define the sets of parameters to be estimated from the model. From what will be the VAR block of parameters, let $\boldsymbol{\beta}$ represent the VAR reduced-form parameters, $\boldsymbol{\Phi_1}, \ldots, \boldsymbol{\Phi_p}$ in Equation (15), $\boldsymbol{\Sigma}$ the VAR covariance matrix from Equation (17), and $\boldsymbol{\mu}$ represent the unconditional means to be estimated (i.e., $\mu_{U.S.}$ and μ_{Dom} in Equation (18)). Further, given the foreign block has more sample observations, the U.S. has T observations, while the domestic block has the first τ observations missing. We define $\boldsymbol{X}$ as the observed sample, where the U.S. has the observations $t = 1, 2, \ldots, T$, and the foreign block $t = \tau+1, \tau+2, \ldots, T$. The real interest rates that are used as observations in the estimation are thus part of $\boldsymbol{X}$. We then define $\hat{\boldsymbol{e}}$ to represent the set of reduced-form residuals from the unobserved part of the sample for the domestic block which comprises of $\boldsymbol{e}_t^D$, $t = 1, 2, \ldots, \tau$, where expanding equation (17), $\boldsymbol{e}_t^D = \boldsymbol{A}_{21}\boldsymbol{\epsilon}_t^F + \boldsymbol{A}_{22}\boldsymbol{\epsilon}_t^D$.

From the correction step, we define $\boldsymbol{\theta}(L)$ to represent all the parameters used in this step (i.e., $\theta(L)_{U.S.}$ and $\theta(L)_{Dom}$). Finally, we define $\hat{\boldsymbol{r}}^{\boldsymbol{*}}$ and $\tilde{\boldsymbol{r}}^{\boldsymbol{*}}$ to represent the preliminary and corrected estimates for r^* for both the U.S. and the economy in the domestic block. Note that we do not construct r^* for the domestic economy for the first τ observations, and so $\hat{\boldsymbol{r}}^{\boldsymbol{*}}$ and $\tilde{\boldsymbol{r}}^{\boldsymbol{*}}$ includes the T estimates of r^* for the U.S. and $T - \tau$ estimates for the domestic economy.

The joint posterior can then be factorised as

$$\begin{aligned}
p(\tilde{\boldsymbol{r}}^*, \theta(L), \hat{\boldsymbol{r}}^*, \hat{\boldsymbol{e}}, \boldsymbol{\beta}, \boldsymbol{\Sigma}, \boldsymbol{\mu}, \mid \boldsymbol{X}) &= \underbrace{p(\tilde{\boldsymbol{r}}^*, \theta(L) \mid \hat{\boldsymbol{r}}^*, \hat{\boldsymbol{e}}, \boldsymbol{\beta}, \boldsymbol{\Sigma}, \boldsymbol{\mu}, \boldsymbol{X})}_{\text{correction step}} \underbrace{p(\hat{\boldsymbol{r}}^* \hat{\boldsymbol{e}}, \boldsymbol{\beta}, \boldsymbol{\Sigma}, \boldsymbol{\mu} \mid \boldsymbol{X})}_{\text{preliminary step}} \\
= \underbrace{p(\tilde{\boldsymbol{r}}^* \mid \theta(L), \hat{\boldsymbol{r}}^*, \hat{\boldsymbol{e}}, \boldsymbol{\beta}, \boldsymbol{\Sigma}, \boldsymbol{\mu}, \boldsymbol{X})}_{\text{construct } \tilde{r}^*} & \underbrace{p(\theta(L) \mid \hat{\boldsymbol{r}}^*, \hat{\boldsymbol{e}}, \boldsymbol{\beta}, \boldsymbol{\Sigma}, \boldsymbol{\mu}, \boldsymbol{X})}_{\text{estimate MA correction parameters}} \\
\underbrace{p(\hat{\boldsymbol{r}}^* \mid \hat{\boldsymbol{e}}, \boldsymbol{\beta}, \boldsymbol{\Sigma}, \boldsymbol{\mu} \mid \boldsymbol{X})}_{\text{construct } \hat{r}^*} & \underbrace{p(\hat{\boldsymbol{e}}, \boldsymbol{\beta}, \boldsymbol{\Sigma}, \boldsymbol{\mu} \mid \boldsymbol{X})}_{\text{estimate VAR}}.
\end{aligned} \tag{A.1}$$

The factorisation in the first line separates the correction and preliminary steps, where the conditioning on the initial estimate in the correction step implies that we can obtain samples from the correction step as long as we can obtain samples from the preliminary step. The second and third line further factorises these joint distributions for the correction and preliminary step respectively. This further factorisation makes clear that as long as one can obtain samples from the posterior distribution of parameters in the VAR model, subsequently constructing the preliminary estimate, $\hat{\boldsymbol{r}}^{\boldsymbol{*}}$, then conditioning on $\hat{\boldsymbol{r}}^{\boldsymbol{*}}$, fitting an MA process and conditioning on both $\hat{\boldsymbol{r}}^{\boldsymbol{*}}$ and the MA parameters means we can construct the corrected estimate, $\tilde{\boldsymbol{r}}^{\boldsymbol{*}}$.

A rough sketch of the sampling scheme is as follows, where we can approximate the joint posterior using a metropolis-within-Gibbs sampling scheme:

1. Obtain draws from $p(\hat{\boldsymbol{e}}, \boldsymbol{\beta}, \boldsymbol{\Sigma}, \boldsymbol{\mu} \mid \boldsymbol{X})$. This can be done in the following sequence

of Gibbs sampling steps:

- Obtain draws from $p(\hat{\boldsymbol{e}} \mid \boldsymbol{\beta}, \boldsymbol{\Sigma}, \boldsymbol{\mu}, \boldsymbol{X})$. As the foreign block is observed, given we are conditioning on the VAR parameters, unconditional means, and most importantly, the VAR covariance matrix of the residuals, allows us to draw the residuals in the domestic block for the missing observations. We use a flat prior on the missing observations.

- Obtain draws from $p(\boldsymbol{\beta}, \boldsymbol{\Sigma} \mid \hat{\boldsymbol{e}}, \boldsymbol{\mu}, \boldsymbol{X})$. Given conditioning on the missing block of residuals in the domestic block, further conditioning on the VAR parameters and means, we can construct the missing observations for the domestic block (i.e., $\boldsymbol{Y_t^D}, t = 1, 2, \ldots, \tau$). Since this would be analogous to conditioning on the missing observations for the domestic block, the problem reduces to estimating a standard Bayesian VAR with T observations (e.g., Banbura et al., 2010). Given we do not have conjugacy due to the block exogenous structure, we sequentially draw from the marginal distributions $p(\boldsymbol{\beta} \mid \boldsymbol{\Sigma}, \hat{\boldsymbol{e}}, \boldsymbol{\mu}, \boldsymbol{X})$ and $p(\boldsymbol{\Sigma} \mid \boldsymbol{\beta}, \hat{\boldsymbol{e}}, \boldsymbol{\mu}, \boldsymbol{X})$. Drawing from this conditional distributions is standard, and we use a relatively standard Normal-Wishart prior with Minnesota-type shrinkage as done in much of the extant literature, and also in some of our previous work with block exogeneity (e.g., see Kamber and Wong, 2020; Morley et al., 2023).

- Draw from $p(\boldsymbol{\mu} \mid \boldsymbol{\beta}, \boldsymbol{\Sigma}, \hat{\boldsymbol{e}}, \boldsymbol{X})$. This can be done using the reparametrisation introduced by Villani (2009). We set a prior of the mean spread being 1% with a standard deviation of 0.3.

2. Construct the preliminary estimate of $\hat{\boldsymbol{r}}^{\boldsymbol{*}}$ since we are conditioning on $\boldsymbol{\beta}, \hat{\boldsymbol{e}}, \boldsymbol{\mu}$, and $\boldsymbol{X}$, this construction is mechanical (e.g., see equation (11)).

3. Draw from $p(\theta(L) \mid \hat{\boldsymbol{r}}^{\boldsymbol{*}}, \hat{\boldsymbol{e}}, \boldsymbol{\beta}, \boldsymbol{\Sigma}, \boldsymbol{\mu}, \boldsymbol{X})$ using a Metropolis step, as described by Morley et al. (2024). The only difference relative to Morley et al. (2024) is that they used a flat (improper) prior on $\theta(L)$. While this works reasonably well for the U.S. in their paper (and in our foreign block), the use of a flat (improper) prior sometimes implies multi-modality in the sum of the MA coefficients for some of the other small open economies we considered. Computationally, the only difference relative to Morley et al. (2024) is that we consider an additional term (the prior) when evaluating the acceptance/rejection probability of the proposed draw. The use of an MA(8) may also induce some overfitting if freely fitted using a flat (improper) prior. We thus use an informative prior on the MA coefficients in order to regularise over the sum of the MA coefficients. We first specified a prior mean for $\theta(1)$ to be 0.7 with standard deviation 0.3. This prior helps in (i) obtaining uni-modality for $\theta(1)$ by pushing draws towards the invertible region and (ii) mitigating some possible overfitting from fitting eight MA parameters. We also set an indicator

function where we reject draws if the MA roots imply non-invertibility. We targeted an acceptance rate of 20-30% when tuning the jump step from our proposal density for the Metropolis step, as suggested by standard practice.

4. Construct the corrected estimate of $\tilde{\boldsymbol{r}}^{*}$. Since we are conditioning on $\hat{\boldsymbol{r}}^{*}$ and $\theta(L)$, this is also mechanical (see equation (13)).

5. Repeat and iterate on the steps above until desired number of MCMC draws are obtained.

We take 50,000 draws from the MCMC chain, burning the first 10,000.

Because steps 2 and 4 are straightforward and step 3 follows directly from Morley et al. (2024), we expand on how to draw from $p(\hat{\boldsymbol{e}}, \boldsymbol{\beta}, \boldsymbol{\Sigma}, \boldsymbol{\mu} \mid \boldsymbol{X})$, which is similar to Morley et al. (2024), but augmented by two slightly non-standard steps.

B.1 Drawing from $p(\hat{\boldsymbol{e}}, \boldsymbol{\beta}, \boldsymbol{\Sigma}, \boldsymbol{\mu} \mid \boldsymbol{X})$

Draw from $p(\hat{\boldsymbol{e}} \mid \boldsymbol{\beta}, \boldsymbol{\Sigma}, \boldsymbol{\mu}, \boldsymbol{X})$

Recall that the first τ observations are missing for the domestic block. Therefore, from equation (17), given $\boldsymbol{e}_t^D = \boldsymbol{A}_{21}\boldsymbol{\epsilon}_t^F + \boldsymbol{A}_{22}\boldsymbol{\epsilon}_t^D$ and $\boldsymbol{e}_t^F = \boldsymbol{A}_{11}\boldsymbol{\epsilon}_t^F$, we need to construct $\boldsymbol{e}_t^D$, $t = 1, 2, \ldots, \tau$ conditional on observing $\boldsymbol{e}_t^F$ with a known covariance matrix, $\boldsymbol{\Sigma}$. Given

$$\begin{bmatrix} \boldsymbol{e}_t^F \\ \boldsymbol{e}_t^D \end{bmatrix} \sim MVN\left(\begin{pmatrix} \mathbf{0} \\ \mathbf{0} \end{pmatrix}, \begin{pmatrix} \boldsymbol{\Sigma}_{\mathbf{11}} & \boldsymbol{\Sigma}_{\mathbf{12}} \\ \boldsymbol{\Sigma}_{\mathbf{21}} & \boldsymbol{\Sigma}_{\mathbf{22}} \end{pmatrix}\right). \tag{A.2}$$

This amounts to drawing from

$$p(\boldsymbol{e}_t^D \mid \boldsymbol{e}_t^F) \sim N(\boldsymbol{\Sigma}_{\mathbf{21}}\boldsymbol{\Sigma}_{\mathbf{11}}^{-\mathbf{1}}\boldsymbol{e}_t^F, \boldsymbol{\Sigma}_{\mathbf{22}} - \boldsymbol{\Sigma}_{\mathbf{21}}\boldsymbol{\Sigma}_{\mathbf{11}}^{-\mathbf{1}}\boldsymbol{\Sigma}_{\mathbf{12}}), \quad t = 1, 2, \ldots, \tau. \tag{A.3}$$

Draw from $p(\boldsymbol{\beta}, \boldsymbol{\Sigma} \mid \hat{\boldsymbol{e}}, \boldsymbol{\mu}, \boldsymbol{X})$

Conditioning on $\hat{\boldsymbol{e}}$ and $\boldsymbol{\mu}$ (i.e., the mean spread), $\boldsymbol{Y}_t^D, t = 1, 2, \ldots, \tau$ is known since it can be constructed mechanically given the second row of equation (16) (i.e., the domestic block). Therefore, we now have T observations of both $\boldsymbol{Y}_t^F$ and $\boldsymbol{Y}_t^D$. Defining ϕ_i^{jk} to be the $(j,k)^{th}$ element in $\boldsymbol{\Phi}_i$ in equation (15), n_F being the number of variables in the foreign block, where $n_F = 2$ in the baseline model and N being the total number of variables in the whole system (i.e., foreign and domestic block), we use a standard Minnesota type prior on the VAR coefficients as per the following:

$$\mathbb{E}[\phi_i^{jk}] = \begin{cases} 0.9, & i = 1, j = k, j \neq 1, j \neq n_F + 1 \\ 0 & \text{otherwise} \end{cases} \tag{A.4}$$

$$Var[\phi_i^{jk}] = \begin{cases} \frac{\lambda^2}{i^2}, & j = k \\ \frac{\lambda^2}{i^2}\frac{\sigma_j^2}{\sigma_k^2}, & \text{otherwise,} \end{cases} \tag{A.5}$$

unless $j \leq n_F$ and $k > n_F$, for which we do not estimate these parameters, but impose zeros due to the block exogeneity assumption. The intuition of the Minnesota prior basically shrinks longer lags more aggressively towards zero to mitigate overfitting. We set σ_i^2 to be the variance of the forecast error when fitting an AR(4) on each series, as per usual practice (e.g., see Carriero et al., 2015). The traditional Minnesota prior shrinks every equation towards a random walk, but we shrink the spread towards an AR(1) with a coefficient of 0.9, rather than the random walk, given the assumption used in calculating the BN decomposition that the VAR is stationary. While the persistence of the spread does help in pinning down the cycle of the real interest rate, we are using the spread to pin down transitory variation in the interest rate, and so a persistent but ultimately transitory process model aligns with our modelling strategy. Note that the interest rate is shrunk towards a random walk since we difference in the real interest rate, and thus all coefficients should have a prior mean of zero in the interest rate equations. The degree of shrinkage is governed by the hyperparameter, λ, which we set to 0.2, consistent with extant work (e.g., see Carriero et al., 2015), and also similar to the Morley et al. (2024) model which we build on.

Let $\boldsymbol{b}_j$ as the VAR coefficients for the j^{th} equation, with

$$\boldsymbol{b}_j = \begin{pmatrix} \phi_1^{j1} \\ \vdots \\ \phi_1^{jk} \\ \vdots \\ \phi_p^{jk} \\ \vdots \\ \phi_p^{jk} \end{pmatrix}, \tag{A.6}$$

where $k = n_F$ if the equation is in the foreign block and $k = N$ if the equation is in the domestic block. Let $\tilde{y}_{i,t} = y_{i,t} - \mu_i$, where the variable is in demeaned form,[17] we can

[17] This mean may or may not be estimated. From equation (18), we estimate a mean for the spread, but impose a zero mean for the change in the real interest rate to impose random walk without drift. Therefore, for the spread, this will be demeaned relative to its mean (which we condition on), and for the change in real interest rate just enters the model directly since we impose a zero mean.

write the j^{th} equation of the VAR as

$$\tilde{y}_{j,t} = \boldsymbol{Y}'_{j,t}\boldsymbol{b}_j + \boldsymbol{e}_{j,t}, \tag{A.7}$$

where $\boldsymbol{Y}_{j,t} = \boldsymbol{w_t}^F$ if the equation is in the foreign block and $\boldsymbol{Y}_{j,t} = \boldsymbol{w_t}$ if the equation is in the domestic block, where we define $\boldsymbol{y}_t^F = [\tilde{y}_{1,t}, \ldots, \tilde{y}_{n_F,t}]'$, $\boldsymbol{y}_t = [\boldsymbol{y}_t^{F\prime}, \tilde{y}_{n_F+1,t}, \ldots, \tilde{y}_{N,t}]'$, $\boldsymbol{w_t}^F = [\boldsymbol{y}_{t-1}^{F\prime}, \ldots, \boldsymbol{y}_{t-p}^{F\prime}]'$, and $\boldsymbol{w_t} = [\boldsymbol{y}'_{t-1}, \ldots, \boldsymbol{y}'_{t-p}]'$.

Stacking the equations,

$$\boldsymbol{y}_t = \boldsymbol{Y}_t\boldsymbol{\beta} + \boldsymbol{e}_t, \tag{A.8}$$

where

$$\boldsymbol{Y}_t = \begin{bmatrix} \boldsymbol{Y}_{1,t} & \boldsymbol{0} & \ldots & \boldsymbol{0} \\ \boldsymbol{0} & \ddots & \ddots & \vdots \\ \vdots & \ddots & \ddots & \vdots \\ \boldsymbol{0} & \ldots & \boldsymbol{0} & \boldsymbol{Y}_{N,t} \end{bmatrix}, \boldsymbol{\beta} = \begin{pmatrix} \boldsymbol{b}_1 \\ \vdots \\ \boldsymbol{b}_N \end{pmatrix}. \tag{A.9}$$

The VAR with block exogeneiety can be estimated using a standard Normal-Wishart prior (e.g., see Zha, 1999; Kamber and Wong, 2020). We specify the standard Normal-Wishart prior as per a regular Bayesian VAR:

$$\boldsymbol{\beta} \sim N(\boldsymbol{\beta_0}, \boldsymbol{V_\beta}), \tag{A.10}$$

$$\boldsymbol{\Sigma} \sim W(\boldsymbol{S}_0, \nu_0), \tag{A.11}$$

where $\boldsymbol{\beta_0}$ follows from equation (A.4) and $\boldsymbol{V_\beta}$ is a diagonal matrix with its diagonal elements as per equation (A.5). We set $\nu_0 = N{+}2$ and $\boldsymbol{S}_0$ to $diag\left((\nu_o - N - 1)\,[\sigma_1^2, \ldots, \sigma_N^2]\right)$ which are the variance of the residuals from an AR(4) regression which we used to set equation (A.5), ensuring the prior implies $\mathbb{E}(\boldsymbol{\Sigma}) = diag\left([\sigma_1^2, \ldots, \sigma_N^2]\right)$ (e.g., see Kadiyala and Karlsson, 1997). Standard results imply we can draw from the conditional distributions

$$p(\boldsymbol{\beta} \mid \boldsymbol{\Sigma}, \hat{\boldsymbol{e}}, \boldsymbol{\mu}, \boldsymbol{X}) \sim MVN(\hat{\boldsymbol{\beta}}, \hat{\boldsymbol{V_\beta}}), \tag{A.12}$$

$$p(\boldsymbol{\Sigma} \mid \boldsymbol{\beta}, \hat{\boldsymbol{E}}, \boldsymbol{\mu}, \boldsymbol{X}) \sim IW(\hat{\boldsymbol{S}}, \hat{\boldsymbol{\nu}}), \tag{A.13}$$

where

$$\begin{aligned} \hat{\boldsymbol{V_\beta}} &= \left(\boldsymbol{V_\beta}^{-1} + \sum_{t=p+1}^{T} \boldsymbol{Y}'_t\boldsymbol{\Sigma}^{-1}\boldsymbol{Y}_t\right)^{-1}, \\ \hat{\boldsymbol{\beta}} &= \hat{\boldsymbol{V_\beta}}\left[\boldsymbol{V_\beta}^{-1}\boldsymbol{\beta}_0 + \sum_{t=p+1}^{T} \boldsymbol{Y}'_t\boldsymbol{\Sigma}^{-1}\boldsymbol{y}_t\right], \end{aligned}$$

and

$$\begin{aligned}\hat{\boldsymbol{S}} &= \boldsymbol{S}_0 + \sum_{t=p+1}^{T} (\boldsymbol{y}_t - \boldsymbol{Y}_t\boldsymbol{\beta})(\boldsymbol{y}_t - \boldsymbol{Y}_t\boldsymbol{\beta})', \\ \hat{\boldsymbol{\nu}} &= T - p + \nu_0.\end{aligned}$$

We can thus obtain draws from $p(\boldsymbol{\beta}, \boldsymbol{\Sigma} \mid \hat{\boldsymbol{e}}, \boldsymbol{\mu}, \boldsymbol{X})$ by sequentially conditioning on and drawing from $p(\boldsymbol{\beta} \mid \boldsymbol{\Sigma}, \hat{\boldsymbol{e}}, \boldsymbol{\mu}, \boldsymbol{X})$ and $p(\boldsymbol{\Sigma} \mid \boldsymbol{\beta}, \hat{\boldsymbol{E}}, \boldsymbol{\mu}, \boldsymbol{X})$, as one would do for a standard Bayesian VAR.

Draw from $p(\boldsymbol{\mu} \mid \boldsymbol{\beta}, \boldsymbol{\Sigma}, \hat{\boldsymbol{e}}, \boldsymbol{X})$

This step is almost identical to Villani (2009), apart from the fact that we adapt his solution slightly given we impose a mean of zero for the change in the interest rate in equation (18). In other words, we estimate a mean for every variable except the change in the interest rate in order for r^* to be a random walk without drift as it is conventionally modelled.

Define

$$\boldsymbol{\mu} = \begin{bmatrix} \mu_{U.S} \\ \mu_{Dom} \end{bmatrix}, \boldsymbol{H} = \begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 0 \\ 0 & 1 \end{bmatrix}, \boldsymbol{Y}_t = \begin{bmatrix} \Delta r^L_{U.S.,t} \\ r^L_{U.S.,t} - r^S_{U.S.,t} \\ \Delta r^L_{Dom,t} \\ r^L_{Dom,t} - r^S_{Dom,t} \end{bmatrix}. \tag{A.14}$$

Combing equations (18) and (15), we can rewrite equation (15) as

$$\begin{aligned}[\boldsymbol{I} - \boldsymbol{\Phi}_1 - \ldots - \boldsymbol{\Phi}_p][\boldsymbol{Y}_t - \boldsymbol{H}\boldsymbol{\mu}] &= \boldsymbol{e}_t, \\ \boldsymbol{Y}_t - \boldsymbol{\Phi}_1\boldsymbol{Y}_{t-1} - \ldots - \boldsymbol{\Phi}_p\boldsymbol{Y}_{t-p} = \boldsymbol{\Phi}_1\boldsymbol{H}\boldsymbol{\mu} + \ldots + \boldsymbol{\Phi}_p\boldsymbol{H}\boldsymbol{\mu} &+ \boldsymbol{e}_t, \\ \tilde{\boldsymbol{Y}}_t = \boldsymbol{\Phi}_1\boldsymbol{H}\boldsymbol{\mu} + \ldots + \boldsymbol{\Phi}_p\boldsymbol{H}\boldsymbol{\mu} &+ \boldsymbol{e}_t, \\ \tilde{\boldsymbol{Y}}_t = \boldsymbol{W}\boldsymbol{\Theta} + \boldsymbol{e}_t,\end{aligned}$$

$$\text{where} \quad \boldsymbol{W} = \begin{bmatrix} 1 & -1 & \ldots & -1 \\ \vdots & \vdots & \ddots & \vdots \\ 1 & -1 & \ldots & -1 \end{bmatrix}, \boldsymbol{\Theta} = \begin{bmatrix} \boldsymbol{\mu}'\boldsymbol{H}' \\ \boldsymbol{\mu}'\boldsymbol{H}'\boldsymbol{\Phi}_1' \\ \vdots \\ \boldsymbol{\mu}'\boldsymbol{H}'\boldsymbol{\Phi}_p' \end{bmatrix}, \tilde{\boldsymbol{Y}}_t = \boldsymbol{Y}_t - \boldsymbol{\Phi}_1\boldsymbol{Y}_{t-1} - \ldots - \boldsymbol{\Phi}_p\boldsymbol{Y}_{t-p}.$$

Define $vec(\boldsymbol{\Theta}') = \boldsymbol{U}\boldsymbol{H}\boldsymbol{\mu}$, where

$$\boldsymbol{U} = \begin{bmatrix} \boldsymbol{I} \\ \boldsymbol{\Phi}_1 \\ \vdots \\ \boldsymbol{\Phi}_p \end{bmatrix}. \tag{A.15}$$

The above amounts to a linear regression with $\boldsymbol{\mu}$ being the estimated parameters. We specify the following prior

$$\boldsymbol{\mu} \sim N(\boldsymbol{\mu}_0, \boldsymbol{V}_\mu). \tag{A.16}$$

We set $\boldsymbol{\mu}_0$ as a vector of ones and $\boldsymbol{V}_\mu = diag([0.3^2, 0.3^2])$, embedding that the mean spread between the short and long rate is 1 percentage point with a one standard deviation between 0.3. The prior implies a mean term spread with a 95% interval of [0.4,1.6] and one standard deviation interval of [0.7,1.3]. This prior is sufficiently uninformative about the spread, but at the same time, avoids freely estimating this value, which may end up having an unrealistically large uncertainty on the mean (including negative values).

We can thus draw from the following conditional distribution

$$p(\boldsymbol{\mu} \mid \boldsymbol{\beta}, \boldsymbol{\Sigma}, \hat{\boldsymbol{e}}, \boldsymbol{X}) \sim N(\hat{\boldsymbol{\mu}}, \hat{\boldsymbol{V}}_{\boldsymbol{\mu}}), \tag{A.17}$$

where

$$\hat{\boldsymbol{V}}_{\boldsymbol{\mu}} = (\boldsymbol{V}_\mu^{-1} + \boldsymbol{H}'\boldsymbol{U}'(\boldsymbol{W}'\boldsymbol{W} \otimes \boldsymbol{\Sigma}^{-1})\boldsymbol{U}\boldsymbol{H})^{-1}, \tag{A.18}$$

$$\hat{\boldsymbol{\mu}} = \hat{\boldsymbol{V}}_{\boldsymbol{\mu}}(\boldsymbol{V}_\mu^{-1}\boldsymbol{\mu}_0 + \boldsymbol{H}'\boldsymbol{U}'vec(\boldsymbol{\Sigma}^{-1}(\tilde{\boldsymbol{Y}}'\boldsymbol{W})]. \tag{A.19}$$

C Informational sufficiency

Two key assumptions underlying our approach to recovering the role of foreign and domestic shocks to domestic r-star are that (i) the U.S. block spans the foreign shocks to r-star and (ii) we span all the shocks driving domestic r-star. These are somewhat testable assumptions, as shown by Forni and Gambetti (2014). Specifically, spanning the space of the shocks amounts to including sufficient information in the VAR so that we can recover the shocks of interest. If our empirical specification is informationally sufficient, this implies that our system includes all the relevant information in order to identify the shocks of interest. If all relevant information is included, any information through additional variables should be irrelevant, and so should provide no marginal information to help forecast the included state variables.

We can therefore run a series of Granger causality tests in order to test for informational sufficiency. We conduct all our Granger causality tests as out-of-sample test, as per Forni and Gambetti (2014). That is, we first took a common sample (if the sample does not align), then split the sample into half. We then test for out-of-sample forecastability with the second half of the sample using a recursively expanding window. We do out-of-sample forecasts with and without the possibly Granger causing variables (i.e., the unrestricted and restricted model respectively). We then collate the one-step ahead out-of-sample forecast errors across the multiple equations and use canonical correlations to test whether the inclusion of the extra variables led to statistically better forecasts (see Gelper and Croux, 2007). Critical values and p-values were obtained via bootstrapping.

If we reject the null, this implies that the restricted model is informationally insufficient, and thus may not span the shocks of interest.

Table A.2: Informational sufficiency tests (p-values)

(a) Foreign block							
(i) Non-U.S. Interest Rates	Australia	Canada	Euro Area	New Zealand	Norway	Sweden	UK
	0.12	0.04	0.96	0.84	0.25	0.57	0.87
(ii) Other Variables							
Global Economic Conditions Indicator	0.18						
U.S. Real Exchange Rate	0.49						

(b) Domestic block							
	Australia	Canada	Euro Area	New Zealand	Norway	Sweden	UK
(i) U.S. Interest Rates	0.00	0.00	0.31	0.52	0.85	0.00	0.28
(ii) Domestic Real Exchange Rate	0.83	0.04	0.61	0.68	0.10	0.30	0.55
(iii) Global Economic Conditions Indicator	0.66	0.86	0.02	0.09	0.79	0.01	0.22
(iv) U.S. Real Exchange Rate	0.62	0.81	0.60	0.30	0.50	0.21	0.25

Notes: Null is no Granger causality. Rejecting of the null thus implies Granger causality, and thus relevant information is omitted. p-values are for single tests, and do not account for any adjustment for multiple testing. If variables are included, we use 4 lags due to quarterly data, and also consistent with our empirical specification.
For panel (a), we test for whether additional information helps forecast the U.S. long-rate and term spread. The restricted model only uses the lags of these two variables.
(i) The unrestricted model also includes the long-rate and spread from the respective open economy.
(ii) The unrestricted model also includes either the global economic conditions indicator by Baumeister et al. (2022) or the U.S. real exchange rate.
For panel (b), we test for whether additional information helps forecast the domestic long-rate and term spread. Apart from (i), the restricted model always includes the lags of the U.S. and respective open economy's long rate and term spread (i.e., 4 variables).
(i) The restricted model only includes the domestic open economy's long rate and term spread. The unrestricted model also includes the U.S. long rate and term spread.
(ii) The unrestricted model also includes the domestic open economy's real exchange rate.
(iii) The unrestricted model also includes the global economic conditions indicator by Baumeister et al. (2022).
(iv) The unrestricted model also includes the U.S. real exchange rate.

Table A.2 presents the results of these informational sufficiency tests.

We first test whether the domestic block in the other seven economies (i.e., the change of the domestic real long-rate and the domestic term spread) Granger causes the U.S. block. If none of the domestic open economies Granger cause the U.S., it would suggest that the block exogeneity structure is a reasonable approximation to model the two economies since it suggests the foreign block at least spans all the U.S. shocks to interest rates. These are under item (i) of panel (a). At a 1% level of significance, there is no evidence that the U.S. block is informationally insufficient. At 5% level of significance, there may be some evidence that the Canadian block may contain information that may span the shocks that drive U.S. interest rates. Nonetheless, given we are doing repeated testing across all seven economies, a Bonforroni correction would imply we need a p-value of under 0.007 in order to reject Granger causality from any of the domestic block at a 5% level of significance. Therefore, applying the Bonforroni correction would mean we cannot reject the null of informational sufficiency at even a 5% level of significance.

In addition, given that our theoretical discussion based on Del Negro et al. (2019) suggests the change of the real exchange rate may include information about the shocks that drive r-star, we also tested whether the first difference of the log U.S real exchange rates Granger cause our U.S. block. We also tested whether the global economic activity index by Baumeister et al. (2022) Granger causes the U.S. block. We were unable to reject the null in either case. Taking as a whole, our results suggest the U.S. block is informationally sufficient. These results are also aligned with results in Morley et al. (2024), who despite using a very large set of variables, found that the dominant piece of information in obtaining r-star estimates using a multivariate BN decomposition is the interest rate term spread.

Next, we tested the domestic block. We first tested whether the foreign block of U.S. interest rates Granger causes the domestic block with just the domestic interest rate and domestic term spread. Item (i) of panel (b) presents these results. We find evidence that the U.S. block Granger causes three out of seven of our open economies. From Evans and Reichlin (1994) and Morley and Wong (2020), we know that one needs to include all sources of useful forecasting information in order to estimate the Beveridge-Nelson cycle (and trend) as these will change the estimate of the BN trend and cycle. Therefore, our results suggest that the U.S. interest rates is needed to help estimate r^* for about half the economies. For the other half, these results also suggest that the role that the U.S. interest rate plays is not so much in estimating r^* (given one can obtain almost identical estimates without the U.S. block), but only plays the role of identifying global shocks through our identification scheme.

We next test whether our specification is sufficient to conclude that we are able to distinguish between shocks to foreign and domestic r^* from the perspective of each of our open economies in question. We take a two-pronged approach to answer this question.

First, we check whether we can plausibly rule any other domestic shocks that may influence domestic r^*. From the simple theoretical framework that we adapt from Del Negro et al. (2019), we know this information is contained in the real exchange rate (e.g., see equation (4) and also (8) in a more specific case). Therefore, if we have omitted relevant information that will span the shocks driving domestic r^*, the domestic real exchange rate should Granger cause the variables in the domestic block. We are unable reject no Granger causality at a 1% level of significance, and once again, cannot reject at a 5% level of significance if we apply the Bonforroni correction for multiple testing, while nonetheless acknowledging the possibility that the Canadian real exchange rate may contain relevant information to determine Canadian r^*. Our results therefore suggest that, apart from perhaps with the exception of Canada, our specification likely spans all the domestic shocks of interest.

Second, we test whether the global economic conditions indicator contains any relevant marginal information for the domestic block. Our motivation for this test is as follows. While the testing in panel (a) provides support that we do span all the U.S. shocks, making the leap of the U.S. block representing foreign shocks requires us to rule out shocks from a third foreign economy from driving domestic r^*. The results of these tests are reported in (iii) of panel (b). The results are mixed. If allowing for the fact that we conduct multiple testing, the evidence is not always clear that the global economic conditions indicator contains relevant marginal information for the small open economy. We are nonetheless cognisant of the possibility that some evidence of the global economic conditions indicator Granger causing some of the domestic blocks suggests shocks from a third economy could be potentially important. To rule out these shocks being from the U.S., we tested whether U.S. real exchange rate Granger causes any of our domestic block and found no evidence, suggesting that any shocks spanned by the global economic conditions indicator, if any, likely reflect foreign shocks from a third economy and not from the U.S.

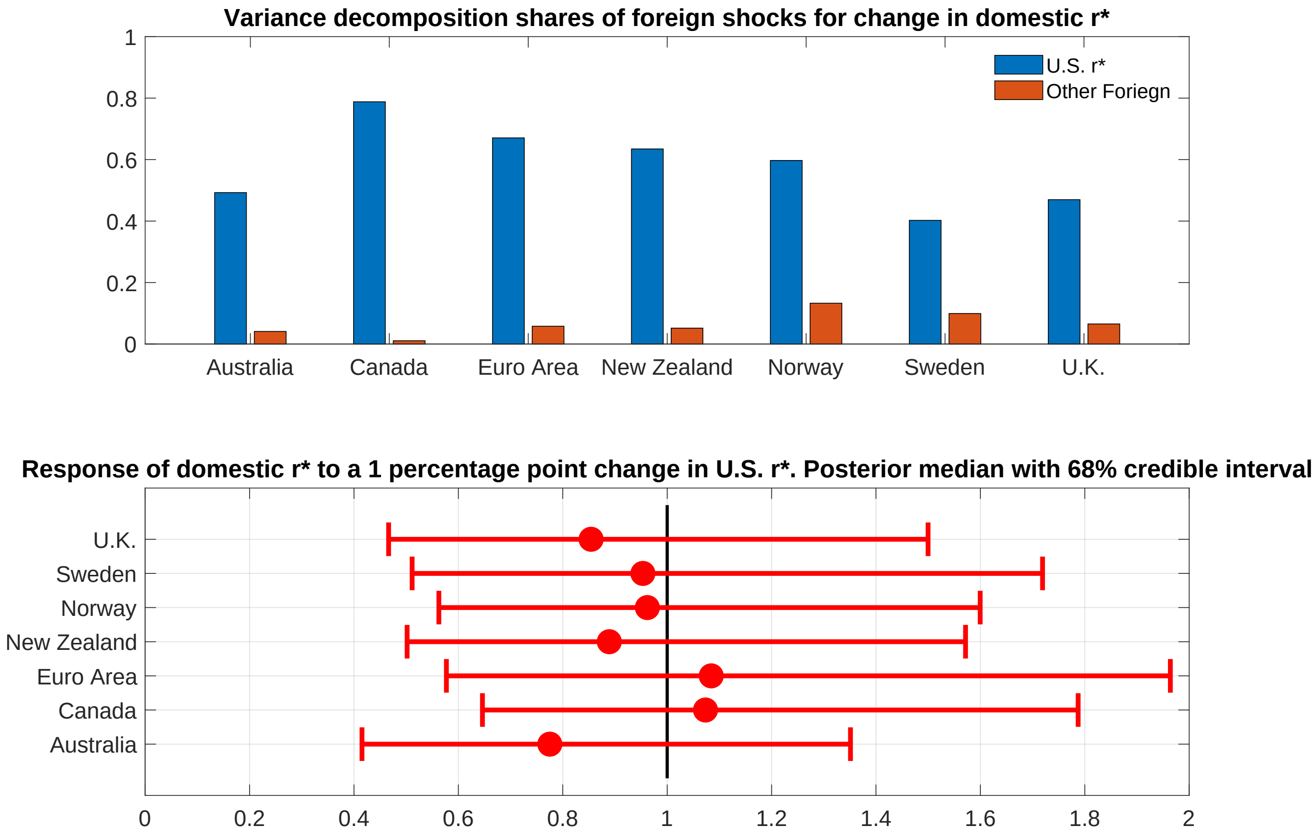
Therefore, on the basis of the Granger causality tests developed by Forni and Gambetti (2014) for informational sufficiency, and taken as a whole, we conclude the following: (i) The foreign block is likely informationally sufficient for the foreign shocks. In the vast majority of cases, the domestic block is probably also sufficient, which suggests the baseline setup is a plausible specification to study the effect of global and foreign shocks; (ii) There is mixed evidence from the global economic activity index whether we have spanned all the foreign shocks. Given the global economic activity index does not span the U.S. shocks, it is more likely that any predictability from the global economic activity index reflects foreign shocks from a non-U.S. source (i.e., a third economy).[18] Given our focus on decomposing foreign and domestic shocks to global r^*, we also augment our foreign block with the global economic activity indicator as a robustness check in the next section to explore the sensitivity of our conclusions to the inclusion of this source of information.

D Robustness

Motivated by the informational sufficiency tests we conducted in Section C of the Appendix, we consider a system where we include the Global economic activity indicator by Baumeister

[18] One could postulate a structure where we have a two block foreign block (i.e., a three block model) where the U.S. interest rates are in one block and global economic activity indicator is another block. Given including the global economic activity indicator in the foreign block with the U.S. interest rates is just a more unrestricted version of this three block structure, we just add the global economic activity indicator into the U.S. block.

Figure A.1: The importance of global r^*, model estimated with GECON



Notes: The top panel presents the share of the variance of the change in the estimated r^* attributed to the foreign shocks. The bottom panel presents the posterior median and 68% credible set of the estimated percent change in domestic r^* in response to a 1 percentage point change in U.S. r^*. The horizontal line marks out the point where domestic r^* changes one-for-one with U.S. r^*.

et al. (2022) in the foreign block. As we already presented in Figure 5, the estimates of r^* are almost identical to our baseline. There are slight differences to the counterfactuals role of foreign shocks, which, if anything, suggest global shocks may have accounted slightly more of the decline in domestic r^* across the seven open economies relative to our baseline. For completeness, Figure A.1 provides a comparison to Figure 3 in the main text to understand if the inclusion of the global economic activity indicator changes the role of foreign shocks.

In general, the results are almost identical to our baseline case. Interestingly, we note that the point estimate of whether domestic r^* changes one for one with the U.S. r^* shifted left to being slightly under one-for-one when we consider the global economic activity indicator. These may suggest that including the global economic activity indicator may be accounting for additional foreign shocks that affect domestic r^* that do not originate from the U.S. Nonetheless, we do not want to over-interpret this result given the credible set still covers very much the same range as in the baseline case, and 1 lies very much in the middle of the credible sets in both specifications. We therefore conclude that despite the mixed testing results that global economic activity indicator may include omitted foreign shocks from our baseline, accounting for these possible omitted foreign shocks has almost no effect to our main results.