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Normalizing the Central Bank's Balance Sheet: Implications for Inflation and Debt Dynamics

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Keywords

Inflation, Government Bonds, Liquidity, Spreads, Maturity, Balance Sheet.

JEL Classification

E40, E61, E62, H21

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Abstract

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1 Introduction

After reducing policy interest rates close to zero, in 2009 the Federal Reserve started to implement unconventional monetary policies. These involved purchases of assets that were not traditionally present in its portfolio. These operations dramatically increased the size of the central bank's balance sheet and expanded its longer-term public debt position.[1] As quantitative easing became more widely utilized in response to the Global Financial Crisis (GFC) around the world, central banks believed most of the increase in their balance sheets should be temporary.[2] Thus, it is not surprising to see that as soon as the economy of the United States started to recover, explicit discussions among policy makers turned to *monetary policy normalization* and how to execute it. This process would require the implementation of policies so that, in the future, the central bank would operate as it did before the GFC.[3] If policies that drastically changed the size and composition of the central bank balance sheet may affect financial markets in different ways than reductions in policy interest rates, then unwinding asset holdings by the central bank is also likely to affect the economy in different ways than raising interest rates.[4] Moreover, given that quantitative easing was unprecedented in size and scale, the *Great Unwinding* would be also equally without precedent. This paper explores such consequences.

During the transition period towards *normalization*, the new conduct of monetary policy would not only involve interest rate management, but also a new set of policies aimed at bringing the central bank's balance sheet back to its pre-GFC level and composition. In 2014 the Federal Open Market Committee (FOMC), through the Policy Normalization Principles and Plans, outlined three key actions in this process: (i) begin increases in short-term interest rates,[5] (ii) reduce the size of the central bank's balance sheet, and (iii) maturity reduction in order to have a composition in

[1]In December 2007, the Federal Reserve securities were entirely Treasury securities, and of these 32.1 percent were Treasury bills (short-term government debt). In December 2014, the Federal Reserve held no short-term Treasury securities; 41 percent of its security holdings were long-maturity mortgage-backed securities and 58.1 percent of its security holdings were long-maturity Treasury notes and bonds.

[2]As Forbes (2021), among others, highlights that this intention to unwind asset purchases was key in signaling that central banks were not going to partly finance government budget deficits.

[3]As early as June 2013, Federal Open Market Committee minutes already include some discussion on policy normalization and the long-run composition of the balance sheet.

[4]Since the implementation of unconventional monetary policies, the literature has tried to quantify their impact on financial markets and real economic activity and has found mixed evidence. We refer to Neely (2015), Dell'Ariccia et al. (2018), Swanson (2021), and reference therein, for more on this topic.

[5]The Federal Open Market Committee (FOMC) took this step in December 2015.

the balance sheet similar to the one prior to the Great Recession.[6] While the effects of running down the balance sheet and changing its composition are not fully implemented and known yet, the main purpose of this paper is to study the macroeconomic consequences of such *monetary policy normalization* process.[7]

Rather than analyzing the quantitative easing policies that led to the substantial expansion of the central bank's balance sheet and significant change in its composition, this paper studies various aspects of the *normalization* process. In particular, we take as given the post Great Recession characteristics of the Federal Reserve's balance sheet and analyze the consequences of the Great Unwinding as outlined by the FOMC Policy Normalization Principles and Plans. More precisely, we provide answers to the following questions. What are the consequences for economic activity, inflation and debt dynamics of having a central bank reduce the size of its balance sheet and its maturity structure? How will changes in the size of the central bank's balance sheet and its maturity structure alter the inherent links between monetary and fiscal policy? What type of rules should the central bank use when the *normalization* process is initiated?

To answer these questions, we consider a cashless flexible price environment where agents, in some markets, face stochastic trading opportunities and limited commitment.[8] Agents have access to nominal public debt, of different maturities, to smooth consumption. These securities can also be used as collateral to secure loans in markets where agents face limited commitment. However, public debt of different maturities do not provide the same collateral services. This different degree of liquidity is reflected in lower bond premia at longer maturities.[9] Other than private agents trading with each other, there is also a government with two different authorities: fiscal and monetary. Both of these trade with the private sector in a multilateral, frictionless and competitive financial and

[6]For more information we refer the reader to the "Policy Normalization Principles and Plans" (Board of Governors, 2014).

[7]The U.S. Federal Reserve reduced its re-investments from 2017 to 2019, but at a modest pace that the balance sheet only declined by about \$750bn to \$3.8tn in 2019.

[8]Given that we are studying monetary policy normalization, considering an environment with reserves is not necessary. This is the case as the normalization process would be one where we have asset run-offs and the central bank would trade long for short-term bonds. For more on this approach to monetary policy normalization, we refer the reader to Rickets et al. (2014) and Bullard (2017).

[9]The differential pledgeability (ability for assets to serve as collateral) among U.S. Government Treasury Securities, which include Bills, Notes and Bonds, is consistent with the haircuts observed by clearing houses. For instance, the ICE Clear U.S. imposes a haircut of 1.75% for Treasury securities that have a maturity less than one year, while it charges a 11.75% haircut for Treasury securities with maturities between 20 to 30 years. Since short and long-term public debt can be used as collateral, these securities carry a liquidity premium.

goods markets. The fiscal authority needs to finance an exogenous stream of expenditures through taxes and the issuance of short and long-term nominal bonds. The fiscal authority also decides the long-run quantity of debt to be issued as well as its composition. Moreover, the operating procedures for fiscal policy consist of a tax rule that responds to public debt and a rule for its target maturity composition. The monetary authority, on the other hand, manages interest rates through a Taylor rule. To highlight the role of premia on public debt, we analyze two specifications. The *traditional* Taylor rule, whereby the central bank only responds to deviations of inflation from its target. We also consider a *modified* Taylor rule, a long the lines of Cúrdia and Woodford (2010), that takes into account bond premia. Finally, in contrast to most of the literature, we consider two additional monetary rules that specify the evolution of the size and composition of the central bank's balance sheet. These operating procedures for monetary policy capture some of the features set out in the Policy Normalization Principles and Plans by the Federal Reserve in 2014. It also helps operationalize the monetary policy normalization process by explicitly considering the central bank's balance sheet size and composition targets. In addition, this approach allows us to conduct a variety of monetary policy normalization counterfactuals.

Our main results are as follows. When public debt does not exhibit premia, the stationary equilibria is unique and the prescriptions for determinacy of equilibria are similar to those found in Leeper (1991), Woodford (1994), Sims (1994), among others. For long-run public debt and its dynamics, what is crucial is the relative holdings of real debt held by the central bank. However, how much public debt is held by the private sector and changes to the composition of the balance sheet do not impact inflation nor debt dynamics. This is not surprising as the economy is Ricardian and all public debt is priced fundamentally. Thus, these assets are perfect substitutes.

When short and long-term bond premia exist, changes in the central bank's balance sheet alter real allocations. In particular, by changing the composition of assets in the central bank's balance sheet, the monetary authority can influence households' portfolio decisions. This is the case as agents face imperfect asset substitutability when trading in frictional and decentralized markets. We also find that the relative premium between short and long-term bonds have first-order consequences for long-run inflation and local stability of steady states. For a given balance sheet size target, when following active monetary policies and to ensure a unique and locally stable steady state inflation rate, it is crucial that the central bank targets a low enough maturity composition of government bonds. When calibrated to the United States, we find that when the central bank holdings of long-term debt are less than 0.5 times than that of their short-term bond holdings, the resulting stationary equilibria is unique and locally determinate. Moreover, the process of balance sheet normalization should respond aggressively to the total amount of debt issued in the economy. We also conducted a series of robustness checks. We find that how liquid long-term bonds matters for monetary policy normalization. The less liquid long-term bonds are, the higher is the value of the composition threshold and the parameter space consistent with unique and stable equilibria is larger. The central bank also requires a lower level of aggressiveness in response to deviations from total debt to its target. In addition, we find that our results are robust to changes in seller mark-ups and search frictions. Finally, regardless of the monetary and fiscal stance, the specifics determining the aggressiveness of the central bank to deviations of public debt to target critically depend on the central bank's balance size target and the fiscal authority's debt and compositional targets. These findings highlight that further coordination between fiscal and monetary authorities is needed when monetary normalization begins.

Under a modified Taylor rule that takes into account of the premium of short-term debt, we find that the long-run inflation is equal to the central bank's target. However, such interest rate policy increases the prevalence of multiple steady states. Thus, the compositional target becomes even more important to deliver desirable equilibria. In addition, these stationary equilibria result in lower welfare. Thus, given that a traditional Taylor rule is less likely to deliver real indeterminacies and yields higher welfare, this policy should be the preferred one when managing interest rates.

This paper is organized as follows. Section 2 follows with a review of the recent literature. Section 3 presents the economic environment. Section 4 characterizes the resulting dynamic equilibrium. Section 5 provides a numerical analysis of the resulting equilibria. Finally, Section 6 concludes.

2 Literature Review

This paper connects with two strands of literature. One that studies unconventional monetary policies when the economic environment has short and long-term government bonds. The other literature that we relate to is the one on monetary policy normalization.

Within the context of the Great Recession, there is now a voluminous empirical literature that has tried to evaluate the effectiveness of unconventional monetary policies enacted by the Federal Reserve.[10] However, fewer theoretical studies have analyzed the inherent trade-offs of a central bank trading public debt of different maturities to implement such policies. A notable exception is Harrison (2011), who considers the New Keynesian segmented financial market framework of Andres et al. (2004), where agents have different preferences over public debt of different maturities. Harrison (2011) shows that to the extent that asset purchase programs reduce long-term interest rates, aggregate demand can be stimulated. Within a similar framework, Chen et al. (2012) consider various nominal and real rigidities and estimate the effects of large scale asset purchase programs. Relative to no intervention, the authors find that U.S. GDP growth increases by less than a third of a percentage point and inflation barely changes. Reis (2017) also studies unconventional monetary policies in a simplified sticky price environment where agents have access to one and two-period nominal government bonds. The author shows that the power of quantitative easing policies to alter real allocations is due to the interest payment on reserves.[11] Similarly, Arce et al. (2019) show that a lean-balance-sheet regime with temporary and prompt quantitative easing achieves similar stabilization and welfare outcomes than that of a large-balance-sheet regime where managing interest rates is the primary adjustment margin. Within a flexible price environment with fiat money, government debt of different maturities, credit and banking, Williamson (2016) shows that when private banks face scarcity of collateralizable wealth, the economy displays an upward-sloping nominal yield curve. Under such scenario, the author finds that central bank purchases of long maturity government debt are always a good idea. Within a similar framework, Williamson (2019) shows that an increase in the central bank's balance sheet can have re-distributive effects and reduce welfare. In contrast, a reverse repo facility at the central bank puts a floor under the interbank interest rate, always improving welfare.

After signs of economic recovery, practitioners discussed three main approaches to undoing unconventional monetary policies enacted during the financial crisis.[12] The literature examining the

[10] We refer to Dell'Ariccia et al. (2018) for an excellent survey.

[11] Reis (2017) notes that reserves are special as they are not substitutable by currency nor by public debt.

[12] These are typically referred as *exit strategies*. One policy option is to leave excess reserves unchanged and pay interest on reserves. Another strategy is to absorb reserves through reverse repos, central bank bills, or term deposits. Finally, another option is to sell assets purchased under quantitative easing programs.

consequences of such exit strategies is small. An important contribution is the work of Armenter and Lester (2017).[13] The authors find that a cap on the volume at the overnight reverse repurchase agreements facility poses a risk to successful monetary policy implementation.[14] Similarly, Berentsen et al. (2018) study the consequences of paying interest on reserves. The authors find that it is optimal if the central bank has full fiscal support. If the central bank has no fiscal support, reducing reserves is optimal. This can be achieved by reserve absorbing operations, which hold the size of the balance sheet constant, or by selling assets, which reduces the size of the balance sheet.[15]

3 The Environment

Time is discrete and there is a continuum of infinitively-lived agents of measure one that discount the future at a rate $\beta \in (0,1)$. As in Lagos and Wright (2005) and Rocheteau and Wright (2005), agents face stochastic trading opportunities in some markets and sequentially trade. Each period has two sub-periods. The first one corresponds to a decentralized and frictional specialized goods market (DM). The other sub-period is a frictionless and competitive frictionless centralized market (CM). In DM agents receive a preference shock that determines whether they are consumers or producers in this market. After preference shocks are realized, DM consumers are randomly and bilaterally matched with DM producers. When trading in DM, other than search frictions, agents also face limited commitment. As a result, producers do not provide unsecured credit. To obtain DM credit, consumers are required to post collateral.[16] However, not all assets have the same pledgeability properties.[17] In the second sub-period, agents enter a centralized, frictionless and competitive market. In CM all agents can produce and consume a general perishable good, re-adjust their portfolio of

[13] These authors provide a model that captures the institutional details of the U.S. money market and study the central bank's ability to increase the policy rate in an environment with large excess reserves.

[14] This risk increases as the target range rises (holding the spread between the interest on overnight excess reserves and the overnight reverse repurchase agreements rates fixed) but falls as the spread widens.

[15] When calibrated to the Swiss franc repo market, the authors find that absorbing reserves using term deposits is equivalent to selling assets and thus unwinding quantitative easing.

[16] We refer to Kiyotaki and Moore (1997) for more on the need to collaterize loans and to Geromichalos, Herrenbrueck, and Salyer (2016) for a micro-foundation of the liquidity provided by short-term bonds.

[17] The asymmetric treatment of the collateral properties among assets is one way, among other alternatives, to generate a liquidity premium. We refer the reader to Venkateswaran and Wright (2014), Berentsen and Waller (2018), Andolfatto and Martin (2018), Canzeroni et al. (2016), among others, for examples where bonds serve as collateral, yielding then a liquidity premium. Other authors such as Herrenbrueck and Geromichalos (2016), Lee et al. (2016) and Domínguez and Gomis-Porqueras (2019), among others, consider secondary over the counter markets to generate such a premium, while Berentsen and Waller (2011) consider a multilateral and competitive market.

short and long-term government nominal bonds, settle their private debts and pay their tax liabilities. Nominal government bonds are the only durable objects in the economy that can help buyers and sellers smooth their consumption.

Preferences and Technologies: Agents have preferences over the consumption of CM goods (x_t), effort to produce CM goods (h_t), consumption of specialized DM goods (q_t), and effort to produce DM goods (e_t). The expected utility of the i-agent is then given by

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left[\ln(x_t) - h_t + \chi_{i,t} \frac{q_t^{1-\xi}}{1-\xi} + \frac{\chi_{i,t} - \chi}{\chi} e_t \right], \tag{1}$$

where $\xi \in (0,1)$ is the inverse of the inter-temporal elasticity of substitution of DM consumption and $\chi_{i,t} = \{0, \chi\}$ is an idiosyncratic time-varying shock. This preference shock is such that when $\chi_{i,t} = 0$, then the i-agent is a DM producer, and when $\chi_{i,t} = \chi > 0$, the i-agent is a DM consumer. This shock is independently distributed across all agents. Finally, $\mathbb{E}_0$ denotes the linear expectation operator with respect to an equilibrium distribution of idiosyncratic agent types.

All DM and CM goods are produced with a linear technology where labor is the only input. The production function is such that one unit of labor yields one unit of output.

Assets: Agents in this economy have access to one period government nominal debt, which we denote by B_t^S. From now we refer to this type of debt as short-term. Agents also have access to a more general portfolio of public nominal debt, which we denote by B_t^L. Following Woodford (2001), this latter debt instrument has a nominal payment structure equal to $\rho^{T-(t+1)}$, where $T > t$ and $0 < \rho < 1$. This asset can be interpreted as a portfolio of infinitely many nominal bonds, with weights along the maturity structure given by $\rho^{T-(t+1)}$.[18] From now on we refer to these as long-term bonds and their price is denoted by Q_t. The nominal interest rate corresponding to short-term public debt purchased at time t is represented by R_t.

Frictions and Trades: In the first sub-period, after agents learn if they are going to be DM consumers (buyers) or DM producers (sellers). These agents then face stochastic trading opportunities. In particular, buyers and sellers are bilaterally matched with probability $\sigma \in [0,1]$. After matching

[18] For example, one-period debt corresponds to $\rho = 0$, while a consol bond is consistent with $\rho = 1$.

takes place, to consume q units of the DM good, buyers can promise the DM seller a payment in the next CM. However, due to limited commitment, the DM buyer can renege on his future payment. This possibility allows assets to be used as collateral. The usual interpretation of such arrangement is that if a borrower reneges on his promise, his assets are seized. This contingency dissuades opportunistic default. Note, however, that as in Rocheteau et al. (2018), one can also describe DM trade as a repurchase agreement, where a buyer getting q units of DM goods sells assets to a seller, who sells them back at prearranged terms in the next CM.[19]

In DM, we further assume that not all assets are equally pledgeable as in Rocheteau et al. (2018) and Dong et al. (2019). More precisely, consistent with U.S. financial markets, we assume that short-term bonds are more pledegable than long-term bonds.

Government

As in Del Negro and Sims (2015), among others, we distinguish between the central bank's balance sheet and the rest of the government budget constraint. Below we describe how these two different institutions implement policy.

Fiscal Authority

This government agency needs to finance an exogenous and constant stream of expenditures, which we denote by G, and outstanding debt interest payments. To finance them, the fiscal authority has access to lump-sum CM taxes, τ_t^{CM}, and the issuance of short and long-term nominal bonds. The corresponding budget constraint for the fiscal authority is then given by

$$\tau_t^{CM} + \phi_t B_t^S + Q_t \phi_t B_t^L + T_t^C = G + R_{t-1}\phi_t B_{t-1}^S + (1+\rho Q_t)\,\phi_t B_{t-1}^L, \tag{2}$$

where $\phi_t \equiv \frac{1}{P_t}$ is the real price of the CM good and T_t^C is the transfer given by the central bank. The real value of all bond issuance is $\phi_t B_t = \phi_t \left(B_t^S + Q_t B_t^L\right)$. We assume that it is bounded above by a sufficiently large constant to avoid Ponzi schemes.

To describe the specific operating procedures for fiscal policy, we follow the specification typically used in the fiscal theory of the price level.[20] In such settings, taxes respond to previously issued

[19]As in Rocheteau et al. (2018), we do not propose a deep theory of repurchase agreements (repos). We refer to Antinolfi et al. (2015) or Gottardi et al. (2015) for more on repos.

[20]We refer to Leeper (1991), Woodford (1994), Sims (1994), among others, for more on this fiscal policy.

public debt. More specifically, we have that

$$\tau_t^{CM} = \gamma_0 + \gamma^S \left(\phi_{t-1} B_{t-1}^S - b^{S*}\right) + \gamma^L \left(\phi_{t-1} Q_{t-1} B_{t-1}^L - b^{L*}\right), \tag{3}$$

where γ_0 determines how taxes are set regardless of the economy's debt structure and γ^S $\left(\gamma^L\right)$ captures how taxes respond to the level of short-term (long-term) bonds. Finally, b^{S*} and b^{L*} represent the real target levels for short and long-term public debt, respectively. From now on, a variable with an asterisk denotes its corresponding target.

The intuition behind equation (3) is that systematic changes in taxes reflect fiscal financing considerations as encapsulated in the outstanding government debt obligations relative to their target levels. This is consistent with the debt sustainability framework commonly used for policy analysis by the IMF.[21]

From now on, we define total real debt, $b_t = \phi_t B_t$, as follows

$$b_t = b_t^S + Q_t b_t^L,$$

where we have that $b_t^S = \phi_t B_t^S$ and $b_t^L = \phi_t B_t^L$. Taking these into account, the fiscal rule can then be written as follows

$$\tau_t^{CM} = \gamma_0 + \gamma_1 \left(b_{t-1} - b^*\right). \tag{4}$$

Implicit in this rule is that the fiscal authority also specifies a constant composition of the new issuance of long and short-term bonds so that $\Omega \equiv \frac{Q_t B_t^L}{B_t^S}$, where the balance sheet composition target is such that $\Omega^* = \frac{Q^* b^{*L}}{b^{*S}}$. In particular, we get $\gamma_1 = \left(\frac{\gamma^S + \gamma^L \Omega}{1+\Omega}\right)$ and the total debt target becomes $b^* = b^{S*} + Q^* b^{L*}$.

Monetary Authority

The central bank also manages interest rates through a Taylor rule. In particular, we have that

$$R_t = \alpha_0 + \alpha_1 \left(\Pi_t - \Pi^*\right), \tag{5}$$

where $\Pi_{t+1} = \frac{\phi_t}{\phi_{t+1}}$ denotes the gross inflation rate, α_0 is a constant that determines how interest rates are set regardless of the economy's inflation rate, while α_1 captures how interest rates respond to inflation rate departures from its target Π^*.

[21] We refer to Article IV and Ghosh et al. (2013) for more on debt sustainability frameworks.

In addition to managing interest rates through a Taylor rule and in contrast to most of the literature, we consider additional monetary rules. More precisely, the central bank specifies the targets regarding the size and composition of its balance sheet. To capture some features of the *Policy Normalization Principles and Plans* outlined by the FOMC in 2014, we consider the following rules

$$b_t^M = \gamma_0^M + \gamma_1^M \left(b_t - b^*\right), \tag{6}$$

$$\frac{1}{(1+\Omega_t^M)} b_t^M = \eta_0^M + \eta_1^M \left(\frac{1}{(1+\Omega)} b_t - \frac{1}{(1+\Omega^*)} b^* \right), \tag{7}$$

where $b_t^M = \theta_t^S b_t^S + Q_t \theta_t^L b_t^L$ represents the monetary authority's total bond holdings and θ_t^S (θ_t^L) is the fraction of all outstanding short-term (long-term) public debt held by the central bank. We denote the composition of public debt in the hands of the monetary authority by $\Omega_{t-1}^M = \frac{Q_{t-1}\theta_{t-1}^L B_{t-1}^L}{\theta_{t-1}^S B_{t-1}^S}$. Taking the operating procedures for fiscal policy into account, the asset composition of the central bank is then given by $\Omega_{t-1}^M = \frac{\theta_{t-1}^L}{\theta_{t-1}^S}\Omega$. γ_0^M is a constant that determines the size of the central bank's balance sheet regardless of the total amount of public debt issued in the economy, while γ_1^M captures how the size of the central bank's balance sheet responds to deviations from total amount of government debt outstanding relative to its target. Similarly, η_0^M is a constant that determines the composition of the central bank's balance sheet regardless of the overall composition of total public debt, while η_1^M captures how composition of the central bank's balance sheet responds to the deviations from composition of debt relative to its target.

The corresponding budget constraint for the monetary authority is then given by

$$T_t^C + \theta_t^S \phi_t B_t^S + \theta_t^L Q_t \phi_t B_t^L = R_{t-1}\theta_{t-1}^S \phi_t B_{t-1}^S + \theta_{t-1}^L \left(1 + \rho Q_t\right) \phi_t B_{t-1}^L. \tag{8}$$

where T_t^C is the transfer that the central bank provides to the fiscal authority.

3.1 Agent's Problem

Given the sequential nature of our environment, we solve the representative agent's problem backwards. Thus, we first solve the CM and then the DM problem.

CM Problem

In this market all agents can produce and consume the CM good, while trading in a frictionless and competitive market. Agents can settle their CM trades with any assets, CM goods or CM labor.

Given a portfolio of nominal government bonds ($\tilde{B}^S_{t-1}$,$\tilde{B}^L_{t-1}$) at the beginning of CM, the problem of a representative agent is as follows

$$W\left(\tilde{B}^S_{t-1}, \tilde{B}^L_{t-1}, \tilde{L}^{S,L}_{t-1}\right) = \max_{x_t, h_t, \tilde{B}^S_t, \tilde{B}^L_t} \left\{\ln(x_t) - h_t + \beta\, V^{DM}\left(\tilde{B}^S_t, \tilde{B}^L_t\right)\right\} \quad \text{s.t.} \tag{9}$$

$$x_t + \phi_t \tilde{B}^S_t + Q_t \phi_t \tilde{B}^L_t + \phi_t \tilde{L}^{S,L}_{t-1} = h_t - \tau^{CM}_t + \phi_t \left(1 + \rho Q_t\right) \tilde{B}^L_{t-1} + \phi_t R_{t-1} \tilde{B}^S_{t-1},$$

where $V^{DM}(\cdot,\cdot)$ is the agent's expected DM value function, $\tilde{B}^S_t$ ($\tilde{B}^L_t$) denotes the agent's short-term (long-term) nominal bond holdings in period t and $\tilde{L}^{S,L}_{t-1}$ represents the nominal payment of an agent that was granted a collateralized loan in DM when both short-term and long-term bonds are pledged. Note that while the initial portfolio of nominal government bonds ($\tilde{B}^S_{t-1}$, $\tilde{B}^L_{t-1}$) is the same across agents, the secured loan ($\tilde{L}^{S,L}_{t-1}$) may be different. This is the case as when agents trade in DM, buyers obtain the good through a secured loan and no bonds change hands between buyers and sellers.

The corresponding first-order conditions are given by

$$\frac{1}{x_t} - 1 = 0, \tag{10}$$

$$-\phi_t + \beta\, \frac{\partial V^{DM}\left(\tilde{B}^S_t, \tilde{B}^L_t\right)}{\partial \tilde{B}^S_t} = 0, \tag{11}$$

$$-\phi_t Q_t + \beta\, \frac{\partial V^{DM}\left(\tilde{B}^S_t, \tilde{B}^L_t\right)}{\partial \tilde{B}^L_t} = 0. \tag{12}$$

The associated envelope conditions are $\frac{\partial W_t}{\partial \tilde{B}^S_{t-1}} = \phi_t R_{t-1}$, $\frac{\partial W_t}{\partial \tilde{B}^L_{t-1}} = \phi_t \left(1 + \rho Q_t\right)$ and $\frac{\partial W_t}{\partial \tilde{L}^{S,L}_{t-1}} = -\phi_t$.

It is important to note that households hold fractions $(1-\theta^S_t)$ and $(1-\theta^L_t)$ of all short and long-term government bonds issued at time t, respectively. Because of quasi-linearity, all agents enter DM with the same portfolio of bond holdings, as they do not depend on past bond holdings. This implies then that all agents after CM hold $\tilde{B}^S_t = \left(1-\theta^S_t\right) B^S_t$ and $\tilde{B}^L_t = \left(1-\theta^L_t\right) B^L_t$. The rest of the public debt is held by the central bank.

DM Problem

At the beginning of each period, agents experience a preference shock. With probability $\frac{1}{2}$ $\left(\frac{1}{2}\right)$ an agent becomes a consumer (producer) in the ensuing DM. Thus, before the shocks are realized, the corresponding value function of an agent with a portfolio of short and long-term bonds $(\tilde{B}^S_{t-1}, \tilde{B}^L_{t-1})$ is given by

$$V^{DM}(\tilde{B}^S_{t-1}, \tilde{B}^L_{t-1}) = \frac{1}{2}\left[V_b^{DM}(\tilde{B}^S_{t-1}, \tilde{B}^L_{t-1}) + V_s^{DM}(\tilde{B}^S_{t-1}, \tilde{B}^L_{t-1})\right],$$

where $V_j^{DM}(\cdot,\cdot)$ is the expected DM utility of an agent of type $j = \{b, s\}$, where subscript b denotes a DM consumer (buyer) and subscript s represents a DM producer (seller).

After the preference shock is realized, consumers and producers are randomly and bilaterally matched. Other than search frictions, agents in this market also face limited commitment. The value of DM consumer with a beginning of period portfolio $(\tilde{B}^S_{t-1}, \tilde{B}^L_{t-1})$ is given by

$$V_b^{DM}(\tilde{B}^S_{t-1}, \tilde{B}^L_{t-1}) = \sigma\left[\chi\frac{q_t^{1-\xi}}{1-\xi} + W\left(\tilde{B}^S_{t-1}, \tilde{B}^L_{t-1}, \tilde{L}^{S,L}_{t-1}\right)\right] + (1-\sigma)W\left(\tilde{B}^S_{t-1}, \tilde{B}^L_{t-1}, 0\right),$$

where q_t denotes the quantity of DM goods purchased and σ is the matching probability. Because of limited commitment agents can renege on their future payments. In order for trade to take place, DM consumers use their bonds as collateral.[22] In particular, short-term bonds are more pledgeable than long-term bonds. Then, the amount of credit extended in DM is $\tilde{L}^{S,L}_{t-1} \leq \zeta^S\hat{B}^S_{t-1} + \zeta^L Q_{t-1}\hat{B}^L_{t-1}$, where ζ^S (ζ^L) represents the relative pledgeability of short (long) term bonds.

Similarly, the expected utility of a DM producer is given by

$$V_s^{DM}(\hat{B}^S_{t-1}, \hat{B}^L_{t-1}) = \left[-q + W\left(\tilde{B}^S_{t-1}, \tilde{B}^L_{t-1}, -\tilde{L}^{S,L}_{t-1}\right)\right] + (1-\sigma)W\left(\tilde{B}^S_{t-1}, \tilde{B}^L_{t-1}, 0\right).$$

The trading protocol in this frictional market is determined by a buyer's take it or leave it offer. Formally, the terms of trade are given by

$$\max_{q_t, \tilde{L}^{S,L}_{t-1}} \left\{\chi\frac{q_t^{1-\xi}}{1-\xi} + W\left(\tilde{B}^S_{b,t-1}, \tilde{B}^L_{b,t-1}, \tilde{L}_{t-1}\right) - W\left(\tilde{B}^S_{b,t-1}, \tilde{B}^L_{b,t-1}, 0\right)\right\} \text{ s.t.}$$

$$\tilde{L}^{S,L}_{t-1} \leq \zeta^S\tilde{B}^S_{b,t-1} + \zeta^L Q_{t-1}\tilde{B}^L_{b,t-1},$$

$$-q_t + W\left(\tilde{B}^S_{s,t-1}, \tilde{B}^L_{s,t-1}, -\tilde{L}^{S,L}_{t-1}\right) \geq W\left(\tilde{B}^S_{s,t-1}, \tilde{B}^L_{s,t-1}, 0\right),$$

[22] As in Kiyotaki and Moore (1997), Bernanke et al. (1999), Iacoviello (2005), Andolfatto and MartíÂn (2018), Berentsen and Waller (2018), among others, because of limited commitment, the loan extended has to be collaterized.

where $\tilde{B}^S_{b,t-1}(\tilde{B}^S_{s,t-1})$ and $\tilde{B}^L_{b,t-1}(\tilde{B}^L_{s,t-1})$ represent the consumer's (producer's) short and long-term nominal bond holdings, respectively. As in Kiyotaki and Moore (1997), the collateral used by DM consumers does not change hands in DM. This is the case as they are used in case of default, which never occurs in equilibrium. The last constraint is the DM seller's incentive compatibility constraint, ensuring that a DM seller trading in DM is no worst off than not when he chooses not to trade. This problem can be further simplified as follows

$$\max_{q_t} \quad \left\{ \chi \frac{q_t^{1-\xi}}{1-\xi} - q_t \right\} \text{ s.t. } q_t = \phi_t \tilde{L}^{S,L}_{t-1} \leq \zeta^S \phi_t \tilde{B}^S_{b,t-1} + \zeta^L \phi_t Q_{t-1} \tilde{B}^L_{b,t-1},$$

delivering the following first-order conditions

$$\chi q_t^{-\xi} - 1 - \lambda_t^{S,L} = 0,$$
$$\lambda_t^{S,L} \left(\zeta^S \phi_t \tilde{B}^S_{b,t-1} + \zeta^L \phi_t Q_{t-1} \tilde{B}^L_{b,t-1} - \phi_t \tilde{L}^{S,L}_{t-1} \right) = 0.$$

The previous terms of trade imply the following DM consumer's envelope condition for short-term debt

$$\frac{\partial V_b^{DM}}{\partial \tilde{B}^S_{b,t-1}} = \sigma \left[\frac{\chi}{q_t{}^\xi} \frac{\partial q_t}{\partial \tilde{L}^{S,L}_{t-1}} \frac{\partial \tilde{L}^{S,L}_{t-1}}{\partial \tilde{B}^S_{b,t-1}} - \phi_t \frac{\partial \tilde{L}^{S,L}_{t-1}}{\partial \tilde{B}^S_{b,t-1}} + \phi_t R_{t-1} \right] + (1-\sigma)\phi_t R_{t-1}.$$

Similarly, for long-term nominal public debt we have that

$$\frac{\partial V_b^{DM}}{\partial \tilde{B}^L_{b,t-1}} = \sigma \left[\frac{\chi}{q_t{}^\xi} \frac{\partial q_t}{\partial \tilde{L}^{S,L}_{t-1}} \frac{\partial \tilde{L}^{S,L}_{t-1}}{\partial \tilde{B}^L_{b,t-1}} - \phi_t \frac{\partial \tilde{L}^{S,L}_{t-1}}{\partial \tilde{B}^L_{b,t-1}} + \phi_t (1+\rho Q_t) \right] + (1-\sigma)\phi_t (1+\rho Q_t).$$

Given the DM optimal terms of trade, the corresponding envelope conditions for the seller are given by

$$\frac{\partial V_s^{DM}}{\partial \tilde{B}^S_{s,t-1}} = \phi_t R_{t-1}, \quad \frac{\partial V_s^{DM}}{\partial \tilde{B}^L_{s,t-1}} = \phi_t \left(1+\rho Q_t\right).$$

It is important to highlight that the DM optimal terms of trade depend whether the DM borrowing constraints bind or are slack. Whether these constraints bind or not is endogenous and will depend on government policies and the liquidity of public debt. If the DM borrowing constraint is slack, an agent can consume the DM first-best ($\chi q_{t+1}^{-\xi} = 1 \ \forall t$). We denote this DM consumption as $\hat{q}$. As a result, short and long-term public debt do not have a liquidity premium ($s^S_{t+1} = s^L_{t+1}$=0 $\forall t$).

If, on the other hand, the DM borrowing constraint is binding, the endogenous value of the liquidity premium on short-term nominal public debt is given by

$$s_{t+1}^S = \frac{\sigma}{2}\left(\frac{\chi}{{q_{t+1}}^{\xi}} - 1\right)\zeta^S, \tag{13}$$

while the long-term nominal public debt liquidity premium is given by

$$s_{t+1}^L = \frac{\sigma}{2}\left(\frac{\chi}{{q_{t+1}}^{\xi}} - 1\right)\zeta^L\, Q_{t+1}, \tag{14}$$

where ζ^S and ζ^L represent the pledgeability of short and long-term pubic debt, respectively.

4 Dynamic Equilibrium

Having characterized the representative agent's optimal decisions, we can now define the corresponding dynamic equilibrium for this economy.

Definition 1 *Given the operating procedures for monetary policy (equations (6), (7) and (5)) and fiscal policy (equation (4) and $B_t^L = B_t^S \Omega$), public spending $\{G\}_{t=0}^{\infty}$ and initial conditions $(B_{-1}^S, B_{-1}^L, \theta_{-1}^S, \theta_{-1}^L)$, a dynamic equilibrium is a sequence of consumptions $\{x_t, q_t\}_{t=0}^{\infty}$, assets and prices $\left\{B_t^S, B_t^L, Q_t, R_t, \phi_t\right\}_{t=0}^{\infty}$ solving the agents' problems and clearing all markets.*

After imposing the agents' optimal decisions and market clearing, it is easy to show that the dynamic equilibrium is characterized by the following system of dynamic equations

$$x_t = 1, \tag{15}$$

$$q_{t+1} = \begin{cases} \hat{q}, & \text{if DM constraint is slack} \\ \left(\frac{b_t}{(1+\Omega)} - \frac{b_t^M}{(1+\Omega_t^M)}\right)\frac{\zeta^S}{\Pi_{t+1}} + \left(\frac{\Omega\, b_t}{(1+\Omega)} - \frac{\Omega_t^M\, b_t^M}{(1+\Omega_t^M)}\right)\frac{\zeta^L}{\Pi_{t+1}}, & \text{if DM constraint binds} \end{cases} \tag{16}$$

$$\Pi_{t+1} = \beta\left(R_t + s_{t+1}^S\right), \tag{17}$$

$$\Pi_{t+1} Q_t = \beta\left(1 + \rho Q_{t+1} + s_{t+1}^L\right), \tag{18}$$

$$T_t^C + b_t^M = \frac{1}{\beta} b_{t-1}^M - \left(s_t^S + s_t^L \frac{\Omega_{t-1}^M}{Q_{t-1}}\right)\frac{b_{t-1}^M}{\Pi_t\left(1 + \Omega_{t-1}^M\right)}, \tag{19}$$

$$\tau_t^{CM} + b_t + T_t^C = G + \frac{1}{\beta} b_{t-1} - \left(s_t^S + s_t^L \frac{\Omega}{Q_{t-1}}\right)\frac{b_{t-1}}{\Pi_t\left(1 + \Omega\right)}, \tag{20}$$

where R_t is determined by the Taylor rule (given by equation (5)), τ_t^{CM} is specified by the fiscal rule (given by equation (3)) and normalization monetary policies, b_t^M and Ω_t^M are given by equations (6) and (7), respectively.

It is important to highlight that the characterization of the dynamic equilibria depends on whether a liquidity premium exits or not. Next we examine these various possibilities.

Case 0: No Liquidity Premium

In this scenario, it is easy to show that the resulting dynamic equilibrium is characterized by the following dynamic system

$$\Pi_{t+1} = \Pi^* + \beta\alpha_1(\Pi_t - \Pi^*),$$

$$(1-\gamma_1^M)b_t = G - \left[\gamma_0 - \left(\frac{1}{\beta}-1\right)\gamma_0^M\right] + \left[\gamma_1 + \left(\frac{1}{\beta}-1\right)\gamma_1^M\right]b^* + \left[\frac{1}{\beta}(1-\gamma_1^M) - \gamma_1\right]b_{t-1}.$$

As with environments that do not include central bank balance sheet policies, the evolution of inflation is independent of the economy's total real debt. It is easy to show that the corresponding steady state is unique and given by

$$\Pi = \Pi^* = \beta\alpha_0, \;\; \& \;\; b = \frac{G - \left[\gamma_0 + \left(\frac{1}{\beta}-1\right)\right]\gamma_0^M + \left[\gamma_1 + \left(\frac{1}{\beta}-1\right)\gamma_1^M\right]b^*}{\gamma_1 - \left(\frac{1}{\beta}-1\right)(1-\gamma_1^M)},$$

which implies that the buyer consumes the first-best in DM, $\hat{q}$, and that the price of the long-term nominal bonds is given by $Q = \frac{\beta}{\Pi - \beta\rho}$.

The corresponding local dynamics are characterized by the following Jacobian

$$J = \begin{pmatrix} \beta\alpha_1 & 0 \\ 0 & \frac{1}{\beta} - \frac{\gamma_1}{(1-\gamma_1^M)} \end{pmatrix},$$

which results in a monetary and a fiscal eigenvalue that are independent of each other. As a result, the normative prescriptions for determinacy of equilibria are similar to those found in Leeper (1991), Woodford (1994), Sims (1994), among others. Nevertheless, it is worth highlighting that the relative real debt held by the public is now relevant for the fiscal eigenvalue.[23] However, it does not affect inflation dynamics. On the other hand, changes to the composition of the balance sheet (either in terms of desired maturity structure target or the specific response to deviations from the target) does not impact inflation nor debt dynamics. This is not surprising as the economy is Ricardian and all

[23] In Leeper (1991), Woodford (1998), among others, the central bank does not hold any public debt. This situation can be thought as one where $\gamma_1^M = 0$.

public debt is priced fundamentally.[24]

For the remainder of the paper and following Leeper (1991), we refer to traditionally active (passive) monetary policy to one that satisfies $\beta\alpha_1 > 1$ ($\beta\alpha_1 < 1$) and to traditionally passive fiscal policy to one that satisfies $\frac{1}{\beta} - \frac{\gamma_1}{(1-\gamma_1^M)} < 1$ and active to policies such that $\frac{1}{\beta} - \frac{\gamma_1}{(1-\gamma_1^M)} > 1$.

Case 1: Liquidity Premium

When holding additional short and long-term government bonds expand agents' DM consumption possibilities, the dynamic equilibria can be summarized by the evolution of inflation and total government debt as follows

$$\Pi_{t+1} = \Pi^* + \beta\alpha_1(\Pi_t - \Pi^*) + \beta s_{t+1}^S$$

$$(1-\gamma_1^M)b_t = G - \left[\gamma_0 - \left(\frac{1}{\beta} - 1\right)\gamma_0^M\right] + \left[\gamma_1 + \left(\frac{1}{\beta} - 1\right)\gamma_1^M\right] b^* + \left[\frac{1}{\beta}(1-\gamma_1^M) - \gamma_1\right] b_{t-1} -$$

$$-\frac{s_t^S}{\Pi_t}\left[(1-\eta_1^M)\frac{b_{t-1}}{1+\Omega} - \eta_0^M + \eta_1^M\frac{b^*}{1+\Omega^*}\right] - \frac{s_t^L}{Q_{t-1}\Pi_t}\left[b_{t-1}\left(\frac{\Omega+\eta_1^M}{1+\Omega} - \gamma_1^M\right) + \eta_0^M - \gamma_0^M + b^*\left(\gamma_1^M - \frac{\eta_1^M}{1+\Omega^*}\right)\right],$$

where the premiums s_t^S and s_t^L are given by equations (13) and (14), respectively and the pricing of long-term bonds is given by

$$\Pi_{t+1}Q_t = \beta\left(1 + \rho Q_{t+1} + s_{t+1}^L\right).$$

In contrast to the environment without a premium on short and long-term debt, the evolution of inflation and total real debt are not independent of each other. Moreover, operating procedures for monetary and fiscal policy both affect the inflation rate. This is the case even when the monetary authority follows a Taylor principle and fiscal policy is passive. This is not surprising as the spread on short-term bonds depends on both fiscal and monetary policies. In addition, the size and composition of the central bank's balance sheet matter for real allocations.

After imposing that the economy is in steady state, it is easy to show that the stationary equilibria is characterized by the following two implicit equations

$$\Pi = \Pi^* + \frac{\beta s^S}{(1-\beta\alpha_1)},$$

$$(1-\gamma_1^M)b = G - \left[\gamma_0 - \left(\frac{1}{\beta} - 1\right)\gamma_0^M\right] + \left[\gamma_1 + \left(\frac{1}{\beta} - 1\right)\gamma_1^M\right] b^* + \left[\frac{1}{\beta}(1-\gamma_1^M) - \gamma_1\right] b-$$

[24] As pointed out by Wallace (1981) and Lucas (1984), when an economy is Ricardian the maturity structure of government debt is totally irrelevant.

$$-\frac{s^S}{\Pi}\left[(1-\eta_1^M)\frac{b_{t-1}}{1+\Omega}-\eta_0^M+\eta_1^M\frac{b^*}{1+\Omega^*}\right]-\frac{s^L}{Q\Pi}\left[b\left(\frac{\Omega+\eta_1^M}{1+\Omega}-\gamma_1^M\right)+\eta_0^M-\gamma_0^M+b^*\left(\gamma_1^M-\frac{\eta_1^M}{1+\Omega^*}\right)\right],$$

where $s^S \geq 0$ and $s^L \geq 0$ are the long-run bond premium associated with short and long-term public debt, respectively. Such spreads depend on the long-run inflation and total debt circulating in the economy. Finally, the long-run price of long-term bonds and the return on short-term are given by

$$Q=\frac{\beta(1+s^L)}{\Pi-\beta\rho} \;\;\&\;\; R=\frac{\Pi}{\beta}-s^S.$$

It is important to highlight that when the premium is not taken into account by the monetary authority when managing interest rates, the liquidity premium leads to deviations of steady state inflation from the central bank's target. As in Domínguez and Gomis-Porqueras (2019), under the traditional Taylor rule, the deviation increases with the premium of short-term public debt s^S. Moreover, under an active (passive) monetary policy, $\beta\alpha_1 > 1$ ($\beta\alpha_1 < 1$), induces steady state inflation rates below (above) the target level Π^*. As we will show later in the paper, when the Taylor rule takes into account the public debt premia, the long-run inflation rate is equal to the target.

In a neighborhood of a steady state, local dynamics are described by the following Jacobian

$$J=\begin{pmatrix} \frac{\beta\alpha_1}{\omega_1}-\beta\omega_2\omega_4 & \beta\omega_4\left[\frac{1}{\beta}-\frac{\gamma_1}{\left(1-\gamma_1^M\right)}-\omega_3\right] \\ \omega_2 & \frac{1}{\beta}-\frac{\gamma_1}{\left(1-\gamma_1^M\right)}-\omega_3 \end{pmatrix},$$

where $\omega_1=1-\beta\frac{\partial s^S}{\partial \Pi}$, $\omega_2=-\frac{1}{\left(1-\gamma_1^M\right)}\left[\left(\frac{\partial s^S}{\partial \Pi}\Pi-s^S\right)Z_b^S+\left(\frac{\partial s^{L,Q}}{\partial \Pi}\Pi-s^{L,Q}\right)Z_b^L\right]\frac{1}{\Pi^2}$, with $s^{L,Q}=\frac{s^L}{Q}$, $Z_b^S=\left(1-\eta_1^M\right)\frac{b}{(1+\Omega)}-\eta_0^M+\eta_1^M\frac{b^*}{(1+\Omega^*)}$ and $Z_b^L=\frac{\Omega b}{(1+\Omega)}-\left(\gamma_0^M-\eta_0^M\right)-\gamma_1^M\left(b-b^*\right)+\eta_1^M\left(\frac{b_t}{(1+\Omega)}-\frac{b^*}{(1+\Omega^*)}\right)$, $\omega_3=\frac{1}{\left(1-\gamma_1^M\right)}\left\{\frac{\partial s^S}{\partial b}\frac{Z_b^S}{\Pi}+\frac{s^S}{\Pi}\frac{1}{1+\Omega}\left(1-\eta_1^M\right)+\frac{\partial s^{L,Q}}{\partial b}\frac{Z_b^L}{\Pi}+\frac{s^{L,Q}}{\Pi}\left(\frac{\Omega+\eta_1^M}{(1+\Omega)}-\gamma_1^M\right)\right\}$, and $\omega_4=\frac{\frac{\partial s^S}{\partial b}}{\omega_1}$.

In terms of local dynamics, generically, the corresponding monetary and fiscal eigenvalues depend on all monetary and fiscal policy rules. In particular, both eigenvalues are affected by the various central bank's balance sheet targets and degree of responses relative to the target gaps. Thus, the specifics of the normalization process are going to be key for inflation and debt dynamics. As those effects cannot be quantified analytically, we examine them numerically.

5 A Numerical Exploration

To gain further insights on the consequences for macroeconomic aggregates and local stability of having different normalization policies, we resort to numerical analysis. To discipline the underlying parameters describing the economy, our calibration strategy is as follows. The period we consider covers from 1985 to 2014. To be closer to the literature that examines monetary and fiscal policy interactions in non-search environments, we set $\sigma = 1$ so that agents can always find a counter-party in DM. To discipline the rest of the parameters, we use historical U.S. data at an annual frequency. In particular, to pin down preference parameters we rely on interest rate data, inflation and GDP obtained from the Federal Research of St. Louis Economic Data (*FRED*). Data on short and long-term bond holdings is collected from various issues of the *Treasury Bulletin* of the United States. Finally, we assume that policy parameters are consistent with active monetary and passive fiscal policies of an economy with no government debt premium.

In our analysis, we define short-term bonds as the marketable interest-bearing public debt maturing within one year by the end of the fiscal period. We calculate the amount of short-term bonds held by the domestic private sector by assuming that the proportion of foreign and international held bonds is evenly distributed across the different maturities.[25] The 3-Month Treasury constant maturity rate is represented by R_t. We then define short-term debt premium in our model, s^S_{t+1}, as the difference between the 20-Year and the 3-Month Treasury constant maturity, while the long-term premium, s^L_{t+1} is defined as the difference between the 20-Year and the 10-Year bond. The data analog for the inflation rate in our model, Π_t, is the Consumer Price Index (all items, 2015 = 100). Finally, nominal quantities of short and long-term bonds privately and domestically held as well as output are converted to real and detrended by the average real growth rate of GDP during that period, 2.2 per cent.[26]

Using the corresponding data on inflation, interest rates and short-term government bond premia, we use equation (17) to derive an implied annual discount factor. We then set β as the average for that period, which yields 0.97. Combining equations (13) and (14), the long-term debt premium satisfies $\frac{s^L_{t+1}}{Q_{t+1}} = \frac{\zeta^L}{\zeta^S} s^S_{t+1}$. Normalizing the pledgeability of short-term public debt so that $\zeta^S = 1$ and imposing the average long and short-term debt premia of our time series into this equation, we can pin down ζ^L as the bond premia differential. This procedure yields $\zeta^L = 0.2045$. Given a grid for

[25] The proportion of privately-held bonds in the hands of foreigners and international investors has been steadily growing over time. This proportion was 17 per cent in 1985 while over 40 per cent in 1999.

[26] In our model, output equals $1 + \frac{1}{2}\sigma q_t$, while in the data is measured by the Gross Domestic Product (GDP).

ξ, we use data on privately and domestically held short and long-term bonds and GDP to construct an implied time series for q_t. For each ξ, we further assume that the highest value of q_t, over this 30-year period, coincides with the efficient one, which is given by $\hat{q} = \chi^{\frac{1}{\xi}}$. This implies a value of χ for each ξ. Then for each pair (ξ, χ) we calculate the mean square error between the implied short and long-term bond premia and the historical U.S. data. The values that minimize such error yield $\xi = 0.2084$ and $\chi = 0.6760$.

For the rest of policy parameters, we make the following assumptions. We set $G = 0.23$, consistent with a federal government spending to GDP ratio of 22 per cent as of 2014 Q4. We set b^* to 0.76, which is consistent with a target of public debt to GDP ratio of 72 per cent. The maturity of the public debt is fixed at $\Omega = \Omega^* = 2.35$, to represent the long-term to short-term public debt ratio. For the fiscal rule, given by (4), we fix the intercept, γ_0, to match a public debt (domestically held) to output ratio of 72 per cent. Note that, in order to match it, γ_0 adjusts with γ_0^M. The slope of the fiscal rule is set so that fiscal policy is passive in an economy without the premium. Specifically, we consider $\frac{\gamma_1}{1-\gamma_1^M} = \frac{1}{\beta} - 1 + 0.0025$, where γ_1 adjusts with different values of γ_1^M.

To discipline the management of interest rates by the central bank, given by equation (5), we fix the intercept to match an annual inflation target of 2 per cent.[27] This implies $\alpha_0 = 1.05$. For the slope that describes the sensitivity to deviations from the inflation target, we assume $\alpha_1 = 1.5$, which is consistent with an active monetary policy (in an economy without premium) that satisfies the Taylor Principle. For the rest of the policy parameters, we consider a range of values consistent with various choices of targets and responses to current economic conditions that characterize the balance sheet normalization process. The resulting parameters for the benchmark model and calibration targets are summarized in Table 1.

Table 1: Calibration Targets

[27] On 25 January 2012, U.S. Federal Reserve Chairman Ben Bernanke set a 2% target inflation rate. Until then, the Federal Open Market Committee (FOMC), did not have an explicit inflation target but regularly announced a desired target range for inflation (usually between 1.7% and 2%).

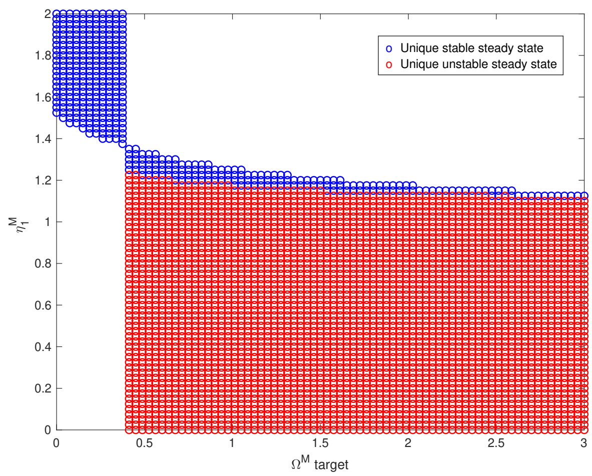
Parameter	Target
$\beta = 0.9735$	Implied by (17) with data on Π, R and s in 1985-2014
$\chi = 0.6760$	Short-term bond premia defined by the difference between the
$\xi = 0.2084$	20-Year and the 3-Month Treasury in 1985-2014
$\zeta^S = 1$	fixed
$\zeta^L = 0.2045$	20-Year and the 10-Year in 1985-2014
$G = 0.2317$	Federal government expenditure of 21.97 % of GDP as of 2014 Q4
$b^* = 0.7576$	Public debt (domestically held) of 71.84 % of GDP as of 2014 Q4
$\Omega^* = 2.3527$	Long-term to short-term public debt (domestically held) as of 2014 Q4
γ_0	Adjusts with γ_0^M to match a public debt (dom. held) of 71.84 % of GDP as of 2014 Q4
γ_1	Adjusts with γ_1^M to ensure passive fiscal policy (without a premium)
$\alpha_0 = 1.0478$	Inflation target of 2 %
$\alpha_1 = 1.5000$	Active monetary policy (without a premium)

To evaluate the effect of the normalization process we consider the following experiment. In 2014 the Federal Reserve had a balance sheet of 4.5 trillion, which corresponds to 45 per cent of GDP. The model counterpart is b^M equal to 0.47. According to FOMC statements, the explicit target size of the balance sheet has been set to be between 2.5 trillion and 3 trillion. In our benchmark, we study the consequences of a reduction to 3 trillion, which implies a target of $b^{M*} = 0.32$. Then, given the size target, we consider different targets for the maturity composition, Ω^{M*}, and responsiveness to deviations from compositional targets, η_1^M, and compute the resulting equilibria. To discipline the range of compositional targets, it is important to note that in 2007 the Federal Reserve had a maturity of long-term bonds that corresponded to $\Omega^{M*} = 2.125$.[28]

Given the parameters in Table 1, we find that the targets and responsiveness of the central bank to size and composition of public debt are important in determining uniqueness and local stability stability properties. More specifically, given the target size of the balance sheet, b^{M*}, then both the maturity compositional target, Ω^{M*}, and the responsiveness to deviations from the compositional target, η_1^M, matter for uniqueness of stationary equilibria and its associated local stability. We find, however, that the degree of responsiveness to target size deviations, γ_1^M, is unimportant. This might be the case as we re-adjust γ_1 so that $\frac{\gamma_1}{1-\gamma_1^M}$ remains constant throughout the various cases. We do so to ensure that the implied eigenvalues in an economy without a premium remain the same across all our simulations. This allows for meaningful comparisons across the various equilibria. Thus, from now on, we fix $\gamma^M = 0.50$. The results of normalizing the central bank's balance sheet when also implementing a standard Taylor Rule are illustrated in Figure 1.

[28]By end of 2014, the Federal Reserve held no short-term treasuries, implying a substantially larger value for Ω^{M*}.

Figure 1: Uniqueness and Stability of Steady States

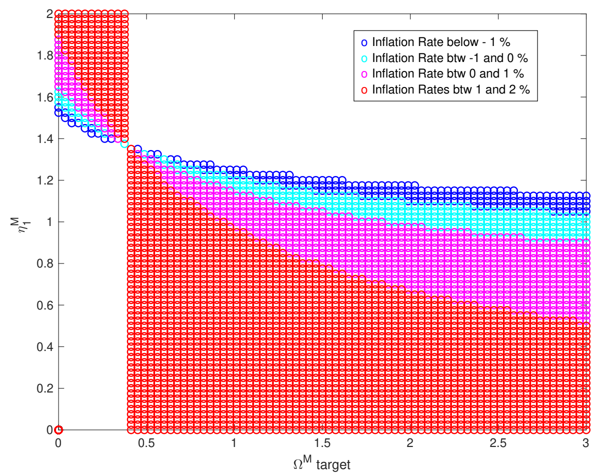


We find that long-term debt holdings by the central bank should be less than 0.5 times of their short-term positions, i.e. Ω^{M*} <0.5. Notice that this value is substantially lower than the 2007 levels, which was around 2.1. In addition, the process of balance sheet normalization should incorporate an aggressive response to deviations of the total debt in the hands of the households relative to its target. In particular, it should respond at least 20% more, i.e. η^M >1.2. We argue that for higher levels of the compositional target, Ω^{M*}, the balance sheet of the central bank increases liquidity in the market. This induces the economy to move to the other side of the liquidity Laffer curve. As a result, the steady state becomes unstable.

Next, we explore the effect of the balance sheet normalization process on long-run inflation. As we previously showed, the long-run inflation remains below its target. But how low are they? Figure 2 reports the net long-run inflation rate.

As shown in Figure 2, for a compositional target Ω^{M*} below 0.5, further reductions deliver lower long-run inflation rates. In particular, for low enough maturity targets, the net inflation rate becomes negative. Moreover, for a given composition target, Ω^{M*}, when the responsiveness to deviations from the compositional target, η_1^M, is larger, the resulting inflation rate increases. As a result, it brings the long-run inflation rate closer to the central bank's target. However, for maturity targets Ω^{M*} above 0.5, the effects are reversed. This asymmetric response is, once again, a reflection of the equilibrium moving to the other side of the liquidity Laffer curve. On that side of the curve, lowering the maturity composition target increases the steady state inflation. Moreover, for a given Ω^{M*}, the higher is the responsiveness η_1^M, long-run inflation becomes lower.

Figure 2: Steady State Inflation Rates



We perform sensitivity analysis around a number of aspects of the economic environment. We first consider a different central bank's debt target. For a lower target $b^{M*} = 0.26$, we find qualitatively similar results.[29] In this new size setting, to ensure unique and local determinate equilibria, the central bank needs a lower compositional target; i.e, Ω^{M*} <0.25. To achieve a similar level of liquidity in the hands of the public, a lower target size of the central bank's balance sheet requires a lower compositional target. We also explore the effect of having different responses to deviations from the inflation target. We find that such changes hardly affect the threshold composition target. Nevertheless, a more aggressive response to inflation leads to a wider range of parameter values for which an equilibrium with premium exists. These long-run equilibria are locally stable. In addition, we also perform sensitivity analysis with respect to changes to the degree of the liquidity of short-term bonds ζ^S and the extent of search frictions σ in the economy. Overall, we find that a lower liquidity (lower ζ^S) and higher search frictions (lower σ) have minimal changes to the threshold level of Ω^{M*} needed to enable stability.[30] However, they do lower the degree of responsiveness to deviations required for stability, and expand the area consistent with a stable stationary equilibrium. We refer the reader to the Appendix for the figures illustrating these various robustness checks.

Summarizing, when the central bank follows a traditional Taylor rule, the process of balance sheet normalization should entail a reduction of the holdings of long-term debt in order to ensure uniqueness and stability. Such reduction should be adequate but not severe if the central bank wants

[29]This corresponds to 25 % of GDP or equivalent to 2.5 trillion.
[30]This could be due to the calibration strategy.

to achieve an inflation rate close to its target.

Next we examine how changes in the economic environment affect the nature of the equilibria. In particular, we consider the role of the liquidity of long-term bonds, the debt management by the fiscal authority, and a modified Taylor rule. In the Appendix, we also study changes to the buyer's bargaining power.

Liquidity of Long-Term Bonds

Here we explore how the liquidity of long-term bonds affects the nature of equilibria. To do this, we re-do our calibration and simulation exercises and we impose that the long-term bonds have a zero liquidity premium. By doing this, we are assuming that the representative long-term bond is illiquid. Figure 3 illustrates this new scenario.

Figure 3a: Stability & Uniqueness, $s^L = 0$

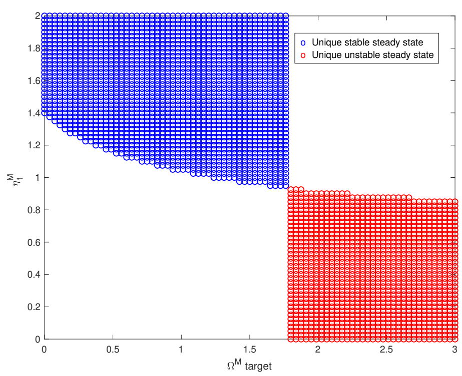
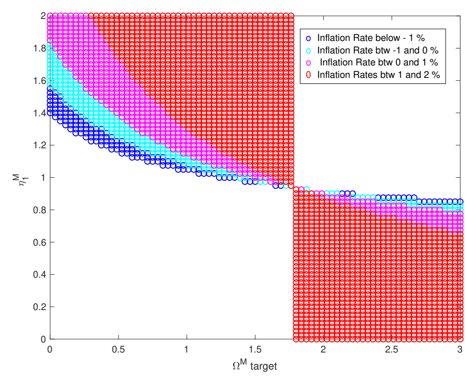


Figure 3b: Inflation Rates, $s^L = 0$



As we see from Figures 3a-3b, when long-term bonds are less liquid, the qualitative nature of the equilibrium remains similar to the benchmark model. However, it is crucial for the value of the threshold maturity target that delivers desirable equilibria. The less liquid long-term bonds are the higher is this threshold and the larger is the parameter space consistent with unique and stable equilibria. Now the threshold maturity is around 1.80. The central bank also requires a lower level of aggressiveness in response to deviations from total debt to its target.

Debt Management by the Fiscal Authority

So far when analyzing balance sheet normalization policies, we have not discussed how policies regarding debt issuance by the fiscal authority change the nature of the equilibria. Taking into account current increased fiscal pressures, we explore the consequences of considering an alternative larger target for total debt. In particular we consider an increase to 75% of GDP (which corresponds to $b^* = 0.79$) relative to the benchmark of 72% (which corresponds to $b^* = 0.7576$). We also analyze a situation where the issuance of long-term bonds by the fiscal authority decreases to $\Omega^* = 2.00$. Figure 4 illustrates our findings for an economy where the central bank follows a traditional Taylor rule.

Figure 4a: Stability & Uniqueness, $b^* = 0.78$

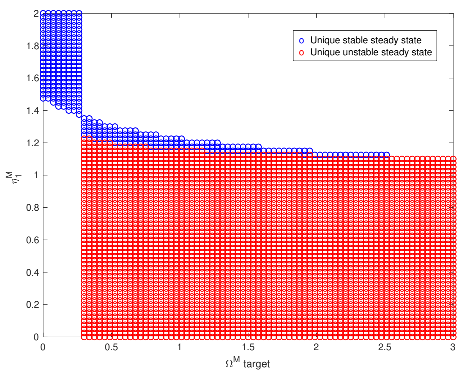


Figure 4b: Inflation Rates, $b^* = 0.78$

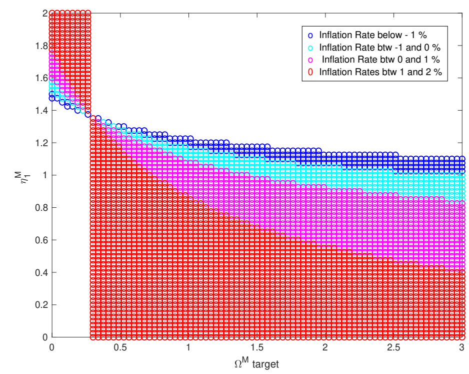


Figure 4c: Stability & Uniqueness, $\Omega^* = 2.00$

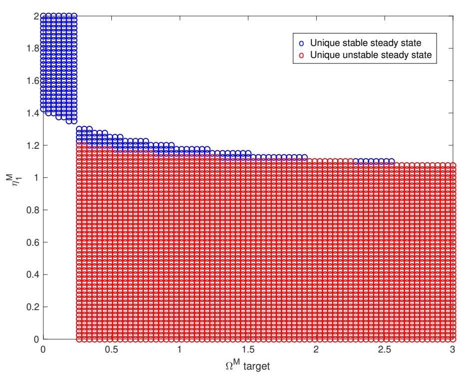
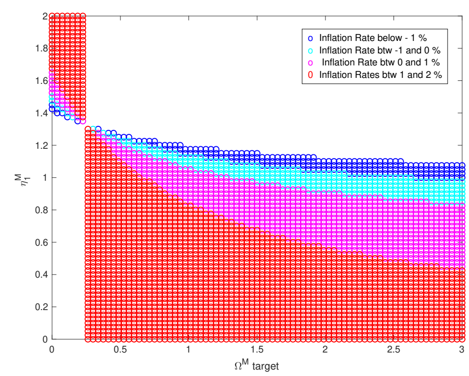


Figure 4d: Inflation Rates, $\Omega^* = 2.00$



As we can see from Figures 4a-4d, the central bank's response to the new fiscal realities would require a lower maturity threshold target and a similar aggressiveness in the response. We again interpret such findings as suggesting that what is relevant is the liquidity held by the public. Moreover, these results also highlight the importance of having even further coordination between monetary and fiscal authorities to achieve desirable equilibria, when the normalization process begins.

Modified Taylor Rule

We now examine alternative interest rate management policies. In particular, we study a monetary authority that sets interest rates so that the premium on the short-term debt is explicitly considered. In particular, we examine

$$R_t = \alpha_0 + \alpha_1 \left(\Pi_t - \Pi^*\right) - \alpha_2 s_t^S, \tag{21}$$

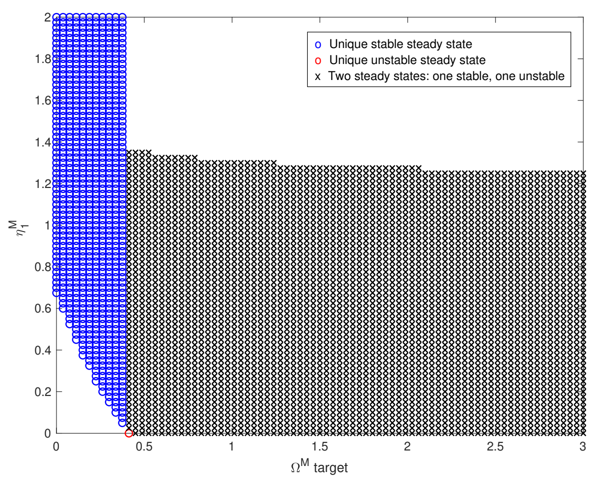
where α_2 measures the central bank's response to observed short-term debt premia s_t^S. Note that when $\alpha_2 = 0$, we recover a *traditional* Taylor rule that does not take into account premia on public debt. Moreover, when $\alpha_2 = 1$, we capture Cúrdia and Woodford's (2010) modified Taylor rule that explicitly takes into account premia.[31]

It is easy to show that the long-run inflation rate under this new policy rule is given by

$$\Pi = \Pi^* + \frac{\beta}{(1-\beta\alpha_1)} \left((1-\alpha_2)s^S\right).$$

When the spread adjustments to interest rate policy settings are equal to the size of the increase in spreads, $\alpha_2 = 1$, the steady state inflation is unique and equal to the target level $\Pi = \Pi^* = \beta\alpha_0$. However, is the steady state government debt unique? Are those steady states locally stable? These answers are illustrated in Figure 5.

Figure 5: Uniqueness and Stability of Steady States



As we can see from Figure 5, as long as the central bank's maturity composition target is low enough, $\Omega^{M*} < 0.5$, the resulting stationary equilibria is unique and locally stable. Above this compositional threshold, we find many normalization policy configurations that deliver multiple steady states. Moreover, we find that the response to deviations from total debt, η_1^M, does not significantly affect the number nor the stability properties of stationary equilibria. However, in the new environment the maturity composition target, Ω^{M*}, is now even more important. This is the case as it can deliver locally stable equilibria and rule out real indeterminacies. In particular, not having

[31] According to McCulley and Toloui (2008) and Taylor (2008), spread adjustments to interest rate policy settings should be equal to the size of the increase in spreads. This situation corresponds to $\alpha_2 = 1$. In a sticky price model, Cúrdia and Woodford (2010) analyze intermediate responses where $\alpha_2 \in (0, 1)$.

a large proportion of long-term debt allows the central bank to eliminate self-fulfilling prophecies consistent with an economy with two equilibria on both sides of the liquidity Laffer curve. It is worth highlighting that these two stationary equilibria have different locally determinacy properties.

As is the case when the central bank follows a traditional Taylor rule, a lower size target value of bond holdings by the central bank, b^{M*}, yields similar qualitative results. Such size reduction implies a lower compositional target that delivers unique and locally determinate equilibria. We refer the reader to the Appendix for more details on these various robustness checks.

In terms of achieving the target inflation, Π^*, the *modified* Taylor rule that takes into account short-term premium ($\alpha_2 = 1$) outperforms the *traditional* Taylor rule ($\alpha_2 = 0$). Is that also true in terms of welfare? We find that this is not the case. Figure 6 reports the DM consumption-equivalent welfare associated with every steady state. We then compare it with the welfare of an economy where the efficient level of DM consumption is attained. For multiple steady states, Figure 6b (Figure 6c) show the welfare costs of the better (worse) steady state.

Figure 6a: Welfare Costs, $\alpha_2 = 0$

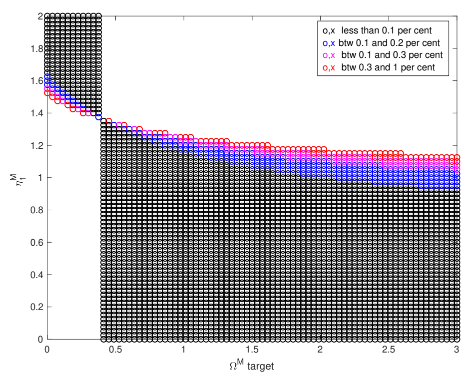


Figure 6b: Welfare Costs, $\alpha_2 = 1$

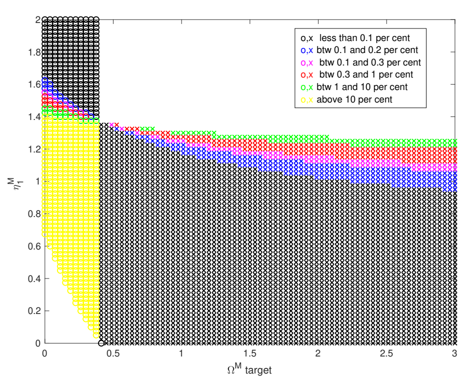
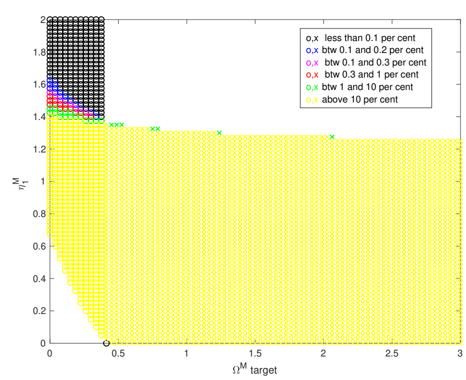


Figure 6c: Welfare Costs, $\alpha_2 = 1$



As we can see from Figures 6a-6c, the modified Taylor rule leads to a larger parameter space consistent with the existence of stationary equilibria. However, the prevalence for multiple equilibria is also much higher. Overall, we find that unique and stable equilibria tend to deliver lower welfare costs when the responsiveness of the maturity composition to current conditions, η_1^M is higher. When multiple steady states exist, one of the steady states has agents facing much lower welfare. Moreover, our numerical findings also suggest that the welfare associated with a *traditional* Taylor rule always outperforms that of the *modified* Taylor rule.[32]

Summarizing, while achieving better inflation targets, the *modified* Taylor leads to many more situations where real indeterminacies are possible. Such multiplicity of steady states allows for increased volatility as one can always construct sunspot equilibria between the various steady states.[33] Moreover, the *modified* Taylor rule delivers lower welfare. In light of these findings, we conclude that following a traditional Taylor rule is a more desirable policy for central bank when managing interest rates.

6 Conclusions

This paper has explored the effects of reducing the overall size of the central bank's balance sheet and changing its composition towards short-term public debt. When public debt does not exhibit premia, the stationary equilibria is unique and the prescriptions for determinacy of equilibria are similar to those found in Leeper (1991), Woodford (1994), Sims (1994), among others. Changes to the composition of the balance sheet (either in terms of desired maturity structure target or the specific response to deviations from its compositional target) does not impact inflation nor debt dynamics. However, once the economy exhibits bond liquidity premia, we find that changes in the central bank's balance sheet have important implications for long-run inflation, real allocations, government debt as well as for uniqueness and local stability of stationary equilibria. In order to ensure a unique and stable steady state, the central bank should target a low enough maturity composition in its portfolio. In our numerical exercise, calibrated to the United States, we find that long-term debt holdings by the central bank should be less than 0.5 times of their short-term positions. Moreover, the process of balance sheet normalization should aggressively respond to the total debt issued in the economy relative to its target. While hitting the inflation target, the modified Taylor rule, that explicitly takes

[32]In the Appendix, we perform sensitivity analysis with respect to the same parameters as with the traditional Taylor rule. We obtain similar results.

[33]See Azariadis (1981), among others, for detailed discussion on sunspot equilibria.

into account spreads, makes the existence of multiple equilibria more likely and delivers lower welfare. Given these findings, the traditional Taylor rules is the preferred policy for managing interest rates when agents in the economy face a liquidity premia. Finally, our findings also show that an adequate balance sheet normalization process critically depends on the size and maturity composition targets of the fiscal authority. These findings highlight that further coordination between fiscal and monetary authorities is needed when monetary normalization begins.

Our analysis and findings are timely. Since the outbreak of the covid-19 pandemic, quantitative easing policies have further expanded the balance sheets of central banks. As the economic effects of the pandemic are becoming more under control, central bankers around the world are focusing on the normalization of monetary policy not only in terms of interest rates but also in terms of balance sheets.

References

[1] Andolfatto, D. and M. Martín (2018). "Monetary Policy And Liquid Government Debt," *Journal of Economic Dynamics and Control*, 89, 183-199.

[2] Antinolfi, G., Carapella, F., Kahn, C., Martin, A., Mills, D. and E. Nosal (2015). "Repos, Fire Sales, and Bankruptcy Policy,"*Review of Economic Dynamics*, 18, 21-31.

[3] Andres, J., López-Salido, J.D. and E. Nelson (2004). "Tobin's Imperfect Asset Substitution in Optimizing General Equilibrium," *Journal of Money, Credit and Banking*, 36(4), 665-90.

[4] Arce, O., Nuño, G., Thaler, D. and C. Thomas (2020). "A Large central bank Balance Sheet? Floor vs Corridor Systems in a New Keynesian Environment," *Journal of Monetary Economics*, 114, 350-367.

[5] Armenter, R. and B. Lester (2017). "Excess Reserves and Monetary Policy Implementation," *Review of Economic Dynamics*, 23, 212-235.

[6] Berentsen, A. and C. Waller (2011). "Outside Versus Inside Bonds: A Modigliani-Miller Type Result for Liquidity Constrained Economies," *Journal of Economic Theory*, 146, 1852-87.

[7] Berentsen, A. and C. Waller (2018). "Liquidity Premiums on Government Debt and the Fiscal Theory of the Price Level," *Journal of Economic Dynamics and Control*, 89, 173-182.

[8] Berentsen, A., Kraenzlin, S. and B. Muller (2018). "Exit strategies for monetary policy," *Journal of Monetary Economics*, 99(C), 20-40.

[9] Bernanke, B., Gertler, M. and S. Gilchrist (1999). "The Financial Accelerator in a Quantitative Business Cycle Framework," in John Taylor and Michael Woodford, eds., Handbook of macroeconomics, Vol. 1C. Amsterdam: Elsevier Science, North-Holland, 1341-93.

[10] Bullard, J. (2017). "A Case for Shrinking the Fed's Balance Sheet," *The Regional Economist*, Federal Reserve Bank of St. Louis, vol. 25(2).

[11] Calligaris, S., Criscuolo, C. and L. Marcolin (2018). "Mark-ups in the Digital Era", OECD Science, Technology and Industry Working Papers, No. 2018/10, OECD Publishing, Paris.

[12] Canzeroni, M., Cumby, R. and B. Diba (2016). "Optimal Money and Debt Management: Liquidity Provision v. Tax Smoothing," *Journal of Monetary Economics*, 83, 39-53.

[13] Chen, H., Cúrdia, V. and A. Ferrero (2012). "The Macroeconomic Effects of Large-Scale Asset Purchase Programmes," *Economic Journal*, 122(564), 289-315.

[14] Cúrdia, V. and M. Woodford (2010). "Credit Spreads and Monetary Policy," *Journal of Money, Credit and Banking,* 42(s1), 3-35.

[15] Dell'Ariccia, G., Rabanal, P. and Sandri, D. (2018). "Unconventional Monetary Policies in the Euro Area, Japan, and the United Kingdom," *Journal of Economic Perspectives*, 32(4), 147-72.

[16] Del Negro, and C. Sims (2015), "When Does a Central Bank's Balance Sheet Require Fiscal Support?," *Journal of Monetary Economics*, 73, 1-19.

[17] Domínguez, B. and P. Gomis-Porqueras (2019). "The Effects of Secondary Markets for Government Bonds on Inflation Dynamics," *Review of Economic Dynamics*, 32, 249-273.

[18] Ghosh, A., Kim, J., Mendoza, E., Ostry, J. and M. Qureshi. (2013) "Fiscal Fatigue, Fiscal Space and Debt Sustainability in Advanced Economies," *Economic Journal*, 123, 4-30.

[19] Forbes, K. (2021). "Unwinding Monetary Stimulus in an Uneven Economy: Time for a New Playbook?," *mimeo.*

[20] Gottardi, P., Maurin, V. and C. Monnet (2019). "A Theory of Repurchase Agreements, Collateral re-use, and Repo Intermediation," *Review of Economic Dynamics*, 33, 30-56.

[21] Harrison, R. (2011). "Asset Purchase Policies and Portfolio Balance Effects: A DSGE Analysis". In Chadha, J. and Holly, S., editors, Interest Rates, Prices and Liquidity: Lessons from the Financial Crisis, chapter 5.

[22] Herrenbrueck, L. and T. Geromichalos (2016). "Monetary Policy, Asset Prices, and Liquidity in Over-the-Counter Markets," *Journal of Money, Credit, and Banking*, 48, 35-79.

[23] Iacoviello, M. (2005). "House Prices, Borrowing Constraints, and Monetary Policy in the Business Cycle," *American Economic Review*, 95, 739-64.

[24] Kiyotaki, N. and J. Moore (1997) "Credit Cycles," *Journal of Political Economy*, 105(2), 211-248.

[25] McCulley, P. and R. Toloui (2008) "Chasing the Neutral Rate Down: Financial Conditions, Monetary Policy, and the Taylor Rule," *Global Central Bank Focus*, PIMCO, February 20.

[26] Lagos, R. and R. Wright (2005). "A Unified Framework for Monetary Theory and Policy Analysis," *Journal of Political Economy*, 113, 463-484.

[27] Leeper, E. (1991). "Equilibria Under 'Active' and 'Passive' Monetary and Fiscal Policies," *Journal of Monetary Economics*, 27, 129-147.

[28] Lee, J., Lee, S., Geromichalos, T. and K. Oikawa (2016) "Over-the-Counter Trade and the Value of Assets as Collateral," *Economic Theory*, 62, 443-475.

[29] Lucas, R. E. (1984). "Money in a Theory of Finance," in Carnegie-Rochester Conference Series on Public Policy, North-Holland, 21, 9-46.

[30] Neely, C. J. (2015). "Unconventional monetary policy had large international effects," *Journal of Banking & Finance* 52, 101-111.

[31] Reis, R. (2017). "QE in the Future: The Central Bank's Balance Sheet in a Fiscal Crisis," *IMF Economic Review*, 65(1), 71-112.

[32] Ricketts, R. and C. Waller (2014). "The Fed's Bloated Balance Sheet: How We Got Here and Why it's Familiar," *The Regional Economist*, Federal Reserve Bank of St. Louis, January.

[33] Rocheteau, G. and R. Wright (2005). "Money in Search Equilibrium, in Competitive Equilibrium, and in Competitive Search Equilibrium," *Econometrica*, 73(1), 175-202.

[34] Sims, C. A. (1994). "A Simple Model for Study of the Determination of the Price Level and the Interaction of Monetary and Fiscal Policy," *Economic Theory*, 4(3), 381-399.

[35] Swanson, E. T. (2021). "Measuring the Effects of Federal Reserve Forward Guidance and Asset Purchases on Financial Markets," *Journal of Monetary Economics*, 118(C), 32-53.

[36] Taylor, J. B. (2008). "Monetary Policy and the State of the Economy." Testimony before the Committee on Financial Services, U.S. House of Representatives, February 26.

[37] Taylor, J. B. (2019). *Reform of the International Monetary System Why and How?*, MIT-Press.

[38] Venkateswaran, V. and R. Wright. (2014) "Pledgeability and Liquidity: A New Monetarist Model of Financial and Macroeconomic Activity," *NBER Macroeconomics Annual*, 28(1), 227-270.

[39] Wallace, N. (1981). "A Modigliani-Miller Theorem for Open-Market Operations," *American Economic Review*, 71(3), 267-74.

[40] Williamson, S. (2016). "Scarce Collateral, the Term Premium, and Quantitative easing," *Journal of Economic Theory*, 164(C), 136-165.

[41] Williamson, S. (2019). "Interest on Reserves, Interbank Lending, and Monetary Policy," *Journal of Monetary Economics*, 101(C), 14-30.

[42] Woodford, M. (1998). "Doing Without Money: Controlling Inflation in a Post-Monetary World," *Review of Economic Dynamics*, 1(1), 173-219.

[43] Woodford, M. (1994). "Monetary Policy and Price Level Determinacy in a Cash-in-advance Economy," *Economic Theory*, 4(3), 345-380.

[44] Woodford, M. (2001). "Fiscal Requirement for Price Stability, " *Journal of Credit, Money and Banking*, 33, 669-728.

Additional Appendix

Sensitivity Analysis for Traditional Taylor Rule

Figure 7a: Stability & Uniqueness, $b^{M*} = 0.26$

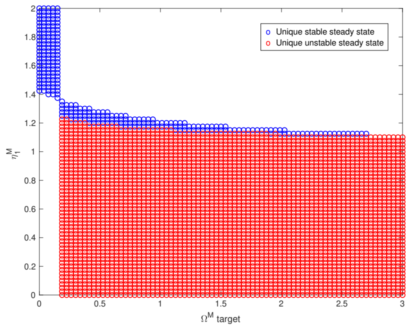


Figure 7b: Inflation Rates, $b^{M*} = 0.26$

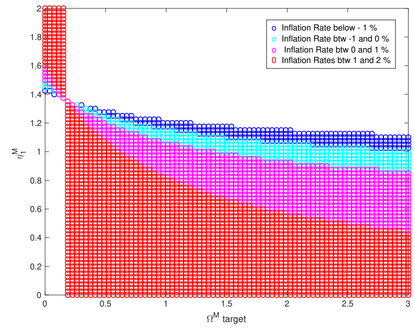


Figure 7c: Stability & Uniqueness, $\alpha_1 = 2.00$

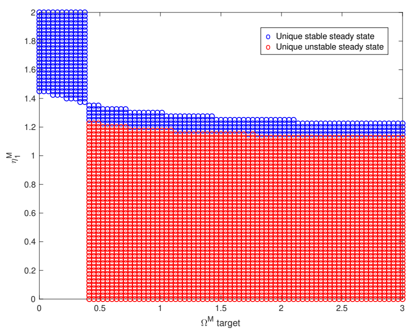


Figure 7d: Inflation Rates, $\alpha_1 = 2.00$

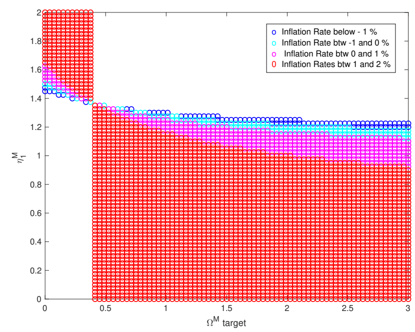


Figure 7e: Stability & Uniqueness, $\zeta^S = 0.50$

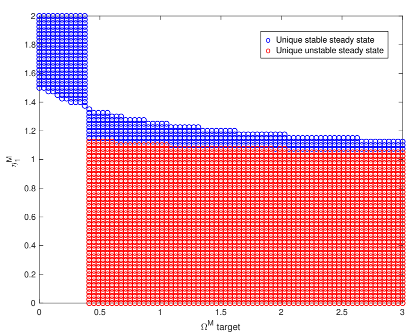


Figure 7f: Inflation Rates, $\zeta^S = 0.50$

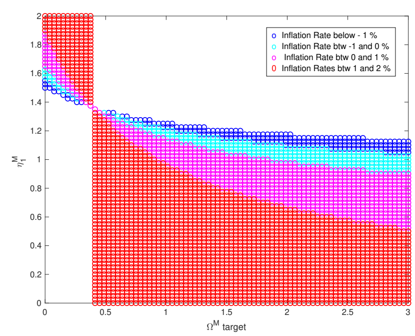


Figure 7g: Stability & Uniqueness, $\sigma = 0.50$

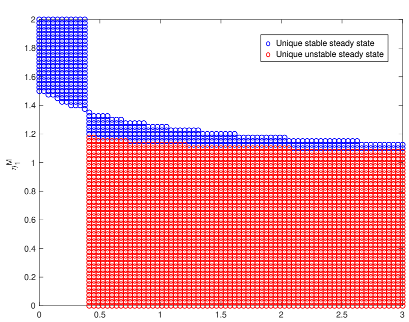
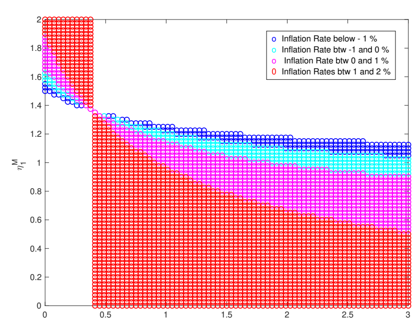


Figure 7h: Inflation Rates, $\sigma = 0.50$



Sensitivity Analysis for Modified Taylor Rule

Figure 8: Sensitivity Analysis for Modified Taylor, $\alpha_2 = 1$

Figure 8a: Stability & Uniqueness, $b^{M*} = 0.26$

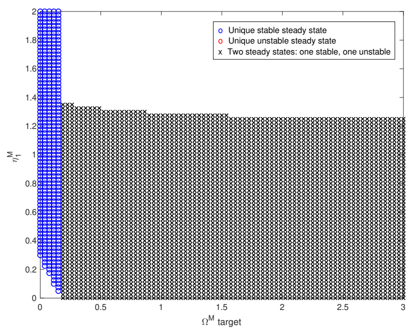


Figure 8b: Stability & Uniqueness, $\alpha_1 = 2.00$

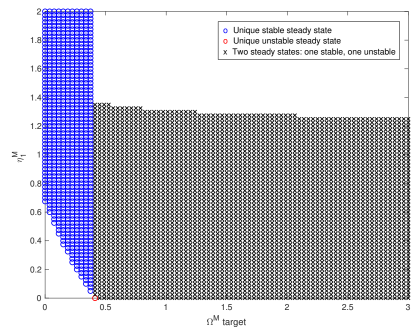


Figure 8c: Stability & Uniqueness, $\zeta^S = 0.50$

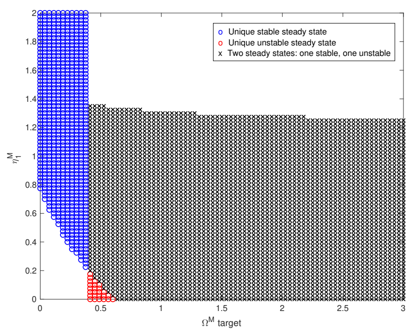
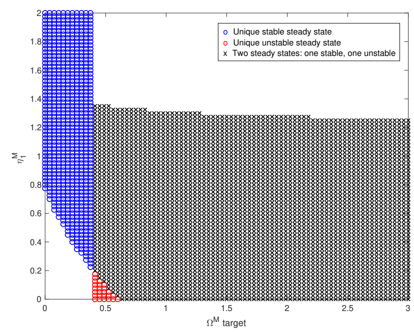


Figure 8d: Stability & Uniqueness, $\sigma = 0.50$



Seller's Bargaining Power

When short and long-term bonds are accepted as collateral and the terms of trade are given by proportional bargaining with buyer's bargaining power equal to ω, we have that

$$\max_{q_t, \tilde{L}^{S,L}_{t-1}} \left\{ \chi \frac{q_t^{1-\xi}}{1-\xi} - \phi_t \tilde{L}^{S,L}_{t-1} \right\} \text{ s.t.}$$

$$\tilde{L}^{S,L}_{t-1} \leq \zeta^S \tilde{B}^S_{b,t-1} + \zeta^L Q_{t-1} \tilde{B}^L_{b,t-1},$$

$$\chi \frac{q_t^{1-\xi}}{1-\xi} - \phi_t \tilde{L}^{S,L}_{t-1} = \omega \left(\chi \frac{q_t^{1-\xi}}{1-\xi} - q_t \right),$$

which delivers the following first-order condition

$$\omega \left\{ \chi q_t^{-\xi} - 1 \right\} - \lambda_t^{S,L} \left[(1-\omega) \chi q_t^{-\xi} + \omega \right] = 0.$$

The endogenous values of the liquidity premium on short and long-term nominal public debt are given by

$$s^S_{t+1} = \left(\frac{\sigma}{2} \frac{\omega \left(\frac{\chi}{q_{t+1}{}^{\xi}} - 1 \right)}{\omega + (1-\omega) \frac{\chi}{q_{t+1}{}^{\xi}}} \right) \zeta^S, \quad \& \quad s^L_{t+1} = \left(\frac{\sigma}{2} \frac{\omega \left(\frac{\chi}{q_{t+1}{}^{\xi}} - 1 \right)}{\omega + (1-\omega) \frac{\chi}{q_{t+1}{}^{\xi}}} \right) \zeta^L \, Q_{t+1}.$$

It is easy to show that, when the DM payment constraint binds, the dynamic equilibrium is characterized by the following system of dynamic equations:

$$x_t = 1, \tag{22}$$

$$\frac{(1-\omega)\chi q_{t+1}^{1-\xi}}{1-\xi} + \omega q_{t+1} = \left[\frac{b_t}{(1+\Omega)} - \frac{b_t^M}{(1+\Omega_t^M)} + \zeta^L \left(\frac{\Omega \, b_t}{(1+\Omega)} - \frac{\Omega_t^M \, b_t^M}{(1+\Omega_t^M)} \right) \right] \frac{1}{\Pi_{t+1}}, \tag{23}$$

$$\Pi_{t+1} = \beta \left(R_t + s^S_{t+1} \right), \tag{24}$$

$$\Pi_{t+1} Q_t = \beta \left(1 + \rho Q_{t+1} + s^L_{t+1} \right), \tag{25}$$

$$T_t^C + b_t^M = \frac{1}{\beta} b^M_{t-1} - \left(s_t^S + s_t^L \frac{\Omega^M_{t-1}}{Q_{t-1}} \right) \frac{b^M_{t-1}}{\Pi_t \left(1 + \Omega^M_{t-1} \right)}, \tag{26}$$

$$\tau_t^{CM} + b_t + T_t^C = G + \frac{1}{\beta} b_{t-1} - \left(s_t^S + s_t^L \frac{\Omega}{Q_{t-1}} \right) \frac{b_{t-1}}{\Pi_t (1+\Omega)}. \tag{27}$$

We now explore numerically how the bargaining power when trading in DM changes the properties of the dynamic equilibria. We set $\omega = 0.63$, which delivers a mark up of 22%. This is consistent with the median markup found by Calligaris et al. (2018). Figure 9 illustrates the equilibria corresponding to various central bank balance sheet policies when sellers have some market power.

Figure 9a: Stability & Uniqueness, $\omega = 0.63$

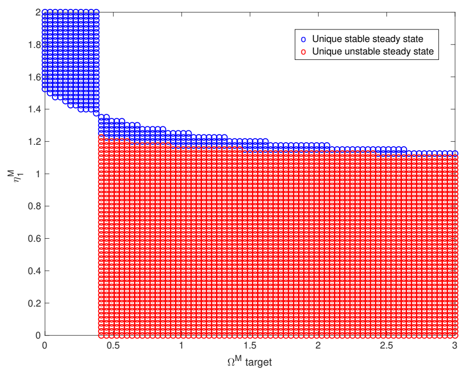
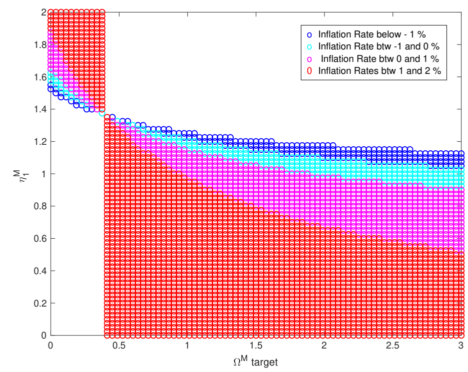


Figure 9b: Inflation Rates, $\omega = 0.63$



As we can see from Figures 9a-9b, the degree of bargaining power does not qualitatively affect the nature of the equilibria. In particular, local stability stability and inflation rates in the economy are quite similar to those found in the benchmark model.