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Keywords

Production networks, Hulten's theorem, disaggregated macroeconomic models

JEL Classification

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Abstract

In a broad class of macroeconomic models with production networks, it is difficult to discern which set of structural relationships between sectors amplify shocks and shape aggregate outcomes. As a remedy, we provide a formula that sidesteps this issue by considering linkages in isolation, thereby quantifying the macroeconomic significance of specific intersectoral relationships. In an application, we specialize our framework to derive a closed-form expression for network spillovers in efficient economies, where network spillovers are defined as the effect of shocks on GDP due to propagation to other sectors. Empirically, we find significant fluctuations in network spillovers for 43 countries between 2000 and 2014, suggesting this channel to be a key driver of macroeconomic outcomes. In a second application, we quantify the gains of having different hypothetical input-output structures, keeping the final expenditure shares of goods and services the same. We find the United States' growth rate would have been almost 20 percent higher per year had its input-output architecture been identical to China's.

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1 Introduction

At least since Leontief (1936), economists have recognized the importance of systematically quantifying interrelationships among sectors of the macroeconomy. These interdependencies are typically captured by structural input-output coefficients, which summarize the proportional input requirements of a given sector. In the 1980s, macroeconomists began building multi-sector models that captured this vast web of relationships formally and, in turn, succeeded in bringing quantitative real business cycle modeling closer to reality (see, for example, Long and Plosser, 1983). More recent developments in macroeconomics have highlighted the importance of specific structural properties of the production network in propagating sector-specific shocks to macroeconomic aggregates (Acemoglu et al., 2012; Atalay, 2017; Baqaee and Farhi, 2019). However, it is often difficult to discern which set of structural relationships are responsible for amplifying microeconomic shocks in these models.

We provide a theoretical framework that sidesteps this issue by considering linkages in isolation. Specifically, we derive an analytical formula that computes the macroeconomic impact of sector-specific shocks under any arbitrary linear transformation of the input-output network. By isolating specific structural relationships, our formula measures the extent to which these linkages amplify shocks. Our point of departure is the foundational theorem of Hulten (1978), which states that in efficient economies and under minimal assumptions, the effect on GDP of a microeconomic productivity shock to sector i is given by i's sales as a fraction of nominal GDP:

$$\frac{d\log Y}{d\log A_i} = \lambda_i.$$

where $d\log Y$ is real GDP growth, $d\log A_i$ is the shock, and λ_i is i's sales share or Domar weight. The power of Hulten's theorem lies in its simplicity: Domar weights envelop a substantial amount of information about an economy's underlying production network, microeconomic elasticities of substitution, and the extent of returns to scale. However, since Domar weights are reduced-form objects, simply observing the sales distribution cannot provide insight into how specific structural interdependencies shape macroeconomic outcomes. For example, two sectors with similar sales can propagate shocks through the economy in very different ways. By contrast, our formula measures exactly the extent of propagation attributed to specific linkages.

Methodologically, our framework compares *actual* Domar weights (which are directly observable) with *counterfactual* Domar weights that correspond to the transformed production network. In its most general form, our formula is defined in terms of both observed and counterfactual Domar weights:

$$\zeta \equiv \sum_{i=1}^{N} \left(\lambda_i - \tilde{\lambda}_i\right) \tag{1}$$

where $\tilde{\lambda}_i$ is sector i's Domar weight under the counterfactual production structure. Since the counterfactual Domar weights are unobservable *ex-ante*, we characterize them in terms of observable equilibrium objects, permitting the computation of ζ. We derive our formula in the context of a standard neoclassical CES production network model à la Atalay (2017), Baqaee and Farhi (2019), and Carvalho et al. (2020). By setting up a parametric model and deriving our formula through this lens, we explicitly show how to map from model primitives to equilibrium objects, highlighting the assumptions required to operationalize our formula.

As proof of concept, we apply our formula to study the role of network spillovers in explaining cross-country differences in economic growth rates. We define network spillovers as the indirect effect of shocks on GDP, that is, through output spillovers to other sectors. Indeed, the second contribution of our paper is the derivation of an exact nonparametric analytical expression that measures network spillovers in efficient economies. Crucially, this expression only requires data on bilateral intermediate sales between sectors, the household's nominal final consumption expenditure, and the pre-shock level of nominal GDP, making its computation straightforward. Remarkably, knowledge of the elasticities of substitution in production and consumption is not required to compute network spillovers, relating this measure to the nonparametric results of the theoretical macro-networks literature (for example Liu, 2019, Bigio and La'O, 2020, and Baqaee and Farhi, 2019, 2020b, 2021b, among others). Network spillovers matter because they explain much of the variation in economies' input-output multipliers (which, following the definition of Baqaee and Farhi (2019), is a measure of the macroeconomy's sensitivity to shocks). Using input-output data for 43 countries between 2000 and 2014, we show empirically that economies with more substantial network spillovers typically have higher economic growth rates, suggesting network spillovers to be a powerful mechanism driving aggregate outcomes.

In a second application, we use our formula to investigate whether economies would gain by having different hypothetical input-output structures, keeping the final expenditure shares of goods and services the same. To this end, we select a reference country and compute all other economies' hypothetical growth rates using the reference country's input-output structure. For the same sample as the previous application, we find that all economies would have had higher economic growth rates had their input-output architecture been identical to China's. For instance, the United States' growth rate would have been, on average, 18.5 percent higher per annum between 2000 and 2014. While we only consider two applications in the paper, we stress that our formula can predict how GDP would respond to shocks given any arbitrary set of structural relationships between sectors. Thus the generality of our approach goes beyond the two aforementioned applications.

Our article relates to the literature on growth accounting and production networks. Hulten (1978) provided the economic rationale for using Domar aggregation to measure changes in aggregate TFP; in the presence of intermediate inputs, sales shares (not value-added) are the correct weights for aggregating microeconomic productivity changes. Hulten's result was in contrast to Solow (1957), who began with an aggregate production function and measured changes in TFP as the residual change in output after accounting for the growth of factor inputs.

Hulten's theorem is also the benchmark result in the macroeconomic literature on production networks.[1] A seminal paper in this literature is Acemoglu et al. (2012), which shows how Domar weights relate to the economy's input-output network and characterizes how idiosyncratic shocks can propagate through linkages, leading to volatility cascades. The Leontief inverse, which plays a central role in the production networks literature, provides detailed information about the direct and indirect linkages between sectors. Domar weights compress this information, preventing an analysis of how productivity shocks interact with the network structure. By contrast, our formula allows us to directly study the relationship between network structure and aggregate outcomes.

Building on Hulten (1978) and Acemoglu et al. (2012), Baqaee and Farhi (2019) show that nonlinearities in production matter quantitatively for a range of macroeconomic phenomena. They show that microeconomic primitives are important for aggregate output beyond the first-order of approximation in economies with intermediate inputs. We limit our attention to efficient economies and first-order approximations where the input-output matrix does not respond endogenously to shocks. Allowing for second-order effects complicates our approach because the primitive input-output objects we consider will no longer coincide with their general equilibrium counterparts. Thus to highlight the key ideas of our approach, we deal only with first-order approximations. Additionally, we abstract from inefficiencies to highlight how our formula applies in the simplest setting, where Hulten's theorem indeed characterizes the macroeconomic impact of microeconomic shocks. Notably, papers studying production networks in inefficient economies have shown that sales shares are insufficient for ag-

[1] See Carvalho (2014) and Carvalho and Tahbaz-Salehi (2019) for a detailed overview of the production networks literature.

gregating microeconomic shocks. For example, Baqaee and Farhi (2020b) show that in the presence of distortions, cost-based Domar weights are the correct statistics.[2] A natural next step is to recharacterize our results in the presence of inefficient equilibria.

Our paper also relates to the recent literature in macroeconomics that has sought to understand how input-output linkages contribute to aggregate volatility (see, for example, Foerster et al., 2011, Acemoglu et al., 2012, Di Giovanni et al., 2014, Acemoglu et al., 2017, Atalay, 2017, Grassi, 2017, Baqaee, 2018, and Altinoglu, 2021).[3] A running theme in this literature is that input-output linkages are a powerful mechanism for generating aggregate volatility. Our empirical results complement this literature by highlighting the outsized role of indirect propagation in dertemining macroeconomic outcomes.

Tangential to our article is the set of papers that use quasi-experiments to investigate the propagation of microeconomic shocks through input-output linkages. For example, Barrot and Sauvagnat (2016), Boehm et al. (2019), and Carvalho et al. (2020) study how natural disasters propagate through input-output linkages, finding disasters cause substantial output losses both up and down supply chains. Relatedly, Caliendo et al. (2017) find that sectoral productivity changes spread through interregional trade linkages across US states. Furthermore, Baqaee and Farhi (2020a); Baqaee et al. (2020), and Barrot et al. (2021) use production network models to quantify the macroeconomic impact of Covid-19, finding heterogeneous supply and demand shocks to propagate through the economy due to intermediate input requirements. By contrast, vom Lehn and Winberry (2021) focus on investment networks, showing that intersectoral investment linkages have driven the decline in labor productivity since the 1980s. Finally, Chahrour et al. (2021) explore the relationship between news media and sectoral labor demand in a network model, finding some sectors make suboptimal hiring decisions in response to news updates. These papers (among others) all strengthen the rationale for studying macroeconomic phenomena through the lens of production network models.

The rest of the paper proceeds as follows. Section 2, sets up the model and characterizes ζ in terms of observables for any linear transformation of the structural input-output matrix. In section 3, we present two applications of our formula. First, we study the role of network spillovers in shaping aggregate outcomes. Then, we investigate whether economies would gain if their production networks were organized like

[2]Other papers that study distortions in network models include Jones (2011), Liu (2019), Bigio and La'O (2020) and Baqaee and Farhi (2021b).

[3]These papers in turn relate to the older macroeconomics literature that studies aggregate volatility in multi-sector models such as Long and Plosser (1983), Horvath (1998, 2000), Dupor (1999) and Shea (2002).

China's. Section 4 concludes. Proofs and supplementary results are relegated to the Appendix.

2 Model

In this section, we set up a general equilibrium model in the spirit of Hulten (1978), Acemoglu et al. (2012), Baqaee and Farhi (2019) and Carvalho et al. (2020) to characterize equation (1) in terms of observable equilibrium objects.[4] The resulting formula is the central focus of the paper. We consider a static economy with N competitive sectors that each produce one distinct product using some combination of labor and intermediate goods. Each sector's output can either be consumed by households as final demand, or by other sectors as an intermediate input. We define a matrix $\boldsymbol{\omega} \equiv [\omega_{ij}]$ that captures each sector's reliance on intermediates from other sectors. For example, an element $\omega_{mk} \geq 0$ captures the importance of good k in the production of good m. If $\omega_{mk} = 0$, then sector m does not rely on sector k's output. We refer to $\boldsymbol{\omega}$ as the *primitive input-output matrix* so as to distinguish it from its general equilibrium counterpart. Notably, each row of $\boldsymbol{\omega}$ sums to less than one due to the fact that sectors also use labor to produce. We also define a matrix $\tilde{\boldsymbol{\omega}} \equiv T(\boldsymbol{\omega})$, where $T(\boldsymbol{\omega})$ denotes some linear transformation of $\boldsymbol{\omega}$ satisfying $\tilde{\omega}_{ij} \geq 0$, $\sum_{j=1}^{N} \tilde{\omega}_{ij} \leq 1$. Furthermore, let $\boldsymbol{\mu} \equiv [\mu_i]$ be a vector that captures the importance of labor in each sector's production. Note that like $\boldsymbol{\omega}$, the vector $\boldsymbol{\mu}$ is a primitive object. Finally, let $\tilde{\boldsymbol{\mu}} \equiv [\tilde{\mu}_i]$ be the vector of labor shares that correspond to the matrix $\tilde{\boldsymbol{\omega}}$. Since $\tilde{\boldsymbol{\omega}}$ can have row sums that are different from $\boldsymbol{\omega}$, the vector $\tilde{\boldsymbol{\mu}}$ correspondingly adjusts so that, collectively, the objects $\boldsymbol{\omega}$, $\tilde{\boldsymbol{\omega}}$, $\boldsymbol{\mu}$, and $\tilde{\boldsymbol{\mu}}$ satisfy $\mu_i \geq 0$, $\tilde{\mu}_i \geq 0$, and $\sum_{j=1}^{N} \tilde{\omega}_{ij} + \tilde{\mu}_i = \sum_{j=1}^{N} \omega_{ij} + \mu_i = 1$ for all i.

Throughout the paper, we refer to the economy defined by the tuple $(\boldsymbol{\omega}, \boldsymbol{\mu})$ as the *actual* (or *observed*) economy. By contrast, the tuple $(\tilde{\boldsymbol{\omega}}, \tilde{\boldsymbol{\mu}})$ defines the *counterfactual* economy. In what follows, the notation $\tilde{X}$ typically denotes an endogenous variable X in the counterfactual economy.[5]

Real GDP in the observed and counterfactual economy is defined as the maximizer of a CES aggregator of final demand for individual goods

[4]Our model is most similar to the CES production network model of Carvalho et al. (2020). However, unlike Carvalho et al. (2020) we do not specify nested CES production technologies in intermediate inputs.

[5]Except for the objects $\tilde{\boldsymbol{\omega}}$ and $\tilde{\boldsymbol{\mu}}$, all other tilde variables are endogenous.

$$Y=\left(\sum_{i=1}^{N}a_i^{\frac{1}{\sigma}}c_i^{\frac{\sigma-1}{\sigma}}\right)^{\frac{\sigma}{\sigma-1}} \quad \text{and} \quad \tilde{Y}=\left(\sum_{i=1}^{N}a_i^{\frac{1}{\sigma}}\tilde{c}_i^{\frac{\sigma-1}{\sigma}}\right)^{\frac{\sigma}{\sigma-1}},$$

where Y is real consumption (or "output"), a_i is the importance of good i in the representative household's consumption bundle, c_i is the final consumption of good i, and σ is the elasticity of substitution between final goods. The price index associated with Y is given by

$$P=\left(\sum_{i=1}^{N}a_i p_i^{1-\sigma}\right)^{\frac{1}{1-\sigma}}$$

where p_i is the price of good i.[6]

Households have log utility over aggregate consumption,

$$U(Y)=\log Y,$$

and maximize their utility subject to $w=\sum_{i=1}^{N}p_i c_i$.[7] Both sides of the budget constraint correspond to nominal GDP in this model since households supply one unit of labor inelastically.

Sectoral production is described by constant returns CES production technologies that transform labor and intermediate goods into output

$$y_i=A_i\left(\mu_i^{\frac{1}{\theta}}l_i^{\frac{\theta-1}{\theta}}+\sum_{j=1}^{N}\omega_{ij}^{\frac{1}{\theta}}x_{ij}^{\frac{\theta-1}{\theta}}\right)^{\frac{\theta}{\theta-1}} \quad \text{and} \quad \tilde{y}_i=A_i\left(\tilde{\mu}_i^{\frac{1}{\theta}}\tilde{l}_i^{\frac{\theta-1}{\theta}}+\sum_{j=1}^{N}\tilde{\omega}_{ij}^{\frac{1}{\theta}}\tilde{x}_{ij}^{\frac{\theta-1}{\theta}}\right)^{\frac{\theta}{\theta-1}}$$

where y_i is sector i's real gross output, A_i is a Hicks-neutral productivity shifter, l_i is labor and x_{ij} is i's use of intermediates from j. In section 3, we impose an explicit structure on the counterfactual input-output network to characterize network spillovers.

Sector i's profits are

$$\pi_i=p_i y_i-wl_i-\sum_{j=1}^{N}p_j x_{ij},$$

[6] Similarly, the price index in the counterfactual economy is

$$\tilde{P}=\left(\sum_{i=1}^{N}a_i\tilde{p}_i^{1-\sigma}\right)^{\frac{1}{1-\sigma}}.$$

[7] $\tilde{U}\left(\tilde{Y}\right)$ is the level of utility and $\tilde{w}$ is the market wage under the counterfactual.

and the goods and labor market-clearing conditions are given by

$$y_i = c_i + \sum_{j=1}^{N} x_{ji} \quad \text{and} \quad l = \sum_{i=1}^{N} l_i = 1$$

for goods $1 \leq i \leq N$.[8] Note that aggregate labor supply l is exogenous, inelastic, and normalized to 1 in both economies.

The competitive equilibrium is defined in the usual way, where all agents take prices as given, and markets for labor and goods $1 \leq i \leq N$ clear.

We must also introduce a few objects that play a central role in our analysis. Firstly, let $\mathbf{b}$ be an $N \times 1$ vector of equilibrium *final expenditure shares* where an element b_i is defined

$$b_i \equiv \frac{p_i c_i}{\sum_{j=1}^{N} p_j c_j}.$$

The denominator of the above expression is nominal GDP and $\sum_{i=1}^{N} b_i = 1$.

Secondly, let $\boldsymbol{\lambda} \equiv [\lambda_i]$ be an $N \times 1$ vector of Domar weights where

$$\lambda_i = \frac{p_i y_i}{\sum_{j=1}^{N} p_j c_j}$$

and $\sum_{i=1}^{N} \lambda_i \geq 1$ in the presence of intermediate inputs. Similarly, under the counterfactual we have $\tilde{\mathbf{b}} \equiv \left[\tilde{b}_i\right]$ and $\tilde{\boldsymbol{\lambda}} \equiv \left[\tilde{\lambda}_i\right]$, both of which are defined as above but with counterfactual prices and quantities $\{\tilde{p}_i\}_{i=1}^{N}$, $\{\tilde{y}_i\}_{i=1}^{N}$, and $\{\tilde{c}_i\}_{i=1}^{N}$. Furthermore, we define the $N \times N$ equilibrium input-output matrix $\boldsymbol{\Omega} \equiv \left[\Omega_{ij}\right]$ where

$$\Omega_{ij} = \frac{p_j x_{ij}}{p_i y_i}.$$

Following Baqaee and Farhi (2019), and Carvalho et al. (2020), the Leontief inverse associated with $\boldsymbol{\Omega}$ is defined

$$\boldsymbol{\Psi} \equiv (I - \boldsymbol{\Omega})^{-1} \equiv \left[\psi_{ij}\right].$$

Intuitively, a typical element of this matrix, ψ_{ij}, captures both the direct and indirect ways through which sector i uses sector j's output.[9] Indeed, the Leontief inverse

[8] Since prices and quantities are endogenous, profits in the counterfactual economy are given by $\tilde{\pi}_i = \tilde{p}_i \tilde{y}_i - \tilde{w}\tilde{l}_i - \sum_{j=1}^{N} \tilde{p}_j \tilde{x}_{ij}$. Similarly, the goods and factor market-clearing conditions are, respectively, $\tilde{y}_i = \tilde{c}_i + \sum_{j=1}^{N} \tilde{x}_{ji}$ and $l = \sum_{i=1}^{N} \tilde{l}_i$.

[9] See Carvalho and Tahbaz-Salehi (2019) for a more detailed discussion of the Leontief inverse matrix

summarizes all production chains in the economy.

In a similar manner, let $\tilde{\mathbf{\Omega}} \equiv \left[\tilde{\Omega}_{ij}\right]$ where

$$\tilde{\Omega}_{ij} = \frac{\tilde{p}_j \tilde{x}_{ij}}{\tilde{p}_i \tilde{y}_i}$$

and $\tilde{\mathbf{\Psi}} \equiv \left(I - \tilde{\mathbf{\Omega}}\right)^{-1} \equiv \left[\tilde{\psi}_{ij}\right]$. Finally, we define $N \times 1$ vectors of equilibrium labor expenditure shares as $\mathbf{\Lambda} \equiv [\Lambda_i]$ and $\tilde{\mathbf{\Lambda}} \equiv \left[\tilde{\Lambda}_i\right]$, where

$$\Lambda_i = \frac{w l_i}{p_i y_i} \quad \text{and} \quad \tilde{\Lambda}_i = \frac{\tilde{w} \tilde{l}_i}{\tilde{p}_i \tilde{y}_i}.$$

The goods market-clearing condition relates the vector of Domar weights to the Leontief inverse via

$$p_i y_i = p_i c_i + \sum_{j=1}^{N} p_i x_{ji} = b_i \left(\sum_{j=1}^{N} p_j c_j \right) + \sum_{j=1}^{N} \Omega_{ji} p_j y_j$$

and thus

$$\boldsymbol{\lambda}' = \mathbf{b}' \mathbf{\Psi}. \tag{2}$$

Equation (2) shows that the Domar weights (λ_i's) encode all weighted paths of any length from sector i to final demand. The economy's intersectoral production network, which is captured by $\mathbf{\Psi}$, is therefore embodied in the Domar weights. As we shall see, equation (2) is directly related to our formula. Domar weights play a central role in our analysis and are the focus of Hulten's (1978) theorem, which we are now in a position to introduce.

THEOREM 1—Hulten (1978): *The first-order macroeconomic impact of a microeconomic productivity shock is*

$$\frac{d \log Y}{d \log A_i} = \lambda_i, \tag{3}$$

where λ_i is the Domar weight of sector i.

Hulten's theorem is the natural starting point for our analysis since it characterizes how real GDP responds to microeconomic productivity shocks in efficient economies with intermediate goods. For our purposes, equation (3) relates to the economy defined by $(\boldsymbol{\omega}, \boldsymbol{\mu})$. Therefore, in the counterfactual economy, we have

and its properties.

$$\frac{d\log\tilde{Y}}{d\log A_i} = \tilde{\lambda}_i.$$

Remarkably, the theorem states that Domar weights are sufficient for characterizing the macroeconomic impact of microeconomic shocks; elasticities of substitution, the extent of returns to scale, and properties of the production network are not required to compute the elasticity in (3). Furthermore, since sales data is readily observable at various levels of disaggregation, calculating the Domar weights is straightforward.

With Hulten's theorem in hand, we now define (in its most general form) the central object of the paper, ζ, which measures the extent to which a counterfactual production network amplifies (or mitigates) productivity shocks relative to the observed production network.

DEFINITION 1: *The general form of the measure ζ is given by*

$$\zeta \equiv \sum_{i=1}^{N}\left(\lambda_i - \tilde{\lambda}_i\right). \tag{4}$$

Using the terminology of Baqaee and Farhi (2019), equation (4) is the difference in the *input-output multipliers* of the actual and counterfactual economies. The economy's input-output multiplier, which is given by $\xi \equiv \sum_{i=1}^{N} \frac{d\log Y}{d\log A_i} = \sum_{i=1}^{N} \lambda_i$, captures the percentage change in real GDP in response to a uniform one-percent increase in productivity.[10] Intuitively, equation (4) measures the role of the *network structure* in determining how the macroeconomy responds to a uniform one-percent increase in technology. This is because any difference between Domar weights $\{\lambda_i\}_{i=1}^{N}$ and $\{\tilde{\lambda}_i\}_{i=1}^{N}$ is attributed solely to dissimilarities in the underlying primitive input-output networks that give rise to the distribution of sales. By taking the difference between the Domar weights $\tilde{\lambda}_i$ and λ_i, ζ measures how the counterfactual network structure amplifies productivity shocks relative to the actual economy. Thus the measure ζ quantifies the macroeconomic importance of the specific structural linkages under study.

The key challenge in measuring ζ is that the counterfactual Domar weights are unobserved. Therefore in order to compute ζ, we must express equation (4) solely in terms of observable objects, which rests on two assumptions.

ASSUMPTION 1: *Consumption weights are equal in the actual and counterfactual*

[10] The input-output multiplier is also related to the "granular residual" of Gabaix (2011), the intermediate input multiplier of Jones (2011) and the measure of network influence of Acemoglu et al. (2012).

economies

$$a_i = \tilde{a}_i \quad \text{for all } i.$$

ASSUMPTION 2: *Exogenous technology levels are equal and normalized to one in the steady-state*

$$(A_1, ..., A_N) = \left(\tilde{A}_1, ..., \tilde{A}_N\right) = (1, ..., 1).$$

Assumption 1 ensures that counterfactual final expenditure shares coincide with their observed counterparts for all final goods in the economy, or $\tilde{b}_i = b_i$ for all i, allowing us to isolate the role of the production network in shaping fluctuations in real GDP. Assumption 2 defines the steady-state condition, where all technology levels are normalized to one. In steady-state, all prices (including the wage) are equal, meaning the objects $\boldsymbol{\omega}$, $\boldsymbol{\mu}$ and $\mathbf{a}$ coincide with $\boldsymbol{\Omega}$, $\boldsymbol{\Lambda}$ and $\mathbf{b}$, respectively. This is also true for the counterfactual economy, that is $\tilde{\boldsymbol{\omega}} = \tilde{\boldsymbol{\Omega}}$, $\tilde{\boldsymbol{\mu}} = \tilde{\boldsymbol{\Lambda}}$ and $\tilde{\mathbf{a}} = \tilde{\mathbf{b}}$. Since the counterfactual network is simply a linear transformation of the observed network, it then follows that $\tilde{\boldsymbol{\omega}} = T(\boldsymbol{\omega}) = T(\boldsymbol{\Omega})$ in steady-state. Therefore, any counterfactual production network can always be expressed in terms of general equilibrium input-output coefficients (as captured by $\boldsymbol{\Omega}$). Assumption 2 also ensures the initial level of real GDP is equal in both economies $(\boldsymbol{\omega}, \boldsymbol{\mu})$ and $(\tilde{\boldsymbol{\omega}}, \tilde{\boldsymbol{\mu}})$, which is important because we are dealing with relative changes in real GDP. Finally, we note that the elasticities of substitution θ and σ need not be equal to the counterfactual elasticities $\tilde{\theta}$ and $\tilde{\sigma}$, however to simplify the notation we set $\theta = \tilde{\theta}$ and $\sigma = \tilde{\sigma}$ but acknowledge that this does not bear materially on our results. For the rest of the paper, we consider Hicks-neutral productivity shocks around the steady-state without loss of generality following Baqaee and Farhi (2019), and Carvalho et al. (2020).

In light of assumptions 1 and 2, the following proposition formalizes the idea that equation (4) can always be characterized in terms of *observable* input-output objects, as long as the transformation $\tilde{\boldsymbol{\omega}} = T(\boldsymbol{\omega})$ satisfies $\tilde{\mu}_i + \sum_{j=1}^{N} \tilde{\omega}_{ij} = 1$ for all i, and $\tilde{\omega}_{ij} \geq 0, \tilde{\mu}_i \geq 0$.

PROPOSITION 1: *For any linear transformation of the matrix $\boldsymbol{\omega}$, the measure ζ is given by*

$$\zeta = \sum_{i,j=1}^{N} b_j \left(\boldsymbol{\Psi} - (I - T(\boldsymbol{\Omega}))^{-1}\right)_{ji}, \tag{5}$$

where $\boldsymbol{\Omega}$ is the observed equilibrium input-output matrix.

Proof. See Appendix.

First note that equation (5) is related to the identity given by (2) which states that $\boldsymbol{\lambda}' = \mathbf{b}'\boldsymbol{\Psi} \equiv \left[\sum_{j=1}^{N} b_j \psi_{ji}\right]$. Indeed, (5) can also be expressed as $\zeta = \sum_{i,j=1}^{N} b_j \psi_{ji} - \sum_{i,j=1}^{N} b_j \tilde{\psi}_{ji}$, where $\tilde{\psi}_{ji}$ is the jith element of counterfactual Leontief inverse. It is then immediately clear that the only component of λ_i and $\tilde{\lambda}_i$ that differs is the production network (as summarized by the Leontief inverse); the final expenditure shares are constant under both $(\boldsymbol{\omega}, \boldsymbol{\mu})$ and $(\tilde{\boldsymbol{\omega}}, \tilde{\boldsymbol{\mu}})$. Remarkably, proposition 1 shows that ζ does not rely on knowledge of the elasticities of substitution θ and σ. Only the observed final expenditure shares and equilibrium input-output matrix are required to measure ζ, making its computation straightforward.

In the empirical applications of the following section, we consider specialized versions of (5). First, we discipline our formula to measure network spillovers in the presence of efficient equilibria. Then, in the second application, we quantify economies' hypothetical gain in GDP growth if their input-output architecture was identical to China's.

3 Empirical Applications

In this section, and as proof of concept, we present two applications of the framework outlined in section 2. First, we show that the extent of output spillovers to neighboring sectors explains patterns in cross-country growth rates for 43 countries between 2000 and 2014. To this end, we specialize equation (5) to provide an exact analytical expression for network spillovers and show how this channel is significant for macroeconomic outcomes. In the second application, for the same sample of countries and time period, we quantify the gains economies would experience by having an input-output structure identical to China's, finding all countries to be better off under China's structure.

3.1 Data

We use the 2016 release of the World Input-Output Database (WIOD) for our empirical analysis (see Timmer et al., 2015 for an overview of the WIOD data). The dataset contains information on gross output, value-added, factor compensation, final expenditures, and intermediate input flows for 43 countries over the period 2000-2014.[11] The WIOD data is disaggregated into 56 sectors based on the International Standard Industrial Classification Revision 4 (ISIC Rev. 4). The block-diagonal of each input-output table captures domestic intermediate input transactions for each country, whereas the off-diagonal relates to the flow of intermediates between countries. For our purposes,

[11]The WIOD also contains a model for the rest of the world, which we omit from our analysis.

we focus on domestic transactions and abstract from international trade.[12] Each element of the equilibrium input-output matrix is given by the nominal expenditure by sector i on sector j's product, as a share of i's total *domestic* nominal expenditure (thus excluding spending on imported inputs). Constructing the input-output matrices this way ensures that factor compensation plus domestic intermediate input expenditure equals nominal GDP for each sector. Additionally, sectoral final expenditure shares are computed as the sum of household and government final consumption expenditure plus gross fixed capital formation as a fraction of nominal GDP. We ignore changes in inventories as these can lead to negative final expenditure shares if inventories are drawn down over the period.

We supplement the WIOD data with data from the Penn World Tables (Version 10.0). Specifically, we use data for real GDP and TFP as well as country-specific economic information used as controls in our regressions. After merging with the WIOD data, we are left with a sample of 43 countries from 2000 to 2014.[13]

3.2 Application I: Network Spillovers and the Macroeconomy

Our first empirical application studies the role of network spillovers in shaping aggregate outcomes, where network spillovers are defined as the effect of shocks on GDP via propagation to other sectors of the economy. In this application, we highlight the importance of input-output linkages emanating from a given sector, and quantify the extent to which these linkages amplify shocks.

3.2.1 Input-output multipliers and GDP growth

Before defining network spillovers formally, we establish a positive empirical relationship between economies' input-output multipliers and the rate of economic growth. Recall from the previous section that the input-output multiplier ξ captures the percentage change in real GDP in response to a uniform one-percent increase in productivity. It is thus a measure of the economy's sensitivity to macroeconomic productivity shocks. Theoretically, a higher input-output multipler means that GDP growth will be higher for a given change in TFP. Therefore, to test this prediction we estimate panel regressions of the form

$$\text{GDP Growth}_{ct} = \alpha + \beta_\xi * \xi_{ct} + \beta_A * \text{TFP Growth}_{ct} + \mathbf{X}_{ct} + \gamma_c + \delta_t + \varepsilon_{ct} \tag{6}$$

[12]See Baqaee and Farhi (2021a) for a detailed treatment of how Hulten's theorem generalizes to open economies.

[13]Only the "rest-of-world" data remains unmatched.

Table 1: Sensitivity to Shocks and GDP Growth

	(1)	(2)	(3)	(4)
ξ_{ct} (Input-output multiplier)	4.793***	2.806***	2.788**	4.021***
	[0.669]	[0.400]	[1.163]	[1.172]
TFP Growth$_{ct}$		1.157***	1.032***	1.054***
		[0.033]	[0.035]	[0.036]
Country FE	No	No	Yes	Yes
Year FE	No	No	Yes	Yes
Controls	No	No	No	Yes
Observations	645	645	645	644
Adjusted R^2	0.072	0.676	0.770	0.787

Notes: This table reports estimates from regression specification (6). The dependent variable in all columns is the annual rate of GDP growth. The independent variable ξ_{ct} is the input-output multiplier, and TFP Growth$_{ct}$ is the annual rate of total factor productivity growth. The regression in column (4) includes a set of control variables consisting of the national population (in millions), average annual hours worked by persons engaged, a human capital index (based on Barro and Lee, 2013), the real internal rate of return on capital, the average depreciation rate of the capital stock, and the national currency/USD exchange rate. The regressions in columns (3) and (4) include country and year fixed effects. Standard errors are reported in brackets. *, **, and *** denote significance at the 10%, 5%, and 1% levels, respectively.

where ξ_{ct} is the input-output multiplier of country c in year t, $\mathbf{X}_{ct}$ is a vector of controls, and γ_c and δ_t are country and time fixed effects, respectively. The coefficient of interest is β_ξ, which captures the average percentage-point increase in real GDP growth for a one-unit increase in the input-output multiplier. Specifying time fixed effects allows us to control for potential common factors affecting GDP growth rates in all countries at a given time. Additionally, the inclusion of country fixed effects controls for country-specific factors determining economic growth rates. We also include aggregate TFP growth to control for cross-country differences in productivity growth.

Table 1 reports the results relating to specification (6). The first column presents the point estimate for β_ξ in the most simple regression, where the input-output multiplier is the sole regressor. First, note that the mean value of ξ across the sample is 1.79, and the standard deviation is 0.20. A back-of-the-envelope calculation implies a one standard deviation increase in the input-output multiplier is associated with a $(0.20 \times 4.793 =)$ 0.96 percentage point (pp) increase in the rate of economic growth. The inclusion of TFP growth in column (2) reduces this estimate to 0.56pp, suggesting that economies with higher input-output multipliers typically have higher rates of TFP growth as

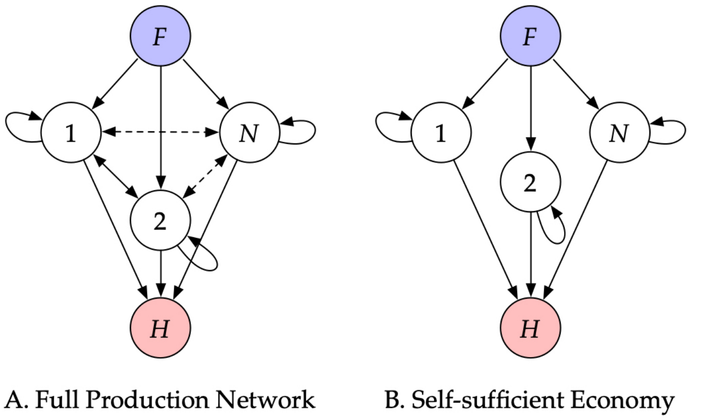

Figure 1: Visual Decomposition of a Production Network
Note: The lilac nodes, F, represent the primary factor (labor) and the red nodes, H, represent the household. Directed arrows depict the flow of inputs/goods.

well. We take the specification in column (4) as our benchmark. Here, the relationship between ξ and GDP growth is robust to the inclusion of country/time fixed effects and a vector of other controls.[14] Here, the point estimate implies that a one-sd increase in the input-output multiplier relates to a 0.80pp increase in the growth rate.[15]

Taken together, the results in Table 1 and Table A.1 establish a positive and statistically/economically significant relationship between economies' input-output multiplier and the rate of economic growth.

3.2.2 Measuring network spillovers

We now specialize the framework outlined in section 2 to characterize network spillovers in efficient economies. Specifically, we diagonalize the primitive input-output matrix $\tilde{\boldsymbol{\omega}} = \text{diag}(\boldsymbol{\omega})$ and characterize counterfactual Domar weights under this structure. As a first step, and to build intuition, we graphically illustrate the concept of network spillovers. Consider the economies shown in Figure 1. Panel A shows a full production network as observed in the data, where all input-output relationships between sectors

[14] The vector of controls comprises national population (in millions), average annual hours worked by persons engaged, a human capital index (based on the average years of schooling index of Barro and Lee, 2013), the real internal rate of return on capital, the average depreciation rate on the capital stock, and the national currency/USD exchange rate.

[15] In Table A1 in the Appendix, we include lags of GDP growth, and TFP growth to regression specification (6). We find the relationship between ξ and GDP growth to be even stronger. Specifically, a one standard deviation increase in ξ is associated with an increase in GDP growth of between 0.62 and 1.11 percentage points, on average. The rationale behind including lags of TFP growth in the regressions comes from Baqaee and Farhi (2019), who show that nonlinearities in production can induce an endogenous response of the input-output network to changes in productivity. If production functions are not Cobb-Douglas, then changes in productivity will be correlated with the input-output multiplier and bias our estimates of β_ξ.

are captured. Under this production structure, shocks to sectors $1,2,...,N$ can propagate to any other sector, affecting final consumption directly and indirectly. By contrast, shocks to sectors in the "self-sufficient economy" of Panel B can only affect GDP directly through the household's demand for good i; propagation to other sectors is thus nonexistent. Network spillovers comprise only the propagation of shocks to other sectors, omitting all direct effects and thereby cutting down the number of paths en route to consumption.

Formally, we define the counterfactual economy as a tuple $(\text{diag}(\boldsymbol{\omega}), \tilde{\boldsymbol{\mu}})$, where the objects $\boldsymbol{\omega}$ and $\tilde{\boldsymbol{\mu}}$ contain primitive input coefficients relating to intermediate inputs and labor, respectively. The matrix $\tilde{\boldsymbol{\omega}} = \text{diag}(\boldsymbol{\omega})$, retains each sectors' reliance on its own product (captured by ω_{ii}), but removes all interdependencies between i and the other sectors of the economy. Specifically, $(\tilde{\boldsymbol{\omega}}, \tilde{\boldsymbol{\mu}})$ relates to the economy shown in Panel B of Figure 1.

Following the discussion of section 2, the structure imposed on the input-output matrix implies constant returns CES technologies of the form

$$\tilde{y}_i = A_i \left(\tilde{\mu}_i^{\frac{1}{\theta}} \tilde{l}_i^{\frac{\theta-1}{\theta}} + \tilde{\omega}_{ii}^{\frac{1}{\theta}} \tilde{x}_{ii}^{\frac{\theta-1}{\theta}} \right)^{\frac{\theta}{\theta-1}}.$$

Given there is no demand for intermediate inputs from other sectors, producers maximize profits given by

$$\tilde{\pi}_i = \tilde{p}_i (\tilde{y}_i - \tilde{x}_{ii}) - \tilde{w} \tilde{l}_i,$$

and the market-clearing conditions are

$$\tilde{y}_i = \tilde{c}_i + \tilde{x}_{ii} \quad \text{and} \quad \sum_{i=1}^{N} \tilde{l}_i = l = 1.$$

for all sectors i. In defining the economy $(\tilde{\boldsymbol{\omega}}, \tilde{\boldsymbol{\mu}})$, we make the following implicit assumption.

ASSUMPTION 3: *Since the input-output parameter ω_{ii} and the labor share parameter $\tilde{\mu}_i$ sum to one in the counterfactual economy*

$$\omega_{ii} + \tilde{\mu}_i = 1,$$

then

$$\tilde{\mu}_i = \mu_i + \sum_{j \neq i}^{N} \omega_{ij}.$$

Put another way, primitive labor share coefficients satisfy $\tilde{\mu}_i \geq \mu_i$ since $(\tilde{\boldsymbol{\omega}}, \tilde{\boldsymbol{\mu}})$ retains the diagonal elements of $\boldsymbol{\omega}$ and shuts down the off-diagonal. However, crucially, assumption 3 does not bear materially on our results since ζ does not depend on the value of the labor shares. With assumptions 1-3 in hand, we are in a position to characterize network spillovers in terms of observable equilibrium objects, which is the substance of the following proposition.

PROPOSITION 2: *Network spillovers are characterized in terms of observable equilibrium objects as*[16]

$$\zeta = \sum_{i,j \neq i}^{N} \frac{\text{Sales}_{ji}}{\text{GDP}} + \sum_{i=1}^{N} b_i \left(\psi_{ii} - \tilde{\psi}_{ii}\right) \tag{7}$$

where $\tilde{\psi}_{ii} = \left(1 - \Omega_{ii}\right)^{-1}$.

Proof. See Appendix.

Firstly, equation (7) is simply a specialized version of equation (5), and can also be expressed as $\zeta = \sum_{i,j=1}^{N} b_j \left(\boldsymbol{\Psi} - \left[I - \text{diag}\left(\boldsymbol{\Omega}\right)\right]^{-1}\right)_{ji}$. However, the expression in proposition 2 is more intuitive since it showcases the two components of network spillovers. The first term on the right-hand side of (7) is the sum of bilateral sales from i (the shocked sector) to j, as a share of nominal GDP. Note that this term echoes Hulten's theorem, which states that total nominal sales as a share of GDP gives the first-order impact of a shock on aggregate output. By contrast, $\frac{\text{Sales}_{ji}}{\text{GDP}}$ captures the impact of a shock on GDP due to the direct and indirect propagation of the shock from sector i to sector j. Intuitively, if $\frac{\text{Sales}_{ki}}{\text{GDP}} > \frac{\text{Sales}_{mi}}{\text{GDP}}$, then the supply chains linking sector k to sector i amplify the shock to i relative to the supply chains linking m to i. Notice that $\frac{\text{Sales}_{ii}}{\text{GDP}}$ is excluded from the first term because this gives the effect on GDP through i, which does not constitute a network spillover.

The term $\sum_{i=1}^{N} b_i \left(\psi_{ii} - \tilde{\psi}_{ii}\right)$ in (7) has a more subtle interpretation. In a sense, this term is an adjustment that corrects for the fact that there are production chains linking sector i to itself indirectly. In other words, the shock impacts i's output due to higher-round feedback from other sectors, which are counted as network spillover effects. The intuition behind the correction is as follows. The diagonal elements of the observed Leontief inverse, ψ_{ii}, capture all direct and indirect exposures from i to itself. By contrast,

[16] Here, ζ can also be expressed as

$$\zeta = \sum_{i=1}^{N} \left(\lambda_i - b_i \tilde{\psi}_{ii}\right).$$

Table 2: Explanatory Power of Network Spillovers

	(1)	(2)
ζ_{ct} (Network spillovers)	1.056^{***}	
	[0.008]	
$\tilde{\xi}_{ct}$ ($\xi_{ct} - \zeta_{ct}$, Direct component)		2.197^{***}
		[0.176]
Observations	645	645
R^2	0.962	0.196

Notes: This table reports estimates for the pooled OLS regression specification (8). The dependent variable is the input−output multiplier ξ. The independent variable ζ_{ct} corresponds to equation (7), and measures each country's network spillovers, whereas $\tilde{\xi}_{ct}$ is the residual from $\xi_{ct} - \zeta_{ct}$. Standard errors are reported in brackets. *, **, and *** denote significance at the 10%, 5%, and 1% levels, respectively.

$\tilde{\psi}_{ii}$ comprises only the exposures stemming from i's use of its own intermediates, given by $1+\Omega_{ii}+\Omega_{ii}^2+... = (1-\Omega_{ii})^{-1}$. The difference $\psi_{ii} - \tilde{\psi}_{ii}$ purges out the latter exposures, retaining only the production chains involving other sectors, which constitute network spillovers.

Crucially, the measure ζ is related to the economy's input-output multiplier ξ through the relationship $\xi = \zeta + \tilde{\xi}$ where $\tilde{\xi}$ is the input-output multiplier in the counterfactual economy defined by $(\tilde{\boldsymbol{\omega}}, \tilde{\boldsymbol{\mu}})$. As $\zeta \to \xi$, aggregate output is increasingly influenced by network spillovers. In the next subsection, we employ the measure ζ to study patterns in the input-output multipliers of countries in the WIOD sample.

3.2.3 Network spillovers and input-output multipliers

Empirically, network spillovers account for most of the variation in economies' input-output multipliers between 2000 and 2014. To show this, we estimate the following pooled OLS regression

$$\xi_{ct} = \alpha + \beta * \zeta_{ct} + \varepsilon_{ct} \tag{8}$$

where ζ_{ct} is the extent of network spillovers for country c in year t, as measured by equation (7). We are primarily interested in the explanatory power of ζ. A high R^2 indicates that year-to-year fluctuations in network spillovers explain the evolution of economies' input-output multipliers. Table 2 reports the results relating to specification (8). The coefficient on ζ_{ct} is highly statistically significant and close to one, meaning input-output multipliers typically increase one-for-one with network spillovers. In a separate

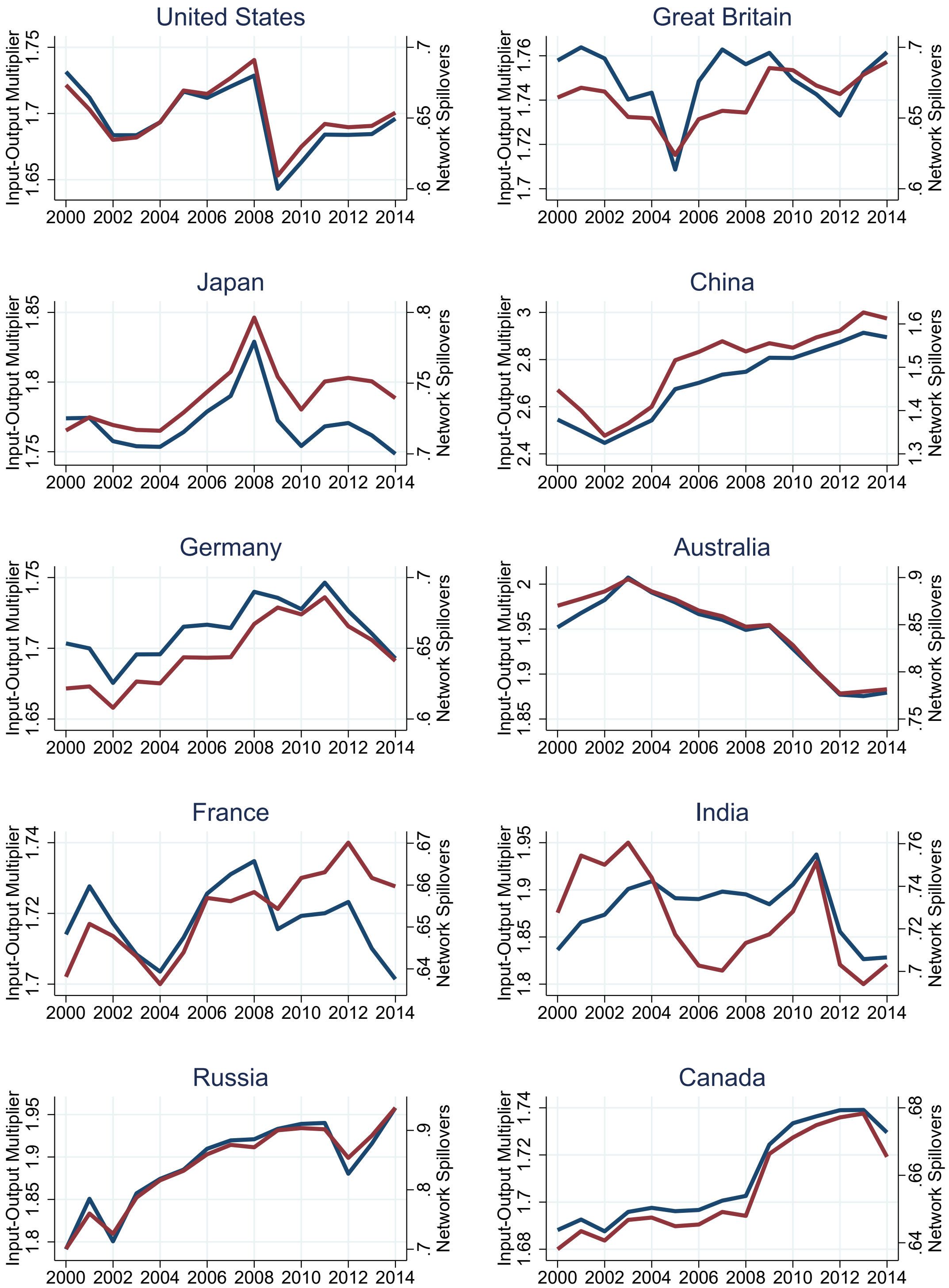

Figure 2: Aggregate Network Spillovers & Input-Output Multipliers

Note: Each panel plots the input-output multiplier (ξ, blue line) against the measure of network spillovers (ζ, red line) for the years 2000 to 2014 using data from the 2016 release of the World Input-Output Database.

regression (column (2) of Table 2), we test the explanatory power of the residual, or "direct" component, $\tilde{\xi}$. While the coefficient on $\tilde{\xi}$ is highly statistically significant, its R^2 value is only 19.6%. By contrast, the regression of input-output multipliers on network spillovers returns an R^2 of 96.2%. *Prima facie*, the results in Table 2, suggest that between 80.4% and 96.2% of the variation in input-output multipliers is "explained" by network spillovers.

The results shown in table 3 manifest themselves graphically in figure 2. For most of the countries shown, it is immediately clear that the evolution of network spillovers (red line) closely tracks the path of input-output multipliers (blue line) from 2000 to 2014. Given that input-output multipliers are highly sensitive to changes in network spillovers, our empirical results suggest that output spillovers are an important channel linking microeconomic shocks to aggregate outcomes. As a sanity check, and to test this hypothesis more directly, we estimate the following specification

$$\begin{aligned}\text{GDP Growth}_{ct} = \alpha + \beta_\zeta * \zeta_{ct} + \sum_{k=0}^{2} \text{TFP Growth}_{c,t-k} \\ + \sum_{m=1}^{2} \text{GDP Growth}_{c,t-m} + \mathbf{X}_{ct} + \gamma_c + \delta_t + \varepsilon_{ct}. \quad (9)\end{aligned}$$

Table A.2 in the Appendix reports the results. Reassuringly, our estimates suggest a positive and statistically significant relationship between network spillovers and GDP growth. Noting that across the sample the standard deviation of ζ is 0.18, the point estimates of Table A.2 imply that a one standard deviation increase in network spillovers is associated with a 0.45 to 0.91pp increase in the economic growth rate, *ceteris paribus*.

3.3 Application II: Does China's input-output structure amplify growth?

We now turn to a second application of the framework outlined in section 2. Specifically, we show how countries' economic growth rates could be higher under different network structures, using the recent growth experience of China as a motivating example.

According to data from the Penn World Tables, China's economy grew (in real terms) at an average annual rate of 6.35 percent between 1978 and 2019. Economists have suggested China's extraordinary growth since the late 1970s can be attributed to a variety of sources: rising participation rates, improvements in human capital, rapid productivity growth, the systematic understatement of inflation in official statistics, governmental reforms acting to reduce distortions, and the transition of labor out of the agriculture sector (Young, 2003; Brandt et al., 2012; Zhu, 2012). We propose another explanation; that China's input-output architecture is particularly powerful at amplifying productivity growth relative to other economies. To test this hypothesis, we specialize the framework developed in section 2 to calculate the gains attributed to China's production structure. Specifically, we ask the following counterfactual question: if some country c had the same input-output structure as China over the period 2000 to 2014, how much higher (or lower) would c's growth rate have been?[17]

To use equation (5) to study this question, we set each country c's primitive input-output network to that of China, or $\tilde{\boldsymbol{\omega}}_{ct} = \boldsymbol{\omega}_{\text{CHN}t}$, where t indexes time. By doing so, we are imposing China's sectoral interrelationships on country c, for each year t. Crucially, the WIOD data (which we use for this application) comprises a common set of sectors for all countries, allowing for a direct substitution of input-output matrices. Our steady-state result in proposition 1 implies $\tilde{\boldsymbol{\omega}}_{ct} = \boldsymbol{\Omega}_{\text{CHN}t}$, meaning we can write

$$\zeta_{ct} = \sum_{i,j=1}^{N} b_{cjt} \left(\Psi_{cjit} - \Psi_{\text{CHN}jit} \right)$$

where $\Psi_{\text{CHN}jit}$ is the jith element of China's Leontief inverse in year t. Here, ζ_{ct} compares country c's production structure to China's, and a value of $\zeta_{ct} < 0$ implies c's economy would experience accelerated growth in response to a productivity increase if its input-output architecture were identical to China's. For this calculation, the final expenditure shares take the values of those in country c since we calculate c's hypothetical gain relative to its observed growth rate. The input-output objects of the above equation are calibrated to the WIOD data, noting that the Leontief inverse matrices are computed in the usual way, $\boldsymbol{\Psi}_{ct} = (I - \boldsymbol{\Omega}_{ct})^{-1}$. Additionally, we assume that productivity growth is equal for every sector of a given country to focus on the role of the network in explaining cross-country differences in growth.[18] As in the previous application, we use productivity data from the Penn World Tables to run this set of counterfactuals. In order to calculate country c's *percentage point* gain in economic growth under China's structure, we compute

[17]Remarkably, China's average real GDP growth rate was 7.7 percent per annum from 2000 to 2014, according to data from the Penn World Tables.

[18]Under heterogeneous sectoral productivity shocks, country c's GDP growth may be influenced by the productivity growth experiences of specific sectors of the economy, which may be different from those of China. Instead, we shut down this channel and focus purely on differences in input-output architecture between countries.

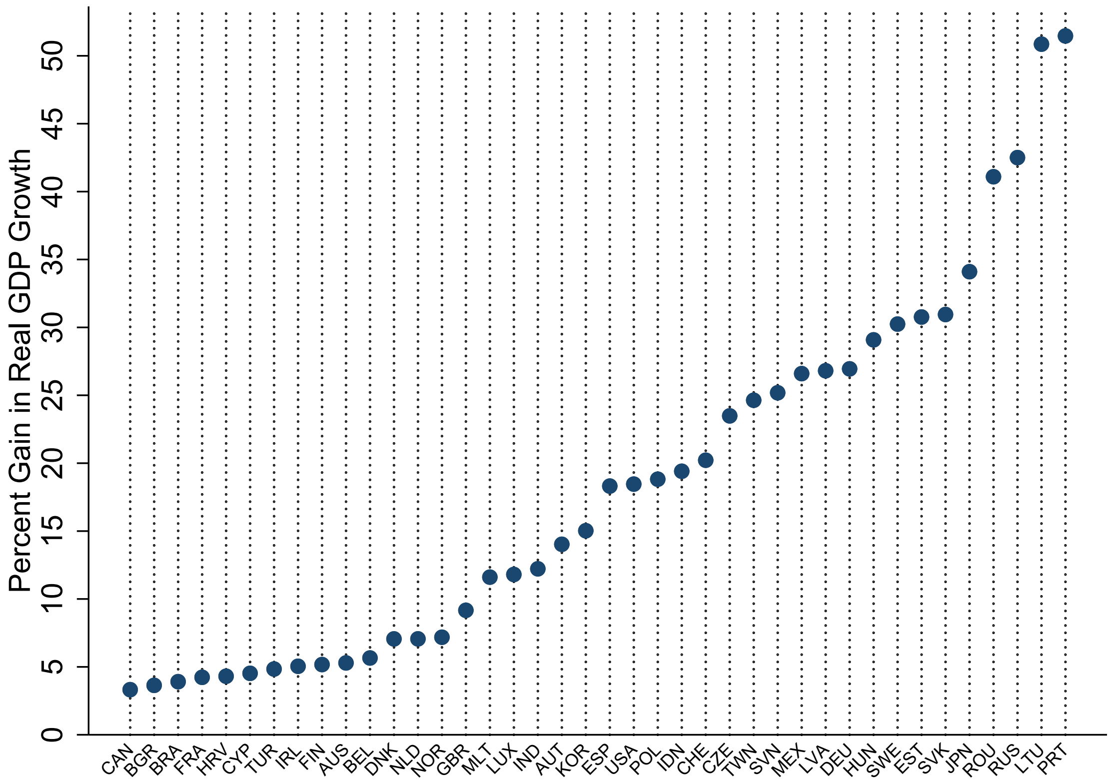

Figure 3: Hypothetical Output Gains of Having China's Input-Output Structure
Note: Each point is computed as in (10), and measures each country's hypothetical annual percentage gain in real GDP growth under China's input-output structure.

$$Z_{ct} = \begin{cases} -\zeta_{ct} \times \Delta \log \bar{A}_c, & \text{if } \Delta \log \bar{A}_c > 0 \\ \zeta_{ct} \times \Delta \log \bar{A}_c, & \text{otherwise} \end{cases}$$

where $\Delta \log \bar{A}_c$ is the average annual rate of TFP growth in country c between 2000 to 2014, and Z_{ct} measures c's percentage point gain (loss) in GDP growth under China's production architecture. A few comments are in order regarding the measure Z_{ct}. First, so Z_{ct} returns the percentage point *gain* under the counterfactual, we compute Z_{ct} as $-\zeta_{ct} \times \Delta \log \bar{A}_c$ for increases in productivity and $\zeta_{ct} \times \Delta \log \bar{A}_c$ for decreases in productivity. Intuitively, if $Z_{ct} = x > 0$ then country c's real GDP growth rate would have been x percentage points higher under China's network structure. Second, we take the average annual rate of TFP growth over the sample period for each country. This is to limit the influence of the global financial crisis on our estimates. Updating the TFP growth rates annually for each country yields qualitatively similar (but quantitatively larger) estimates. Finally, to compare our results across countries, we average Z_{ct} over the sample period and calculate the average annual *percent* gain in GDP growth as

$$\frac{\bar{Z}_c}{\Delta \log \bar{Y}_c} \tag{10}$$

where $\Delta \log \bar{Y}$ is the average rate of economic growth in country c between 2000 to 2014. Importantly, when computing this counterfactual, we place no restriction on the elasticities of substitution in consumption (σ) and production (θ). Specifically, we need not assume that country c retains the same values of these elasticities under the counterfactual, therefore reducing the information required to compute (10).

Figure 3 plots $\frac{\bar{Z}_c}{\Delta \log \bar{Y}}$ for each country in the WIOD sample.[19] Remarkably, the figure shows positive growth gains for every country, though substantial heterogeneity in these gains across countries. For example, Canada, Bulgaria, Brazil, France, Croatia, Cyprus, and Turkey are on the modest end of the spectrum, with a gain of less than 5 percent. On the other hand, Sweden, Estonia, Slovakia, Japan, Romania, Russia, Lithuania, and Portugal all experience gains of over 30 percent. Additionally, our results suggest the United States' average annual growth rate would have been 18.5 percent higher under China's production structure, or 2.32 percent per annum (the actual average annual US growth rate was 1.96 percent). Our results suggest China's input-output structure greatly amplifies productivity growth relative to the other countries in the sample. However, we stop short of tracing this finding to particular structural features of the Chinese economy, leaving this task for future work.

4 Conclusion

It is often difficult to understand which set of intersectoral linkages amplify shocks in multi-sector macroeconomic models with production networks. As a remedy, we provide a theoretical framework that measures the macroeconomic importance of specific structural linkages. Methodologically, we set up a standard neoclassical production network model and derive an analytical formula that computes the macroeconomic impact of sector-specific shocks under any arbitrary linear transformation of the production network.

In an application and as proof of concept, we discipline our formula to derive an exact closed-form expression for network spillovers in efficient economies. Empirically, we show that (for a sample of 43 countries) the extent of network spillovers fluctuated substantially over 2000 to 2014, suggesting output spillovers to be a driver of macroeconomic outcomes. In a second application, we quantify the gains of having different

[19] We omit Italy and Greece since these countries had very low average real GDP growth rates over the sample period (0.18% and 0.08%, respectively).

network structures. As an example, we show that all countries in the WIOD sample would have experienced accelerated economic growth had their input-output structure been identical to China's between 2000 and 2014. In particular, when China's input-output network is hypothetically substituted for that of the United States, we calculate the US GDP growth rate to be 18.5 percent higher per annum.

While our theoretical results apply to economies where distortions are nonexistent, a natural next step would be to characterize our formula in economies with inefficiencies, taking the models of Jones (2011), Liu (2019), Baqaee and Farhi (2020b), and Bigio and La'O (2020) as a starting point. Such an analysis would illuminate how distortions interact with input-output linkages, extending our results beyond the benchmark provided in this paper.

Appendix A. Proofs

PROOF OF PROPOSITION 1: From the first-order condition with respect to intermediate consumption x_{ij}, we get the following expression for the input-output coefficient Ω_{ij}

$$\Omega_{ij} = \frac{p_j x_{ij}}{p_i y_i} = \omega_{ij} A_i^{\theta-1} p_i^{\theta-1} p_j^{1-\theta}.$$

Similarly, the first-order condition with respect to i's labor use implies

$$\Lambda_i = \frac{w l_i}{p_i y_i} = \mu_i A_i^{\theta-1} p_i^{\theta-1} w^{1-\theta}.$$

From the household's optimization problem, we get

$$c_j = w a_j p_j^{-\sigma} P^{\sigma-1},$$

Additionally, the first-order conditions for labor and intermediate inputs imply

$$l_j = p_j^{\theta} y_j A_j^{\theta-1} \mu_j w^{-\theta} \tag{11}$$

and

$$x_{ji} = p_j^{\theta} y_j A_j^{\theta-1} \omega_{ji} p_i^{-\theta}. \tag{12}$$

Plugging (11) and (12) into j's production function gives the following price equation

$$p_j^{1-\theta} = A_j^{\theta-1}\left(\mu_j w^{1-\theta} + \sum_{i=1}^{N} \omega_{ji} p_i^{1-\theta}\right).$$

Imposing the steady-state condition that $(A_1,...,A_N) = \left(\tilde{A}_1,...,\tilde{A}_N\right) = (1,...,1)$, and solving for prices in the actual and counterfactual economies, we get

$$\mathbf{p}^{1-\theta} = (I-\boldsymbol{\omega})^{-1}\boldsymbol{\mu} w^{1-\theta} \quad \text{and} \quad \tilde{\mathbf{p}}^{1-\theta} = (I-\tilde{\boldsymbol{\omega}})^{-1}\tilde{\boldsymbol{\mu}}\tilde{w}^{1-\theta},$$

where $(I-\boldsymbol{\omega})^{-1}\boldsymbol{\mu} = \mathbf{1}$, ensuring $\tilde{p}_i = \tilde{w}$ and $p_i = w$ for all i. From the price index we have $P = \left(w^{1-\sigma}\sum_{i=1}^{N} a_i\right)^{\frac{1}{1-\sigma}}$, and using the fact that $\sum_{i=1}^{N} a_i = 1$, in steady-state $P = w$. By a similar argument $\tilde{P} = \tilde{w}$. Using the condition $c_i = wa_i p_i^{-\sigma} P^{\sigma-1}$, we can write $c_i = wa_i w^{-\sigma} w^{\sigma-1} = a_i$. Similarly, for the counterfactual economy, $\tilde{c}_i = \tilde{w}a_i\tilde{w}^{-\sigma}\tilde{w}^{\sigma-1} = a_i$, which uses the fact that $a_i = \tilde{a}_i$ for all i. Thus, in steady-state we have $c_i = \tilde{c}_i$, and from the consumption aggregators

$$Y = \tilde{Y} = \left(\sum_{i=1}^{N} a_i^{\frac{1}{\sigma}} a_i^{\frac{\sigma-1}{\sigma}}\right)^{\frac{\sigma}{\sigma-1}} = \left(\sum_{i=1}^{N} a_i\right)^{\frac{\sigma}{\sigma-1}} = 1.$$

Using the expression $\Omega_{ij} = \omega_{ij} A_i^{\theta-1} p_i^{\theta-1} p_j^{1-\theta}$ and imposing the steady-state condition, we get $\Omega_{ij} = \omega_{ij} w^{\theta-1} w^{1-\theta} = \omega_{ij}$ and similarly, $\tilde{\Omega}_{ij} = \tilde{\omega}_{ij}$ for all j,i. Finally, since $\tilde{\boldsymbol{\omega}} = T(\boldsymbol{\omega})$ and $\boldsymbol{\omega} = \boldsymbol{\Omega}$ in steady-state, we have $\tilde{\boldsymbol{\omega}} = T(\boldsymbol{\omega}) = T(\boldsymbol{\Omega})$. Finally, since $\zeta = \sum_{i,j=1}^{N} b_j\left(\psi_{ji} - \tilde{\psi}_{ji}\right)$, we arrive at equation (5).

Q.E.D.

PROOF OF PROPOSITION 2: We will get to our result in four steps. The first step is to establish that

$$\frac{d\log Y}{d\log A_i} = \lambda_i$$

and characterize λ_i in terms of the derivatives of prices. We then compute

$$\frac{d\log \tilde{Y}}{d\log A_i} = \tilde{\lambda}_i$$

in terms of counterfactual equilibrium objects $\tilde{\mathbf{b}}$ and $\tilde{\boldsymbol{\Omega}}$. Thirdly, we characterize $\tilde{\mathbf{b}}$ and $\tilde{\boldsymbol{\Omega}}$ in terms of observables. Finally, to complete the proof we compute χ in terms of observables.

Throughout the proof, we will make use of the steady-state condition that $(A_1,...,A_N) =$

$\left(\tilde{A}_1, ..., \tilde{A}_N\right) = (1, ..., 1)$.

STEP 1: In the proof of proposition 1, we derived the following expression for the input-output coefficient Ω_{ij}

$$\Omega_{ij} = \frac{p_j x_{ij}}{p_i y_i} = \omega_{ij} A_i^{\theta-1} p_i^{\theta-1} p_j^{1-\theta}. \tag{13}$$

Similarly, we have the following expression for Λ_i

$$\Lambda_i = \frac{w l_i}{p_i y_i} = \mu_i A_i^{\theta-1} p_i^{\theta-1} w^{1-\theta}. \tag{14}$$

Now, from the consumption aggregator

$$Y = \left(\sum_{j=1}^{N} a_j^{\frac{1}{\sigma}} c_j^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}},$$

the change in aggregate output with respect to changes in individual final demands is

$$d \log Y = \sum_{j=1}^{N} \frac{\partial \log Y}{\partial \log c_j} d \log c_j,$$

or,

$$d \log Y = \sum_{j=1}^{N} Y^{\frac{1-\sigma}{\sigma}} a_j^{\frac{1}{\sigma}} c_j^{\frac{\sigma-1}{\sigma}} d \log c_j. \tag{15}$$

Earlier we derived

$$c_j = w a_j p_j^{-\sigma} P^{\sigma-1} \tag{16}$$

and log differentiation of (16), gives

$$d \log c_j = d \log w + d \log a_j - \sigma d \log p_j (\sigma - 1) d \log P. \tag{17}$$

Plugging (17) into (15), and taking the derivative with respect to $\log A_i$, gives

$$\frac{d \log Y}{d \log A_i} = \sum_{j=1}^{N} Y^{\frac{1-\sigma}{\sigma}} a_j^{\frac{1}{\sigma}} c_j^{\frac{\sigma-1}{\sigma}} \left(\frac{d \log w}{d \log A_i} - \sigma \frac{d \log p_j}{d \log A_i} - (1-\sigma) \frac{d \log P}{d \log A_i} \right). \tag{18}$$

Additionally, substituting (16) into (18), and noting that

$$\frac{d \log P}{d \log A_i} = \sum_{j=1}^{N} P^{\sigma-1} a_j p_j^{1-\sigma} \frac{d \log p_j}{d \log A_i}, \tag{19}$$

equation (18) becomes

$$\frac{d \log Y}{d \log A_i} = \sum_{j=1}^{N} P^{\sigma-1} a_j p_j^{1-\sigma} \times \left[\frac{d \log w}{d \log A_i} - \sigma \frac{d \log p_j}{d \log A_i} - (1-\sigma) \left(\sum_{m=1}^{N} P^{\sigma-1} a_m p_m^{1-\sigma} \frac{d \log p_m}{d \log A_i} \right) \right]. \tag{20}$$

The first-order conditions for labor and intermediate inputs imply

$$l_j = p_j^{\theta} y_j A_j^{\theta-1} \mu_j w^{-\theta} \tag{21}$$

and

$$x_{ji} = p_j^{\theta} y_j A_j^{\theta-1} \omega_{ji} p_i^{-\theta}. \tag{22}$$

Pluging (21) and (22) into j's production function gives the following price equation

$$p_j^{1-\theta} = A_j^{\theta-1} \left(\mu_j w^{1-\theta} + \sum_{i=1}^{N} \omega_{ji} p_i^{1-\theta} \right). \tag{23}$$

Log differentiating (23), implies

$$d \log p_j = -p_j^{\theta-1} A_j^{\theta-1} \left(\mu_j w^{1-\theta} + \sum_{i=1}^{N} \omega_{ji} p_i^{1-\theta} \right) d \log A_j + p_j^{\theta-1} A_j^{\theta-1} \mu_j w^{1-\theta} d \log w + \sum_{i=1}^{N} p_j^{\theta-1} A_j^{\theta-1} \omega_{ji} p_j^{1-\theta} d \log p_i. \tag{24}$$

From (13) and (14), equation (24) can be written as

$$d \log p_j = -\left(\Lambda_j + \sum_{i=1}^{N} \Omega_{ji} \right) d \log A_j + \Lambda_j d \log w + \sum_{i=1}^{N} \Omega_{ji} d \log p_i,$$

and noting that $\Lambda_j + \sum_{i=1}^{N} \Omega_{ji} = 1$, we get

$$d \log p_j = -d \log A_j + \Lambda_j d \log w + \sum_{i=1}^{N} \Omega_{ji} d \log p_i.$$

Solving for $d\log p_j$, gives

$$d\log p_j = \sum_{k=1}^{N} \psi_{jk}\Lambda_k d\log w - \sum_{k=1}^{N} \psi_{jk} d\log A_k.$$

Thus,

$$\frac{d\log p_j}{d\log A_i} = \sum_{k=1}^{N} \psi_{jk}\Lambda_k \frac{d\log w}{d\log A_i} - \psi_{ji}. \tag{25}$$

Now, multiplying both sides of equation (14) by p_j and dividing by w, gives

$$b_j = \frac{p_j c_j}{w} = p_j^{1-\sigma} a_j P^{\sigma-1}, \tag{26}$$

and plugging (25) and (26) into (20), gives

$$\frac{d\log Y}{d\log A_i} = \sum_{j=1}^{N} b_j$$
$$\times \left[\frac{d\log w}{d\log A_i} - \sigma \left(\sum_{k=1}^{N} \psi_{jk}\Lambda_k \frac{d\log w}{d\log A_i} - \psi_{ji} \right) - (1-\sigma) \left(\sum_{m=1}^{N} b_m \left(\sum_{k=1}^{N} \psi_{mk}\Lambda_k \frac{d\log w}{d\log A_i} - \psi_{mi} \right) \right) \right].$$

which simplifies to

$$\frac{d\log Y}{d\log A_i} = \frac{d\log w}{d\log A_i} - \sum_{j=1}^{N}\sum_{k=1}^{N} b_j \psi_{jk}\Lambda_k \frac{d\log w}{d\log A_i} + \sum_{j=1}^{N} b_j \psi_{ji}.$$

Noting that $\sum_{j=1}^{N} b_j \psi_{jk} = \lambda_k$

$$\frac{d\log Y}{d\log A_i} = \frac{d\log w}{d\log A_i} - \sum_{k=1}^{N} \lambda_k \Lambda_k \frac{d\log w}{d\log A_i} + \lambda_i.$$

Therefore, we recover Hulten's theorem:

$$\frac{d\log Y}{d\log A_i} = \lambda_i.$$

We now characterize λ_i in terms of derivatives of prices. From equation (24), we have

$$\frac{d\log p_j}{d\log A_i} = \sum_{k=1}^{N} \Omega_{jk} \frac{d\log p_k}{d\log A_i} + \Lambda_j \frac{d\log w}{d\log A_i} - \frac{d\log A_j}{d\log A_i}, \tag{27}$$

where (27) is a consequence of $\sum_{k=1}^{N} \Omega_{jk} + \Lambda_j = 1$. Plugging (27) into (20) and simplifying, gives

$$\lambda_i = \frac{d\log w}{d\log A_i} - \sum_{m=1}^{N} b_m \Lambda_m \frac{d\log w}{d\log A_i} - \sum_{m=1}^{N}\sum_{k=1}^{N} b_m \Omega_{mk} \frac{d\log p_k}{d\log A_i} + \sum_{m=1}^{N} b_m \frac{d\log A_m}{d\log A_i},$$

or,

$$\lambda_i - b_i = \frac{d\log w}{d\log A_i} - \sum_{m=1}^{N} b_m \Lambda_m \frac{d\log w}{d\log A_i} - \sum_{m=1}^{N}\sum_{k=1}^{N} b_m \Omega_{mk} \frac{d\log p_k}{d\log A_i} \tag{28}$$

where the left-hand side of the above equation is simply intermediate sales from sector i to all other sectors, as a fraction of nominal GDP. Furthermore, since

$$\frac{d\log w}{d\log A_i} = \frac{d\log Y}{d\log A_i} + \frac{d\log P}{d\log A_i} = \lambda_i + \sum_{m=1}^{N} b_m \frac{d\log p_m}{d\log A_i},$$

equation (28) becomes

$$\sum_{m=1}^{N} \lambda_m \Omega_{mi} = \lambda_i + \sum_{m=1}^{N} b_m \frac{d\log p_m}{d\log A_i} - \sum_{m=1}^{N} b_m \Lambda_m \frac{d\log w}{d\log A_i} - \sum_{m=1}^{N}\sum_{k=1}^{N} b_m \Omega_{mk} \frac{d\log p_k}{d\log A_i}.$$

Using the fact that $\lambda_i = \sum_{m=1}^{N} b_m \psi_{mi}$, we get

$$\sum_{m=1}^{N} \lambda_m \Omega_{mi} = \sum_{m=1}^{N} b_m \psi_{mi} + \sum_{m=1}^{N} b_m \frac{d\log p_m}{d\log A_i} - \sum_{m=1}^{N} b_m \Lambda_m \frac{d\log w}{d\log A_i} - \sum_{m=1}^{N}\sum_{k=1}^{N} b_m \Omega_{mk} \frac{d\log p_k}{d\log A_i}$$

Thus, we get

$$\lambda_i \Omega_{ii} = b_i \psi_{ii} + b_i \frac{d\log p_i}{d\log A_i} - b_i \Lambda_i \frac{d\log w}{d\log A_i} - \sum_{k=1}^{N} b_i \Omega_{ik} \frac{d\log p_k}{d\log A_i},$$

or,

$$\lambda_i \Omega_{ii} = b_i \psi_{ii} - b_i \Lambda_i \frac{d\log w}{d\log A_i} + b_i \left(\frac{d\log p_i}{d\log A_i} - \sum_{k=1}^{N} \Omega_{ik} \frac{d\log p_k}{d\log A_i} \right). \tag{29}$$

From equation (27), we have

$$\frac{d\log p_i}{d\log A_i} - \sum_{k=1}^{N} \Omega_{ik} \frac{d\log p_k}{d\log A_i} = \Lambda_i \frac{d\log w}{d\log A_i} - \frac{d\log A_i}{d\log A_i}. \tag{30}$$

Substituting (30) into (29), gives

$$\lambda_i \Omega_{ii} = b_i \psi_{ii} - b_i \Lambda_i \frac{d \log w}{d \log A_i} + b_i \left(\Lambda_i \frac{d \log w}{d \log A_i} - 1 \right)$$

and simplifying, gives

$$\lambda_i \Omega_{ii} = b_i \left(\psi_{ii} - 1 \right). \tag{31}$$

STEP 2: We now move onto characterizing $\tilde{\lambda}_i$ in terms of $\tilde{\mathbf{b}}$ and $\tilde{\boldsymbol{\Omega}}$. As a first observation, we note that as long as $\tilde{\omega}_{ii} = \omega_{ii}$, we will have $\tilde{\Omega}_{ii} = \Omega_{ii}$ in equilibrium. This is a consequence of sector i's first-order condition with respect to its own intermediates,

$$\tilde{\Omega}_{ii} = \frac{\tilde{p}_i \tilde{x}_{ii}}{\tilde{p}_i \tilde{y}_i} = A_i^{\theta-1} \omega_{ii}. \tag{32}$$

From (11), it is easy to verify that $\Omega_{ii} = A_i^{\theta-1} \omega_{ii}$. Also from sector i's optimization, we get

$$\tilde{\Lambda}_i = \tilde{\mu}_i A_i^{\theta-1} \tilde{p}_i^{\theta-1} \tilde{w}^{1-\theta}. \tag{33}$$

The first-order conditions also imply that

$$\tilde{x}_{ii} = \tilde{y}_i A_i^{\theta-1} \omega_{ii} \quad \text{and} \quad \tilde{l}_i = \tilde{p}_i^{\theta} \tilde{y}_i A_i^{\theta-1} \tilde{\mu}_i \tilde{w}^{-\theta}.$$

Plugging the expressions for $\tilde{x}_{ii}$ and $\tilde{l}_i$ into sector i's *counterfactual* production function implies

$$\tilde{p}_j^{\theta-1} = A_j^{\theta-1} \tilde{\mu}_j \tilde{w}^{1-\theta} \left(1 - A_j^{\theta-1} \omega_{jj} \right)^{-1}. \tag{34}$$

Total (log) differentiation of (34), gives

$$\begin{aligned} d \log \tilde{p}_j = {} & \tilde{p}_j^{\theta-1} A_j^{\theta-1} \tilde{\mu}_j \tilde{w}^{1-\theta} \left(1 - A_j^{\theta-1} \omega_{jj} \right)^{-1} d \log \tilde{w} \\ & - \tilde{p}_j^{\theta-1} \tilde{\mu}_j \tilde{w}^{1-\theta} A_j^{\theta-1} \left(1 - A_j^{\theta-1} \omega_{jj} \right)^{-2} d \log A_j \end{aligned} \tag{35}$$

and substituting $\tilde{\Lambda}_j$ and $\tilde{\Omega}_{jj}$ into (35)

$$d \log \tilde{p}_j = \tilde{\Lambda}_j \left(1 - \Omega_{jj} \right)^{-1} d \log \tilde{w} - \tilde{\Lambda}_j \left(1 - \Omega_{jj} \right)^{-2} d \log A_j. \tag{36}$$

Noting that $\tilde{b}_j = \tilde{p}_j^{1-\sigma} a_j \tilde{P}^{\sigma-1}$ and

$$\frac{d\log\tilde{Y}}{d\log A_i}=\sum_{j=1}^{N}\tilde{P}^{\sigma-1}a_j\tilde{p}_j^{1-\sigma}\left[\frac{d\log\tilde{w}}{d\log A_i}-\sigma\frac{d\log\tilde{p}_j}{d\log A_i}-(1-\sigma)\left(\sum_{m=1}^{N}\tilde{P}^{\sigma-1}a_m\tilde{p}_m^{1-\sigma}\frac{d\log\tilde{p}_m}{d\log A_i}\right)\right], \tag{37}$$

plugging the derivative of (36) with respect to $\log A_i$ into (37), gives

$$\frac{d\log\tilde{Y}}{d\log A_i}=\frac{d\log\tilde{w}}{d\log A_i}-\sum_{m=1}^{N}\tilde{b}_m\tilde{\Lambda}_m(1-\Omega_{mm})^{-1}\frac{d\log\tilde{w}}{d\log A_i}$$
$$+\sum_{m=1}^{N}\tilde{b}_m\tilde{\Lambda}_m(1-\Omega_{mm})^{-2}\frac{d\log A_m}{d\log A_i}. \tag{38}$$

Equation (38) can also be written

$$\frac{d\log\tilde{Y}}{d\log A_i}=\frac{d\log\tilde{w}}{d\log A_i}-\sum_{m=1}^{N}\tilde{b}_m\tilde{\Lambda}_m\tilde{\Psi}_{ii}\frac{d\log\tilde{w}}{d\log A_i}+\sum_{m=1}^{N}\tilde{b}_m\tilde{\Lambda}_m\tilde{\Psi}_{mm}^2\frac{d\log A_m}{d\log A_i}.$$

Noting that $\tilde{\lambda}_m=\tilde{b}_m\tilde{\Lambda}_m$

$$\frac{d\log\tilde{Y}}{d\log A_i}=\frac{d\log\tilde{w}}{d\log A_i}-\sum_{m=1}^{N}\tilde{\lambda}_m\tilde{\Lambda}_m\frac{d\log\tilde{w}}{d\log A_i}+\sum_{m=1}^{N}\tilde{b}_m\tilde{\Lambda}_m\tilde{\Psi}_{mm}^2\frac{d\log A_m}{d\log A_i},$$

and

$$\frac{d\log\tilde{Y}}{d\log A_i}=\sum_{m=1}^{N}\tilde{b}_m\tilde{\Lambda}_m\tilde{\Psi}_{mm}^2\frac{d\log A_m}{d\log A_i}. \tag{39}$$

The assumption of constant returns to scale in production implies that

$$\tilde{\Lambda}_m+\tilde{\Omega}_{mm}=1,$$

equation (39) therefore becomes

$$\frac{d\log\tilde{Y}}{d\log A_i}=\tilde{b}_i\tilde{\Psi}_{ii}. \tag{40}$$

Equation (40) thus characterizes $\tilde{\lambda}_i$ in terms of $\tilde{b}_i$ and Ω_{ii}.

STEP 3: We must first establish that goods prices are equal in the actual and counterfactual economies. Recall that our price equation in the actual is

$$p_i^{1-\theta} = A_i^{\theta-1}\mu_i w^{1-\theta} + A_i^{\theta-1}\sum_{j=1}^{N}\omega_{ij}p_j^{1-\theta}.$$

Thus, in the actual economy, we have (at the steady-state, where $A_i = 1$ for all i)

$$p_i^{1-\theta} - \sum_{j=1}^{N}\omega_{ij}p_j^{1-\theta} = \mu_i w^{1-\theta}$$

Writing the above equation in matrix form and solving for prices, we have

$$\mathbf{p}^{1-\theta} = (I-\boldsymbol{\omega})^{-1}\boldsymbol{\mu}w^{1-\theta}.$$

Noting that $\sum_{j=1}^{N}\omega_{ij} + \mu_i$, we can write

$$(I-\boldsymbol{\omega})^{-1}\boldsymbol{\mu} = \mathbf{1},$$

thus

$$\mathbf{p}^{1-\theta} = \mathbf{1}w^{1-\theta}.$$

and $p_i = w$ for all i in steady-state.
Recall that our price equation in the counterfactual economy is

$$\tilde{p}_i^{1-\theta} = A_i^{\theta-1}\tilde{\mu}_i\tilde{w}^{1-\theta}\left(1 - A_i^{\theta-1}\omega_{ii}\right)^{-1}$$

Imposing the steady-state condition that $A_i = 1$ for all i

$$\tilde{p}_i^{1-\theta} = \tilde{\mu}_i\tilde{w}^{1-\theta}(1-\omega_{ii})^{-1}$$

and noting that $\tilde{\mu}_i + \omega_{ii} = 1$. Therefore, $\tilde{\mu}_i = 1 - \omega_{ii}$, so

$$\tilde{p}_i^{1-\theta} = (1-\omega_{ii})\,\tilde{w}^{1-\theta}(1-\omega_{ii})^{-1}$$

$$\tilde{p}_i^{1-\theta} = \tilde{w}^{1-\theta}$$

therefore, $\tilde{p}_i = \tilde{w}$ for all i. Now, from the actual economy's price equation we have

$$w^{1-\theta} = \mu_i^{-1}A_i^{1-\theta}p_i^{1-\theta} - \mu_i^{-1}\sum_{j=1}^{N}\omega_{ij}p_j^{1-\theta}.$$

Under the counterfactual, solving for the wage, we get

$$\tilde{w}^{1-\theta} = \tilde{\mu}_i^{-1} A_i^{1-\theta} \tilde{p}_i^{1-\theta} - \tilde{\mu}_i^{-1} \tilde{p}_i^{1-\theta} \omega_{ii}$$

Now, we set $w^{1-\theta} = \tilde{w}^{1-\theta}$:

$$\mu_i^{-1} A_i^{1-\theta} p_i^{1-\theta} - \mu_i^{-1} \sum_{j=1}^{N} \omega_{ij} p_j^{1-\theta} = \tilde{\mu}_i^{-1} A_i^{1-\theta} \tilde{p}_i^{1-\theta} - \tilde{\mu}_i^{-1} \tilde{p}_i^{1-\theta} \omega_{ii}$$

Imposing the steady-state condition that $A_i = 1$ for all i

$$\mu_i^{-1} p_i^{1-\theta} - \mu_i^{-1} \sum_{j=1}^{N} \omega_{ij} p_j^{1-\theta} = \tilde{\mu}_i^{-1} \tilde{p}_i^{1-\theta} - \tilde{\mu}_i^{-1} \tilde{p}_i^{1-\theta} \omega_{ii}$$

We've established that $p_i = w$ and $\tilde{p}_i = \tilde{w}$ for all i, so

$$\mu_i^{-1} w^{1-\theta} - \mu_i^{-1} \sum_{j=1}^{N} \omega_{ij} w^{1-\theta} = \tilde{\mu}_i^{-1} \tilde{w}^{1-\theta} - \tilde{\mu}_i^{-1} \tilde{w}^{1-\theta} \omega_{ii}$$

$$\mu_i^{-1} w^{1-\theta} \left(1 - \sum_{j=1}^{N} \omega_{ij}\right) = \tilde{\mu}_i^{-1} \tilde{w}^{1-\theta} (1 - \omega_{ii})$$

Finally, noting that $1 - \omega_{ii} = \tilde{\mu}_i$ and $1 - \sum_{j=1}^{N} \omega_{ij} = \mu_i$

$$\mu_i^{-1} \mu_i w^{1-\theta} = \tilde{\mu}_i^{-1} \tilde{\mu}_i \tilde{w}^{1-\theta},$$

thus $w = \tilde{w}$ in steady-state. This then ensures that $p_i = \tilde{p}_i$, $P = \tilde{P}$, $Y = \tilde{Y}$ and $b_i = \tilde{b}_i$. Additionally, since Ω_{ii} is price invariant, we get $\Omega_{ii} = \tilde{\Omega}_{ii}$ (i.e. because $\Omega_{ii} = \tilde{\Omega}_{ii} = A_i^{\theta-1} \omega_{ii}$).

STEP 4: We now compute ζ in terms of observables. As a consequence of our results in step 3 of the proof, it is immediate to show that

$$\zeta = \lambda_i - b_i \tilde{\psi}_{ii}. \tag{41}$$

First note that the market clearing condition for good i is given by $y_i = c_i + \sum_{j=1}^{N} x_{ji}$. Multiplying both sides of this equation by p_i and dividing by nominal GDP gives $\lambda_i = b_i + \sum_{j=1}^{N} \lambda_j \Omega_{ji}$, which can also be written as

$$\lambda_i = b_i + \sum_{j \neq i}^{N} \lambda_j \Omega_{ji} + \lambda_i \Omega_{ii}$$

Substituting (31) into the above equation gives

$$\lambda_i = b_i + \sum_{j \neq i}^{N} \lambda_j \Omega_{ji} + b_i \left(\psi_{ii} - 1 \right).$$

And therefore,

$$\zeta = \sum_{i=1}^{N} b_i \left(\psi_{ii} - \tilde{\psi}_{ii} \right) + \sum_{i,j \neq i}^{N} \frac{\text{Sales}_{ji}}{\text{GDP}}$$

Q.E.D.

Appendix B. Supplementary Results

Table A.1: Sensitivity to Shocks and GDP Growth, Robustness

	(1)	(2)	(3)	(4)
ξ_{ct} (Input-output multiplier)	5.530***	3.078***	3.299***	3.749***
	[1.199]	[1.121]	[1.233]	[1.244]
TFP Growth$_{ct}$	1.055***	1.054***	1.045***	1.036***
	[0.036]	[0.033]	[0.035]	[0.035]
TFP Growth$_{c,t-1}$	0.238***	-0.190***	-0.162***	-0.200***
	[0.035]	[0.052]	[0.057]	[0.059]
TFP Growth$_{c,t-2}$			0.014	0.109**
			[0.034]	[0.054]
GDP Growth$_{c,t-1}$		0.391***	0.376***	0.413***
		[0.038]	[0.041]	[0.044]
GDP Growth$_{c,t-2}$				-0.097**
				[0.043]
Country FE	Yes	Yes	Yes	Yes
Year FE	Yes	Yes	Yes	Yes
Controls	Yes	Yes	Yes	Yes
Observations	601	601	558	558
Adjusted R^2	0.806	0.838	0.842	0.843

Notes: This table presents estimates for regression specification (6) with the inclusion of lags of GDP growth and TFP growth. All regressions include the same set of control variables as in Table 1. Standard errors are reported in brackets. *, **, and *** denote significance at the 10%, 5%, and 1% levels, respectively.

Table A.2: GDP Growth and Network Spillovers

	(1)	(2)	(3)	(4)	(5)
ζ_{ct} (Network spillovers)	5.050***	2.997***	2.674**	2.498**	2.759**
	[0.722]	[0.430]	[1.293]	[1.232]	[1.377]
TFP Growth$_{ct}$		1.159***	1.050***	1.051***	1.032***
		[0.033]	[0.036]	[0.033]	[0.035]
TFP Growth$_{c,t-1}$				-0.203***	-0.216***
				[0.052]	[0.059]
TFP Growth$_{c,t-2}$					0.094*
					[0.054]
GDP Growth$_{c,t-1}$				0.402***	0.426***
				[0.037]	[0.044]
GDP Growth$_{c,t-2}$					-0.087**
					[0.043]
Country FE	No	No	Yes	Yes	Yes
Year FE	No	No	Yes	Yes	Yes
Controls	No	No	Yes	Yes	Yes
Observations	645	645	644	601	558
Adjusted R^2	0.069	0.675	0.784	0.837	0.841

Notes: This table presents estimates for regression specification (9). The dependent variable is contemporaneous real GDP growth. The independent variable ζ_{ct} corresponds to equation (7) and is a measure of network spillovers. Regressions (3), (4), and (5) include the same set of control variables as in Table 1. Standard errors are reported in brackets. *, **, and *** denote significance at the 10%, 5%, and 1% levels, respectively.

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