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Keywords

Non-stationary productivity shocks, TFP, investment specific technology shocks, trend shocks, DSGE modelling, state space model.

JEL Classification

E2, E3

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1 Introduction

How important are global or international productivity trend shocks for small open economies? How relevant are total factor productivity (TFP) and investment specific technology (IST) as international and country specific sources for productivity advancements? In this paper, we addresses these question by estimating a novel state-space model for the UK economy. It allows us to uncover the dynamic effects and the empirical relevance of domestic and international productivity shocks and differentiates between IST and TFP types of these innovations. International productivity shocks are identified as shocks originating in the US economy. The joint identification of IST and TFP types of international and domestic productivity shocks, which is a strength of our empirical setup, gives important insights into the dynamic transmission of exogenous changes in productivity and and their relevance for aggregate fluctuations.

The left subplot of Figure 1 shows UK and US real consumption and country specific labor productivity which is often used as a proxy for TFP. Both US and UK consumption exhibit a positive long run trend and the figure suggests this may be driven by improvements in TFP. However, based on the figure, it remains an open question whether the improvements UK TFP are driven by country specific shocks or international forces. The right subplot in Figure 1 shows UK and US real investment as well as the corresponding inverse of the relative prices of investment (RPI). A candidate explanation for the positive long run trend in US and UK real investment, according to Greenwood et al. (1997) and Fisher (2006), is a continuous improvement in investment specific technology. The latter can be proxied for by the inverse of the relative price of investment goods, measured as the consumption deflator over the GDP deflator. Consistent with the notion that the positive trend in US and UK investment may be driven by improvements in IST, Figure 1 shows an increase in the inverse RPI of these countries. This discussion highlights the need for a better understanding of the driving forces behind long run trends in UK macroeconomic aggregates and the quantification of the role played by international and UK-specific IST and TFP shocks. This paper sheds light onto the role of international IST and TFP shocks and their domestic counterparts for UK aggregate fluctuations.

The strength of our empirical methodology is that it allows for the joint identification of international and domestic non-stationary TFP and IST shocks. In particular, we consider a state space representation that is a linear Gaussian model and estimate it, given Minnesota-type prior distributions for the parameters, using Gibbs sampling and simulation smoothing techniques. The international and domes-

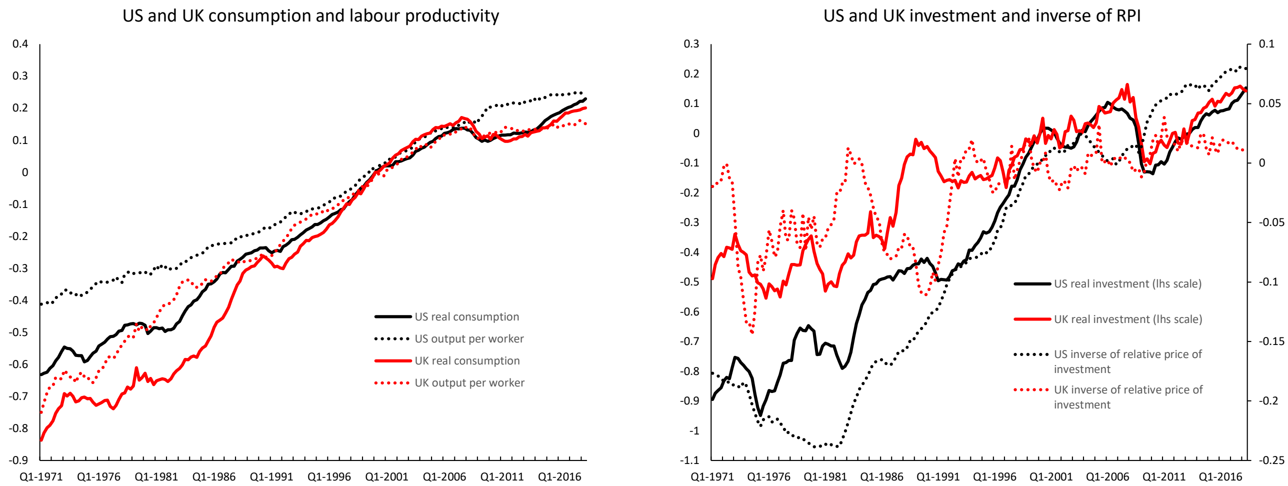

Figure 1: **UK and US macroeconomic aggregates.** All series are in natural logs and normalized to zero at 2000Q1.

tic non-stationary productivity shocks are identified during the estimation process together with cyclical shocks that can also explain variations in macroeconomic variables. The model includes four unobserved variables — US TFP growth, US IST growth, UK TFP growth and UK IST growth — that display a stochastic trend. These four I(1) trends are the long-term driving forces of the observables while the remaining shocks can account for cyclical variation in the observables. Our decomposition between trend and cyclical components in the vector of observables is agnostic as it is informed by economic theory. In particular, it is informed by the relationships between non-stationary and cyclical components implied by standard two-country dynamic stochastic general equilibrium models, as in Backus et al. (1994) and Heathcote and Perri (2002).

For a 1971Q1-2018Q2 sample, we document that both positive international TFP and IST shocks generate a strong expansion in UK and US output, consumption, investment and labor productivity. An international or global IST shock further leads to a persistent decline in the relative price of investment goods in both economies. In response to either of the two international technology shocks, the trade balance is not significantly different from zero. The exchange rate appreciates in the medium run which is consistent with a lack of risk sharing and the fact that in the medium run potential supply expands relatively more in the US than in the UK. We document that the international IST and TFP shocks explain about 16% and 9% respectively of fluctuations in UK GDP at business cycle frequencies. These two international non-stationary technology shocks compete with two non-stationary domestic technology shocks as well as other cyclical shocks in explaining variations in macroeconomic aggregates.[1] Our findings suggest that both international technology shocks are relevant components for understanding UK business cycles, however UK specific non-stationary technology and cyclical shocks account for the bulk of the volatility in the data. This holds particularly for the UK specific TFP shock, which accounts for 42% of fluctuations in GDP, while the contribution of the country specific IST shock is very small (3%).

We also estimate our state space model taking each of the remaining G7 countries — Canada, France, Italy, Germany and Japan — as the domestic economy. We find that the overall results are consistent with those for the UK economy in the sense that for all countries considered the two international productivity shocks play a relevant role in explaining aggregate fluctuations. Together the two international shocks explain between 20% and 38% of variations in GDP.

The two international technology shocks are not only important for the UK economy on average, but also if we inspect their role during the Great Recession and the following recovery. Particularly international IST shocks have contributed substantially to the contraction after 2007 while the TFP counterpart was much less important. The international IST shock has played an important supportive role also for GDP growth during the subsequent recovery, while the international TFP shock became a substantial dampening force. The domestic TFP shock also helped with the recovery, albeit its role was much smaller than the one of the international shocks. The contribution of the domestic IST shock is negligible over this episode.

We show that the empirically documented dynamic patterns of non-stationary international IST and TFP shocks can be captured by a standard two-country model dynamic stochastic general equilibrium (DSGE) model. An impulse response function (IRF) matching exercise shows consistency between the empirical and model-implied responses and demonstrates the influence of key parameters, such as those determining the responsiveness of the labor supply and investment to shocks, on the model's ability to generate empirically plausible results. In particular, the model generates, in response to both technology shocks, strong co-movement in macroeconomic aggregates and a muted response in relative quantities and prices.

The absolute and relative importance of TFP and IST shocks has been the focus of several researchers, yet most studies confine themselves to assessing the shocks' importance for US business cycles.[2] Benati (2014) finds evidence for the notion that US IST and TFP are not co-integrated and are best thought of

[1]The international technology shocks are also relevant drivers of US GDP. The IST shock explains 49% and the TFP 15% of fluctuations in US GDP at business cycle frequencies.

[2]See e.g. Fisher (2006), Schmitt-Grohe and Uribe (2011) and Justiniano et al. (2011).

as independent processes. Our econometric setup is informed by this evidence in that we jointly identify IST and TFP shocks, but do not impose any co-integrating relationships between these two technology shocks.

A growing recent literature is concerned with the open economy aspects of technology shocks. Ireland (2013) identifies non-stationary shocks to TFP and IST that are co-integrated between the US and the Euro Area in an estimated two-country dynamic stochastic general equilibrium model. This work points to important differences between these regions regarding the effects of IST and TFP shocks in the 1970s and 1990s. Our analysis also focuses on international IST and TFP shocks, yet we are agnostic in that we apply a minimum of structure for their empirical identification and do not rely on co-integration relationships. Mandelman et al. (2011) discuss the ability of IST shocks in a standard international real business cycle model to reconcile the model's predictions with the data. Importantly, they provide evidence for a common IST shock across countries in that they document that US and rest of the world IST processes are co-integrated. Guerron-Quintana (2013) estimates a small open economy DSGE model on data from a set of advanced small open economies and assesses the importance of a common international non-stationary productivity shock. The common non-stationary shock is found to be of particular importance during the Great Recession, and accounting for between 10% and 19% of the variance of output over his 1980Q1-2010Q4 sample. We instead focus on the UK economy and differentiate between international or global IST and TFP shocks while applying a minimum of structure in our estimation.

Dogan (2019) documents in the context of a two-country two-sector international real business cycle framework that permanent US IST shocks are important for Mexican business cycle dynamics. We also stress the importance of international IST shocks for aggregate fluctuations in small open economies. In her model however, non-stationary IST shocks do not compete with permanent TFP shocks in explaining aggregate fluctuations. The international transmission of country specific TFP shocks has received considerable attention, see e.g. Corsetti et al. (2008b), Benigno and Thoenissen (2008), Enders and Müller (2009) and Klein and Linnemann (2021). Our novel empirical approach allows us to add to this literature by jointly identifying TFP and IST shocks and, importantly, we quantify the transmission of international productivity shocks to small open economies.

The paper proceeds as follows. Section 2 introduces a two-country international business cycle model that is used to inform a state space model that empirically establishes the relevance of international productivity shocks for the UK economy. Section 3 provides an overview of the data and Section 4 introduces the state space model. Section 5 discusses the empirical results based on the state space model and shows these are consistent with the implications of the structural model developed in Section 2. Section 7 concludes.

2 Structural Model

We consider a two-country flexible price, international business cycle model similar in structure to Backus et al. (1994), with incomplete financial markets as in Heathcote and Perri (2002) and more recently Bodenstein et al. (2018) and variants of it are widely used in the literature.[3] In each of the two countries, firms produce a specialised tradable good that is used in the production of final consumption and investment goods. Households consume a final good which is a composite of home and foreign-produced goods. The basket of consumption and investment goods reflects a preference for domestically produced goods, i.e. there is home-bias in consumption and investment. Households in both countries are able to smooth consumption across time by trading in one-period non-state contingent bonds. Both economies are subject to the same non-stationary TFP and IST shocks, following the closed-economy framework by Justiniano et al. (2011).

The rather general theoretical framework developed in this section is subsequently used to inform a state space model developed in Section 4. In particular, we derive restrictions from the theoretical model in Section 2.6. These are subsequently employed to inform the state space model which is used to empirically establish the relevance of the two international non-stationary technology shocks.

2.1 Households

The representative household in the domestic economy derives utility from the consumption of final goods, c_t, and disutility from supplying labor, n_t according to preferences described by the expected utility function

$$E_0 \sum_{t=0}^{\infty} \beta^t \left\{ \frac{1}{1-\sigma} (c_t)^{1-\sigma} - \frac{\phi_0}{1+\phi} (\hat{n}_t)^{1+\phi} \right\} \tag{1}$$

where the discount factor is denoted by $0 < \beta < 1$. The parameters $\sigma \geq 1$ and $\phi > 0$ are coefficient of relative risk aversion and the inverse of the Frisch elasticity, respectively. $\phi_0 > 0$ determines the dis-utility of labour. Since we will later on introduce non-stationary productivity shocks that imply that certain variables exhibit a stochastic trend, it is convenient to denote stationary variables with a hat. Households

[3] See for example Chari et al. (2002), Corsetti et al. (2008a), Kamber et al. (2017)

maximize their expected utility given by equation (1) with respect the following flow budget constraint

$$c_t + p_{F,t}b_t = (1+\hat{r}_{t-1})p_{F,t}b_{t-1} + w_t\hat{n}_t + \pi_t,$$

where $p_{F,t}$ denotes the price of the foreign-produced intermediate good relative to the domestic final good, $\frac{P_{F,t}}{P_t}$, and where $w_t\hat{n}_t$ denotes the representative household's wage income and π_t is the dividend income received due to ownership of firms. Households are able to smooth consumption risk by holding non-state contingent bonds, b_t, denominated in terms of the foreign-produced intermediate good, that pay a quarterly yield of $\hat{r}_t$. [4]

2.2 Final Goods Producers

Final goods, used for consumption, c_t, and for the production of investment goods, x_t, are produced by combining home and foreign-produced intermediate goods according to a constant elasticity of substitution (CES) technology

$$c_t = \left[\eta^{\frac{1}{\theta}}\left(c_{H,t}\right)^{\frac{\theta-1}{\theta}} + (1-\eta)^{\frac{1}{\theta}}\left(c_{F,t}\right)^{\frac{\theta-1}{\theta}}\right]^{\frac{\theta}{\theta-1}} \qquad \text{and} \qquad x_t = \left[\eta^{\frac{1}{\theta}}\left(x_{H,t}\right)^{\frac{\theta-1}{\theta}} + (1-\eta)^{\frac{1}{\theta}}\left(x_{F,t}\right)^{\frac{\theta-1}{\theta}}\right]^{\frac{\theta}{\theta-1}},$$

where $c_{H,t}$ and $x_{H,t}$ are domestic consumption and investment of home-produced intermediate goods, and $c_{F,t}$ and $x_{F,t}$ denote domestic consumption and investment of foreign intermediate goods. The parameter $0 < \eta < 1$ determines the share of home and foreign produced intermediate goods. The representative agent has a home-bias and the real exchange rate can deviate from purchasing power parity if η is greater than the relative size of the home country, N. The parameter $\theta > 0$ governs the elasticity of substitution between home and foreign-produced goods.

2.3 Investment Goods Producers

Investment goods producers in the home economy purchase final goods x from final goods producers at price P_t and transform these into investment goods i_t. The latter are then sold at price $P_{I,t}$ to domestic intermediate goods producers. Hence, investment goods producers maximize profits

$$\pi_t^I = P_{I,t}i_t - P_t x_t,$$

[4] As in Schmitt-Grohe and Uribe (2003) we allow the interest rate on domestically held foreign bonds to differ from the rate applicable to foreign agents by a small debt-elastic premium. Specifically, $(1+\hat{r}_t) = (1+\hat{r}_t^*)e^{-\phi_b b_t}$, the premium decreases with the net foreign asset position of the home country. This small bond holding cost eliminates the unit root in bond holdings and closes the model.

subject to the constraint

$$i_t = v_t x_t,$$

so that final goods x_t are turned into investment goods using the time-varying investment specific technology, v_t. Profit maximization links the relative price of investment goods to investment specific technology

$$\frac{P_{I,t}}{P_t} = p_{I,t} = v_t^{-1}.$$

Investment specific technology v_t is an international non-stationary process which affects the relative price of investment goods in both, the domestic and the foreign economy.

2.4 Intermediate Goods Producing Firms

Domestic intermediate goods producing firms produce country-specific output goods, y_t, that are used in the production of final consumption and investment goods. These firms maximize cash-flow

$$\pi_t = p_{H,t} y_t - w_t \hat{n}_t - p_{I,t} i_t,$$

where $p_{H,t}$ denotes the price of the home-produced intermediate good relative to the domestic final good, $\frac{P_{H,t}}{P_t}$, subject to the representative firm's production function and the capital accumulation constraint. The firm produces output goods according to a standard Cobb-Douglas production function

$$y_t = k_{t-1}^{\alpha} (z_t \hat{n}_t)^{1-\alpha}, \qquad 0 < \alpha < 1,$$

where k_{t-1} denotes physical capital services, $\hat{n}_t$ is hours worked and z_t is a non-stationary international productivity process that affects labor productivity in both, the home and foreign economy. The parameter α determines the share of capital in production. The capital accumulation constraint is given by

$$k_t = (1-\delta) k_{t-1} + \left(1 - S\left(\frac{i_t}{i_{t-1}}\right)\right) i_t. \tag{2}$$

where the function $S(\cdot)$ captures investment adjustment costs with $S(1) = S'(1) = 0$ and $S'' > 0$ as in Christiano et al. (2005).[5]

[5] A prime denotes differentiation.

2.5 The Foreign Economy and Market Clearing

Sections 2.1 to 2.4 outline the structure of the domestic economy. An analogous set of equations applies to the foreign economy, so that the two economies are symmetric and both subject to the same international TFP and IST shocks.

The model is closed by the following market clearing conditions for home and foreign-produced intermediate goods

$$Ny_t = N[c_{H,t} + x_{H,t}] + (1-N)[c^*_{H,t} + x^*_{H,t}]$$

and

$$(1-N)y^*_t = N[c_{F,t} + x_{F,t}] + (1-N)[c^*_{F,t} + x^*_{F,t}],$$

where N denotes the relative size of the domestic economy and expressions with a star refer to variables corresponding to the foreign economy. Total production of home and foreign goods must equal total home and foreign uses of the goods. The bond market clearing condition is the current account which is derived by accounting for firms' profit in their domestic households' budget constraint. Clearing of bond and goods markets then implies

$$c_t + x_t + p_{F,t}b_t = (1 + \hat{r}_{t-1})p_{F,t}b_{t-1} + p_{H,t}y_t.$$

2.6 Disentangling Trend and Cyclical Components

The model dynamics are driven by the two international non-stationary technology processes on TFP, z_t, and IST, v_t, which hence affect both the home and domestic economy. Mandelman et al. (2011) provide evidence that IST shocks are co-integrated between the US and rest of the World and Benati (2014) shows that IST and TFP processes are not co-integrated within the US. Hence, we model the driving forces that affect both countries of the model as two international non-stationary processes. Both the home and foreign economy are hit by the same non-stationary shock to TFP and IST.

We assume the underlying processes to be first-difference stationary so that $\hat{\Gamma}^z_t = \frac{z_t}{z_{t-1}}$ and $\hat{\Gamma}^v_t = \frac{v_t}{v_{t-1}}$ are given by

$$\hat{\Gamma}^z_t = \rho_z \hat{\Gamma}^z_{t-1} + \hat{\epsilon}_{z,t} \qquad \text{and} \qquad \hat{\Gamma}^v_t = \rho_v \hat{\Gamma}^v_{t-1} + \hat{\epsilon}_{v,t},$$

where $\hat{\epsilon}_{z,t}$ and $\hat{\epsilon}_{v,t}$ are i.i.d. with mean zero and constant standard deviations.

The non-stationary technology processes z_t and v_t govern the trends of model variables. For each variable we can determine the particular combination of these two technology processes, such that dividing all variables by their respective trend, yields a stationary model. In this sense, our theoretical model can be informative about the behavior of international stochastic trends across model variables, which in turn motivates key assumptions made for the design of the empirical model in Section 4 on the relationship between observables and technology processes. In particular, we can derive a formulation

$$y_t = \hat{y}_t z_t v_t^{\frac{\alpha}{1-\alpha}} \tag{3}$$

which disentangles non-stationary output, y_t, into a stationary component, $\hat{y}_t$, and the underlying trend component driven by TFP and IST. Recall that variables with a 'hat' are stationary and as such, we can interpret $\hat{y}_t$ as the cyclical component of output. Output, consumption and labor productivity share a common trend, so that the non-stationary and the cyclical components of consumption and labor productivity can be disentangled analogous to equation (3). For investment the relationship between trend and cyclical components is given by

$$i_t = \hat{i}_t z_t v_t^{\frac{1}{1-\alpha}} \tag{4}$$

and for the relative price of investment by

$$p_{I,t} = \hat{p}_{I,t} z_t v_t^{\frac{1}{1-\alpha}}. \tag{5}$$

Given the symmetry of our model, the relationships shown in equations (3) to (5) for output, consumption, labor productivity, investment and the real price of investment, hold for both, foreign and domestic variables. These relationships will be used in the in the following sections to empirically identify structural trends by informing long-run restrictions in the state space model.[6]

[6]It is noteworthy that the derived relationships between non-stationary variables, the corresponding cyclical component and the trend are unaffected by a number of widely used extensions to this baseline framework. This holds for example for adding price rigidities (see e.g. Benigno and Thoenissen (2003)), search and matching friction (see e.g. Bodenstein et al. (2018)), or variable capacity utilisation (see e.g. Görtz and Tsoukalas (2013)).

3 Data

The main focus in our study is on the UK and US economies. We consider quarterly data over the horizon 1971Q1-2018Q2. Output, y_t is real GDP per capita, consumption, c_t is private final consumption expenditures and investment, i_t, is gross fixed capital formation. These variables are in logs and real per capita units. Labor productivity, lp_t, is defined as log real per capita GDP minus log hours worked per capita. The real price of investment, rpi_t, is defined as the deflator for gross fixed capital formation over the GDP deflator. The share of real net UK exports, nx_t, is defined as the difference of real exports and real imports divided by real GDP. All data mentioned previously is obtained from the OECD Quarterly National Accounts. The effective real exchange rate for the UK, ex_t, is obtained from the Bank for International Settlements and puts the national currency in relation to a broad basket of foreign currencies.

4 The Empirical Model

In this section, we introduce a state space model that is subsequently estimated and we discuss the identification of shocks to the trend of productivity. In particular, we outline how we separately identify the contribution of trend shocks IST and TFP and how these can be disentangled from cyclical components in explaining the variations in macroeconomic aggregates.

The identification of several non-stationary shocks is a complicated task that determines – to a large extend – the format of the empirical model. The latter is a state-space model of the following form

$$\zeta_t = C + A\xi_t, \tag{6}$$

$$\xi_t = B\xi_{t-1} + \omega_t, \tag{7}$$

where ζ_t denotes the vector of the observable variables. We employ as observables: US real GDP growth, US real consumption growth, US real investment growth, US real labor productivity growth, US relative prices of investment growth, UK real GDP growth, UK real consumption growth, UK real investment, UK real labor productivity growth, UK relative prices of investment growth, UK net-trade as % of GDP and the UK real exchange rate. The variable ξ_t summarizes the vector of the state variables which includes: US TFP growth, US IST growth, US cyclical GDP, US cyclical consumption, US cyclical investment, US cyclical labor productivity, US cyclical relative prices of investment, UK TFP growth, UK IST growth, UK cyclical GDP, UK cyclical consumption, UK cyclical investment, UK cyclical labor productivity, UK cyclical relative prices of investment, UK net-trade as % of GDP and the UK real exchange rate. The vector C captures a constant in the relationship between observable and state variables and the vector of errors, ω_t, is normally distributed with zero mean and constant Σ covariance matrix ($\omega_t \sim N(0, \Sigma)$).

The matrix A maps the state vector into the set of the observable variables. In our exercise, the matrix A also contributes to the identification of the structural trends by preserving the long-run restrictions discussed in Section 2.6. Consistent with those, we impose the restrictions

$$
\begin{aligned}
\Delta \ln y_t^* &= \Delta \ln y^* + \Delta \ln \hat{y}_t^* + \Delta \ln z_t^* + \frac{\alpha}{1-\alpha} \Delta \ln v_t^* \\
\Delta \ln c_t^* &= \Delta \ln c^* + \Delta \ln \hat{c}_t^* + \Delta \ln z_t^* + \frac{\alpha}{1-\alpha} \Delta \ln v_t^* \\
\Delta \ln i_t^* &= \Delta \ln i^* + \Delta \ln \hat{i}_t^* + \Delta \ln z_t^* + \frac{1}{1-\alpha} \Delta \ln v_t^* \\
\Delta \ln lp_t^* &= \Delta \ln lp^* + \Delta \hat{lp}_t^* + \Delta \ln z_t^* + \frac{\alpha}{1-\alpha} \Delta \ln v_t^* \\
\Delta \ln rpi_t^* &= \Delta \ln rpi^* + \Delta \ln \hat{rpi}_t^* - \Delta \ln v_t^* \\
\Delta \ln y_t &= \Delta \ln y + \Delta \ln \hat{y}_t + \underbrace{\Delta \ln z_t^* + \frac{\alpha}{1-\alpha} \Delta \ln v_t^*}_{\text{Foreign Trends}} + \underbrace{\Delta \ln z_t + \frac{\alpha}{1-\alpha} \Delta \ln v_t}_{\text{Domestic Trends}} \\
\Delta \ln c_t &= \Delta \ln c + \Delta \ln \hat{c}_t + \Delta \ln z_t^* + \frac{\alpha}{1-\alpha} \Delta \ln v_t^* + \Delta \ln z_t + \frac{\alpha}{1-\alpha} \Delta \ln v_t \\
\Delta \ln i_t &= \Delta \ln i + \Delta \ln \hat{i}_t + \Delta \ln z_t^* + \frac{1}{1-\alpha} \Delta \ln v_t^* + \Delta \ln z_t + \frac{1}{1-\alpha} \Delta \ln v_t \\
\Delta \ln lp_t &= \Delta \ln lp + \Delta \ln \hat{lp}_t + \Delta \ln z_t^* + \frac{\alpha}{1-\alpha} \Delta \ln v_t^* + \Delta \ln z_t + \frac{\alpha}{1-\alpha} \Delta \ln v_t \\
\Delta \ln rpi_t &= \Delta \ln rpi + \Delta \ln \hat{rpi}_t - \Delta \ln v_t^* - \Delta \ln v_t \\
\ln nx_t &= \ln nx + \ln \hat{nx}_t \\
\ln ex_t &= \ln ex + \ln \hat{ex}_t
\end{aligned} \tag{8}
$$

where, in our context, variables with (without) a star relate to the US (UK). A variable without a hat indicates a non-stationary observable, while the corresponding variable with the hat denotes the stationary cyclical component. Δ indicates the first-difference of a variable so that $\Delta x_t = x_t - x_{t-1}$ and variables without a time subscript stand for historical averages. The paper focusses on how the UK, as a small open economy, is affected by international and domestic technology shocks. We use the US to capture international technology shocks. This is also apparent from the restrictions (8), since the international shocks, z_t^* and v_t^*, affect both, the UK and US observables, while the domestic shocks, z_t and v_t, only affect the UK observables. In comparison to the structural model introduced in Section 2, we include the domestic trend shocks for the UK economy in the empirical model. In this sense, the restrictions in equation (8) are agnostic in that they give the empirical model a way out to explain movements through an additional channel.[7] Matrix B governs the dynamics of the state vector and is also subject to small open economy block-recursive restrictions ensuring that domestic disturbances have no (lag) effects on the foreign economy.

The empirical model employed in this study shares many features with those proposed by Crump et al. (2016), Negro et al. (2017), Del Negro et al. (2019) and Johannsen and Mertens (2021). Similar to the latter studies, our procedure disentangles the trend and cyclical component of the observed series during the estimation. In our case, the trend cycle decomposition, as well as, the relationships between the observable vector and the set of international stochastic trends are pinned down by the economic theory that forms the core of the modern DSGE literature (see Fisher (2006), Justiniano et al. (2011), Schmitt-Grohe and Uribe (2011) and Ireland (2013) among others). Estimating the state space model hence allows us to establish the relevance of non-stationary UK and international TFP shocks, z_t and z_t^*, non-stationary UK and international IST shocks, v_t and v_t^*, and stationary cyclical shocks, ω_t, as drivers for variations in the set of UK observables.

4.1 Estimation of the State State Model

The estimation algorithm is reviewed here only briefly, with the discussion regarding all the important details and necessary steps to take place in Section B.1 of the Appendix. Our decision to use economic theory to determine the long-run relationships among the set of the observed variables allows us to "map" our empirical results to the structural model discussed earlier and identify the theoretical transmission mechanism that explain any stylized facts emerging from the empirical exercises. In addition, the reliance on the theoretically implied long-run relationships also simplifies the estimation of the state space model. Given the value for the capital share, α, the state-space model summarized by the equations (6) and (7) is a linear Gaussian model that we estimate using Gibbs sampling and simulation smoothing techniques (Carter and Kohn (1994) and Durbin and Koopman (2002)).[8] In other words, the posterior distribution is approximated by sampling parameters $\mu = (vec(B)', vec(\Sigma)')'$ and states $\xi_{1:T}$ sequentially from their

[7] To be precise, by allowing for the presence of domestic trends – in addition to foreign trends – we do not force the empirical model to explain the low frequency dynamics of the domestic UK data by using the US trends. In other words, we do not bias upwards the importance of the US TFP and IST non-stationary shocks to the UK economy. The theoretical model abstracts from the UK specific TFP and IST non-stationary shocks as they are not the key focus of this paper due to the vast attention that they have received from the literature mentioned earlier.

[8] We set α to 0.3 which is a standard in the literature.

conditional posterior distributions

$$p(\xi_{1:T}|\zeta_{1:T},\mu) \propto p(\xi_{1:T}|\mu)\, p(\zeta_{1:T}|\xi_{1:T},\mu) \quad (9)$$

$$p(\mu|\zeta_{1:T},\xi_{1:T}) \propto p(\xi_{1:T}|\mu)\, p(\zeta_{1:T}|\xi_{1:T},\mu) \underbrace{N\left(vec(\tilde{B}),\tilde{\Sigma}_B\right) IW\left((\varkappa+1+d\xi)\,\tilde{\Sigma},\varkappa\right)}_{\text{Parameters' Prior Distribution}} \quad (10)$$

respectively.[9] The sampling scheme could be summarized in two steps, namely:

- For $i = 1, ..., N_{simulations}$
  - Draw $\mu^{(i)}$ form $p\left(\mu|\zeta_{1:T},\xi_{1:T}^{(i-1)}\right)$.

    Given $\xi_{1:T}$, this step collapses to a BVAR Gibbs sampling draw (Kadiyala and Karlsson (1997a), Koop and Korobilis (2010)). However, due to the “small open economy” and “trend exogeneity” restrictions the draws are obtained using seemingly unrelated regression (SUR) schemes as in Zha (1999) and Justiniano and Preston (2010), among others.
  - Draw $\xi_{1:T}^{(i)}$ from $p\left(\xi_{1:T}|\zeta_{1:T},\mu^{(i)}\right)$.

    The unobserved state vector $\xi_{1:T}$ is derived using the smoother proposed by Durbin and Koopman (2002).

Given the large scale of the estimated model, the use of prior information as a vehicle to shrink the space of the estimated parameters is unavoidable. As a result, the use of Minnesota type prior distribution for the VAR parameter vector seems a natural choice (Sims (1980), Doan et al. (1984a)). Finally, the vector of constants C in equation (6) is set equal to the average historical value of the observable vector.

4.2 Trend Shocks Identification

Similar to the work of Crump et al. (2016), Negro et al. (2017), Del Negro et al. (2019) and Johannsen and Mertens (2021), the identification of the trend (international and domestic) shocks takes place during the estimation of the model. This paragraph provides only a high-level description of how the identification works. The vector of the observable variables, ζ_t, contains ten variables that grow over time, and two that are stationary. On the other hand, the state vector, ξ_t, and the stability restrictions imposed on B, indicate that four variables (US TFP growth, US IST growth, UK TFP growth, and UK IST growth) out of sixteen variables display a stochastic trend. These four trends are the I(1) driving forces of the

[9]Information about prior and posterior moments can be found in Appendix B.1.

observed data, while the remaining twelve variables describe the cyclical variation of ζ_t. The matrix A disciplines the exact decomposition between the trend and cyclical component of the individual data series in the observable vector. For our exercise, the matrix A is pinned down by the economic theory that is a fundamental block of the DSGE literature concerned with stochastic trends, as detailed in the discussion above.

5 The Role of International Technology Shocks

In this section, we use the estimated state space model to study the transmission and relevance of non-stationary international technology shocks, originating in the US, on a set of UK macroeconomic aggregates.

5.1 Transmission and Importance for the UK Economy

We first focus on the dynamics implied by the international TFP shock which are shown in Figure 2. In the following, in all figures, variables with a * are associated with US responses and variables without a star correspond to the UK.

The international shock to the growth rate of TFP triggers a broad based expansion in the US. Output, consumption, investment and labor productivity rise. Our empirical methodology also allows us to examine the response of the UK economy to this shock. Dynamics for the UK mirror, to a large extent, the behavior seen for the US economy. The international shock leads to a strong and persistent expansion in macroeconomic aggregates which is significant at 68% and even 90% confidence levels. The shock transmits with approximately equal speed through the economies and affects UK output and consumption almost as strongly as the corresponding US quantities. In terms of investment the response in the UK is somewhat weaker than in the US. Any movements in the trade balance are insignificant. This is consistent with the fact that the exogenous part of the technology employed in both countries shares a common component, meaning that a common improvement in TFP leads to an expansion of the potential supply in both economies. This feature leaves no room for "risk-sharing" — i.e. goods or/and asset trade dynamics — and agents in both economies are left only with the option to consume these additional resources. This lack of risk sharing associated with an international TFP shock is particularly important in light of adverse shocks as demand would need to contract sharply (i.e. not smoothing) to adjust to the permanent loss of the potential output. Consistent with the lack of risk sharing, the

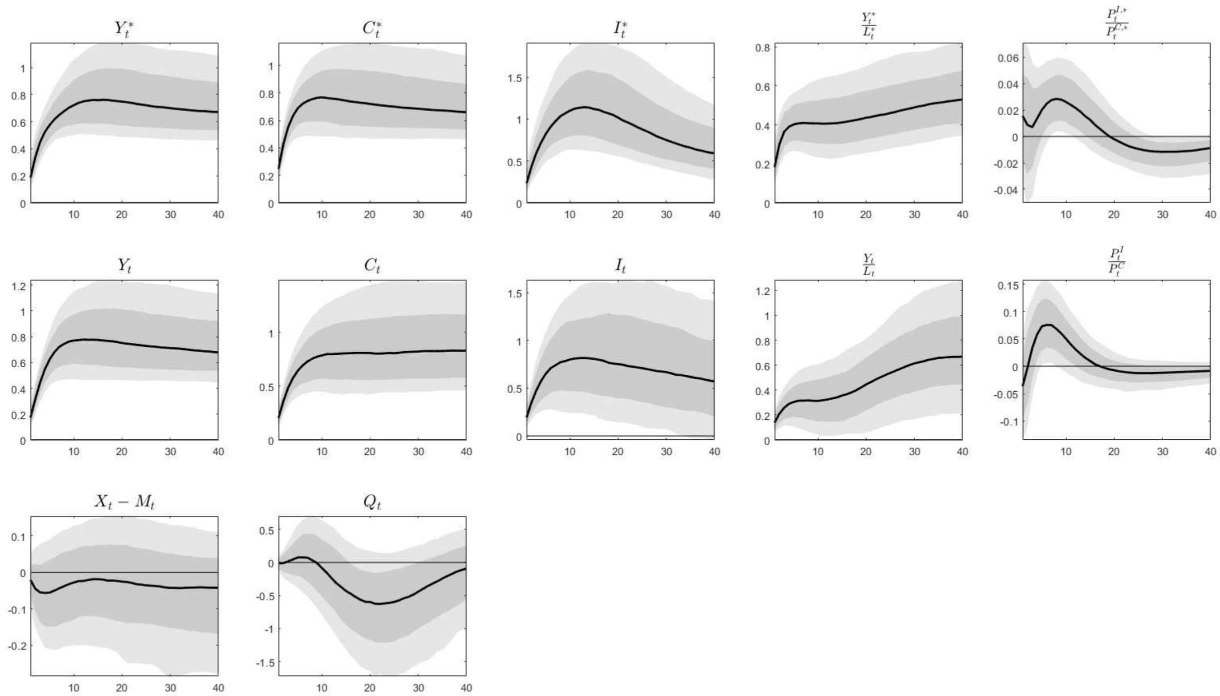

Figure 2: **Responses to a non-stationary international TFP shock.** Variables with a * denote the response of the US economy, variables without a * denote UK variables. Light (dark) grey denotes 90% (86%) confidence bands. Units on the y-axis are percentage deviations.

exchange rate remains muted in the first three years, while it appreciates significantly after about four years. This seems to reflect the fact that potential supply expands relatively more in the US than in the UK, as indicated by the somewhat stronger investment response in the US.

A strength of our empirical setup is that it allows us to jointly identify international TFP and IST shocks. Jointly identifying these shocks gives a broader picture of the effects of international productivity shocks, including the relative importance of TFP and IST trends. Figure 3 shows that an international IST shock leads to an expansion in output, consumption and investment in the US and the UK. Qualitatively, the response of the macroeconomic aggregates is very similar to those to an international TFP shock. A notable difference is that the boom in output, consumption and investment is driven by a strong and persistent decline in the relative price of investment. These responses are consistent with the evidence for US IST shocks e.g. in Fisher (2006) and the decline in the relative price of investment clearly distinguishes this shock from the response to the TFP shock. The international IST shock transmits strongly also through to the UK economy. As in the case of the TFP shock, international IST is a part of the UK production technology and increases the potential supply in both economies which limits trade (risk-sharing) dynamics. In response to the international IST shock the trade balance does not move

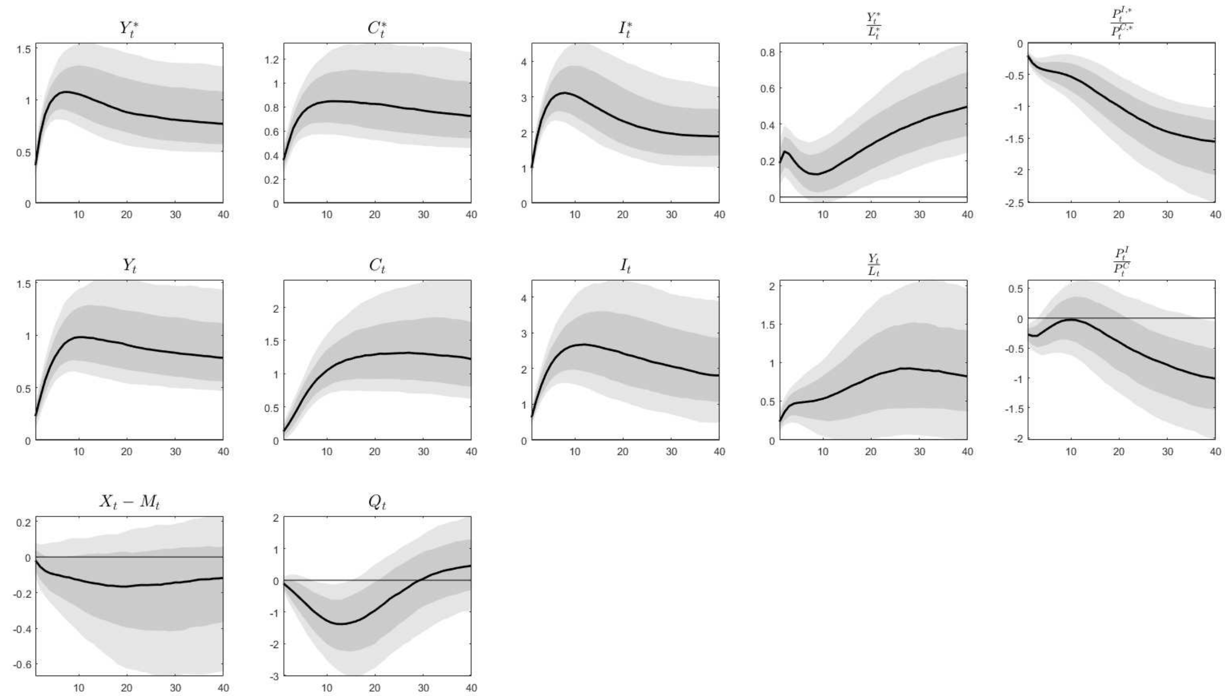

Figure 3: **Responses to a non-stationary international IST shock.** Variables with a * denote the response of the US economy, variables without a * denote UK variables. Light (dark) grey denotes 90% (86%) confidence bands.

significantly, which is consistent with the notion in e.g. Glick and Rogoff (1995) and Bussière et al. (2010) who argue that international shocks should not substantially affect current accounts. The real exchange rate is unresponsive for about the first year after the shock, while it appreciates significantly about two years latter. As in the case of the international TFP shock, the delayed appreciation of the real exchange rate seems to reflect the fact that potential supply in the US expands by more than in the UK and again the difference is driven by capital dynamics.[10]

Both non-stationary international technology shocks generate a strong expansion of all macroeconomic aggregates in the UK. This implies, in principle, that they may be important contributors to UK business cycles. Figures 4 and 5 show the associated forecast error variance decompositions and allow for a more formal inspection of the shocks' importance for aggregate fluctuations. International technology shocks of either kind, TFP or IST, are rather important for fluctuations in US GDP. From Figure 5 it is evident that the international IST shock explains about 50% of the fluctuations in US GDP at business cycle frequencies and Figure 4 shows that the international TFP shock accounts for about 15% of the variations

[10] Impulse responses to the domestic UK TFP and IST shocks are shown in Appendix A.1, which we jointly identify with the international technology shocks. The UK TFP shock triggers an expansion in all macroeconomic aggregates. The UK IST shock drives up investment and labor productivity, but does not move output and consumption significantly.

in US GDP.

It is interesting though that both international technology shocks also account for a substantial share in the forecast error variance of UK GDP. They compete with the domestic IST and TFP shocks as well as the cyclical shocks. The international IST shock explains about 16%, and the international TFP shock about 10% of the fluctuations in UK GDP. Both shocks also account for about 10% of the FEVD of UK consumption. The IST shock accounts for about 12% in the investment and 10% in the labor productivity forecast error variance. Given that the two international non-stationary shocks compete with a large number of other shocks in explaining variables' forecast error variance, these international technology shocks play a non-negligible role in explaining variation in macroeconomic aggregates, where the IST shock is somewhat more important than the TFP shock.

Both international technology shocks are an important component for understanding UK business cycles. Table 1 summarizes the median forecast error variance decomposition for all shocks. From this table it is evident that UK TFP shocks are about three times more important than their international counterparts for explaining variations in the UK observables (28% vs. 9%). For the IST shock, it is the reverse: the international IST innovations are substantially more important than those specific to the UK economy (16% vs. 3%). Overall, international and domestic non-stationary technology shocks account for about three quarters of the overall fluctuations in GDP. The remaining share of variance is explained by the cyclical shocks. The latter are even more important for fluctuations in investment and consumption, accounting for 78% and 51% of the variations in these variables.

Table 1: Forecast error variance decomposition

	international TFP	international IST	UK TFP	UK IST	all cyclical
US Output	14.86	48.81	n.a.	n.a.	36.33
US Consumption	28.09	41.02	n.a.	n.a.	30.88
US Investment	5.43	65.63	n.a.	n.a.	28.95
US Labor Productivity	16.02	15.06	n.a.	n.a.	68.92
US Rel. Price of Investment	4.16	35.86	n.a.	n.a.	59.98
UK Output	9.87	16.00	42.27	3.40	28.46
UK Consumption	8.49	10.23	28.43	1.87	50.98
UK Investment	1.29	11.64	5.74	3.43	77.90
UK Labor Productivity	3.77	10.27	43.14	6.49	36.32
UK Rel. Price of Investment	2.22	6.63	2.95	11.84	76.36
Trade Balance	2.36	9.51	4.12	6.47	77.55
Real Exchange Rate	2.82	15.72	7.07	1.59	72.80

Having established the relevance of international technology shocks for the UK on average, we now want go further and inspect their role for variations in UK GDP during particular episodes. In particular,

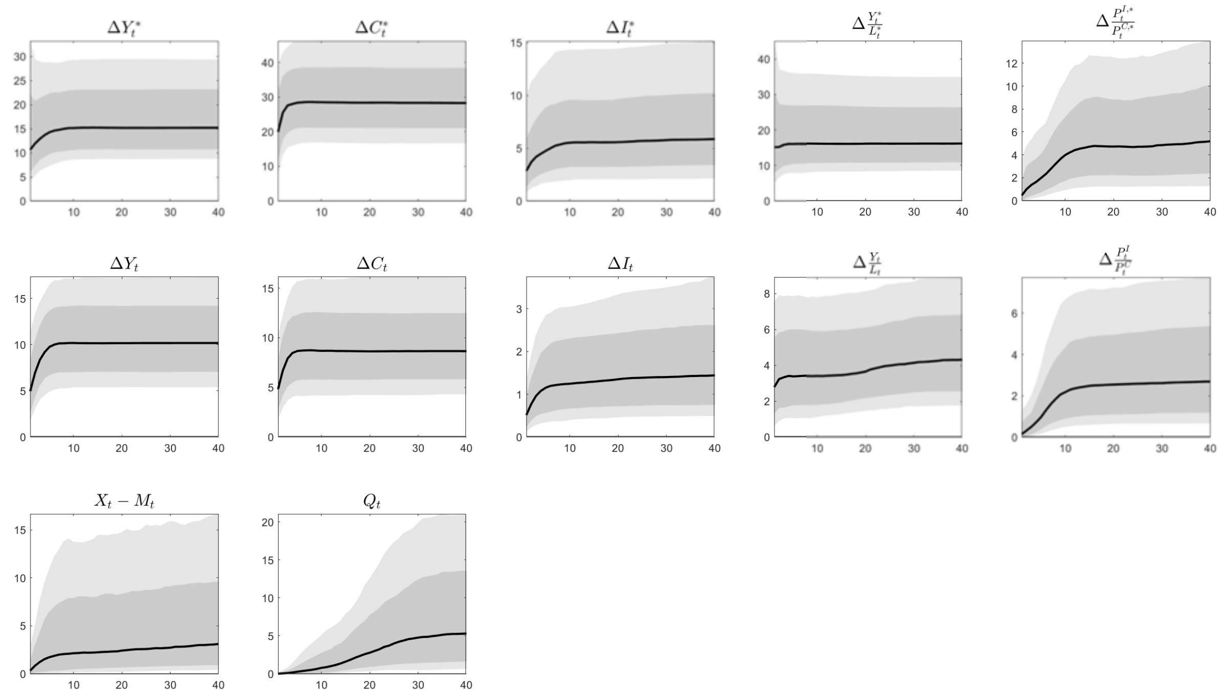

Figure 4: **US TFP Shock — Forecast Error Variance Decomposition.** Variables with a * denote the FEVD of US variables and variables without a * denote the FEVD of UK variables. Light (dark) grey denotes 90% (86%) confidence bands.

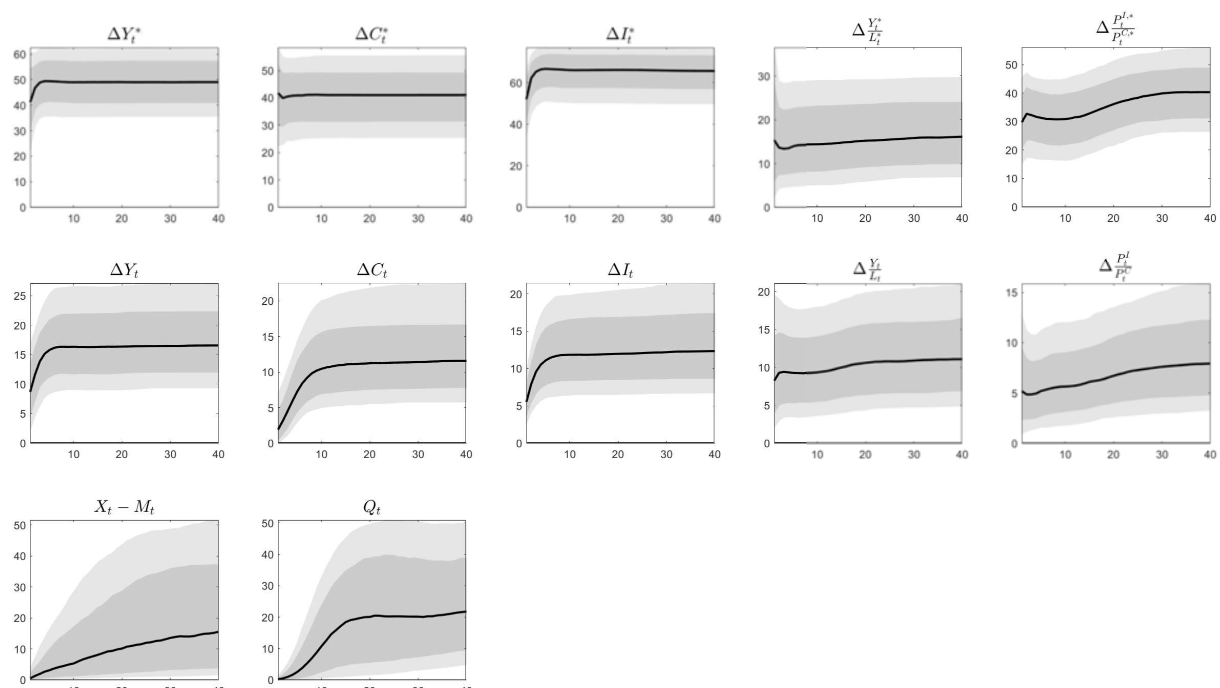

Figure 5: **US IST Shock — Forecast Error Variance Decomposition.** Variables with a * denote the FEVD of US variables and variables without a * denote the FEVD of UK variables. Light (dark) grey denotes 90% (86%) confidence bands.

we turn to inspecting the role of the different technology shocks during the Great Recession and the following recovery. Figure 6 shows the historical decomposition of UK GDP growth during this episode.[11] Guerron-Quintana (2013), who analyzes the impact of international non-stationary TFP shock on a set of advanced small open economies (not including the UK) finds that the international shock significantly contributed to the Great Recession in these economies. Figure 6 illustrates that for the UK, it is in particular the international IST shock that has made a substantial contribution to the economy's contraction after 2007. It was also driven by a decline in UK specific TFP and to a much lesser extent by a decline in international TFP. From mid-2011, the international IST shock turns into a substantial force for GDP growth which lasts for several years.Without the strong positive contribution of the IST shock, the recovery after the financial crisis would have been substantially slower. At the same time the international TFP shock now substantially dampens GDP growth. During this episode, the two international technology shocks are the two most important drivers of variations in UK GDP. Also the UK-specific TFP shock and the cyclical (stationary) shocks contributed to the strong contraction during the recession and particularly the former contributed to the following recovery. The UK-specific IST shock plays virtually no role during this episode. These results stress that to understand the driving forces of the 2007 financial crisis and the following recovery in the UK, it is important to account for international as well as UK-specific shocks.

5.2 Evidence from Other Economies

The finding that both non-stationary international technology shocks are important for driving variations in macroeconomic aggregates is not confined to the UK economy. In particular, we consider five other G7 countries as domestic economies, one at a time, relative to the US economy. What somewhat limits direct comparisons between other G7 economies and the UK are the different data samples available. The individual time horizons considered are 1964Q1-2016Q4 for Canada, 1980Q2-2017Q4 for France, 1991Q2-2017Q4 for Germany, 1995Q1-2017Q4 for Italy and 1994Q2-2016Q4 for Japan, which are limited by data availability considerations. The data for these countries has been obtained from the same sources as outlined in Section 3.

Figure 7 reports the forecast error variance decompositions of GDP for Canada, Japan, Germany, France and Italy when they are considered, one at a time, as domestic economies. International IST shocks are an important driver of the business cycle, explaining around 21%, 12%, 27% and 15% of the GDP volatility in Canada, Japan, Italy and France. The exception is Germany, where the international

[11] We provide the historical decomposition for the entire sample in Appendix A.1.

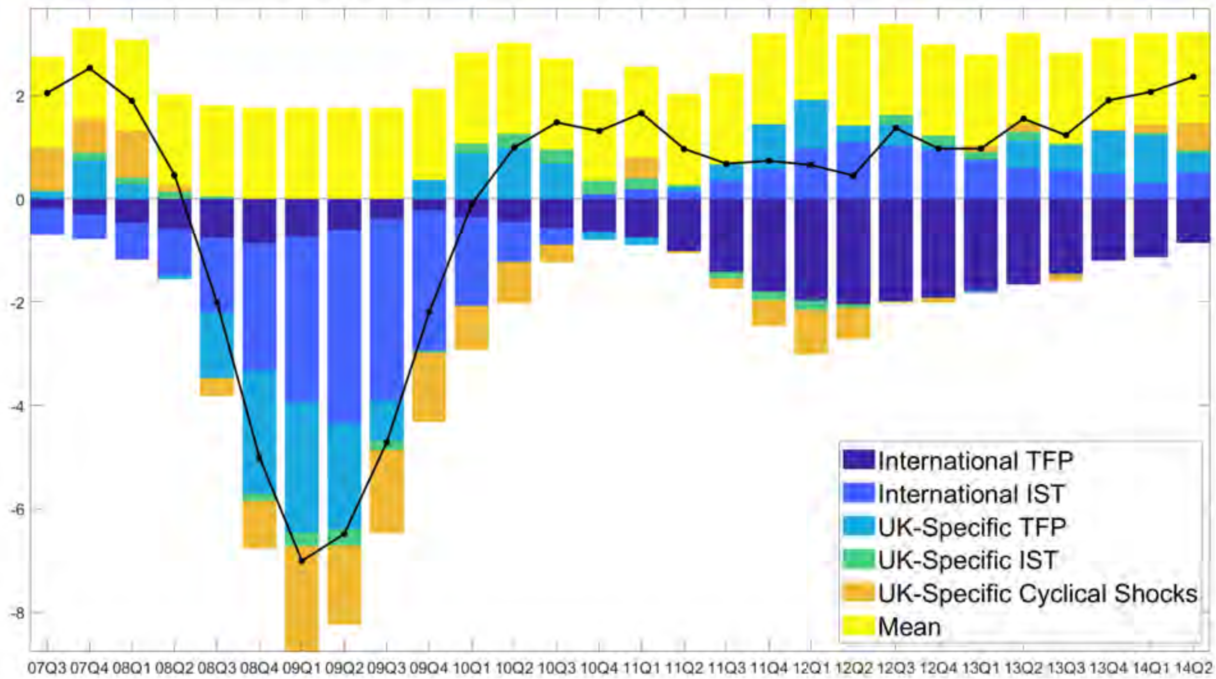

Figure 6: **Historical Decomposition of UK GDP growth.** The figure shows the historical shock decomposition of GDP growth (solid black line) during and after the Great Recession.

TFP shock is much more important (29%) than its IST counterpart (3.5%). The international TFP shock is also important for explaining aggregate fluctuations in Canada and France (17% and 23%) and to a lesser extent in Japan (8%) and Italy (8%). Overall, for all considered countries the international productivity shocks play an important role for explaining aggregate fluctuations, explaining together between about 20% and 38% of variations in GDP.[12] In this sense are these results consistent with those for the UK discussed in the sections above. Variations in the importance of these productivity shocks across countries will be driven by differences in the structure of the economies, the proximity to the US and in time horizons considered.

6 Reconciling the Structural Model with the Data

Having shown the importance of international TFP and IST trend shocks in the data, we now return to the DSGE model that we used to motivate our shock identification. In particular, we ask if the structural model developed in Section 2 can match the impulse responses from the state space model. To answer this question, we use a minimum distance strategy, as in Bodenstein et al. (2018), that minimizes the

[12] Appendix A.2 reports the variance decomposition for all variables and all countries.

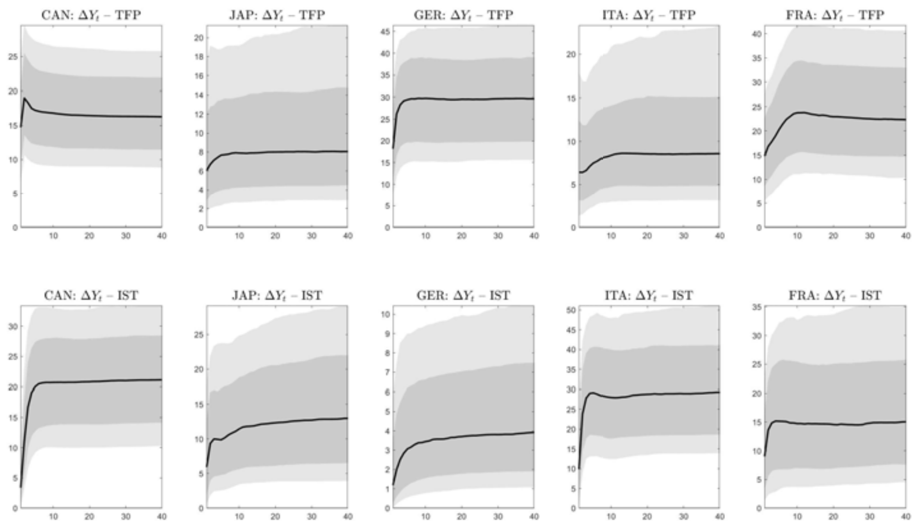

Figure 7: **GDP Forecast Error Variance Decomposition for Selected Economies.** The figure shows the FEVD of GDP for selected domestic economies. In each case, the US is the foreign economy. Light (dark) grey denotes 90% (86%) confidence bands. Units on the y-axis are percentage deviations.

distance between the impulse responses of the state space model, estimated on UK data, and the impulse responses of the stationary DSGE model. Before we can apply the minimum distance strategy, we need to make choices on functional forms used in the structural model presented in Section 2 and calibrate some parameters.

6.1 Functional Forms, Calibration and Stationarization

Capital accumulation, as given by equation (2), is subject to investment adjustment costs. We specify the functional form of these cost to be

$$S\left(\frac{i_t}{i_{t-1}}\right) = \frac{\psi}{2}\left(\frac{i_t}{i_{t-1}} - 1\right)^2,$$

as in Christiano et al. (2005). The parameter $\psi > 0$ governs the degree of investment adjustment costs.

Using our minimum distance strategy, we distinguish between calibrated and estimated parameters. The calibrated parameters are listed in the top half of Table 3. The steady-state trend growth in both economies is normalized to unity and we assume a discount rate of 1% per quarter in both countries. The share of capital in output, α, is set to 0.3, which is the same value used in the identification of TFP and IST trends in the emprical model. in The depreciation rate, δ is set to 0.025. These are all standard in the literature. The relative size of the UK in our two country model, N, is defined as the sample average of UK to US GDP and subsequently set to 0.175. The openness parameter, $\gamma = 0.24$, which determines *inter alia* the degree of consumption home-bias, is set according to the sample average of imports to GDP in the UK. The parameter ϕ_b denotes a small bond holding cost that rules out a unit root in bond holdings. The model dynamics are virtually invariant to a range of small values of this parameter and we set it to be 0.01.

The non-stationary model outlined in Section 2 can be stationarized by dividing non-stationary variables by their trend. Specifically output, consumption, goods to be transformed into investment, bond holdings and real wages share a common trend and can be stationarized according to: $\hat{y}_t = \frac{y_t}{z_t v_t^{\frac{\alpha}{1-\alpha}}}$, $\hat{c}_t = \frac{c_t}{z_t v_t^{\frac{\alpha}{1-\alpha}}}$, $\hat{x}_t = \frac{x_t}{z_t v_t^{\frac{\alpha}{1-\alpha}}}$, $\hat{b}_t = \frac{b_t}{z_t v_t^{\frac{\alpha}{1-\alpha}}}$, $\hat{w}_t = \frac{w_t}{z_t v_t^{\frac{\alpha}{1-\alpha}}}$. The Lagrange multiplier for the household's optimization problem can be stationarized as follows: $\hat{\lambda}_t = \lambda_t (z_t v_t^{\frac{\alpha}{1-\alpha}})^{\sigma}$. Let $p_{I,t}$ denote the relative price of investment goods, $P_{I,t}/P_t$ and q_t Tobin's Q. These two relative prices become stationary when multiplied by the IST trend, $\hat{p}_{I,t} = p_{I,t} v_t$ and $\hat{q}_t = q_t v_t$, respectively. The capital stock and investment become stationary after dividing by the capital specific trend: $\hat{k}_t = \frac{k_t}{z_t v_t^{\frac{1}{1-\alpha}}}$ and $\hat{i}_t = \frac{i_t}{z_t v_t^{\frac{1}{1-\alpha}}}$. Since the model is rather standard, we summarize the complete set of equations for the stationary model in Table 2.[13]

6.2 Minimum Distance IRF Matching

Given the values of the calibrated parameters — stacked in the vector Θ^c — we estimate the remaining parameters — stacked in the vector Θ^e — by minimising the distance between the empirical impulse response functions from the state space model for the UK, denoted by G, and the impulse response function implied the theoretical model, denoted by $G(\Theta^c, \Theta^e)$

$$\hat{\Theta}^e = argmin_{\Theta^e} [G - G(\Theta^c, \Theta^e)]'^{-1} [G - G(\Theta^c, \Theta^e)]. \tag{11}$$

We minimize the objective (11) over the first twelve periods after the shock. Specifically, we match the model response with those of the state space model for output, consumption, investment, labor productivity and the relative price of investment goods for both the UK and the US. We do not attempt to match the dynamics of UK net trade or the effective real exchange rate.

Matching the impulse responses of the model to the IRFs of both international TFP and IST shocks, inevitably results in two different sets of estimated parameters. To avoid this, we focus on matching the

[13]The model equations are divided into optimality conditions pertaining to the household, equations [i] to [iv]; firms [v] to [xiv]; market clearing conditions, [xv] to [xvii]; relative prices, [xviii] to [xxiii] and shock processes,[xxviii] to [xxix]. We remark with (hf) those model equations that are identical for in both home and foreign economies, except for their notation (foreign variables being denoted with an $*$). A model equation applying only to the home economy is remarked by (h) and one applying only to the foreign economy with (f).

Table 2: Equilibrium conditions and constraints of the stationary model

Description	Model equation	
Households		
FOC c_t (hf)	$\hat{c}_t^{-\sigma} = \hat{\lambda}_t$	[i]
FOC n_t (hf)	$\phi_0(\hat{n}_t)^{\phi} = \hat{\lambda}_t \hat{w}_t$	[ii]
FOC b_t (h)	$\hat{\lambda}_t = \beta \hat{\lambda}_{t+1} \left(\frac{1}{\hat{\Gamma}^y_{t+1}} \right)^{\sigma} (1+\hat{r}_t) \frac{r\hat{e}r_{t+1}}{r\hat{e}r_t}$	[iii]
FOC b_t^* (f)	$\hat{\lambda}_t^* = \beta \hat{\lambda}^*_{t+1} \left(\frac{1}{\hat{\Gamma}^{y_{t+1}}} \right)^{\sigma} (1+\hat{r}_t^*)$	[iv]
Firms		
Prod fn (hf)	$\hat{y}_t = \left(\frac{1}{\hat{\Gamma}^k_t} \right)^{\alpha} (\hat{k}_{t-1})^{\alpha} \hat{n}_t^{1-\alpha}$	[v]
FOC x_t (hf)	$1 = \hat{q}_t \left[\left(1 - \frac{\psi}{2} \left(\frac{\hat{x}_t}{\hat{x}_{t-1}} \hat{\Gamma}^k_t - 1 \right)^2 \right) - \psi \left(\frac{\hat{x}_t}{\hat{x}_{t-1}} \hat{\Gamma}^k_t - 1 \right) \frac{\hat{x}_t}{\hat{x}_{t-1}} \hat{\Gamma}^k_t \right] + \beta E_t \hat{q}_{t+1} \frac{\hat{\lambda}_{t+1}}{\hat{\lambda}_t} \frac{1}{\hat{\Gamma}^k_{t+1}} \left(\frac{1}{\hat{\Gamma}^y_{t+1}} \right)^{\sigma-1} \psi \left(\frac{\hat{x}_{t+1}}{\hat{x}_t} \hat{\Gamma}^k_{t+1} - 1 \right) \left(\frac{\hat{x}_{t+1}}{\hat{x}_t} \hat{\Gamma}^k_{t+1} \right)^2$	[vi]
KC (hf)	$\hat{k}_t = (1-\delta) \hat{k}_{t-1} \frac{1}{\hat{\Gamma}^k_t} + \left[1 - \frac{\psi}{2} \left(\frac{\hat{x}_t}{\hat{x}_{t-1}} \hat{\Gamma}^k_t - 1 \right)^2 \right] \hat{x}_t$	[vii]
FOC n_t (h)	$(1-\alpha) \hat{p}_{H,t} \frac{\hat{y}_t}{\hat{n}_t} = \hat{w}_t$	[viii]
FOC n_t^* (f)	$(1-\alpha) \hat{p}^*_{F,t} \frac{\hat{y}^*_t}{\hat{n}^*_t} = \hat{w}^*_t$	[ix]
FOC k_t (h)	$\hat{q}_t = \beta E_t \frac{\hat{\lambda}_{t+1}}{\hat{\lambda}_t} \left(\frac{1}{\hat{\Gamma}^y_{t+1}} \right)^{\sigma-1} \left[\hat{q}_{t+1} (1-\delta) \frac{1}{\hat{\Gamma}^k_{t+1}} + \hat{p}_{H,t+1} \alpha \frac{\hat{y}_{t+1}}{\hat{k}_t} \right]$	[xiii]
FOC k_t (f)	$\hat{q}^*_t = \beta E_t \frac{\hat{\lambda}*_{t+1}}{\hat{\lambda}^*_t} \left(\frac{1}{\hat{\Gamma}^{y_{t+1}}} \right)^{\sigma-1} \left[\hat{q}^*_{t+1} (1-\delta) \frac{1}{\hat{\Gamma}^{k_{t+1}}} + \hat{p}^*_{F,t+1} \alpha \frac{\hat{y}^*_{t+1}}{\hat{k}^*_t} \right]$	[xiv]
Market clearing		
AUC (h)	$\hat{y}_t = (1-(1-N)\gamma) (\hat{p}_{H,t})^{-\theta} (\hat{c}_t + \hat{x}_t) + (1-N)\gamma \left(\hat{p}^*_{H,t} \right)^{-\theta} (\hat{c}^*_t + \hat{x}^*_t)$	[xv]
AUC (f)	$\hat{y}_t = (1-N)\gamma) \left(\hat{p}^*_{F,t} \right)^{-\theta} (\hat{c}^*_t + \hat{x}^*_t) + N\gamma (\hat{p}_{F,t})^{-\theta} (\hat{c}_t + \hat{x}_t)$	[xvi]
CA (h)	$\hat{c}_t + \hat{x}_t + \hat{b}_t = (1+r_{t-1}) \frac{r\hat{e}r_t}{r\hat{e}r_{t-1}} \hat{b}_{t-1} \frac{1}{\hat{\Gamma}^y_t} + \hat{p}_{H,t} \hat{y}_t$	[xvii]
Relative prices		
	$(\hat{p}_{H,t})^{\theta-1} = (1-(1-N)\gamma) + (1-N)\gamma \hat{T}_t^{1-\theta}$	[xviii]
	$(\hat{p}_{F,t})^{\theta-1} = (1-(1-N)\gamma) \hat{T}_t^{1-\theta} + (1-N)\gamma$	[xix]
	$\left(\hat{p}^*_{H,t} \right)^{\theta-1} = N\gamma + (1-N\gamma) \hat{T}_t^{1-\theta}$	[xx]
	$\left(\hat{p}^*_{F,t} \right)^{\theta-1} = N\gamma \hat{T}_t^{1-\theta} + (1-N\gamma)$	[xxi]
	$r\hat{e}r_t^{1-\theta} = \frac{N\gamma + (1-N\gamma)\hat{T}_t^{1-\theta}}{(1-(1-N)\gamma) + (1-N)\gamma \hat{T}_t^{1-\theta}}$	[xxii]
UIP	$(1+\hat{r}_t) = (1+\hat{r}^*_t) e^{-\phi_b \hat{b}_t}$	[xxiii]
TFP trend	$\hat{\Gamma}^z_t = \frac{z_t}{z_{t-1}}$	[xxiv]
IST trend	$\hat{\Gamma}^v_t = \frac{v_t}{v_{t-1}}$	[xxv]
y growth (hf)	$\hat{\Gamma}^y_t = \hat{\Gamma}^z_t (\hat{\Gamma}^v_t)^{\frac{\alpha}{1-\alpha}}$	[xxvi]
k growth (hf)	$\hat{\Gamma}^k_t = \hat{\Gamma}^y_t \hat{\Gamma}^v_t$	[xxvii]
Shock processes		
	$\ln \hat{\Gamma}^z_t = \rho_z \ln \hat{\Gamma}^z_{t-1} + \hat{\epsilon}_{z,t}$	[xxviii]
	$\ln \hat{\Gamma}^v_t = \rho_v \ln \hat{\Gamma}^v_{t-1} + \hat{\epsilon}_{v,t}$	[xxix]

Notes: (h) = home, (f) = foreign, (hf) = applies to home and foreign, FOC = first order condition, KC = capital accumulation constraint, CA = current account, AUC = adding up constraint

Table 3: Calibrated and estimated model parameters

Parameter	Description	Value
Calibrated parameters		
β	discount rate	0.99
N	relative size of UK to the US	0.175
γ	openness	0.24
α	share of capital in GDP	0.3
δ	depreciation rate	0.025
ϕ_b	bond holding cost	0.01
Estimated parameters (international TFP shock)		
ρ_z	Persistence of international TFP shock	0.3263
σ_z	Standard deviation of international TFP shock	0.4701
ϕ	UK Inverse of Frisch elasticity	2.8796
ϕ^*	US Inverse of Frisch elasticity	0.4251
θ	Trade elasticity	0.5647
ψ	UK investment adjustment cost	10.2206
ψ^*	US investment adjustment cost	3.7844
σ	Constant relative risk aversion parameter	5.8540

impulse response to a shock to the trend in TFP, which are summarized in the bottom half of Table 3. We then impose those estimated parameters on the model to ascertain to what extent the model can reproduce the dynamics of the trend shock to IST.

Figure 8 shows the impulse responses of the UK and the US in response to an international non-stationary TFP shock. The black solid line shows the median impulse response based on the state space model and the grey shaded areas are the 90% and 86% confidence bands. The red solid line shows the responses based on the structural model. Overall, the structural model is able to capture the salient features of the international TFP shock in both the US and the UK. In particular, it generates the broad based expansion in output, consumption and investment as well as the rise in labor productivity that emerged from our empirical exercise. As implied by the data, the structural model generates an extremely muted response for net trade following an international shock. Given the estimated parameters, the model generates a small but persistent real appreciation.

Table 3 reports the estimated model parameters. The UK model economy differs from the US economy in that the model needs a significantly smaller Frisch elasticity (inverse of the parameter ϕ) to match UK data than is required to match the dynamics of US variables. This suggests that the UK labor supply is relatively less responsive to shocks than the US economy. Another key point of departure between the two economies is the investment adjustment cost parameter. The empirical results based on the state

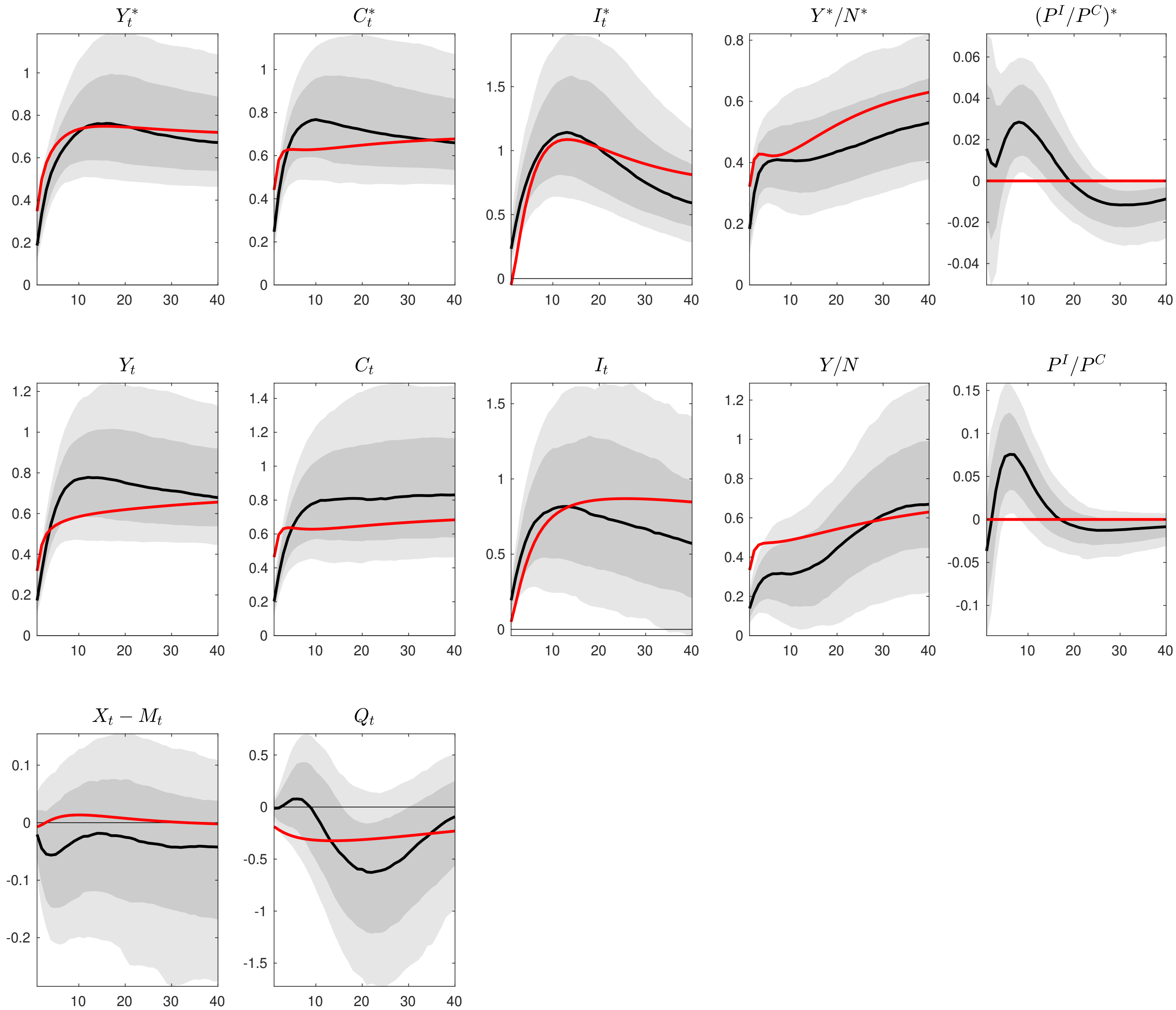

Figure 8: **Responses to a non-stationary international TFP shock — DSGE model and state space model.** The red line denotes the impulse response of the DSGE model. The median response from the estimated state space model is given by the solid black line. The grey shaded areas show the **90**% and **86**% confidence bands.

space model imply that US investment rises by more in response to the international TFP shock than investment in the UK. Hence, to match these dynamics, the DSGE model requires an adjustment cost parameter that is larger for the UK than for the US. Considering this set of estimates suggests that the UK economy reacts somewhat slower to international non-stationary TFP shocks, both in terms of labor supply and investment, than the US. As a result, the DSGE model generates a data-congruent small but persistent real appreciation.

Figure 9 shows the impulse responses of the US and the UK in response to a non-stationary international IST shock. The structural model used to generate Figure 9 uses the model parameters estimated on the TFP shock, except for the persistence and shocks size of the IST process, which is chosen to optimise the overall fit.[14] Overall, the DSGE model also captures the salient features of the IST shock in both the UK and the US. The model is able to capture the broad based expansion in GDP, consumption and investment, along with the persistent decline in the relative price of investment goods. Similar to the case of an international TFP shock, model generates only a very muted response in the trade balance following an international IST shock. Given the structural differences between the US and the UK economy implied by our estimated model parameters, the DSGE model generates a real appreciation in response to an international non-stationary IST shock.

In summary, a simple flexible price, two-country DSGE model comes close to matching the macroeconomic dynamics implied by shocks to the international trend in TFP and IST. Using a minimum distance strategy to estimate some of the model's key parameters suggests that some of differences in the response to international shocks across the US and the UK economies are due to more rapid adjustment of the US labor market as well as lower costs associated with adjusting the rate of investment in the US.

7 Conclusion

In this paper, we set out to determine the role and importance of international productivity shocks in small open economies. In particular, we consider international non-stationary shocks to total factor productivity and investment specific technology that originate in the the United States and analyze their effects on the economy of the United Kingdom. One of the key contributions of this paper, that sets it apart from the literature, is our novel simultaneous identification of international TFP and IST shocks and UK-specific TFP and IST shocks. We show that for the UK economy, both of the international productivity shocks

[14] In this impulse response matching exercise, the only parameters we optimize over are the persistence and standard deviation of the international IST shock, all other parameters are carried over from the TFP shock matching exercise.

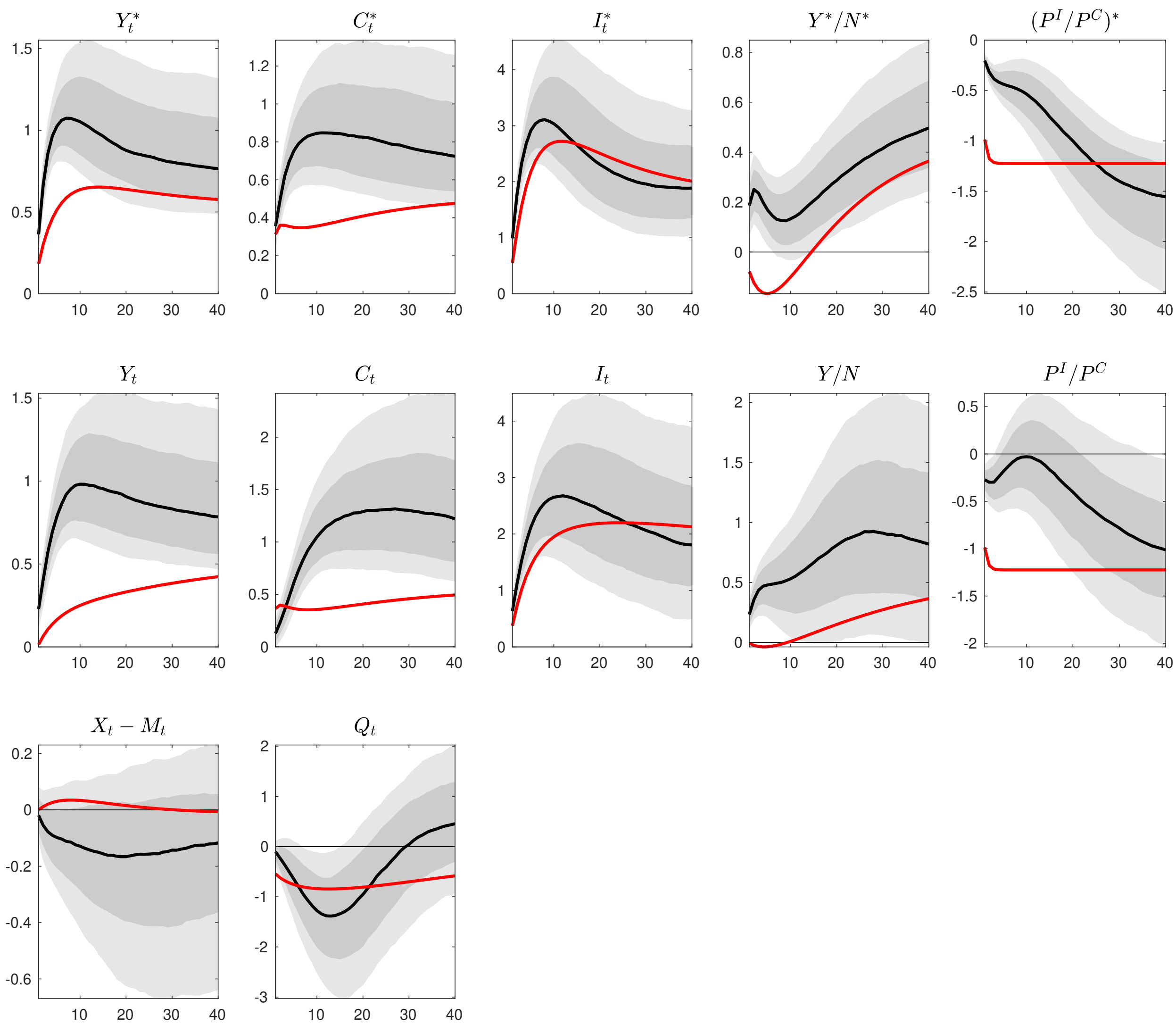

Figure 9: **Responses to a non-stationary international IST shock — DSGE model and state space model.** The red line denotes the impulse response of the DSGE model. The median response from the estimated state space model is given by the solid black line. The grey shaded areas show the **90%** and **86%** confidence bands. The model parameters are the same as in Table 3 except for autocorrelation and standard deviation of the IST shock process: $\rho_v = 0.1943$ and $\sigma_v = 0.9864$.

lead to persistent expansions in GDP and its components. In terms of contributions to the business cycle, we find that both international shocks contribute about a quarter to the variability of GDP, with the international IST shock playing a more pertinent role than the international TFP shock. The notion that international technology shocks are non-negligible drivers of aggregate fluctuations is confirmed also when estimating our model, one at a time, for the other G7 economies. Looking at the historical composition of UK GDP growth during the Great Recession, we find that the international IST shock played an important role in the downturn as well as in the subsequent upturn. The international TFP shock, on the other hand, contributed to the recession and has acted as a drag on the recovery in subsequent years. We further show that the dynamics implied in the data are consistent with those of international TFP and IST shocks in a simple two-country DSGE model.

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Appendix (for online publication)

A Additional Empirical Evidence

A.1 Additional Empirical Evidence for the UK

Figures 10 and 11 show the responses of the US and UK economies to international non-stationary TFP and IST shocks, respectively. These figures are based on an alternative VAR specification which we consider to inspect the responses of imports and exports. In comparison to our baseline specification, these two variables replace UK consumption and the trade balance. We estimate a twelve-variable VAR which does not include the trade balance — this variable is shown in Figures 10 and 11 as the difference of the responses of exports and imports. The results from this alternative VAR specification are overall consistent with the ones in Figures 2 and 3. The additional insight we gain from this exercise is an understanding of the dynamics in imports and exports. Behind the insignificant response of the trade balance, reported from the baseline specification in Section 5, is an approximately equal expansion in imports and exports.

Figures 12 and 13 show the responses of the UK economy to non-stationary UK TFP and IST shocks, respectively. Following a positive non-stationary TFP shock, output per worker (Y/L) rises and the real exchange rate appreciates. GDP and its domestic components expand. Net trade, on the other hand is somewhat counter-cyclical, but not statistically significant. GDP and its components expand by more in response to a shock to the trend of domestic TFP than they do in response to a shock to the trend of international TFP (see Figure 2). This corresponds to the observation in Table 1 that domestic non-stationary TFP shocks contribute about 4 times as much to the variance of GDP than do non-stationary international TFP shocks.

Table 1 also suggests that contribution of UK non-stationary IST shocks to the variance of GDP and its components is relatively small. Indeed, Figure 13 shows that a non-stationary UK IST shock has only a small and statistically insignificant effect on GDP and its components. A non-stationary IST shock causes a persistent fall in the relative price of investment goods. Labour productivity rises and does so persistently. The increase in investment that corresponds to the fall in the relative price of investment goods raises the capital stock over time and thus improves output per worker. Neither the real exchange rate nor the trade balance respond in a significant manner.

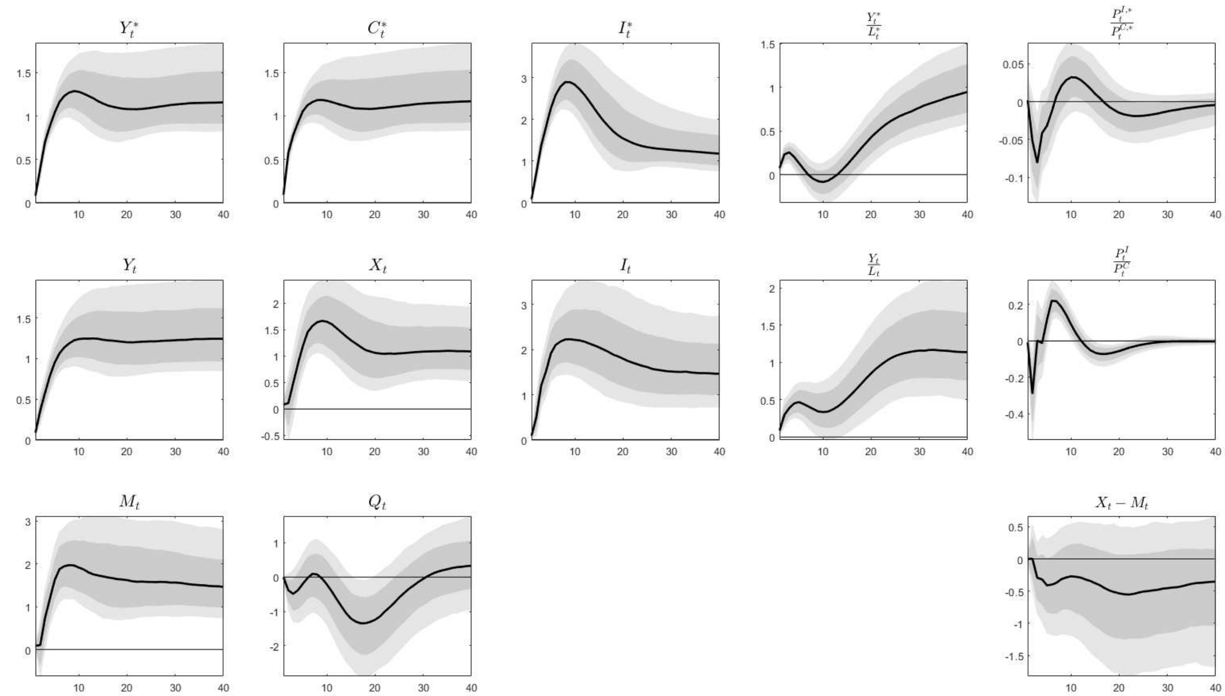

Figure 10: **Responses to a non-stationary international TFP shock. Alternative VAR.** Variables with a * denote the response of the US economy, variables without a * denote UK variables. Light (dark) grey denotes 90% (86%) confidence bands. Units on the y-axis are percentage deviations.

Figure 14 shows the historical decomposition for UK GDP growth over the entire sample.

A.2 Additional Empirical Evidence for Other Economies

Figures 15 and 16 show the forecast error variance decomposition for a VAR system in which either Canada, Japan, Germany, Italy or France is considered as the domestic economy. Figure 15 summarizes the contribution of the foreign non-stationary TFP shocks and Figure 16 the corresponding statistics of the foreign non-stationary IST shock. Overall, these results are broadly consistent with the ones for the UK economy.

B Estimation of the Empirical Model

This appendix provides information about the prior moments or the VAR parameters and the Gibbs sampler steps.

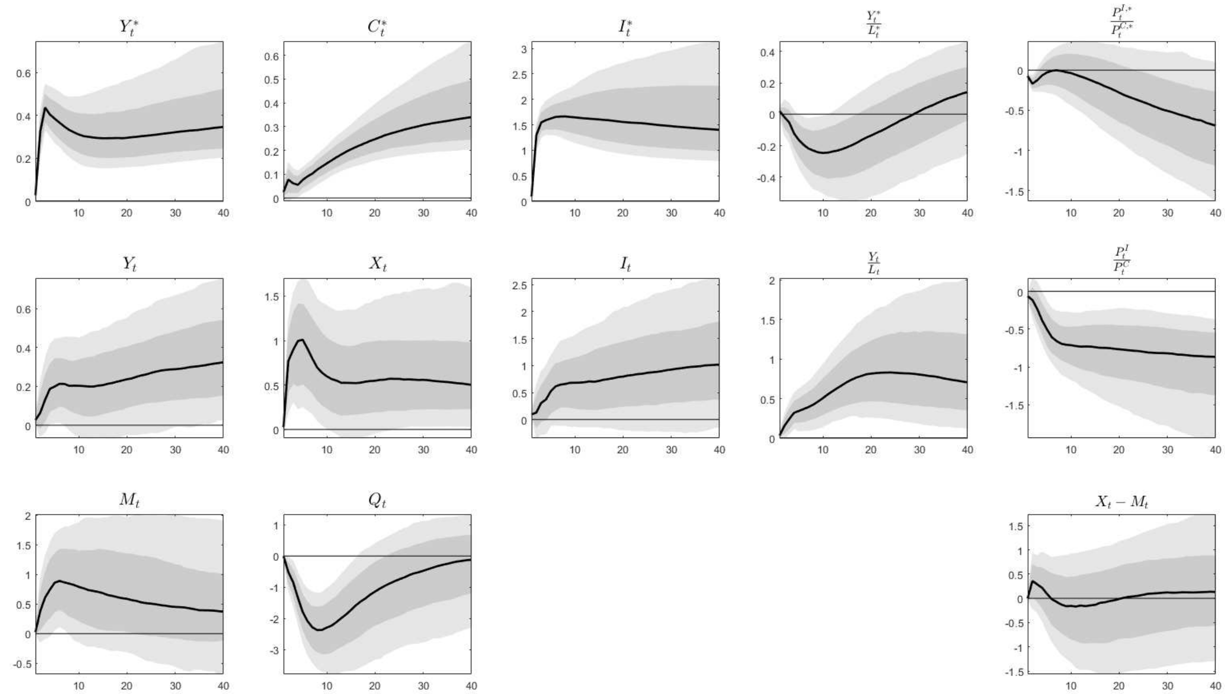

Figure 11: **Responses to a non-stationary international IST shock. Alternative VAR.** Variables with a * denote the response of the US economy, variables without a * denote UK variables. Light (dark) grey denotes 90% (86%) confidence bands. Units on the y-axis are percentage deviations.

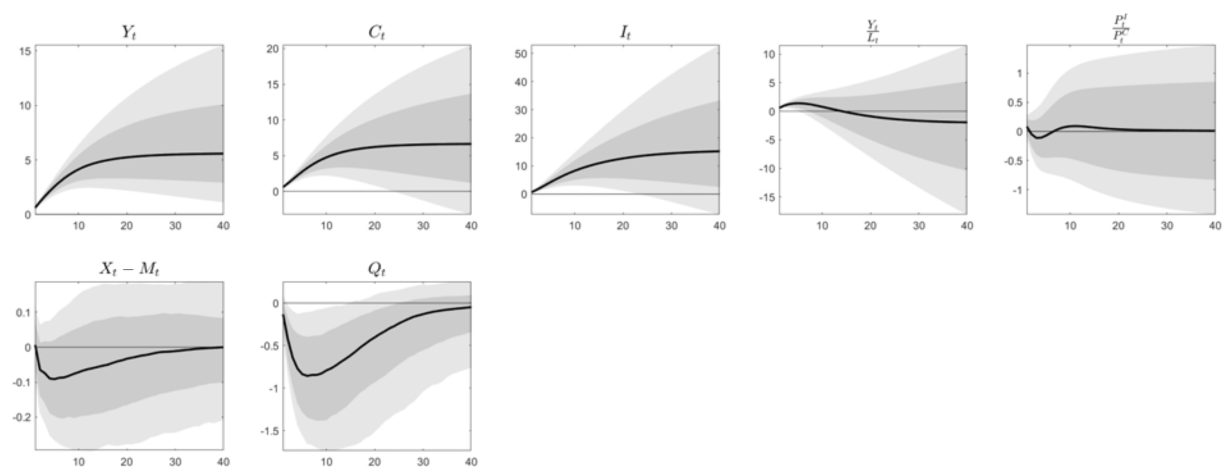

Figure 12: **Responses to a non-stationary UK TFP shock.** Light (dark) grey denotes 90% (86%) confidence bands. Units on the y-axis are percentage deviations.

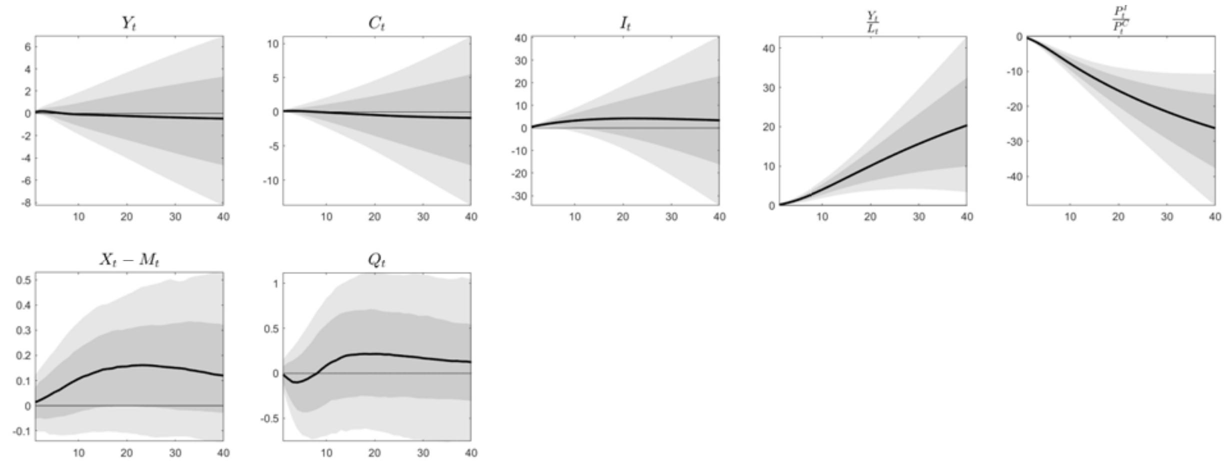

Figure 13: **Responses to a non-stationary UK IST shock.** Light (dark) grey denotes 90% (86%) confidence bands. Units on the y-axis are percentage deviations.

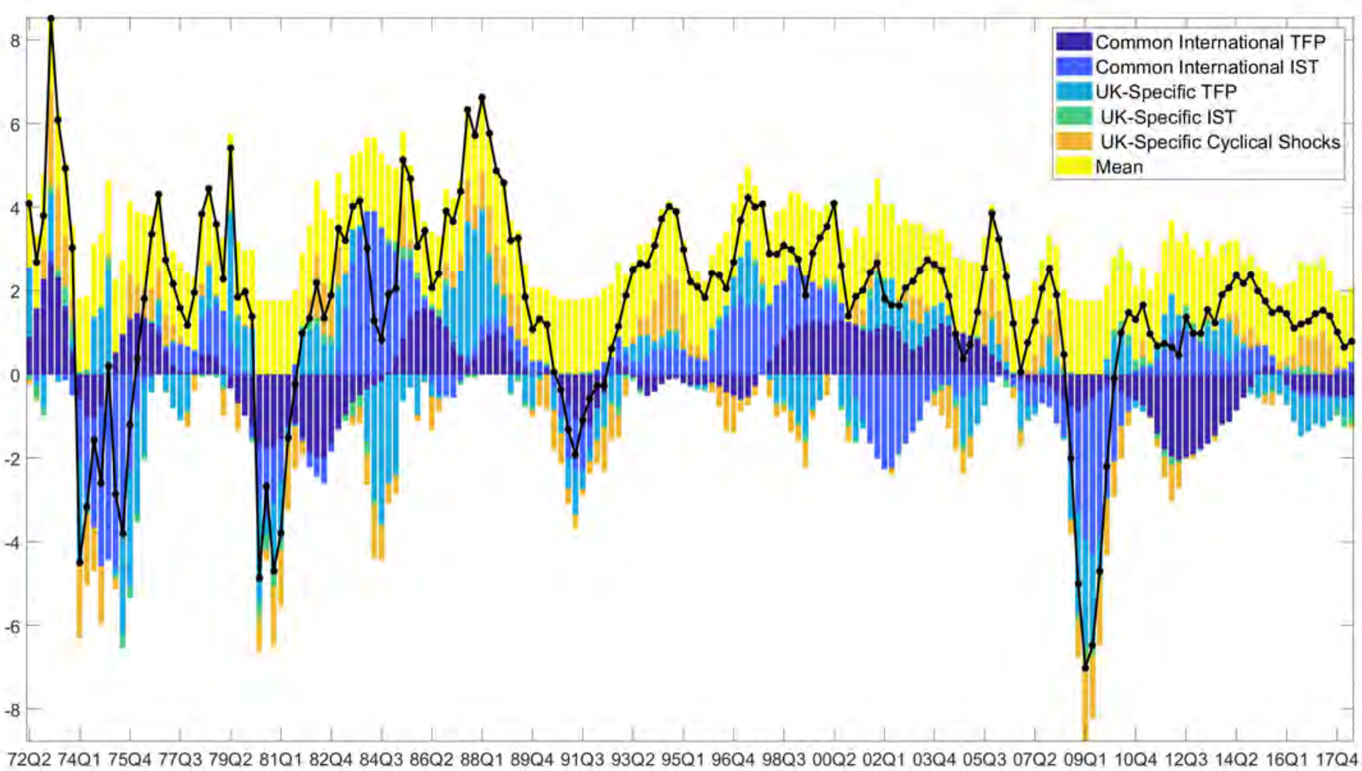

Figure 14: **Historical Decomposition of UK GDP growth.**

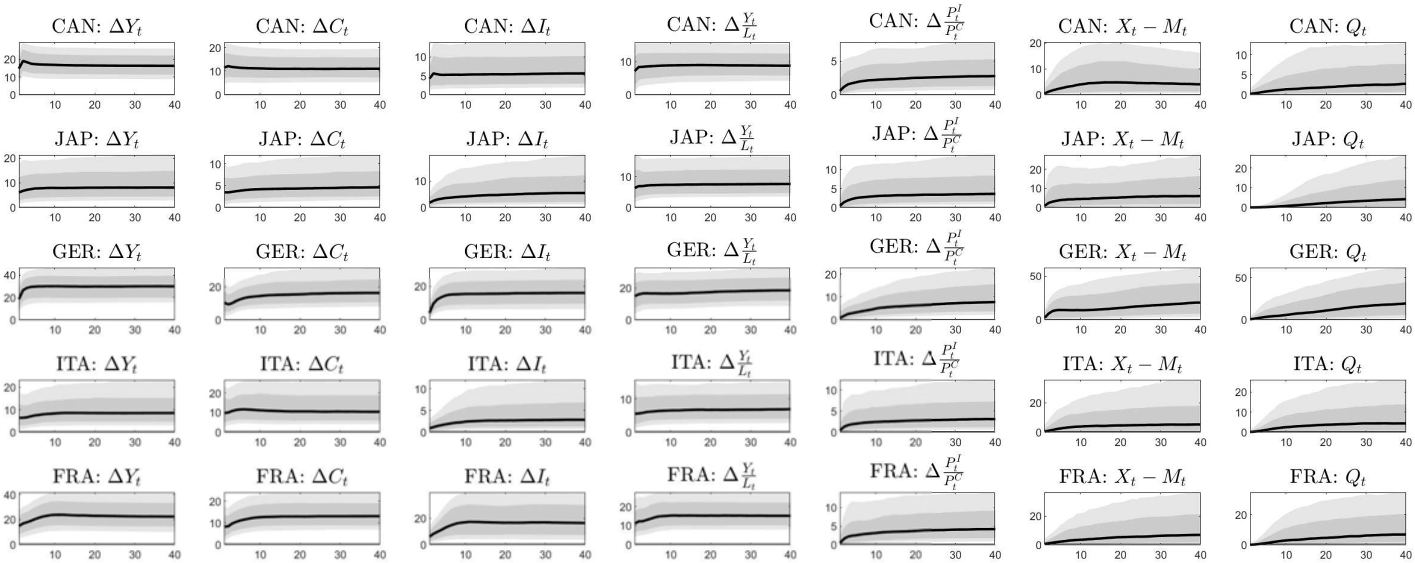

Figure 15: **International TFP Shock — Forecast Error Variance Decomposition for Selected Countries.** For each country the US has been considered as the foreign economy. Light (dark) grey denotes 90% (86%) confidence bands.

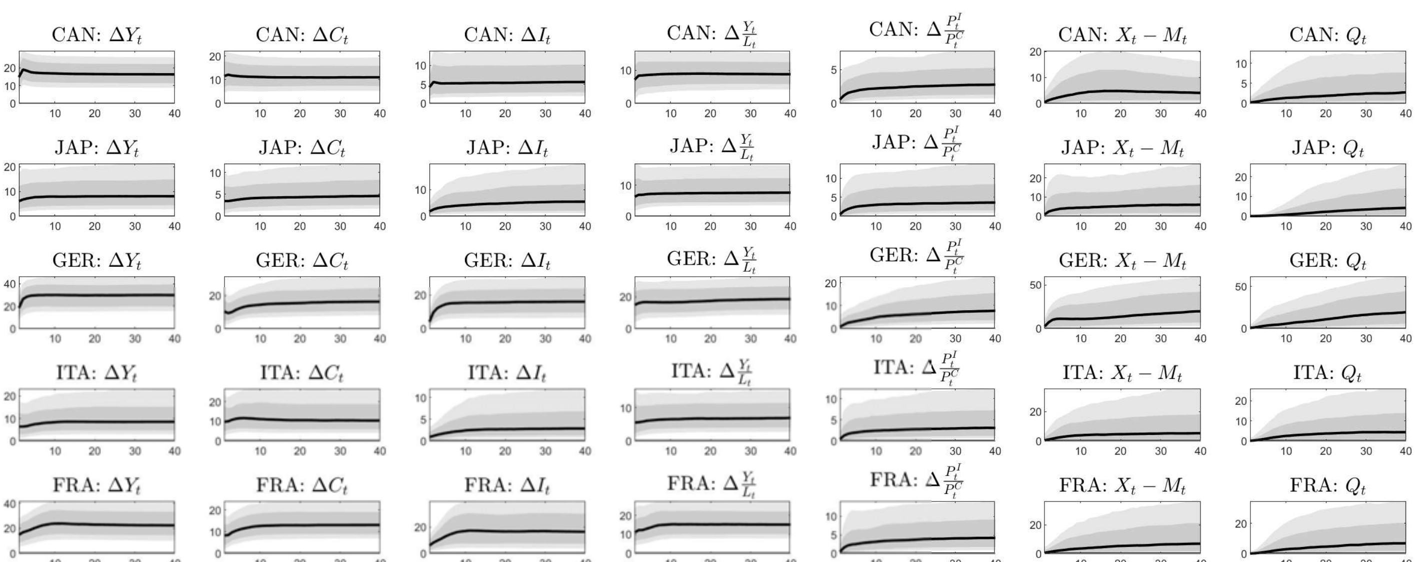

Figure 16: **International IST Shock — Forecast Error Variance Decomposition for Selected Countries.** For each country the US has been considered as the foreign economy. Light (dark) grey denotes 90% (86%) confidence bands.

B.1 Prior & Posterior VAR Parameter Moments

As proposed by Zha (1999) we exploit the block recursive structure of the VAR model – due to the small open economy restrictions and homogeneity of trends – and break the likelihood into $m = 6$ (two for the foreign trends, one for the foreign economy, two for the domestic trends and one for the domestic economy) blocks. As explained in Zha (1999), this is extremely convenient property as it allows to draw parameters block by block and preserving the zero restrictions. The VAR model can take the following form

$$\xi_t = \begin{bmatrix} \Delta \ln z_t^* \\ \Delta \ln v_t^* \\ \xi_t^* \\ \Delta \ln z_t \\ \Delta \ln v_t \\ \xi_t^{domestic} \end{bmatrix} = \begin{bmatrix} \psi_t^1 & & & & & \\ & \psi_t^2 & & & & \\ & & \psi_t^3 & & & \\ & & & \psi_t^4 & & \\ & & & & \psi_t^5 & \\ & & & & & \psi_t^6 \end{bmatrix} \begin{bmatrix} B^1 \\ B^2 \\ B^3 \\ B^4 \\ B^5 \\ B^1 \end{bmatrix} + \begin{bmatrix} \omega_t^1 \\ \omega_t^2 \\ \omega_t^3 \\ \omega_t^4 \\ \omega_t^5 \\ \omega_t^6 \end{bmatrix}$$

Each block can be expressed as

$$\xi_t^j = B^j \psi_t^j + \omega_t^j, \;\; \omega_t^j \sim \mathcal{N}(\mathbf{0}, \Sigma^j)$$

where ψ_t^j contains lags of ξ_t^j and it can also contains contemporenaous and lag values of ξ_t^{j-1}, ω_t^j is a $d\omega^j \times 1$ vector of reduced-form errors that is normally distributed with zero mean and covariance matrix Σ^j. The regression-equation representation of this system is

$$\Xi^j = \Psi^j B^j + U^j \tag{12}$$

where $\Xi = [\xi_{h+1}^j, ..., \xi_T^j]$ is a $d\xi^j \times T$ matrix, $\Psi^j = \begin{bmatrix} \Xi_{-h}^j & \Xi^{j-1} & \Xi_{-h}^{j-1} \end{bmatrix}$ is a $(Hd\xi^j + (H+1)\, d\xi^{j-1}) \times T$ matrix containing the h-th lag of Ξ^j, contemporaneous and lag values Ξ^{j-1}, $B^j = [B_1^j, ..., B_H^j]$ is a $d\xi^j \times (Hd\xi^j + (H+1)\, d\xi^{j-1})$ matrix, and $\Omega^j = [\omega_{h+1}^j, ..., \omega_T^j]$ is a $d\xi^j \times T$ matrix of disturbances.

The Bayesian estimation of VAR models has become standard in empirical macroeconomics. Specifically, we use a Minnesota-type prior (Doan et al., 1984b; Litterman, 1986). It is assumed that the prior distribution of the VAR parameters has a Normal-Wishart conjugate form

$$\beta^j | \Sigma^j \sim \mathcal{N}(\tilde{\beta}^j, \Sigma^j \otimes \tilde{F}^j), \;\; \Sigma^j \sim \mathcal{IW}(\kappa, \tilde{\Sigma}^j), \tag{13}$$

where β^j is obtained by stacking the columns of B^j, $\tilde{\beta}^j = vec\left(\tilde{B}^j\right)$ and $\tilde{\Sigma}_B^j = \Sigma^j \otimes \tilde{F}^j$. In contrast to

Litterman (1986), the covariance matrix Σ^j in the prior described in Equation (13) is not replaced by an estimated and thus known (diagonal) counterpart. Therefore, sampling from the conditional posterior distributions described below requires Gibbs sampling (see also Mumtaz and Zanetti, 2012). The (Minnesota) prior moments of β^j are given by

$$\mathbb{E}[(B_h^j), i, k] = \begin{cases} \delta_i^j & i = k, h = 1 \\ 0 & \text{otherwise} \end{cases} \quad , \quad Var[(B_h), i, h] = \lambda \left(\sigma_i^j\right)^2 / \left(\sigma_k^j\right)^2,$$

and, as outlined in Bańbura et al. (2010), they can be constructed using the following T_D^j dummy observations

$$Y_D^j = \begin{pmatrix} \frac{diag(\delta_1\sigma_1,...,\delta_{d\xi^j}\sigma_{d\xi^j})}{\lambda} \\ 0_{d\xi^j \times \left(Hd\xi^j + (H+1)d\xi^{j-1}\right)} \\ \cdots\cdots\cdots \\ diag(\sigma_1, ..., \sigma_{d\xi^j}) \\ \cdots\cdots\cdots \\ 0_{1\times d\xi^j} \end{pmatrix} \text{and } X_D^j = \begin{pmatrix} \frac{J_P \otimes diag(\sigma_1,...,\sigma_{d\xi^j})}{\lambda} \\ 0_{d\xi^j \times \left(Hd\xi^j + (H+1)d\xi^{j-1}\right)} \\ \cdots\cdots\cdots \\ 0_{1\times Hd\xi^j} \end{pmatrix},$$

where $J_P = diag(1, 2, ..., H)$ and $diag$ denotes the diagonal matrix. The prior moments in Equation (13) are functions of Y_D^j and X_D^j, $\tilde{B}^j = Y_D^j X_D^{j\prime} \left(X_D^j X_D^{j\prime}\right)^{-1}$, $\tilde{F}^j = (X_D X_D')^{-1}$, $\tilde{\Sigma}^j = (Y_D^j - \tilde{B}^j X_D^j)(Y_D^j - \tilde{B}^j X_D^j)'$ and $\kappa^j = T_D^j - d\xi^j H$. Finally, the hyper-parameter λ controls the tightness of the prior and our baseline choice is $\lambda = 2$.

Since the normal-inverse Wishart prior is conjugate, the conditional posterior distribution of this model is also normal-inverse Wishart (Kadiyala and Karlsson, 1997b)

$$\beta^j | \Sigma^j, \Xi^j, Z^j \sim \mathcal{N}(\bar{\beta}^j, \Sigma^j \otimes \bar{F}^j), \;\; \Sigma^j | \Xi^j, Z^j \sim \mathcal{IW}(\bar{v}^j, \bar{S}^j),$$

where variables with a bar denote the parameters of the posterior distribution. Defining $\hat{B}^j$ and $\hat{U}^j$ as the OLS estimates from Equation (12), the parameters of the conditional posterior distribution can be computed as $\bar{B}^j = \left(\left(\tilde{F}^j\right)^{-1} \tilde{S}^j + \Xi^j \Psi^{j\prime}\right) \left(\left(\tilde{F}^j\right)^{-1} + \left(\Psi^j \Psi^{j\prime}\right)^{-1}\right)^{-1}$, $\bar{F}^j = \left(\left(\tilde{F}^j\right)^{-1} + \left(\Psi^j \Psi^{j\prime}\right)^{-1}\right)^{-1}$, $\bar{\kappa}^j = \kappa^j + T$, and $\bar{S}^j = \hat{B}^j + \tilde{B}^j \left(\tilde{F}^j\right)^{-1} \left(\tilde{B}^j\right)' + S_0 + \hat{U}^j \left(\hat{U}^j\right)' - \bar{B}^j \left(\bar{F}^j\right)^{-1} \left(\bar{B}^j\right)'$. Lastly, as in Mumtaz and Zanetti (2012), the values of the persistence parameter δ_i^j and the error standard deviation σ_i^j of the AR(1) model are obtained from its OLS estimation.

B.2 Gibbs Sampler Steps

Given C, A and starting values for B and Σ

For i = 1,...,Nsimulations:

1. Use Durbin and Koopman (2002) algorithm to draw ξ_t^i conditional on ζ_t, C, A, B^{i-1} and Σ^{i-1}

2. For j = 1,...,m

   - Draw $\left(B^j\right)^i$ and $\left(\Sigma^j\right)^i$ from Normal-Inverse-Wishart distribution using the moments discussed in the previous section conditional on ζ_t, ξ_t^i, C, A, $\left(B^{j-1}\right)^i$ and $\left(\Sigma^{j-1}\right)^i$

3. If the maximum absolute value of B is less than one move to step 1 otherwise repeat step 2

Discard the Nburn first draws and use the remaining Nsimulations-Nburn draws to infer the statistics.