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Identifying High-Frequency Shocks with Bayesian Mixed-Frequency VARs*†

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We contribute to research on mixed-frequency regressions by introducing an innovative Bayesian approach. Based on a new "high-frequency" identification scheme, we provide novel empirical evidence of identifying uncertainty shock for the US economy. As main findings, we document a "temporal aggregation bias" when we adopt a common low-frequency model instead of estimating a mixed-frequency framework. The bias is amplified when we identify a higher frequency shock.

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1 Introduction

The co-movements between macroeconomic and financial time series have been predominantly studied using vector autoregressive (VAR) models (Sims 1980). VARs are usually estimated by relying on a common low sampling frequency. For instance, the business cycle fluctuations are investigated considering quarterly or monthly data. As argued by Ghysels (2016), forecasting and structural shock identification could be potentially misspecified because we ignore that some data, for example, financial series, are available at a higher frequency.

For this reason, mixed-frequency vector autoregressive (hereafter MF-VAR) models have become popular in recent years. These tools can produce more accurate and reliable forecasting and structural analysis avoiding the issues associated with temporal aggregation (see Marcellino 1999, Foroni, Ghysels and Marcellino 2013, Foroni and Marcellino 2016, among others).

We can consider a simple example: a financial uncertainty measure, e.g. VIX, observed at a daily frequency and US business cycle variables (such as inflation and industrial production) published monthly.

How can we identify the VIX shock on macroeconomic variables without ignoring the different sampling frequency? Or generalizing, *how can we identify the impact of a high-frequency shock on low-frequency variables?* We address this research question providing a twofold contribution.

First, on the methodological side, we introduce a new high-frequency identification strategy based on Bayesian Mixed-Frequency VARs. Our approach is inspired by Götz, Hecq and Smeekes (2016) and Ghysels (2016) that discuss how Bayesian techniques could improve the estimation of models in case of different data sampling. In details, we estimate a MF-VAR using a prior of a Normal Inverse Wishart form that is implemented by adding a set of auxiliary dummies to the system as discussed by Götz et al. (2016).

Second, on the empirical side, we apply this high-frequency identification framework by estimating a stacked MF-VAR à la Ghysels (2016) to study the impact of the financial uncertainty shock, proxied by the VIX, on the US business cycle. We provide a shred of new evidence about the role of financial uncertainty in explaining macroeconomic variables. In this investigation, we focus on the "temporal aggregation bias" induced by relying on a common low-frequency Bayesian VAR (hereafter CF-VAR).

Our main findings suggest how aggregating the high-frequency VIX to the low-frequency of the macroeconomic variables of interest (i.e. monthly) could lead to biased responses. In particular, these reactions document more serious recessionary effects on the business cycle when different sampling frequencies are ignored. Our results are robust to different specifications. The estimation sample is for data from 1990-2019, however we also study the "temporal aggregation bias" including the current economic crisis due to the COVID-19 pandemic. In this case, our findings show fewer recessionary effects when we rely on a mixed-frequency analysis.

Our work bridges the literature that studies MI(xed) DA(ta) S(ampling) regressions with the burgeoning research investigating the impact of financial uncertainty on the business cycle. MIDAS models are mainly employed to provide forecasting and in particular nowcasting analyses (Kuzin, Marcellino and Schumacher 2011, Foroni and Marcellino 2014, Huber, Koop, Onorante, Pfarrhofer and Schreiner 2020, among others). Few articles rely on estimating a mixed-frequency model to identify an economic shock. Ferrara and Guérin (2018), Casarin, Foroni, Marcellino and Ravazzolo (2018), and Bacchiocchi, Bastianin, Missale and Rossi (2020) provide interesting evidence by adopting a mixed-frequency strategy for the identification of the uncertainty shock. Ferrara and Guérin (2018) and Bacchiocchi et al. (2020) rely on a frequentist VAR estimation, while Casarin et al. (2018) propose a Bayesian multi-country Markov-Switching model.

Our approach can be compared to the recent contribution in adopting Bayesian techniques in a MIDAS framework. Eraker, Chiu, Foerster, Kim and Seoane (2014) introduce a Gibbs sampler in the Bayesian estimation of a MF-VAR, assuming that the high-frequency realizations of the low-frequency data are missing. Schorfheide and Song (2015) and Schorfheide and Song (2020) employ Bayesian techniques to estimate a state-representation introducing a numerical approximation of the marginal data density of a linear Gaussian MF-VAR.

In another approach, Berger, Morley and Wong (2020) and McCracken, Owyang and Sekhposyan (2021) apply a shrinkage prior in a stacked MF-VAR. Meanwhile, Mogliani and Simoni (2020) introduce a novel penalized Bayesian MIDAS approach in a high-dimensional environment. However, all these Bayesian contributions employ the mixed-frequency models only for forecasting analysis. Cimadomo, Giannone, Lenza, Monti and Sokol (2020) introduce three strategies for dealing with mixed-frequency in a VAR framework (state-space, blocking, and cube-root BVARs) to nowcast and study the propagation of the US GDP. They provide evidence of how mixed-frequency BVARs are an important and powerful tool for both forecasting and structural analysis. As for the latter, Cimadomo et al. (2020) focus on the transmission of a low-frequency shock (the GDP one) on both low- and high-frequency variables.

Our approach is different from the above-mentioned studies for both the methodological framework and the shock identification strategy. Technically, we impose a

Natural conjugate prior which is tailored to take into account the mixed-frequency nature of the data (in the spirit of Ghysels 2016). Then, the use of Bayesian shrinkage allows the researchers to identify the impact of high-frequency (e.g. daily/weekly) shocks on common low-frequency variables avoiding the "curse of the dimensionality". In particular, this approach is useful and more appropriate to study shock identification in case of a large mismatching between high and low frequency (for example, between daily and monthly) and when more endogenous variables are included.
Last but not least, our empirical results corroborate the macro-finance literature that discusses how an increase of uncertainty is followed by a contraction in real activity (Bloom 2009, Caggiano, Castelnuovo and Groshenny 2014, Leduc and Liu 2016, Basu and Bundick 2017, Alessandri and Mumtaz 2019, among others). In particular, our findings are connected with the current research about the macroeconomic effects of COVID-19-induced financial uncertainty (see Baker, Bloom, Davis, Kost, Sammon and Viratyosin 2020, Caggiano, Castelnuovo and Kima 2020, Leduc and Liu 2020, among others). However, while the aforementioned studies rely only on a common frequency estimation, we document empirical evidence about the recent pandemic crisis using a MIDAS model.

The rest of the paper is organized as follows. Section 2 introduces the Bayesian Mixed-Frequency VAR approach. Section 3 describes the empirical analysis details: data and identification strategy. Section 4 shows the empirical evidence with robustness checks. Concluding remarks are in Section 5.

2 Bayesian Mixed Frequency VAR Approach

We estimate a stacked Mixed-frequency Vector Autoregressive model (MF-VAR) à la Ghysels (2016). Let us consider $Kh = 1$ high-frequency variable ($y^{(m)}_{t-i/m}$) (e.g.

observed daily or weekly) and a vector of Kl variables sampled at a lower frequency (e.g. monthly), i.e. $X_t = \left(x_{1,t}, \ldots, x_{Kl,t}\right)'$, which are observed every m fixed periods. The reduced-form representation of the MF-VAR can be written as follows:

$$Z_t = \sum_{\ell=1}^{p} A_\ell Z_{t-\ell} + c + u_t \tag{1}$$

where $Z_t = (y^{(m)'}_{t-(m-1)/m}, \ldots, y^{(m)'}_{t-1/m}, y^{(m)'}_{t}, X'_t)'$ is the K-dimensional vector of endogenous variables, with $K = Kl + (Kh \times m)$, which follows a stacked skip-sampled process, c is a $K \times 1$ vector of intercepts and $u_t \sim \mathcal{N}(0, \Sigma)$ is a $K \times 1$ vector of error terms, with a variance-covariance matrix (Σ) that is not assumed to be diagonal.[1] The model in equation (1) can be estimated via OLS at the cost of obtaining imprecise estimates of the MF-VAR coefficients in case of a large number of parameters and a relatively small sample size.[2]

To deal with a potential parameters proliferation, we estimate the MF-VAR in equation (1) by adopting Bayesian estimation techniques. In particular, we build on the work of Götz et al. (2016) that performs Granger causality testing in MF-VAR using a Bayesian approach. This methodology, which in turn adapts the approach of Sims and Zha (1998) and Bańbura, Giannone and Reichlin (2010) to data sampled at different frequencies, consists of imposing a Natural Conjugate prior on the MF-VAR coefficients by augmenting the system in equation (1) with a set of *ad-hoc* artificial observations.

[1] The order of appearance of high- and low-frequency variables in the stacked vector Z_t depends on the empirical strategy (see Ghysels 2016). In our baseline model specification, the high-frequency variable (i.e. the VIX) is placed before the block of low-frequency variables (e.g. the macroeconomic aggregates) (see Section 3.2).

[2] As shown by the study of Foroni, Marcellino and Schumacher (2015), unrestricted lag polynomials in MIDAS regressions can be estimated via OLS. The authors find that unrestricted regressions perform better than standard MIDAS models (which are generally estimated through a non-linear least squares approach, see i.e. Ghysels, Sinko and Valkanov 2007) for small differences in sampling frequencies.

Following Ghysels (2016) and Götz et al. (2016), the prior distributions of the MF-VAR coefficients in A_ℓ (i.e. a_{ij}^{ℓ}, for $\ell = 1, \ldots, p$), are centered around a restricted MF-VAR(1). In particular, AR(1) priors tailored for the mixed-frequency nature of the data are imposed as follows:

$$\begin{bmatrix} y_{t-(m-1)/m}^{(m)} \\ \vdots \\ y_{t-1/m}^{(m)} \\ y_t^{(m)} \\ X_t \end{bmatrix} = \begin{bmatrix} 0 & \ldots & \rho_H & 0 \\ \vdots & \ddots & \vdots & \vdots \\ 0 & \ldots & \rho_H^{m-1} & 0 \\ 0 & \ldots & \rho_H^{m} & 0 \\ 0 & \ldots & 0 & \operatorname{diag}(\rho_L^m) \end{bmatrix} \begin{bmatrix} y_{t-1-(m-1)/m}^{(m)} \\ \vdots \\ y_{t-1-1/m}^{(m)} \\ y_{t-1}^{(m)} \\ X_{t-1} \end{bmatrix} + v_t \tag{2}$$

where $\rho = (\rho_H, \rho_L)$ denotes the prior mean respectively for the high- and low-frequency variables, with $\rho_L = \rho_{x_1}, \ldots, \rho_{x_{Kl}}$. Equivalently, the AR(1) prior for the MF-VAR coefficients can be set as follows:

$$\mathbb{E}(a_{ij}^{\ell}) = \begin{cases} \rho_H^{m+i-j} & \text{if } i \leq m \ \ \& \ \ j = m \ \ \& \ \ \ell = 1 \\ \rho_L^{m} & \text{if } i = j \ \ \& \ \ i > m \ \ \& \ \ \ell = 1 \\ 0 & \text{otherwise} \end{cases} \tag{3}$$

In line with Götz et al. (2016), we specify the uncertainty around the prior means similarly to the CF-VAR:

$$VAR(a_{ij}^{\ell}) = \begin{cases} \phi \frac{\lambda^2 \sigma_H^2}{\ell^2 \sigma_L^2} & \text{if } i \leq m \ \ \& \ \ j > m \\ \phi \frac{\lambda^2 \sigma_L^2}{\ell^2 \sigma_H^2} & \text{if } i > m \ \ \& \ \ j \leq m \\ \phi \frac{\lambda^2 \sigma_{i,L}^2}{\ell^2 \sigma_{j,L}^2} & \text{if } i > m \ \ \& \ \ j > m \ \ \& \ \ i \neq j \\ \frac{\lambda^2}{\ell^2} & \text{otherwise} \end{cases} \tag{4}$$

where λ controls the tightness of the prior distributions around the specifications in equations (2) and (3), the ratio σ_i/σ_j, for $i, j = (H, L)$, accounts for the different scales of the high- and low-frequency variables and ϕ controls, for each VAR equation, the standard deviation of the prior on lags associated to the variables different from the dependent one (e.g. in case of MF-VAR, it controls the influence of low-frequency variables on the high-frequency ones and vice versa) (see Götz et al. 2016).[3]

As in CF-VAR models, augmenting the system in equation (1) with a set of dummy observations is equivalent to imposing a Natural conjugate prior for the MF-VAR coefficients.[4] Before describing the construction of the artificial observations for the mixed-frequency case, let us write the model in equation (1) in compact matrix notation:

$$Z = \underline{Z}B + U \tag{5}$$

where $Z = (Z_1, \ldots, Z_T)'$, $\underline{Z} = (\underline{Z}_1, \ldots, \underline{Z}_T)'$, with $\underline{Z}_t = (\underline{Z}'_{t-1}, \ldots, \underline{Z}'_{t-\ell}, 1')$, $U = (u_1, \ldots, u_t)'$ and $B = (A_1, \ldots, A_p, c)'$. In line with Bańbura et al. (2010), the Natural conjugate prior can be imposed by augmenting the model in equation (5) with a set of artificial observations, Y_d and X_d, that is $Z^* = \underline{Z}^* B + U^*$, where $Z^* = (Z', Y_d')'$ and $\underline{Z}^* = (\underline{Z}', X_d')'$. While the set of dummy observations for the lagged endogenous variables (X_d) are constructed as in Bańbura et al. (2010), to match the moments in equations (2)-(4), we specify Y_d as follows:

[3]Notice that the specifications of the prior means and variances in equations (2)-(4) are tailored to the case of Kl low-frequency variables and $Kh = 1$ high-frequency variable. However, these specifications can be easily modified to handle more than one high-frequency variable.

[4]The Natural conjugate prior is related to the Minnesota prior with $\phi = 1$, that is, for each VAR equation, there is no distinction between the lags associated to the dependent variable and those related to the independent ones (see Sims and Zha 1998, Bańbura et al. 2010).

$$
\underset{[(Kp+1)+K]\times K}{Y_d} = \left(\begin{array}{ccccc}
\multicolumn{5}{c}{\mathbf{0}_{[(m-1)\times Kh]\times K}} \\
\frac{\rho_H \sigma_H}{\lambda} & \dots & \frac{\rho_H^{m-1}\sigma_H}{\lambda} & \frac{\rho_H^{m}\sigma_H}{\lambda} & \mathbf{0}_{1\times Kl} \\
\mathbf{0}_{Kl\times 1} & \dots & \mathbf{0}_{Kl\times 1} & \mathbf{0}_{Kl\times 1} & \mathrm{diag}\left(\frac{\rho_L^{m}\sigma_L}{\lambda}\right)_{Kl\times Kl} \\
\multicolumn{5}{c}{\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots} \\
\multicolumn{5}{c}{\mathbf{0}_{K(p-1)\times K}} \\
\multicolumn{5}{c}{\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots} \\
\multicolumn{5}{c}{\mathrm{diag}(\sigma_{1,H}, \dots, \sigma_{m,H}, \sigma_{1,L}, \dots, \sigma_{Kl,L})_{K\times K}} \\
\multicolumn{5}{c}{\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots} \\
\multicolumn{5}{c}{\mathbf{0}_{1\times K}}
\end{array}\right) \tag{6}
$$

where the artificial data in the first block (i.e. those for the first lag) are formed such that they reflect the prior belief on the restricted MF-VAR, while the other blocks are constructed in line with Bańbura et al. (2010).

It is important to notice how the order of the high-frequency variable is relevant in this framework. The researcher can easily adapt this approach just modifying consistently the position of the dummy observations according to the order of the high-frequency variable.

In the empirical application, we follow the suggestions of Ghysels (2016) and we set the prior mean of the high-frequency variable equal to zero, that is $\rho_H = 0$.[5] In line with Alessandri and Mumtaz (2017), the coefficients associated with the first lag of the low-frequency variables are centered around the OLS estimates of the coefficients obtained from an AR(1) fitted to each endogenous variable over a

[5] This choice is also in line with the empirical analysis described in Cimadomo et al. (2020), where the prior mean of the high-frequency proxy of uncertainty (i.e. the Economic Policy Uncertainty Index of Baker, Bloom and Davis 2016) is centered around zero.

training sample. The hyperparameter that controls for the overall tightness around the prior (λ) is selected by maximizing the marginal likelihood of the model.[6] We set the scaling factors σ_i, σ_j , for $i, j = (H, L)$, using the standard deviation of the residuals from AR(m) and AR(1) regressions estimated for y_t^m and x_t, respectively (see Götz et al. 2016). Finally, we impose a diffuse prior on the intercept (c).

To make inference, we then proceed as in the common frequency VARs with Natural conjugate prior. In particular, the conditional posterior distributions for the MF-VAR coefficients (B and Σ) can be written as follows:

$$
\begin{aligned}
&B|\Sigma, Y \sim \mathcal{N}\Big(B^*,\ \Sigma \otimes (\underline{Z}^{*'} \underline{Z}^*)^{-1}\Big) \\
&\Sigma|B, Y \sim \mathcal{IW}\Big(S^*,\ v^*\Big)
\end{aligned} \tag{7}
$$

where $B^* = (\underline{Z}^{*'}\underline{Z}^*)^{-1}\underline{Z}^{*'}Z^*$ is the OLS estimate of the augmented regression, while $S^* = (Z^* - \underline{Z}^*\tilde{B})'(Z^* - \underline{Z}^*\tilde{B})$ and v^* are, respectively, the scale parameter and the degrees of freedom of the Inverse Wishart distribution, with $\tilde{B}$ being a draw of the MF-VAR coefficients and v^* set equal to the number of observations in the augmented regression. In the empirical illustration, we focus on the structural analysis, hence we rely on the Gibbs sampler to simulate the posterior distribution of the MF-VAR coefficients.[7] In particular, we set the number of draws equal to $15,000$ and we discard the first $10,000$ as burn-in draws.

[6] In our study, the selection of the optimal overall tightness of the prior (λ) is based on Carriero, Kapetanios and Marcellino (2012), which suggest selecting λ over a grid of values. In particular, we use the following grid: $\lambda \in \{0.01, 0.05, 0.1, 0.2, 0.5, 1, 1.5, 2, 3\}$ (see also Del Negro and Schorfheide 2004, Del Negro and Schorfheide 2011).

[7] The code used in this paper are an adaptation of the Haroon Mumtaz's code for the estimation of a Bayesian CF-VAR (which are available on his website).

3 Empirical Analysis

3.1 Data

We assess the effects of high-frequency financial uncertainty shocks on a set of US business cycle variables over the sample period 1990M1-2019M12.[8] The macroeconomic variables are sampled at a monthly frequency. In detail, we use the industrial production index (IP) and the consumer price index (CPI) as proxies of real economic activity and prices, respectively. The set of endogenous variables includes the real personal consumption expenditures (PCE).[9] The short-term interest rate is proxied by the effective federal funds rate (FFR).[10]

The high-frequency financial uncertainty shock is proxied by the daily VIX data (see Figure 1). We conduct two empirical exercises. First, we estimate the MF-VAR in equation (1) fitted to weekly series of VIX and the above mentioned monthly macroeconomic variables. In line with Ferrara and Guérin (2018), the

[8]In the baseline specification, we exclude the COVID-19 era from the sample and we estimate the MF-VAR using data up to December 2019. During the recent months, a number of authors have developed VAR-based strategies to deal with the extreme observations reported by several U.S. macroeconomic variables after March 2020, for both nowcasting and structural analysis (see Lenza and Primiceri 2020, Schorfheide and Song 2020, among others). For example, Lenza and Primiceri (2020) introduce breaks in shock variances and down-weigh the impact of the pandemic observations on the parameters estimates. As structural analysis concerns, they find that the impulse responses of the estimated VAR with breaks in shock variances over a sample including also COVID-19 period (e.g. up to May 2020) are similar to those of a homoschedastic VAR with the sample excluding the pandemic time. However, as a robustness check, we also estimate the MF-VAR over the 1990M1-2020M11 time span (whose results are discussed in Section 4.2).

[9]The real personal consumption expenditure is computed by applying the personal consumption expenditures (price) on the nominal series.

[10]In the baseline specification we select the endogenous variables according to Caggiano, Castelnuovo and Pellegrino (2017) which estimate the impact of uncertainty shock (proxied by an unexpected increase in the VIX) on GDP deflator, real GDP, real investment, real consumption, and federal funds rate through the estimation of a non-linear (common-frequency) quarterly VAR. Unlike Caggiano et al. (2017), since in our empirical application the MF-VAR includes monthly series of business cycle variables, we rely on industrial production (instead of the real GDP) as a proxy of real economic activity and we exclude the investment, whose observations are only available at a quarterly frequency. The motivation of this choice is due to our focus on the miss-match between weekly (daily) and monthly series.

weekly observations on VIX are constructed such that each month contains four weeks (see Figure 2).[11] In a second empirical exercise, we replace the weekly series of VIX with daily observations by following the approach proposed by Götz et al. (2016). In particular, the series is constructed by assuming that each month contains 20 daily observations.[12]

Since we seek to investigate potential bias arising from the aggregation of high-frequency variables (i.e. VIX) into lower-frequency series, we also estimate a common-frequency (CF) VAR where all the endogenous variables (including the VIX) are observed monthly. In particular, we aggregate the daily VIX series (see Figure 1) to a monthly frequency by averaging out the observations over each month.[13]

We conduct several robustness checks on the specification of the MF-VAR (see Section 4.2). First, we augment the baseline specification by the unemployment rate (UNEMP.RATE) and by the 10-year treasury constant maturity rate (10YR-TB) (as a measure of the long-term interest rate).[14] In a second robustness check, we

[11]Following Ferrara and Guérin (2018), the daily observations on VIX are rearranged at a weekly frequency as follows. Given a number of traded days within each month (D_t), we compute the four weekly observations by considering the days $D_t - 15$, $D_t - 10$, $D_t - 5$ and D_t as observations for week 1, week 2, week 3 and week 4, respectively. We thank Laurent Ferrara and Pierre Guérin for sending us detailed information on the construction of the weekly series of VIX used in Ferrara and Guérin (2018).

[12]In their empirical application, Götz et al. (2016) construct a daily series of bipower variation of the S&P500 stock index by considering that each month has 20 observations. In case of more than 20 observations within a certain month, the authors suggest disregarding the corresponding amount of days at the beginning of the month. For example, March 2019 has 21 traded days. Hence, to obtain the daily series of VIX, we discard the first observation, e.g. that of 1 March 2019.

[13]As a robustness check, in the two empirical exercises, the aggregation of the VIX to a monthly frequency is also carried out by averaging, respectively, the four weekly observations (i.e. those constructed as in Ferrara and Guérin 2018) and the twenty daily observations (i.e. those constructed as in Götz et al. 2016) over each month. The results obtained using these two aggregation schemes are qualitatively and quantitatively similar to those described in the rest of the paper and they are available upon request.

[14]It is worth mentioning that both the federal funds rate and the 10-year treasury constant maturity rate are available at a daily frequency. However, since the focus of the empirical analysis is on the identification of uncertainty shocks through the use of real-time proxies of financial uncertainty, we use the monthly series for both the federal funds rate and the 10-year treasury constant maturity rate.

replace the federal funds rate and the 10-year treasury rate with the shadow short rate proposed by Wu and Xia (2016).[15]

For both mixed-frequency and common-frequency VARs, the variables are adequately transformed to induce stationarity. In particular, we take the first difference of the log transformation of prices (CPI), industrial production (IP), and real consumption (PCE), while the proxy of financial uncertainty (VIX) and the federal funds rate (FFR) enter the model in levels.[16] Furthermore, the unemployment rate (UNEMP.RATE), the 10-Year treasury constant maturity rate (10YR-TB), and the shadow short rate proposed by Wu and Xia (2016) (SHADOW RATE) are taken in levels (see Section 4.2). The lag length is set equal to three.[17] Following Ferrara and Guérin (2018), to ensure comparison across the models, the common-frequency (monthly) VAR is estimated using the same lag length of the mixed-frequency VAR.[18] The data are seasonally adjusted and downloaded from the Federal Reserve Bank of St. Louis (FRED) Database unless indicated otherwise.

3.2 Identification strategy

The relationship between the reduced-form residuals (obtained by estimating the model in equation (1)) and the structural disturbances can be written as follows:

[15] The Wu-Xia shadow rates series is available at https://sites.google.com/view/jingcynthiawu/shadow-rates.

[16] The results are qualitatively similar when estimating the models with variables entering in log-levels (i.e. CPI, IP, PCE) and levels (i.e. VIX, FFR). Results are available upon request.

[17] The Akaike Information Criteria (AIC) for the MF-VAR with weekly VIX and monthly macroeconomic variables indicates an optimal lag length equal to two-three. However, we impose an one-quarter lag on the MF-VAR processes. As a further robustness check, we estimate both the MF-VAR and the CF-VAR with, respectively, six and twelve lags (see Section 4.2). The MF-VAR with daily VIX includes also three lags. The results with different lag structures (available upon request) are qualitatively similar.

[18] This choice is also confirmed by the AIC that suggests an optimal lag length equal to three-four for the monthly CF-VAR specifications.

$$u_t = A_0 \varepsilon_t \tag{8}$$

where A_0 contains the contemporaneous effects of the structural shocks (ε_t) on the endogenous variables, with $\varepsilon_t \sim \mathcal{N}(0, I_K)$. To identify the high-frequency uncertainty shocks, we compute the Cholesky decomposition of the reduced-form residual covariance matrix, $\Sigma = A_0 A_0'$, imposing a recursive ordering of the elements in A_0.

For the sake of simplicity, in the rest of this section, we describe only the first empirical exercise (i.e. that using weekly VIX).[19] Following Caggiano et al. (2017) and Ferrara and Guérin (2018), we order the endogenous variables in the baseline specification as follows:

$$Z_t = \left[VIX'_{t-3/4}, VIX'_{t-2/4}, VIX'_{t-1/4}, VIX'_t, X'_t\right]' \tag{9}$$

where $VIX_{t-i/4} = \left[VIX'_{t-3/4}, \ldots, VIX'_t\right]$, for $i = 1, \ldots, 4$, is the vector containing the series of VIX, respectively, for the first, second, third, and fourth week, while $X_t = \left[CPI'_t, IP'_t, PCE'_t, FFR'_t\right]$ is the block of monthly business cycle variables. Notice that, according to the specification in equation (9) where the weekly observations of VIX are aligned to the lowest sampling frequency, the stacked vector of endogenous variables evolves according to a standard monthly VAR.

The ordering of the variables in the macro-block (X_t) is standard in the VAR literature. The slow-moving variables (CPI, IP, and PCE) are placed before the fast-moving ones (FFR). This implies that monetary policies depend on the real activities.

[19] The MF-VAR fitted to the daily series of VIX and to the monthly business cycle variables is estimated using the same empirical strategy of that used in case of weekly VIX (i.e. same specification, priors, and identification strategy).

Moreover, in line with Ferrara and Guérin (2018), the weekly series of VIX are placed before the macro-block, with an ordering of the intra-month observations that are consistent with the timing of data release (i.e. publication lags).[20] This has two implications. First, we allow for a contemporaneous effect of uncertainty shocks on real economic activities and monetary policies. This ordering is also consistent with Leduc and Liu (2016), Basu and Bundick (2017) and, more recently, Caggiano et al. (2020), among others. Furthermore, this ordering implies that a shock occurring in a certain week has an impact on the corresponding weekly series of VIX and the following weeks. As stated by Ferrara and Guérin (2018), this is a plausible assumption given that i.e. the observation of the VIX related to the second week is released always after the observation of the first week. Hence, a financial uncertainty shock occurring at week 2 affects only the VIX observed during that week and the weeks after.

We calibrate the size of the uncertainty shock considering different impact scales to investigate how the magnitude of the uncertainty is important. In particular, we identify a 5σ of the VIX shock estimated over the sample period 1990M1-2019M12. Our decision is motivated by the recent work of Caggiano et al. (2020) that estimates the effects of global uncertainty (proxied by an exogenous increase in the VIX) on global financial conditions and world industrial production.[21] As in Caggiano et al. (2020), the size of the shock is set by comparing the values of the VIX observed during its peak (that is on 16 March 2020) with the value reported in the previous

[20] As a robustness check, we repeat the empirical exercise by ordering the VIX last in the vector of endogenous variables (Z_t) (see Section 4.2). As discussed by Ghysels (2016), the Cholesky decomposition seems a natural identification scheme for MF-VAR. However, we rely on a Bayesian approach that makes our methodology flexible to be used in alternative identification patterns. For example, the researchers can adopt it to identify more shocks at the same time as well as combining different identification schemes.

[21] See Caggiano et al. (2020) for a discussion on the use of the VIX as a proxy of global uncertainty.

month (18 February 2020) (see Figure 3).[22] However, we report in Appendix results with different scale values (see Appendix A, Figure A.1). As can be seen from the charts, the results are qualitatively similar.

4 Empirical Evidence

4.1 Results

Figures 4-7 show the high-frequency shock (VIX) identification on monthly macroeconomic variables (CPI, IP, PCE, and FFR) providing results from the baseline specification. The estimated model is the MF-VAR(3) over the 1990M1-2019M12 time span.[23]

The orthogonalized impulse responses, computed over a 36-month forecast horizon, are equal to 5σ VIX shocks (see Section 3.2). For the variables entering the models in first-order difference of log transformation (IP, CPI, and PCE), the impulse responses are computed as the cumulative sum of those obtained for the log changes.

Unless specified otherwise, all the figures show the posterior median response (red line) with the 68% (red shading) and 90% (gray shading) credibility intervals obtained from the estimation of the MF-VARs.

In the first exercise, see Figure 4, we estimate the baseline mixed-frequency VAR to identify the impact of VIX shocks at a weekly frequency on the consumer price index, industrial production index, real personal consumption expenditures,

[22] As stated by Caggiano et al. (2020), only 90 percent of the increase in the VIX observed between mid-February and mid-March 2020 can be attributed to the coronavirus outbreak. Hence, given that the value reported on March, 16 is 5.6 times larger than the value observed in the previous month, the size of the COVID-19-induced financial uncertainty shock is set as follows: $5.6 \times 0.9 = 5.04 \approx 5$ (see Caggiano et al. 2020, for further details).

[23] Information on the convergence of the Gibbs sampler algorithm are reported in Appendix B.

and effective federal funds rate. In particular, Figure 4 shows the responses of the business cycle variables to uncertainty shocks occurring in each of the four weeks.[24]

At a first glance, we can observe how an unexpected increase in high-frequency financial uncertainty is followed by a negative effect on the real economic activity, prices, real consumption, and federal funds rate. These results corroborate the findings in both theoretical and empirical literature. For example, Leduc and Liu (2016) and Basu and Bundick (2017) discuss how the uncertainty shock resembles a negative demand shock relying on DSGE and VAR models. Furthermore, in line with the empirical evidence reported in Ferrara and Guérin (2018) for the US, we find a different response of the low-frequency variables, depending on the timing of the shocks within the month. In particular, Figure 4 reveals a different magnitude of the response diminishing from week 1 to week 4.[25] As discussed by Ferrara and Guérin (2018), these results can be explained by the high degree of persistence and by the typical hump-shaped response of macroeconomic variables to uncertainty shocks (see also Baker et al. 2016).[26]

Furthermore, we compare the weekly impulse responses from the estimation of the MF-VAR model with those obtained from a CF-VAR.[27] In particular, we aggregate the high-frequency impulse responses of the macroeconomic variables by computing their mean (see Figure 5).[28] As shown in Figure 5, we find evidence of difference

[24]Notice that, as mentioned before, the impulse responses are scaled such that the size of the shocks occurring in each of the four weeks is equal to 5σ VIX shocks estimated over the sample period 1990M1–2019M12.

[25]Similar results are also reported by the study of Bacchiocchi et al. (2020), which finds that the response of the Federal Funds Target rate to uncertainty shocks is stronger in the first month than late in the quarter (although the responses are not statistically significant).

[26]Moreover, Ferrara and Guérin (2018) argue that if economic agents take decisions at a high-frequency, it is plausible to expect that shocks occurring late in the month might have different short-term impacts with respect to shocks taking place early in the month.

[27]As stated before, the standard VAR is estimated using the same lag structure of that used for the estimation of the MF-VAR (i.e. 3 lags) (see Section 3).

[28]Foroni and Marcellino (2016) and, more recently, Bacchiocchi et al. (2020) provide a discussion

in the responses of the low-frequency variables to uncertainty shocks. In particular, for all the macroeconomic variables, the magnitude of the responses is smaller in MF-VAR (almost half) than that obtained from the estimation of a common low-frequency VAR. What is striking in Figure 5 is that the difference in the magnitude of the responses obtained from both mixed-frequency and common-frequency VAR is relevant not only over a short horizon, but also in a longer run. Moreover, we find a less amount of uncertainty around the posterior median estimates, with credibility intervals for MF-VAR much tighter than those reported in case of common-frequency VAR. This finding is also supported by the evidence of Foroni and Marcellino (2016) about a reduction in the uncertainty when relying on a mixed-frequency approach.[29]

We can discuss how the impulse responses are leading a "temporal aggregation bias" with a visual inspection of both Figure 4 and Figure 5. First, as also documented by Ghysels (2016) in case of high-frequency shock, the first week response is more relevant than the one in the last week that seems to die out quickly (see Figure 4). Second, we note also how the CF-VAR responses are different from the ones of the MF-VAR, even in longer horizons (see Figure 5). This suggests how the "temporal aggregation bias" is an important aspect to consider when deciding to rely on a common or mixed-frequency framework.

In the second high-frequency identification exercise, we consider the VIX at a daily frequency. This variable is available in days and the aforementioned aggregation to a weekly frequency is likely to lead to a an additional "temporal aggregation bias"

on the comparison between mixed-frequency and common-frequency VAR, respectively, in the case of the parameter-driven model (i.e. state-space representation) and stacked MF-VAR. However, the mean is one of the possible ways to aggregate high-frequency responses.

[29]The authors study the identification of monetary policy shocks in US by estimating a MF-VAR fitted to monthly and quarterly data. In particular, comparing results from a mixed-frequency model and a common-frequency VAR, Foroni and Marcellino (2016) find a reduction in the uncertainty around the estimates when using a mixed-frequency data sampling approach. Moreover, they find differences in the magnitude of the responses, particularly striking for interest rates.

in our analysis. The daily series of VIX is constructed such that each month contains 20 daily observations (see Section 3.1). As in case of weekly frequency, the size of the shocks occurring in each of the 20 days is equal to 5σ VIX shocks estimated over the period 1990M1-2019M12.

In particular, in Figure 6 we report the posterior median estimates of the daily impulse responses to VIX shocks obtained from the estimation of a MF-VAR(3) fitted to the daily series of VIX and to the monthly macroeconomic variables.[30] Figure 6 documents an interesting pattern that describes the evolution over time for each variable. Similarly to the results obtained by estimating the model using weekly observations on VIX, we find that the magnitude of responses of CPI, PCE, FFR and, to less extend, IP is larger in the first days of the month than that reported late in the month.

In Figure 7, we compare the aggregated daily impulse responses of the US business cycle variables with those computed by estimating a CF-VAR.[31] In general, we still find less severe responses to the uncertainty shock for the macroeconomic variables in case of MF-VAR (less than half) than in case of common low-frequency VAR. The results are statistically significant in almost all macroeconomic variables. These findings reveal a stronger evidence of "temporal aggregation bias" than that found in case of weekly frequency, both in terms of magnitude in the response (with differences also at longer horizons) and of uncertainty around the estimates.

[30]Notice that Figure 6 shows only the median responses of the macroeconomic variables to daily uncertainty shocks. The set of daily impulse responses with the 68% and 90% credibility intervals are available upon request.

[31]The aggregated impulse responses are obtained by averaging out the daily responses.

4.2 Robustness

This section describes a number of empirical exercises implemented to assess the robustness of the results produced by the baseline model.[32] The results are shown in Figures 8-14. Unless specified otherwise, in each figure, we report the posterior median of the aggregated high-frequency impulse responses (from the MF-VAR) (red line) with the 68% (red shading) and 90% (gray shading) credibility intervals, together with the responses obtained from the estimation of a CF-VAR (blue lines).

Number of Lags. Figure 8 documents the aggregated impulse responses estimated at a weekly frequency using either 6 (Panel a) or 12 lags (Panel b).[33] The evidence of "temporal aggregation bias" is also confirmed when increasing the lag length. In particular, we find that while the responses of the MF-VAR are statistically significant over the whole forecast horizon, the uncertainty around the estimates tends to become larger in case of CF-VAR.

Endogenous Variables. In Figure 9, we report the aggregated weekly impulse responses obtained from a MF-VAR when the unemployment rate and the 10-year treasury constant maturity rate are included in the set of endogenous variables. As can be seen from the charts, we still find evidence of a "temporal aggregation bias" when including additional macroeconomic variables. In particular, both the unemployment rate and the long-term interest rate also report a lower magnitude (almost half) in the impulse responses of the MF-VAR (see Figure 9). Moreover, by introducing these two variables, we note how the response of industrial production converges more quickly to zero than in the baseline specification.

Figure 10 shows the aggregated weekly responses when replacing FFR with the

[32] We show robustness checks for the baseline specification including weekly VIX. Only when we include the COVID-19 period, we provide evidence with both weekly and daily frequencies.

[33] The use of lags equal to (or greater than) 12 is a common choice in a VAR fitted to monthly variables.

shadow short rate à la Wu and Xia (2016). The empirical findings are still robust. The response of the shadow short rate shows a "temporal aggregation bias" similar to the one reported with FFR and 10-year treasury bill.

VIX ordered last. Figure 11 shows the aggregated weekly impulse responses of the US macroeconomic variables (included in the baseline specification) obtained by computing the Cholesky decomposition of the reduced-form residuals covariance matrix with the VIX ordered last in the vector of the endogenous variables. As shown by Figure 11, the results are qualitatively and quantitatively similar to those described in Section 4.1 (i.e. with the VIX ordered first).

Including COVID-19 period. We extend the sample including the period subsequent to the COVID-19 outbreak, repeating the estimation of the baseline model over the sample 1990M1-2020M11. We identify the shock by relying on both weekly and daily frequencies. As in the previous empirical exercises, the size of the shocks occurring in each of the four weeks (or in each of the 20 days) is equal to 5σ VIX shocks estimated over the period 1990M1-2020M11.

Figure 12 reports the aggregated weekly responses. We document the aggregation bias shown in the baseline model. Interesting to note how the response both of industrial production and of consumption show a quick decrease followed by an increase around period 5 and, after that, another less severe decrease.

Figures 13-14 provide evidence of the daily responses. In particular, we find interesting results in case of higher frequency. For almost all macroeconomic variables, we document how the responses reach their peak at the mid of month (around 10 days) (see Figure 13). The aggregated daily responses (obtained from MF-VAR) still show the aggregation bias as in the case of the baseline specification (see Figure 14).

However, we need to consider the criticisms about the inclusion of the time after March 2020 as a significant caveat when reading these results. Lenza and Primiceri (2020) suggest to model the change in shock volatility in order to account for the exceptionally large macroeconomic variation during the pandemic crisis. They propose to re-scale the standard deviation of the March shocks by an unknown parameter as well in April and May with other unknown parameters as done in Giannone, Lenza and Primiceri (2015). We leave in our future research agenda to explore this further issue.

5 Concluding Remarks

We contribute to the literature on mixed-frequency regressions by introducing an innovative Bayesian approach to identify high-frequency shocks. This methodology is inspired by Götz et al. (2016) and Ghysels (2016) that suggest Bayesian techniques to improve the estimation of models with different data sampling. A Normal Inverse Wishart prior is imposed in estimating a MF-VAR by adding a set of auxiliary dummies. Relying on this new methodology, we provide novel empirical evidence of a financial uncertainty shock (VIX) identification for the US economy. When we estimate a CF-VAR instead of a MF-VAR, with weekly and daily frequencies, we find a "temporal aggregation bias". Moreover, we document an amplified bias when a higher frequency shock is identified. The mixed-frequency and common low-frequency responses differ consistently across horizons. We extend our empirical investigation by including the recent pandemic crisis induced by the COVID-19. These results show an amplified "temporal aggregation bias" providing an interesting policy interpretation. The mixed-frequency approach suggests less severe recessionary effects on the macroeconomic variables and we can also disentangle accurately the responses along weeks and days.

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Appendices

Appendix A. Multi-σ shocks

In this appendix, we report the impulse responses of the U.S. business cycle variables obtained from the estimation of the baseline MF-VAR(3) (whose results are reported in Section 4.1) using different sizes of the VIX shock. Figure A.1 shows the posterior median responses, respectively, to 1σ, 5σ, and 10σ VIX shocks estimated over the period 1999M1-2019M12.[34]

For all the macroeconomic variables, the red line (i.e. response to 5σ shock) is the same of that reported in Figure 5. Since the impulse responses are simply re-scaled, the use of different sizes of the VIX shock leads to a similar shape in the (negative) response profiles of the U.S business cycle variables.

Appendix B. Convergence diagnostics

In this appendix, we assess the convergence of the Gibbs sampler algorithm performed in the estimation of the baseline MF-VAR(3) using both the weekly and the daily series of VIX (see Section 4.1). In particular, following Primiceri (2005), we compute the autocorrelation function of the retained draws (i.e. 5000 replications) for the MF-VAR coefficients (slope coefficients and the intercepts) in B and for the elements entering the residual covariance matrix Σ (see equations (1) and (5)). As reported in Primiceri (2005), low autocorrelation of the draws increases the efficiency of the algorithm.

Figure B.1 shows the 20-th order sample autocorrelation computed for the 200 MF-VAR parameters (slope coefficients and intercepts) (upper panel) and for the 64 parameters in the residual covariance matrix (lower panel), obtained from the estimation of the MF-VAR(3) fitted to weekly VIX and monthly macroeconomic variables. As can be see from the charts, the autocorrelations remain below 0.1 (in absolute value) for both the VAR parameters and the residual covariance matrix, suggesting that the retained draws are almost independent.

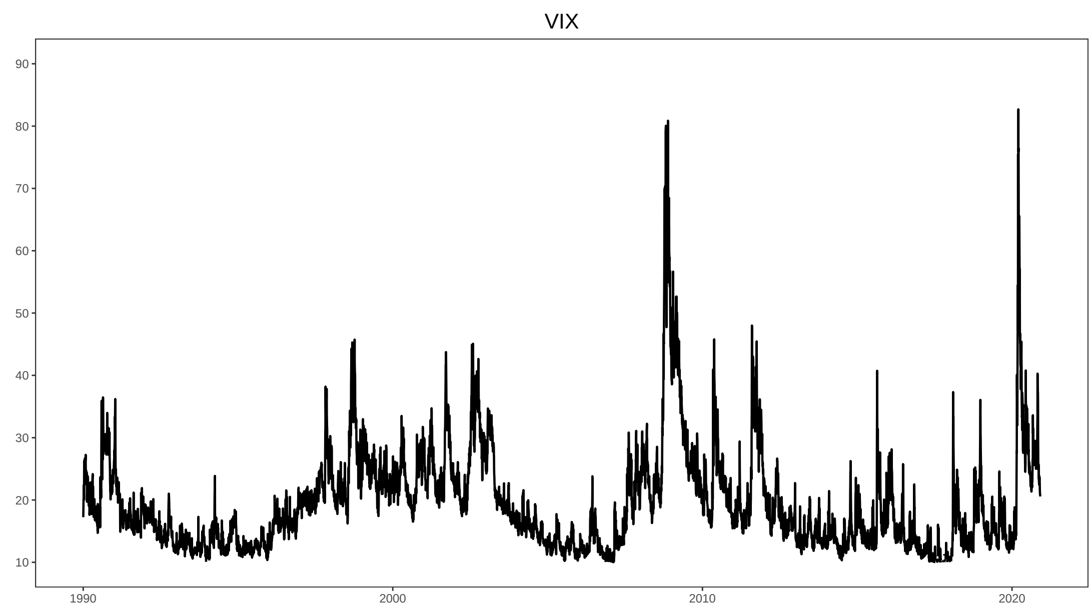
[34] It is important to notice that, while 5σ is the size of the shock used in the empirical application (see also Section 3.2), the choice of the other two magnitudes (i.e. 1σ and 10σ) is arbitrary.

Similar results are obtained when computing the autocorrelation functions for the parameters obtained from the estimation of the MF-VAR(3) using the daily series of VIX (see Figure B.2).

Thus, there is evidence of convergence of the Gibbs sampler algorithm in both the empirical exercises (i.e. using either weekly or daily series of VIX).

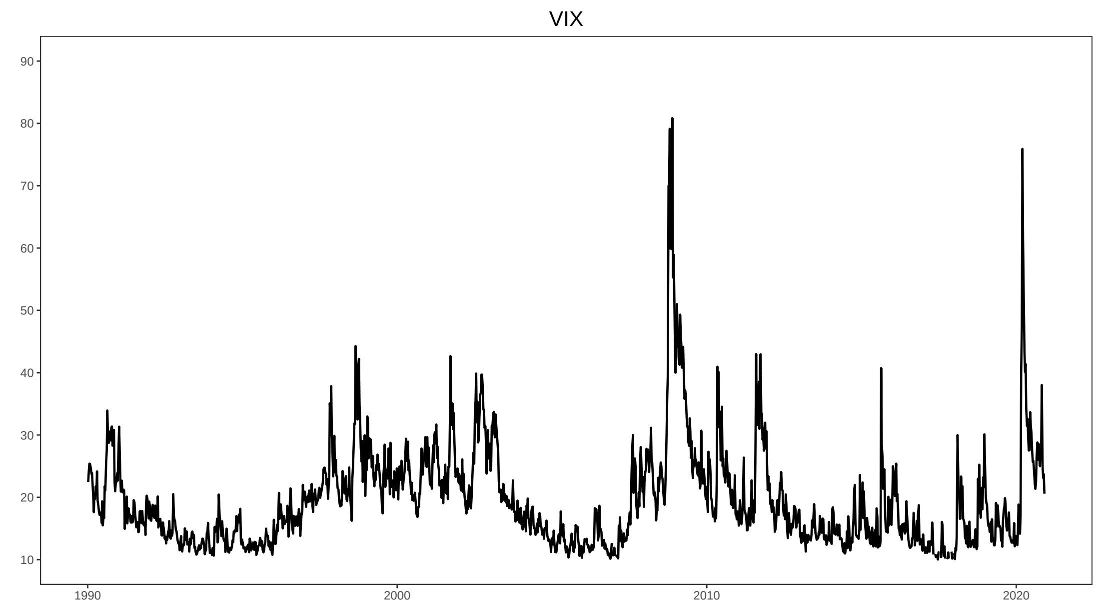
List of Figures

Figure 1: VIX (daily frequency). 1999M1-2020M11.



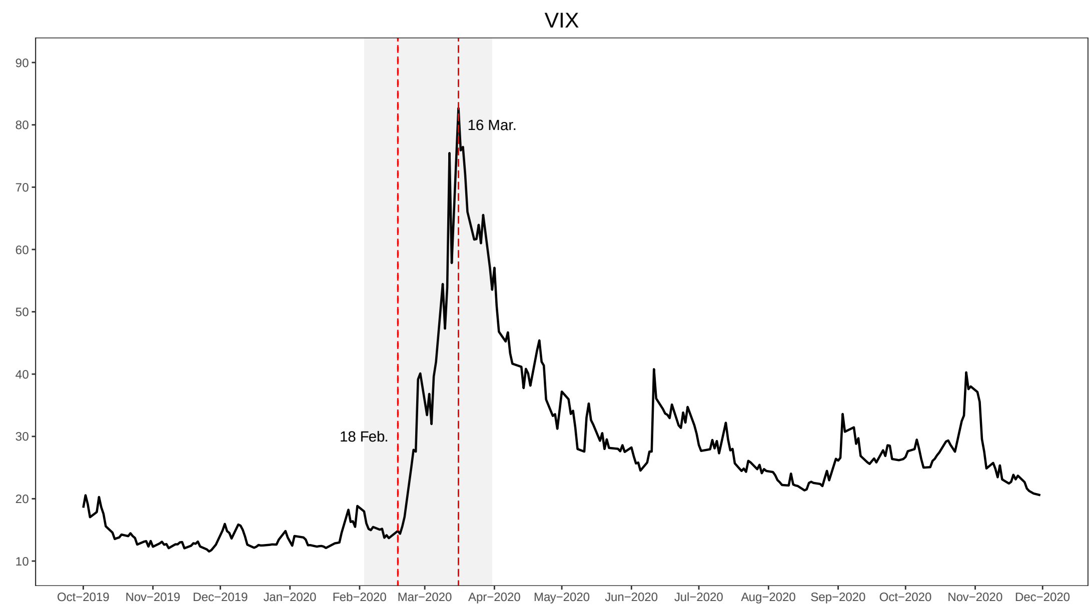
Notes. The chart shows the VIX at a daily frequency over the period 1990M1-2020M11.

Figure 2: VIX (weekly frequency). 1990M1-2020M11.



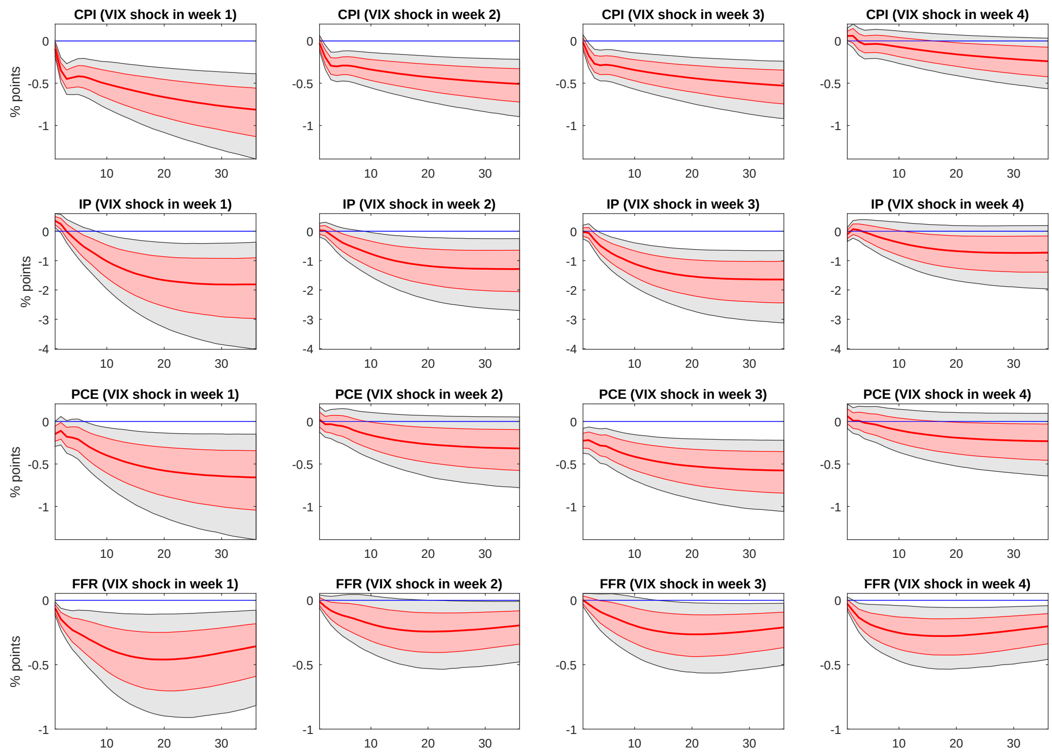
Notes. The chart shows the VIX at a weekly frequency over the period 1990M1-2020M11. The weekly series of VIX is constructed by following the suggestions of Ferrara and Guérin (2018). In particular, the daily observations on VIX are rearranged at a weekly frequency as follows. Given a number of traded days within each month (D_t), the four weekly observations are computed by considering the days $D_t - 15$, $D_t - 10$, $D_t - 5$, and D_t as observations for week 1, week 2, week 3, and week 4, respectively.

Figure 3: Calibration of the size of the financial uncertainty shock.



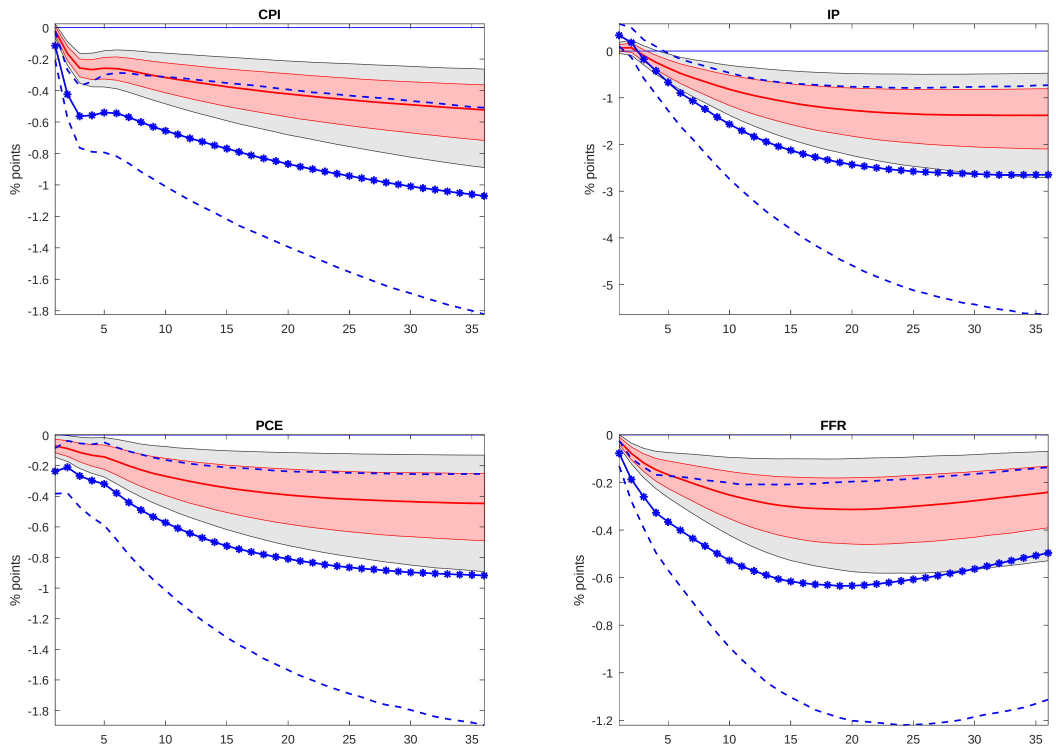
Notes. The chart shows the VIX at a daily frequency over the period 2019M10-2020M11. The vertical red dashed lines correspond to the peak of the VIX observed on 16 March 2020 (the VIX is equal to 82.69) and to the value of the VIX registered one month before, that is on 18 February 2020 (with a value equal to 14.83). Information on the calibration of the size of the uncertainty shock are reported in Section 3.2.

Figure 4: Weekly responses of the U.S. macroeconomic variables from the baseline MF-VAR(3) estimated over 1990M1-2019M12.



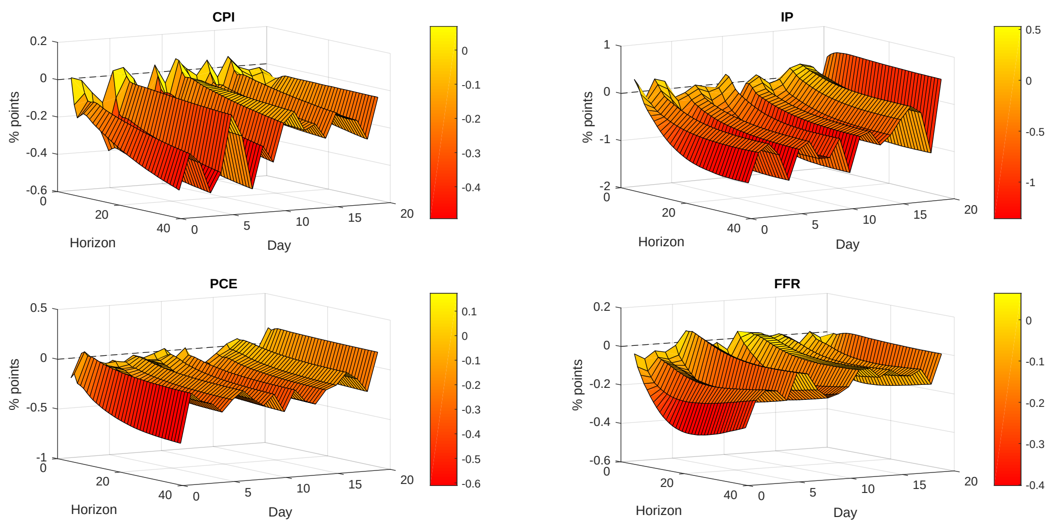
Notes. Impulse responses of the level of U.S. consumer price index (CPI), industrial production index (IP), real personal consumption expenditures (PCE), and effective federal funds rate (FFR) in percentage points, computed over a 36-month forecast horizon. Each row displays the response of the variable of interest to shocks occurring in week 1, week 2, week 3, and week 4. The size of the shocks occurring in each of the four weeks is equal to 5σ VIX shocks estimated over the period 1990M1-2019M12. Each chart shows the median response (red line) with 68% (red shading) and 90% (gray shading) credibility intervals obtained from the estimation of the MF-VAR(3) (with variables ordered as specified in equation (9)).

Figure 5: Aggregated responses of U.S. business cycle variables to weekly financial uncertainty shocks from the baseline MF-VAR(3) estimated over 1990M1-2019M12.



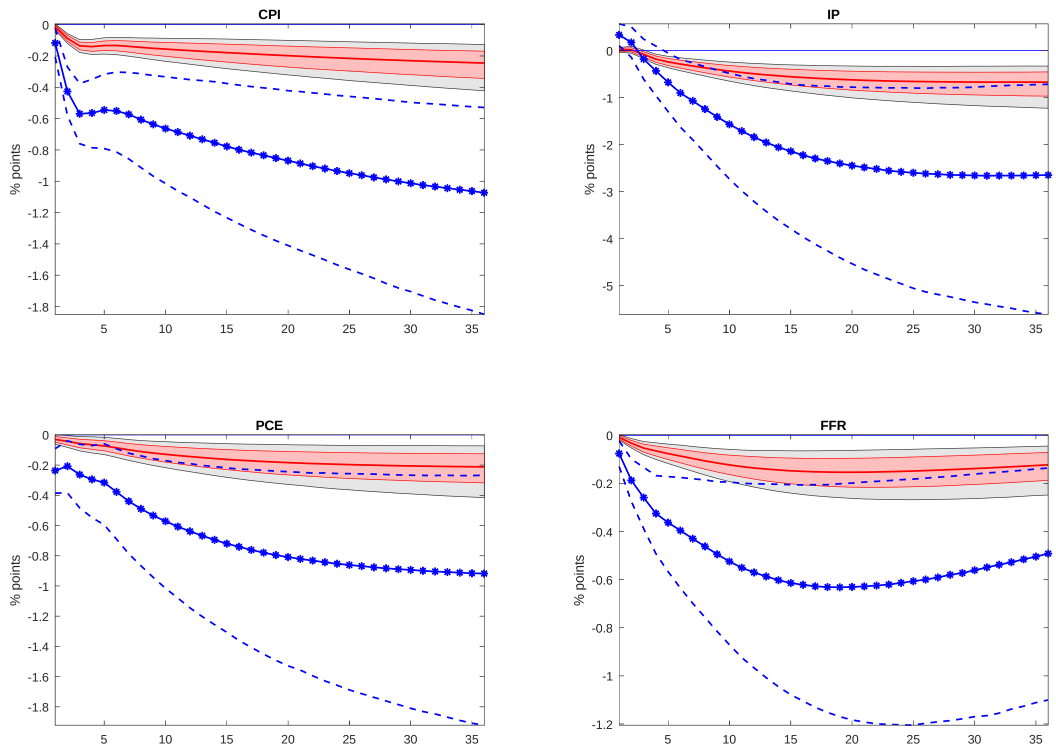
Notes. Aggregated weekly impulse responses (in levels) of consumer price index (CPI), industrial production (IP), real personal consumption expenditures (PCE), and effective federal funds rate (FFR) in percentage points, computed over a 36-month forecast horizon. The size of the shocks occurring in each of the 4 weeks is equal to 5σ VIX shocks estimated over the period 1990M1-2019M12. In each chart, the impulse responses from the MF-VAR are aggregated by averaging out the weekly responses. Each chart shows the median response (red line) with 68% (red shading) and 90% (gray shading) credibility intervals obtained from the estimation of the MF-VAR(3) (see equation (9)). The median impulse response from a common-frequency VAR (blue line with asterisk) and the corresponding 90% credibility intervals (blue dashed line) are also reported.

Figure 6: Responses of U.S. business cycle variables to daily financial uncertainty shocks from the baseline MF-VAR(3) estimated over 1990M1-2019M12.



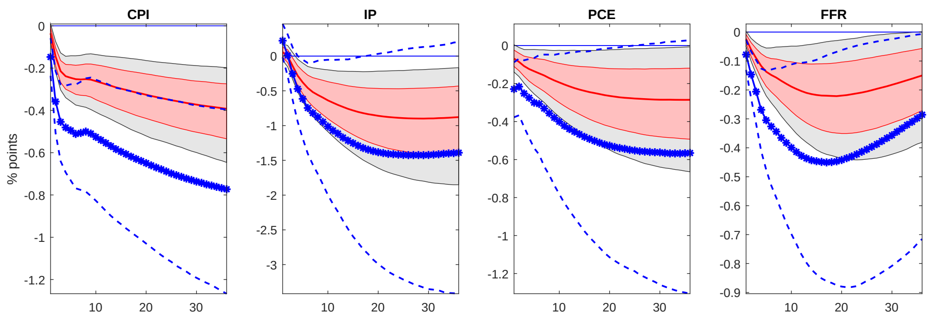
Notes. Median responses (in levels) of consumer price index (CPI), industrial production index (IP), real personal consumption expenditures (PCE), and effective federal funds rate (FFR) in percentage points, computed over a 36-month forecast horizon. The impulse responses are obtained by estimating the baseline MF-VAR(3) using daily series (i.e. 20 observations in each month) of VIX. The size of the shocks occurring in each of the 20 days is equal to 5σ VIX shocks estimated over the period 1990M1-2019M12. Each chart displays the daily responses (x-axis), the 36-month forecast horizon (y-axis), and the magnitude of the responses (z-axis).

Figure 7: Aggregated responses of U.S. business cycle variables to daily financial uncertainty shocks from the baseline MF-VAR(3) estimated over 1990M1-2019M12.

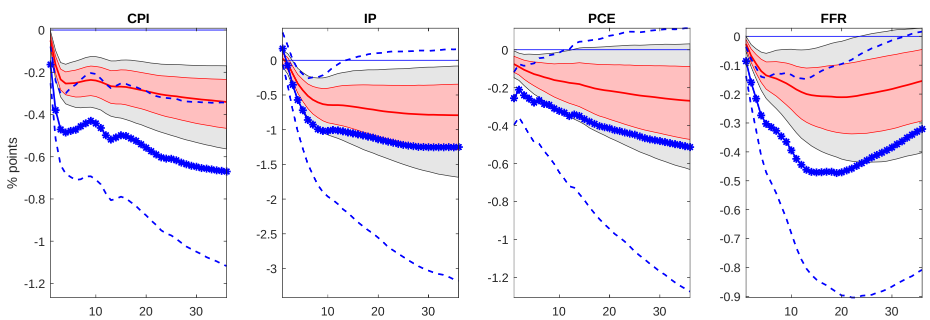


Notes. Aggregated daily impulse responses (in levels) of consumer price index (CPI), industrial production index (IP), real personal consumption expenditures (PCE), and effective federal funds rate (FFR) in percentage points, computed over a 36-month forecast horizon. The size of the shocks occurring in each of the 20 days is equal to 5σ VIX shocks estimated over the period 1990M1-2019M12. In each chart, the impulse responses from the MF-VAR are aggregated by averaging out the daily responses. Each chart shows the median response (red line) with 68% (red shading) and 90% (gray shading) credibility intervals obtained from the estimation of the baseline MF-VAR(3). The median impulse response from a common-frequency VAR (blue line with asterisk) and the corresponding 90% credibility intervals (blue dashed lines) are also reported.

Figure 8: Responses of U.S. business cycle variables to weekly financial uncertainty shocks from the baseline MF-VAR estimated over 1990M1-2019M12. Different lags.



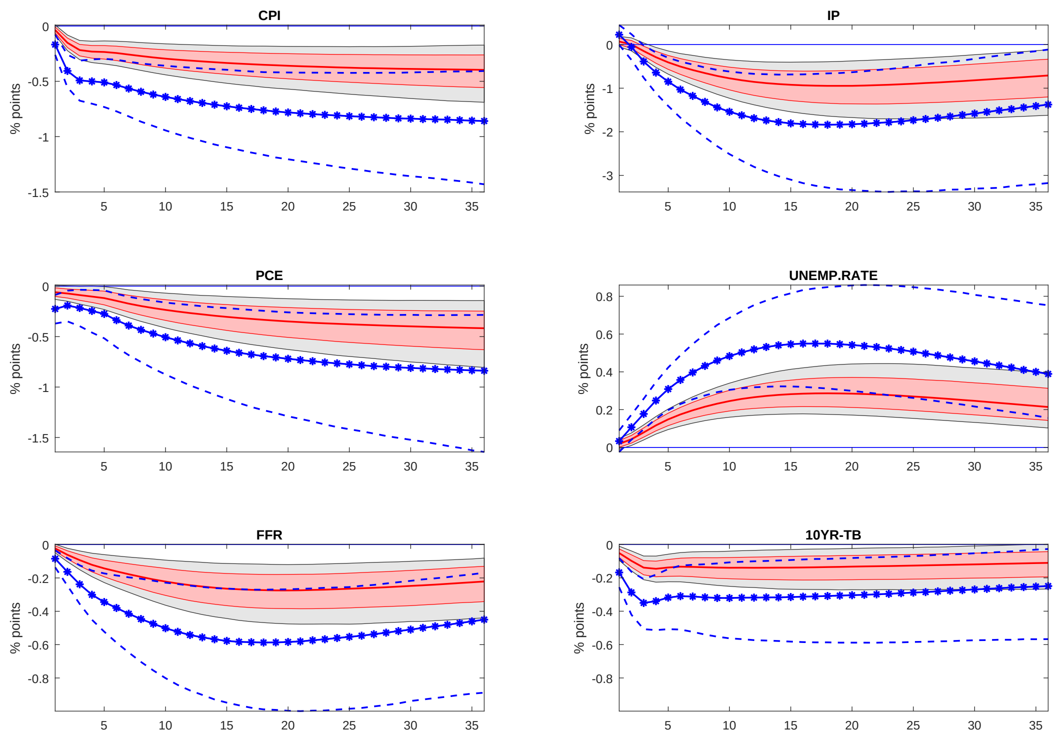
(a) Panel a. Mixed-Frequency VAR with 6 lags.



(b) Panel b. Mixed-Frequency VAR with 12 lags.

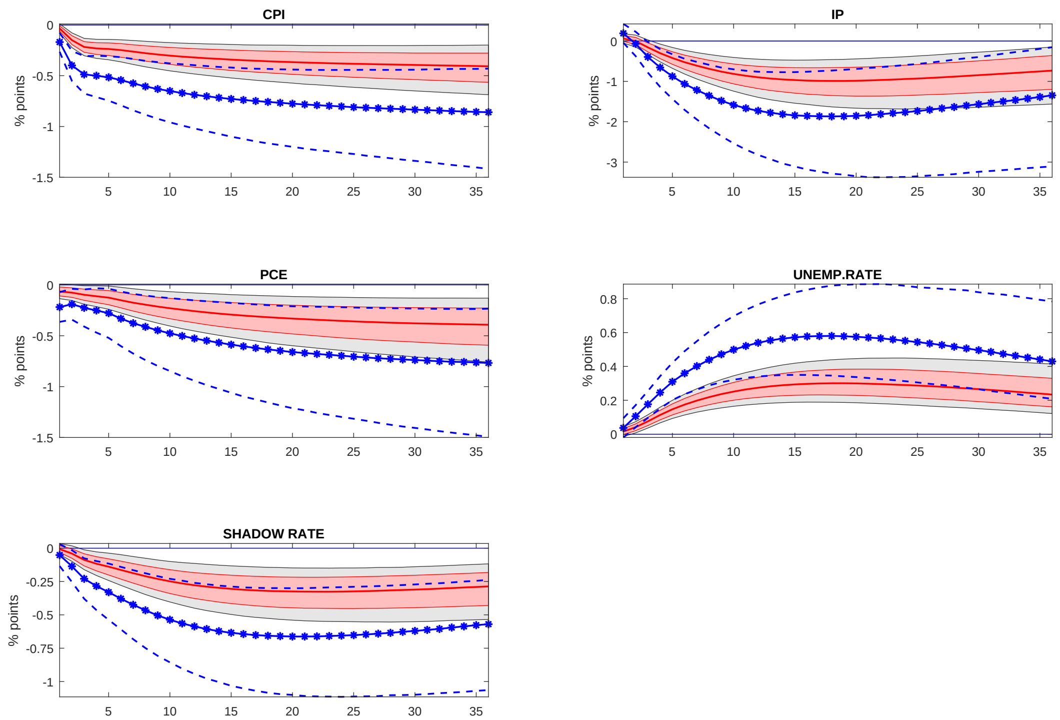
Notes. Aggregated weekly impulse responses (in levels) of consumer price index (CPI), industrial production index (IP), real personal consumption expenditures (PCE), and effective federal funds rate (FFR) in percentage points, computed over a 36-month forecast horizon. The size of the shocks occurring in each of the 4 weeks is equal to 5σ VIX shocks estimated over the period 1990M1-2019M12. In each chart, the impulse responses from the MF-VAR are aggregated by averaging out the weekly responses. Each chart shows the median response (red line) with 68% (red shading) and 90% (gray shading) credibility intervals obtained from the estimation of MF-VAR(6) (panel a) and MF-VAR(12) (panel b) (see equation (9)). The median impulse response from a common-frequency VAR (blue line with asterisk) and the corresponding 90% credibility intervals (blue dashed line) are also reported.

Figure 9: Responses of U.S. business cycle variables to weekly financial uncertainty shocks from a MF-VAR(3) estimated over 1990M1-2019M12. Extended set of variables.



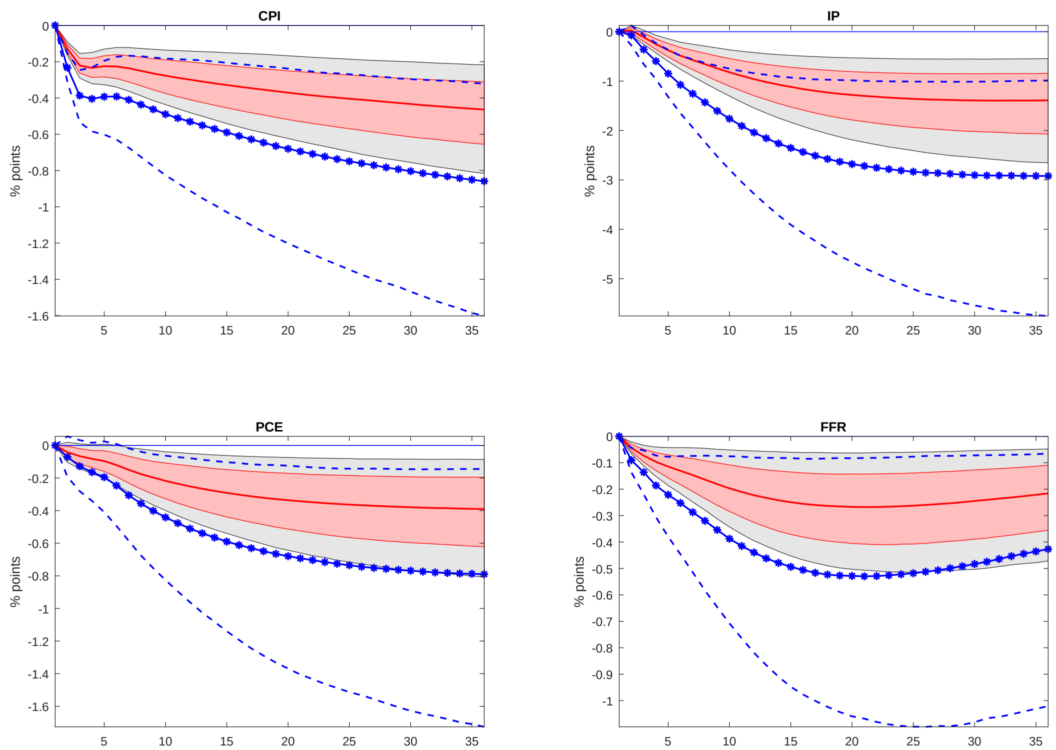
Notes. Aggregated weekly impulse responses (in levels) of the selected macroeconomic variables in percentage points, computed over a 36-month forecast horizon. The shocks are identified using a Cholesky decomposition of the reduced-form residual covariance matrix (see Section 3.2) with variables ordered as follows: weekly VIX, consumer price index (CPI), industrial production index (IP), real personal consumption expenditures (PCE), unemployment rate (UNEMP.RATE), effective federal funds rate (FFR), and 10-Year treasury constant maturity rate (10YR-TB). The size of the shocks occurring in each of the 4 weeks is equal to 5σ VIX shocks estimated over the period 1990M1-2019M12. In each chart, the impulse responses from the MF-VAR are aggregated by averaging out the weekly responses. Each chart shows the median response (red line) with 68% (red shading) and 90% (gray shading) credibility intervals obtained from the estimation of a MF-VAR(3). The median impulse response from a common-frequency VAR (blue line with asterisk) and the corresponding 90% credibility intervals (blue dashed lines) are also reported.

Figure 10: Responses of U.S. business cycle variables to weekly financial uncertainty shocks from a MF-VAR(3) estimated over 1990M1-2019M12. Shadow short rate.



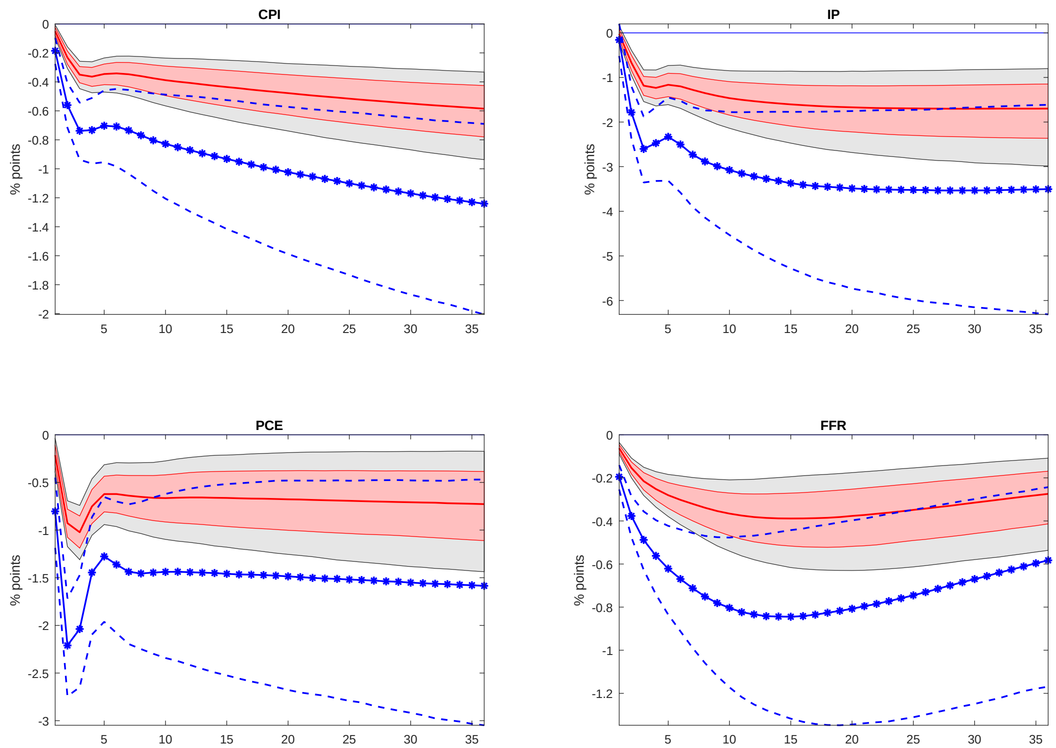
Notes. Aggregated weekly impulse responses (in levels) of the selected macroeconomic variables in percentage points, computed over a 36-month forecast horizon. The shocks are identified using a Cholesky decomposition of the reduced-form residual covariance matrix (see Section 3.2) with variables ordered as follows: weekly VIX, consumer price index (CPI), industrial production index (IP), real personal consumption expenditures (PCE), unemployment rate (UNEMP.RATE), and shadow short rate (SHADOW RATE). The size of the shocks occurring in each of the 4 weeks is equal to 5σ VIX shocks estimated over the period 1990M1-2019M12. In each chart, the impulse responses from the MF-VAR are aggregated by averaging out the weekly responses. Each chart shows the median response (red line) with 68% (red shading) and 90% (gray shading) credibility intervals obtained from the estimation of a MF-VAR(3). The median impulse response from a common-frequency VAR (blue line with asterisk) and the corresponding 90% credibility intervals (blue dashed lines) are also reported.

Figure 11: Responses of U.S. business cycle variables to weekly financial uncertainty shocks from a MF-VAR(3) estimated over 1990M1-2019M12. VIX ordered last.



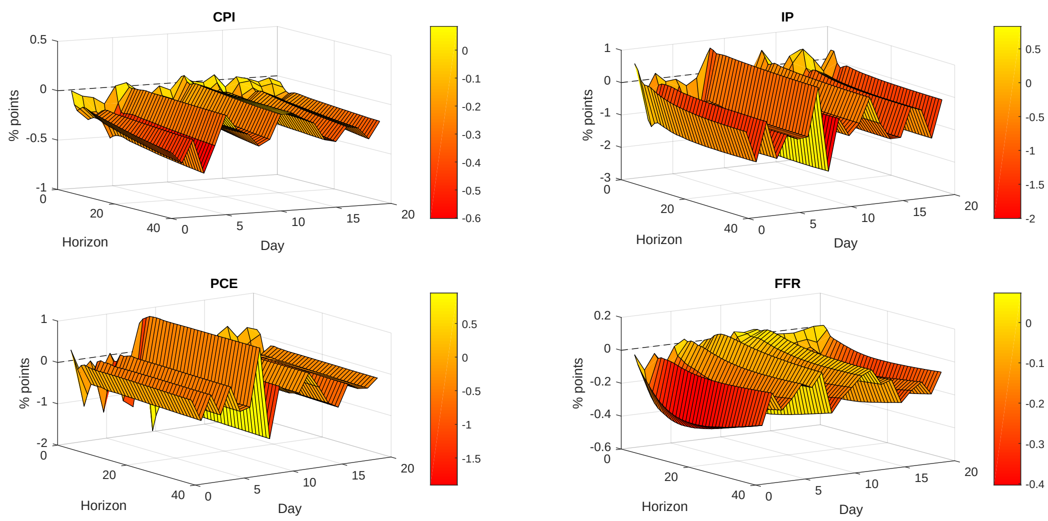
Notes. Aggregated weekly impulse responses (in levels) of the selected macroeconomic variables in percentage points, computed over a 36-month forecast horizon. The shocks are identified using a Cholesky decomposition of the reduced-form residual covariance matrix with variables ordered as follows: consumer price index (CPI), industrial production index (IP), real personal consumption expenditures (PCE), effective federal funds rate (FFR), and weekly VIX. The size of the shocks occurring in each of the 4 weeks is equal to 5σ VIX shocks estimated over the period 1990M1-2019M12. In each chart, the impulse responses from the MF-VAR are aggregated by averaging out the weekly responses. Each chart shows the median response (red line) with 68% (red shading) and 90% (gray shading) credibility intervals obtained from the estimation of a MF-VAR(3). The median impulse response from a common-frequency VAR (blue line with asterisk) and the corresponding 90% credibility intervals (blue dashed lines) are also reported.

Figure 12: Aggregated responses of U.S. business cycle variables to weekly financial uncertainty shocks from a MF-VAR(3) estimated over 1990M1-2020M11.



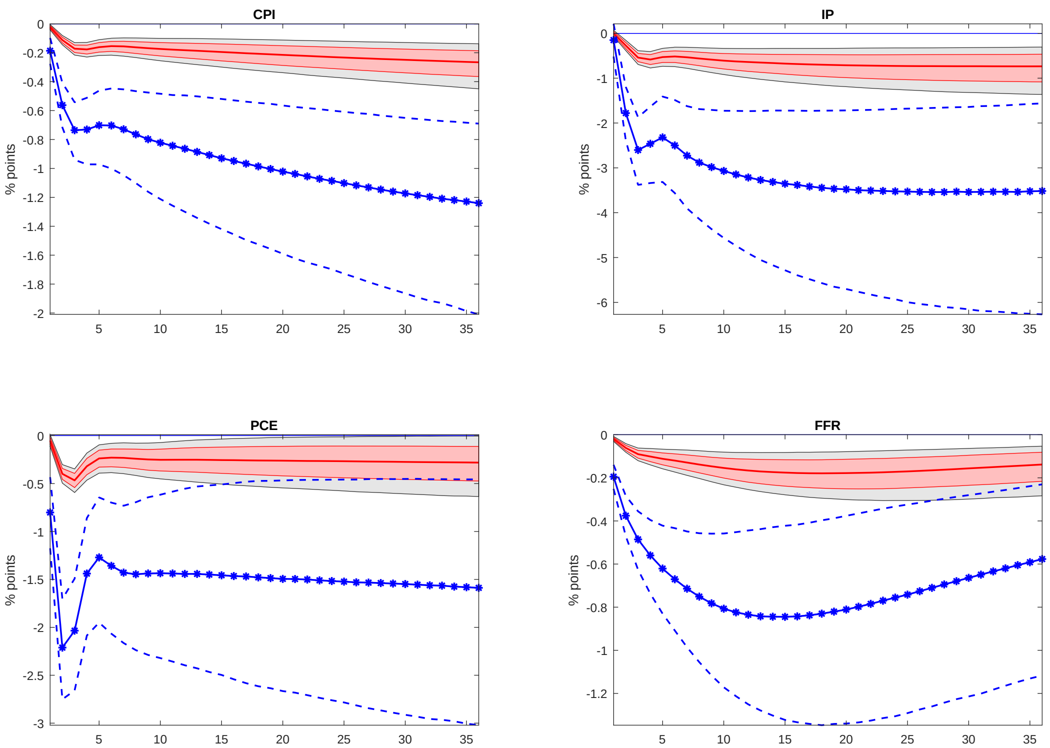
Notes. Aggregated weekly impulse responses (in levels) of consumer price index (CPI), industrial production index (IP), real personal consumption expenditures (PCE), and effective federal funds rate (FFR) in percentage points, computed over a 36-month forecast horizon. The size of the shocks occurring in each of the 4 weeks is equal to 5σ VIX shocks estimated over the period 1990M1-2020M11. In each chart, the impulse responses from the MF-VAR are aggregated by averaging out the weekly responses. Each chart shows the median response (red line) with 68% (red shading) and 90% (gray shading) credibility intervals obtained from the estimation of the MF-VAR(3) (see equation (9)). The median impulse response from a common-frequency VAR (blue line with asterisk) and the corresponding 90% credibility intervals (blue dashed lines) are also reported.

Figure 13: Responses of U.S. business cycle variables to daily financial uncertainty shocks from a MF-VAR(3) estimated over 1990M1-2020M11.



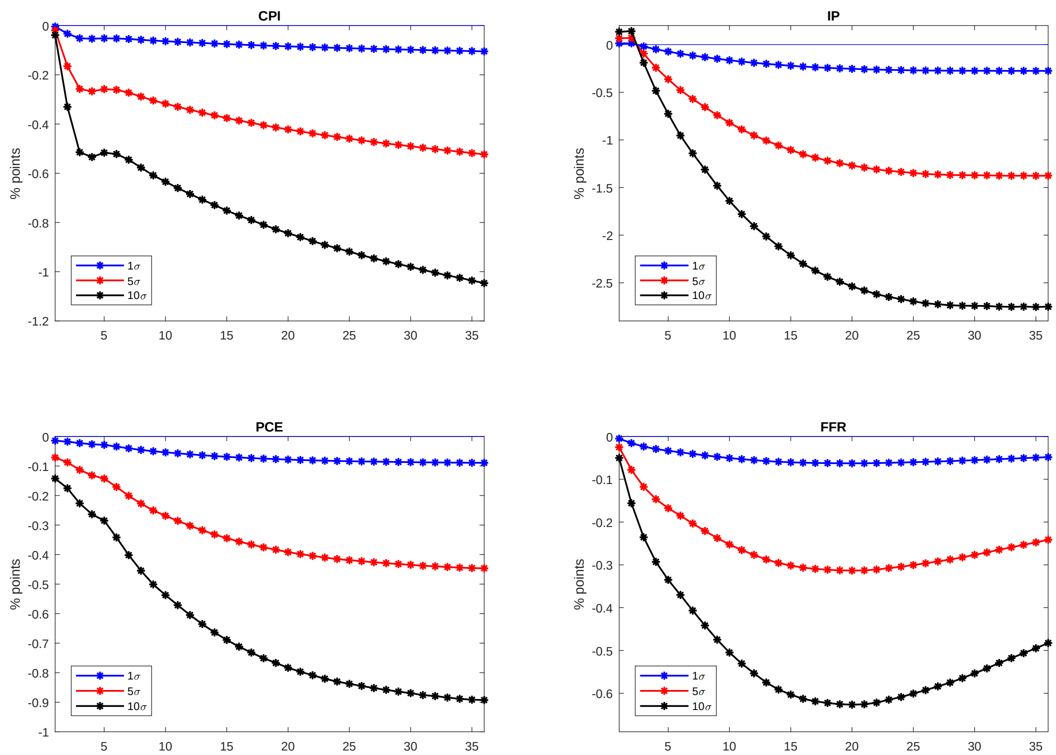
Notes. Median responses (in levels) of consumer price index (CPI), industrial production index (IP), real personal consumption expenditures (PCE), and effective federal funds rate (FFR) in percentage points, computed over a 36-month forecast horizon. The impulse responses are obtained by estimating the baseline MF-VAR(3) using daily series (i.e. 20 observations in each month) of VIX. The size of the shocks occurring in each of the 20 days is equal to 5σ VIX shocks estimated over the period 1990M1-2020M11. Each chart displays the daily responses (x-axis), the 36-month forecast horizon (y-axis), and the magnitude of the responses (z-axis).

Figure 14: Aggregated responses of U.S. business cycle variables to daily financial uncertainty shocks from a MF-VAR(3) estimated over 1990M1-2020M11.



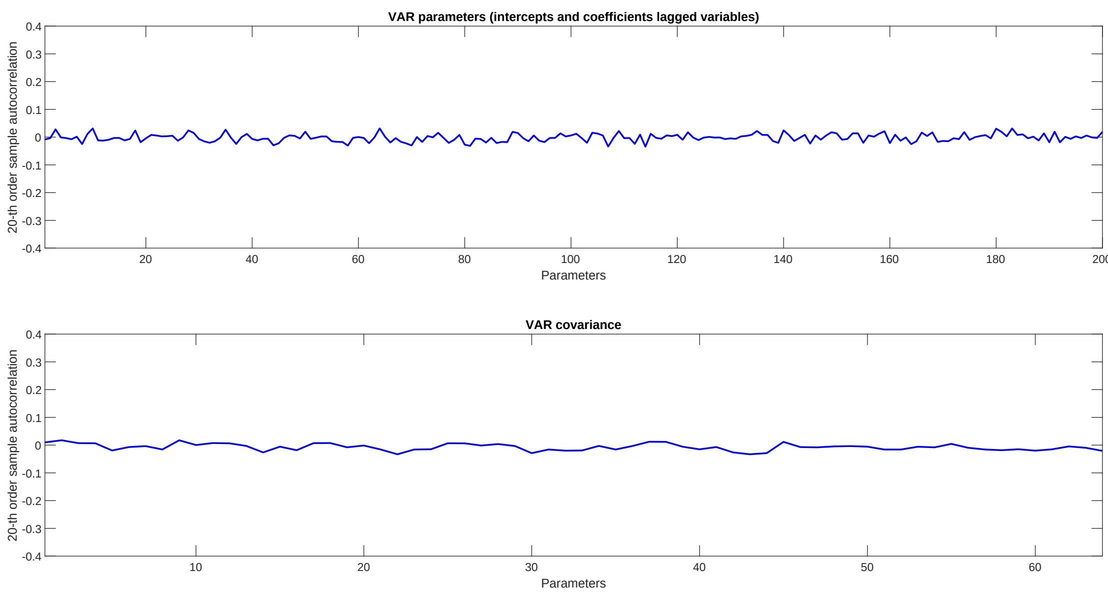
Notes. Aggregated daily impulse responses (in levels) of consumer price index (CPI), industrial production index (IP), real personal consumption expenditures (PCE), and effective federal funds rate (FFR) in percentage points, computed over a 36-month forecast horizon. The size of the shocks occurring in each of the 20 days is equal to 5σ VIX shocks estimated over the period 1990M1-2020M11. In each chart, the impulse responses from the MF-VAR are aggregated by averaging out the daily responses. Each chart shows the median response (red line) with 68% (red shading) and 90% (gray shading) credibility intervals obtained from the estimation of the baseline MF-VAR(3). The median impulse response from a common-frequency VAR (blue line with asterisk) and the corresponding 90% credibility intervals (blue dashed lines) are also reported.

Figure A.1: Aggregated median responses of U.S. business cycle variables to weekly financial uncertainty shocks from a MF-VAR(3) estimated over 1990M1-2019M12. Multi-σ shocks.



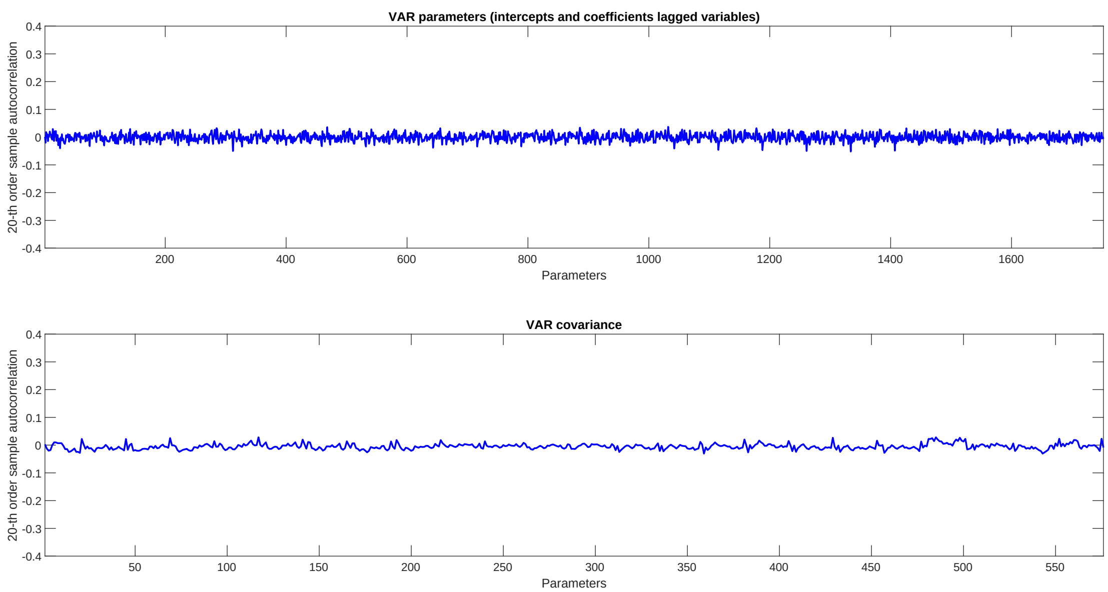
Notes. Posterior median of the aggregated weekly impulse responses (in levels) of consumer price index (CPI), industrial production index (IP), real personal consumption expenditures (PCE), and effective federal funds rate (FFR) in percentage points, computed over a 36-month forecast horizon. The sizes of the shocks occurring in each of the 4 weeks are calibrated to be 1σ (blue line), 5σ (red line), and 10σ (black line) VIX shocks estimated over the period 1990M1-2019M12. In each chart, the impulse responses from the MF-VAR are aggregated by averaging out the weekly responses.

Figure B.1: 20-th order sample autocorrelation for VAR coefficients and residual covariance matrix from a MF-VAR(3) using weekly VIX.



Notes. 20-th order sample autocorrelation of the retained draws (i.e. 5000). The autocorrelation functions are computed for the 200 MF-VAR parameters (slope coefficients and intercepts) (upper panel) and for the 64 parameters in the residual covariance matrix (lower panel), obtained from the estimation of the MF-VAR(3) fitted to weekly VIX and monthly macroeconomic variables (see equations (1) and (5)).

Figure B.2: 20-th order sample autocorrelation for VAR coefficients and residual covariance matrix from a MF-VAR(3) using daily VIX.



Notes. 20-th order sample autocorrelation of the retained draws (i.e. 5000). The autocorrelation functions are computed for the 1752 MF-VAR parameters (slope coefficients and intercepts) (upper panel) and for the 576 parameters in the residual covariance matrix (lower panel), obtained from the estimation of the MF-VAR(3) fitted to daily VIX and monthly macroeconomic variables (see equations (1) and (5)).