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Richard Dennis
University of Glasgow
Centre for Applied Macroeconomic Analysis, ANU

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JEL Classification

E32, E71

Address for correspondence:

(E) cama.admin@anu.edu.au

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Persistence and Scarring in a Non-linear New Keynesian Model with Experienced-Based-Expectations

Richard Dennis*
University of Glasgow and CAMA

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Abstract

Experienced-based-expectations allow the outcomes people experience to shape their views regarding future outcomes. We describe three forms of experienced-based-expectations and show how they can be applied in general equilibrium. The three expectations processes differ according to the nature of the information people use to form expectations and according to how well people understand their economic environment. In the context of a non-linear new Keynesian business cycle model, we show that experienced-based-expectations generally lead to increased volatility and sustained persistence, akin to scarring, relative to rational expectations. Through this expectations channel, periods of sustained bad outcomes, such as systematically low aggregate technology shocks, lead to persistently lower inflation. Changes in the inflation target have a greater effect on behavior when expectations are formed using outcomes on endogenous variables than when they are formed using outcomes on the shocks.

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*Address for Correspondence: Adam Smith Business School, University of Glasgow, Main Building, University Avenue, Glasgow G12 8QQ; email: richard.dennis@glasgow.ac.uk.

1 Introduction

This paper studies a mechanism whereby increased macroeconomic volatility, sustained persistence, and economic scarring are generated endogenously. The mechanism operates through beliefs, driven by learning behavior whereby people's beliefs about future outcomes are determined through an interaction between their knowledge of the economy and the economic outcomes they have observed and experienced. When peoples' knowledge of the economy's structure is incomplete and their information sets reflect the shocks they happen to have experienced, then beliefs can depart importantly from those implied by rational expectations. The mechanism we describe is a form of learning that involves probability densities being updated following the arrival of new information, and can be applied to both linear and non-linear modeling environments.

The 1930s saw a generation of people experience the poverty and hardship caused by the great depression. This experience is widely understood to have had lasting effects on behavior. For example, Malmendier and Nagel (2009) show that people's investment decisions and attitudes to risk are affected for decades by their exposure to stock returns and inflation earlier in life. Similarly, Malmendier and Nagel (2016) establish that people's inflation expectations are affected by their inflation experiences. Further, Giuliano and Spilimbergo (2010) find that people that experienced recessions when younger are more likely to see success as the outcome of luck rather than hard work and to favor government redistribution. More recently, the global financial crisis, the European debt crisis, and the lockdowns associated with the Covid-19 pandemic represent massive macroeconomic shocks that have affected much of the world. The mechanism described in this paper allows these experiences to become embedded for sustained periods within people's information sets giving rise to beliefs that can put non-negligible probability on such events reoccurring. Through this mechanism, extreme events—both positive and negative—can have enduring consequences for people's behavior, consequences over-and-above those coming from the event's direct impact on a household's income and wealth.

Drawing on the idea that beliefs are shaped by experiences, we describe three forms of experienced-based-expectations and show how they can be applied to general equilibrium models. These three forms all involve decisionmakers using newly obtained information to revise probability density functions, but differ in the nature of the information obtained and on the assumed knowledge about the underlying model. We term these three expectations processes type-I, type-II, and type-III experienced-based-expectations. With type-I experienced-based-expectations decisionmakers know the economy's structure, but not the processes for the shocks. They observe a finite sequence of shocks, and form expectations by using these observed shocks to infer their probability density function. Under type-II experienced-based-expectations decisionmakers do not know the processes for the shocks and they also don't fully know the economy's structure, but they do know which variables are state variables and which are not. They are endowed with a finite sequence of realized outcomes, which they use to infer conditional probability density functions. Finally, with type-III experienced-based-expectations decisionmakers do not know the economy's structure, the shock process, and nor do they know which variables are state variables. Endowed with a finite sequence of realized outcomes, decisionmakers infer unconditional probability density functions and use these to form expectations. For each of these three processes, expectations are formed using probability density functions that are shaped by events that have actually occurred, when those events may be extremely improbable under rational expectations.

We study these three forms of experienced-based-expectations in a simple non-linear New Keynesian business cycle model, contrasting the results to rational expectations. The model is one in which firms are monopolistically competitive and firm's pricing decisions are subject to Rotemberg-style (Rotemberg, 1982) adjustment costs while households make consumption and leisure decisions and have external consumption habits (Abel, 1990). Monetary policy to conducted according to a non-linear Taylor-type rule. The model allows for aggregate technology shocks and monetary policy shocks and has an endogenous state variable in the form of last period's aggregate consumption. With this model providing a framework for analysis, we consider three counter-factual experiments. The first experiment examines the behavior of the model and the speed at which learning occurs following an unanticipated permanent increase in the central bank's inflation target. The second experiment supposes that people have experienced a sustained period during which policy shocks have kept monetary policy loose. The final experiment, supposes that the economy has been subject to a sustained period of large adverse aggregate technology shocks.

Our main results are as follows. The set of experiences used for form expectations has an important impact on economic outcomes. However, as the set of experiences gets larger and tends to an infinite history, type-I and type-II experienced-based-expectations converge on rational expectations. The same cannot be said for type-III experienced-based-expectations, for which the lack of a model to guide the expectation-formation process hinders tremendously people's ability to learn. Following an exogenous unanticipated increase in the inflation target, inflation outcomes naturally increase, but there is a large and sustained gap between rational expectations and experienced-based-expectations. This gap in inflation outcomes is reflected also in the setting for the nominal interest rate. In this experiment, monetary policy remains loose relative to rational expectations for a sustained period.

A temporary easing of monetary policy generated through policy shocks that keep policy loose leads to a period of sustained higher inflation. With type-I experienced-based-expectations, which forms expectations using observed shocks, the outcome is an ongoing period of higher inflation and tighter monetary policy relative to rational expectations. This effect is naturally magnified by type-III experienced-based-expectations, but does not arise for type-II experienced-based-expectations because the underlying equilibrium relationship between inflation as the state variables is unchanged. Lastly, when the economy is impacted by a sequence of adverse technology shocks, such that the economy experiences a sustained decline in output, it is type-I and type-III (in particular) experienced-based-expectations for which the consequences are most evident. While the effects are relatively small, outcomes for inflation and the nominal interest rate are low relative to rational expectations, and remain low for an extended period. As earlier, for this experiment type-II experienced-based-expectations behave similarly to rational expectations because the model's underlying relationships are unchanged.

The approaches to expectation formation studied in this paper depart from rational expectations by allowing recently observed outcomes to drive the expectations that people form. Related departures from rational expectations can be found elsewhere in the literature. Notably, Veldkamp and Venkateswaran (2019) study a model that generates persistent stagnation when expectations are formed using what are termed here as type-I experienced-based-expectations. More generally, the literatures on ambiguity aversion (Gilboa and Schmeidler, 1989), risk-sensitive preferences (Hansen and Sargent, 1995), diagnostic expectations (Bordalo, Gennaioli, and Shleifer, 2018), and rare disasters (Rietz, 1988) all represent departures from rational expectations that modify the perceived distribution of the shocks in ways that are similar to type-I experienced-based-expectations. These literatures often use the risk/uncertainty generated by the expectation formation process to study asset-market phenomena. In addition, the literature on real-time learning (Evans and Honkapohja, 2001) is related to our work, having close similarity to type-II experienced-based-expectations.

The remainder of this paper is organized as follows. In the next section we describe the general non-linear framework within which the experienced-based-expectations mechanism can operate. In section 3 we present and illustrate the three forms of experienced-based-expectations, and use a simple model to compare them to rational expectations. Section 4 develops a simple non-linear new Keynesian model that we use as a laboratory to study the effects of experienced-based-expectations. Section 5 presents benchmark simulation results, considers three counter-factual experiments, and compares the results to rational expectations. Section 6 discusses some related mechanisms that similarly depart from rational expectations. Section 7 concludes. Appendices describe the non-parametric tools we use for density approximation, present the non-linear new Keynesian model in greater detail, show how the model is solved under the different expectations processes, and dig deeper into the effect of habit formation.

2 A general non-linear framework

Many non-linear dynamic stochastic general equilibrium models can be expressed in the form:

$$E_t \left[\mathbf{f} \left(\mathbf{x}_t, \mathbf{y}_t, \mathbf{x}_{t+1}, \mathbf{y}_{t+1}, \varepsilon_{t+1} \right) \right] = \mathbf{0}, \tag{1}$$

where $\mathbf{f}$ is a vector of (smooth) functions, $\mathbf{x}_t$ is an $n_x \times 1$ vector of state variables, $\mathbf{y}_t$ is an $n_y \times 1$ vector of non-predetermined (jump) variables, ε_t is an $s \times 1$ vector of *i.i.d.* $[\mathbf{0}, \mathbf{\Omega}]$ innovations, and E_t is the mathematical expectations operator conditional upon knowing the model and information up to and including period t (i.e. rational expectations).

As is well-known, the solution to equation (1) can be expressed in the form of two functions:

$$\begin{aligned} \mathbf{x}_{t+1} &= \mathbf{g} \left(\mathbf{x}_t, \varepsilon_{t+1} \right), & (2) \\ \mathbf{y}_t &= \mathbf{h} \left(\mathbf{x}_t \right), & (3) \end{aligned}$$

the first describing the equilibrium law-of-motion for the state variables and the second representing the decision rules for the jump variables. Recognizing that the state variables can be partitioned into endogenous predetermined variables, $\mathbf{z}_t$ and (exogenous) shocks, $\mathbf{u}_t$, equations (2)—(3) can be re-expressed as:

$$\begin{aligned} \mathbf{u}_{t+1} &= \mathbf{g}_u \left(\mathbf{u}_t, \varepsilon_{t+1} \right), & (4) \\ \mathbf{z}_{t+1} &= \mathbf{g}_z \left(\mathbf{z}_t, \mathbf{u}_t \right), & (5) \\ \mathbf{y}_t &= \mathbf{h} \left(\mathbf{z}_t, \mathbf{u}_t \right). & (6) \end{aligned}$$

Equation (4) describes the stochastic processes for the shocks, which often takes the form of a stationary VAR(1) process. Equation (5) summarizes the equilibrium law-of-motion for $\mathbf{z}_t$ while equation (6) contains equilibrium decision rules.

Now, let's consider the process by which expectations are formed in this model. Take the representative j'th decision (or jump) variable, $j \in \{1, ..., n_y\}$, i.e.,

$$y_t^j = h^j\left(\mathbf{z}_t, \mathbf{u}_t\right). \tag{7}$$

Forming the expectation $E_t y_{t+1}^j$ requires evaluating the integral:

$$E_t y_{t+1}^j = \int \cdots \int h^j\left(\mathbf{g}_z\left(\mathbf{z}_t, \mathbf{u}_t\right), \mathbf{u}_{t+1}\right) p\left(\mathbf{u}_{t+1}|\mathbf{u}_t\right) d\mathbf{u}_{1,t+1} \cdots d\mathbf{u}_{s,t+1}, \tag{8}$$

where $p\left(\mathbf{u}_{t+1}|\mathbf{u}_t\right)$ is the conditional probability density function that governs the shocks. In order to evaluate equation (8) agents must know the equilibrium law-of-motion, the equilibrium decision rule, and the probability density function for the shocks. Under rational expectations these information requirements are not a problem because these objects are all assumed to be known.

3 Experienced-based-expectations

In this section we describe a sequence of increasingly more pronounced departures from rational expectations. We characterize these departures as experienced-based-expectations because the processes all involve agents forming expectations based on the particular history of observations they have experienced. Underlying experienced-based-expectations is the idea that agents are endowed with a history of observations, $\{\mathbf{y}_t, \mathbf{z}_t, \mathbf{u}_t\}_{t=1}^T$, and that they form their expectations of future outcomes based, to a greater or lesser extent, on this history. We will call the set of observations used to form expectations the experience set.

In what follows, we consider three forms of experienced-based-expectations which we term experienced-based-expectations of types I, II, and III. We close the section with a simple illustration of how these expectations are formed and how they compare to rational expectations.

3.1 Type-I

With type-I experienced-based-expectations we relax the assumption that agents know the conditional probability density function, $p\left(\mathbf{u}_{t+1}|\mathbf{u}_t\right)$. Instead, agents use their history of observations, in particular the history of the shocks, $\{\mathbf{u}_t\}_{t=1}^T$, to construct an approximate conditional density, $\widehat{p}(\mathbf{u}_{t+1}|\mathbf{u}_t)$. Based on their approximate conditional density, agents solve the model to arrive at the associated functions characterizing equilibrium:

$$\mathbf{z}_{t+1} = \widehat{\mathbf{g}}_z(\mathbf{z}_t, \mathbf{u}_t), \tag{9}$$

$$\mathbf{y}_t = \widehat{\mathbf{h}}(\mathbf{z}_t, \mathbf{u}_t). \tag{10}$$

In this equilibrium, expectations of future non-predetermined variables are formed according to:

$$\widehat{E}_t^I y_{t+1}^j = \int \cdots \int \widehat{h}^j(\widehat{\mathbf{g}}_z(\mathbf{z}_t, \mathbf{u}_t), \mathbf{u}_{t+1})\, \widehat{p}(\mathbf{u}_{t+1}|\mathbf{u}_t)\, d\mathbf{u}_{1,t+1} \cdots d\mathbf{u}_{s,t+1}, \tag{11}$$

$j \in \{1, ..., n_y\}$. Equation (11) assumes that agents know the model—up to the shock processes—and that they form expectations using the equilibrium law-of-motion for the endogenous state variables and the equilibrium decision rules for the jump variables, where these equilibrium relationships are consistent with the agents' (experienced-based) approximate conditional probability density function.

3.2 Type-II

For an arbitrary non-predetermined variable, say y_t^j, $j \in \{1, ..., n_y\}$, the rational expectation of y_{t+1}^j given $\mathbf{z}_t$ and $\mathbf{u}_t$, can be expressed as:

$$E_t y_{t+1}^j = \int y_{t+1}^j p\left(y_{t+1}^j|\mathbf{z}_t, \mathbf{u}_t\right) dy_{t+1}^j, \tag{12}$$

where $p\left(y_{t+1}^j|\mathbf{z}_t, \mathbf{u}_t\right)$ is the probability density for y_{t+1}^j, conditional upon the state variables, $\mathbf{z}_t$ and $\mathbf{u}_t$. With rational expectations this conditional probability distribution function is known to agents, because agents know the model, its parameters, and the probability density function for the shocks.

With type-II experienced-based-expectations we relax the assumption that agents know $p\left(y_{t+1}^j|\mathbf{z}_t, \mathbf{u}_t\right)$. Instead, we assume that they approximate this conditional probability distribution function based on the outcomes $\{\mathbf{y}_t, \mathbf{z}_t, \mathbf{u}_t\}_{t=1}^T$ that they observe, or have observed earlier in life. Accordingly, expectations are formed by evaluating the integral in equation (12) but where $p\left(y_{t+1}^j|\mathbf{z}_t, \mathbf{u}_t\right)$ is approximated from a finite sample of observed outcomes. This approximation yields $\widehat{p}\left(y_{t+1}^j|\mathbf{z}_t, \mathbf{u}_t\right)$, from which the expectation is:

$$\widehat{E}_t^{II} y_{t+1}^j = \int y_{t+1}^j \widehat{p}\left(y_{t+1}^j|\mathbf{z}_t, \mathbf{u}_t\right) dy_{t+1}^j. \tag{13}$$

Note that equation (13) represents a much stronger departure from rational expectations than equation (12). With equation (12) agents are assumed to not know the conditional density for the shocks. In contrast, with equation (13) agents are assumed to not know the equilibrium law-of-motion for the states, the equilibrium decision rules, and the conditional density for the shocks. These three objects are all subsumed within the approximated density, $\widehat{p}\left(y_{t+1}^{j}|\mathbf{z}_t,\mathbf{u}_t\right)$.

3.3 Type-III

The third form of experienced-based-expectations that we examine assumes that decision-makers do not use any conditioning information from the model when forming expectations. Expectations are therefore based on the integral:

$$\widehat{E}_t^{III}y_{t+1}^{j}=\int y_{t+1}^{j}\widehat{p}\left(y_{t+1}^{i}\right)dy_{t+1}^{j}, \tag{14}$$

$j\in\{1,...,n_y\}$, where $\widehat{p}\left(y_{t+1}^{j}\right)$ is an approximation to the unconditional density for y_{t+1}^{j}. This unconditional density is approximated based on the outcomes that decisionmakers have experienced, $\left\{y_t^j\right\}_{t=1}^{T}$. We term this mechanism type-III experienced-based-expectations, but note that it is based on a deep ignorance regarding the underlying economic model and represents an extreme departure from rational expectations. In particular, the implied learning does not deliver the conditional densities useful for accurate forecasting, leading to systematic and sustained forecast errors. Moreover, unlike type-I and type-II experienced-based-expectations, type-III experienced-based-expectations do not approach rational expectations as $T\uparrow\infty$.

3.4 An illustration

To provide a simple illustration, suppose that data on u_t are generated by the process:

$$u_{t+1}=\rho_u u_t+\eta_{t+1}, \tag{15}$$

where $\eta_t\sim i.i.d.\ N\left[0,\sigma_\eta^2\right]$ and $\rho_u\in(0,1)$. If this process is known, as is assumed to be the case under rational expectations, then decisionmakers can exploit the known conditional and unconditional density functions, which are given by:

$$p\left(u_{t+1}|u_t\right) = \frac{1}{\sqrt{2\pi}\sigma_\varepsilon}\exp\left[-\frac{1}{2}\left(\frac{u_{t+1}-\rho u_t}{\sigma_\eta}\right)^2\right], \tag{16}$$

$$p(u_{t+1}) = \frac{\sqrt{1-\rho^2}}{\sqrt{2\pi}\sigma_\varepsilon}\exp\left[-\frac{1-\rho^2}{2}\left(\frac{u_{t+1}}{\sigma_\eta}\right)^2\right], \tag{17}$$

respectively. Rational expectations are based on equation (16). In contrast, with experienced-based-expectations decisionmakers are endowed with a finite sample of T observations, $\{u_t\}_{t=1}^T$, that they use to approximate either $p\left(u_{t+1}|u_t\right)$ or $p\left(u_{t+1}\right)$. In the former case, this approximation will condition upon the state variable, u_t, but otherwise employ no knowledge of equation (15). For this simple illustration, where we are focusing only upon a shock process and there is no decision variable(s), type-I and type-II experienced-based-expectations are equivalent.

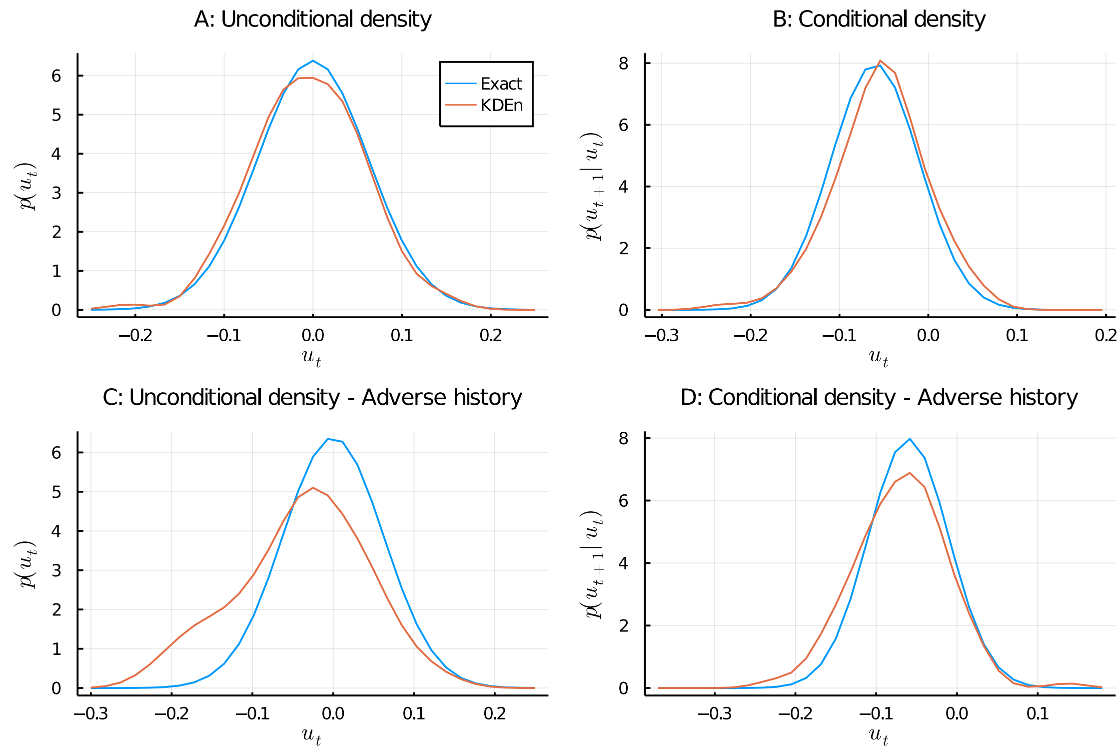

Figure 1: Probability densities under rational expectations and experienced-based-expectations.

Figure 1 illustrates the densities under consideration. All four panels are based on an experience set consisting of a sample of 120 observations simulated from equation (15) with

$\rho = 0.6$ and $\sigma_\eta = 0.05$. Panels A and B show unconditional and conditional densities, respectively, for rational expectations and experienced-based-expectations. The densities for experienced-based-expectations are computed using a kernel density estimator (normal kernel); results for other density estimators are shown in Appendix A. The conditional densities assume $u_t = -0.1$. By way of contrast, panels C and D are based on the same sample, but where the final 40 observations have been subjected to a negative shock, capturing the idea that the sample has been subject to a sequence of adverse shocks. Where the densities for rational expectations are unaffected by the adverse sequence, the densities for all three types of experienced-based-expectations are affected, with the unconditional (type-III) and conditional densities (type-I/II) assigning more probability mass to negative outcomes.

If we look just at the conditional mean, then for these parameter values, and with $u_t = -0.1$, the rational conditional expectation of u_{t+1} is $E_t[u_{t+1}|u_t] = 0.6 \times -0.1 = -0.06$ and the rational unconditional expectation is $E_t u_{t+1} = 0.0$. In contrast, the type-I/II experienced-based expectation, given $u_t = -0.1$, equals -0.0541 and the type-III experienced-based expectation equals -0.0054. For the sample containing the sequence of adverse shocks these experienced-based-expectations equal -0.0700 and -0.0387, respectively. Experienced-based-expectations lead to forecasted values for u_{t+1} that are lower, both conditionally and unconditionally, than rational expectations when the sequence of adverse outcomes is experienced.

4 A non-linear new Keynesian model

We present a model that is populated by three types of agents: a unit-continuum of households, a unit-continuum of firms, and a central bank. Households value consumption and leisure, and have preferences that reflect external consumption habits. Production is characterized by monopolistic competition and Rotemberg (1982) sticky prices. The central bank conducts monetary policy according to a simple Taylor-type rule.

4.1 Households

The representative household buys consumption goods, c_t, supplies their labor, h_t, to firms in a perfectly competitive market, and saves by holding risk-free one-period nominal bonds, b_t, that are in zero-net supply. With C_t denoting period-t aggregate consumption, the household's decision problem is to choose $\{c_t, h_t, b_{t+1}\}_{t=0}^{\infty}$ to maximize:

$$U_t = E_t \left[\sum_{t=0}^{\infty} \beta^t \left(\frac{(c_t - \gamma C_{t-1})^{1-\sigma}}{1-\sigma} - \mu \frac{h_t^{1+\chi}}{1+\chi} \right) \right], \tag{18}$$

where $\sigma > 0$, $\chi > 0$, $\mu > 0$, $0 < \beta < 1$, and $0 < \gamma < 1$, subject to the flow budget constraint:

$$c_t + Q_t \frac{b_{t+1}}{P_t} = w_t h_t + \frac{b_t}{P_t} + D_t, \tag{19}$$

in which P_t is the aggregate price level, w_t is the real wage, Q_t is the nominal price of a bond, and D_t is the aggregate dividend households receive from owning equity in firms.

4.2 Firms

Production for the i'th firm, $i \in [0, 1]$, is given by the linear technology:

$$y_t(i) = e^{z_t} h_t(i), \tag{20}$$

where z_t is an aggregate technology shock that follows the process:

$$z_{t+1} = \rho_z z_t + \varepsilon_{t+1}, \tag{21}$$

where $0 < \rho_z < 1$ and $\varepsilon_t \sim i.i.d.\ N[0, \sigma_\varepsilon^2]$.

With all firms employing labor from a perfectly competitive market at real wage w_t, their real marginal costs, ω_t, when producing efficiently are given by:

$$\omega_t = \frac{w_t}{e^{z_t}}. \tag{22}$$

Let $p_t(i)$ denote the i'th firm's price relative to the aggregate price, π_t denote aggregate inflation, and λ_t denote the marginal utility of consumption. Then the decision problem for the i'th firm is to choose $\{p_t(i)\}_{t=0}^{\infty}$ to maximize:

$$\Gamma_t(i) = E_t \left[\sum_{t=0}^{\infty} \left(\beta^t \frac{\lambda_t}{\lambda_0} \left((p_t(i) - \omega_t) y_t(i) - \frac{\phi}{2} \left(\frac{p_t(i)(1+\pi_t)}{p_{t-1}(i)} - 1 \right)^2 Y_t \right) \right) \right], \tag{23}$$

subject to the demand curve:

$$y_t(i) = p_t(i)^{-\epsilon} Y_t, \tag{24}$$

where $\epsilon > 1$ is the price elasticity of demand and $\phi > 0$ governs the cost of changing prices.

As is typical in this literature, the goods produced by the unit-continuum of firms are aggregated costlessly through the constant elasticity of substitution technology (Dixit and Stiglitz, 1977):

$$Y_t = \left[\int_0^1 y_t(i)^{\frac{\epsilon-1}{\epsilon}} di \right]^{\frac{\epsilon}{\epsilon-1}}. \tag{25}$$

Finally, we assume that the cost that firms pay to change prices is received by workers in addition to their wages (the payment is subsumed within D_t). A consequence of this assumption is that the resource constraint for this economy equates aggregate consumption and aggregate output: $C_t = Y_t$.

4.3 Central bank

The gross nominal interest rate is related to the price of a bond according to:

$$1 + R_t = \frac{1}{Q_t}, \tag{26}$$

where R_t is the net nominal interest rate. The central bank's policy instrument is the net nominal interest rate, and we assume that it sets its policy instrument such that the gross nominal interest rate satisfies the Taylor-type rule:

$$1 + R_t = \frac{1+\bar{\pi}}{\beta} \left(\frac{1+\pi_t}{1+\bar{\pi}} \right)^{\theta_\pi} \left(\frac{Y_t}{Y_{t-1}} \right)^{\theta_y} e^{m_t}, \tag{27}$$

where $\bar{\pi}$ denotes the inflation target, $\phi_\pi > 1$ and $\phi_y > 0$ are policy feedback coefficients, and m_t represents a monetary policy shock. The monetary policy shock is assumed to follow the process:

$$m_{t+1} = \rho_m m_t + \eta_{t+1}, \tag{28}$$

where $0 < \rho_m < 1$ and $\eta_t \sim i.i.d.\ N[0, \sigma_\eta^2]$.

4.4 Key equations

Solving the optimization problems described above and aggregating across households and firms, the key equations that emerge are the following (see Appendix B):

$$
\begin{aligned}
(Y_t-\gamma Y_{t-1})^{-\sigma} &= \beta E_t\left[\frac{(Y_{t+1}-\gamma Y_t)^{-\sigma}(1+R_t)}{(1+\pi_{t+1})}\right], && (29)\\
Y_t &= e^{z_t}H_t, && (30)\\
\pi_t(1+\pi_t) &= \frac{1-\epsilon}{\phi}+\frac{\epsilon}{\phi}\omega_t+\beta E_t\left[\frac{(Y_{t+1}-\gamma Y_t)^{-\sigma}Y_{t+1}\pi_{t+1}(1+\pi_{t+1})}{(Y_t-\gamma Y_{t-1})^{-\sigma}Y_t}\right], && (31)\\
\omega_t &= \frac{H_t^{\chi}}{e^{z_t}(Y_t-\gamma Y_{t-1})^{-\sigma}}, && (32)\\
1+R_t &= \frac{1+\bar{\pi}}{\beta}\left(\frac{1+\pi_t}{1+\bar{\pi}}\right)^{\theta_\pi}\left(\frac{Y_t}{Y_{t-1}}\right)^{\theta_y}e^{m_t}, && (33)
\end{aligned}
$$

with the shock processes given by equations (21) and (28).

Equation (29) is the consumption-Euler equation for the case of external consumption habits. Equation (30) is the aggregate production function. Equation (31) is the forward-looking new Keynesian Phillips curve. The equilibrium expression for real marginal costs is given in equation (32). Lastly, equation (33) is the monetary policy rule. It is straightforward to use equations (30) and (32) to substitute Y_t and ω_t from the model, leaving a system of three non-linear equations. This model has three state variables, one endogenous state variable, Y_{t-1}, and two exogenous state variables, z_t and m_t.

5 Results

In this section we study the behavior of the non-linear new Keynesian model with experienced-based-expectations, contrasting its behavior to rational expectations. We first use stochastic simulation to establish a benchmark. The benchmark simulation allows us to compute the volatility associated with the different expectation-formation processes. Subsequently, we consider three counter-factual experiments. In the first experiment we change the inflation target that enters the policy rule and examine how long it takes for realized inflation outcomes to closely reflect the new target value. The second experiment explores the situation in which monetary policy has been historically loose due to a sequence of monetary policy shocks that have pulled down the policy rate. The final experiment considers the case where the economy has been subject to a sequence of adverse aggregate technology shocks. To better separate the signal from the noise, for each of these counter-factual experiments we perform multiple simulation runs and present results that average across the runs.

5.1 Parameterization

Our first step to analyzing this non-linear model is to parameterize it. We do not estimate the model, as the model is non-linear and methods for estimating such models are prohibitively slow. Moreover, how to estimate such a model with experienced-based-expectations is an open question. Instead, because our goal is to illustrate the role of experienced-based-expectations as a mechanism for generating volatility and persistence, we parameterize the model using conventional values from the literature, as summarized in Table 1.

Table 1: Model Parameters

Name	Symbol	Value	Name	Symbol	Value
Discount factor	β	0.99	Inflation target	$\overline{\pi}$	0.005
Utility curvature	σ	2.0	Policy response to inflation	θ_π	1.5
Habits	γ	0.8	Policy response to growth	θ_y	0.25
Labor supply elasticity	χ	5.0	Technology persistence	ρ_z	0.95
Price elasticity of demand	ϵ	11.0	Policy shock persistence	ρ_m	0.8
Price rigidity	ϕ	80.0	Technology innovation s.d.	σ_ε	0.007
			Policy innovation s.d.	σ_η	0.0035

We set the discount factor, β, to 0.99, implying an annualized real interest rate of just over 4% p.a. The curvature of the utility function with respect to consumption, σ, is set to 2.0, consistent with the estimate in Levin, Onatski, Williams, and Williams (2005), while slightly higher than the estimate from Smets and Wouters (2007). In line with the estimates in Dennis (2009), we set the habit formation parameter, γ, to 0.8. The Frisch labor-supply elasticity, χ, is set to 5.0, based on the estimates from Christiano, Trabandt, and Walentin (2011). Drawing on Basu and Fernald (1997) and Ireland (2001), we set the price elasticity of demand, ϵ, to 11.0, implying an average mark-up of 10 percent, and the price-rigidity, ϕ, to 80.0. This value for ψ is a bit lower than the estimate of around 100 found by Gust, Herbst, López-Salido, and Smith (2017), but higher than the 59.11 value assumed in Gavin, Keen, Richter, and Throckmorton (2015).

In the monetary policy rule, we assume that the inflation target is 2.0 at an annualized rate and, accordingly, set $\overline{\pi}$ to 0.005. For the remaining coefficients in the monetary policy rule, we set θ_π to 1.5, so that the Taylor principle holds, and θy to 0.25, which implies that the nominal interest rate is raised by one percentage point in response to a one percent increase in annualized output growth. The coefficient values in the two shocks are largely standard. For the aggregate technology shock, we follow convention (see Faia (2008) and the references therein) and set the persistence parameter, ρ_z, to 0.95 and the standard deviation for the technology innovation, σ_η, to 0.007. For the monetary policy shock we set ρ_m to 0.8 and σ_η to 0.0035, drawing on the estimates in Rabanal and Rubio-Ramírez (2005).

5.2 Comparison to rational expectations

In this section we compare the outcomes generated from the three forms of experienced-based-expectations to those from rational expectations. To this end we perform a stochastic simulation, generating model-data for 1000 periods (after excluding a burn-in period of 100 observations) using the same shock-sequences for all four expectations-processes. For the nominal interest rate and inflation, the data are annualized and expressed in percentage points. For each of the experienced-based-expectations, people are assumed to use the most recent 120 observations (equivalent to a span of 30 years) to form expectations. The final 300 periods of data simulated from each expectations-process are plotted in Figure 2.

Two points regarding Figure 2 deserve emphasis. First, with an experience set of 120 observations, the departures from rational expectations are often quite large and occur for sustained periods. These departures are noticeable for all variables, but can perhaps be seen most clearly in inflation and the nominal interest rate. Second, type-III experienced-based-expectations generate behavior that is very different to the other expectation-mechanisms, which are relatively similar by comparison.

Table 2: Standard Deviations

Variable	R.E.	EBE-I	EBE-II	EBE-III
Output	0.0309	0.0313	0.0338	0.0176
labor	0.0127	0.0129	0.0090	0.0290
Inflation	4.3986	4.1986	4.9464	6.3939
Nominal rate	5.2655	5.1759	5.3467	9.3709

In Table 2 we summarize the volatility for each of the model's main variables, for each of the four expectations-processes, in terms of unconditional standard deviations. While there are exceptions, such as the decline in labor volatility associated with type-II experienced-based-expectations and the decline in output volatility associated with type-III experienced-based-expectations, the main finding is that experienced-based-expectations leads to greater

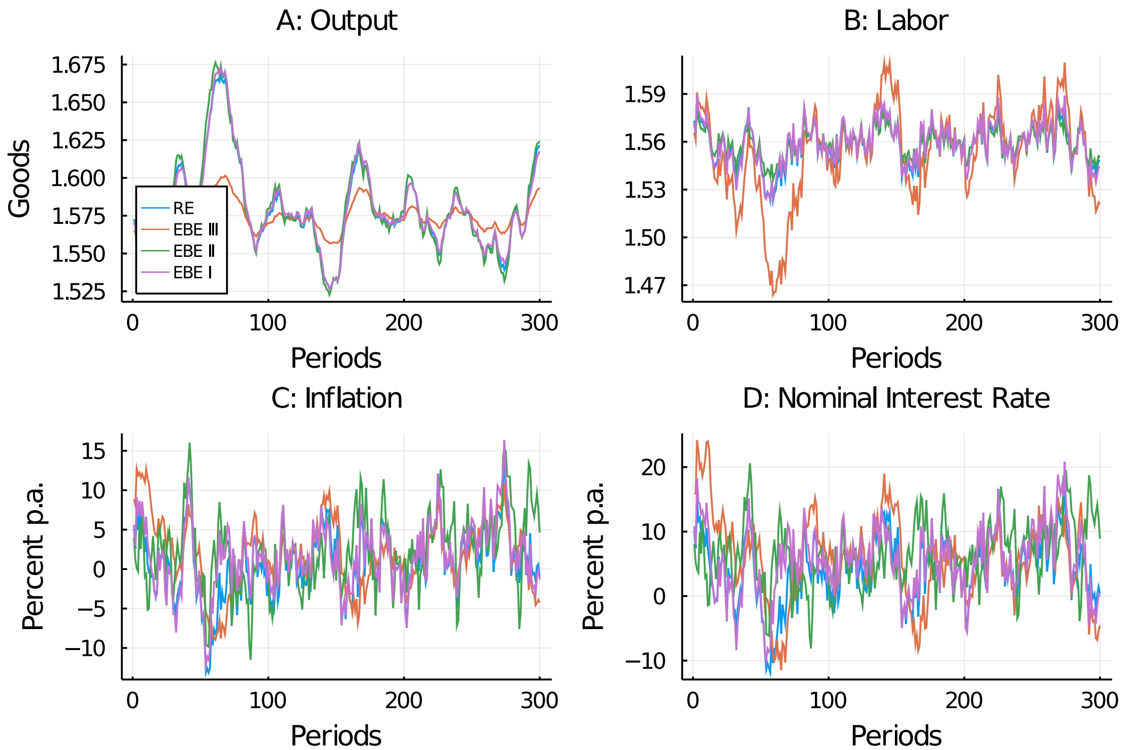

Figure 2: Simulation using an experience set of 120 observations.

volatility than rational expectations. As noted earlier, type-I experienced-based-expectations are most closely related to rational expectations in the sense that both assume that the model's structure is known. This similarity is reflected in Table 2 where the volatilities produced are generally similar. For this model, Table 2 shows that type-II experienced-based-expectations lead to noticeably greater volatility for both inflation and the nominal interest rate. This greater volatility reflects the de-anchoring that occurs when people form expectations with an incomplete understanding of the underlying economic model. Related increases in volatility are shown to occur for type-III experienced-based-expectations.

Before leaving this section we consider an additional simulation that increases the length of the experience set from 120 observations to 5000 periods. With a sample of 5000 observations used to form expectations we expect experienced-based-expectations to be similar to rational expectations. Looking at Figure 3 we can see that this is indeed the case—at least for type-I and type-II experienced-based-expectations. In contrast, people's ignorance about the model embedded in type-III experienced-based-expectations means that expectations formed this way do not converge to rational expectations as the size of the experience set

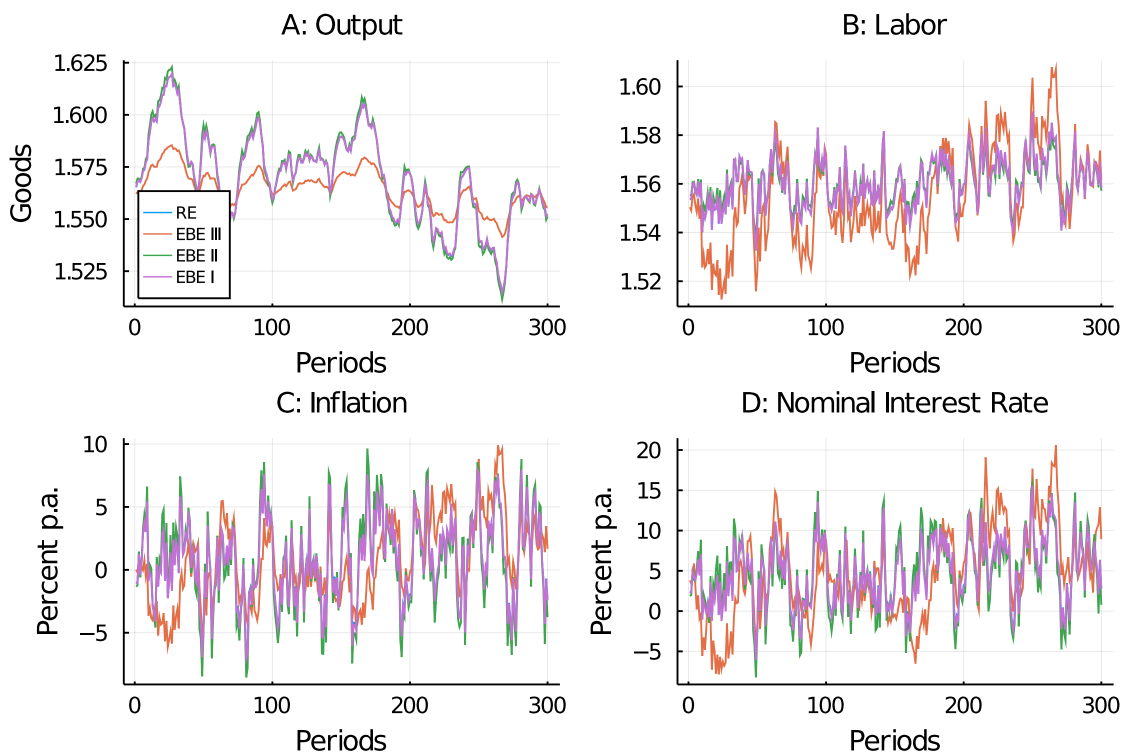

Figure 3: Simulation using an experience set of 5000 observations.

increases. We continue to examine type-III experienced-based-expectations in what follows, but recognize that it is not a mechanism that allows a model's dynamics to be learned and is fundamentally different in that respect to type-I and type-II experienced-based-expectations.

5.3 Permanent change to the inflation target

Here we suppose that the central bank makes an unannounced, exogenous, change to the inflation target, raising it from 2% p.a. to 4% p.a. As previously, we assume that people form expectations using the most recent 120 observations, and we compare the results for experienced-based-expectations to those for rational expectations. From an initial sample comprising observations for 120 periods, we simulate data for an additional 160 periods, with the change in the inflation target occurring 20 periods into the simulation. We repeat this exercise 115 times and present average simulation paths in Figure 4.

Panels A and B display the paths for output and labor, respectively. Although there is variation in these variables according to how expectations are formed this variation is relatively muted, a consequence of the fact that the classical dichotomy is close to holding and

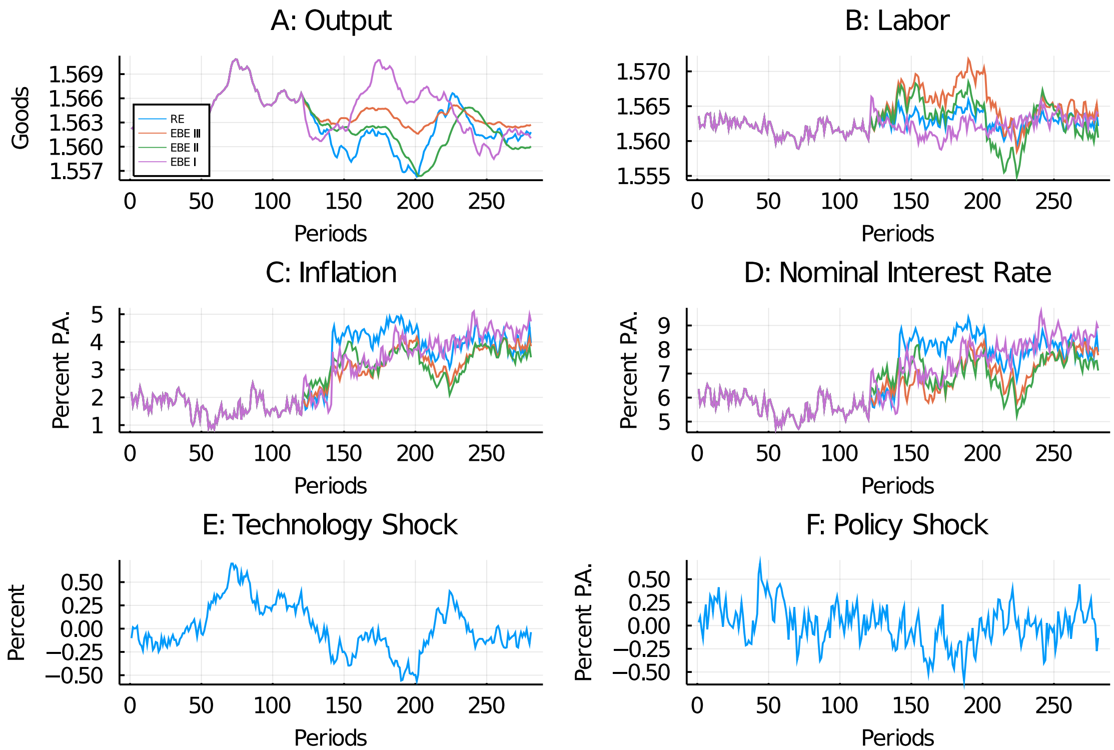

Figure 4: An unanticipated permanent change in the inflation target.

there is relatively little interaction between the real and nominal sides of the model. Panels C and D illustrate the paths for inflation and the nominal interest rate. Under rational expectations, although the change in the inflation target is unanticipated, once the change occurs actual inflation and the nominal interest rate respond quickly and proportionately to the change in the target. With expectations formed according to type-I experienced-based-expectations, the model's behavior is very similar to that with rational expectations, a consequence of the fact that other than the shocks processes, the structure of the model is known under type-I experienced-based-expectations. Much greater differences are observed when expectations are formed using either type-II or type-III experienced-based-expectations. For both of these expectation-formation processes, the change in the inflation target generates ongoing gradual changes to observed inflation and observed nominal interest rates as decisionmakers only gradually learn that the inflation target has changed. Convergence to the new higher inflation consistent with the increased target takes considerable time as the change is clouded by uncertainty regarding the conditional densities for output and inflation (unconditional densities in the case of type-III experienced-based-expectations), although learning occurs more rapidly for type-II experienced-based-expectations than for type-III experienced-based-expectations.

5.4 Temporarily loose monetary policy

In our second experiment we consider the effects of a temporary period of sustained loose monetary policy, modeled as a sequence of low realizations for the monetary policy shock. Specifically, for each expectation-process we endow agents with an experience set containing 120 periods of observed outcomes, where the final 40 outcomes for the policy shock have been artificially lowered by 50 basis points so that the average monetary policy shock during this period is -2 percent at an annualized rate. We then simulate learning outcomes for an additional 140 periods, repeat the entire exercise 115 times, and report average outcomes in Figure 5.

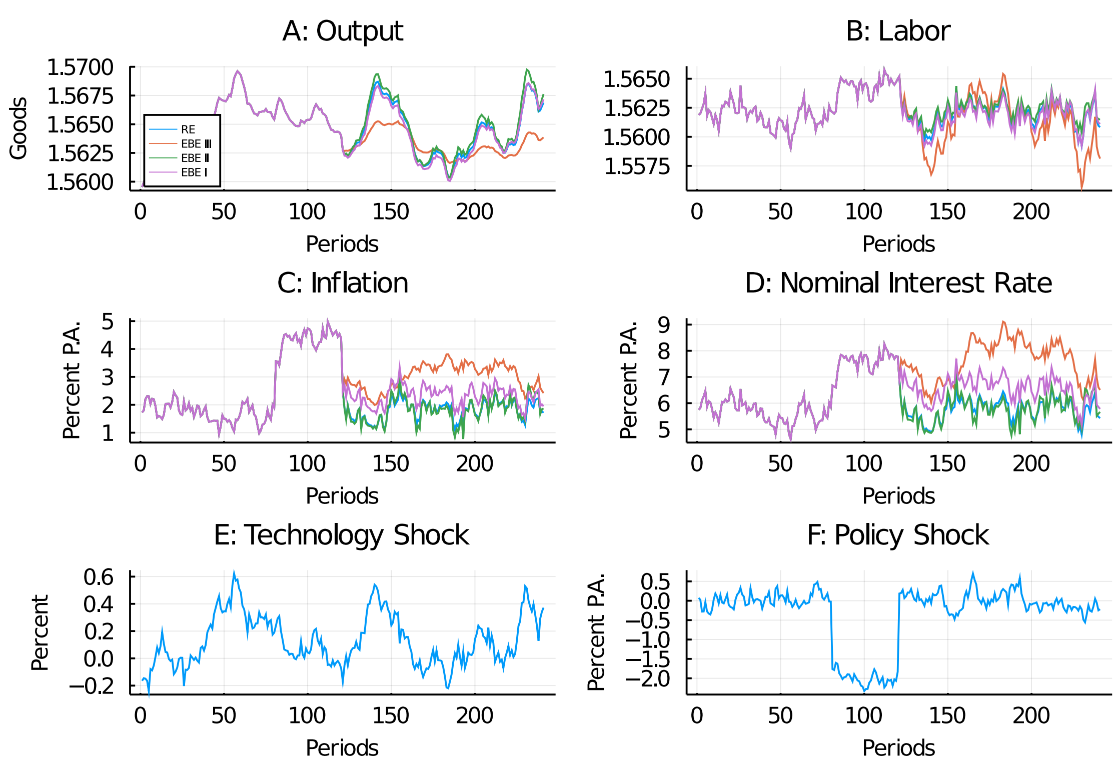

Figure 5: Temporarily loose monetary policy.

Panel F in Figure 5 shows the average path for the monetary policy shock, clearly illustrating the period during which the monetary policy shocks were systematically low. The effect of this period of loose monetary policy can be seen most prominently in the behavior of inflation (panel C) and the nominal interest rate (panel D), which are buoyed by the shock. It may seem counter-intuitive that the shock causes the nominal interest rate to rise, but the policy rule directs interest rates to rise following an increase in inflation and although the nominal interest rate rises the real interest rate does not. Instead, the real interest rate remains largely unchanged, which is why the shock does not lead to higher output (panel A).

Looking now at how the period of loose monetary policy shocks affects the economy across the various expectations processes, it is immediately clear that type-II experienced-based-expectations behaves similarly to rational expectations. The reason for this similarity is that the realizations for the shocks do not alter how the decision variables relate to the state variables in equilibrium, which is the key relationship being learned under type-II experienced-based-expectations. Turning to the model's behavior under type-I experienced-based-expectations, although output (panel A) and labor (panel B) behave similarly to rational expectations, the same cannot be said for inflation (panel C) and the nominal interest rate (panel D), which are slow to return to normal following the period of temporary stimulus. Although a return to noral does eventually occur, it takes considerable time and produces sustained departures from rational expectations that are akin to economic scarring. In this case, the period of loose monetary policy shocks gets built into people's experience, causing them to build loose monetary policy shocks into their expectation-formation process, which produces persistently high inflation and interest rates. Even greater and more persistent departures from rational expectations are produced by type-III experienced-based-expectations, where the absence of a model in the expectations process leads to large and sustained departures from rational expectations.

5.5 Temporarily adverse technology

Our final experiment relates to the process for technology. In this experiment we consider the effects of a period of sustained low outcomes for aggregate technology, modeled as a sequence of low realizations for the aggregate technology shock. Much like the experiment above for monetary policy shocks, for each of the expectations processes we endow people with 120 periods of observed outcomes, of which the final 40 periods for the technology shock have been artificially lowered by 3 percent. We then simulate learning outcomes for an additional 140 periods, repeat the entire exercise 115 times, and report average outcomes in Figure 6.

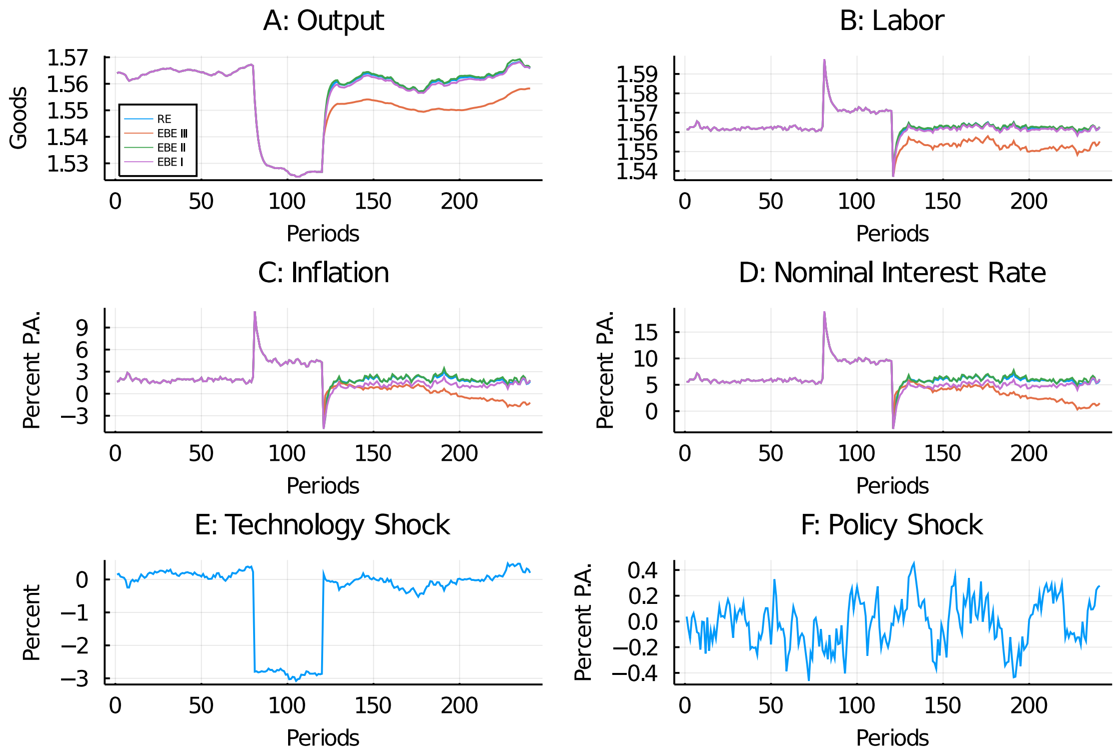

Figure 6: Temporarily adverse aggregate technology.

Several features of Figure 6 are worth noting. First, the period of temporarily adverse technology shocks (panel E) leads to a notable decline in output (penal A) while at the same time causing spikes in the behavior of labor (panel B), inflation (panel C), and the nominal interest rate (panel D). These spikes are a consequence of the habit formation interacting with the (adverse) technology process. More specifically, habit formation implies that it is the quasi-difference of output that matters for labor supply and firms' pricing, and the adverse technology process generates spikes in the quasi-difference of output when the adverse technology shock begins and ends. Through the monetary policy rule, the behavior of inflation gets reflected in the nominal interest rate (panel D). In appendix D we show that these spikes in labor, inflation, and the nominal interest rate are not present when the habit formation parameter is set to zero ($\gamma = 0.0$).

Looking at the model's behavior under the different expectations processes, much like subsection 5.4, type-II experienced-based-expectations behaves very similarly to rational expectations. However, sustained departures from rational expectations are present under type-I experienced-based-expectations, where the sequence of adverse technology shocks gets built into people's expectation-formation process, causing them to place higher probability on adverse technology outcomes. As for the monetary policy shock, the departures from rational expectations are sufficienctly long-lived that they appear muck like economic scarring. Type-III experienced-based-expectations behave very differently to the other processes because the absence of a model the expectation formation process leads to weak anchoring and very slow convergence back to baseline.[1]

5.6 The effects of experience on learning

In this subsection we consider the effect on learning of agents being endowed with a smaller experience set. Specifically, we shorten the length of the experience set to just 60 observations. With this smaller experience set we repeat the exercises in subsections 5.4 and 5.5 and report the results in Figures 7 and 8, respectively, below.

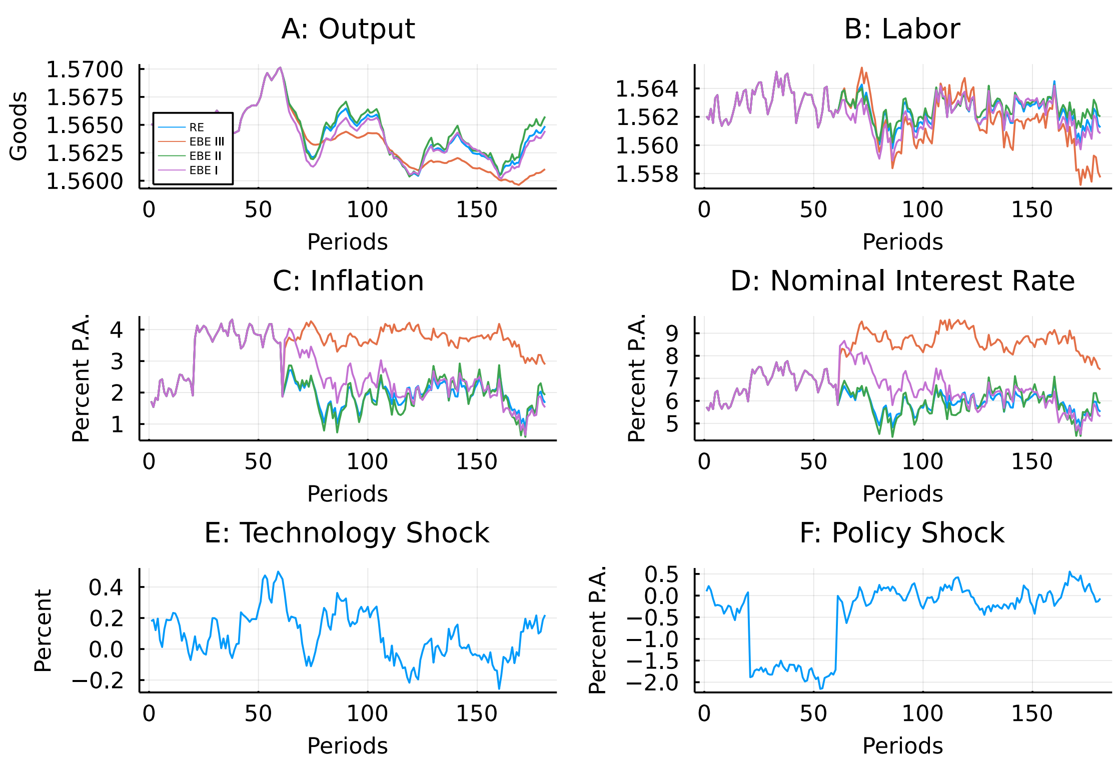

Figure 7: Temporarily loose monetary policy with an experience set of 60 observations.

Qualitatively, the results in Figure 7 are similar to those in Figure 5, but underlying the

[1]We have extended the length of the simulation to determine that, indeed, type-III experienced-based-expectations do eventually return to baseline.

similarity are some important differences. First, with the experience set dominated by the history of loose monetary policy shocks, the initial differences between type-I (and type-III) experienced-based-expectations and rational expectations are magnified. At the same time, with decision-makers employing a shorter experience set the history of loose policy shocks recedes from memory more rapidly. As a consequence, the gap between type-I experienced-based-expectations and rational expectations narrows more quickly as time passes. These effects can be seen most clearly in the behavior of inflation (panel C) and the nominal interest rate (panel D).

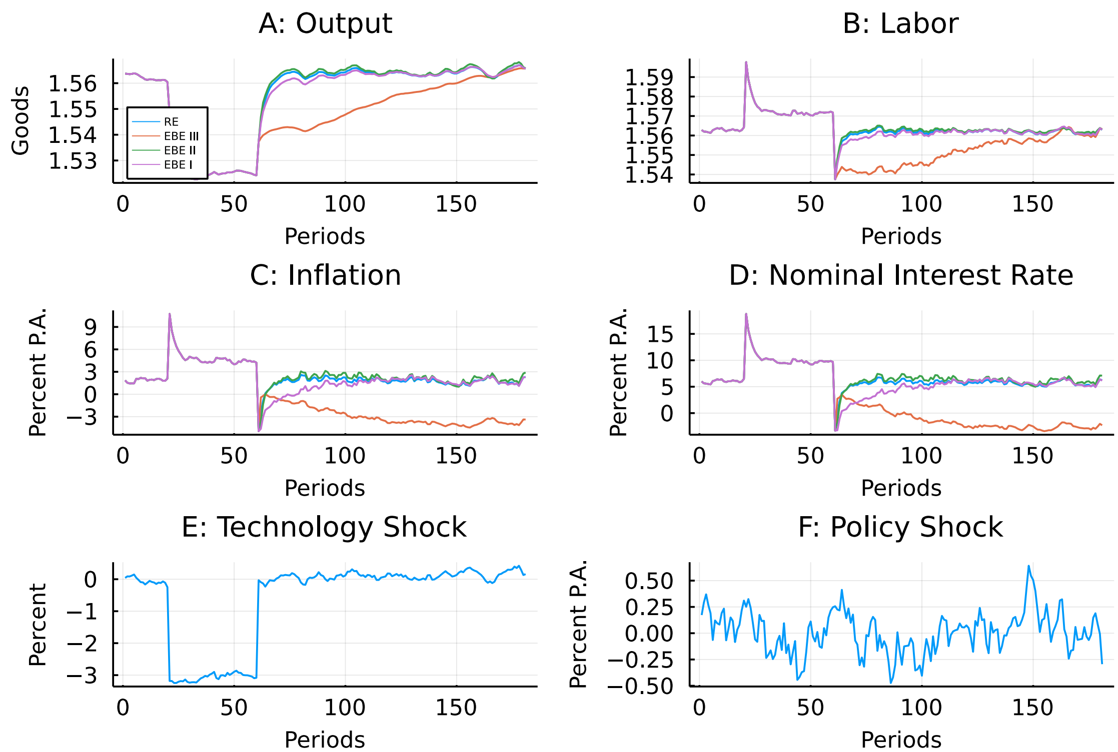

Figure 8: Temporarily adverse technology with an experience set of 60 observations.

The results in Figure 8 reinforce those from Figure 7. Focusing on the behavior of type-I and type-II experienced-based-expectations, we see that with a shorter experience set the initial effect on the economy of the adverse technology shocks is increased while the gap between experienced-based-expectations and rational expectations closes more rapidly over time. Further, because the underlying relationships linking economic outcomes to the state variables have not changed, type-II experienced-based-expectations behave similarly to rational expectations while type-I experienced-based-expectations, which are driven by the realized shocks, generate meaningful differences. Finally, type-III experienced-based-expectations exhibit large and sustained departures from rational expectations, especially for inflation and the nominal interest rate (panels C and D, respectively).

6 Related departures from rational expectations

In this section we briefly discuss a few departures from rational expectations that are related to experienced-based-expectations to a greater or lesser extent. There are others, but here we focus on diagnostic expectations, ambiguity aversion, rare disasters, and real-time learning. The first three share similarities with type-I experienced-based-expectations while the fourth has similarities with type-II experienced-based-expectations.

For illustrative purposes, we will present the various departures in the context of a shock process that has the following autoregressive form:

$$x_{t+1} = \rho x_t + \upsilon_{t+1}, \tag{34}$$

where $0 < \rho < 1$ and $\upsilon_t \sim i.i.d.\ N[0, \sigma_\upsilon^2]$. Denote the conditional density for x_{t+1} given x_t by $p(x_{t+1}|x_t)$ where, under the assumptions above, this conditional density is the density for a normal distribution.

6.1 Diagnostic expectations

Diagnostic expectations are motivated on Kahneman and Tversky's (1972) representativeness heuristic. The heuristic suggests that people will assign excess probability or mass to events whose likelihood has increased in light of current news. So if current economic growth is high, then people will tend to over-estimate the probability that next period's economic growth will be high.

One implementation of the representativeness heuristic has beliefs formed according to the distorted probability density:

$$p_\theta\left(x_{t+1}|x_t\right) = p\left(x_{t+1}|x_t\right)\left(\frac{p\left(x_{t+1}|x_t\right)}{p\left(x_{t+1}|\rho x_{t-1}\right)}\right)^\theta \frac{1}{X}, \tag{35}$$

where $\theta \geq 0$ is the diagnostic parameter, $p_\theta\left(x_{t+1}|x_t\right)$ is the diagnostic density, and X is a scale factor that ensures that the diagnostic density integrates to one. Clearly, equation (35) simplifies to rational expectations in the case that $\theta = 0$. When $\theta > 0$, equation (35) assigns higher probability to outcomes that are more representative or diagnostic given the observed outcome.

Bordalo, Gennaioli, and Shleifer (2018) show for an AR(1) process with normally distributed shocks (i.e., equation 34) that diagnostic expectations cause decisionmakers to behave as if the underlying shock process were actually:

$$x_{t+1} = \rho x_t + \rho\theta\upsilon_t + \upsilon_{t+1}, \tag{36}$$

i.e., an ARMA(1,1) process.

Equation (36) implies that the effect of diagnostic expectations for the standard case of an AR(1) shock is to raise the unconditional variance of the shock, and to impart transitory persistence, or momentum, through the MA(1) term. Because diagnostic expectations alters the probability density function for the shock process, it operates through a mechanism similar to type-I experienced-based-expectations.

6.2 Ambiguity aversion

To illustrate the connection to ambiguity aversion (Gilboa and Shmeidler, 1989), consider the following situation. Suppose an agent gets utility from x_t and that their expected discounted lifetime utility satisfies the recursion:

$$V(x_t) = u(x_t) + \beta \int V(x_{t+1}) p(x_{t+1}|x_t) dx_{t+1}, \tag{37}$$

where $u(x_t)$ is a smooth and concave period-utility function that satisfies the Inada (1965) conditions and $V(x_t)$ is a value function (that is also smooth and concave).

Suppose, further, that the agent is concerned that the conditional density for x_{t+1} may instead be some other density, $q(x_{t+1}|x_t)$, a worst-case density, whose distance from $p(x_{t+1}|x_t)$ is constrained by the entropy condition:

$$I(p, q|x_t) = \int log\left(\frac{q(x_{t+1}|x_t)}{p(x_{t+1}|x_t)}\right) \frac{q(x_{t+1}|x_t)}{p(x_{t+1}|x_t)} p(x_{t+1}|x_t) dx_{t+1}. \tag{38}$$

To combine this entropy condition with an infinite-horizon discounted Bellman equation, Hansen and Sargent (1995, 2007) place a constraint on expected discounted entropy:

$$E_t\left[\sum_{s=0}^{\infty} \beta^s I(p, q|x_{t+s})\right] \leq \varpi. \tag{39}$$

It is convenient to define:

$$m_{t+1} = \frac{q(x_{t+1}|x_t)}{p(x_{t+1}|x_t)}, \tag{40}$$

then the worst-case conditional density that the agent needs to be concerned about is the solution to the decision problem:

$$V(x_t) = min_{m_{t+1}} \left[u(x_t) + \beta \int \left(m_{t+1}V(x_{t+1}) + \alpha m_{t+1} log(m_{t+1}) \right) p(x_{t+1}|x_t)dx_{t+1} \right], \quad (41)$$

with the additional constraint:

$$\int m_{t+1}p(x_{t+1}|x_t)dx_{t+1} = 1. \quad (42)$$

The solution to equations (41)—(42) gives the worst-case density:

$$q(x_{t+1}|x_t) = p(x_{t+1}|x_t)\frac{e^{-\frac{V(x_{t+1})}{\alpha}}}{\int e^{-\frac{V(x_{t+1})}{\alpha}} p(x_{t+1}|x_t)dx_{x+1}}, \quad (43)$$

where $\alpha > 0$ is the Lagrange multiplier on the entropy constraint, which approaches rational expectations in the limit as $\alpha \uparrow \infty$.

This worst-case density slants or distorts the conditional density used to form expectations by assigning lower mass to outcomes that deliver high value and higher mass to outcomes that deliver low value. For this reason ambiguity-averse decisionmakers are sometimes described as being pessimistic. Like diagnostic expectations, ambiguity aversion alters the conditional density for the shock process and in this regard it shares similarities with type-I experienced-based-expectations. However, where ambiguity aversion causes the decisionmaker to put greater probability on shocks that it fears will happen, type-I experienced-based-expectations causes the decisionmaker to put greater probability on shocks that have actually happened.

6.3 Rare disasters

The connections between experienced-based-expectations and the rare disasters literature (Reitz, 1988, Barro, 2006; 2009, Barro and Ursua 2012) are reasonably clear. In the rare disasters literature the shocks that enter the model come from a mixed distribution. For a typical shock process (often that for technology) the shock, u_t, would be composed of two terms:

$$u_t = x_t + z_t, \quad (44)$$

where the process for x_t is given by equation (34). For a standard implementation of rare disasters applied to technology, such as Barro (2006), the process for z_t is taken to be:

$$z_t = \left\{ \begin{array}{cc} 0, & \text{with probability } p \\ \log(1-b), & \text{with probability } 1-p, \end{array} \right\} \quad (45)$$

where b is the contraction proportion. An alternative is to have b be the realization from a power law.

Equation (45) shows that the rare disasters literature operates by altering the distribution of the shock processes used to form expectations. Unlike ambiguity-aversion, which slants the shock-distribution toward adverse outcomes, the rare disasters literature achieves something similar using a two-part distribution, where the second part skews the distribution toward disastrous events. Rare disasters share similarities with type-I experienced-based-expectations insomuch as expectations are being formed using a modified shock-process. However, type-I experienced-based-expectations reflects disastrous events only to the extent that they have been observed and, contrary to the rare disasters literature, they allow periods of unusually good outcomes as well and periods of unusually bad outcomes.

6.4 Real-time learning

There are close connections between experienced-based-expectations and the real-time learning literature pioneered and popularized by Evans and Honkapohja, and summarized in their book, Evans and Honkapohja (2001). Real-time learning starts with a perceived law-of-motion (PLM) that relates endogenous variables to the state variables, estimates the coefficients in this PLM using regression, or some form of filtering (such as the Kalman filter), and uses the estimated PLM to form expectations. These expectations are combined with the model to arrive at the actual law-of-motion (ALM), which is used to generate next-period's observation.

Real-time learning is based on estimation using observed outcomes for endogenous variables and, in this sense, it is most closely related to experienced-based-expectations of type-II. The difference between experienced-based-expectations and real-time learning is that the real-time learning focuses almost exclusively on linear models and on modeling the conditional mean whereas experienced-based-expectations can be applied equally to linear and non-linear models and approximates the entire conditional density, not just the conditional mean.

7 Conclusions

This paper has shown how to integrate experienced-based-expectations, which allow the outcomes people experience to guide their expectations, into dynamic, stochastic, general equilibrium models. Three types of experienced-based-expectations are described and analyzed, with each type differing according to the experience set drawn upon to form expectations and according to people's knowledge of the model's underlying structure. In the context of a non-linear new Keynesian business cycle model, the paper considered three experiments and analyzed how the model behaves under the three types of experienced-based-expectations relative to rational expectations.

In response to an exogenous unanticipated increase in the inflation target, we showed that type-I experienced-based-expectations behaved very similarly to rational expectations while type-II (and type-III) experienced-based-expectations produced substantial departures from rational expectations, an ongoing period during which inflation and the nominal interest rate were low. The differences between type-I and type-II experienced-based-expectations were due to their differing assumptions in regard to people's understanding of the model. Our second two experiments involved people experiencing periods of unusually loose monetary policy shocks and unusually adverse technology shocks, respectively. For each of these experiments we found that type-I experienced-based-expectations produced substantial and sustained deviations from rational expectations, not unlike scarring. These same departures did not occur with type-II experienced-based-expectations because the adverse shocks do not change the model's underlying structure. We further showed that when expectations are formed using a shorter experience set that the departures from rational expectations are initially larger, but dissipate more quickly as time passes.

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Appendix A: Density estimation

This appendix describes methods that can be used to estimate densities and conditional densities from a sample. Our intention is to simply document these methods and show how they can be used, not to derive them or establish by proof their theoretical foundation. We illustrate these methods using data simulated from a simple univariate AR(1) process, extending the results presented in Section 2.

Kernel density estimation

A standard method for approximating a density is to use kernel density estimation (Rosenblatt (1956) and Parzan (1962)), which involves building up a density by assigning a little bit of mass around each observation in a sample. The amount of mass that is assigned is governed by the kernel, which is itself a density, and the region around an observation that receives mass is governed by the bandwidth. For a given kernel, the probability estimate is given by:

$$p(y) = \frac{1}{n}\sum_{i=1}^{n} K_h(y - y_i), \tag{A1}$$

where $K_n(y - y_i)$ is the kernel and h is the bandwidth or smoothing parameter. A very common kernel, the one we use in this paper, is the Gaussian kernel:

$$K_h(y - y_i) = \frac{1}{\sqrt{2\pi}h} e^{-\frac{1}{2}\left(\frac{y-y_i}{h}\right)^2}. \tag{A2}$$

To approximate the conditional density we use linear regression to estimate the conditional mean and then overlay this conditional mean with the kernel-based density constructed from the residuals. We determine the bandwidth, h, according to the plug-in formula from Silverman (1986). The consistency of equation (A2), which ensures that the kernel estimated density converges to rational expectations as the experience set gets longer, is proven in Parzan (1962).

An alternative to estimate the conditional density is to use the Nadaraya-Watson (Nadaraya (1964) and Watson (1964)) kernel regression estimator:

$$p(y|x) = \frac{\sum_{i=1}^{n} K_{h_y}(y-y_i)K_{h_x}(x-x_i)}{\sum_{i=1}^{n} K_{h_x}(x-x_i)}. \tag{A3}$$

However, we found this estimator to be quite sensitive to individual observations and to the choice of bandwidths, h_y and h_x. In the figure below we use the Sheather and Jones (1991) method to determine these bandwidths.

Fourier series approximation

An alternative to kernel density estimation is to use Fourier series methods, as described in Kronmal and Tarter (1968), which use cosine approximation. With this cosine approximation the probability approximated using a m'th order polynomial is given by:

$$p_m(y) = \frac{\bar{c}_0}{2} + \sum_{k=1}^{m} \bar{c}_k cos\left(k\pi \frac{y-a}{b-a}\right), \tag{A4}$$

where $y \in [a,b]$, and the coefficients are determined according to:

$$\bar{c}_k = \frac{2}{(b-a)n} \sum_{i=1}^{n} cos\left(k\pi \frac{y_i - a}{b-a}\right). \tag{A5}$$

Just as for kernel density estimation, we can approximate the conditional density by using linear regression to capture the conditional mean and then overlay that mean with the cosine-approximated density produced from the residuals. The consistency of equation (A4), which ensures that the density estimated using Fourier series converges to rational expectations as the experience set gets longer, is proven in Kronmal and Tarter (1968).

We found the kernel density methods and the Fourier series methods to give very similar densities, but preferred to use the kernel methods because they are guaranteed to produce non-negative probabilities and we found them numerically faster.

Figure 9 below repeats the exercise undertaken in Section 2, but adds the Fourier series approximation to the panel on unconditional density approximation and the Fourier series approximation and the Nadaraya-Watson estimator to the panel on conditional density approximation.

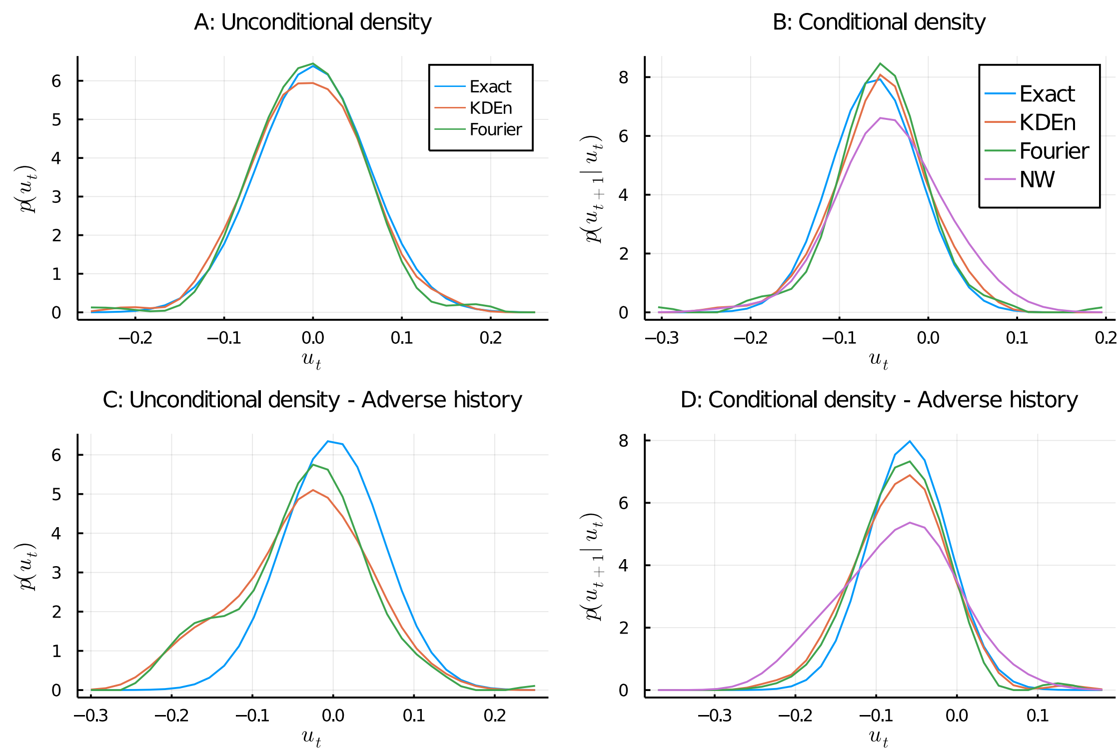

Figure 9: Densities under rational expectations and experienced-based-expectations

Appendix B: The model

This appendix presents the decision problems for households and firms and derives the model's aggregated first-order conditions that are presented in the main text. The notation follows the main text.

Household's problem

The decision problem for the representative household is to choose $\{c_t, h_t, b_{t+1}\}_{t=0}^{\infty}$ to maximize:

$$U_t = E_t \left[\sum_{t=0}^{\infty} \beta^t \left(\frac{(c_t - \gamma C_{t-1})^{1-\sigma}}{1-\sigma} - \mu \frac{h_t^{1+\chi}}{1+\chi} \right) \right],$$

where $\sigma > 0$, $\chi > 0$, $\mu > 0$, $0 < \beta < 1$, and $0 < \gamma < 1$, subject to the flow budget constraint:

$$c_t + Q_t \frac{b_{t+1}}{P_t} = w_t h_t + \frac{b_t}{P_t} + D_t.$$

The Lagrangian for the problem is:

$$\mathcal{L}_t = E_t\left[\sum_{t=0}^{\infty}\beta^t\left(\frac{(c_t-\gamma C_{t-1})^{1-\sigma}}{1-\sigma}-\mu\frac{h_t^{1+\chi}}{1+\chi}+\lambda_t\left(w_t h_t+\frac{b_t}{P_t}+D_t-c_t-Q_t\frac{b_{t+1}}{P_t}\right)\right)\right],$$

where λ_t is a Lagrange multiplier, and the first-order conditions are:

$$\begin{aligned}
\frac{\partial \mathcal{L}_t}{\partial c_t} &: \quad (c_t-\gamma C_{t-1})^{-\sigma}-\lambda_t=0,\\
\frac{\partial \mathcal{L}_t}{\partial h_t} &: \quad \mu h_t^{\chi}-w_t\lambda_t=0,\\
\frac{\partial \mathcal{L}_t}{\partial b_{t+1}} &: \quad -\frac{Q_t}{P_t}\lambda_t+\beta E_t\left[\frac{\lambda_{t+1}}{P_{t+1}}\right]=0.
\end{aligned}$$

After substituting out the Lagrange multiplier, we obtain:

$$\begin{aligned}
\mu h_t^{\chi} &= w_t\left(c_t-\gamma C_{t-1}\right)^{-\sigma},\\
(c_t-\gamma C_{t-1})^{-\sigma} &= \beta E_t\left[\frac{(c_{t+1}-\gamma C_t)^{-\sigma}}{Q_t\left(1+\pi_{t+1}\right)}\right].
\end{aligned}$$

Firm's problem

With all firms hiring labor from a perfectly competitive market at real wage w_t, the cost minimization problem for the i'th firm, $i\in[0,1]$, is given by the Lagrangian:

$$\mathcal{L}(i) = w_t h_t(i)+\omega_t(i)\left(y_t(i)-e^{z_t}h_t(i)\right).$$

The first order conditions are:

$$\begin{aligned}
\frac{\partial \mathcal{L}_t(i)}{\partial h_t(i)} &: \quad w_t-e^{z_t}\omega_t(i)=0,\\
\frac{\partial \mathcal{L}_t(i)}{\partial \omega_t(i)} &: \quad y_t(i)-e^{z_t}h_t(i)=0,
\end{aligned}$$

which imply that real marginal costs for the i'th firm, $\omega_t(i)$, in equilibrium satisfy:

$$\omega_t(i)=\frac{w_t}{e^{z_t}},$$

and are the same for all firms.

To determine the firm's optimal price, the decision problem for the i'th firm is to choose $\{p_t(i)\}_{t=0}^{\infty}$ to maximize its equity-value:

$$\Gamma_t(i)=E_t\left[\sum_{t=0}^{\infty}\left(\beta^t\frac{\lambda_t}{\lambda_0}\left(\left(p_t(i)-\omega_t\right)p_t(i)^{-\epsilon}Y_t-\frac{\phi}{2}\left(\frac{p_t(i)\left(1+\pi_t\right)}{p_{t-1}(i)}-1\right)^2 Y_t\right)\right)\right].$$

Differentiating with respect to $p_t(i)$, the first-order condition is:

$$\frac{\partial \Gamma_t(i)}{\partial p_t(i)} : (1-\epsilon)\, p_t(i)^{-\epsilon} Y_t + \epsilon \omega_t p_t(i)^{-(1+\epsilon)} Y_t - \phi \left(p_t(i)^{1-\epsilon} \left(\frac{p_t(i)\,(1+\pi_t)}{p_{t-1}(i)} - 1 \right) \left(\frac{1+\pi_t}{p_{t-1}(i)} \right) Y_t \right)$$
$$: +\phi \mathrm{E}_t \left[\frac{\lambda_{t+1}}{\lambda_t} p_{t+1}(i)^{1-\epsilon} Y_{t+1} \left(\frac{p_{t+1}(i)\,(1+\pi_{t+1})}{p_t(i)} - 1 \right) \left(\frac{p_{t+1}(i)\,(1+\pi_{t+1})}{p_t(i)^2} \right) \right] = 0,$$

which simplifies to:

$$(1-\epsilon)\, Y_t + \epsilon \omega_t Y_t - \phi \pi_t \,(1+\pi_t)\, Y_t + \phi \mathrm{E}_t \left[\frac{\lambda_{t+1}}{\lambda_t} Y_{t+1} \pi_{t+1} \,(1+\pi_{t+1}) \right] = 0,$$

once we recognize that in a symmetric equilibrium $p_t(i) = 1$, $\forall\ t,\ i$.

Aggregation across households and firms

From the household's problem we obtain the aggregate expressions:

$$\mu H_t^{\chi} = w_t \,(C_t - \gamma C_{t-1})^{-\sigma}, \tag{B1}$$

$$(C_t - \gamma C_{t-1})^{-\sigma} = \beta E_t \left[\frac{(C_{t+1} - \gamma C_t)^{-\sigma}}{Q_t \,(1+\pi_{t+1})} \right], \tag{B2}$$

$$C_t = Y_t, \tag{B3}$$

and from the firm's problem we obtain, after aggregation:

$$Y_t = e^{z_t} H_t, \tag{B4}$$

$$\omega_t = \frac{w_t}{e^{z_t}}, \tag{B5}$$

$$\pi_t \,(1+\pi_t) = \frac{1-\epsilon}{\phi} + \frac{\epsilon}{\phi} \omega_t + \beta E_t \left[\frac{(C_{t+1} - \gamma C_t)^{-\sigma} Y_{t+1} \pi_{t+1} \,(1+\pi_{t+1})}{(C_t - \gamma C_{t-1})^{-\sigma} Y_t} \right]. \tag{B6}$$

Equation (B4) corresponds to equation (30) in the text. Substituting equation (B3) into equation (B2) and noting the relationship between the nominal bond price and its yield, equation (26), gives equation (28) in the text. Substituting equations (B1) and (B3) into equation (B5) gives equation (32) in the text. Finally, substituting equation (B3) into (B6) gives equation (31) in the text.

Appendix C: Model solution

Here we briefly summarize how the model is solved under each of the expectations processes.

Rational expectations

Our method for solving the model under rational expectations is very standard. The model has three state variables. Decisions rules are approximated using Chebyshev polynomials (5'th order in all three variables) and the integrals associated with conditional expectations are approximated using Gauss-Hermite quadrature (9 quadrature points). The domain for each of the shocks is taken to be plus/minus three unconditional standard deviations while that for output is its nonstochastic steady state $\pm$ 0.2 (which allows the same domain to be used for all experiments).

For convenience, we define:

$$C_{t+1}^{*} = \frac{(C_{t+1} - \gamma C_t)^{-\sigma}}{(1 + \pi_{t+1})}, \tag{C1}$$

$$\pi_{t+1}^{*} = (C_{t+1} - \gamma C_t)^{-\sigma} Y_{t+1} \pi_{t+1} (1 + \pi_{t+1}). \tag{C2}$$

Type-I

With type-I experienced-based-expectations the solution method parallels that for rational expectations above. Decision rules are approximated using Chebyshev polynomials and quadrature is used to approximate expectations. However, while the Gauss-Hermite nodes are employed for this quadrature (again, 9 quadrature points) the weights associated with each quadrature point are determined using a (normal) kernel density estimator, as described in Appendix A. The order for the Chebyshev polynomials and the domain for the state variables are the same as those documented earlier for rational expectations.

Type-II

Determining outcomes with type-II experienced-based-expectations involves the straightforward application of a non-linear equation solver. Given the observed state and the sample of outcomes available, expectations in the consumption-Euler equation ($E_t^{II}\left[C_{t+1}^{*}\right]$) and the Phillips curve ($E_t^{II}\left[\pi_{t+1}^{*}\right]$) are formed by applying the (normal) kernel density estimator described in Appendix A to the variables in equations (C1) and (C2). Then the following non-linear system is solved:

$$\begin{aligned}
\mu H_t^{\chi} &= w_t \left(C_t - \gamma C_{t-1}\right)^{-\sigma}, \\
\left(C_t - \gamma C_{t-1}\right)^{-\sigma} &= \beta \frac{E_t^{II}\left[C_{t+1}^{*}\right]}{Q_t}, \\
C_t &= Y_t, \\
Y_t &= e^{z_t} H_t, \\
\omega_t &= \frac{w_t}{e^{z_t}}, \\
\pi_t \left(1+\pi_t\right) &= \frac{1-\epsilon}{\phi} + \frac{\epsilon}{\phi}\omega_t + \beta \frac{E_t^{II}\left[\pi_{t+1}^{*}\right]}{\left(C_t - \gamma C_{t-1}\right)^{-\sigma} Y_t}.
\end{aligned}$$

Type-III

Much like type-II experienced-based-expectations, outcomes with type-III experienced-based-expectations are determined using a non-linear equation solver. Given the observed state and the sample of outcomes available, simple averages are taken of the variables in equations (C1) and (C2) to form the expectations in the consumption-Euler equation ($E_t^{III}\left[C_{t+1}^{*}\right]$) and the Phillips curve ($E_t^{III}\left[\pi_{t+1}^{*}\right]$). Then current-period outcomes are obtained by solving the non-linear system:

$$\begin{aligned}
\mu H_t^{\chi} &= w_t \left(C_t - \gamma C_{t-1}\right)^{-\sigma}, \\
\left(C_t - \gamma C_{t-1}\right)^{-\sigma} &= \beta \frac{E_t^{III}\left[C_{t+1}^{*}\right]}{Q_t}, \\
C_t &= Y_t, \\
Y_t &= e^{z_t} H_t, \\
\omega_t &= \frac{w_t}{e^{z_t}}, \\
\pi_t \left(1+\pi_t\right) &= \frac{1-\epsilon}{\phi} + \frac{\epsilon}{\phi}\omega_t + \beta \frac{E_t^{III}\left[\pi_{t+1}^{*}\right]}{\left(C_t - \gamma C_{t-1}\right)^{-\sigma} Y_t}.
\end{aligned}$$

Appendix D: Expectations and habit formation

In the Appendix we demonstrate that the spikes observed in labor, inflation, and the nominal interest rate in Figure 6 are due to habit formation, consistent with the explanation given in the text. We demonstrate this by resolving the model and recreating the dynamics following sequences of loose monetary policy shocks and adverse technology shocks, but with γ, the habit formation parameter, set to zero. The results from these simulations are shown below in Figures 10 and 11.

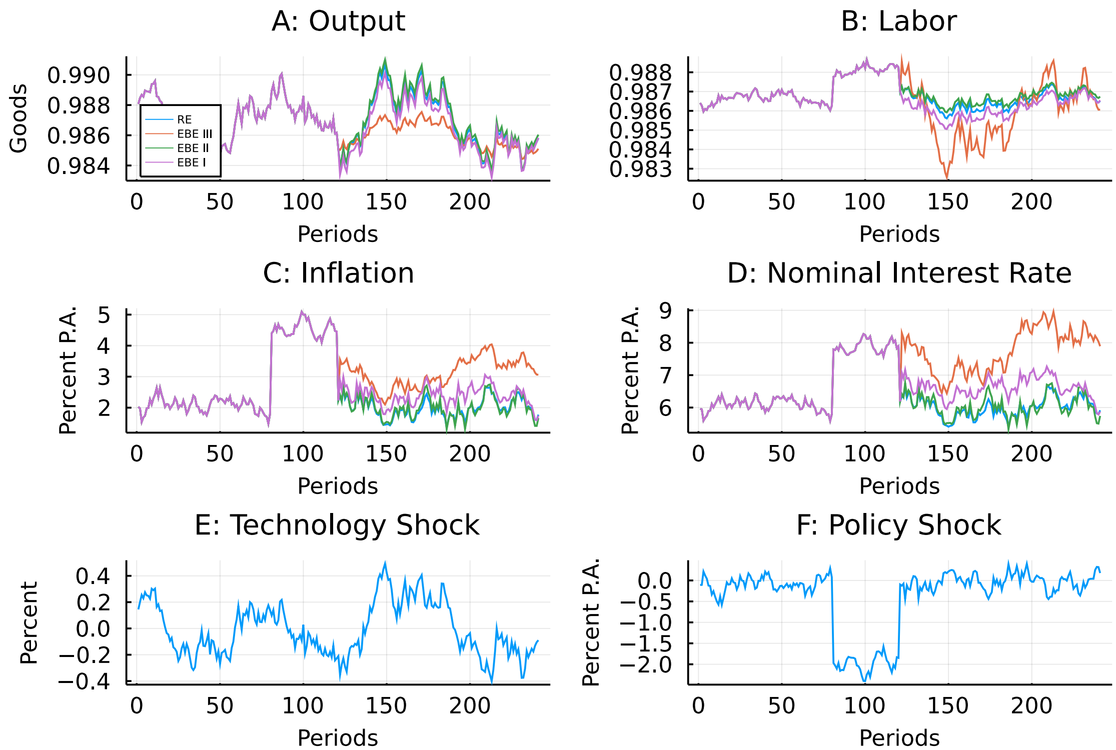

Figure 10: Temporarily adverse aggregate policy shocks with $\gamma = 0.0$.

Comparing Figure 10 to Figure 6 it is apparent that habit formation has relatively little impact on how experienced-based-expectations affect the model following a sequence of loose monetary policy shocks. The same cannot be said for the case where there is an adverse sequence of technology shocks. Relative to Figure 7, Figure 11 does not display the spikes to labor, inflation, and the nominal interest rate seen in Figure 7, demonstrating the those observed spikes are due to the quasi-differencing introduced by habit formation.

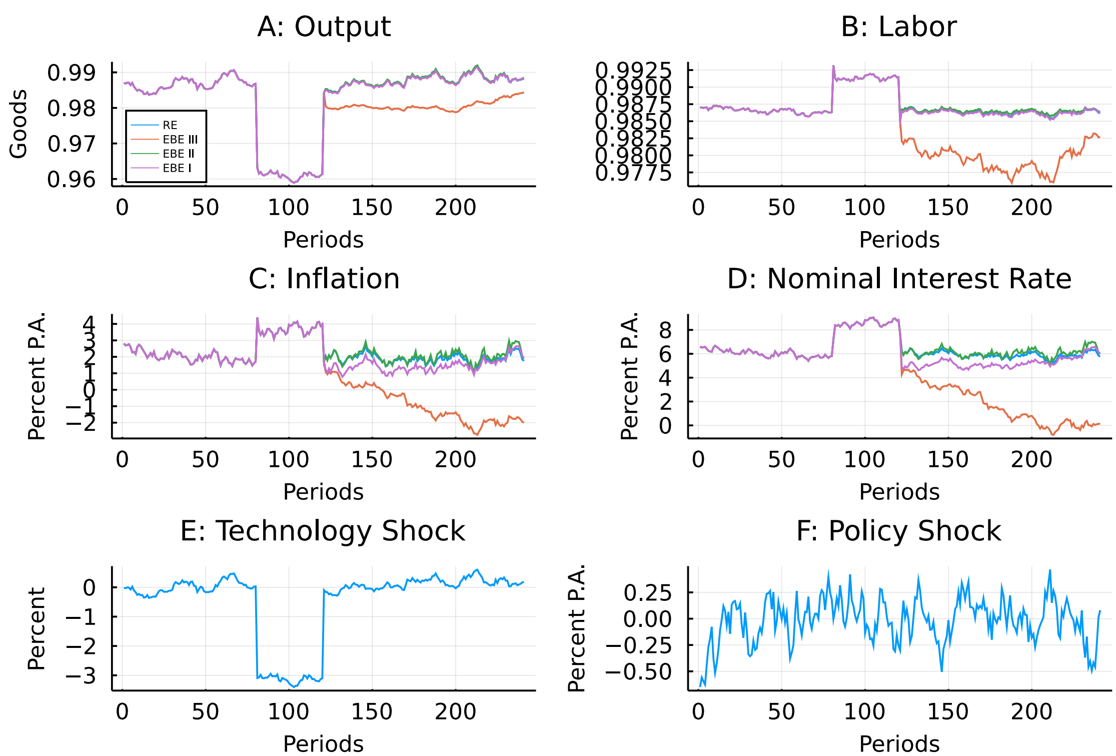

Figure 11: Temporarily adverse aggregate technology shocks with $\gamma = 0.0$.