Crawford School of Public Policy

CAMA

Centre for Applied Macroeconomic Analysis

Children matter: Global imbalances and the economics of demographic transition

---

CAMA Working Paper 13/2022
February 2022

Tsendsuren Batsuuri
Crawford School of Public Policy, ANU
Centre for Applied Macroeconomic Analysis, ANU

Abstract

This paper investigates the effect of child dependency on the economy and external imbalances under an asymmetric demographic and productivity transition within a life-cycle model. It embeds dependent children within a two-country model with lifecycle features to examine child dependency's effect on the economy and external imbalances. Specifically, the paper compares the effects of the same fertility and mortality shocks across models with and without children. Simulations show that child dependency changes both the steady-state and the transition dynamics under a demographic shock. The paper finds that while child dependency changes the direction of the impact of the fertility transition on external imbalances in the short run, it changes the magnitude of the effects in the long run. Furthermore, the model comparison shows that parameters must be chosen differently across models with and without child dependency to start from the same interest rate in the steady-state. Different calibration affects the magnitude of the transition dynamics of different models. These findings illustrate the importance of considering child dependency in studies that seek to explain the historical contribution of demographic changes to external imbalances, and suggest to approach studies that use models without child dependency for this purpose with caution.

Keywords

Global imbalances,Trade imbalances, Demographic transition, Life-cycle model

JEL Classification

D15, E21, E22, E43, E62, F21, F41, J11

Address for correspondence:

(E) cama.admin@anu.edu.au

ISSN 2206-0332

Children matter: Global imbalances and the economics of demographic transition *

Tsendsuren Batsuuri**

February 2, 2022

Abstract: This paper investigates the effect of child dependency on the economy and external imbalances under an asymmetric demographic and productivity transition within a life-cycle model. It embeds dependent children within a two-country model with life-cycle features to examine child dependency's effect on the economy and external imbalances. Specifically, the paper compares the effects of the same fertility and mortality shocks across models with and without children. Simulations show that child dependency changes both the steady-state and the transition dynamics under a demographic shock. The paper finds that while child dependency changes the direction of the impact of the fertility transition on external imbalances in the short run, it changes the magnitude of the effects in the long run. Furthermore, the model comparison shows that parameters must be chosen differently across models with and without child dependency to start from the same interest rate in the steady-state. Different calibration affects the magnitude of the transition dynamics of different models. These findings illustrate the importance of considering child dependency in studies that seek to explain the historical contribution of demographic changes to external imbalances, and suggest to approach studies that use models without child dependency for this purpose with caution.

Keywords: Global imbalances,Trade imbalances, Demographic transition, Life-cycle model

JEL Codes: D15, E21, E22, E43, E62, F21, F41, J11

*I gratefully acknowledge the financial support provided by the Australian Research Council Centre of Excellence in Population Ageing Research (CE170100005). I am grateful to Warwick McKibbin, Robert Mendelsohn, Creina Day, Larry Liu, and Gan-Ochir Doojav for their valuable suggestions and discussions on this work. I would also like to thank the ACDE Ph.D. student seminar participants and the CEPAR annual workshop participants for their questions and helpful comments. I am also grateful to Yale University for providing financial support and giving me complete access to its facilities and resources under the Fox international fellowship. Lastly, I want to thank the fellow students of my Peer-Review Group at Yale University for reading this work and giving valuable feedback to improve the writing.

**Centre for Applied Macroeconomic Analysis, Crawford School of Public Policy, Australian National University. E-mail: tsendsuren.batsuuri@anu.edu.au

1 Introduction

Trade and current account imbalances across major economies have become a persistent feature of the global economy. Figure 1 shows the increasing trade imbalances of the US, China, and G6 (UK, France, Germany, Italy, Canada, and Japan) since the 1980s. A growing literature has attempted to understand the underlying causes. As emphasized by Blanchard and Milesi-Ferretti (2012), such research must be interpreted with caution due to its potential impact on the nature and scope of policy responses. This paper highlights some of the issues that arise in quantifying the effects of demographics on historical external imbalances using calibrated life-cycle models that do not consider the impact of changes in child dependency.

Traditional theories of trade á la David Ricardo and Heckscher-Ohlin may explain why we trade but fail to answer why there is a persistent imbalance across countries. To explain imbalances, economists rely on intertemporal theories of trade which see imbalances as a means of smoothing consumption across time in the face of non-smooth income. However, the neoclassical theory, based on an infinite horizon representative agent, falls short in explaining China's recent experience, as the standard model would predict China to have a large trade deficit driven by high investment and consumption given high productivity growth. Despite the high investment rate, China was in fact able to have a trade surplus thanks to its even higher savings rate (see the second figure in Figure 1). Hence, understanding diverging savings rates across countries is thought to be the key to understanding trade imbalances (Gourinchas & Rey, 2014). The failure of the standard model to account for changes in savings, and hence in predicting the trade imbalances, opens room for alternative theories.

Furthermore, the timing of the growing imbalances in these economies has led economists to attribute some of the external imbalances to unprecedented yet asymmetric demographic transitions, namely different speed and timing in the falling fertility rates and increasing life expectancy in these countries. From around the 1970s, the child dependency ratio has fallen in these economies while the old age dependency ratio has increased (see Figure 2). Notably, China is experiencing a rapid and profound demographic change, with China's child dependency ratio* falling by almost three quarters from the early 1980s to 2020. Meanwhile, with the dramatic fertility decline and the increasing life expectancy at old age, the old age dependency ratio in China is expected to surpass the US level by 2040 (UN, 2019).

Prompted by these facts, a growing literature aims to investigate the factors behind the persistent trade deficits of the US using calibrated life-cycle models. For example, by extending a tractable life-cycle model proposed by Gertler (1999) to two-country settings, Ferrero (2010) analyzed the contribution of productivity, demographics, and fiscal policy differentials on the trade imbalance between US and G6. He attributes the persistent US trade deficit against G6 between the 1980s to 2005 to its lower life expectancy and higher productivity. Other papers found institutions such as social security (Eugeni, 2015; Niemeläinen, 2021) and financial systems (Caballero et al., 2008; Coeurdacier et al., 2015)

*In this paper, the child dependency ratio is defined as the ratio of population aged between 0-19 to between 20-64. Furthermore, the term child dependency has been used to describe the number of children per worker and the economic dependence of children on workers who provide for their consumption and take time out of work to raise children.

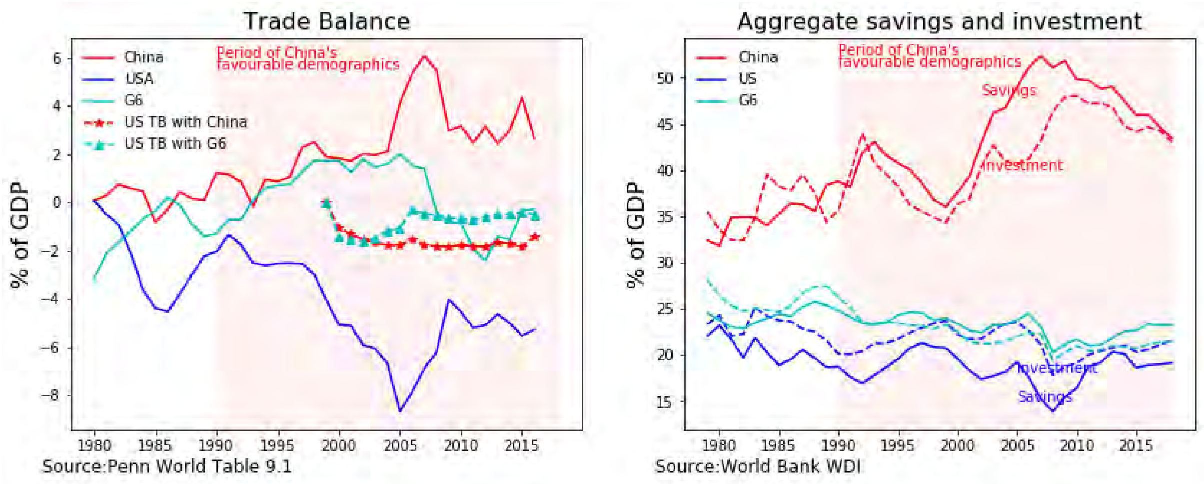

Figure 1: External imbalances

to be important in determining changes in aggregate savings and trade imbalance during the demographic transition. For example, using the model similar to Ferrero (2010), Niemeläinen et al. (2020, Chapter 3 & 4) examined the trade imbalances between the US and China, allowing different pension coverage, variable labour supply, and exchange rates. She found that the high savings in China, which drive Chinese trade surpluses, can be attributed to its low pension coverage in the face of increasing life expectancy. On the other hand, Coeurdacier et al. (2015) emphasizes credit constraints along with demographic change and productivity growth differentials as other mechanisms whereby diverging rates of savings occur between the US and China, which underlies its trade imbalances.

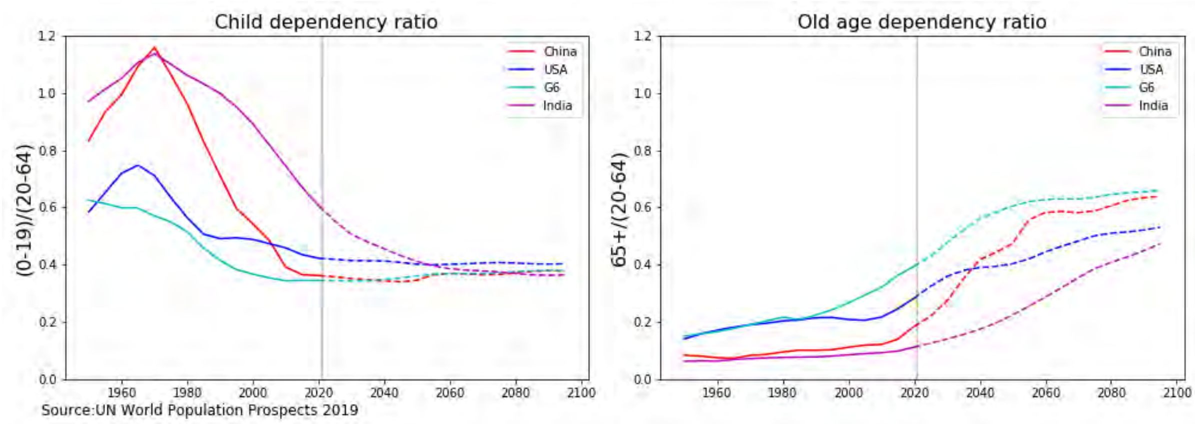

Figure 2: Child and old age dependency

Despite being the most prominent feature of the global demographic transition, as shown in Figure 2, the effect of changes in child dependency on aggregate savings has been neglected altogether by the research mentioned above. In the frameworks used by Ferrero (2010); Niemeläinen (2021); Coeurdacier et al. (2015), the population dynamics start with the working-age population. Neglecting child dependency is a serious shortcoming if one aims to understand the effect of demographic changes (Bommier & Lee, 2003). At the micro and macro levels, empirical evidence points to dependent children's significance in affecting savings. Micro-level studies estimate the consumption cost of children to fall in the range between 10 and 40 percent of household's income or budget (Donni, 2015; Letablier et al., 2009). However, these estimates do not include the time cost of children. Econometric studies find that the number of children can affect labour force participation, particularly women's (Bloom et al., 2009a; Lundborg et al., 2017). Bloom et al. (2009a) finds that each additional child reduces the female labour supply on average by two years. On the other hand, macro-level empirical studies find child dependency to be significant in determining aggregate savings (Leff, 1969; Masson et al., 1998; Park & Shin, 2009; Curtis et al., 2015, 2017; Lugauer et al., 2019; Higgins & Williamson, 1997) and capital flow (Lührmann, 2003; Chinn & Prasad, 2003).

The main research question addressed in this paper is the consequence of neglecting consumption and time cost of children when examining the effect of demographic changes on the economy and external imbalances within a life-cycle model. This paper incorporates the consumption and time cost of children in a way similar to Barro and Becker (1989) ** in a two-country tractable life-cycle model á la Gertler (1999) and compares the effects of same fertility and mortality transitions between models with and without child dependency. Previous modeling studies have shown that child dependency depresses the savings of young workers (Brooks, 2003), and it changes both the steady-state and the transition dynamics associated with demographic change (Bryant et al., 2004). The contribution of the current paper is to embed consumption and time cost of children within a tractable life-cycle model with realistic demography that is solved on an annual basis to investigate the consequence of neglecting child dependency.

The paper finds that child dependency matters for at least two reasons. First, within a non-Ricardian environment (as in life-cycle settings) where the timing of taxes and transfers do matter for aggregate savings and investment, the existence of consumption and time cost of children matters for the aggregate savings and capital per effective labour during the fertility and longevity transitions. Simulations show that child dependency changes the direction of the impact of the fertility transition on external imbalances in the short run. In contrast,in the long-run, it changes the magnitude of the effect. Second, even when there is no fertility transition and no change in child dependency, the existence of child dependency changes the steady-state capital per effective labour, the interest rate, and the distribution of assets between workers and retirees. Workers who bear the cost of raising children both in terms of time and money end up with lower savings and a lower share of aggregate assets in the economy. These differences in the steady-state and the distribution of assets matter for models that intend to investigate the effect of demographic transitions on the economy and on external imbalances.

Table 7 shows the parameter values and calibration strategy of three different studies that use a life-cycle framework to explain historical trade and current account imbal-

**However, the current paper abstracts from endogenous fertility and intergenerational transfers by assuming that the parents do not choose the number of children nor care about their children's utility but pay a fixed amount per child for consumption and allocate time out of work to raise children. Introducing children's consumption into parent's utility reduces the life-cycle motive of savings and makes the savings behavior of the individual closer to the behavior of a representative agent with infinite horizon (Weil, 1989).

ance of the US. The calibration strategy used in these papers includes setting the key parameters that determine savings to target savings or interest rates in the data for a specific year. For example, Ferrero (2010) calibrated his subjective time preference rate to match the interest rate in the 1970s. By using the same parameter values, Niemeläinen (2021) gets a much lower interest rate in the steady-state for the fixed labour supply case due to the change in the model structure where she considers pension. However, it is unclear whether Niemeläinen (2021) targets the interest rate even though she is doing historical analysis. On the other hand, Coeurdacier et al. (2015) used a calibration strategy where key parameters that determine individual savings are chosen to target a close match between model predicted savings rate and the household savings rate in the data. Even though these papers conduct sensitivity analysis, sensitivity analysis may not be adequate for at least two reasons. First, model misspecification (e.g. omission of child dependency) may result in differences in parameter values across models much wider than the range considered in the sensitivity analysis. Second, sensitivity analysis for each parameter separately may not capture the interaction between parameters. In the case of Coeurdacier et al. (2015), both subjective time preference rate and the elasticity of intertemporal substitution are chosen at low levels, which increases the credit constraint mechanism artificially.

This paper shows that incorporating children's consumption and time costs will reduce steady-state aggregate savings and increase the steady-state interest rate. Using the same calibration strategy as in Ferrero (2010); Niemeläinen (2021); Coeurdacier et al. (2015), the different steady-states will result in the need to calibrate the subjective time preference to be higher in the model that considers the cost of children to target the same interest rate and savings in the data. The differences in the value of the calibrated parameters lead to differences in the dynamic impact of fertility and longevity transitions even though the latter does not affect the child dependency rates. Furthermore, the existence of child dependency increases the importance of credit constraints, thereby reducing the need to choose intertemporal elasticity of substitution and subjective time preference rate to be set at unconventional levels, as in Coeurdacier et al. (2015), which itself affects the dynamics. In other words, the models with and without child dependency represent different economies at the micro-level, which cause differences in terms of their response to demographic shocks even if they correspond to the same aggregate outcome at a certain point in time.

The paper contains six sections, including the introduction section. Section 2 presents the model used for simulation, with section 3 providing some analytical explanation of the external imbalance determinants. Section 4 outlines the calibration strategy and the description of the quantitative experiment; section 5 and 6 reports the steady-state and the dynamic analysis respectively, and section 7 concludes. In addition, a separate appendix section provides steady-state equations of the model together with the description of the solution method and mathematical proofs of the relationship between the agent's marginal propensity to consume, the life expectancy, and the interest rate.

2 Model

The analysis in section 6 is based on an extension of the closed economy life-cycle model proposed by Gertler (1999) to a two-country model with four different cohorts, the children, young and mature workers, and retirees. The production side of the model is kept simple as in Ferrero (2010); Niemeläinen (2021); Coeurdacier et al. (2015) with the assumption of a representative firm producing a single consumption good using capital and labour. The model also considers government debt (as in Ferrero (2010)) and pension policy (as in Niemeläinen (2021)). The pension is modeled as a pay-as-you-go system where taxes on current workers are distributed among current retirees. The countries, referred to as home and foreign, are initially identical but later differ due to their different demographic transition shocks.

Furthermore, it is a deterministic model with no aggregate uncertainty. Agents correctly foresee prices in the economy except only in the first period after an exogenous shock. Both economies have the same production technology and produce a single good traded internationally with no transaction costs. The structure of the home country is described in detail in sections 2.1 to 2.5. All equations apply to the foreign country except when upper index f refers to variables for foreign countries.

2.1 Population dynamics

Population dynamics are exogenous in the model. A set of probabilities that govern the transition from one life stage to another, given in table 1, generates population dynamics in the model. These probabilities are independent of the age of an individual to facilitate aggregation. Time is discrete and goes forever, i.e., $t = 0, 1, 2, ...$ There are four age groups of individuals: children, young workers, mature workers, and retirees.

Table 1: Transition probabilities from period t-1 to t

	Children (t)	Young worker (t)	Mature worker (t)	Retiree (t)	Deceased (t)
Children (t-1)	$1-z$	z	0	0	0
Young worker (t-1)	0	1-θ	θ	0	0
Mature worker (t-1)	0	0	1-ω	ω	0
Retiree (t-1)	0	0	0	1-$\gamma_{t-1,t}$	$\gamma_{t-1,t}$
Deceased (t-1)	0	0	0	0	1

Let N_t^c denote the stock of children at time t. At time t, $(z + n_t - 1)N_{t-1}^c$ identical children are born which implies that the stock of children grow by n_t at time t.

$$N_t^c = (z + n_t - 1)N_{t-1}^c + (1 - z)N_{t-1}^c = n_t N_{t-1}^c \tag{1}$$

Let N_t^y denote the stock of young workers at time t. The stock of young workers at time t consists of children who became young workers at the beginning of time t (end of t-1) and young workers who didn't become a mature worker at the beginning of time t (end of t-1).

$$N_t^y = zN_{t-1}^c + (1 - \theta)N_{t-1}^y \tag{2}$$

Let's also denote children to younger worker ratio at time t by ψ_t^c:

$$\psi_t^c = \frac{N_t^y}{N_t^c} \tag{3}$$

The ratio of younger worker to children ratio evolves according to:

$$\psi_t^c = \frac{z}{n_t} + \frac{(1-\theta)(\psi_{c,t-1})}{n_t} \tag{4}$$

We can also define the growth rate of young workers as a function of the growth rate of children at time t as:

$$n_t^y = \frac{N_t^y - N_{t-1}^y}{N_{t-1}^y} = \frac{\psi_t^c(1+n_t)}{\psi_{t-1}^c} \tag{5}$$

Let N_t^m denote stock of mature or older workers at time t. The stock of older workers at time t consist of young workers who became mature at the beginning of time t (end of t-1) and mature workers who didn't retire at the beginning of time t (end of t-1).

$$N_t^m = \theta N_{t-1}^y + (1-\omega)N_{t-1}^m \tag{6}$$

Let's denote ratio of older to younger workers at time t by ψ_t^1:

$$\psi_t^1 = \frac{N_t^m}{N_t^y} \tag{7}$$

The ratio of older to younger workers evolves according to:

$$\psi_t^1 = \frac{\theta}{n_t^y} + \frac{(1-\omega)\psi_{t-1}^1}{n_t^y} \tag{8}$$

Let N_t^r denote the stock of retirees at time t. The stock of retirees at time t consists of older workers who retired at the beginning of time t (end of time t-1) and retirees who survived at the beginning of time t (end of time t-1).

$$N_t^r = \omega N_{t-1}^m + (1-\gamma_t)N_{t-1}^r \tag{9}$$

Let's denote the ratio of retirees to total number of mature and young workers at time t by ψ_t^2:

$$\psi_t^2 = \frac{N_t^r}{N_t^y + N_t^m} \tag{10}$$

The ratio of retirees to total number of older and young workers evolves according to:

$$\psi_t^2 = \frac{\omega\psi_{t-1}^1}{(\psi_t^1+1)n_t^y} + \frac{(1-\gamma_t)(\psi_{t-1}^1+1)\psi_{t-1}^2}{(\psi_t^1+1)n_t^y} \tag{11}$$

The growth rate of the workforce (total number of young and mature workers) can be estimated as:

$$n_t^{wf} = \frac{(N_t^m + N_t^y)}{(N_{t-1}^m + N_{t-1}^y)} - 1 = \frac{(1+\psi_t^1)n_t^y}{1+\psi_{t-1}^1} - 1 \tag{12}$$

labour force in the economy as equal to number of mature workers and number of young workers in same effective units and it is given as:

$$N_t = \xi_t(1 - \frac{\lambda^c}{\psi_t^c})N_t^y + N_t^m = (\xi_t(1 - \frac{\lambda_t^c}{\psi_t^c}) + \psi_t^1)N_t^y \tag{13}$$

Growth rate of the economy's labour force in efficiency unit:

$$n_t^{wf\xi} = \frac{\xi_t(1 - \frac{\lambda^c}{\psi_t^c})N_t^y + N_t^m}{\xi_{t-1}(1 - \frac{\lambda^c}{\psi_{t-1}^c})N_{t-1}^y + N_{t-1}^m} - 1 = \frac{(\xi_t(1 - \frac{\lambda_t^c}{\psi_t^c}) + \psi_t^1)n_t^y}{\xi_{t-1}(1 - \frac{\lambda^c}{\psi_{t-1}^c}) + \psi_{t-1}^1} \tag{14}$$

Because labour supply of young workers depend on child dependency ratio, it is given as:

$$N_t^{y\xi} = \xi_t(1 - \frac{\lambda^c}{\psi_t^c})N_t^y \tag{15}$$

Growth rate of effective labour supply of young workers are equal to:

$$n_t^{y\xi} = \frac{N_t^{y\xi} - N_{t-1}^{y\xi}}{N_{t-1}^{y\xi}} = \frac{\xi_t(1 - \frac{\lambda^c}{\psi_t^c})n_t^y}{\xi_{t-1}(1 - \frac{\lambda^c}{\psi_{t-1}^c})} \tag{16}$$

2.2 Households

There are four different age groups of individuals: retirees, mature workers, young workers, and dependent children. All age groups, except children, maximize their respective lifetime utility subject to their intertemporal budget constraints. Individuals maximize a non-expected utility function proposed by Farmer (1990), which provides a certainty-equivalent decision rule in the face of income risk that arises due to stochastic transition probability from one life stage to another. Different age individuals have different intertemporal budget constraints. The superscript 'i' denotes the individual type: 'r' for retirees, 'm' for mature workers, 'y' for young workers, and 'c' for children.

$$V_t^i = \Big[(C_t^i)^\rho + \beta_i \mathbb{E}(V_{t+1}|i)^\rho\Big]^{\frac{1}{\rho}} \tag{17}$$

where

$$\beta_i = \begin{cases} (1 - \gamma_{t,t+1})\beta & \text{if i = r} \\ \beta & \text{if i = m,y} \end{cases}$$

$$\mathbb{E}(V_{t+1}|i) = \begin{cases} V_{t+1}^r & \text{if i = r} \\ (1 - \omega)V_{t+1}^m + \omega V_{t+1}^r & \text{if i = m} \\ (1 - \theta)V_{t+1}^y + \theta V_{t+1}^m & \text{if i = y} \end{cases}$$

2.2.1 Children

Children do not make any decisions in the economy, and each child consumes a fixed portion of labour income. Workers pay for aggregate child consumption proportional to their labour productivity. Workers give away a part of their labour income for child consumption every period. Because the workers in this model supply labour inelastically, the specification of children's consumption in this way works as a lump-sum tax. Furthermore, because the wage rate is increasing at the rate of labour productivity growth, children's resources are also growing at the same rate as the wage increase. Every period, a worker gives away $t^c\psi_t^3$ portion of his/her labour income for child consumption. How much a worker gives away for child consumption depends on child dependency ratio (children to effective labour force ratio) given by ψ_t^3.

2.2.2 Young workers

This section provides the solution and the derivation of a young worker's decision rule. In addition to notation for whether a person is a retiree, a mature worker, or a young worker, we can differentiate individuals by their date of birth and the period when he/she transitioned to his/her next life stage. For example, 'j' indicates when people become young workers, 'k' denotes when they become mature workers, and 's' denotes when they become retirees. Individuals differ by their accumulated assets, depending on how much time an individual spent in each life stage.

Young workers have lower productivity than mature workers as well as lower labour supply due to the need to take care of children as in Lau (2014). Like mature workers, young workers also have to transfer a portion of their labour income for child consumption and pay taxes for pension and government expenditure.

When transitioning from a dependent child to a young worker, a young worker starts without any assets. This means $A_t^{yj} = 0$ when j and t correspond to same date. A young worker supplies 1 unit of labour inelastically and consumes out of assets and labour income. However, young workers' productivity level is lower than the productivity of mature workers due to inexperience. Besides, young workers have to take some time out raising children. Therefore, a young worker's actual labour supply equals their labour endowment minus time spent raising children. Because young workers' have lower productivity than mature workers, they receive a fraction of mature workers' wage corresponding to their labour productivity level represented, here, by ξ . Young workers also transfer a portion of their labour income given by $\xi W_t t^c$ for child consumption and pay taxes.

With the above constraints reflected, a young worker's problem is to maximize lifetime utility:

$$V(A_t|yj) = \max_{\{C_t, A_{t+1}\}} \left\{ [C_t^{yj}]^\rho + \beta \mathbb{E}_t V(A_{t+1}^{yj})^\rho \right\}^{\frac{1}{\rho}} \tag{18}$$

subject to an intertemporal budget constraint:

$$A_{t+1}^{yj} = R_t A_t^{yj} + \xi(1 - \frac{\lambda^c}{\psi_{c,t}})(W_t(1 - \psi_t^3 t^c) - T_t^w) - C_t^{yj} \tag{19}$$

where expected next period lifetime value function is

$$\mathbb{E}_t V(A_{t+1}^{yj}) = (1-\theta)V(A_{t+1}^{yj}) + \theta V(A_{t+1}^{mjk)}) \tag{20}$$

First order necessary condition for the decision problem of a young worker gives the consumption euler equation of the form:

$$(1-\theta)C_{t+1}^{yj} + \theta\Lambda_{2,t+1}C_{t+1}^{mjk} = \Big(R_{t+1}\Omega_{2,t+1}\beta\Big)^{\sigma} C_t^{yj} \tag{21}$$

where $\Lambda_{2,t+1}= \epsilon_{2,t+1}^{\frac{\sigma}{\sigma-1}}$ Furthermore, the consumption function for a young worker is conjectured as:

$$C_t^{yj} = \epsilon_{2,t}\pi_t\Big(R_t A_t^{yj} + H_t^{yj} + S_t^{yj}\Big) \tag{22}$$

Solution to a young worker's decision problem gives the evolution of marginal propensity to consume out of wealth for a young worker:

$$\epsilon_{2,t}\pi_t = 1 - \beta^{\sigma}(R_{t+1}\Omega_{2,t+1})^{\sigma-1}\frac{\epsilon_{2,t}\pi_t}{\epsilon_{2,t+1}\pi_{t+1}} \tag{23}$$

where

$$\Omega_{2,t+1} = (1-\theta) + \theta\epsilon_{2,t+1}^{1/(\sigma-1)} \tag{24}$$

$$H_t^{yj} = \xi(1-\frac{\lambda^c}{\psi_{c,t}})(W_t(1-\psi_t^3 t^c) - T_t^w) + (1-\theta)\frac{H_{t+1}^{yj}}{R_{t+1}\Omega_{2,t+1}} + \theta\frac{H_{t+1}^{mjk}\epsilon_{2,t+1}^{\frac{1-\rho}{\rho}}}{R_{t+1}\Omega_{2,t+1}} \tag{25}$$

Social security wealth for a young worker is defined as the present value of expected pension income by a young worker:

$$S_t^{yj} = (1-\theta)\frac{S_{t+1}^{yj}}{R_{t+1}\Omega_{2,t+1}} + \theta\frac{S_{t+1}^{mjk}\epsilon_{2,t+1}^{1/(\sigma-1)}}{R_{t+1}\Omega_{2,t+1}} \tag{26}$$

2.2.3 Mature workers

Mature workers supply one unit of labour inelastically and receive wage income of W_t at the end of each period. However, workers need to transfer a portion of their labour income ($\psi_t^3 t^c W_t$) for child consumption which depends on child dependency ratio (children to total labour force ratio given by ψ_t^3) as well as has to pay taxes. Workers save and consume from assets and remaining labour income after transfer to children and taxes. Workers face an uncertain time of retirement with $1-\omega$ is the probability to stay in the workforce in period t+1 of a mature worker at t. At the same time, ω is the probability of retiring at the beginning of the next period. The retirement risk will not complicate the mature workers' decision rule since they are risk-neutral by assumption.

$$V(A_t|mjk) = \max_{\{C_t, A_{t+1}\}}\Big\{[C_t^{mjk}]^{\rho} + \beta\mathbb{E}_t V(A_{t+1}^{mjk})^{\rho}\Big\}^{\frac{1}{\rho}} \tag{27}$$

subject to one's intertemporal budget constraint:

$$A_{t+1}^{mjk} = R_t A_t^{mjk} + (W_t(1-\psi_t^3 t^c) - T_t^w) - C_t^{mjk} \tag{28}$$

where expected next period lifetime value function is the weighted sum of lifetime value function of being a worker at t+1 and being a retiree at t+1.

$$\mathbb{E}_t V(A_{t+1}^{mjk}) = (1-\omega)V(A_{t+1}^{mjk}) + \omega V(A_{t+1}^{rjks}) \tag{29}$$

As in the retirees' case, workers' assets are measured at the beginning of each period while the wage and the interest are paid at the end of each period. A worker's consumption expenditure is also undertaken at the end of each period. Therefore, the assets of a mature worker evolve according to:

$$A_{t+1}^{mjk} = R_t A_t^{mjk} + (W_t(1-\psi_t^3 t^c - T_t^w) - C_t^{mjk} \tag{30}$$

First order necessary condition for the decision problem of a mature worker gives the consumption euler equation of the form:

$$(1-\omega)C_{t+1}^{mjk} + \omega\Lambda_{t+1}C_{t+1}^{rjks} = \Big(R_{t+1}\Omega_{t+1}\beta\Big)^{\sigma} C_t^{mjk} \tag{31}$$

where $\Lambda_{t+1} = \epsilon_{t+1}^{\frac{\sigma}{1-\sigma}}$

Furthermore, the consumption function for a mature worker is conjectured as:

$$C_t^{mjk} = \pi_t\Big(R_t A_t^{mjk} + H_t^{mjk} + S_t^{mjk}\Big) \tag{32}$$

Combining the Euler equation with the conjectured consumption function yields the solution to a mature worker's decision problem in the form of the evolution of marginal propensity to consume out of wealth for a mature worker as:

$$\pi_t = 1 - \beta^{\sigma}(R_{t+1}\Omega_{t+1})^{\sigma-1}\frac{\pi_t}{\pi_{t+1}} \tag{33}$$

where

$$\Omega_{t+1} = (1-\omega) + \omega\epsilon_{t+1}^{1/(1-\sigma)} \tag{34}$$

Human wealth is defined as the present value of expected lifetime labour income which is the discounted sum of wages. The human wealth of a mature worker evolves according to:

$$H_t^{mjk} = (W_t(1-\psi_t^3 t^c) - T_t^w) + (1-\omega)\frac{H_{t+1}^{mjk}}{R_{t+1}\Omega_{t+1}} \tag{35}$$

Social security wealth for a mature worker is defined as the present value of expected pension income a mature worker gets if retired next period:

$$S_t^{mjk} = (1-\omega)\frac{S_{t+1}^{mjk}}{R_{t+1}\Omega_{t+1}} + \omega\frac{S_{t+1}^{rjks}\epsilon_{t+1}^{\frac{\rho-1}{\rho}}}{R_{t+1}\Omega_{t+1}} \tag{36}$$

2.2.4 Retirees

Retirees do not work and consume out of their asset income. A retiree's decision problem is to choose consumption in each period and assets for the next period to maximize his/her lifetime utility given by a recursive value function:

$$V(A_t|rjks) = \max_{\{C_t, A_{t+1}\}}\Big\{[C_t^{rjks}]^{\rho} + \beta(1-\gamma_{t,t+1})\mathbb{E}_t V(A_{t+1}^{rjks})^{\rho}\Big\}^{\frac{1}{\rho}} \tag{37}$$

subject to an intertemporal budget constraint:

$$A_{t+1}^{rjks} = \frac{R_t}{1-\gamma_{t-1,t}} A_t^{rjks} - C_t^{rjks} + \frac{P_t}{N_t^r} \tag{38}$$

with expected lifetime utility next period being:

$$\mathbb{E}_t V(A_{t+1}^{rjks}) = V(A_{t+1}^{rjks}) \tag{39}$$

where ρ is associated with intertemporal elasticity of substitution σ with $\sigma = \frac{1}{1-\rho}$. β is the subjective discount rate or time preference rate. C_t^{rjks} is the consumption at t of the retiree born in period j, become a mature worker at period k, and retired at period s. $V(A_t|r)$ is the recursive form of a retiree's lifetime value function with an asset of A_t at time t. P_t is the aggregate pension expenditure that is divided among retirees equally.

Since retirees can die at the end of the current period, the discount rate for the expected lifetime utility from the next period includes retirees' survival probability $1-\gamma_t$. The assumed value function and associated intertemporal budget constraint imply that the retirees do not value leaving bequests. Retirees are constrained to have 0 or positive assets at the end of their life. Furthermore, to eliminate unintentional bequest, either positive or negative, due to uncertain time of death, it is assumed there exists a fair-insurance company as in Yaari (1965) and Blanchard (1985). Retirees turn their assets to a mutual fund that invests the proceeds at the beginning of each period. The surviving retirees receive the fund and interest earnings at the end of each period in proportion to their initial contribution. The existence of an insurance fund makes the return on assets of a retiree who turns his wealth to this mutual fund $\frac{R_t}{1-\gamma_{t-1,t}}$ where $R_t = (1-\delta) + r$ and r being market interest rate.

Assets are measured at the beginning of each period, while interest is paid at the end of each period. Retirees also receive pension benefits equal to total pension expenditure divided by the number of retirees $\frac{P_t}{N_t^r}$ at the end of each period. The intertemporal budget constraint of a retiree who faces the probability of death at the end of the current period becomes:

$$A_{t+1}^{rjks} = \frac{R_t}{1-\gamma_{t-1,t}} A_t^{rjks} - C_t^{rjks} + \frac{P_t}{N_t^r} \tag{40}$$

First order necessary condition for the decision problem of a retiree gives the consumption euler equation of the form:

$$C_{t+1}^{rjks} = C_t^{rjks} (\beta R_{t+1})^{\sigma} \tag{41}$$

Furthermore, the consumption function for a retiree is conjectured as:

$$C_t^{rjks} = \epsilon_t \pi_t \Big(\frac{R_t}{1-\gamma_{t-1,t}} A_t^{rjks} + S_t^{rjks} \Big) \tag{42}$$

Combining the Euler equation with the conjectured consumption function yields the solution to a retiree's decision problem in the form of the evolution of marginal propensity to consume out of wealth for a retiree as:

$$\epsilon_t \pi_t = 1 - \beta^{\sigma} R_{t+1}^{\sigma-1} (1-\gamma_{t,t+1}) \frac{\epsilon_t \pi_t}{\epsilon_{t+1} \pi_{t+1}} \tag{43}$$

Social security wealth for a retiree who are alive at t is defined as the present value of expected pension income over the retirement:

$$S_t^{rjks} = \frac{P_t}{N_t^r} + (1-\gamma)\frac{S_{t+1}^{rjks}}{R_{t+1}} \tag{44}$$

2.2.5 Aggregation

Because every person in retirement has the same life expectancy regardless of when he/she is retired or being born, they all have the same planning horizon implying everyone has the same marginal propensity to consume out of wealth. Therefore, it is straightforward to aggregate consumption of retirees as:

$$C_t^r = \epsilon_t \pi_t (R_t A_t^r + S_t^r) \tag{45}$$

with A_t^r equal to total assets held by retirees at time t. Similarly, everyone who is a mature worker also faces the same probability of retiring the next period making their planning horizon the same regardless of their age. Therefore, the aggregate consumption of mature workers are:

$$C_t^m = \pi_t (R_t A_t^m + H_t^m + S_t^m) \tag{46}$$

where A_t^m is total assets held by mature workers at time t and H_t^m is the present value of total lifetime labour income of those who are a mature worker at time t. Aggregate consumption of young workers is also:

$$C_t^y = \epsilon_{2,t} \pi_t (R_t A_t^y + H_t^y + S_t^y) \tag{47}$$

Where A_t^y is total assets held by young workers at time t and H_t^y is the present value of total lifetime labour income of those who are a young worker at time t.

As for consumption by children, since each child consumes $t^c W_t$ amount in every period, aggregate consumption by children becomes

$$C_t^c = t^c W_t N_t^c \tag{48}$$

Because, as shown in equation 49 and 50, workers discount the future at a higher rate due to finite work and lifetime, they do not capitalize the stream of future taxes fully, whether it is for child consumption or government spending. In other words, the increase in taxes reduces the present value of lifetime human wealth less in this framework than in the infinitely lived representative agent case causing the workers to internalize the future taxes partially. Thus, the introduction of child consumption does not crowd-out workers' consumption one to one, and the presence of child consumption increases aggregate consumption.

The aggregate, present value of the human wealth of mature workers who are alive at time t evolves according to:

$$H_t^m = (W_t(1-\psi_t^3 t^c) - T_t^w) N_t^m + (1-\omega)\frac{H_{t+1}^m}{n_{t+1}^m R_{t+1} \Omega_{t+1}} \tag{49}$$

whereas, aggregate, present value of human wealth of young workers who are alive at time t evolves according to:

$$H_t^y = \xi(1 - \frac{\lambda^c}{\psi_t^c})(W_t(1 - \psi_t^3 t^c) - T_t^w)N_t^y + (1-\theta)\frac{H_{t+1}^y}{R_{t+1}\Omega_{2,t+1}n_{t+1}^{y\xi}} + \theta\frac{H_{t+1}^m \epsilon_{2,t+1}^{\frac{1-\rho}{\rho}}}{R_{t+1}\Omega_{2,t+1}n_{t+1}^m} \quad (50)$$

Aggregate lifetime social security wealth evolve according to:

$$S_t^y = (1-\theta)\frac{S_{t+1}^y}{R_{t+1}\Omega_{2,t+1}n_{t+1}^y} + \theta\frac{S_{t+1}^m \epsilon_{2,t+1}^{1/(\sigma-1)}}{R_{t+1}\Omega_{2,t+1}n_{t+1}^m} \quad (51)$$

$$S_t^m = (1-\omega)\frac{S_{t+1}^m}{R_{t+1}\Omega_{t+1}n_{t+1}^m} + \omega\frac{S_{t+1}^r \epsilon_{t+1}^{\frac{\rho-1}{\rho}}}{R_{t+1}\Omega_{t+1}n_{t+1}^r} \quad (52)$$

$$S_t^r = P_t + (1-\gamma)\frac{S_{t+1}^r}{R_{t+1}n_{t+1}^r} \quad (53)$$

with the economy-wide consumption function being

$$C_t = \pi_t\Big[A_t R_t\Big(1+(\epsilon_t-1)\lambda_t^1+(\epsilon_{2,t}-1)\lambda_t^2\Big)+\epsilon_{2,t}(H_t^y+S_t^y)+\epsilon_t S_t^r+H_t^m+S_t^m\Big]+t^c W_t N_t^c \quad (54)$$

where $\lambda_t = A_t^r/A_t$ is the fraction of total assets in the economy held by retirees, $\lambda_{2,t} = A_t^y/A_t$ is the fraction of total assets in the economy held by young workers.

Evolution of capital distribution between different generations follows:

$$\lambda_{t+1}^1 + \frac{\omega}{1-\theta}\lambda_{t+1}^2 = \frac{P_t - \epsilon_t\pi_t S_t^r + (1-\omega)R_t\lambda_t^1 A_t(1-\epsilon_t\pi_t)}{A_{t+1}} + \omega \quad (55)$$

$$\lambda_{t+1}^2 = \frac{(1-\theta)((1-\epsilon_{2,t}\pi_t)\lambda_t^2 R_t A_t + \xi(1-\frac{\lambda^c}{\psi_t^c})(W_t(1-\psi_t^3 t^c) - T_t^w)N_t^y - \epsilon_{2,t}\pi_t(H_t^y + S_t^y))}{A_{t+1}} \quad (56)$$

2.3 Government

In each period, government levies a lump-sum tax and incurs debt to finance its wasteful spending and pays retirees . The period budget constraint of the government is:

$$B_{t+1} = R_t B_t + G_t + P_t - T_t \quad (57)$$

where

$$T_t = T_t^w N_t \quad (58)$$

The intertemporal budget constraint of the government is:

$$\sum_{v=0}^{\infty}\frac{T_{t+v}}{\Pi_{s=1}^v R_{t+z}} = R_t B_t + \sum_{v=0}^{\infty}\frac{G_{t+v}+P_{t+v}}{\Pi_{s=1}^v R_{t+z}} \quad (59)$$

In the model, it is assumed that debt to GDP ratio ($\overline{b_y}Y_t$), government spending to GDP ratio ($\overline{g_y}Y_t$) and the pension expenditure to GDP ratio ($\overline{p_y}Y_t$) is fixed and taxes adjust endogenously to satisfy the government's intertemporal budget constraint.

Comparing equation 59 with equations 49 and 50 shows that the discount rate used by the workers is different from the discount rate the government uses. Workers discount rate is higher, including parameters that reflect finite work (presence of θ and ω) and lifetime (presence of Ω). In other words, the government expenditure does not crowd-out private consumption to the extent it does in the infinite horizon representative agent economy because workers only partially internalize the future tax burden. Therefore, taxes and debts matter in this life-cycle economy than in the economy populated by representative agents with infinite horizons. In addition, because of different marginal propensities to consume out of the wealth of different age cohorts, redistribution of wealth across cohorts also have an additional effect on aggregate consumption and savings separate from its effect due to change in timing of taxes and transfers as in the model of Gertler (1999).

2.4 Firms

The production structure of the economy is kept simple as in Ferrero (2010); Niemeläinen (2017); Coeurdacier et al. (2015) to make it comparable and contrast how child dependency and age-earnings profile changes the results found in previous papers. It is assumed that there is a representative firm producing output with capital and labour with a technology represented by a production function:

$$Y_t = (X_t N_t)^{\alpha} K_t^{1-\alpha} \tag{60}$$

with labour augmenting technology growing at an exogenous rate of x

$$X_t = (1 + x_{l,t})X_t \tag{61}$$

I considered capital adjustment cost with same specification as in Ferrero (2010); Niemeläinen (2021). However, the simulations showed that the adjustment costs do not change the results much within a perfect foresight model. Within a perfect foresight model, agents know the demographic and productivity transition paths except only in the first period. Therefore, the model's results with capital adjustment cost differ from the model without adjustment cost only in the first period. In the absence of capital adjustment costs, the capital stock in the economy evolves according to:

$$K_{t+1} = (1 - \delta)K_t + I_t \tag{62}$$

First-order conditions of the firm's problem give the demand for labour and capital:

$$W_t = \alpha \frac{Y_t}{N_t} \tag{63}$$

$$r_t = (1 - \alpha)\frac{Y_t}{K_t} \tag{64}$$

2.5 Characterization of equilibrium in the world economy

The competitive world equilibrium is defined as a sequence of endogenous quantity and price variables given the sequence of exogenous predetermined variables, and the initial values of all the predetermined variables such that in each country i) households maximize their utility subject to their budget constraints, ii) firms maximize their profits subject to their technology constraints, and iii) all markets clear.

Total assets in the home country is equal to the value of total capital stock in the home country, government debt and net foreign asset holdings of the home country.

$$A_t = K_t + B_t + F_t \tag{65}$$

Foreign assets evolve according to

$$F_{t+1} = R_t F_t + NX_t \tag{66}$$

which links the goods and asset markets. Trade deficits and surpluses change net foreign asset holdings.

The aggregate capital stock evolves according to

$$K_{t+1} = (1-\delta)K_t + I_t \tag{67}$$

The trade balance is determined by the difference between aggregate output and aggregate expenditures which include both private and government consumption and investment expenditure.

$$NX_t = Y_t - (C_t + I_t + G_t) \tag{68}$$

Free flow of assets between economies ensures equalization of gross interest rate R_t between countries. In equilibrium, net asset holdings of the two countries cancel each other out.

$$F_t + F_t^f = 0 \tag{69}$$

Current account is the trade balance plus net interest payment on foreign assets.

$$CA_t = NX_t + (R_t - 1)F_t \tag{70}$$

The total labour force in the economy equals the number of mature workers plus the number of young workers multiplied by their labour productivity. labour market equilibrium is when the total supply of labour equals the total demand for labour.

$$N_t = \xi(1 - \frac{\lambda^c}{\psi_t^c})N_t^y + N_t^m = (\xi(1 - \frac{\lambda^c}{\psi_t^c}) + \psi_t^1)N_t^y \tag{71}$$

All quantity variables are normalized by population and productivity growth to estimate the model's steady state. The steady-state and the transition dynamics are solved using the non-linear Newton method. Government debt is assumed to be exogenous and a fixed share of GDP to close the model. The appendix section B provides a description of the solution method and the algorithm used to solve the model. Julliard (2005) also offers further explanation of the solution method.

3 Determinants of external imbalances

A perfectly integrated capital market implies that capital per effective labour is equalized across countries in each period in the absence of capital adjustment cost.

$$\kappa = \kappa^f \tag{72}$$

Since output per effective labour is only a function of capital per effective labour, output per effective labour is also equalized across countries in each period.

$$y = y^f \tag{73}$$

When the government expenditure per effective labour is equal across countries, the differences in consumption and investment completely characterize the trade balance between countries 74.

$$nx_t = \frac{-1}{(1+\Psi_t^r)}\left[(c_t - c_t^f) + (i_t - i_t^f)\right] \tag{74}$$

where Ψ_t^r reflects relative country sizes as expressed by:

$$\Psi_t^r = \frac{X_t N_t}{X_t^f N_t^f} \tag{75}$$

which evolves according to:

$$\Psi_{t+1}^r = \Psi_t^r \frac{n_{t+1}^{f,wf\xi} x_{l,t+1}^f}{n_{t+1}^{wf\xi} x_{l,t+1}} \tag{76}$$

Small alphabets denote variables expressed in effective labour units. As in Ferrero (2010) we can denote a variable that is showing the difference between variable in the home country relative to the foreign country as $z_{R,t} = z_t - z_t^f$. Then equation 74 becomes:

$$nx_t = \frac{-1}{(1+\Psi_t^r)}\left[c_{R,t} + i_{R,t}\right] \tag{77}$$

Further, consumption differentials can be written as:

$$c_{R,t} = c_{R,t}^c + c_{R,t}^y + c_{R,t}^m + c_{R,t}^r \tag{78}$$

Different from the model in Ferrero (2010), consumption differences stem from not only the differences in consumption of the workers and the retirees but also from the differences in consumption by the children, which depend on child dependency ratio and cost per child. The importance of this mechanism grows when one considers China. Despite examining the trade imbalances of China, which experienced a sharp drop in its child dependency ratio, Niemeläinen (2017) has also neglected the role of this mechanism. Child consumption differences can be written as:

$$c_{R,t}^c = \left(D_t w_t \frac{\psi_t^4}{\psi_t^c} - D_t^f w_t^f \frac{\psi_t^{4,f}}{\psi_t^{c,f}}\right) \tag{79}$$

With the same children to labour force ratio, consumption differences between children across countries depend only on the cost per child:

$$c^c_{R,t} = \left(D_t - D^f_t\right) w_t \frac{\psi^4_t}{\psi^c_t} \tag{80}$$

However, the effect of child dependency on trade balance works through the consumption of children and its impact on workers' consumption. Higher consumption costs of children will reduce consumption by the workers. However, it doesn't reduce workers' consumption one to one because workers discount the future more. Therefore, even though the relative consumption of workers in a country with high child consumption may be less, it is not enough to offset the higher consumption of children, resulting in higher aggregate consumption, which causes a country to run a trade deficit.

In addition to the consumption cost of children, dividing the work-life stage into two stages allow us to introduce credit constraints as in Coeurdacier et al. (2015). Credit constraint limits the effect of productivity growth on consumption growth by lowering access to future human wealth. Coeurdacier et al. (2015) argues that the credit constraint is one of the critical factors underlying the divergence of savings between the US and China. Consumption differentials can be written for young workers as:

$$c^y_{R,t} = R_t\left(\epsilon_{2,t}\pi_t\lambda^2_t a_t - \epsilon^f_{2,t}\pi^f_t\lambda^{2,f}_t a^f_t\right) + (\epsilon_{2,t}\pi_t(h^y_t + s^y_t) - \epsilon^f_{2,t}\pi^f_t\zeta(h^{y,f}_t + s^{y,f}_t)) \tag{81}$$

for mature workers as:

$$c^m_{R,t} = R_t\left(\pi_t(1-\lambda_t-\lambda^2_t)a_t - \pi^f_t(1-\lambda^f_t-\lambda^{2,f}_t)a^f_t\right) + (\pi_t(h^m_t + s^m_t) - \pi^f_t(h^{m,f}_t + s^{m,f}_t)) \tag{82}$$

for retirees as:

$$c^r_{R,t} = R_t\left(\epsilon_t\pi_t\lambda_t a_t - \epsilon^f_t\pi^f_t\lambda^f_t a^f_t\right) + (\epsilon_t\pi_t s^r_t - \epsilon^f_t\pi^f_t s^f_t) \tag{83}$$

As can be seen from equations 81 to 83, not only differences in the MPCs but also in age-earnings profile (ξ) that affects the human wealth of young workers and the credit constraint (ζ) is important in determining consumption differentials.

Another determinant of the trade imbalances across countries is the differences in investment growth. Capital accumulation equation in 67 provides clues to the investment differentials. When there are no capital adjustment costs in the symmetric steady-state, capital per effective labour force is equalized across countries. Therefore investment differentials reflect the differences in the growth of the effective labour force.

$$i_{R,t} = (n^{wf\xi}_t - n^{wf\xi,f}_t + x_{l,t} - x^f_{l,t})\kappa_t \tag{84}$$

If economy-wide labour productivity growth is the same across countries, then investment differentials depend only on the differences in labour force growth. However, when you take into account age-dependent labour productivity and labour cost of children, relative differences in labour force growth during the demographic transition becomes more complex than just the difference between the growth of the working-age population as assumed in Ferrero (2010).

Specifically, in a country with a declining fertility rate and an age-earnings profile higher at mature age on average, the labour force growth will be higher than the growth rate of the young workers during the medium term. At the same time, when children require time out of work, a declining fertility rate will also have a short-run positive impact on labour force participation. However, both of these channels' effects will be transitory and depend on country-specific circumstances that determine the age-earnings profile and children's time cost.

4 Calibration

The experiment in this paper is purely hypothetical, and it uses parameter values commonly used in the literature, mainly for the US economy. A summary is given below for each parameter. The current paper highlights issues with calibration using historical values. Section 5 reports results of sensitivity analysis to illustrate the importance of different parameters on the steady-state of the model. The section below discusses possible strategies for calibrating critical parameters that determine the effect of demographic transition, particularly changes in the child dependency ratio, on external imbalances.

Consumption cost of children: In the baseline simulation, the paper assumes the consumption cost of children t^c to be equal to an average estimated value commonly found by empirical papers (see Letablier et al. (2009) for a review of the literature) as the share of annual income spent for children. Estimating the consumption cost of children is challenging, and the estimates differ quite a bit depending on the assumption and the methodology (Letablier et al., 2009). Therefore, a sensitivity analysis is conducted in section 5 to understand the implication of different values for the steady-state of the model.

labour cost of children: Similarly, to calibrate λ_c, findings from econometric studies are taken as guidance. Specifically, a study by Bloom et al. (2009b) finds that each additional child causes two years out of work for the mother. Therefore, one can calibrate the parameter λ_c to generate an outcome where young workers must spend 1-2 years out of work per child. Section 5 presents the result from a sensitivity analysis for this parameter for the steady-state of the model.

Shape of the age-earnings profile: The age-earnings profile works as a weight given to different age cohorts for the growth of the workforce. The national transfer accounts database provides age-earnings profiles by country. In addition, Coeurdacier et al. (2015) provides more detailed data on the empirical age-earnings profile for both China and the US. Baseline simulation assumes the shape of the profile to be increasing at an older age and the same across countries. As highlighted by Coeurdacier et al. (2015), the shape of the age-earnings profile matters more when the young workers face credit constraints. Specifically, a profile with an increasing income at a later age will induce a greater consumption motive at a young age. In addition, in the presence of an age-earnings profile, the growth of the effective labour force diverges from the growth of the workforce multiplied by economy-wide productivity growth. Then the question of how this discrepancy is taken into account when explaining the past arises. In studies that use Solow residual as a proxy for productivity growth as in Ferrero (2010), productivity growth measurements may incorrectly capture the changes in effective labour force growth due to age-earnings profile as economy-wide productivity growth.

Table 2: Demographic and household parameters

Parameter	Values	Description
ω	0.039	Probability to retire
θ	0.048	Probability to become mature worker
z	0.05	Probability to become young worker
ξ	0.75	Productivity of young workers relative to mature workers
γ_{ss}	0.08	Initial mortality rate
n_{ss}	1.028	Initial population growth rate
β	0.99 0.946	Time preference rate (The rest, NC)
σ	0.55 0.34	Household intertemporal elasticity of substitution (The rest, NC3)

Table 3: Child dependency parameters

Parameter	Values	Description
t^c	0.25	Consumption cost of children as a share of wage
λ_c	0.23	Time cost of children

Table 4: Production parameters

Parameter	Values	Description
α	0.667	labour share in the final output
δ	0.1	Capital stock depreciation rate
x_{lss}	1.028	Initial productivity growth rate

Table 5: Other parameters

Parameter	Values	Description
Ψ^r_{ss}	1	Relative size of countries at the steady state
p_y	0.06	Share of pension expenditure in GDP
b_y	0.2	Government debt to GDP ratio
g_y	0.2	Government expenditure to GDP ratio
ζ	1	Share of capitalized value of labour income of the young workers that can be borrowed

5 Steady-state analysis

This section reports the results from sensitivity analysis concerning the steady-state of the model. Figure 3 shows the steady-state impact of the consumption cost of children (t^c) on the output per effective labour, interest rate and the distribution of assets at the steady-state. It shows that the higher consumption cost of children corresponds to lower output per effective labour and higher interest rate reflecting lower capital per effective labour. Not only do aggregate savings and assets in the economy decline in absolute value in the presence of the consumption cost of children, but also its distribution among the adult population changes. The bottom half of figure 3 shows that the asset holdings of workers differ markedly between models with and without child consumption. Higher the consumption cost of children, lower the share of assets held by workers and higher the share of assets by retirees'. Specifically, the share of assets held by retirees can increase by almost 16 percentage points. In comparison, young and mature workers can be reduced by around seven percentage points each when the consumption cost of children increases from 0 to 50% of workers' income.

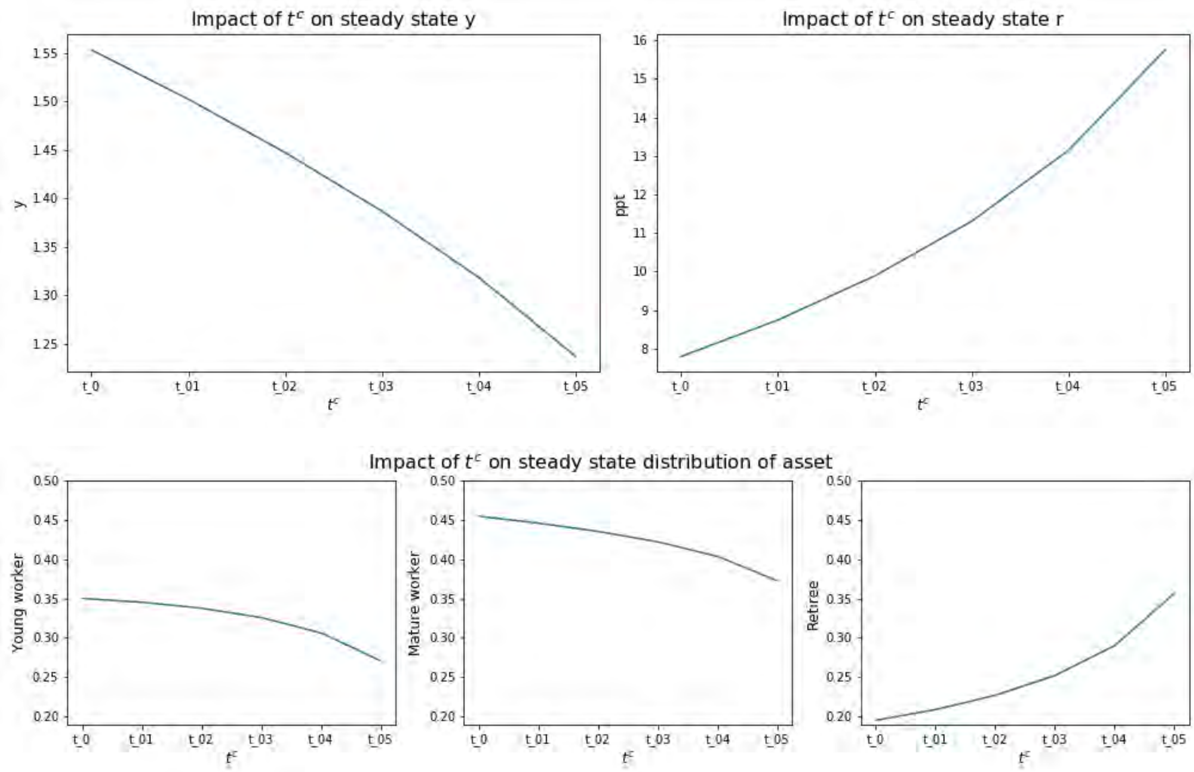

Figure 3: Steady state impact of consumption cost of children

Figure 4 shows the steady-state impact of the time cost of children. Unlike the consumption cost of children, the paper assumes that only young workers (20-39 age) spend time out of work to care for children. Like the consumption cost of children, the time cost of children reduces output per effective labour. It increases the interest rate reflecting lower capital per effective labour at the steady state. In contrast to the consumption cost of children, the time cost of children only reduces the share of assets held by young workers as they are the ones who bear the time cost of children. Specifically, the share of assets held by retirees and mature workers can increase by more than 6 and 5 percentage points each. In comparison, young workers' share can decline by more than 12 percentage points when the time cost of children increases from 0 to 30 % of young workers' available labour supply.

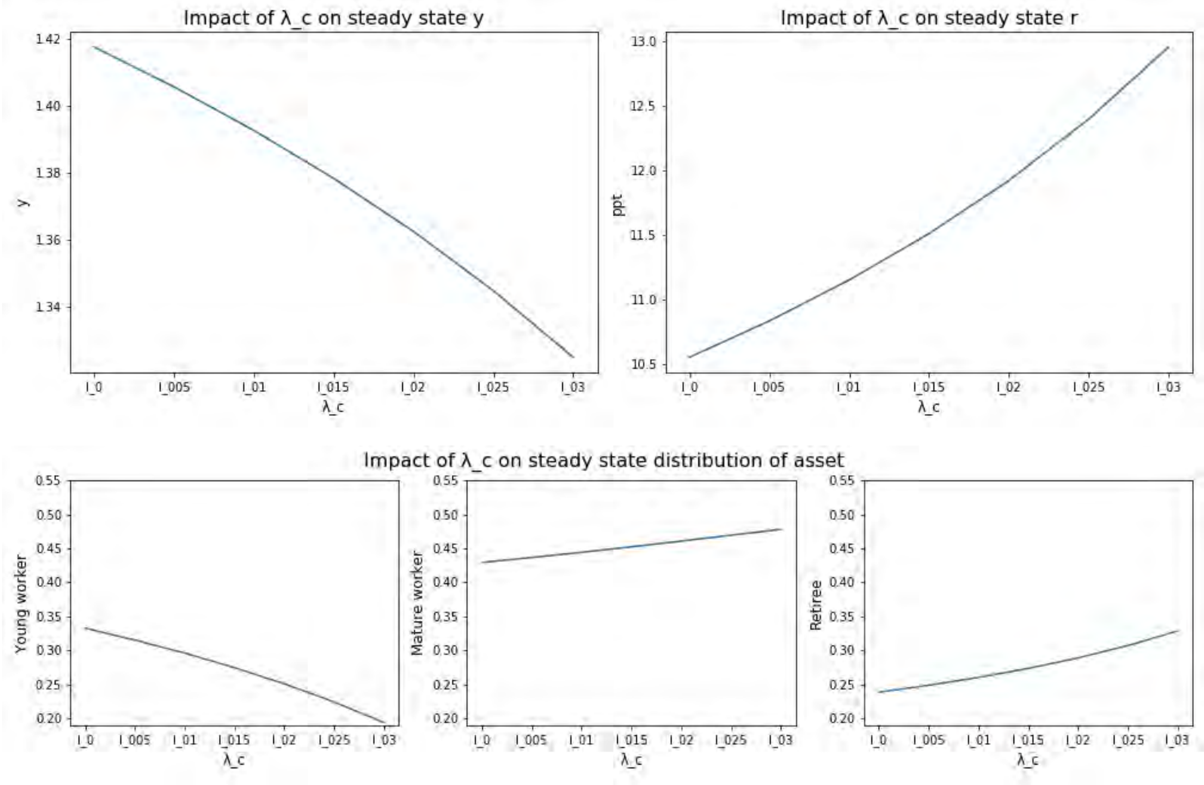

Figure 4: Steady state impact of time cost of children

Two other critical parameters determining the interest rate at the steady-state are subjective time preference rate and the elasticity of intertemporal substitution. Figure 5 reports the steady-state impact of sigma (elasticity of intertemporal substitution). Higher sigma implies a greater motive for consumption smoothing, and it is associated with higher savings (lower MPCs) and capital per effective labour. In turn, this translates to higher output per effective labour and a lower interest rate. Higher sigma is also associated with a higher share of assets held by young workers who have the longest horizon and greater consumption smoothing motive. However, compared to children's time and consumption cost, the effect of sigma on the distribution of assets is modest. It increases the share of assets held by young workers while reducing the share of assets held by mature workers by around two percentage points each when sigma rises from 0.3 to 0.6. In contrast, the share of assets held by retirees decline when sigma increases, but the magnitude of impact is almost negligible.

The effect of subjective time preference rate (beta) on output per effective labour and interest rate is comparable to sigma. Figure 7 reports the steady-state impact of different values for parameter beta. Higher beta, implying lower discounting of the future, is associated with higher savings (lower MPCs) and higher capital per effective labour.

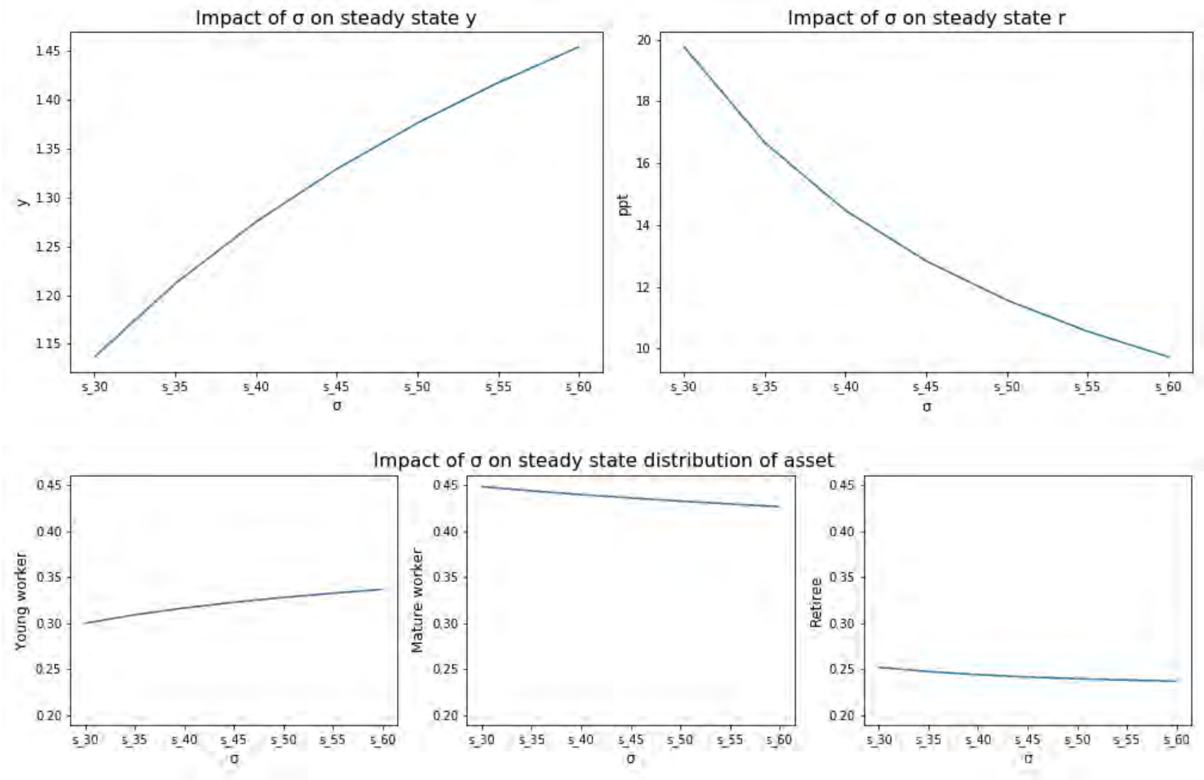

Figure 5: Steady state impact of elasticity of intertemporal substitution

It translates to higher output per effective labour and a lower interest rate. Like lower sigma, lower beta is also associated with a lower share of assets held by young workers and a higher share of assets held by retirees and mature workers. The magnitude of the effect is also comparable to the impact of sigma. It ranges from less than 1 to 3 percentage points.

To summarize, the effect of child consumption and the time cost of children on aggregate savings and interest rate is similar to the impact of taxes and transfers within the life-cycle model. In a life-cycle model, discount rates used by cohorts differ from the interest rate. Specifically, all cohorts discount the future more because of finite life. In addition, workers discount their future income even more to take into account retirement. These additional discounts mean that the present value of the reduction in child consumption stream is more than the present value of the increase in human wealth due to reduced child consumption or time cost.

Second, the time and consumption cost of children and the subjective time preference rate and the elasticity of intertemporal substitution change the consumption of different age groups and affect the distribution of assets. By choosing lower values for beta and sigma than conventional, the models such as Coeurdacier et al. (2015) may end up selecting a micro-situation where younger workers, who have a longer planning horizon, consume more, thereby unjustly increasing the importance of their credit constraint mechanism. The steady-state analysis here shows that incorporating the cost of children may not only eliminate the need for setting beta and sigma to unconventionally low values, but it may increase the importance of the credit constraint channel.

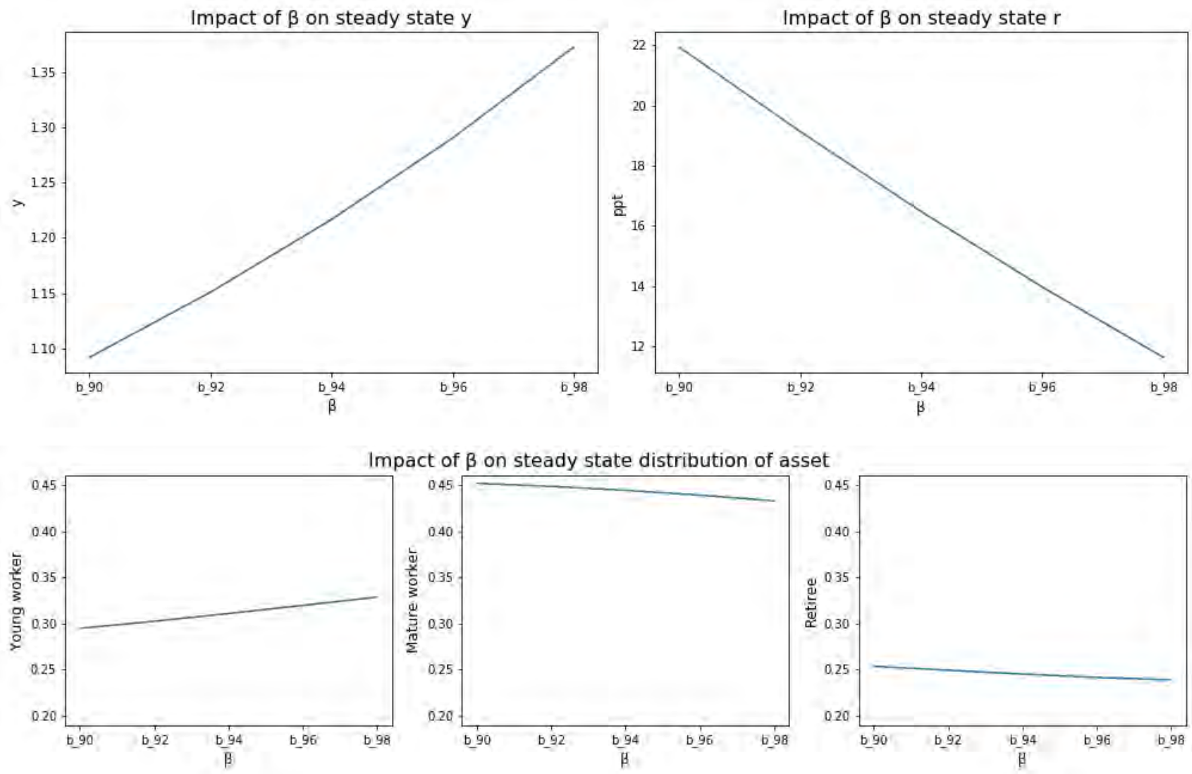

Figure 6: Steady state impact of subjective time preference rate

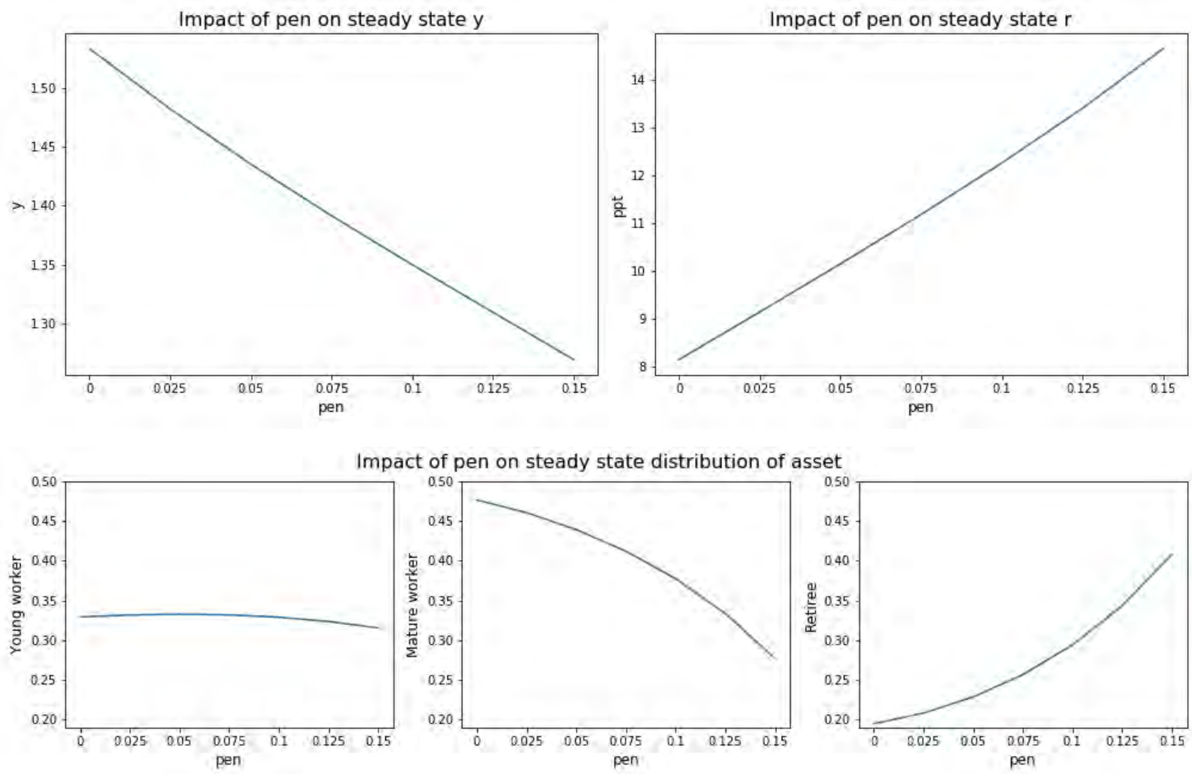

Figure 7: Steady state impact of PAYG pension

6 Simulation results

To understand the effect of consumption and time costs of children on the sign and the magnitude of the impact of a demographic shock on external balances, an asymmetric fertility (section 6.1) and mortality (section 6.2) shock is introduced into the model described in section 2 in sequence. Then, to contrast the fertility shock with a productivity shock, an asymmetric productivity shock is introduced into the model in section 6.3 to contrast its effect with that from a fertility shock on external imbalances.

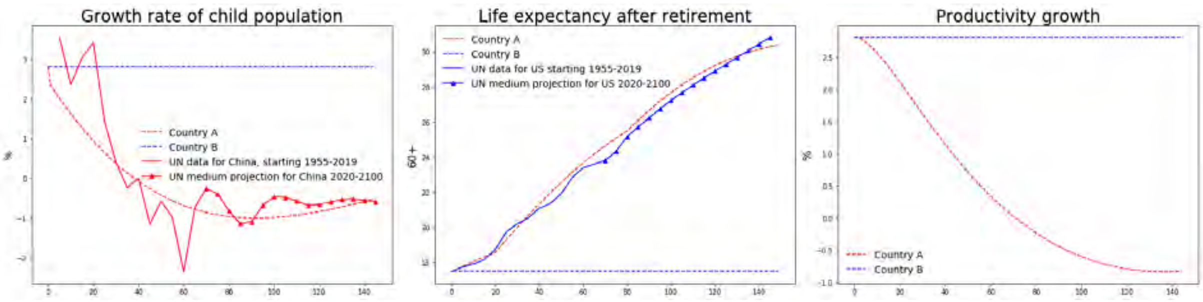

Table 6: Model shocks and data

The two countries are identical, and the trade balance is zero initially. The paper compares the results from a model with and without child dependency to isolate the children's impact on aggregate savings and external imbalances. Real-world examples are taken as guidance when choosing the magnitude of the demographic shocks. Figure 6 compares the fertility and mortality shocks introduced into the model with actual data for China and the US. In the fertility scenario, country A experiences a relative fertility decline, as shown in the first column of figure 6. In the longevity transition, country A experiences a relative increase in life expectancy at old age, as shown in the second column of figure 6. In the third scenario, country A experiences a relative decline in its economy-wide productivity growth as shown in the last column of figure 6. The sections 6.1, 6.2 and 6.3 analyze the impact of these three transition scenarios on external imbalances of both countries in sequence.

6.1 Fertility decline

Figure 8 shows the population dynamics under the fertility transition. Fertility changes affect the population's age structure, starting first by changing the child dependency ratio and the age structure of the workforce before changing the old-age dependency ratio. Specifically, a decline in fertility reduces the child dependency ratio, increases the mature to young worker ratio, and increases the old-age dependency ratio.

As pointed out earlier, the model's steady-state with and without child dependency is different under the same parameter values. The paper calibrated the model without children in three different ways. Model NC2 has the same elasticity of intertemporal substitution (σ) and subjective time preference rate (β) as the model Base, which considers child dependency. Consequently, model NC2 has a different steady-state interest rate. Models NC and NC3 have different β (NC) and σ (NC3) respectively to target the

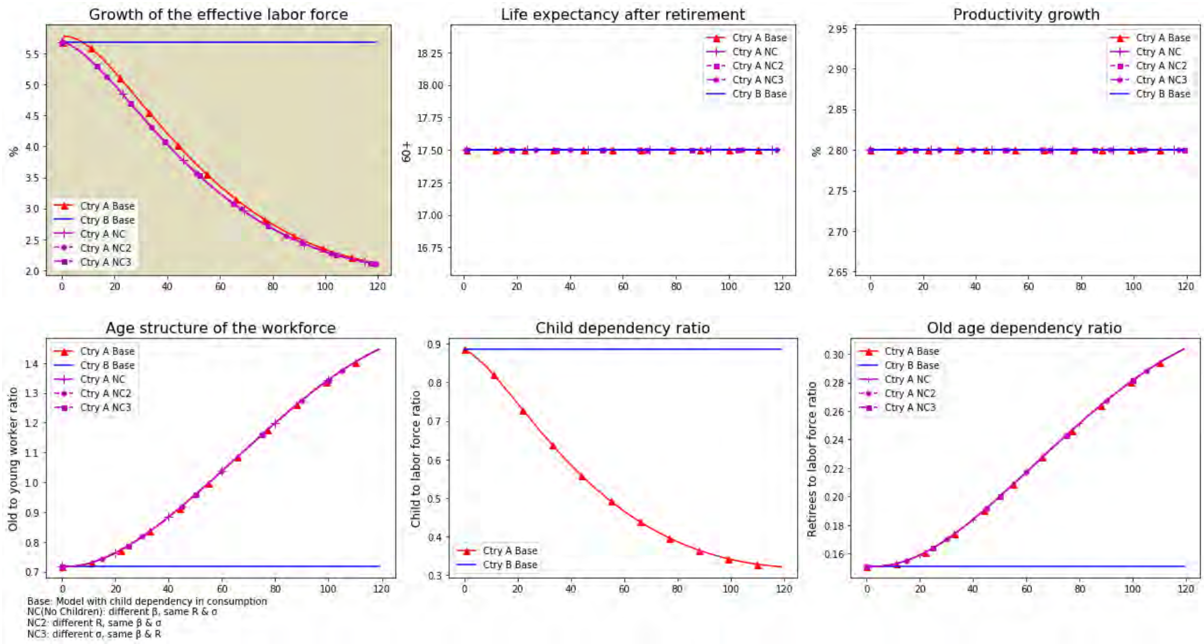

Figure 8: Population dynamics after the fertility shock

same interest rate at the steady-state as in the model Base. Specifically, model NC has a lower β than the rest, while model NC3 has a lower sigma. In other words, the presence of the consumption cost of children reduces the steady-state savings and increases the steady-state interest rate. Therefore, the model without children needs to be calibrated with lower beta (higher discounting of the future) and lower sigma (lower elasticity of intertemporal substitution) to generate the same interest rate as the model Base.

The declining fertility rate reduces the growth of the effective labour force for a given path of economy-wide productivity growth. However, as shown in the first plot of figure 8, the growth of the effective labour force diverges between models with and without the time cost of children. Specifically, the effective labour force can grow under a fertility decline in the medium run due to declining child dependency in models with the time cost of children. Moreover, during a fertility transition, the growth of the effective labour force falls less in the model with children's time cost than the model without one.

Figure 9 compares the effect of fertility decline across models with and without child dependency on aggregate savings, investment, and trade balance in the medium-run (top panel) and the long-run (bottom panel). Comparing results from different models show that children's time and consumption costs may change the sign of the impact of the fertility decline on aggregate savings, investment, and trade balance in the medium run. In contrast, it changes the magnitude of the long-run impact, particularly for the country under the fertility shock.

One can explain the short to medium-run dynamics by looking at the change in the marginal propensity to consume out of wealth for different age cohorts and the capitalized value of different types of wealth, reflecting changing factor prices and changing transfers between different generations. Under a fertility decline, the interest rate declines, reflecting lower growth in the effective labour force while the wage rate increases. The decrease in the interest rate is the largest in the model that considers children because the declin-

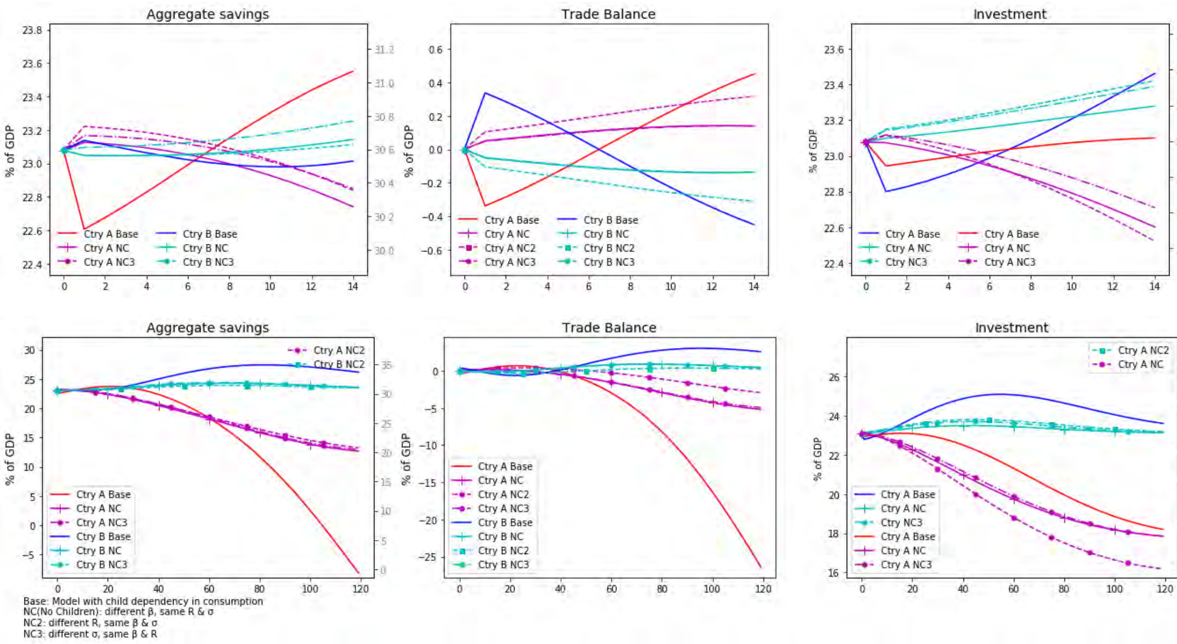

Figure 9: Aggregate savings, investment and trade balance after the fertility shock

ing child dependency also affects aggregate savings under a fertility shock. Consequently, when σ is less than 1, a decline in the interest rate reduces the marginal propensity to consume (MPC) out of wealth for all cohorts. Across models, the magnitude of the fall in the MPCs is the largest for the model with children, while among generations, the fall is the largest for the young workers who have the most extended planning horizon.

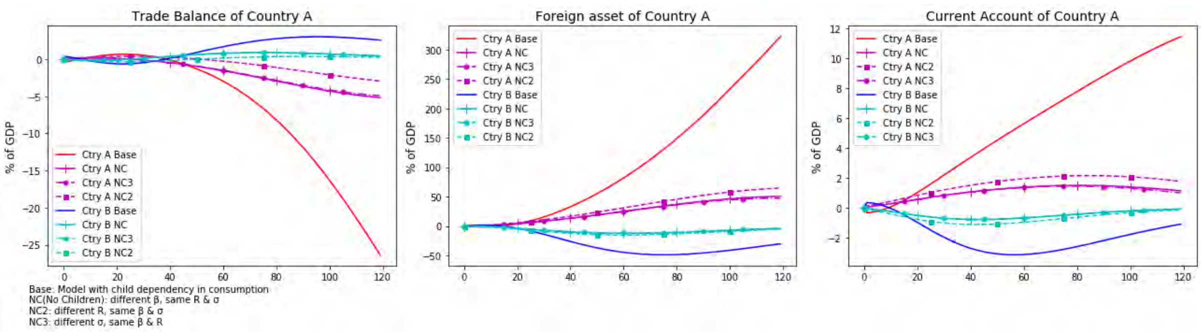

Figure 10: External balances after the fertility shock

One can explain the different short to medium-run dynamics across models with and without child dependency by looking at the differences in the dynamic response of the different types of wealth to the fertility shock. The bottom part of figure 11 shows the human wealth for young and mature workers and the social security wealth of retirees as a percent of GDP. The decline in the interest rate increases the capitalized value of the forward-looking wealth by reducing the discount rate. In addition to the effect of the interest rate on human wealth, child dependency will further increase the human wealth (which shows human wealth after all taxes and transfers to children) of workers in country A by reducing transfer to children. The impact of the fertility reduction on aggregate consumption can be different in the short-run from its long-run impact because it simultaneously changes the current and future dependency rates. When the demographic transition completes in the long run, a country with lower child dependency will have higher aggregate savings in the long run. However, at the beginning of the transition, when the workers expect the fertility rate to fall further, the increase in the human wealth of young and mature workers could raise consumption before the consumption by the children declines. The size and path of the fertility shock and the consumption cost of children simultaneously determine the magnitude of fertility decline's impact on aggregate consumption in short to medium run.

In contrast, the long-run dynamics are affected more by the aggregation effect driven by the changing relative size of different age cohorts. The bottom part of figure 11 shows that in the long run, the human wealth of young workers as a percent of GDP declines while the human wealth of mature workers increases, reflecting the increase in the size of mature workers relative to the young workers. Consequently, in the long run, population aging pushes consumption up in country A by increasing the share of retirees with higher consumption propensities. Thus, country A's savings decline in the long run while it increases in country B. Across all models, investment falls in country A, driven by the decline in its workforce's growth, while it increases in country B, reflecting an increase in savings and capital flow to country B.

The bottom half of figure 9 shows that trade imbalance is driven to a larger extent by savings change in the long run. Country A experiences a larger decline in its savings than its investment which underlies its trade deficit. Figure 10 shows that neglecting the consumption cost of children will result in significant differences in the magnitude of fertility shock on external imbalances in the long run, particularly for the country under fertility shock. The model with child dependency is associated with a higher increase in foreign assets and current account position, allowing higher trade deficits in the long run. Trade deficits are twice higher in the model with children than without children in the long run.

Figure 11 compares factor prices across different model setups under the same fertility shock. As expected, the change in factor prices is the largest in the model with child dependency because it introduces an additional mechanism where child dependency affects aggregate savings. Figure 11 also shows that the interest rate can increase in the model with children in the short run, reflecting an increase in the growth of the effective labour force in the short run. On the other hand, simulations show that the different calibrations also affect the dynamics of the factor prices under the fertility shock. Comparing the dynamics of the factor prices under the fertility shock in three different NC models is not straightforward. Specifically, because the steady-state interest rate is low in the NC2 model, the percentage change in the interest rate relative to its steady-state or initial value is higher in the NC2 model than in the NC and NC3 models. However, if we compare the change in factor prices in absolute terms, then the change in factor prices among the models without children is the highest under lower σ value as shown in figure 11. Overall, declining fertility results in decline in the interest rate and an increase in wage rate in the long run across all models for a given path of productivity growth. The exact magnitude of the effect depends on parameter values and the model specification.

Figure 12 and 13 shows the effect of fertility transition on the distribution of assets and marginal propensity to consume of different age cohorts. The first observation is that

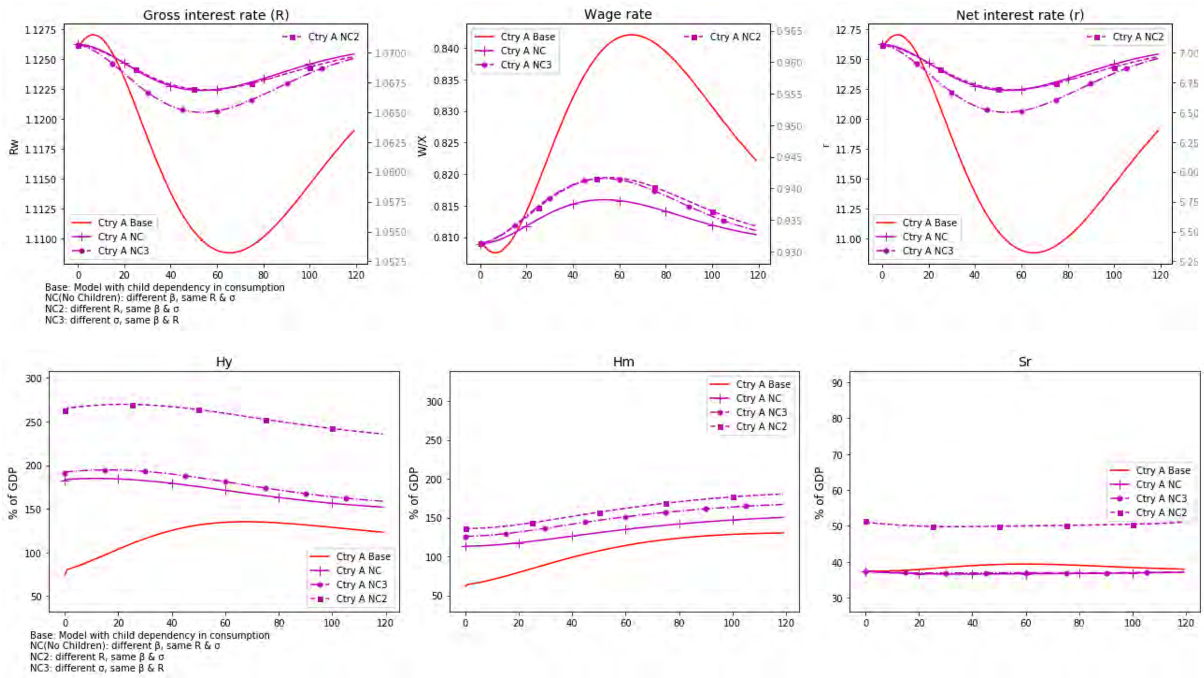

Figure 11: Factor prices and wealth dynamics after the fertility shock

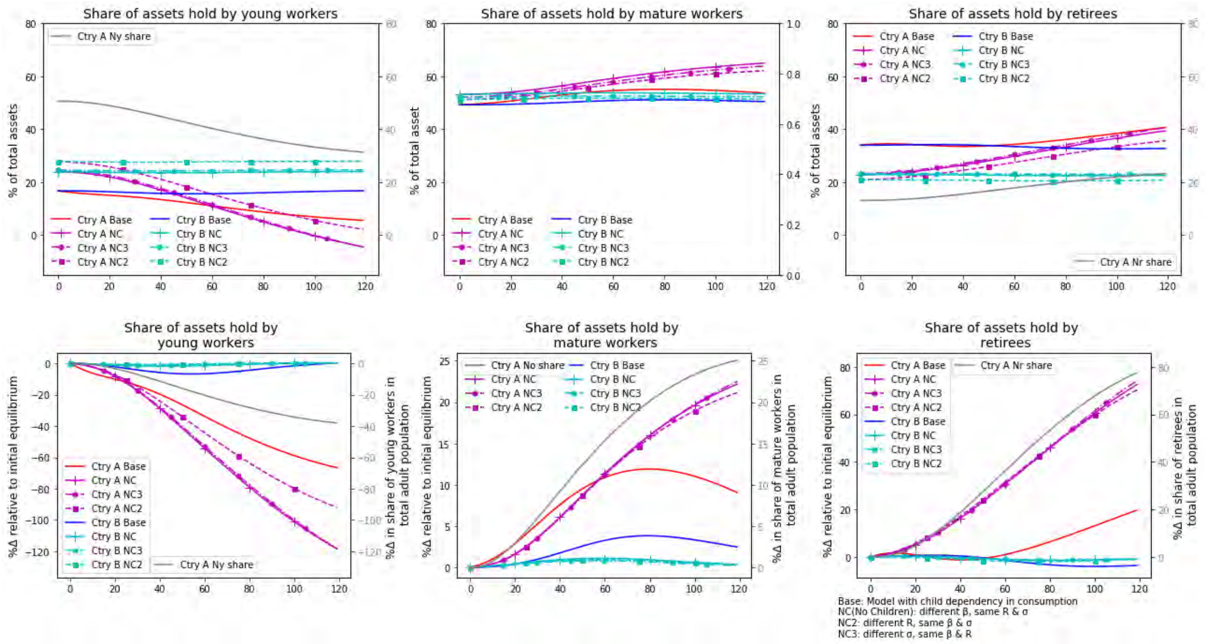

Figure 12: Distribution of assets after the fertility shock

the consumption cost of children changes the distribution of assets across age groups. Specifically, the share of assets held by retirees is larger in the model with children, while that of workers is lower than the model without children. In addition, marginal propensity to consume (opposite of savings rate) is different across all models. As shown in section C, MPCs have a negative relationship with β and σ, while they have a positive relationship with interest rate.

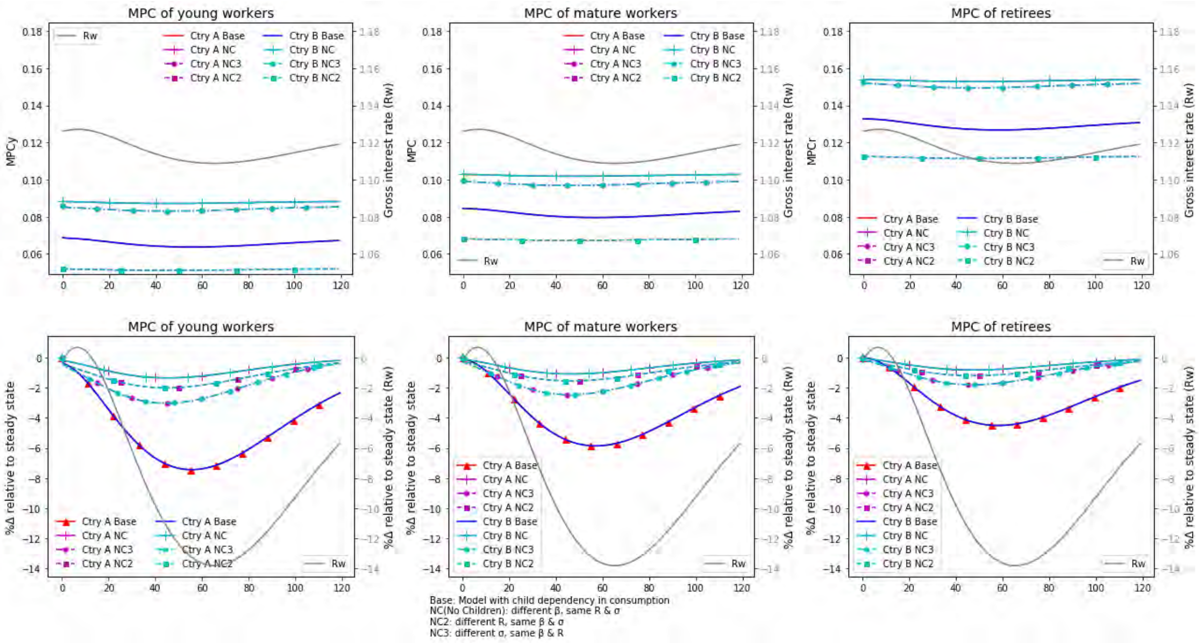

Figure 13: MPCs after the fertility shock

Figure 12 makes it clear that the change in the distribution of assets is driven mainly by the aggregation effect, the changing relative size of different age cohorts. Blue lines show that the share of assets held by different age groups is relatively constant in country B, whose population age structure is stable. Any changes are due to change in factor prices. The share of assets held by young workers in country A declines more than the change in their population share, reflecting greater consumption by these groups under the fertility transition. On the other hand, the share of assets held by retirees increases less than the increase in the percentage of retirees in the adult population, reflecting the decline in interest rate. In other words, retirees reduce their consumption less relative to the decrease in interest rate. Figure 13 confirms this by showing that despite falling in line with the reduction in the interest rate, retirees' MPCs react the least to changes in interest rate compared to workers reflecting their shorter planning horizon.

To summarize, the presence of time and the consumption cost of children changes the sign of the effect of fertility decline on aggregate savings and trade balance in the short run. In contrast, it changes the magnitude of the long-run impact. By adding additional mechanisms, the presence of children's time and consumption costs reduces the factor prices the most compared to models without children across different parameter specifications, which affects the magnitude of the impact of fertility shock on external imbalances across countries. This section also shows that different calibrations and model specifications correspond to a different economy at the micro-level, despite representing the same economy at the aggregate level. Furthermore, simulation results indicate that while the differences could be minor in the short run, it is quite large in the long run.

6.2 Longevity

Unlike fertility shocks, longevity shocks do not affect the child dependency ratio and workforce's age structure, as shown in figure 20. However, it changes the old-age dependency ratio. Because a longevity shock doesn't affect the growth of the effective labour force in the models in this paper, investment dynamics across the two countries follow the same path, as shown in figure 15. The absence of blue lines in figure 15 indicates that the blue and red lines overlap.

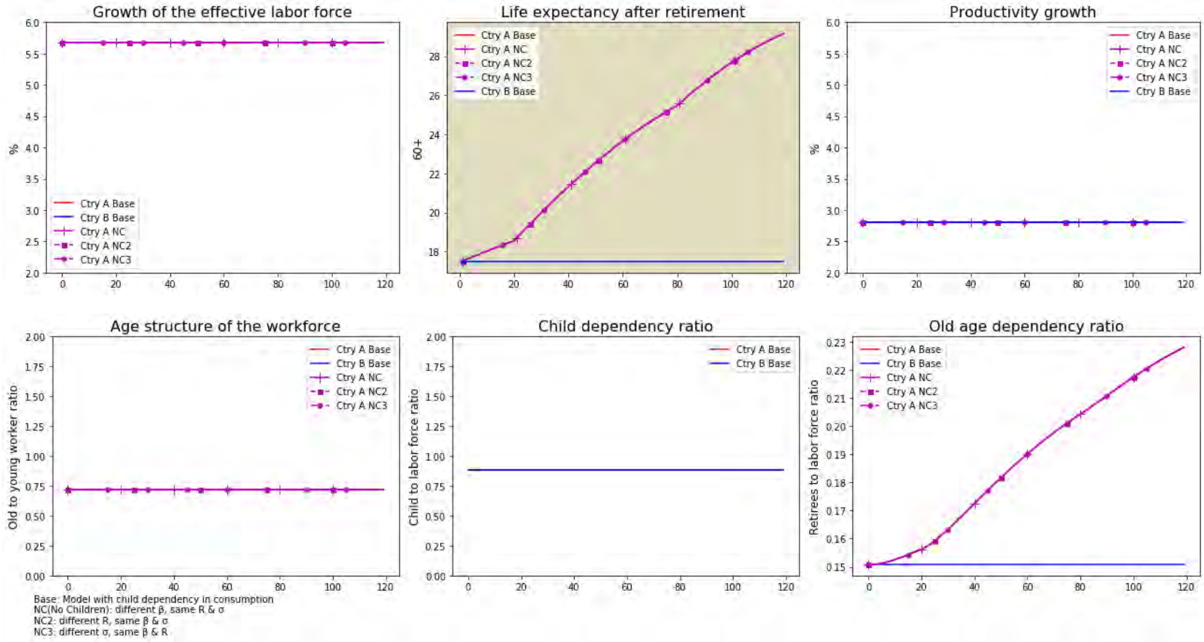

Figure 14: Population dynamics after the longevity shock

Figure 15 shows the effect of longevity shock on aggregate savings, investment, and trade balance of the two countries in the short and long run. Because investment dynamics are the same across countries, differences in savings alone drive trade imbalances between the countries. Aggregate savings increase in response to longevity shock in country A in short to medium run as the households in country A anticipate a more extended retirement period. Thus, country A runs a trade surplus in the short to medium run when it invests parts of its increased savings in foreign assets.

Across all models, except NC2, trade imbalances display similar patterns in response to longevity shock both in the short and long run, if not counting minor differences in magnitude. The model with child dependency predicts country A to have a larger trade surplus in the short run and a larger trade deficit in the long run. However, the observed differences are minimal compared to the effect of fertility shock. Interestingly, in model NC2, longevity shock increases the magnitude and lengthens the horizon over which country A accumulates foreign assets through a trade surplus. Because in the NC2 model, the steady-state world interest rate is low, country A needs to increase its savings more for far longer to accumulate a certain target level of wealth to finance its consumption for a more extended retirement period. Figure 16 shows this where despite country A having a higher trade surplus for far longer in model NC2, the foreign asset

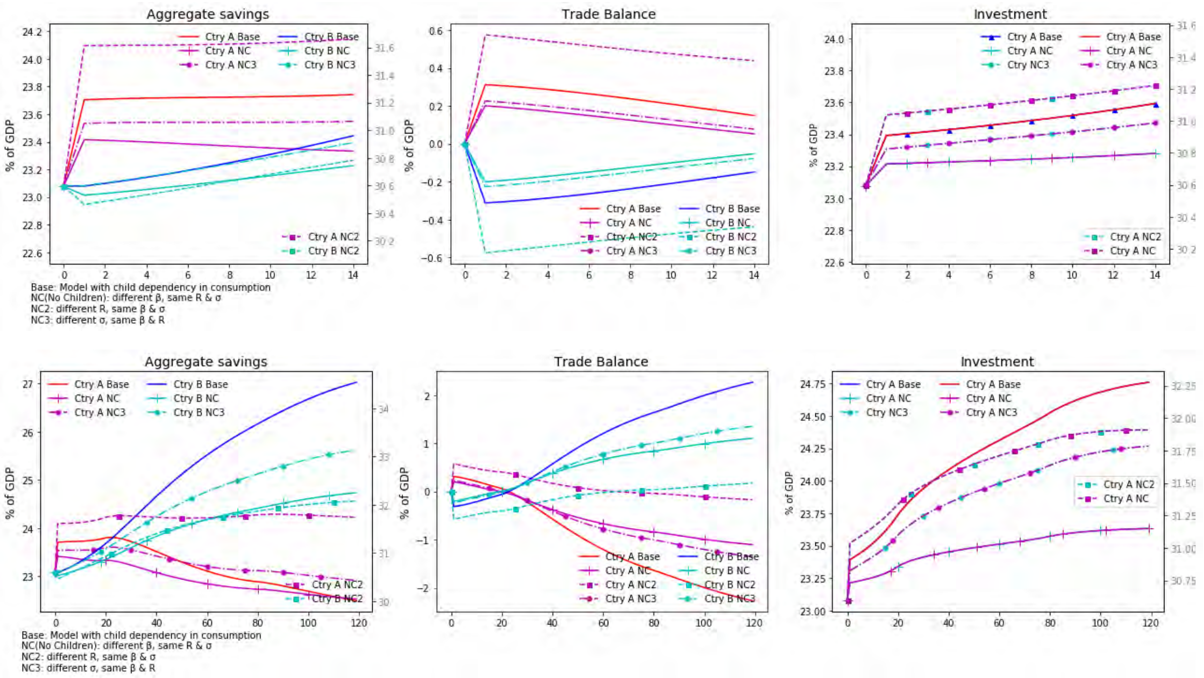

Figure 15: Aggregate savings, investment and trade balance after the longevity shock

and current accounts of country A are very similar to other models.

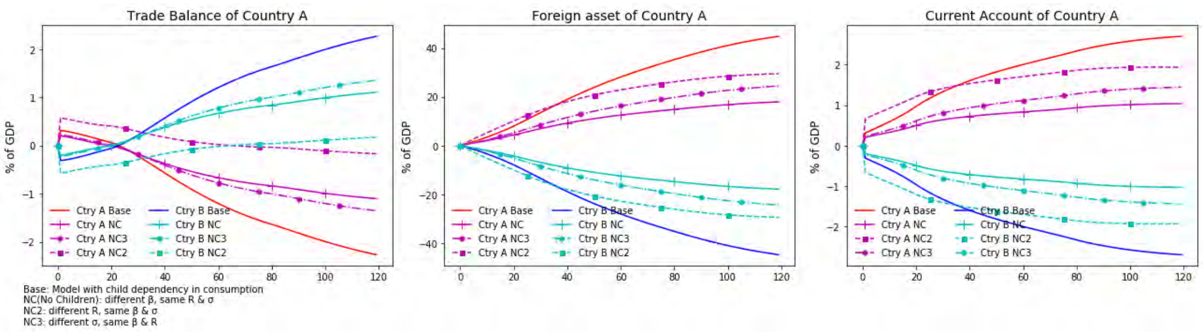

Figure 16: External balances after the longevity shock

In contrast to fertility shock, the effect of longevity shock on interest rate is modest and comparable across models with and without child dependency. Figure 17 shows that lower beta is associated with the smallest decline in interest rate under a longevity transition. It also indicates that longevity shock on factor prices is the largest in the model with children due to a higher share of assets being held by retirees in the model with children.

The dynamics of the share of assets held by different age groups display a similar pattern to the fertility shock. Longevity shock also changes the share of different age groups in the adult population, affecting the share of assets held by different age groups through the aggregation effect. Specifically, the share of assets held by retirees increases much more than the increase in the share of retirees in the adult population, as shown in figure 18. In contrast, the share of assets held by young workers declines much more

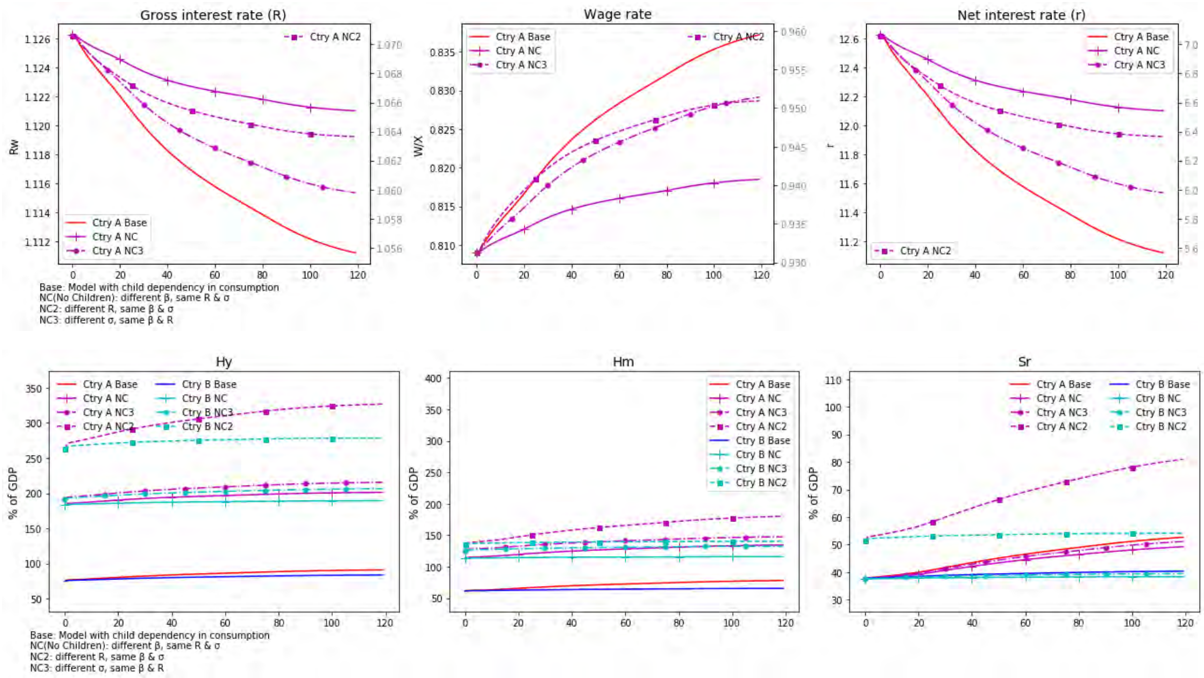

Figure 17: Factor prices after the longevity shock

than their share in the adult population. A more pronounced reduction in retirees' consumption reflects the fact that the retirees' MPCs decline the most in response to longevity shock and young workers' the least, as shown in Figure 19.

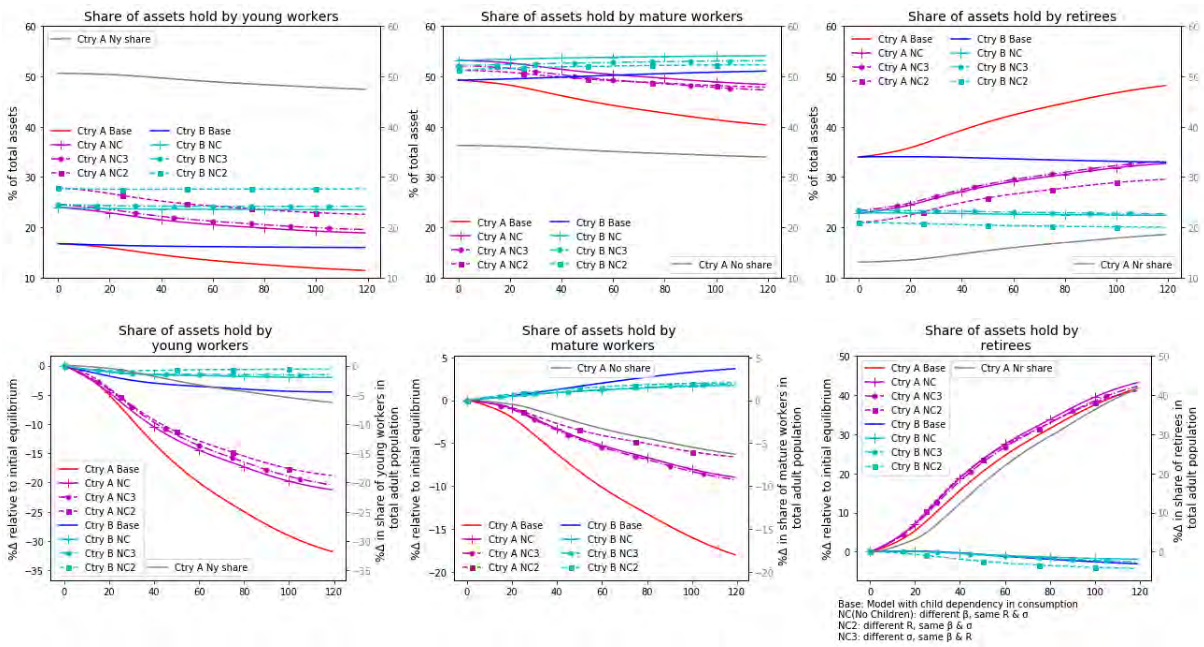

Figure 18: Distribution of assets after the longevity shock

Overall, the effect of longevity shock on aggregate savings, investment, and trade balance across models with and without child dependency with the same steady-state

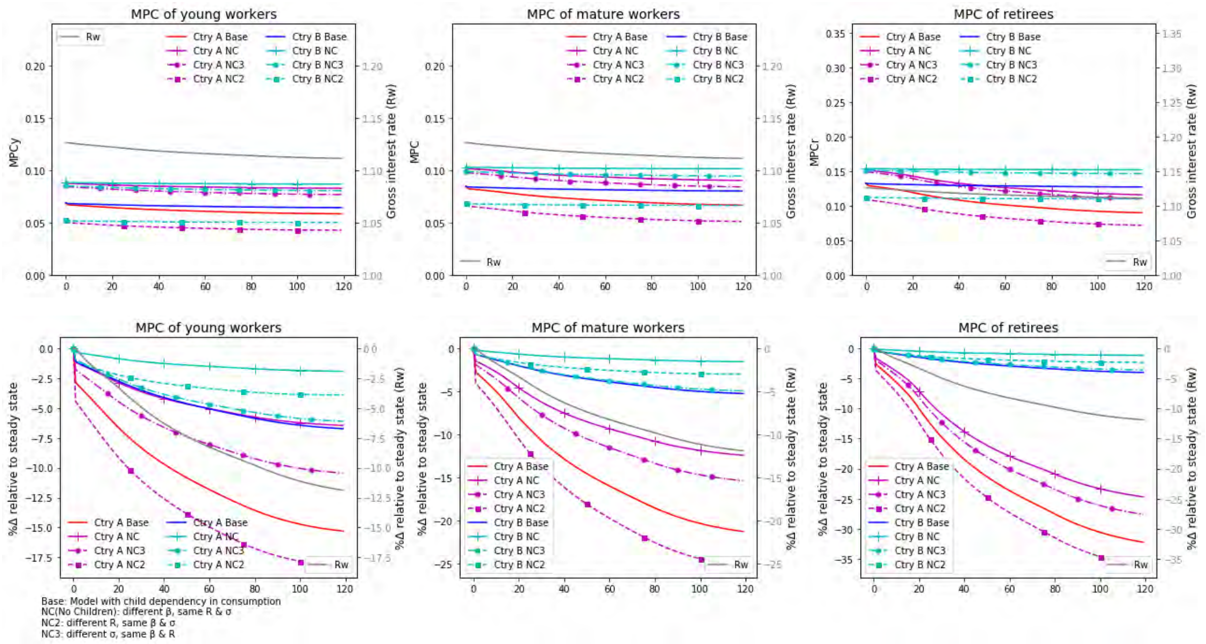

Figure 19: MPCs after the longevity shock

interest rate is quite comparable. The model with child dependency predicts a larger impact of the longevity shock on trade imbalance across countries for a given steady-state interest rate. Furthermore, savings differences alone determine the trade balances as longevity shock is assumed not to affect the growth of the effective labour force. In reality, longer life expectancy may extend the retirement age or increase labour supply by retirees. Then longevity shock will also affect the investment dynamics to the extent increased labour force by the retirees requires new capital investments. The model can simulate this scenario, but the current paper omits this to simplify the analysis. Second, in a low-interest-rate environment, as in NC2, a country under longevity shock has to incur a trade surplus for longer at a much higher rate than if the interest rate was higher. It implies that, as the interest rate declines globally, the demographic transition may lead to larger trade imbalances across countries.

6.3 Decline in productivity

This section analyzes the effect of productivity shock on external imbalances within the framework developed in this paper for two reasons. First, it is difficult to differentiate the impact of demographic dividends from economy-wide productivity growth in some models. Economy-wide productivity decline will have the same effect on the growth of the effective labour force as the same magnitude decline in the fertility rate. Therefore, its impact on investment differentials will be the same as the decline in fertility for a given interest rate path. However, unlike fertility declines, productivity changes do not affect the population's age structure. Therefore it will have a different impact on savings. Because both the savings and investment differentials determine trade imbalances across countries, productivity shock will have a different effect on trade imbalance than a fertility shock. Second, globally, productivity growth is declining, and it is suspected to be one of the main reasons behind the declining interest rate. Productivity differentials are also one of the main reasons behind capital flows between countries. Therefore, it is interesting to see the impact of productivity changes on external imbalances within this framework to compare its effect to demographic changes.

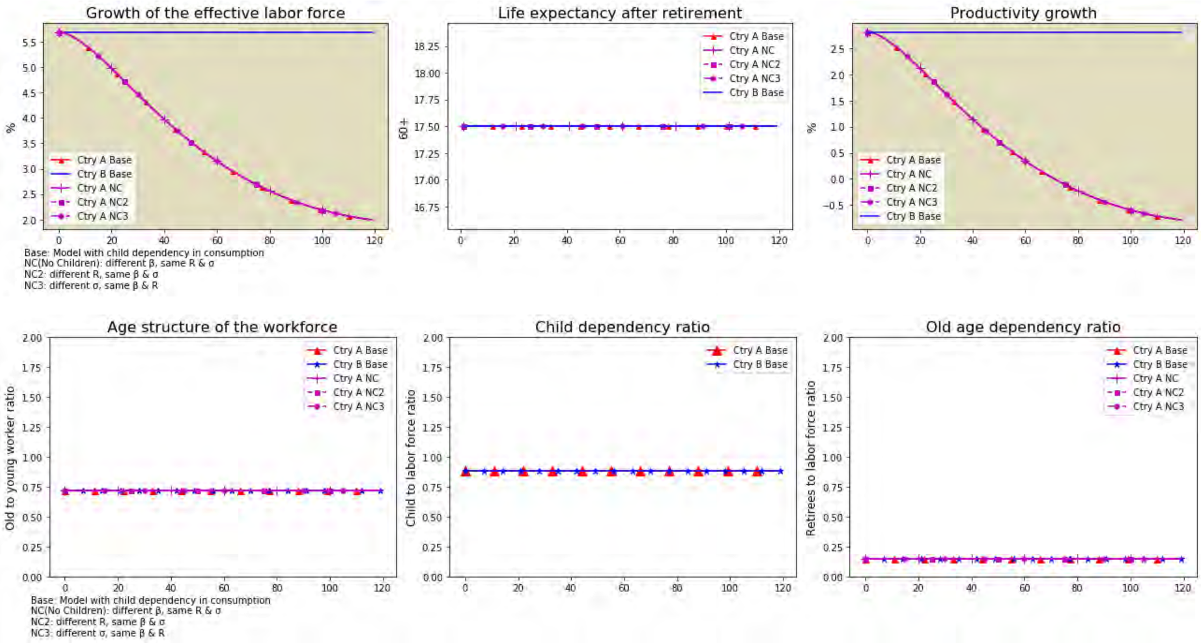

Figure 20: Population dynamics after the productivity shock

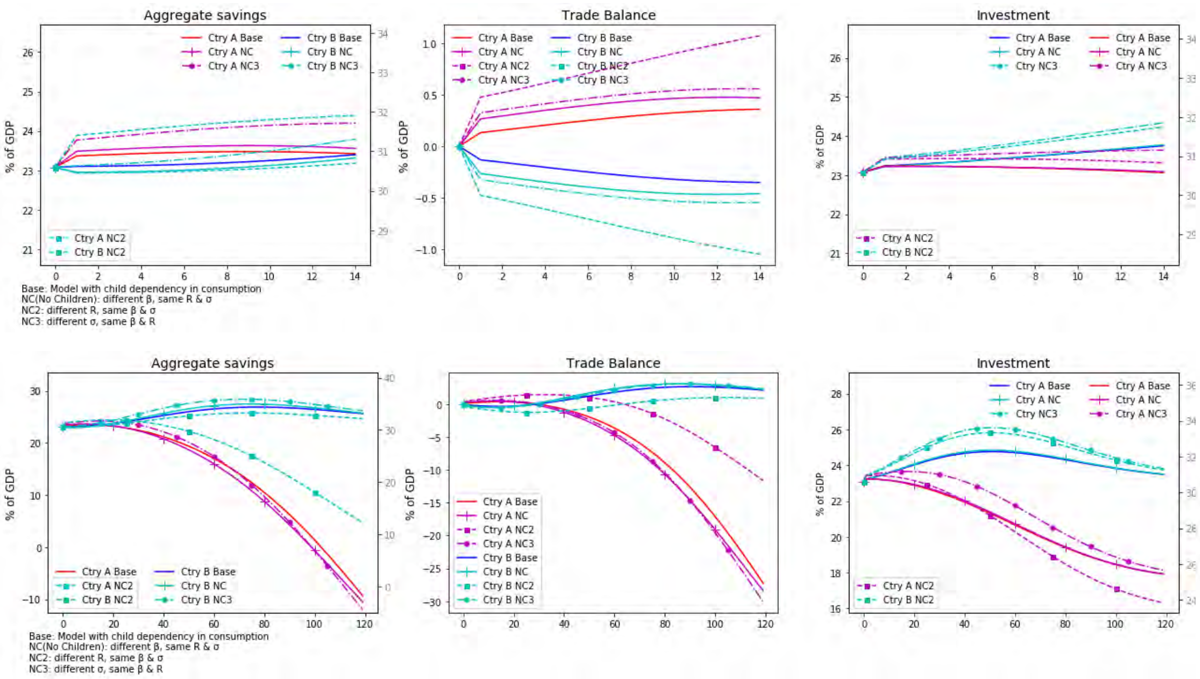

Figure 21: Aggregate savings, investment and trade balance after the productivity shock

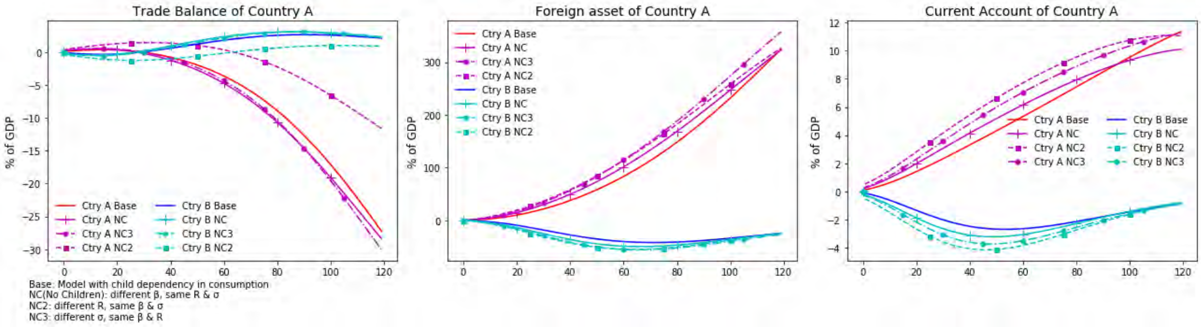

Figure 22: External balances after the productivity shock

Different from fertility and longevity shocks,productivity shocks do not alter the population age structure. However, productivity shocks do affect the growth of the effective labour force (growth of workforce times growth of productivity), which affects the investment rate. Factor prices determine the change in aggregate savings under the productivity shock. Because there is no change in the population's age structure, there is no effect from aggregation. Lower productivity growth in country A and associated lower interest rates increase savings in both countries (see figure 22) by reducing MPCs of all age groups, which has a positive association with an interest rate (when σ is less than 1). Investment in country B increases more than the increase in its savings,resulting in country B to have trade deficits in the short to medium run. Again, the world interest rate affect the magnitude and the duration of the trade imbalance between countries. Figure 22 shows that in model NC2, country A has to run a trade surplus and country B has to run a trade deficit much longer compared to other models because of the low-interest rate.

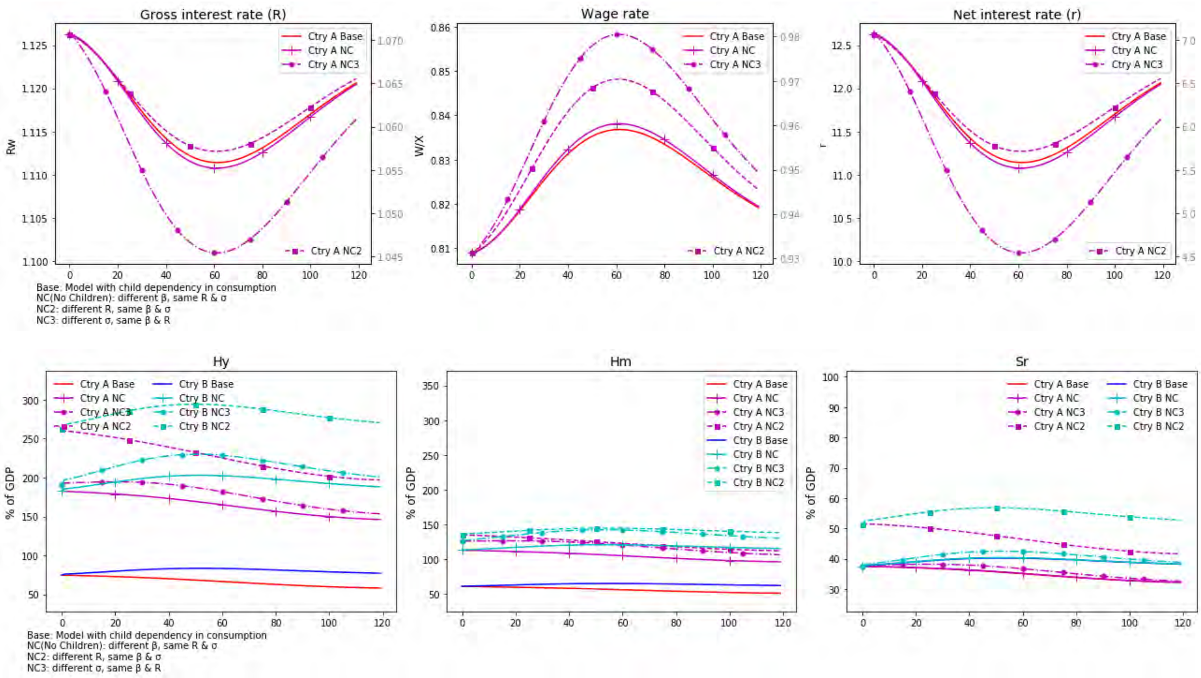

Figure 23: Factor prices after the productivity shock

At the micro-level, contrasting to fertility shock, productivity decline does not alter asset distribution much (see figure 24). Any minor changes are associated with changes in factor prices, with retirees' share of assets increasing slightly in country A. An opposite pattern can be observed in country B, where higher investment increases wage rate and thus increase the share of assets held by workers. However, any impact on the distribution of assets of the productivity shock is quite negligible. The same can be said of the effect of productivity decline on the MPCs (see figure 25), where declining interest rate reduces MPCs but by very small.

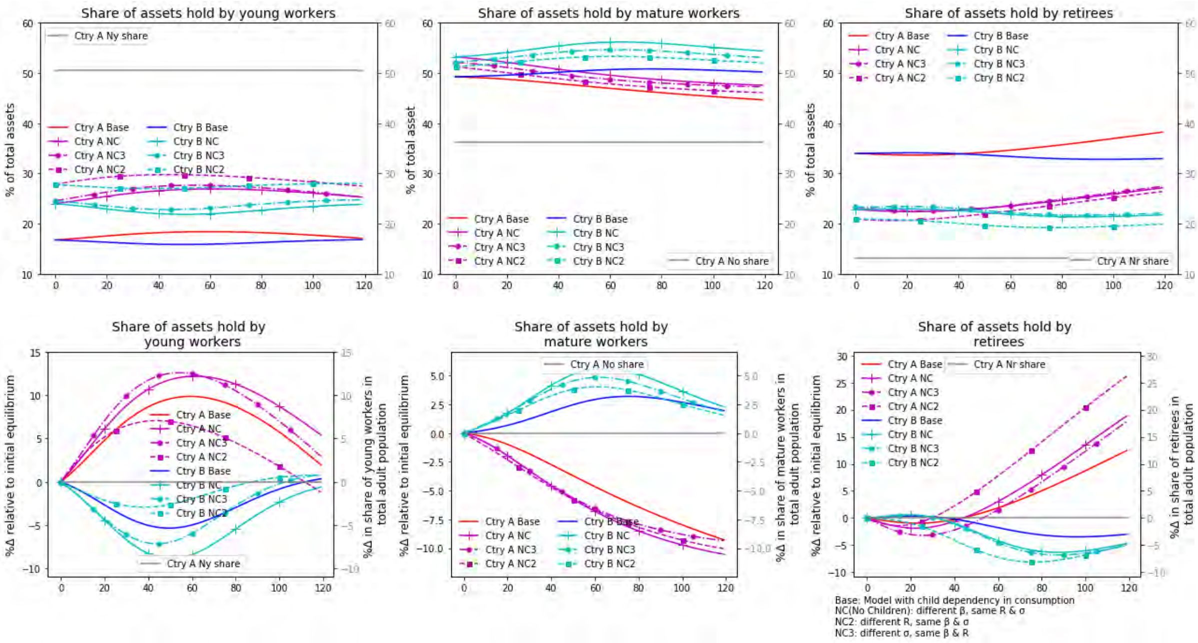

Figure 24: Distribution of assets after the productivity shock

Overall, smaller R, sigma, and beta increase trade imbalances across countries, as shown in figure 22. However, the observed dynamics of the trade imbalance across models with and without children with the same steady-state interest rate under a productivity shock are almost negligible. In contrast, when the interest rate is low, the trade surplus of country A and the trade deficit of country B has to run larger for longer. In other words, in a low-interest-rate environment, it takes time for a country to build its foreign asset in anticipation of an aging population or lower productivity.

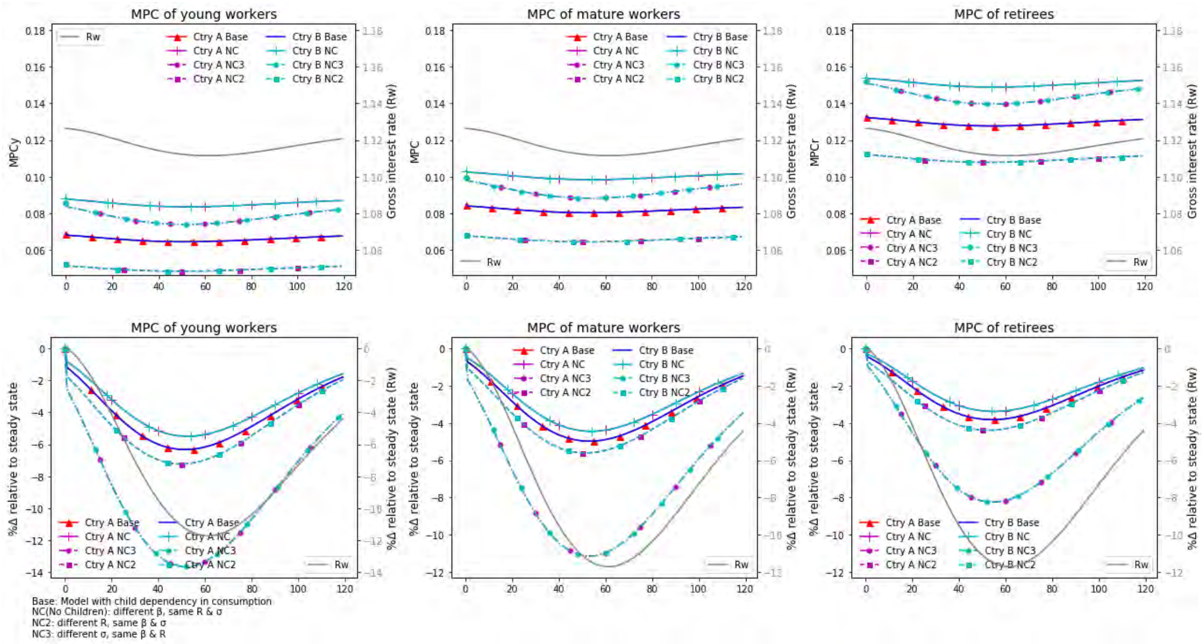

Figure 25: MPCs after the productivity shock

7 Conclusion

Understanding the causes behind the growing and persistent external imbalances across major economies has become a primary endeavor of policymakers and economists. Despite being the most prominent manifestation of the current global demographic transition, most papers neglect the effect of changing child dependency rates on external imbalances. Given the policy importance of correctly interpreting the causes behind global imbalances, this paper investigates the consequence of ignoring children when examining the impact of the demographic transition on external imbalances. To this end, it adds children's consumption and time costs to a tractable life-cycle model of two symmetric countries. To contrast, it then compares the demographic transition's impact on external imbalance across models with and without children.

Two main insights are drawn from this exercise. First, the presence of time and the consumption costs of children amplifies the effect of fertility transitions on factor prices and the external imbalances by adding additional mechanisms where changes in child dependency affect the aggregate savings. The results show that child dependency changes the sign and magnitude of a fertility transition on aggregate savings and external imbalances in the short and medium run. Though the difference is minor in the short to medium run, it is quite large in the long run.

Second, because the model with children corresponds to lower savings and higher interest rates at the steady-state, models with and without children need to be calibrated with different parameter values determining the steady-state interest rate. This implies that studies that use a calibrated life-cycle model without children may indirectly choose lower values for subjective time preference rate (β) and lower values for elasticity of intertemporal substitution (σ). The paper further shows that the different parameter values correspond to a different economy at the micro-level. Specifically, lower β and

σ reduce the share of assets held by young workers who have the longest horizon while increasing retirees' share. Differences in composition and parameter values change the model's dynamics even under longevity and productivity shocks that do not change child dependency.

Overall, the simulations in this paper demonstrate that a country with a declining fertility rate, increasing longevity, and declining productivity, all relative to its trading partner, will have a trade surplus in the medium run and a trade deficit in the very long run. However, the prevailing world interest rate determines the duration over which a country runs a trade surplus before running a trade deficit during its demographic transition. Specifically, when the world interest rate is low, the trade surplus of a country with a declining fertility rate, increasing life expectancy, and declining productivity may continue for far longer than if the world interest rate was higher. Within a life-cycle framework, it is even possible for a country with positive foreign assets to run a trade surplus in the very long run when the interest rates fall below the growth rate of the effective labour force.

To sum up, this paper highlights some of the issues in quantifying the effect of the demographic transition on the external imbalance in the past using calibrated life-cycle models that do not take child dependency into consideration. The absence of critical features underlying the demographic transition (e.g., child dependency) raises concerns that the effect of demographic transition hasn't been captured fully in previous studies. It is therefore important to be cautious in interpreting results from such studies. Further work is needed to develop a unified framework encompassing all the crucial features concerning the demographic transition to quantify the contribution of demographic transition to external imbalances that can better inform policymakers.

References

Barro, R. J., & Becker, G. S. (1989). Fertility choice in a model of economic growth. *Econometrica: journal of the Econometric Society*, 481–501.

Blanchard, O., & Milesi-Ferretti, G. M. (2012). (why) should current account balances be reduced? *IMF Economic Review*, *60*(1), 139–150.

Bloom, D. E., Canning, D., Fink, G., & Finlay, J. E. (2009a). Fertility, female labor force participation, and the demographic dividend. *Journal of Economic growth*, *14*(2), 79–101.

Bloom, D. E., Canning, D., Fink, G., & Finlay, J. E. (2009b). Fertility, female labor force participation, and the demographic dividend. *Journal of Economic growth*, *14*(2), 79–101.

Bommier, A., & Lee, R. D. (2003). Overlapping generations models with realistic demography. *Journal of Population Economics*, *16*(1), 135–160.

Brooks, R. (2003). Population aging and global capital flows in a parallel universe. *IMF staff papers*, *50*(2), 200–221.

Bryant, R. C., Arbatli, E., Faruqee, H., McKibbin, W. J., & Velculescu, D. (2004). *Fertility declines and youth dependency: implications for the global economy.* Brookings Institution.

Caballero, R. J., Farhi, E., & Gourinchas, P.-O. (2008). An equilibrium model of" global imbalances" and low interest rates. *American economic review*, *98*(1), 358–93.

Chinn, M. D., & Prasad, E. S. (2003). Medium-term determinants of current accounts in industrial and developing countries: an empirical exploration. *Journal of international economics*, *59*(1), 47–76.

Coeurdacier, N., Guibaud, S., & Jin, K. (2015). Credit constraints and growth in a global economy. *American Economic Review*, *105*(9), 2838–81.

Curtis, C. C., Lugauer, S., & Mark, N. C. (2015). Demographic patterns and household saving in china. *American Economic Journal: Macroeconomics*, *7*(2), 58–94.

Curtis, C. C., Lugauer, S., & Mark, N. C. (2017). Demographics and aggregate household saving in japan, china, and india. *Journal of Macroeconomics*, *51*, 175–191.

Donni, O. (2015). Measuring the cost of children. *IZA World of Labor*.

Eugeni, S. (2015). An olg model of global imbalances. *Journal of International Economics*, *95*(1), 83–97.

Farmer, R. E. (1990). Rince preferences. *The Quarterly Journal of Economics*, *105*(1), 43–60.

Ferrero, A. (2010). A structural decomposition of the us trade balance: Productivity, demographics and fiscal policy. *Journal of Monetary Economics*, *57*(4), 478–490.

Gertler, M. (1999). Government debt and social security in a life-cycle economy. In *Carnegie-rochester conference series on public policy* (Vol. 50, pp. 61–110).

Gourinchas, P.-O., & Rey, H. (2014). External adjustment, global imbalances, valuation effects. In *Handbook of international economics* (Vol. 4, pp. 585–645). Elsevier.

Higgins, M., & Williamson, J. G. (1997). Age structure dynamics in asia and dependence on foreign capital. *Population and Development Review*, 261–293.

Julliard, M. (2005). *Dynare: A program for the resolution and simulation of dynamic models with forward looking variables through the use of relaxation algorithm. cepremap, 2005.*

Lau, S.-H. P. (2014). Fertility and mortality changes in an overlapping-generations model with realistic demography. *Economic Modelling*, *38*, 512–521.

Leff, N. H. (1969). Dependency rates and savings rates. *The American Economic Review*, *59*(5), 886–896.

Letablier, M.-T., Luci, A., Math, A., & Thévenon, O. (2009). The costs of raising children and the effectiveness of policies to support parenthood in european countries: a literature review.

Lugauer, S., Ni, J., & Yin, Z. (2019). Chinese household saving and dependent children: Theory and evidence. *China Economic Review*, *57*, 101091.

Lührmann, M. (2003). Demographic change, foresight and international capital flows. *MEA Discussion Papers*, *38*.

Lundborg, P., Plug, E., & Rasmussen, A. W. (2017). Can women have children and a career? iv evidence from ivf treatments. *American Economic Review*, *107*(6), 1611–37.

Masson, P. R., Bayoumi, T., & Samiei, H. (1998). International evidence on the determinants of private saving. *The World Bank Economic Review*, *12*(3), 483–501.

Niemeläinen, J. (2017). External imbalances between china and the us: a dynamic analysis with a life-cycle model. *HECER discussion papers*(417).

Niemeläinen, J. (2021). External imbalances between china and the united states: A dynamic analysis with a life-cycle model. *Journal of International Economics*, 103493.

Niemeläinen, J., et al. (2020). Determinants of external imbalances between large economies.

Park, D., & Shin, K. (2009). Saving, investment, and current account surplus in developing asia.

UN. (2019). World population prospects 2019.

Weil, P. (1989). Overlapping families of infinitely-lived agents. *Journal of public economics*, *38*(2), 183–198.

Appendices

A The steady state equations

Equations from 85 to 89 provides steady-state equations for age structure of the population. In the steady-state, population age structure is constant with different age groups growing at the same rate n.

Young worker to children ratio, equation 85, depends on population growth rate and the two transition probabilities that matter for children and young workers.

$$\psi^c = \frac{z}{n + \theta - 1} \tag{85}$$

Similarly, the mature to young worker ratio, equation 86, depends on the population growth rate and two transition probabilities that matter for young workers and mature workers.

$$\psi^1 = \frac{\theta}{n + \omega - 1} \tag{86}$$

The old-age dependency ratio, equation 87, depends not only on the population growth rate and all the transition probabilities from one life stage to another but also on the mortality rate given by γ.

$$\psi^2 = \frac{\psi^1 \omega}{(1 + \psi^1)(n + \gamma - 1)} \tag{87}$$

Equation 88 provides the children to workforce ratio.

$$\psi^3 = \frac{1}{\psi^1 \psi^c + \xi(\psi^c - \lambda_c)} \tag{88}$$

Equation 89 provides the young worker to workforce ratio.

$$\psi^4 = \frac{1}{\psi^1 + \xi \left(1 - \frac{\lambda_c}{\psi^c}\right)} \tag{89}$$

What is important to notice here is that, since z, θ and, ω is fixed, reflecting the average length of each life stage, the age structure of the population below retirement age only depends on population growth rate. In an integrated model where the initial population growth rate is assumed to be the same, the child dependency ratio and the workforce's age structure are constrained to be the same. Therefore, simulation models that assume the same initial population growth rate should start their simulation from a period where the child dependency ratio and workforce's age structure were not vastly different.

Equations 90 to 102 describes equations characterizing household side of the economy. Because of population and productivity growth, the model has a steady-state only in de-trended variables. Small alphabets denote the variable de-trended by population and productivity growth. Specifically, a small alphabet v denotes a variable $\frac{V}{XN}$.

Section C provides mathematical proofs of the relationship between marginal propensity to consume out of lifetime wealth for young and mature workers and the mortality rate for the steady-state. It also provides proof that the additional discount rates (Ω and Ω_2) used by young and mature workers are greater than 1.

Equation 90 provides marginal propensity to consume out of wealth for a retiree:

$$\epsilon\pi = 1 - (1-\gamma)\beta^{\sigma} R_w^{\sigma-1} \tag{90}$$

Equation 91 provides marginal propensity to consume out of wealth for a mature worker:

$$\pi = 1 - \beta^{\sigma}(R_w\Omega)^{\sigma-1} \tag{91}$$

where, mature worker's additional discount rate Ω that reflect his/her finite life is given in equation 92:

$$\Omega = 1 - \omega + \omega\,\epsilon^{\frac{1}{1-\sigma}} \tag{92}$$

Equation 93 provides marginal propensity to consume out of wealth for a young worker:

$$\epsilon_2\pi = 1 - \beta^{\sigma}(R_w\Omega_2)^{\sigma-1} \tag{93}$$

where, the young worker's additional discount rate Ω_2 that reflect his/her finite life is given in equation 94:

$$\Omega_2 = 1 - \theta + \theta\,\epsilon_2{}^{\frac{1}{\sigma-1}} \tag{94}$$

Aggregate household consumption is given in equation 95:

$$c = \pi\Big[aR_w(1-(1-\epsilon)\lambda-(1-\epsilon_2)\lambda^2) + s^m + h^m + \epsilon s^r + \epsilon_2\zeta(h^y+s^y)\Big] + \frac{\psi^4 w t^c}{\psi^c} \tag{95}$$

Human wealth of mature workers are given by equation 96:

$$h^m = \frac{R_w\Omega\psi^1(w(1-\psi^3 t^c)-\tau)}{(R_w\Omega-(1-\omega)x_l)(\psi^1+\xi(1-\frac{\lambda_c}{\psi^c}))} \tag{96}$$

Human wealth of young workers are given by equation 97:

$$h^y = \frac{R_w\Omega_2(1-\frac{\lambda_c}{\psi_c})\xi(w(1-\psi^3 t^c)-\tau)}{(\psi^1+(1-\frac{\lambda_c}{\psi_c})\xi)(R_w\Omega_2-x_l(1-\theta))} + \frac{\theta h^m x_l{}^{\dagger}\epsilon_2^{\frac{1}{\sigma-1}}}{(R_w\Omega_2-x_l(1-\theta))} \tag{97}$$

Social security:

$$s^y = \frac{\theta x_l\epsilon_2^{\frac{1}{\sigma-1}} s^m}{R_w\Omega_2 - x_l(1-\theta)} \tag{98}$$

$$s^m = \frac{x_l\epsilon^{\frac{1}{\sigma-1}}\omega s^r}{(R_w\Omega - x_l(1-\omega))} \tag{99}$$

$$s^r = \frac{R\overline{p_y}y}{(R_w - x_l(1-\gamma))} \tag{100}$$

Share of assets held by retirees at the steady state is given by:

$$\lambda = \frac{\omega x_l n(1 - \frac{\lambda^2}{1-\theta}) + py - \epsilon\pi s^r}{(x_l n - (1-\epsilon\pi)(1-\omega)R_w)} \tag{101}$$

Share of assets held by young workers' at the steady state is given by:

$$\lambda^2 = \frac{(1-\theta)(\psi^4(1 - \frac{\lambda_c}{\psi_c})\xi(w(1-\psi^3 t^c) - \tau) - \epsilon_2\pi(h^y + s^y))}{a(x_l n - (1-\epsilon_2\pi)(1-\theta)R_w)} \tag{102}$$

Output:

$$y = \kappa^{1-\alpha} \tag{103}$$

$$w = y\,\alpha \tag{104}$$

Government:

$$b = \frac{\overline{p_y}y + \overline{g_y}y - \tau}{x_l n - R_w} \tag{105}$$

The government debt is assumed to be equal to a constant fraction of output at the steady-state, which closes the model.

$$x_l n b = \overline{b_y}y \tag{106}$$

The capital accumulation equation shows that investment is a constant fraction of capital stock at the steady state to compensate for population and productivity growth rate.

$$\kappa = \frac{i}{x_l n + \delta - 1} \tag{107}$$

Assets are held in capital stock, government bonds and foreign assets.

$$a = b + \kappa + f \tag{108}$$

In an efficient steady-state, where $R_w > x_l n$, a country with a positive foreign asset position will have a negative trade balance reflecting the difference between domestic savings and investment.

$$f = \frac{nx}{x_l n - R_w} = \frac{y(1-g) - (c+i)}{x_l n - R_w} \tag{109}$$

and will have a positive current account balance given as:

$$ca = nx\Big(\frac{x_l n - 1}{x_l n - Rw}\Big) \tag{110}$$

B Solution method

From a mathematical point of view, the detrended model described in section 2 can be thought of as system of non-linear equations that can be written in the form:

$$f(y_{t+1}, y_t, y_{t-1}, u_t) = 0 \tag{111}$$

where

- y is the vector of endogenous variables with dimension n
- u is the vector of perfectly anticipated exogenous shocks except the one that occurring in period 1. Shocks can occur in any period, however, only the period 1 shocks come as surprise.
- f is a real valued continuous function defined as f: $\mathbb{R}^{3n+q} \to \mathbb{R}^n$
- The model admits a steady state when $f(\bar{y}, \bar{y}, \bar{y}, \bar{u}) = 0$.
- $t = 1, ...T$, with initial condition y_0 given for the state variables and $y_T = \bar{y}$.

The steady-state of the model is solved using the Newton algorithm, with the following steps summarizing its solution steps:

1. For a given exogenous variables, provide initial guess of the endogenous variables $y^{(0)}$. Upper indice (i) denote $(i)^{th}$ guess or iteration.
2. Update the guess value by solving the equation for next guess $y^{(i+1)}$:
$$F(y^{(i)}) + JF(y^{(i)})(y^{(i+1)} - y^{(i)}) = 0 \tag{112}$$
where $JF(y) = \frac{\partial F(y)}{\partial y}$ is the jacobian matrix of F.
3. Stop the iterations if.
$$\|F(y^{(i)})\| < \epsilon \tag{113}$$
where ϵ is the tolerance level close to zero.

Newton method is also used to solve the model's dynamic path but take advantage of the sparsity of the Jacobian matrix of the dynamic model. Description below summarizes the solution steps :

1. It is assumed that the model reaches its steady-state in a finite period T after a disturbance.
2. Then the dynamic solution is found when the endogenous variables satisfy all the dynamic equations given by first-order conditions and the budget constraints for each period until it reaches the steady-state at period T starting from given initial values for state variables. This problem is known as two boundary value problem in the literature.

3. The system of equations can be written in the form of a function F: f: $\mathbb{R}_+{}^{nT} \rightarrow \mathbb{R}^{\mathrm{nT}}$ such that:

$$F(y) = 0 \tag{114}$$

   y includes all the endogenous variables in all periods until the terminal period T. Values for deep parameters and initial values for the state variables and the terminal conditions of the endogenous variables need to be known to solve the function.

4. The non-linear system of equations is approximated by its first derivative stored in the Jacobian matrix. The Jacobian matrix is then solved using the Newton-Rapson algorithm.

The dimension of the problem increases with the number of endogenous variables and the periods. For example, for a system with n endogenous variables and T periods, the number of unknowns and equations to be solved is equal to nT. Because first-order conditions and the budget constraints are expressed only in first lag or lead, the Jacobian matrix of the dynamic model is sparse, meaning most of its columns consist of zeros. In this case, specialized algorithms can be used to reduce the size of the problem and solve the system faster.

C Proofs

The savings rate declines as individual ages, reflecting their horizon over which consumption decisions are made. Which implies that $\epsilon_2 < 1$ and $\epsilon > 1$. It is possible to prove it, at least for the steady-state of the model.

Getting ratio of the MPCs of the retirees and mature workers at the steady state provides following ratio.

$$\frac{1-\gamma}{\Omega^{\sigma-1}} = \frac{(\epsilon\pi - 1)}{\pi - 1} \tag{115}$$

If retirees have lower MPC than mature workers implying $\epsilon < 1$, then.

$$1-\gamma > \Omega^{\sigma-1} \tag{116}$$

and

$$1-\gamma > \left[(1-\omega) + \omega\epsilon^{\frac{1}{1-\sigma}}\right]^{\sigma-1} \tag{117}$$

Since $0 < \gamma < 1$, and when $0 < \sigma < 1$, the last condition holds only if $\epsilon > 1$ which contradicts our guess.

Similarly, the ratio of the MPCs of the young worker and the mature worker at the steady-state provides the following ratio.

$$\left(\frac{\Omega_2}{\Omega}\right)^{\sigma-1} = \frac{(\epsilon_2\pi - 1)}{\pi - 1} \tag{118}$$

If young workers have higher consumption rate than mature workers implying $\epsilon_2 > 1$, then.

$$\left[(1-\theta) + \theta\epsilon_2^{\frac{1}{\sigma-1}}\right]^{\sigma-1} < \left[(1-\omega) + \omega\epsilon^{\frac{1}{1-\sigma}}\right]^{\sigma-1} \tag{119}$$

For values of $0 < \sigma < 1$, this equality implies

$$\left[(1-\theta) + \theta\epsilon_2^{\frac{1}{\sigma-1}}\right] > \left[(1-\omega) + \omega\epsilon^{\frac{1}{1-\sigma}}\right] \tag{120}$$

Since we know that $\epsilon > 1$ and values of $0 < \sigma < 1$, the right-hand side of the equation is greater than one while the left-hand side should be less than 1 when ϵ_2 is greater than one, which contradicts our initial assumption. Therefore, it proves that ϵ_2 is less than 1.

We can establish the relationship between the MPCs of workers and life expectancy. The life expectancy parameter enters workers MPCs through Ω for mature workers and Ω_2 for the young worker.

In the steady state, Marginal propensity to consume (MPC) of mature worker is given by:

$$\pi = 1 - \beta^{\sigma}(R\Omega)^{\sigma-1} \tag{121}$$

where

$$\Omega = (1-\omega) + \omega\epsilon^{\frac{1}{1-\sigma}} \tag{122}$$

where ϵ is the ratio of MPC of retirees to MPC of mature workers:

$$\epsilon = \frac{1 - \beta^\sigma R^{\sigma-1}(1-\gamma)}{\pi} \tag{123}$$

With a function F of γ and π given as:

$$F(\gamma, \pi) = \pi - 1 + \beta^\sigma[\Omega(\gamma, \pi)R]^{\sigma-1} \tag{124}$$

where

$$\Omega(\gamma, \pi) = (1-\omega) + \omega\Big(\frac{(1-(1-\gamma)\beta^\sigma R^{\sigma-1})}{\pi}\Big)^{\frac{1}{1-\sigma}} \tag{125}$$

the effect of mortality rate γ on MPC of mature workers can be deduced by using the implicit function theorem:

$$\frac{\partial \pi}{\partial \gamma} = -\frac{\partial F(\gamma, \pi)/\partial \gamma}{\partial F(\gamma, \pi)/\partial \pi} \tag{126}$$

Where

$$\frac{\partial F}{\partial \gamma} = \beta^\sigma(\sigma-1)\Omega^{\sigma-2}R^{\sigma-1}(\omega(\frac{1}{1-\sigma})(\epsilon)^{\frac{1}{1-\sigma}-1})(\frac{\beta^\sigma R^{\sigma-1}}{\pi}(1)) < 0 \tag{127}$$

$$\frac{\partial F}{\partial \pi} = 1 + \beta^\sigma(\sigma-1)\Omega^{\sigma-2}R^{\sigma-1}\omega(\frac{1}{1-\sigma}\epsilon^{\frac{1}{1-\sigma}-1})\Big(-\frac{(1-(1-\gamma)\beta^\sigma R^{\sigma-1})}{\pi^2}\Big) > 0 \tag{128}$$

Therefore,

$$\frac{\partial \pi}{\partial \gamma} = -\frac{-}{+} > 0 \tag{129}$$

In other words, higher mortality increases MPCs by lowering the length of the horizon over which consumption decisions are made.

Similarly, we can deduce the sign of the effect of mortality rate on MPC of young workers as follows:

Mortality rates do not enter equation of the MPC of young workers directly.

$$MPC^y = 1 - \beta^\sigma(R\Omega_2)^{\sigma-1} \tag{130}$$

However, through Ω_2 which links the MPC of young workers to the MPC of mature workers, life expectancy can influence the MPC of young workers.

$$\Omega_2 = (1-\theta) + \theta(\frac{MPC^y}{1-\beta^\sigma(R\Omega)^{\sigma-1}})^{\frac{1}{\sigma-1}} \tag{131}$$

where

$$\Omega(\gamma, \pi) = (1-\omega) + \omega\Big(\frac{(1-(1-\gamma)\beta^\sigma R^{\sigma-1})}{\pi}\Big)^{\frac{1}{1-\sigma}} \tag{132}$$

For a function G of γ and MPC^y given as:

$$G(\gamma, MPC^y) = MPC^y - 1 + \beta^\sigma(R\Omega_2(MPC^y, \gamma))^{\sigma-1} \tag{133}$$

the effect of mortality rate on the MPC of young workers can be deduced using, again, the implicit function theorem:

$$\frac{\partial MPC^y}{\partial \gamma} = -\frac{\partial G(\gamma, MPC^y)/\partial \gamma}{\partial G(\gamma, MPC^y)/\partial MPC^y} \tag{134}$$

And the sign of the two partial derivative is:

$$\frac{\partial G(\gamma, MPC^y)}{\partial \gamma} > 0 \tag{135}$$

$$\frac{\partial G(\gamma, MPC^y)}{\partial \gamma} < 0 \tag{136}$$

$$\frac{\partial MPC^y}{\partial \gamma} = -\frac{+}{-} > 0$$

D Calibration comparison

Table 7: Calibration comparison (US)

Parameters	Paper 1[1]	Paper 2[2]	Paper 3[3]
Initial Gross interest rate (R_{ss})	1.04		1.016[4] 1.059[4]
Growth of workforce (n)	0.01	0.015	0.015
Productivity growth (x_l)	0.015	0.015	0.014
Capital depreciation rate (δ)	0.1	0.1	0.2
Capital share (α)	1/3	0.28	1/3
Time discount rate (β)	0.98	0.91	0.98
Elasticity of intertemporal substitution (σ)	0.5	0.32	0.5
Relative productivity of young workers (ξ)	N/A	0.79[5]	N/A
Relative productivity of retirees (ξ_2)	N/A	0.28[5]	0.58
Consumption elasticity (v)	N/A	N/A	0.8
Government expenditure (g)	0.2	N/A	0.15
Government debt (b)	0.26	N/A	0.3
Social security (s)	N/A	N/A	0.04
Bequest	N/A	0.19	N/A
Credit constraint	N/A	0.16	N/A

[1] Ferrero (2010) Discount factor targets a 5% real interest rate in 1970.
[2] Coeurdacier et al. (2015) β, σ and other key parameters are calibrated to match household savings data in 1988.
[3] Nieméläinen et al. (2020) In addition to following the previous literature, demographic and productivity parameters chosen to match the average between China and the US between 1980 and 2015.
[4] Fixed and variable labour supply
[5] Calculated based on given age-earnings profile.