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Interlocking Margins: A Framework on The Interaction of Offshoring and Outsourcing Decisions

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Abstract

I develop a multi-country general equilibrium model on global sourcing which considers individual firm's decisions on outsourcing as well as offshoring. These decisions are closely connected as more extensive offshoring provides incentives for further integration of inputs. The firm-level decisions aggregate to produce gravity style equations of trade flows between countries, and intra-firm transactions.

Keywords

international trade, offshoring, outsourcing

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1 Introduction

With the emergence of global supply chains in the recent decades, firms must make increasingly complicated sourcing decisions. Among the numerous intermediate inputs and tasks required to manufacture a final good, firms decide in which countries they are performed. At the same time, while firms may choose to outsource - purchasing from outside suppliers - ready-to-use intermediate inputs, they may also choose to produce such inputs themselves, at the expense of fixed cost spent on additional production capacity, or the cost of integrating such an outside supplier.

This paper focuses on the outsourcing decisions of heterogeneous manufacturers. Specifically, how firms' outsourcing and offshoring decisions are connected and affected by other factors such as comparative advantage and suppliers' bargaining power. Outsourcing an input leads to savings in fixed costs, while on the other hand it results in relatively higher variable cost, as outside suppliers capture profits in such transactions. At the same time, offshoring, or foreign sourcing, of inputs involves similar considerations as firms incur a fixed cost access cheaper inputs provided by foreign suppliers.

In this multi-country, general equilibrium model, lead firms not only choose which inputs to offshore, but also which ones to outsource. A firm maximises profit through its sourcing decisions along the intensive margin of outsourcing, i.e. the expenditure share on outsourced inputs, as well as the extensive margin of outsourcing, i.e. the number of input varieties outsourced. Not only are the decisions on these two margins related, the firm's decisions on outsourcing also interact with the firm's decisions on offshoring. In other words, there is complementarity between the margins of outsourcing and the extensive margin of offshoring, i.e. the number and the types of countries that a firm sources from.

A more productive firm tends to integrate more of its inputs through two mechanisms. Firstly, the cost saving benefit of lowered variable cost from integration is magnified by the larger volume of sales made by more productive firms. At the same time, these firms also have more to gain from integration as they source from relatively more productive countries: integrating in such countries enables a firm to fully exploit the comparative advantages they offer.

The intensive margin of outsourcing is more sensitive to a source country's comparative advantage than the extensive margin: if outsourcing becomes more appealing as a sourcing mode, a firm not only outsources more inputs, it will also increase its expenditure on already outsourced ones.

2 Literature

This model adds to the works on the sourcing behaviours of heterogeneous firms in international trade. While Melitz (2003) notably introduces firm heterogeneity to an international trade model, earlier works such as Helpman et al. (2004) and Eaton et al. (2011) examine how firm heterogeneity shapes multinational firm's decisions to enter foreign markets. Tintelnot (2017) further develops a model on export platform which also demonstrates how fixed cost of foreign investment and export platforms can transmit a shock to third countries through multinational firm's production decisions. In terms of sourcing strategies, Antràs and De Gortari (2020) consider a multistage production model which predicts that relatively downstream tasks are allocated to relatively central locations. Johnson and Moxnes (2019) use numerical optimisation to explore the elasticities of trade flows to changes in comparative advantage in a multistage global value chain framework. Perhaps most closely related to this paper is Antràs et al. (2017), who incorporate elements of workhorse trade models of Melitz (2003) and Eaton and Kortum (2002) to construct a multi-country general equilibrium model that explains firm's importing behaviours with firm productivity and comparative advantages across countries. While retaining its focus on the extensive margin of offshoring, this model incorporates an additional dimension of considerations where firms must choose whether to outsource an input or to produce it in-house.

This paper is also related to the literature on the boundaries of firms and property rights in the global value chain. Following the property-rights framework pioneered by the works of Grossman and Hart (1986) and Hart and Moore (1990), Antràs (2005) uses a two-country model to show that the presence of incomplete contracts compels lead firms to conduct foreign direct investment for new and unstandardised products. Grossman and Helpman (2002) propose a general equilibrium model where an industry is either entirely vertically integrated or specialised, depending on the searching cost of outsourcing, bargaining power of firms, and input specialisation. Along the global value chains, the relative position of a supplier in the value chain and the elasticity of demand for the final goods relative to the degree of substitution among inputs together determine the optimal outsourcing pattern in Antràs and Chor (2013) and Alfaro et al. (2019). This paper, whilst incorporating the bargaining powers of suppliers and the relative elasticity of demand, abstracts from supplier hold up problems and instead focuses on how comparative advantages shape the patterns of integration in a non-sequential production process.

One may draw parallel between this model and the literature on the relationship between international trade and labour market outcomes. Helpman and Itskhoki (2010) examine the implications of labour market conditions on trade flows in a two-country, two-sector model:

lower labour market friction can confer on a country comparative advantage relative to its trading partner. Eckel and Yeaple (2017) demonstrate that larger firms tend to search for and attract better workers, gaining disproportionate amount of resources and creating distortions in the labour market. Autor et al. (2020) find that the most productive firms, who benefit from globalisation and technological progressions and gain additional market share, also tend to have relatively lower share of labour value added. One potential explanation is that those firms increasingly undertake outsourcing, as Goldschmidt and Schmieder (2017) find that outsourcing leads to lower wages in outsourced jobs in German. While this model focuses on manufacturing firms and the optimal sourcing decisions of intermediate inputs, firm heterogeneity plays a similarly crucial role and shapes the outcome on the supply side market.

The contemporaneous work of Chor and Ma (2020) shares the same focus on a multinational firm's decision of integration vs outsourcing. However there are a couple of crucial differences. Firstly, Chor and Ma (2020) focus on how contracting frictions and the presence of bilateral holdup problems shape and distort outsourcing patterns, while I abstract from the details of holdup problems and focus on the effects of comparative advantages, trade costs, and the extensive margin of offshoring. This is also related to the second point of difference, as final-good producers in Chor and Ma (2020) take as given the set of countries that they source from. In other words, those firms do not choose from which markets to source their inputs, a key mechanism in the Antràs et al. (2017) framework that my model retains to track the extensive margin of offshoring and how it interacts with the margins of outsourcing.

Another similar, contemporaneous work is Boehm et al. (2020): they adopt a sourcing model for multinational corporations to explain how foreign sourcing affects domestic employment in a novel firm-level dataset, while my emphasis here is on the interactions between offshoring and outsourcing behaviours in general. This means that I can separately identify the effects of sourcing foreign inputs at arms length, and within the firm's boundary. Additionally, their structural model combines the fixed cost of integrating a specific input with the fixed cost of sourcing from a certain country, while this model distinguishes between these types of fixed costs to allow for more nuanced decisions on outsourcing, generating more intricate firm behaviours.

For the rest of the paper, Section 3 sets up the theoretical framework in terms of preferences and production technology. Section 4 analyses the firm behaviours in this framework and aggregates the model to general equilibrium. And Section 5 proposes an applicable estimation strategy.

3 Theoretical Framework

3.1 Preferences

There are J countries, indexed by $i, j = 1, 2, ..., J$. There are two sectors, a manufacturing sector that I will focus on, and a large outside sector. The consumers in country i derives utility from consuming manufacturing sector goods through

$$U_i = \Big(\int_{\omega \in \Omega_i} q_i(\omega)^{(\sigma-1)/\sigma} d\omega \Big)^{\sigma/(\sigma-1)}, \quad \sigma > 1,$$

where $\omega \in \Omega_i$ refers to the manufactured goods variety available in country i. The manufacturing market demand term which naturally arises from the CES preference is

$$B_i = \frac{1}{\sigma}\Big(\frac{\sigma}{\sigma-1}\Big)^{1-\sigma} E_i P_i^{\sigma-1},$$

where E_i is country i's expenditure on manufactured goods, and P_i is the ideal price index of manufactured goods in country i.

Production requires labour which is associated with wage rate w_i in country i. The outside sector is large enough to pin down the wage rate w_i. The manufacturing sector expenditure is a fixed share, η, of total income.

3.2 Production

Final good manufacturers engage in monopolistic competition. They each draw a core productivity, φ, from a country-specific distribution $H_i(\varphi)$. These firms combine a continuum of firm-specific intermediate inputs $v \in [0, 1]$ to produce final goods

$$q_i(\varphi) = \Big(\int_0^1 x_i(v, \varphi)^{\frac{\rho-1}{\rho}} \, dv \Big)^{\frac{\rho}{\rho-1}}, \tag{1}$$

where ρ is the elasticity of substitution and $x_i(v, \varphi)$ is the amount of input v that firm φ acquires. Inputs from different countries are perfect substitutes. For simplicity, I assume final goods are prohibitively costly to trade across borders to focus on the firm's sourcing behaviours.

Intermediate inputs can be designed and produced within an individual firm's boundary ("integration"), or outsourced to an outside supplier. All such intermediate inputs are produced with labour: the price of input v in country j is hence associated with unit labour requirement $a_j(v, \varphi)$ that is country-, firm- and variety- specific. I will also approach the model from the perspective of a country i firm.

An outside supplier in country j sells input v to firm φ in country i at the price $z^O_{ij}(v,\varphi) = \tau_{ij} w_j a_j(v,\varphi) m_j$, where τ_{ij} is the iceberg trade cost between the two countries, w_j is the wage rate in country j, and $m_j \geq 1$ is a country-specific markup applied to the cost of this input. On the other hand, firm φ can produce the input itself in country j at the cost of $z^I_{ij}(v,\varphi) = \tau_{ij} w_j a_j(v,\varphi)$.

I denote $\mathcal{J}^O_i(\varphi)$ as the set of countries that firm φ outsources to, the "outsourcing set", and $\mathcal{J}^I_i(\varphi)$ as the set of countries that the firm operate in to produce intermediate inputs within its boundary, the "integrated sourcing set". The price that firm φ based in country i pays when purchasing input v from an outside source is then

$$z^O_i\big(v,\varphi;\mathcal{J}^O_i(\varphi)\big) = \min_{j\in\mathcal{J}^O_i(\varphi)}\{z^O_{ij}(v,\varphi)\}.$$

And if the firm chooses to design and produce input v by itself, it faces a unit cost of

$$z^I_i\big(v,\varphi;\mathcal{J}^I_i(\varphi)\big) = \min_{j\in\mathcal{J}^I_i(\varphi)}\{z^I_{ij}(v,\varphi)\}.$$

The unit cost of the final good that firm φ based in country i produces can then be expressed as

$$c_i(\varphi) = \frac{1}{\varphi}\Big(\int_0^{s_i(\varphi)} z^O_i\big(v,\varphi;\mathcal{J}^O_i(\varphi)\big)^{1-\rho}dv + \int_{s_i(\varphi)}^1 z^I_i\big(v,\varphi;\mathcal{J}^I_i(\varphi)\big)^{1-\rho}dv\Big)^{\frac{1}{1-\rho}} \equiv \frac{1}{\varphi}\mathcal{P}_i(\varphi), \quad (2)$$

where $s_i(\varphi)$ is a cutoff point above which all inputs are produced in-house and the elasticity of input substitution ρ is larger than 1.

4 Charaterising Firm Behaviours

In order to source from country j, a firm in country i incurs a fixed entry cost f_{ij}. This can be interpreted as the cost of obtaining information or establishing relationship unique to country j. And in order to produce an input v in-house, a country i firm incurs a fixed cost of $F^I_i(v)$. This cost may be the R&D investment required to produce input v in-house, or the cost of integrating a supplier of this input. $F^I_i(v)$ is assumed to be linearly increasing in input cost $z_i(v,\varphi)$.

For example, if input v is produced abroad in country j and lead firm φ's boundary, the firm incurs both types of fixed costs on v, f_{ij} and $F^I_i(v)$. While the cost of integration is input specific, market entry cost is incurred only once: if the firm then outsources input v' to the same country j, it doesn't have to pay f_{ij} again, and no integration fixed cost is required for an outsourced input.

The firm's profit is then

$$\pi_i(\varphi) = B_i c_i(\varphi)^{1-\sigma} - w_i \sum_{j \in \mathcal{J}_i(\varphi)} f_{ij} - w_i \int_{s_i(\varphi)}^{1} F_i^I(v)dv, \tag{3}$$

where the second term corresponds to the fixed costs incurred to source from all countries in the firm's sourcing set, $\mathcal{J}_i(\varphi) = \mathcal{J}_i^O(\varphi) \cup \mathcal{J}_i^I(\varphi)$, whilst the third term is the fixed cost associated with integrating inputs of measure $[s_i(\varphi), 1]$.

In order to fully characterise firm behaviours, I specify the distribution of input costs.

Assumption 1. *A supplier's productivity draw follows a country-specific Pareto distribution.*

A country j supplier's productivity draw, $1/a_j(v,\varphi)$, is distributed Pareto over $[T_j, +\infty)$ with country-specific shifter T_j, such that the distribution of input cost is

$$G_{ij}^O(z) = Pr(z_{ij}^O \leq z) = T_j(\tau_{ij} w_j m_j)^{-\theta} z^{\theta}, \quad \theta > 0,$$

the distribution function of outsourced input costs faced by firm φ in country i is then[1]

$$G_i^O(z) = Pr(z_i^O \leq z) = \prod_{j \in \mathcal{J}_i^O(\varphi)} T_j(\tau_{ij} w_j m_j)^{-\theta} z^{\theta} \equiv \Phi_i^O(\varphi) z^{J^O \theta},$$

where J^O is the number of countries in the outsourcing set $\mathcal{J}_i^O(\varphi)$. Similarly, the distribution of costs for in-house inputs is $G_i^I(z) = \prod_{j \in \mathcal{J}_i^I(\varphi)} T_j(\tau_{ij} w_j)^{-\theta} z^{\theta} \equiv \Phi_i^I(\varphi) z^{J^I \theta}$, where J^I is the number of countries in the integrated sourcing set $\mathcal{J}_i^I(\varphi)$.

The terms $\Phi_i^O(\varphi)$ and $\Phi_i^I(\varphi)$ are similar in spirit to the *sourcing capability* in Antràs et al. (2017). I will refer to $\Phi_i^I(\varphi)$ as the *integrated sourcing capability*: it increases as more countries are added to the sourcing sets, as the included countries become more productive through T_j, as the wages in these countries decrease through w_j, or as a country's iceberg trade cost to i decreases through τ_{ij}. And I will refer to $\Phi_i^O(\varphi)$ as the *outsourcing capability*: in addition to the characteristics above, outsourcing capability will increase as the markup rate in one or more of these countries in the outsourcing set decreases through m_j. Separating these two types of sourcing capabilities not only enables the model to more clearly identify the factors shaping a firm's outsourcing decisions, but also allows for more varied counterfactual scenarios to be examined.

[1] $Pr(z_i^O \leq z) = 1 - Pr(z_i^O > z) = 1 - (1 - \prod_{j \in \mathcal{J}_i^O(\varphi)} Pr(z_{ij}^O < z))$

4.1 Threshold Input

With a continuum of inputs, lead firm φ can order its inputs in ascending order with respect to their cost, such that it is indifferent between outsourcing and integrating the threshold input variety $s_i(\varphi)$. Let the cost of threshold input $s_i(\varphi)$ be $\tilde{z}_i(\varphi)$: any input with a realised unit price below $\tilde{z}_i(\varphi)$ is outsourced, while the rest are integrated. The magnitude of $\tilde{z}_i(\varphi)$ is then an indication of firm φ's margin of outsourcing. The section focuses on the various properties of this threshold input cost.

Consider the firm's profit maximisation problem with respect to $\tilde{z}_i(\varphi)$: as $\tilde{z}_i(\varphi)$ increases, a larger range of inputs is outsourced, leading to higher unit cost and variable cost while reducing the fixed cost investment required to source integrated inputs. Using Equation 3 and the distributional assumptions made earlier, I can express the firm's unit cost in terms of $\tilde{z}_i(\varphi)$,

$$c_i(\tilde{z}_i(\varphi), \varphi) = \frac{1}{\varphi}\Big(J^O\theta\Phi_i^O \int_0^{\tilde{z}_i(\varphi)} z^{J^O\theta-\rho}dz + J^I\theta\Phi_i^I \int_{\tilde{z}_i(\varphi)}^{\infty} z^{J^I\theta-\rho}dz\Big)^{\frac{1}{1-\rho}}. \tag{4}$$

Here I impose an assumption on the firm's sourcing set.

Assumption 2. *A firm's outsourcing set is identical to its integrated sourcing set, i.e.* $\mathcal{J}_i^O(\varphi) = \mathcal{J}_i^I(\varphi) = \mathcal{J}_i(\varphi)$.

Given $\tilde{z}_i(\varphi)$, a firm's profit $\pi_i(\varphi)$ increases in both $\Phi_i^O(\varphi)$ and $\Phi_i^I(\varphi)$. This means that once the fixed entry fee f_{ij} is paid for any country j, a firm has the incentive to include j in both the outsourcing and integrated sourcing set, as a larger number of countries leads to higher sourcing capabilities, which in turn imply lower unit cost. From this point on, I will use $J = J^O = J^I$. I will also refer to $\mathcal{J}_i(\varphi)$ as the firm's sourcing set.

Solving for the expression of threshold input cost that maximises firm φ's profit,[2]

$$\tilde{z}_i(\varphi) = \Big(\frac{w_i F_i^I(s_i(\varphi))}{B_i(\sigma-1)c_i(\varphi)^{-\sigma}\varphi^{-1}\frac{J\theta}{\rho-1}\mathcal{P}_i(\varphi)^{\rho}(\Phi_i^I(\varphi)-\Phi_i^O(\varphi))}\Big)^{\frac{1}{J\theta-\rho}}. \tag{5}$$

Holding the market demand level B_i constant, and assuming that $\theta > \rho$, in other words when productivity draws are not too widely dispersed, I can establish the properties of the threshold input, and accordingly the range of inputs outsourced. It is immediately obvious that $\tilde{z}_i(\varphi)$ increases as the fixed cost associated with integration increases. In the following sections I will examine how outsourcing behaviours interact with the lead firm's productivity, the comparative advantage of countries in the sourcing set, and the extensive margin of offshoring.

[2]Derived in Section A.1.

Productivity

The core productivity of a firm affects the threshold input cost in two ways. Firstly, higher φ means a lower unit cost through Equation 2: this allows a firm to use more outsourced inputs so that it can save on the fixed costs of integration. This implies that higher productivity discourages integration by increasing $\tilde{z}_i(\varphi)$. Secondly, firms with higher core productivity tend to have more sales, which serves to amplify the benefit of lower variable cost from using more integrated inputs. In this way, higher productivity decreases $\tilde{z}_i(\varphi)$. From Equation 5, one can find the derivative with respect to φ,[3]

$$\frac{\partial \tilde{z}_i(\varphi)}{\partial \varphi} = \Big(\frac{w_i F_i^I(s_i(\varphi))}{B_i(\sigma-1)\frac{J\theta}{\rho-1}\mathcal{P}_i(\varphi)^\rho(\Phi_i^I(\varphi)-\Phi_i^O(\varphi))}\Big)^{\frac{1}{J\theta-\rho}} \frac{(c_i(\varphi)^\sigma\varphi)^{\frac{1}{J\theta-\rho}-1}}{J\theta-\rho} c_i(\varphi)^\sigma(\overbrace{1}^{\varphi \text{ on unit cost}} \underbrace{-\sigma}_{\text{sales}}), \tag{6}$$

where it is apparent that the second effect dominates, and more productive firms tend to produce more inputs in-house. The magnitude of this effect depends on the elasticity of demand for the final good, as more elastic final demand further increases the incentive for lead firms to reduce unit cost.

Comparative Advantage

To understand how threshold input cost responds to changes in comparative advantage of one of the countries in the sourcing set, it is helpful to rely on Assumption 2 and re-write Equation 5 as

$$\tilde{z}_i(\varphi) = \Big(\frac{w_i F_i^I(s_i(\varphi))}{B_i(\sigma-1)c_i(\varphi)^{-\sigma}\varphi^{-1}\frac{J\theta}{\rho-1}\mathcal{P}_i(\varphi)^\rho\Phi_i^I(\varphi)(1-\prod_{j\in\mathcal{J}_i(\varphi)} m_j^{-\theta})}\Big)^{\frac{1}{J\theta-\rho}}. \tag{7}$$

With an improvement in country j's comparative advantage, which can manifest itself as an increase in T_j, a decrease in w_j, or an decrease in τ_{ij}, the threshold input cost decreases. A firm's incentive to integrate increases as it sources from more productive countries, represented as an increase in $\Phi_i^I(\varphi)$: higher sourcing capabilities increase the benefit from utilising their full potentials through integration. As a result, an improvement in the comparative advantage of one of the countries in the sourcing set leads to lower threshold input cost. At the same time, with a higher markup rate in one or more countries, represented as an increase in m_j, integration picks up as it leads to more saving in variable cost as markup is removed. [4]

[3] More details in Section A.2.1.

[4] More details in Section A.2.2

The Margin of Offshoring

Another consideration is the extensive margin of offshoring: as more countries are included in its sourcing set, the threshold input cost decreases, and integration expands. This comes down to two channels that arise from the above expression. Firstly, including more countries directly increases J, which decreases the value of $\tilde{z}_i(\varphi)$.[5] This effect reflects the improved sourcing potential of the firm. Secondly, with markup rate $m_j > 1 \ \forall j$, $\prod_{j \in \mathcal{J}_i(\varphi)} m_j^{-\theta}$ decreases as more countries are included in $\mathcal{J}_i(\varphi)$: the cost saving potential of switching from outsourcing to integration increases, as integration can overcome the markup rate in all the countries in $\mathcal{J}_i(\varphi)$.

Extending to Unit Cost

With the unit cost $c_i(\varphi)$ expressed as a function of the threshold input cost, a change in comparative advantage, e.g. better comparative advantage as τ_{ij} decreases, affects the unit cost through two distinct channels. The improvement in comparative advantage, and consequently sourcing capabilities, directly lowers unit cost. At the same time, the threshold input cost $\tilde{z}_i(\varphi)$ decreases as firm φ switches to more in-house production activities in order to further exploit the sourcing capabilities on offer: the larger extent of integration also lowers the unit cost.[6]

4.2 Input Expenditure Share

Firm φ's expenditure share on outsourced inputs can be written as a function of threshold input cost,[7]

$$\chi_i^O(\varphi) = \mathcal{P}_i(\varphi)^{\rho-1} \Phi_i^O(\varphi) \frac{J\theta}{J\theta + 1 - \rho} \tilde{z}_i(\varphi)^{J\theta+1-\rho}, \tag{8}$$

which is an increasing function in $\tilde{z}_i(\varphi)$ as long as $\theta > \rho$. This expenditure share inherits many properties of the threshold inputs cost discussed above, as it's decreasing in core productivity, and decreasing in individual country's comparative advantage.

In addition to that, the $\Phi_i^O(\varphi)$ and $\tilde{z}_i(\varphi)$ terms provide two separate channels through which the extensive margin of outsourcing can adjust. If outsourcing capability decreases, the expenditure share on outsourced inputs will decrease as the cost of these inputs are now higher than before, while at the same time, the range of outsourced inputs decreases through

[5]Derived in Section A.2.3

[6]Derived in Section A.2.4

[7]Derived in Section A.3.

lower $\tilde{z}_i(\varphi)$ as outsourced inputs are now relatively less attractive than integrated inputs, leading to even lower $\chi_i(\varphi)$. More concretely, consider a decrease in m_j, the markup rate of country j. The expenditure share will change through the two margins of outsourcing,

$$\frac{\partial \chi_i^O(\varphi)}{\partial m_j} = \frac{J\theta}{J\theta+1-\rho}\mathcal{P}_i(\varphi)^{\rho-1}\Phi_i^O(\varphi)(\underbrace{-\theta m_j^{-1}\tilde{z}_i(\varphi)^{J\theta+1-\rho}}_{\text{internal margin }<0}+\overbrace{(J\theta+1-\rho)\frac{\partial \tilde{z}_i(\varphi)}{\partial m_j}}^{\text{external margin }<0}) < 0$$

On the other hand, if the comparative advantage of a country increases, i.e. τ_{ij} decreases, the expenditure share on outsourced inputs will decrease despite experiencing two effects going in separate directions. With an improved comparative advantage for country j in the outsourcing set, outsourced inputs from country j will account for a larger portion of firm φ's expenditure. At the same time, better comparative advantage expands the margin of integration, resulting in a smaller share of expenditure on outsourced inputs, an effect that dominates the former. The size of the net effect relies on the dispersion of productivity draws, θ, the extensive margin of outsourcing, J, and the elasticity of substitution among inputs ρ,

$$\frac{\partial \chi_i^O(\varphi)}{\partial \tau_{ij}} = \chi_i^O(\varphi)\tau_{ij}^{-1}(\underbrace{\frac{\theta(J\theta+1-\rho)}{J\theta-\rho}}_{\text{effect through}\tilde{z}_i(\varphi),>0} - \overbrace{\frac{J\theta^2-\theta\rho}{J\theta-\rho}}^{\text{direct effect},<0}) > 0.$$

Similarly, the expenditure share on integrated inputs is

$$\chi_i^I(\varphi) = \mathcal{P}_i(\varphi)^{\rho-1}\frac{J\theta}{J\theta+1-\rho}\Phi_i^I(\varphi)(\bar{z}_i^{J\theta+1-\rho} - \tilde{z}_i^{J\theta+1-\rho}), \tag{9}$$

and it decreases in τ_{ij},

$$\frac{\partial \chi_i^I(\varphi)}{\partial \tau_{ij}} = \mathcal{P}_i(\varphi)^{\rho-1}\frac{J\theta}{J\theta+1-\rho}\theta\tau_{ij}^{-1}(\underbrace{-\bar{z}_i^{J\theta+1-\rho}(\varphi)+(1}_{\text{direct effect},<0}\overbrace{-\frac{J\theta+1-\rho}{J\theta-\rho})\tilde{z}_i^{J\theta+1-\rho}(\varphi)}^{\text{effect through}\tilde{z}_i(\varphi),<0}) < 0.$$

This is expected as when comparative advantage improves, or when τ_{ij} decreases, firm φ will choose to integrate relatively more inputs.

4.3 Imports

A firm imports both outsourced and integrated inputs. Consider the share of outsourced inputs purchased from country j: for any input v, firm φ will import it from country j if its price is lower than any alternative offered by the countries in the firm's sourcing set for outsourced inputs, in other words, $z_{ij}(v,\varphi) < z_{is}(v,\varphi), \forall s \neq j \in \mathcal{J}_i^O(\varphi)$. Integrating this probability over density function $G_{ij}(z)$ yields the share of expenditure on outsourced inputs from country j, [8]

$$im_{ij}^O(\varphi) = \varepsilon_j (m_j)^{-\theta} \tilde{z}_i(\varphi)^\theta - \frac{1}{J} \Phi_i^O \tilde{z}_i(\varphi)^{J\theta},$$

where $\varepsilon_j = T_i(\tau_{ij} w_j)^{-\theta}$ reflects country j's comparative advantage before markup is applied. Similarly, the cost share of integrated inputs from country j is derived as

$$im_{ij}^I(\varphi) = \varepsilon_j(\bar{z}_i(\varphi)^\theta - \tilde{z}_i(\varphi)^\theta) - \frac{1}{J}\Phi_i^I(\bar{z}_i(\varphi)^{J\theta} - \tilde{z}_i(\varphi)^{J\theta}),$$

where $\bar{z}_i(\varphi)$ denotes the most expensive input cost in the sourcing set $\mathcal{J}_i^I(\varphi)$.[9] The share of firm φ's inputs sourced from country j is then

$$im_{ij}(\varphi) = \chi_i^O(\varphi) im_{ij}^O(\varphi) + \chi_i^I(\varphi) im_{ij}^I(\varphi). \tag{10}$$

Import share $im_{ij}(\varphi)$ responds to a change in country j's comparative advantage, e.g. an increase in τ_{ij}, through the share of imported integrated inputs, ${\partial im_{ij}^I(\varphi)}/{\partial \tau_{ij}}$, the share of imported outsourced inputs, ${\partial im_{ij}^O(\varphi)}/{\partial \tau_{ij}}$, and the margin of outsourcing, ${\partial \chi_i^O(\varphi)}/{\partial \tau_{ij}}$ as well as ${\partial \chi_i^I(\varphi)}/{\partial \tau_{ij}}$,[10]

$$\frac{\partial im_{ij}(\varphi)}{\partial \tau_{ij}} = \chi_i^O(\varphi) \underbrace{\frac{\partial im_{ij}^O(\varphi)}{\partial \tau_{ij}}}_{<0} + im_{ij}^O(\varphi) \underbrace{\frac{\partial \chi_i^O(\varphi)}{\partial \tau_{ij}}}_{>0} + \chi_i^I(\varphi) \underbrace{\frac{\partial im_{ij}^I(\varphi)}{\partial \tau_{ij}}}_{<0} + im_{ij}^I(\varphi) \underbrace{\frac{\partial \chi_i^I(\varphi)}{\partial \tau_{ij}}}_{<0}.$$

Assume country j is and remains in firm φ's sourcing set. If its comparative advantage deteriorates through a higher τ_{ij}, firm φ will decrease the proportion of inputs that it integrates, as well as the proportion of inputs imported from country j. For example, for countries that supply relatively more outsourced inputs, a deterioration in their comparative advantage will lead to a drop in their trade volume which is partially offset by lead firms adjusting their input mix towards more outsourcing.

On the other hand, import share $im_{ij}(\varphi)$ also responds to a change in third country comparative advantage, e.g. an increase in τ_{ik}, through both the margins of offshoring and

[8] Derived in Section A.4.1

[9] $\bar{z}_i(\varphi)$ is bounded above by $\max_{k \in \mathcal{J}_i(\varphi)}\{\tau_{ik} w_k T_k^{-1}\}$.

[10] Further derivation in Section A.4.2.

outsourcing,

$$\frac{\partial im_{ij}(\varphi)}{\partial \tau_{ik}} = \chi_i^O(\varphi)\underbrace{\frac{\partial im_{ij}^O(\varphi)}{\partial \tau_{ik}}}_{>0} + im_{ij}^O(\varphi)\underbrace{\frac{\partial \chi_i^O(\varphi)}{\partial \tau_{ik}}}_{>0} + \chi_i^I(\varphi)\underbrace{\frac{\partial im_{ij}^I(\varphi)}{\partial \tau_{ik}}}_{>0} + im_{ij}^I(\varphi)\underbrace{\frac{\partial \chi_i^I(\varphi)}{\partial \tau_{ik}}}_{<0}.$$

While a deterioration in country k's comparative advantage predictably increases trade flows with country j, this change is augmented by an increasing range of outsourcing by firm φ, thus altering the mix of integrated and outsourced inputs sourced from country j. In other words, compared to a country that supplies relatively more integrated inputs, a country that supplies relatively more outsourced inputs benefits more from worsened comparative advantage in a third country.

The total volume of imported inputs can be expressed as a function of the firm's variable cost,

$$IM_{ij}(\varphi) = im_{ij}(\varphi)(\sigma - 1)B_i c_i(\varphi)^{1-\sigma}. \tag{11}$$

4.4 General Equilibrium

Using Equation 3 , the free entry condition for manufacturing firms is

$$\int_{\underline{\varphi_i}}^{\infty} \left(B_i c_i(\varphi)^{1-\sigma} - w_i \sum_{j \in \mathcal{J}_i(\varphi)} f_{ij} - w_i \int_{s_i(\varphi)}^{1} F_i^I(v)dv \right) dH_i(\varphi) = w_i f_{ei},$$

where f_{ei} is the sunk entry cost into the industry, and $\underline{\varphi_i}$ is the lowest productivity level that would compel a firm to remain active in the industry.

By appealing to Proposition 4 of Antràs et al. (2017), I can verify that there exists a unique market demand level that satisfies the free entry condition for each country and hence find the equilibrium measure of firms in the manufacturing sector in country i, $N_i(1-H_i(\underline{\varphi_i}))$, where

$$N_i = \frac{\eta L_i}{\sigma\left(f_{ei} + \int_{\underline{\varphi_i}}^{\infty} \sum_{j \in \mathcal{J}_i(\varphi)} f_{ij}\ dH_i(\varphi) + \int_{\underline{\varphi_i}}^{\infty} \int_{s_i(\varphi)}^{1} F_i^I(v)dv\ dH_i(\varphi)\right)}. \tag{12}$$

The aggregate volume of trade between countries i and j can be obtained by aggregating firm level import of inputs over the distribution of core productivity among active firms,

$$IM_{ij} = N_i \int_{\underline{\varphi_i}}^{\infty} IM_{ij}(\varphi)\ dH_i(\varphi) = N_i B_i (\sigma - 1)\Lambda_{ij}, \tag{13}$$

where

$$\Lambda_{ij} = \int_{\underline{\varphi_i}}^{\infty} \frac{IM_{ij}(\varphi)}{B_i(\sigma-1)}\, dH_i(\varphi).$$

With this observation, I can rewrite Equation 13 in a similar fashion to the gravity equation in Antràs et al. (2017),

$$IM_{ij} = \frac{E_i}{P_i^{1-\sigma}/N_i} \frac{Q_j}{\sum_k \frac{E_k}{P_k^{1-\sigma}/N_k} \tau_{kj}^{-2\theta} \Lambda'_{kj}} \tau_{ij}^{-2\theta} \Lambda'_{ij}, \tag{14}$$

where Λ'_{ij} is a modified form of Λ_{ij}, $Q_j = \sum_k M_{kj}$ is country j's total input production and the ideal manufacturing price index in country i, $P_i = \left(N_i \int_{\tilde{z}_i(\varphi)}^{\infty} p_i(\varphi)^{1-\sigma}\, dH_i(\varphi) \right)$.[11]

5 Estimation

Consider a firm-level dataset of home country, country i, firms. They import from J countries through either arms-length or intrafirm transactions.

5.1 Comparative Advantages

Using Equation 11, I estimate a country j's outsourcing potential, $T_j(\tau_{ij} w_j m_j)^{-\theta}$. Plugging Equation 8 into Equation 11, focusing only on imported inputs from outside suppliers, and taking logs yield

$$\log(IM_{ij}^O(\varphi) - IM_{ii}^O(\varphi)) = \log(T_j(\tau_{ij} w_j m_j)^{-\theta} - A_i^O) + \log(C_0(\varphi)) + \log\epsilon_j(\varphi), \tag{15}$$

where $IM_{ij}^O(\varphi)$ is the amount of outsourced inputs firm φ's imports from country j, A_i^O is a constant representing the home country's outsourcing potential, $C_0(\varphi)$ is a firm-specific fixed effect, and $\epsilon_j(\varphi)$ is a firm-country specific shock.[12]

Similarly, the integrated sourcing potential, $T_j(\tau_{ij} w_j)^{-\theta}$, can be estimated from the volume of firm φ's import of integrated inputs. The country-specific markup rate, m_j, is the log difference between the outsourcing potential and integrated sourcing potential of that country.

The estimated sourcing potentials can be used to estimate the dispersion of productivity, θ. Following the approach in Antràs et al. (2017), θ can be recovered by regressing the log of sourcing potentials on log of wage, among other relevant factors. Here only integrated

[11] Further derivation for this expression is found in section A.4. $\Lambda'_{ij} = \frac{\Lambda_{ij}}{\tau_{ij}^{-2\theta}}$.

[12] More formally, from Equation 11 one can derive $IM_{ij}^O(\varphi) = \chi_i(\varphi) im_{ij}^O(\varphi)(\sigma-1)B_i c_i(\varphi)^{1-\sigma}$.

sourcing potentials are used so that the markup rate, m_j, does not enter the estimation equation.

5.2 Fixed Costs

In order to estimate the fixed costs of offshoring and integration, I simulate firm behaviours to match moments, ϕ, from the data. Specifically, the share of importers among all home firms and the proportion of intrafirm transactions to all input costs.

The first moment is conducive to the estimation of the fixed cost of offshoring. With J countries in the framework, the fixed cost of sourcing is a vector of size J by 1, $f = (f_{i1}, f_{i2}, f_{i3}, ...)$. The cost of domestic sourcing, f_{ii}, is set to 0 as all firms use a non-zero amount of domestic inputs. The share of importers, $\hat{\phi}_1$ is simply the number of importers divided by the number of all active firms. The second moment is relevant to the fixed cost of outsourcing. The scale of the cost of integration, $F_i(v)$, affects a firm's incentive to integrate an input and hence its expenditure on outsourced inputs. From the simulated model, the expenditure share on outsourced inputs is derived as

$$\hat{\phi}_2 = \int_0^\infty \frac{\chi_i(\varphi) c_i(\varphi)^{1-\sigma}}{\int_0^\infty c_i(\varphi)^{1-\sigma}\ dH_i(\varphi)}\ dH_i(\varphi). \tag{16}$$

The simulated moments is a 2 by 1 vector, $\hat{\phi} = \{\hat{\phi}_1, \hat{\phi}_2\}$.

The simulated environment has a large number of firms, each with its core productivity, φ, drawn from a Pareto distribution. The estimation process is to find a set of parametres $\delta = (B_i, F_i, f)$ to minimise

$$\hat{\delta} = \arg\min_{\delta}(\phi - \hat{\phi}). \tag{17}$$

5.3 Simulation

Conditional on its revealed core productivity, a lead firm solves its profit maximisation problem by choosing the countries from which to source inputs. The sourcing potentials of these countries will in turn inform the firm's outsourcing decision.

With J countries to choose from, a firm's profit optimisation is a discrete combinatorial problem with 2^N potential solutions. Following the works of Jia (2008) and Eckert and Arkolakis (2017), I use an algorithm that iteratively reduces the dimensionality of the problem to quickly arrive at a solution.

Denote $\pi_i(\mathcal{J}, \varphi)$ as firm φ's profit when choosing the sourcing set $\mathcal{J}$, and denote $\mathcal{J}_{\setminus k}$ as sourcing set $\mathcal{J}$ but with country k removed, while $\mathcal{J}_{+k}$ is sourcing set $\mathcal{J}$ with country k

added. The firm's optimal sourcing strategy lies between an upper bound $\bar{\mathcal{J}}_0$ where it sources from all countries, and a lower bound $\underline{\mathcal{J}_0}$ where it sources from only the home country.

Starting from the initial upper bound $\bar{\mathcal{J}}_0$ with all countries included, one can arrive at a new upper bound candidate $\bar{\mathcal{J}}$ by dropping any country k such that $\pi_i(\mathcal{J}_{\setminus k}, \varphi) - \pi_i(\mathcal{J}, \varphi) > 0$. And this process is repeated on the new upper bound candidate until there is no profitable deviation from dropping any country from the sourcing set.

Similarly, starting from the initial lower bound $\underline{\mathcal{J}_0}$ with only the home country included, a new lower bound candidate $\underline{\mathcal{J}}$ can be derived by adding any country k such that $\pi_i(\mathcal{J}_{+k}, \varphi) - \pi_i(\mathcal{J}, \varphi) > 0$. This is repeated until we reach $\underline{\mathcal{J}}$ where no profitable deviation exists from adding an additional country to the set.

The firm's optimal sourcing strategy is immediately arrived at if $\bar{\mathcal{J}}$ and $\underline{\mathcal{J}}$ coincide with each other. And if they don't, the two iterated bounds provide a narrower range to search for the optimality, a marked improvement over the initial range of 2^N potential solutions.

6 Conclusion

In this paper I develop a theoretical model on firms' sourcing patterns that studies how firms' outsourcing and offshoring decisions are connected to each other in the presence of fixed costs. While they involve inherently distinct concerns, offshoring decisions determine the level of productivities available to the lead firms to exploit through their outsourcing or integration decisions. As a result, firms with access to more productive trade partners tend to integrate more inputs in order to fully utilise the lower input prices on offer.

This interaction between the margins of outsourcing and offshoring also affects aggregate trade volumes, which can be of policy relevance. Countries that supply relatively more integrated inputs can benefit more from trade liberalisation and improved productivities than countries that supply relatively more outsourced inputs.

There are many avenues for further extension to this research. At the expense of straining data requirements, a more detailed setup for integration fixed costs can introduce additional textures to the model and its predictions. While in this paper I abstract from intricate mechanisms of contracting frictions and instead use catch-all variable m_j to proxy bargaining shares between the lead firm and its suppliers, it would be natural to consider incorporating such mechanisms into the model, and align more closely to the property rights literature. Variations in the contractibility of inputs can affect the magnitude of the potential benefit from integration and, as a result, the strength of the link between outsourcing and offshoring margins.

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A Calculations for Outsourcing Decisions

A.1 Threshold Input Cost

Take first order derivative of Equation 3 with respect to $\tilde{z}_i(\varphi)$ to obtain

$$\frac{\partial \pi_i(\varphi)}{\partial \tilde{z}_i(\varphi)} = B_i(1-\sigma)c_i(\varphi)^{-\sigma}\frac{\partial c_i(\tilde{z}_i(\varphi),\varphi)}{\partial \tilde{z}_i(\varphi)} + w_i F_i^I,$$

the first term represents a drop in profit as a result of higher variable cost from using more outsourced inputs, and the second term shows the increase in profit from lower fixed production costs.

Note unit cost $c_i(\varphi)$ as stated in Equation 2 can be rewritten as, after taking into account the price associated with the threshold input and the distribution assumption,

$$c_i(\tilde{z}_i(\varphi),\varphi) = \frac{1}{\varphi}\Big(\int_0^{\tilde{z}_i(\varphi)} z^{1-\rho} dG_i^O(z) + \int_{\tilde{z}_i(\varphi)}^{\infty} z^{1-\rho} dG_i^I(z)\Big)^{\frac{1}{1-\rho}}$$

$$= \frac{1}{\varphi}\Big(\int_0^{\tilde{z}_i(\varphi)} z^{1-\rho}\Phi_i^O(\varphi) J\theta z^{J\theta-1} dz + \int_{\tilde{z}_i(\varphi)}^{\infty} z^{1-\rho}\Phi_i^I(\varphi) J\theta z^{J\theta-1} dz\Big)^{\frac{1}{1-\rho}}.$$

Apply Leibniz's rule to differentiate this expression with respect to $\tilde{z}_i(\varphi)$, with the parenthesis (φ) suppressed here for brevity,

$$\frac{\partial c_i(\tilde{z}_i,\varphi)}{\partial \tilde{z}_i} = \frac{1}{\varphi}\frac{1}{1-\rho}\Big(J\theta\Phi_i^O\int_0^{\tilde{z}_i} z^{J\theta-\rho}dz + J\theta\Phi_i^I\int_{\tilde{z}_i}^{\infty} z^{J\theta-\rho}dz\Big)^{\frac{1}{1-\rho}-1}\Big(J\theta\Phi_i^O\tilde{z}_i^{J\theta-\rho} - J\theta\Phi_i^I\tilde{z}_i^{J\theta-\rho}\Big)$$

$$= \varphi^{-1}\frac{J\theta}{1-\rho}\mathcal{P}_i^{\rho}(\Phi_i^O - \Phi_i^I)\tilde{z}_i^{J\theta-\rho}.$$

Plug the above expression into the first order condition to arrive at

$$\frac{\partial \pi_i(\varphi)}{\partial \tilde{z}_i(\varphi)} = B_i(1-\sigma)c_i(\varphi)^{-\sigma}\varphi^{-1}\frac{J\theta}{1-\rho}\mathcal{P}_i(\varphi)^{\rho}(\Phi_i^O(\varphi) - \Phi_i^I(\varphi))\tilde{z}_i(\varphi)^{J\theta-\rho} + w_i F_i^I,$$

which can be re-arranged into Equation 5.

A.2 Properties of Threshold Input Cost

A.2.1 Productivity

Note the contributions of partial derivatives $\frac{\partial \pi_i(\varphi)}{\partial c_i(\varphi)}$ and $\frac{\partial c_i(\varphi)}{\partial \tilde{z}_i(\varphi)}$ in Equation 5,

$$\tilde{z}_i(\varphi) = \Big(\frac{w_i F_i^I(v)}{\underbrace{B_i(\sigma-1)c_i(\varphi)^{-\sigma}}_{\frac{\partial \pi_i(\varphi)}{\partial c_i(\varphi)}} \underbrace{\varphi^{-1}\frac{J\theta}{\rho-1}\mathcal{P}_i(\varphi)^{\rho}(\Phi_i^I(\varphi)-\Phi_i^O(\varphi))}_{\frac{\partial c_i(\varphi)}{\partial \tilde{z}_i(\varphi)}}} \Big)^{\frac{1}{J\theta-\rho}}.$$

Differentiating $\tilde{z}_i(\varphi)$ with respect to φ,

$$\frac{\partial \tilde{z}_i(\varphi)}{\partial \varphi} = \Big(\frac{w_i F_i^I(v)}{B_i(\sigma-1)\frac{J\theta}{\rho-1}\mathcal{P}_i(\varphi)^{\rho}(\Phi_i^I(\varphi)-\Phi_i^O(\varphi))} \Big)^{\frac{1}{J\theta-\rho}} \frac{1}{J\theta-\rho}(c_i(\varphi)^{\sigma}\varphi)^{\frac{1}{J\theta-\rho}-1}\Big(\varphi(-\sigma)\varphi^{-\sigma-1}\mathcal{P}_i(\varphi)^{\sigma}+c_i(\varphi)^{\sigma}\Big)$$

$$= \Big(\frac{w_i F_i^I(v)}{B_i(\sigma-1)\frac{J\theta}{\rho-1}\mathcal{P}_i(\varphi)^{\rho}(\Phi_i^I(\varphi)-\Phi_i^O(\varphi))} \Big)^{\frac{1}{J\theta-\rho}} \frac{(c_i(\varphi)^{\sigma}\varphi)^{\frac{1}{J\theta-\rho}-1}}{J\theta-\rho} c_i(\varphi)^{\sigma}(\underbrace{-\sigma}_{\text{sales}} + \overbrace{1}^{\varphi \text{ on unit cost}})$$

A.2.2 Comparative Advantage

$$\frac{\partial \tilde{z}_i}{\partial \tau_{ij}} = \tilde{z}_i \frac{\theta}{J\theta-\rho}\tau_{ij}^{-1} > 0$$

$$\frac{\partial \tilde{z}_i}{\partial m_j} = -\tilde{z}_i \frac{\theta}{J\theta-\rho}(1-\prod_{j\in\mathcal{J}_i(\varphi)} m_j^{-\theta})m_j^{-1}\prod_{j\in\mathcal{J}_i(\varphi)} m_j^{-\theta} < 0$$

A.2.3 Margin of Offshoring

Differentiate $\tilde{z}_i(\varphi)$ with respect to J,

$$\frac{\partial \tilde{z}_i}{\partial J} = -\frac{1}{J\theta-\rho}J^{-1}\tilde{z}_i - \theta(J\theta-\rho)^{-2} ln(\tilde{z}_i^{J\theta-\rho})\tilde{z}_i \quad < 0$$

A.2.4 Extending to Unit Cost

The change in unit cost can be expressed as

$$\frac{\partial c_i(\varphi)}{\partial \tau_{ij}} = ...(-\theta\tau_{ij}^{-1} - (\tilde{z}_i^{J\theta+1-\rho} - (1-\prod_{j\in\mathcal{J}_i(\varphi)} m_j^{-\theta})\tilde{z}_i(\varphi)^{J\theta+1-\rho})^{-1}(1-\prod_{j\in\mathcal{J}_i(\varphi)} m_j^{-\theta})(J\theta+1-\rho)\tilde{z}_i^{J\theta+1}\frac{\partial \tilde{z}_i(\varphi)}{\partial \tau_{ij}}).$$

A.3 Expenditure Share

Let $x_i(v,\varphi)$ be the quantity of input v purchased and used by firm φ, then using the property of the price index in Equation 2,

$$x_i(v,\varphi) = \frac{c_i(\varphi)}{\mathcal{P}_i(\varphi)}\Big(\frac{z_i(v,\varphi)}{\mathcal{P}_i(\varphi)}\Big)^{-\rho},$$

where $z_i(v,\varphi)$ refers to the unit cost of variety v regardless of its sourcing status. Integrating expenditure share over the range of inputs that are outsourced yields

$$\int_0^{s_i(\varphi)} \frac{x_i(v,\varphi)z_i(v,\varphi)}{c_i(\varphi)}dv = \mathcal{P}_i(\varphi)^{\rho-1}\int_0^{s_i(\varphi)} z_i(v,\varphi)^{1-\rho}dv$$
$$= \mathcal{P}_i(\varphi)^{\rho-1}\int_0^{\tilde{z}_i(\varphi)} z^{1-\rho}dG_i^O(z) = \mathcal{P}_i(\varphi)^{\rho-1}J\theta\Phi_i^O(\varphi)\int_0^{\tilde{z}_i(\varphi)} z^{J\theta-\rho}dz.$$

A.4 Imports

A.4.1 Import share

Consider firm φ's sourcing set $J_i(\varphi)$, and denote $J_{i,-j}(\varphi)$ as the sourcing set with country j removed. The probability of at least one supplier in sourcing set $J_{i,-j}(\varphi)$ provides a particular input at z is given as $Pr(z_{i,-j} < z) = \Phi_{i,-j}(\varphi)z^{(J-1)\theta}$. Integrate $(1 - Pr(z_{i,-j} < z))$, the probability that none of these suppliers can provide this input at a price lower than z, over the density function of $G_{ij}(z)$ yields the expression in the text.

A.4.2 Response to changes in comparative advantage

The change in the import share of outsourced inputs is

$$\frac{\partial im_{ij}^O}{\partial \tau_{ij}} = \theta\tau_{ij}^{-1}(-im_{ij}^O + \underbrace{\frac{\theta}{J\theta-\rho}(\varepsilon_j m_j^{-\theta}\tilde{z}^{\theta} - \Phi_i^O\tilde{z}^{J\theta})}_{\partial\tilde{z}_i}) < 0.$$

And for the change in the import share of integrated inputs,

$$\frac{\partial im_{ij}^I}{\partial \tau_{ij}} = \theta\tau_{ij}^{-1}(-im_{ij}^I - \underbrace{\frac{\theta}{J\theta-\rho}(\varepsilon_j \tilde{z}_i^{\theta} - \Phi_i^I\tilde{z}_i^{J\theta})}_{\partial\tilde{z}_i}) < 0.$$

The first terms of both expressions are consistent with common sense in that firm φ will source less from country j if its comparative advantage weakens. On the other hand, the role of the margin of outsourcing, $\tilde{z}_i$, reinforces this direct effect of comparative advantage on the import share of integrated inputs, and partially offset the effect on the import share of outsourced inputs.

Using the definitions of distribution functions, $G^I_{ij}(z)$, $G^O_{ij}(z)$, $G^I_i(z)$, and $G^O_i(z)$, it is apparent that both underlined terms that work through $\tilde{z}_i$ are positive.

Firstly, note that $\frac{\partial \tilde{z}_i}{\partial \tau_{ij}}$ is positive. In the case of im^O_{ij}, it is weekly increasing in $\tilde{z}_i$, as an expanding range of outsourcing leads to more inputs being outsourced, and countries that are competitive as outside suppliers end up with larger import share. In the case of im^I_{ij}, it is weakly decreasing in $\tilde{z}_{ij}$ as lower sourcing capabilities leads to integration, which results in less competitive countries losing their import share of integrated inputs.

A.5 General Equilibrium

With the assumption that $\mathcal{J}^O_i(\varphi) = \mathcal{J}^I_i(\varphi) = \mathcal{J}_i(\varphi)$, and with $(1 - \chi_i(\varphi))$ derived in the same way as in section A.3, Equation 11 can be rewritten as,

$$IM_{ij}(\varphi) = \mathcal{P}_i(\varphi)^{\rho-1} \frac{J^2\theta^2}{J\theta + 1 - \rho} \Phi^I_i(\varphi)^2 \prod_{j \in \mathcal{J}_i(\varphi)} m_j^{-2\theta} (\sigma - 1) B_i c_i(\varphi)^{\sigma-1}$$

$$\Big(\tilde{z}_i(\varphi)^{J\theta+1-\rho} \int_0^{\tilde{z}_i(\varphi)} \Psi^O_{i,-j}(\varphi, z) z^{J\theta-1} dz + (\bar{z}_i(\varphi)^{J\theta+1-\rho} - \tilde{z}_i(\varphi)^{J\theta+1-\rho}) \int_{\tilde{z}_i(\varphi)}^{\bar{z}_i(\varphi)} \Psi^I_{i,-j}(\varphi, z) z^{J\theta-1} dz\Big),$$

where $\Phi^I_i(\varphi)^2$ is by definition $\prod_{j \in \mathcal{J}^I_i(\varphi)} T_j^2 (\tau_{ij} w_j)^{-2\theta}$, further rearrange to obtain

$$IM_{ij}(\varphi) = T_j^2 (\tau_{ij} w_j)^{-2\theta} \prod_{j \in \mathcal{J}_i(\varphi)} m_j^{-2\theta} (\sigma - 1) \frac{J^2\theta^2}{J\theta + 1 - \rho} B_i c_i(\varphi)^{\sigma-\rho} \varphi^{1-\rho}$$

$$\Big(\tilde{z}_i(\varphi)^{J\theta+1-\rho} \int_0^{\tilde{z}_i(\varphi)} \Psi^O_{i,-j}(\varphi, z) z^{J\theta-1} dz + (\bar{z}_i(\varphi)^{J\theta+1-\rho} - \tilde{z}_i(\varphi)^{J\theta+1-\rho}) \int_{\tilde{z}_i(\varphi)}^{\bar{z}_i(\varphi)} \Psi^I_{i,-j}(\varphi, z) z^{J\theta-1} dz\Big),$$