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#### **Keywords**

Optimal simple rules, welfare analysis, monetary policy, dual mandate

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# Welfare gains in a small open economy with a dual mandate for monetary policy\*

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#### **Abstract**

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#### 1 Introduction

Almost three decades after adopting inflation targeting as its principal objective, the monetary policy framework of the Reserve Bank of New Zealand (RBNZ) underwent a paradigm shift in March 2019. The new Monetary Policy Remit lays out the overarching goals of the Monetary Policy Committee as "...maintaining a stable general level of prices over the medium term and supporting maximum sustainable employment." (RBNZ, 2019). Following this historic amendment to the legislation, the RBNZ joined the Federal Reserve Bank of the United

<sup>\*</sup>The views expressed herein are those of the authors, and do not necessarily represent the official views of the Reserve Bank of New Zealand or the New Zealand Treasury. We have benefitted from insightful comments by James Graham, Vivien Lewis, Nicholas Sander, Dennis Wesselbaum as well as participants at the Reserve Bank-Treasury workshop on 'Fiscal and monetary policy in the wake of COVID' in June 2021.

States and the Reserve Bank of Australia as the only central banks to be entrusted with a dual mandate for monetary policy: an employment stabilisation objective coupled with the traditional price stability goal.<sup>1</sup>

This paper demonstrates that the second monetary policy objective of stabilising the labour market - or in broader terms, resource utilisation - works to the benefit of society. Using a medium-scale open-economy general equilibrium model calibrated to New Zealand data, we find that additionally stabilising resource utilisation improves social welfare at any given level of inflation stabilisation. The welfare gains from stabilising resource utilisation are particularly striking when the response to inflation is weak. The second highlight of our findings is that monetary policy responses to the labour market variables such as the employment gap or the unemployment rate gap, that are more in line with the activity indicators stipulated in actual central bank legislation, yield better welfare outcomes than reacting to the output gap. However, within the range of monetary policy settings we consider, the welfare gains from stabilising resource utilisation decline to mild levels when the central bank is already very sensitive to inflation.

The social desirability of stabilising resource utilisation is robust across a wide variety of model specifications. The only exception is when the economy is driven solely by powerful cost-push shocks to price and wage inflation that generate monetary policy trade-offs between stabilising inflation and stabilising resource stabilisation. In this extreme case, we find that the stabilisation of resource utilisation diminishes welfare, while strict inflation targeting improves welfare. However, in the overall picture, when the business cycle is driven by a wider range of demand-type and supply-type disturbances, the additional monetary policy response to resource utilisation implied by the dual mandate enhances social welfare.

Our main findings for the open economy parallel the results of Debortoli, Kim, Lindé, and Nunes (2019) who establish that placing a high weight on activity stabilisation in the central bank's loss function is optimal in the closed-economy model of Smets and Wouters (2007) for the United States. Apart from the open-economy dimension of our model, our contribution is quite distinct from their work in several other aspects. The legislative foundations for monetary policy formulation in the United States, Australia and New Zealand direct the central bank to target employment, rather than aggregate output which is the focus of Debortoli et al. (2019). We explicitly model equilibrium unemployment. Finally, Debortoli et al. (2019) adopt a linear-quadratic approach which involves the minimisation of a reduced-form central bank loss function defined as a weighted combination of the volatilities of inflation and economic activity, subject to a linear economy. In contrast, our assessment of welfare is in the tradition established by Kollmann (2002) and Schmitt-Grohé and Uribe (2007). Using a second-order

<sup>&</sup>lt;sup>1</sup>In the case of the United States, since 1977, the Federal Reserve has operated under a mandate from Congress to "...promote effectively the goals of maximum employment, stable prices, and moderate long term interest rates" (Steelman, 2011). In section 10(2) of the Reserve Bank Act 1959, the monetary policy mandate of the Reserve Bank of Australia is to "...best contribute to: a. the stability of the currency of Australia; b. the maintenance of full employment in Australia; and c. the economic prosperity and welfare of the people of Australia." (Reserve Bank of Australia, 2020). On the other hand, in Sweden, the Sveriges Riksbank Act 1988 states that the objective of the Riksbank's monetary policy shall be to maintain price stability. The government bill that introduced the Act states that "...the Riksbank shall also, without prejudice to the price-stability objective, support the objectives of the general economic policy with the aim to achieve sustainable growth and high employment". The Monetary Policy Report of the Riksbank interprets the monetary policy mandate as 'flexible inflation targeting' (Svensson, 2014).

approximation of the model around the non-stochastic steady state, we evaluate how the household's expected lifetime utility is affected by a set of simple and implementable monetary policy rules; specifically, by systematically varying the response coefficients on inflation and resource utilisation. Thus, unlike Debortoli et al. (2019), the welfare criterion that we adopt is household utility rather than a reduced-form central bank loss function.<sup>2</sup> However, despite these numerous differences with the framework of Debortoli et al. (2019), the flavour of the results remains similar: the stabilisation of resource utilisation - be it the employment gap, the unemployment rate gap, or the output gap - by the central bank is beneficial for society.

In modelling unemployment in the small open economy, our framework is similar to that of Bodenstein, Kamber, and Thoenissen (2018) and Christiano, Trabandt, and Walentin (2011). These authors use variants of the search-and-matching frictions in the tradition of Mortensen and Pissarides (1994) to model unemployment. Unlike them, we choose the Galí (2011) specification wherein nominal wage frictions give rise to equilibrium unemployment. Corbo and Strid (2020) opt for a similar strategy in their estimated small open economy model for Sweden. It is important to note that the other papers mentioned in this context are positive in spirit, and hence use model solutions based on a first order approximation around the steady state. In contrast, our paper assesses social welfare and as a consequence, relies on the second order approximation of the model which, *ceteris paribus*, incorporates a much more extensive model solution than the first order approximation. Since our small open economy model already incorporates a rich array of other endogenous frictions, the parsimony of the Galí (2011) specification of unemployment makes it particularly appealing.

Another aspect of our paper that links it to Kollmann (2002), besides the methodological similarities, is the focus on the open economy. However, he considers a more limited set of structural disturbances in a relatively smaller model whereas ours is a medium-scale DSGE model that is a prototype of models used in central banks, and hence, is augmented with a wider array of frictions and structural shocks. Kollmann (2002) also does not model unemployment, and specifies only detrended output as the measure of resource utilisation in the monetary policy rule. Since we model unemployment in our framework, we are able to compare the performances of a triad of welfare-relevant measures of resource utilisation; the gaps between output, employment or the unemployment rate and their analogues in a counterfactual world without nominal rigidities.

Analysing the design of monetary policy using a second order approximation of the model, also distinguishes our paper from a vast open-economy literature that instead adopts the linear-quadratic approach. Some papers in this literature determine the monetary policy settings that optimise a second order approximation of the utility function subject to linearised competitive equilibrium conditions, as opposed to a second order approximation, *e.g.* Corsetti, Dedola, and Leduc (2020, 2010), Fujiwara and Wang (2017) and De Paoli (2009). Other authors minimise a reduced-form loss function that involves weighted combinations

<sup>&</sup>lt;sup>2</sup>The part of the optimal policy analysis by Debortoli et al. (2019) that uses the estimated medium-scale model of Smets and Wouters (2007) relies on the reduced-form loss function. They also demonstrate the relevance of their main result in the context of a second order approximation of household utility in a simplified version of the model, again using the linear-quadratic framework.

of the volatilities of target variables, again subject to a linearised economy, *e.g.* Jacob and Munro (2018), Adolfson et al. (2014) and Justiniano and Preston (2010).

The rest of the paper will unfold as follows. In the next section, we lay out the structure of the small open economy, and also explain our strategies for solving the model. In Section 3, we present the welfare implications of altering the central bank responses to resource utilisation and inflation. Section 4 evaluates the robustness of our baseline results, and highlights the extreme case in which powerful cost-push shocks drive the business cycle. We summarise our conclusions in Section 5.

## 2 A small open economy model with unemployment

The world consists of two countries, the home country being infinitesimally small when compared to the foreign country. We will refer to the home country as a small open economy (SOE) and we will focus on deriving the optimality conditions for this region. The labour market is set up in the spirit of Galí, Smets, and Wouters (2012) and Galí (2011). The openeconomy dimension of the model is based on Adolfson, Laséen, Lindé, and Villani (2007). Hence, the exposition for these segments of the model closely follows that of the original papers. As in Christiano, Trabandt, and Walentin (2011) and Mandelman, Rabanal, Rubio-Ramirez, and Vilán (2011), the model incorporates permanent neutral and investment-specific technical progress emanating from stochastic sources, and the model equations are later stationarised around a balanced growth path. The subscript t denotes the time period between dates t and t+1, and  $\mathbb{E}_t$  denotes the conditional expectations operator. The steady-state values of the variables are denoted by an upper bar. Variables in the counterfactual economy without nominal rigidities are indicated using a circumflex $\hat{\cdot}$ . The comprehensive derivation of the model is presented in Jacob and Özbilgin (2021).

#### 2.1 Production of intermediate goods

Three categories of firms operate in the SOE; domestic, importing and exporting firms. The intermediate domestic firms produce a differentiated good, using capital and labour inputs, which they sell to a final good producer who uses a continuum of these intermediate goods in her production. The importing firms, in turn, transform a homogenous good, bought in the world market, into a differentiated import good, which they sell to the domestic households. The exporting firms pursue a similar scheme. The exporting firms buy the domestic final good and differentiate it by brand naming. Each exporting firm is thus a monopoly supplier of its specific product in the world market.

#### 2.1.1 Domestic firms

The domestic firms are of two types. The first buys the aggregated labour (N) from the competitive employment agencies at the nominal wage (W) and rents the capital stock (K(s)) from the household at the nominal rate  $(R^k)$  to produce intermediate goods (Y(s)) which it sells to a final goods producer at a price  $(P_d(s))$ . There is a continuum indexed by  $s \in [0,1]$  of

these intermediate goods producers, each of which is a monopoly supplier of its own good and is competitive in the markets for production inputs. The second type of firm transforms the intermediate products into a homogenous final good, which is used for consumption and investment by the households.

The production function of the final goods firm takes the form

$$Y_{t} = \left[ \int_{0}^{1} Y_{t}(s)^{(\xi_{d,t}-1)/\xi_{d,t}} \partial s \right]^{\xi_{d,t}/(\xi_{d,t}-1)}, \ \xi_{d,t} > 1$$
 (1)

where  $\xi_d$  is a time-varying demand elasticity that follows the law of motion  $\xi_{d,t} = \bar{\xi}_d^{1-\rho_{pd}} \xi_{d,t-1}^{\rho_{pd}} \exp \vartheta_t^{pd}, \ \rho_{pd} \in (0,\ 1)\ , \ \vartheta_t^{pd} \sim \text{i.i.d.}\ \mathcal{N}(0,\sigma_{pd})\ .$  Profit-maximisation by the final goods firm leads to the following first order condition

$$Y_t(s) = \left[\frac{P_{d,t}(s)}{P_{d,t}}\right]^{-\xi_{d,t}} Y_t, \tag{2}$$

where  $P_{d,t} = \left[ \int_0^1 P_{d,t} \left( s \right)^{1-\varsigma_{d,t}} \partial s \right]$ .

The intermediate goods firm *s* rents capital and labour from the household at the and combines the two factors in a constant returns to scale Cobb-Douglas aggregate to produce output.

$$Y_t(s) = \varepsilon_t^{neu} \left[ K_{t-1}(s) \right]^{\alpha} \left[ \varepsilon_t^a N_t(s) \right]^{1-\alpha}$$
(3)

 $\alpha \in [0,1]$  governs the share of capital in the production function.  $\varepsilon^a$  is labour-augmenting technological progress and its growth rate, defined as  $\gamma_t^a \equiv \varepsilon_t^a/\varepsilon_{t-1}^a$ , evolves according to  $\gamma_t^a = (\bar{\gamma}^a)^{1-\rho_a} (\gamma_{t-1}^a)^{\rho_a} \exp \vartheta_t^a, \rho_a \in (0, 1), \ \vartheta_t^a \sim i.i.d. \ \mathcal{N}(0, \sigma_a).$  The production function is also stimulated by a stationary factor-neutral technology shock,  $\varepsilon^{neu}$  , which follows  $\varepsilon^{neu}_t =$  $(\bar{\varepsilon}^{neu})^{1-\rho_{neu}}(\varepsilon_{t-1}^{neu})^{\rho_{neu}}\exp\vartheta_t^{neu},\rho_{neu}\in(0,1),\ \vartheta_t^{neu}\sim i.i.d.\ \mathcal{N}(0,\sigma_{neu})$ . Nominal rigidities in price-setting are introduced using quadratic adjustment costs à la Rotemberg (1982); profits are penalised when the change in the firm's prices is different from a combination of lagged aggregate inflation and steady-state inflation. Our decision to embed Rotemberg-type costs instead of using staggered price-setting in the style of Calvo (1983) to model nominal rigidities is motivated by the presence of time-varying price elasticities in Equation 2 that generate exogenous movements in the price mark-up. However, as noted by Andreasen (2012), a time-varying price elasticity or mark-up precludes an exact recursive formulation of the Calvo price-setting equations necessary for the non-linear approximation of the model. The Rotemberg (1982) specification is particularly convenient for us, given its compatibility with time-varying elasticities of goods and labour demand in the non-linear model solution. Importantly, as we will see in Section 4, the presence of time-varying elasticities that act as cost-push shocks to price and wage inflation have important implications for the monetary policy mandate.<sup>3</sup>

<sup>&</sup>lt;sup>3</sup>Exogenous time-variation in price and wage elasticities are also found to be important driving forces of price and wage inflation in standard empirical DSGE models, *e.g.* Smets and Wouters (2007) and Adolfson et al. (2007). Alternatively, as in Andreasen (2012), we could use Calvo-style price-setting by avoiding price elasticity shocks and instead, introducing shocks to fixed costs that are embedded in the production function. However, in that case, we would not have analogous shocks to the wage-setting problem detailed in Section 2.1.1; while Andreasen

The firm faces the following profit maximisation programme:

$$\max_{\substack{K_{t-1}(s), N_{t}(s) \\ P_{d,t}(s)}} \mathbb{E}_{t} \sum_{\tau=0}^{\infty} \beta^{\tau} \frac{\Lambda_{t+\tau}}{P_{c,t+\tau}} \begin{pmatrix} P_{d,t+\tau}(s) Y_{t+\tau}(s) - W_{t+\tau} N_{t+\tau}(s) - R_{t+\tau}^{k} K_{t+\tau}(s) \\ -\frac{\chi_{d}}{2} \left( \frac{P_{d,t+\tau}(s)}{\bar{\tau}_{d}^{1-\iota_{d}} \bar{\tau}_{d,t+\tau-1}^{\iota_{d}} P_{d,t+\tau-1}(s)} - 1 \right)^{2} P_{d,t+\tau} Y_{t+\tau} \end{pmatrix}$$

subject to the production technology constraint in Equation 3 and the demand constraint in Equation 2.  $\Lambda$  is the marginal utility of income,  $P_c$  is the price of consumption and  $\beta \in (0,1)$  is the subjective discount factor. The parameters  $\chi_d \geq 0$  measures the degree of costly price adjustment and  $\iota_d \in [0,1]$  indicates the degree of indexation to lagged inflation. We will focus on the optimality conditions in a symmetric equilibrium where all firms set the same prices. The optimal choice of labour and capital input yield

$$W_{d,t}N_t = (1 - \alpha)RMC_{d,t}Y_t,\tag{4}$$

and

$$R_{d,t}^k K_{t-1} = \alpha R M C_{d,t} Y_t, \tag{5}$$

where  $W_d$  and  $R_d^k$  are the respective factor payments deflated by the output deflator  $P_d$ . The output-based real marginal cost is given as

$$RMC_{d,t} = \frac{1}{\varepsilon_t^{neu} \alpha^{\alpha} (1 - \alpha)^{1 - \alpha}} \left( R_{d,t}^k \right)^{\alpha} \left( \frac{W_{d,t}}{\varepsilon_t^a} \right)^{1 - \alpha}.$$
 (6)

Optimal price-setting implies that movements in the real marginal cost reflect in movements in price inflation  $\pi_d$ , with the price adjustment cost parameter moderating the strength of the relationship:

$$\frac{\pi_{d,t}}{\bar{\pi}_{d}^{1-\iota_{d}} \pi_{d,t-1}^{\iota_{d}}} \chi_{d} \left( \frac{\pi_{d,t}}{\bar{\pi}_{d}^{1-\iota_{d}} \pi_{d,t-1}^{\iota_{d}}} - 1 \right) = (1 - \xi_{d,t}) + \xi_{d,t} RM C_{d,t} 
+ \beta \mathbb{E}_{t} \frac{\Lambda_{t+1}}{\Lambda_{t} \pi_{c,t+1}} \frac{Y_{t+1}}{Y_{t}} \frac{\pi_{d,t+1}^{2}}{\bar{\pi}_{d}^{1-\iota_{d}} \pi_{d,t}^{\iota_{d}}} \chi_{d} \left( \frac{\pi_{d,t+1}}{\bar{\pi}_{d}^{1-\iota_{d}} \pi_{d,t}^{\iota_{d}}} - 1 \right).$$
(7)

#### 2.1.2 Importing firms

The import sector consists of firms that buy a homogenous good in the world market at the foreign-currency price  $P^*$ . The imported good is absorbed by aggregator firms that produce the final consumption and investment imported goods that are sold to the household. The aggregator firms face the following production functions:

$$C_{m,t} = \left[ \int_0^1 C_{m,t}(s)^{(\xi_{m,t}-1)/\xi_{m,t}} \, \partial s \right]^{\xi_{m,t}/(\xi_{m,t}-1)} \tag{8}$$

(2012) abstracts from nominal wage rigidities, wage-setting frictions are crucial for introducing unemployment into our model. Quadratic adjustment costs allows us to model price- and wage-setting frictions symmetrically, even while retaining conventional shocks understood to be important drivers of the business cycle.

and

$$I_{m,t} = \left[ \int_{0}^{1} I_{m,t} (s)^{(\xi_{m,t}-1)/\xi_{m,t}} \partial s \right]^{\xi_{m,t}/(\xi_{m,t}-1)}, \ \xi_{m,t} > 1$$
 (9)

where  $\xi_m$  is a time-varying demand elasticity that follows the law of motion  $\xi_{m,t} = \bar{\xi}_m^{1-\rho_{pm}} \xi_{m,t-1}^{\rho_{pm}} \exp \vartheta_t^{pm}, \; \rho_{pm} \in (0,\,1)\,,\; \vartheta_t^{pm} \sim i.i.d.\,\mathcal{N}(0,\sigma_{pm})\,.$  Profit-maximisation by the perfectly competitive aggregator firms for consumption and investment imports yield the following first order conditions:

$$C_{m,t}\left(s\right) = \left\lceil \frac{P_{m,t}\left(s\right)}{P_{m,t}} \right\rceil^{-\xi_{m,t}} C_{m,t},\tag{10}$$

and

$$I_{m,t}(s) = \left[\frac{P_{m,t}(s)}{P_{m,t}}\right]^{-\xi_{m,t}} I_{m,t},$$
(11)

where  $P_{m,t} = \left[ \int_0^1 P_{m,t}(s)^{1-\xi_{m,t}} \, \partial s \right]$ . The profit maximisation programme for the monopolistic intermediate goods importer is given by

$$\max_{P_{m,t}(s)} \mathbb{E}_{t} \sum_{\tau=0}^{\infty} \beta^{\tau} \frac{\Lambda_{t+\tau}}{P_{c,t+\tau}} \begin{bmatrix} P_{m,t+\tau}(s) C_{m,t+\tau}(s) + P_{m,t+\tau}(s) I_{m,t+\tau}(s) \\ -\frac{\chi_{m}}{2} \left( \frac{P_{m,t+\tau}(s)}{\bar{\pi}_{m}^{1-\iota m} \pi_{m,t+\tau-1}^{\iota m} P_{m,t+\tau-1}(s)} - 1 \right)^{2} P_{m,t+\tau} \left( C_{m,t+\tau} + I_{m,t+\tau} \right) \\ -ner_{t+\tau} P_{t+\tau}^{*} C_{m,t+\tau}(s) - ner_{t+\tau} P_{t+\tau}^{*} I_{m,t+\tau}(s) \end{bmatrix}$$

$$\chi_{m} \geq 0, \, \iota_{m} \in [0,1],$$

subject to the demand constraints in Equations 10 and 11. On the revenue side, the importer's profits are affected by the consumption and investment goods sold to the household at the home currency-denominated import price. On the other hand, import costs are affected by the nominal exchange rate ner that is defined as the home currency price of one unit of foreign currency as the procurement price of the good is denominated in foreign currency.<sup>4</sup> In addition, profits are influenced by nominal import price rigidities that are modelled using quadratic adjustment costs, analogous to the case of domestic sales pricing that we presented earlier. We further define the output-based real exchange rate as  $rer_d \equiv ner P_d/P^*$  and the relative price of imports in terms of the price of home output as  $rpm_d \equiv P_m/P_d$ . The ratio of these two international relative prices represents the effective real marginal cost of the importer. The optimality condition for import prices implies that the presence of price rigidities moderate the pass-through of exchange rate fluctuations into import price inflation.

$$\frac{\pi_{m,t}}{\bar{\pi}_{m}^{1-\iota_{m}}\pi_{m,t-1}^{\iota_{m}}}\chi_{m}\left(\frac{\pi_{m,t}}{\bar{\pi}_{m}^{1-\iota_{m}}\pi_{m,t-1}^{\iota_{m}}}-1\right) = (1-\xi_{m,t}) + \xi_{m,t}\frac{rer_{d,t}}{rpm_{d,t}} + \beta\mathbb{E}_{t}\frac{\Lambda_{t+1}}{\Lambda_{t}\pi_{c,t+1}}\frac{(C_{m,t+1}+I_{m,t+1})}{(C_{m,t}+I_{m,t})}\frac{\pi_{m,t+1}^{2}}{\bar{\pi}_{m}^{1-\iota_{m}}\pi_{m,t}^{\iota_{m}}}\chi_{m}\left(\frac{\pi_{m,t+1}}{\bar{\pi}_{m}^{1-\iota_{m}}\pi_{m,t}^{\iota_{m}}}-1\right).$$
(12)

<sup>&</sup>lt;sup>4</sup>Hence, a rise in the exchange rate indicates a depreciation of the home currency.

#### 2.1.3 Exporting firms

The exporting firms buy the final domestic good and differentiate it by brand naming, and sell the continuum of differentiated goods to the households in the foreign market. The nominal marginal cost is thus the price  $(P_d)$  of the domestic good. Each exporting firm s faces the following demand for its product.

$$Y_{x,t}(s) = \left(\frac{P_{x,t}(s)}{P_{x,t}}\right)^{-\xi_{x,t}} Y_{x,t},\tag{13}$$

where  $Y_{x,t}$  is the aggregate volume of exports from the SOE and  $P_{x,t}(s)/P_{x,t}$  is the price of the individual exported variety to the aggregate export price.  $\xi_{x,t}$  is the elasticity of export demand to the relative price which follows a stochastic process given as

 $\xi_{x,t} = \bar{\xi}_x^{1-\rho_{px}} \xi_{x,t-1}^{\rho_{px}} \exp \vartheta_t^{px}, \; \rho_{px} \in (0,\,1)\,, \; \vartheta_t^{px} \sim i.i.d.\,\mathcal{N}(0,\sigma_{px})\,.$  Since the exports are priced in foreign currency, profits have to converted back to the SOE currency by using the nominal exchange rate. Akin to the case of price-setting for domestic and import sales, export profits are also diminished by quadratic adjustment costs. The profit maximisation programme is given as

$$\max_{P_{x,t+\tau}(s)} \mathbb{E}_{t} \sum_{\tau=0}^{\infty} \beta^{\tau} \frac{\Lambda_{t+\tau}}{P_{c,t+\tau}} \begin{bmatrix} ner_{t+\tau} P_{x,t+\tau}(s) Y_{x,t+\tau}(s) - P_{d,t+\tau} Y_{x,t+\tau}(s) \\ -\frac{\chi_{x}}{2} \left( \frac{P_{x,t+\tau}(s)}{\bar{\pi}_{x}^{1-\iota_{x}} \pi_{x,t+\tau-1}^{\iota_{x}} P_{x,t+\tau-1}(s)} - 1 \right)^{2} ner_{t+\tau} P_{x,t+\tau} Y_{x,t+\tau} \end{bmatrix}$$

$$\chi_{x} \geq 0, \, \iota_{x} \in [0,1],$$

subject to the constraint in Equation 13. We define the relative price of exports in terms of the foreign price level as  $rpx=P_x/P^*$ . The optimality condition for export prices suggests that export price inflation covaries inversely with the effective foreign-currency price markup  $rer_d rpx$ ,

$$\frac{\pi_{x,t}}{\bar{\pi}_{x}^{1-\iota_{x}}\pi_{x,t-1}^{\iota_{x}}}\chi_{x}\left(\frac{\pi_{x,t}}{\bar{\pi}_{x}^{1-\iota_{x}}\pi_{x,t-1}^{\iota_{x}}}-1\right) = (1-\xi_{x,t}) + \frac{\xi_{x,t}}{rer_{d,t}rpx_{t}} + \beta\mathbb{E}_{t}\frac{\Lambda_{t+1}}{\Lambda_{t}}\frac{ner_{t+1}}{ner_{t}\pi_{c,t+1}}\frac{Y_{x,t+1}}{Y_{x,t}}\frac{\pi_{x,t+1}^{2}}{\bar{\pi}_{x}^{1-\iota_{x}}\pi_{x,t}^{\iota_{x}}}\chi_{x}\left(\frac{\pi_{x,t+1}}{\bar{\pi}_{x}^{1-\iota_{x}}\pi_{x,t}^{\iota_{x}}}-1\right).$$
(14)

Finally, the aggregate exports of the SOE to the foreign economy is given as

$$Y_{x,t} = rpx^{-\eta_x}Y_t^*,\tag{15}$$

where  $Y_{x,t}$  is the volume of exports from the SOE,  $Y_t^*$  is foreign output, foreign price level and  $\eta_x > 0$  is the price elasticity of export demand.

#### 2.2 Aggregation of goods

Final goods producers are assumed to be perfectly competitive. The final consumption and investment goods (C, I) are assumed to be given by CES indices of domestically-produced

 $(C_d, I_d)$  and imported goods  $(C_m, I_m)$  according to

$$C_{t} = \left[ (1 - m_{c})^{\frac{1}{\eta_{c}}} C_{d,t}^{\frac{\eta_{c} - 1}{\eta_{c}}} + m_{c}^{\frac{1}{\eta_{c}}} C_{m,t}^{\frac{\eta_{c} - 1}{\eta_{c}}} \right]^{\frac{\eta_{c}}{\eta_{c} - 1}}, \ m_{c} \in [0, 1], \ \eta_{c} > 0, \tag{16}$$

and

$$I_{t} = \varepsilon_{t}^{ist} \left[ (1 - m_{i})^{\frac{1}{\eta_{i}}} I_{d,t}^{\frac{\eta_{i} - 1}{\eta_{i}}} + m_{i}^{\frac{1}{\eta_{i}}} I_{m,t}^{\frac{\eta_{i} - 1}{\eta_{i}}} \right]^{\frac{\eta_{i}}{\eta_{i} - 1}}, \ m_{i} \in [0, 1], \ \eta_{i} > 0.$$
 (17)

As in Christiano, Trabandt, and Walentin (2011),  $\varepsilon_t^{ist}$  is an investment-specific technology shock that stimulates the production of investment goods, and it follows a random walk with drift. This implies that the growth rate of investment-specific technology defined as  $\gamma_t^{ist} \equiv \varepsilon_t^{ist}/\varepsilon_{t-1}^{ist}$  evolves according to the stationary process  $\gamma_t^{ist} = \left(\bar{\gamma}^{ist}\right)^{1-\rho_{ist}} \left(\gamma_{t-1}^{ist}\right)^{\rho_{ist}} \exp \vartheta_t^{ist}, \rho_{ist} \in (0,1), \ \vartheta_t^{ist} \sim i.i.d.\ \mathcal{N}(0,\sigma_{ist})$ . As noted in the earlier section, the prices of the domestic and imported bundles are given by  $P_d$  and  $P_m$  respectively. The cost minimisation programmes that faces the final goods producers for consumption and investment are respectively:

$$\min_{C_{d,t}, C_{m,t}} P_{d,t} C_{d,t} + P_{m,t} C_{m,t}$$

subject to Equation 16 and

$$\min_{I_{d,t}, I_{m,t}} P_{d,t} I_{d,t} + P_{m,t} I_{m,t}$$

subject to Equation 17. Domestic sales are given as

$$C_{d,t} = (1 - m_c) \left(\frac{P_{d,t}}{P_{c,t}}\right)^{-\eta_c} C_t, \ I_{d,t} = (1 - m_i) \left(\frac{P_{d,t}}{\varepsilon_t^{ist} P_{i,t}}\right)^{-\eta_i} \frac{I_t}{\varepsilon_t^{ist}}$$
(18)

and import demand is derived as

$$C_{m,t} = m_c \left(\frac{P_{m,t}}{P_{c,t}}\right)^{-\eta_c} C_t, \ I_{m,t} = m_i \left(\frac{P_{m,t}}{P_{i,t} \varepsilon_t^{ist}}\right)^{-\eta_i} \frac{I_t}{\varepsilon_t^{ist}}.$$
 (19)

The Lagrange multipliers associated with the above cost-minimisation programmes, *i.e.* the aggregate price deflators for consumption  $(P_c)$  and investment  $(P_i)$ , are combinations of the domestic and import sales prices:

$$P_{c,t} = \left[ (1 - m_c) P_{d,t}^{1 - \eta_c} + m_c P_{m,t}^{1 - \eta_c} \right]^{\frac{1}{1 - \eta_c}}$$
(20)

and

$$P_{i,t} = \frac{1}{\varepsilon_i^{ist}} \left[ (1 - m_i) P_{d,t}^{1 - \eta_i} + m_i P_{m,t}^{1 - \eta_i} \right]^{\frac{1}{1 - \eta_i}}.$$
 (21)

#### 2.3 Household preferences and constraints

The model assumes a large representative household with a continuum of members represented by the unit square and indexed by a pair  $(i,j) \in [0,1] \times [0,1]$ . The first dimension, indexed by  $i \in [0,1]$ , represents the type of labour service in which a given household mem-

ber is specialised. The second dimension, indexed by  $j \in [0,1]$ , determines her disutility from work. The latter is given by  $\varepsilon_t^l \Theta_t j^{\varphi_n}$  if she is employed and zero otherwise.  $\varepsilon_t^l > 0$  is a labour supply shock which follows the law of motion  $\varepsilon_t^l = \left(\bar{\varepsilon}^l\right)^{1-\rho_l} \left(\varepsilon_{t-1}^l\right)^{\rho_l} \exp \vartheta_t^l, \rho_l \in (0,1)$ ,  $\vartheta_t^l \sim i.i.d. \mathcal{N}(0,\sigma_l)$ .  $\Theta_t$  is a preference shifter that is taken as given by each individual household and defined below, and  $\varphi_n \geq 0$  is a parameter determining the shape of the distribution of work disutilities across individuals.

Individual utility in every period is assumed to be given by  $\log \tilde{C}_t(i,j) - \mathbf{1}(i,j) \varepsilon_t^l \Theta_t j^{\varphi_n}$ , where  $\tilde{C}_t(i,j) \equiv C_t(i,j) - h_c \ddot{C}_{t-1}$ , with  $h_c \in [0,1]$  and  $\ddot{C}_{t-1}$  denoting (lagged) aggregate consumption taken as given by each household.  $\mathbf{1}(i,j)$  is an indicator function taking a value equal to one if individual (i,j) is employed in period t and zero otherwise. In addition, consumption risk is shared perfectly among household members, implying that in every period  $C_t(i,j) = C_t$  for all  $(i,j) = C_t \in [0,1] \times [0,1]$ . Thus, the utility of the household can be expressed as the integral over its members' utilities, that is:

$$\mathbb{U}_{t}\left(C_{t}, N_{t}(i)\right) \equiv \log \tilde{C}_{t} - \varepsilon_{t}^{l} \Theta_{t} \int_{0}^{1} \int_{0}^{N_{t}(i)} j^{\varphi_{n}} \partial j \partial i$$

$$= \log \tilde{C}_{t} - \varepsilon_{t}^{l} \Theta_{t} \int_{0}^{1} \frac{N_{t}(i)^{1+\varphi_{n}}}{1+\varphi_{n}} \partial i,$$
(22)

where  $N_t(i) \in [0,1]$  denotes the employment rate in period t among workers specialised in type i labour and  $\tilde{C}_t(i,j) \equiv C_t - h_c \ddot{C}_{t-1}$ . The endogenous preference shifter  $\Theta_t$  is defined as

$$\Theta_t \equiv \frac{Z_t}{C_t - h_c \ddot{C}_{t-1}},\tag{23}$$

where  $Z_t$  evolves over time according to

$$Z_t = Z_{t-1}^{1-v_z} \left( \ddot{C}_t - h_c \ddot{C}_{t-1} \right)^{v_z}. \tag{24}$$

Thus,  $Z_t$  can be interpreted as a 'smooth' trend for (quasi-differenced) aggregate consumption. This preference specification implies a consumption externality on individual labour supply: during aggregate consumption booms when habit-adjusted consumption rises above trend, individual and household-level marginal disutility from work goes down, at any given level of employment. The main role of the endogenous preference shifter  $\Theta_t$  is to reconcile the existence of a long-run balanced growth path with an arbitrarily small short-term wealth effect that is governed by the size of the parameter  $v_z \in [0,1]$ . The smaller is  $v_z$ , the wealth effect on labour supply declines, augmenting the positive comovement of the labour force, consumption and the wage over the business cycle. Under these preferences, the household-relevant marginal rate of substitution between consumption and employment for type i workers in period t is given by:

$$MRS_t(i) \equiv -\mathbb{U}_{N(i),t}/\mathbb{U}_{C,t} = \varepsilon_t^l \Theta_t \tilde{C}_t N_t(i)^{\varphi} = \varepsilon_t^l Z_t N_t(i)^{\varphi}, \tag{25}$$

where the last equality is satisfied in a symmetric equilibrium with  $\ddot{C}_t = C_t$ . As in Erceg, Henderson, and Levin (2000), perfectly competitive 'employment agencies' aggregate the specialised labour-varieties (N(i)) from the households into a homogenous labour input (N) using a constant elasticity of substitution technology, and sell it to the firm. The employment agencies return to the household a labour-type specific nominal wage (W(i)). The demand for the specific labour type from the employment agency is given as

$$N_t(i) = \left(\frac{W_t(i)}{W_t}\right)^{-\xi_{n,t}} N_t, \tag{26}$$

and the elasticity of labour demand to the idiosyncratic wage relative to the aggregate wage index (W) is time-varying, and follows a stochastic process akin to that of analogous price elasticities in the goods market:  $\xi_{n,t} = \bar{\xi}_n^{1-\rho_n} \xi_{n,t-1}^{\rho_n} \exp \vartheta_t^n, \rho_n \in (0, 1), \vartheta_t^n \sim i.i.d. \mathcal{N}(0, \sigma_n)$ . We position nominal wage-setting frictions, another crucial building block of the labour market in the tradition of Galí et al. (2012) and Galí (2011), in the period budget constraint of the household that we present in Equation 27.

$$C_{t} + \frac{P_{i,t}}{P_{c,t}} I_{t} + \frac{B_{t}}{P_{c,t}} + \frac{ner_{t}NFA_{t}}{P_{c,t}} = \frac{\int_{0}^{1} W_{t}(i)N_{t}(i)\partial i}{P_{c,t}} - \frac{W_{t}N_{t}}{P_{c,t}} \int_{0}^{1} \frac{\chi_{w}}{2} \left( \frac{W_{t}(i)}{(\pi_{c,t-1}\gamma_{t-1})^{\iota_{w}} (\bar{\pi}_{c}\bar{\gamma})^{1-\iota_{w}} W_{t-1}(i)} - 1 \right)^{2} \partial i + \frac{R_{t}^{k}}{P_{c,t}} K_{t-1} + \frac{R_{t-1}}{P_{c,t}} B_{t-1} + \frac{ner_{t}R_{t-1}^{*}\Xi_{t-1}}{P_{c,t}} NFA_{t-1} + \frac{DIV_{t}}{P_{c,t}}.$$
(27)

The household receives total wage income  $\int_0^1 W(i)N(i)\partial i$  from all its working members. Mirroring the nominal frictions that impede the price-setting decisions of firms that we presented in the previous section, we introduce nominal wage rigidities by stipulating that quadratic adjustment costs à la Rotemberg (1982) reduce household income when it optimally chooses the labour-type specific wage. In particular, when the growth of the idiosyncratic nominal wage deviates from a combination of past headline inflation  $(\pi_{c,t-1})$  and trend economic growth  $(\gamma_{t-1})$  and their steady-state levels  $(\bar{\pi}_c\bar{\gamma})$ . The parameter  $\chi_w \geq 0$  measures the strength of the penalty to income for a given deviation of the idiosyncratic wage inflation, while the wage indexation parameter  $\iota_w \in [0,1]$  quantifies the dependence of wage inflation on lagged price inflation.

The rest of the budget constraint is fairly standard. In every period, the household purchases the final goods for consumption (C) and investment (I), private domestic currency-denominated nominal bonds (B), and private foreign currency-denominated nominal bonds (NFA). On the income side, nominal net returns  $(R^k)$  are obtained from renting out physical

<sup>&</sup>lt;sup>5</sup>Note that, unlike Galí et al. (2012) and Galí (2011), we do not use Calvo-style wage-setting. This is due to the incompatibility between the objective of deriving the recursive Calvo equations for our non-linear model solution, with that of using the time-varying wage elasticity that we need to account for the implications of inefficient supply shocks. In contrast, Galí et al. (2012) rely on a linearised model solution that combine a Calvo-style wage Phillips curve with time-varying wage markup shocks. See also Section 2.1.1 on price-setting for domestic sales and Footnote 3.

capital  $(K_{t-1})$  to the firms in the previous period, gross nominal interest returns  $(R_{t-1}B_{t-1})$  from the domestic financial asset holdings and  $(ner_tR_{t-1}^*\Xi_{t-1}NFA_{t-1})$  from the foreign financial asset holdings, and finally, dividends (DIV) that accrue to the households due to its ownership of the firms. The gross interest rates  $R_t$  and  $R_t^*$  are set by the SOE and the foreign central banks. In the spirit of Adolfson et al. (2007), a risk premium term  $\Xi_t = \varepsilon_t^{uip} \exp{-\kappa_{nfa}} \left(\frac{ner_tN\ddot{F}A_t}{P_{d_t}Y_t} - \frac{ner_tN\ddot{F}A}{P_{d_t}Y}\right)$  is appended to the foreign interest rate. It has an endogenous component so that the parameter  $\kappa_{nfa} > 0$  governing its elasticity to the aggregate stock of the foreign bonds  $N\ddot{F}A_t$ . This endogenous part of the risk premium ensures that the incomplete markets model has a unique steady state and hence can be solved using standard perturbation methods. The risk premium also has an exogenous component  $\varepsilon^{uip}$  that follows  $\gamma_t^{uip} = (\bar{\gamma}^{uip})^{1-\rho_{uip}} \left(\gamma_{t-1}^{uip}\right)^{\rho_{uip}} \exp{\vartheta_t^{uip}}, \rho_{uip} \in (0,1), \ \vartheta_t^{uip} \sim i.i.d.\ \mathcal{N}(0,\sigma_{uip})$ . Investment is transformed to installed physical capital according to the law of motion

$$K_{t} = (1 - \delta) K_{t-1} + \varepsilon_{t}^{mei} \left[ I_{t} - \frac{\phi_{inv}}{2} I_{t-1} \left( \frac{I_{t}}{I_{t-1}} - \overline{\gamma}_{inv} \right)^{2} \right].$$
 (28)

The new physical capital stock is the sum of the undepreciated capital stock, with  $\delta \in [0, 1]$  determining the rate of depreciation, and newly acquired investment goods. It is assumed that the transformation of investment goods into physical capital is costly. The quadratic cost function we use is adapted from Mandelman et al. (2011). If the adjustment cost parameter  $\phi_{inv}>0$ , the incurred cost rises with the gap betwen the growth rate of investment and its steady-state value of  $\overline{\gamma}_{inv}$ . Finally, as in Justiniano, Primiceri, and Tambalotti (2011), a stationary shock  $\varepsilon_t^{mei}$  that stimulates the marginal efficiency of capital is assumed to affect capital accumulationm, and follows  $\varepsilon_t^{mei}=\left(\bar{\varepsilon}^{mei}\right)^{1-\rho_{mei}}\left(\varepsilon_{t-1}^{mei}\right)^{\rho_{mei}}\exp\vartheta_t^{mei}, \rho_{mei}\in(0,1)$ ,  $\vartheta_t^{mei}\sim i.i.d. \mathcal{N}(0,\sigma_{mei})$ . The optimisation programme that faces the household is:

$$\max_{\substack{B_t, NFA_t, W_t(i), \\ C_t, K_t, I_t}} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \varepsilon_t^c \mathbb{U}\left(C_t, N_t(i)\right)$$

subject to Equations 26, 27, and 28. Along with these constraints, the optimisation programme is supplemented with an end-point restriction for asset holdings, so that the household does not engage in Ponzi schemes.

As in the case of firms, we will focus on a symmetric equilibrium. The optimal choice of consumption implies an inverse relation between the marginal utility of income and habit-adjusted consumption  $C_t - h_c C_{t-1}$ .

$$\frac{\varepsilon_t^c}{C_t - h_c C_{t-1}} = \Lambda_t \tag{29}$$

<sup>&</sup>lt;sup>6</sup>See Schmitt-Grohé and Uribe (2003) for a discussion of the unit root problem in open-economy models of debt accumulation and the various modelling features that can be used to address the issue.

The first order condition for domestic bonds ties down the inter-temporal flow of the marginal utility of income to the *ex ante* real interest rate.

$$\Lambda_t = \beta \mathbb{E}_t \Lambda_{t+1} \frac{R_t}{\pi_{c,t+1}} \tag{30}$$

Combining the above optimality condition with the analogous condition for foreign currency bonds (not presented) yields uncovered interest rate parity; the nominal exchange rate adjusts to equalise the expected real returns in both currencies

$$\mathbb{E}_{t}\Lambda_{t+1}\frac{R_{t}}{\pi_{c,t+1}} = \mathbb{E}_{t}\Lambda_{t+1}\frac{ner_{t+1}}{ner_{t}}\frac{R_{t}^{*}}{\pi_{c,t+1}}\varepsilon_{t}^{uip}\exp{-\kappa_{nfa}\left(\frac{ner_{t}NFA_{t}}{P_{d,t}Y_{t}} - \frac{\overline{ner}\overline{N\ddot{F}A}}{\bar{P}_{d}\bar{Y}}\right)}.$$
 (31)

The first order condition for physical capital links the marginal value of capital, *i.e.* Tobin's Q, to the expected consumption-based rental rate of capital  $(R_c^k)$  and the expected marginal value of undepreciated capital.

$$TQ_t = \beta \mathbb{E}_t \frac{\Lambda_{t+1}}{\Lambda_t} \left( R_{c,t+1}^k + TQ_{t+1} \left( 1 - \delta \right) \right)$$
(32)

The optimal choice of investment suggests that investment growth rises above its trend level when the marginal value of installed capital exceeds the procurement price of new investment goods relative to that of consumption. The presence of investment adjustment costs slows the investment response.

$$\varepsilon_{t}^{mei} \left[ 1 - \phi_{inv} \left( \frac{I_{t}}{I_{t-1}} - \overline{\gamma}_{inv} \right) \right] \\
-\beta \mathbb{E}_{t} \frac{\Lambda_{t+1}}{\Lambda_{t}} \frac{TQ_{t+1}}{TQ_{t}} \varepsilon_{t+1}^{mei} \left[ \frac{\phi_{inv}}{2} \left( \frac{I_{t+1}}{I_{t}} - \overline{\gamma}_{inv} \right)^{2} - \left( \frac{I_{t+1}}{I_{t}} \right) \phi_{inv} \left( \frac{I_{t+1}}{I_{t}} - \overline{\gamma}_{inv} \right) \right] = \frac{P_{i,t}}{P_{c,t} T Q_{t}}.$$
(33)

Finally, optimal wage-setting implies that wage inflation dips below its trend level when the marginal rate of substitution between employment and consumption (see Equation 25) exceeds the CPI-based real wage  $(W_c)$ .

$$\frac{\pi_{t}^{w}}{(\pi_{c,t-1}\gamma_{t-1})^{\iota_{w}}(\pi_{c}\bar{\gamma})^{1-\iota_{w}}}\chi_{w}\left(\frac{\pi_{t}^{w}}{(\pi_{c,t-1}\gamma_{t-1})^{\iota_{w}}(\bar{\pi}_{c}\bar{\gamma})^{1-\iota_{w}}}-1\right) = \xi_{n,t}\frac{\varepsilon_{t}^{l}Z_{t}N_{t}^{\varphi_{n}}}{W_{c,t}}+(1-\xi_{n,t}) + \beta\mathbb{E}_{t}\frac{\Lambda_{t+1}}{\Lambda_{t}}\frac{N_{t+1}}{N_{t}}\frac{1}{\pi_{c,t+1}}\frac{(\pi_{t+1}^{w})^{2}}{(\pi_{c,t}\gamma_{t})^{\iota_{w}}(\bar{\pi}_{c}\bar{\gamma})^{1-\iota_{w}}}\chi_{w}\left(\frac{\pi_{t+1}^{w}}{(\pi_{c,t}\gamma_{t})^{\iota_{w}}(\bar{\pi}_{c}\bar{\gamma})^{1-\iota_{w}}}-1\right).$$
(34)

The incorporation of wage adjustment costs and indexation to lagged price inflation moderates movements in wage inflation. The next section explains how the assumption of incomplete nominal wage adjustment is crucial for the modelling of unemployment in our framework.

#### 2.3.1 Introducing unemployment

Consider an individual specialised in type i labour and disutility of work  $\Theta_t j^{\varphi_n}$ . Using household welfare as a criterion, and taking as given current labour market conditions as summarised by the prevailing wage for her labour type, that individual will find it optimal to

participate in the labour market in period t if and only if the utility value of the consumption-based real wage is high enough to compensate the worker for the disutility arising from supplying labour. More formally, the labour market participation condition is given by:

$$\frac{1}{C_t - h_c C_{t-1}} \frac{W_t(i)}{P_{c,t}} \ge \varepsilon_t^l \Theta_t j^{\varphi_n}. \tag{35}$$

Evaluating the previous condition at the symmetric equilibrium, and letting the marginal supplier of type i labour can be denoted by  $L_t(i)$ , we have:

$$\frac{W_t(i)}{P_{c,t}} = \varepsilon_t^l Z_t L_t(i)^{\varphi_n},\tag{36}$$

since  $\Theta_t = \frac{Z_t}{C_t - h_c C_{t-1}}$ . In a symmetric equilibrium, the participation condition is given as:

$$W_{c,t} = \varepsilon_t^l Z_t L_t^{\varphi_n}. \tag{37}$$

When workers have monopoly power due to the unique labour types they supply, and in addition, nominal wages are sticky, the equilibrium real wage may be higher than that required to ensure that all potential labour market participants are employed. That is, labour supply may exceed labour demand, and hence, there exists unemployment. Formally,

$$U_t = 1 - \frac{N_t}{L_t}. (38)$$

Using the participation condition in Equation 37 to substitute out the real wage in Equation 34, and then imposing the definition of unemployment given in Equation 38, we establish a direct link between nominal wage inflation and the rate of unemployment. In the non-linear wage Phillips Curve that we present below, it is evident that a higher unemployment rate, *ceteris paribus*, places downward pressure on nominal wage increases.

$$\frac{\pi_{t}^{w}}{(\pi_{c,t-1}\gamma_{t-1})^{\iota_{w}}(\pi_{c}\bar{\gamma})^{1-\iota_{w}}}\chi_{w}\left(\frac{\pi_{t}^{w}}{(\pi_{c,t-1}\gamma_{t-1})^{\iota_{w}}(\bar{\pi}_{c}\bar{\gamma})^{1-\iota_{w}}}-1\right) = \xi_{n,t}\left(1-U_{t}\right)^{\varphi_{n}} + \left(1-\xi_{n,t}\right) \\
+\beta\mathbb{E}_{t}\frac{\Lambda_{t+1}}{\Lambda_{t}}\frac{N_{t+1}}{N_{t}}\frac{1}{\pi_{c,t+1}}\frac{\left(\pi_{t+1}^{w}\right)^{2}}{\left(\pi_{c,t}\gamma_{t}\right)^{\iota_{w}}(\bar{\pi}_{c}\bar{\gamma})^{1-\iota_{w}}}\chi_{w}\left(\frac{\pi_{t+1}^{w}}{(\pi_{c,t}\gamma_{t})^{\iota_{w}}(\bar{\pi}_{c}\bar{\gamma})^{1-\iota_{w}}}-1\right).$$
(39)

#### 2.3.2 Goods market clearing and the balance of payments

Intermediate goods are sold at consumption and investment in home and abroad and are also expended on the nominal wage and price adjustment costs.<sup>7</sup>

$$Y_{t} = C_{d,t} + I_{d,t} + Y_{x,t}$$

$$+ W_{d,t} N_{t} \frac{\chi_{w}}{2} \left( \frac{\pi_{t}^{w}}{(\pi_{c,t-1}\gamma_{t-1})^{\iota_{w}} (\bar{\pi}_{c}\bar{\gamma})^{1-\iota_{w}}} - 1 \right)^{2} + rpm_{d,t} \left( C_{m,t} + I_{m,t} \right) \frac{\chi_{m}}{2} \left( \frac{\pi_{m,t}}{\bar{\pi}_{m}^{1-\iota_{m}} \pi_{m,t-1}^{\iota_{m}}} - 1 \right)^{2}$$

$$+ rer_{d,t} rpx_{t} Y_{x,t} \frac{\chi_{x}}{2} \left( \frac{\pi_{x,t}}{\bar{\pi}_{x}^{1-\iota_{x}} \pi_{x,t-1}^{\iota_{x}}} - 1 \right)^{2} + Y_{t} \frac{\chi_{d}}{2} \left( \frac{\pi_{d,t}}{\bar{\pi}_{d}^{1-\iota_{d}} \pi_{d,t-1}^{\iota_{d}}} - 1 \right)^{2} + \varepsilon_{t}^{g}.$$

$$(40)$$

Residual demand is absorbed by an exogenous spending shock  $\varepsilon^g$  such that  $\varepsilon_t^g = (\bar{\varepsilon}^g)^{1-\rho_g} \left(\varepsilon_{t-1}^g\right)^{\rho_g} \exp \vartheta_t^g, \rho_g \in (0,\,1)\,, \ \vartheta_t^g \sim i.i.d.\ \mathcal{N}(0,\sigma_g)\,.$  Aggregating the households' budget constraint and the firms' profits in the SOE, and then imposing the goods market clearing condition in Equation 40, we arrive at the flow of net foreign assets. The balance of payments condition implies that an excess of export revenue over import expenditures reflects in an improved net foreign position for the SOE. After defining  $NFA_d \equiv nerNFA/P_d$ , we express the external balance as

$$NFA_{d,t} = \frac{R_{t-1}^* \Xi_{t-1}}{\pi_t^*} \frac{rer_{d,t}}{rer_{d,t-1}} NFA_{d,t-1} + rer_{d,t} rpx_t Y_{x,t} - rer_{d,t} \left( C_{m,t} + I_{m,t} \right). \tag{41}$$

#### 2.3.3 Domestic monetary policy

The central bank responds to macroeconomic developments, by moving the gross nominal policy rate (R) from its steady-state level  $(\bar{R})$  according to the rule given below:

$$\frac{R_t}{\bar{R}} = \left(\frac{R_{t-1}}{\bar{R}}\right)^{r_r} \left(\mathbb{E}_t \frac{\pi_{c,t+1}}{\bar{\pi}_c}\right)^{(1-r_r)r_\pi} X_t^{(1-r_r)r_x}, \ r_r \in [0,1), \ r_\pi > 1, \ r_x \ge 0.$$
 (42)

Interest rate movements exhibit some history-dependence with  $r_r \in [0,1)$  measuring the inertial influence from past interest rate settings. The first target variable is the expected deviation of CPI inflation from its steady-state level  $(\bar{\pi}_c)$ . The second target variable is a measure of resource utilisation (X), which indicates the gap between the level of economic activity and the analogous level in a counterfactual version of the economy wherein nominal frictions are absent and monetary policy has no real effects.

The policy rule in Equation 42 is the cornerstone of the welfare analysis that we present in Section 3. In our computational experiments, we experiment with different measures of resource utilisation; the output gap  $\left(Y/\hat{Y}\right)$ , the employment gap  $\left(N/\hat{N}\right)$  and the inverse of the unemployment rate gap  $\left(\hat{U}/U\right)$ . For each measure, we will vary the monetary response coefficients  $r_{\pi}$  and  $r_{x}$  to assess the implications for social welfare.

For expositional reasons, the Rotemberg-type cost functions have been deflated by the price of domestic output  $(P_d)$ . Recall that we have defined several relative prices; the real exchange rate  $rer_d \equiv ner P_d/P^*$ , the relative price of imports  $rpm_d \equiv P_m/P_d$  and the relative price of exports  $rpx = P_x/P^*$ .

#### 2.3.4 Foreign economy

The foreign economy is characterised by a closed-economy New Keynesian model growing at the same rate as the SOE. The common parameters are assumed to be the same for both economies. Output  $(y^*)$  is a linear function of employment and on the demand-side, is absorbed by consumption demand  $(c^*)$  and price adjustment costs. Market clearing implies

$$y_t^* = c_t^* + y_t^* \frac{\chi_d}{2} \left( \frac{\pi_t^*}{(\bar{\pi}^*)^{1-\iota_d} (\pi_{t-1}^*)^{\iota_d}} - 1 \right)^2.$$
 (43)

The economy's utility function is logarithmic in consumption and linear in employment. Optimal choice of consumption implies

$$\frac{1}{c_t^*} = \beta \mathbb{E}_t \frac{1}{c_{t+1}^*} \frac{1}{\gamma_{t+1}} \frac{R_t^*}{\pi_{t+1}^*}.$$
 (44)

The foreign economy's price-setting equation mirrors those of the SOE's domestic sales price equation. The first order condition for prices implies that the dynamics of inflation  $(\pi^*)$  are given by

$$\frac{\pi_t^*}{(\bar{\pi}^*)^{1-\iota_d} (\pi_{t-1}^*)^{\iota_d}} \chi_d \left( \frac{\pi_t^*}{(\bar{\pi}^*)^{1-\iota_d} (\pi_{t-1}^*)^{\iota_d}} - 1 \right) = (1 - \bar{\xi}^*) + \bar{\xi}^* y_t^* 
+ \beta \mathbb{E}_t \frac{c_t^*}{c_{t+1}^*} \frac{y_{t+1}^*}{y_t^*} \frac{1}{\pi_{t+1}^*} \frac{(\pi_{t+1}^*)^2}{(\bar{\pi}^*)^{1-\iota_d} (\pi_t^*)^{\iota_d}} \chi_d \left( \frac{\pi_{t+1}^*}{(\bar{\pi}^*)^{1-\iota_d} (\pi_t^*)^{\iota_d}} - 1 \right).$$
(45)

Monetary policy behaviour in the foreign economy is quite different from that specified in the domestic economy. The central bank follows a rule, systematically moving the interest rate  $(R^*)$  from its long run level in response to the expected deviation of price inflation from its long-run level, and output from its natural level. The response coefficients are fixed, rather than optimally chosen to maximise a welfare criterion.

$$\frac{R_t^*}{\bar{R}^*} = \left(\frac{R_{t-1}^*}{\bar{R}^*}\right)^{r_r^*} \left(\mathbb{E}_t \frac{\pi_{t+1}^*}{\bar{\pi}^*}\right)^{(1-r_r^*)r_\pi^*} \left(\frac{Y_t^*}{\hat{Y}_t^*}\right)^{(1-r_r^*)r_y^*}, \ r_r^* \in [0,1), \ r_\pi^* > 1, \ r_y^* \ge 0.$$
 (46)

#### 2.4 Model solution and parameterisation

Several models of optimal monetary policy rely on the linear-quadratic framework that involves the minimisation of a reduced-form quadratic central bank loss function subject to a linear economy. Debortoli et al. (2019) use the estimated linear model of Smets and Wouters (2007) in order to conduct their optimal monetary policy exercises. On the other hand, Ilbas (2012) and Kam, Lees, and Liu (2009), directly estimate linearised models of optimal monetary policy where the weights in the reduced-form loss functions appear in the optimal monetary policy rules. In contrast, we approximate the model solution to the second order around its non-stochastic steady-state, rather than using the log-linearised solution. The

model solution is implemented in the open-source Matlab toolbox DYNARE (see Adjemian et al., 2011).

Table 2.1: Calibration of structural parameters

Parameter		
Symbol	Description	Value
,		
$h_c$	External habit	0.75
$\varphi_n$	Inverse of Frisch elasticity	2
$\phi_{inv}$	Investment adjustment cost	5
$\chi_d$	Domestic sales price adjustment cost	50
$\iota_d$	Domestic sales price indexation	0.5
$\chi_m$	Import sales price adjustment cost	50
$\iota_m$	Import sales price indexation	0.5
$\chi_x$	Export sales price adjustment cost	50
$\iota_x$	Export sales price indexation	0.5
$\chi_w$	Nominal wage adjustment cost	50
$\iota_w$	Nominal wage indexation	0.5
$\eta_c$	Price elasticity of consumption imports	0.75
$\eta_i$	Price elasticity of investment imports	0.75
$\eta_x$	Price elasticity of exports	0.75
$\nu_z$	Wealth effect parameter	0.001
$\alpha$	Share of capital	1/3
$\beta$	Discount factor	0.99
δ	Rate of capital depreciation	0.025
$\beta$ $\delta$ $\bar{\xi}_d, \bar{\xi}_m, \bar{\xi}_x, \bar{\xi}_n, \bar{\xi}^*$	S.S. elasticities of goods and labour demand	10
$\kappa_{nfa}$	Cost of adjusting net foreign assets	0.001
$m_c$	Consumption import share	0.194
$m_i$	Investment import share	0.344
$\bar{\pi}_d, \bar{\pi}_m, \bar{\pi}_x, \bar{\pi}_w, \bar{\pi}$	* S.S. price and wage gross inflations	1
$ar{\gamma}_a$	S.S. gross growth rate of labour-augmenting technology	1.004
$\bar{\gamma}_{ist}$	S.S. gross growth rate of investment-specific technology	1.002

However, while the richer approximation of the model better supports our assessment of social welfare, it also poses computational challenges for a full-information estimation of the model parameters. The added dimension of monetary policy design makes it even more complex. Hence, we choose to calibrate the deep parameters. The values that we pick for the deep parameters are very standard, and well within the range of estimates for New Zealand presented in Jacob and Munro (2018), Kamber et al. (2016), Albertini, Kamber, and Kirker (2012) and Justiniano and Preston (2010). The long-run import shares of consumption and investment are drawn from the input-output tables published by Statistics New Zealand. The AR(1) coefficients of the two permanent shocks to labour-augmenting and investment-specific technology are set at 0.30 and 0.15, in line with Justiniano, Primiceri, and Tambalotti (2011). All other persistence coefficients of the shocks are set at 0.75 and the standard deviations of all the shocks, permanent as well as transitory, are fixed at 0.01. In Tables 2.1 and 2.2, we present the calibrated values for the structural parameters and shock parameters respectively.

Table 2.2: Calibration of shock parameters

Parameter	•	
Symbol	Description	Value
$ ho_a$	AR(1) unit root labour-augmenting technology shock	0.30
$ ho_{ist}$	AR(1) unit root investment-specific technology shock	0.15
$ ho_{neu}$	AR(1) stationary neutral technology shock	0.75
$ ho_g$	AR(1) exogenous (govt.) spending shock	0.75
$ ho_c$	AR(1) consumption impatience shock	0.75
$ ho_{mei}$	AR(1) marginal efficiency of investment shock	0.75
$ ho_{uip}$	AR(1) UIP risk premium shock	0.75
$ ho_l$	AR(1) labour supply shock	0.75
$ ho_n$	AR(1) wage markup shock	0.75
$\rho_d$	AR(1) domestic sales price markup shock	0.75
$\rho_m$	AR(1) import sales price markup shock	0.75
$\rho_x$	AR(1) export sales price markup shock	0.75
$\sigma_a$	St. dev. unit root labour-augmenting technology shock	0.01
$\sigma_{ist}$	St. dev. unit root investment-specific technology shock	0.01
$\sigma_{neu}$	St. dev. stationary neutral technology shock	0.01
$\sigma_g$	St. dev. exogenous (govt) spending shock	0.01
$\sigma_c$	St. dev. consumption impatience shock	0.01
$\sigma_c$	St. dev. marginal efficiency of investment shock	0.01
$\sigma_{uip}$	St. dev. UIP risk premium shock	0.01
$\sigma_l$	St. dev. labour supply shock	0.01
$\sigma_n$	St. dev. wage markup shock	0.01
$\sigma_d$	St. dev. domestic sales price markup shock	0.01
$\sigma_m$	St. dev. import sales price markup shock	0.01
$\sigma_x$	St. dev. export sales price markup shock	0.01

## 3 The welfare implications of stabilising resource utilisation

Given the values we have assigned for the model parameters, we now study how household welfare is affected by the monetary policy settings in the small open economy. In particular, we investigate how altering the policy response coefficients on inflation  $(r_{\pi})$  and resource utilisation  $(r_x)$  in the Taylor-type rule in Equation 42, will influence unconditional lifetime utility.

#### 3.1 A set of simple and implementable monetary policy rules

The monetary policy rules that we focus on are constrained in several ways. The rule is 'simple' in the spirit of Kollmann (2008) and Schmitt-Grohé and Uribe (2007) because it targets a limited set of variables - inflation and resource utilisation - mimicking actual central bank behaviour. In contrast, a *fully optimal* monetary policy rule, popularly known as the Ramsey rule, would stipulate that the interest rate responds to all the state variables in the economy, and hence is less empirically appealing. Since we only consider policy responses that satisfy a unique rational expectations solution of the model, the rules we consider are also 'imple-

mentable' in the terminology used by Schmitt-Grohé and Uribe (2007). To this end, the domains of the optimised policy coefficients for inflation and resource utilisation are restricted to  $r_{\pi} \in [1.01,\ 2.5]$  and  $r_{x} \in [0,\ 2.5]$  respectively, while the interest rate smoothing coefficient  $r_{r}$  is fixed at 0.5. Since the lower bound on the inflation coefficient exceeds unity, the model solution is determinate. The domains that we set for the response coefficients subsume the ranges of estimates presented for several countries in the empirical literature. The restricted grids for the coefficients also aid the numerical searches that we rely on to assess the impact of monetary policy design on welfare. We experiment with 3 distinct measures of resource utilisation - the output gap  $\left(Y/\hat{Y}\right)$ , the employment gap  $\left(N/\hat{N}\right)$  and the inverse of the unemployment rate gap  $\left(\hat{U}/U\right)$ . For this reason, the exercise is computationally intensive. The counterfactual economy that we use as a benchmark to assess the changes in welfare in our numerical experiments is the version of the model that abstracts from nominal rigidities and shocks to the price and wage mark-ups (see also Section 2.3.3).

#### 3.2 The welfare metric

The welfare metric that we consider is the 'consumption equivalent variation' (CEV), the constant percentage change that is needed to be made to the representative household's consumption in every period and the states of the world so that it is indifferent between the allocation under a particular policy configuration, *i.e.* a set of monetary policy response coefficients,  $vis \ \grave{a} \ vis$  the benchmark economy where monetary policy is absent. In other words, the CEV balances welfare in the counterfactual economy and under the monetary policy settings that we experiment with.

The traditional CEV involves consumption compensations defined on the consumption in the current period. The presence of habits in the utility function as in our case (see Equation 22) complicates the calculation of CEV significantly. In fact, in order to calculate the traditional CEV in the presence of habits, one has to rely on simulations, and solve for the CEV using non-linear techniques for every period in the simulations across time horizons. Indeed, the horizon and the number of simulation periods need to be very large to get reliable results. Given that we work on large numerical grids of monetary policy response coefficients in the baseline model as well as many other specifications, this approach quickly becomes computationally expensive.

A second option to calculate the CEV is to exploit the fact that under all the specifications we consider, the sticky-price economy, and the benchmark, flexible-price economy, share the same non-stochastic steady state. In this case, one can compute two auxiliary CEVs for each economy relative to the non-stochastic steady state. The difference between the two approximates the CEV between the sticky- and the flexible-price economies remarkably well. Finally, the CEV can be defined over *habit-corrected* consumption as in Otrok (2001). We computed the three variants of the CEV for randomly-selected monetary policy rules, and

<sup>&</sup>lt;sup>8</sup>Estimates of the monetary policy response to detrended output in DSGE models are typically very small while the estimated responses to inflation are much higher, across countries over time samples. For New Zealand, the average estimates are typically between 0.03 and 0.20 (Jacob and Munro, 2018, Kamber et al., 2016, Justiniano and Preston, 2010 and Kam et al., 2009). The estimated inflation coefficients in these studies are usually higher, between 1.9 and 2.33.

confirmed that the quantitative differences in results were negligible in our case. The qualitative implications for welfare are very similar. For this reason, the rest of this paper focuses on the habit-corrected measure of the *CEV*. We will now detail its derivation.

The stationarised form of the utility function is given as

$$\mathscr{U}_t = \varepsilon_t^c \log \left( c_t - h_c \frac{c_{t-1}}{\gamma_t} \right) - \varepsilon_t^c \varepsilon_t^l \frac{z_t}{\left( c_t - h_c \frac{c_{t-1}}{\gamma_t} \right)} \frac{n_t^{1+\varphi_n}}{1+\varphi_n}, \tag{47}$$

where the preference shifter is given as  $z_t = \left(\frac{z_{t-1}}{\gamma_t}\right)^{1-v_z} \left(c_t - h_c \frac{c_{t-1}}{\gamma_t}\right)^{v_z}$ . Defining habit-corrected consumption as  $\ddot{c}_t = c_t - h_c \frac{c_{t-1}}{\gamma_t}$ , substituting in  $z_t$  into the utility function and simplifying,

$$\mathscr{U}_t = \varepsilon_t^c \log \ddot{c}_t - \varepsilon_t^c \varepsilon_t^l \left(\frac{z_{t-1}}{\gamma_t}\right)^{1-v_z} \ddot{c}_t^{v_z - 1} \frac{n_t^{1+\varphi_n}}{1+\varphi_n}.$$
 (48)

Let the expected lifetime utility of the household under the policy settings we want to consider be defined as

$$\mathcal{V}_{t} \equiv \mathbb{E}_{t} \sum_{\tau=0}^{\infty} \beta^{\tau} \left( \varepsilon_{t+\tau}^{c} \log \ddot{c}_{t+\tau} - \varepsilon_{t+\tau}^{c} \varepsilon_{t+\tau}^{l} \left( \frac{z_{t+\tau-1}}{\gamma_{t+\tau}} \right)^{1-v_{z}} \ddot{c}_{t+\tau}^{v_{z}-1} \frac{n_{t+\tau}^{1+\varphi_{n}}}{1+\varphi_{n}} \right). \tag{49}$$

We denote the unconditional lifetime utility of the agent in the counterfactual economy as  $\mathcal{V}_t^{\mathcal{F}}$ . The CEV balances the welfare functions in economies with and without monetary policy in the following manner:

$$\mathcal{V}_{t}^{\mathcal{F}} \equiv \mathbb{E}_{t} \sum_{\tau=0}^{\infty} \beta^{\tau} \left( \varepsilon_{t+\tau}^{c} \log \left[ (1 + CEV) \ddot{c}_{t+\tau} \right] - \varepsilon_{t+\tau}^{c} \varepsilon_{t+\tau}^{l} \left( \frac{z_{t+\tau-1}}{\gamma_{t+\tau}} \right)^{1-v_{z}} \left[ (1 + CEV) \ddot{c}_{t+\tau} \right]^{v_{z}-1} \frac{n_{t+\tau}^{1+\varphi_{n}}}{1 + \varphi_{n}} \right). \tag{50}$$

Expanding the expression on the right hand side,

$$\mathcal{V}_{t}^{\mathcal{F}} \equiv \mathbb{E}_{t} \sum_{\tau=0}^{\infty} \beta^{\tau} \varepsilon_{t+\tau}^{c} \log \left(1 + CEV\right) + \mathbb{E}_{t} \sum_{\tau=0}^{\infty} \beta^{\tau} \varepsilon_{t+\tau}^{c} \log \ddot{c}_{t+\tau} \\
-\mathbb{E}_{t} \sum_{\tau=0}^{\infty} \beta^{\tau} \left( \varepsilon_{t+\tau}^{c} \varepsilon_{t+\tau}^{l} \left( \frac{z_{t+\tau-1}}{\gamma_{t+\tau}} \right)^{1-v_{z}} \left[ \left(1 + CEV\right) \ddot{c}_{t+\tau} \right]^{v_{z}-1} \frac{n_{t+\tau}^{1+\varphi_{n}}}{1 + \varphi_{n}} \right).$$
(51)

Since  $\mathbb{E}_t \sum_{\tau=0}^\infty arepsilon_{t+ au}^c = 0$ , this expression reduces to

$$\mathcal{V}_{t}^{\mathcal{F}} \equiv \frac{\log\left(1 + CEV\right)}{1 - \beta} + \mathbb{E}_{t} \sum_{\tau=0}^{\infty} \beta^{\tau} \varepsilon_{t+\tau}^{c} \log \ddot{c}_{t+\tau} 
- (1 + CEV)^{v_{z}-1} \mathbb{E}_{t} \sum_{\tau=0}^{\infty} \beta^{\tau} \left( \varepsilon_{t+\tau}^{c} \varepsilon_{t+\tau}^{l} \left( \frac{z_{t+\tau-1}}{\gamma_{t+\tau}} \right)^{1-v_{z}} \ddot{c}_{t+\tau}^{v_{z}-1} \frac{n_{t+\tau}^{1+\varphi_{n}}}{1 + \varphi_{n}} \right).$$
(52)

<sup>&</sup>lt;sup>9</sup>The presence of permanent labour-augmenting and investment-specific technology shocks implies that the model variables have to be stationarised. Lower case letters are used to represent stationarised variables. See Jacob and Özbilgin (2021) for more details on stationarisation,

Define  $V_t^1 \equiv \mathbb{E}_t \sum_{\tau=0}^{\infty} \beta^{\tau} \varepsilon_{t+\tau}^c \log \ddot{c}_{t+\tau}$  and  $V_t^2 \equiv -\mathbb{E}_t \sum_{\tau=0}^{\infty} \beta^{\tau} \varepsilon_{t+\tau}^c \varepsilon_{t+\tau}^l \left( \frac{z_{t+\tau-1}}{\gamma_{t+\tau}} \right)^{1-v_z} \ddot{c}_{t+\tau}^{v_z-1} \frac{n_{t+\tau}^{1+\varphi_n}}{1+\varphi_n}$ , so that  $V_t = V_t^1 + V_t^2$ . In terms of unconditional expectations,

$$\mathbb{E}\mathcal{V}_{t}^{\mathcal{F}} \equiv \frac{\log\left(1 + CEV\right)}{1 - \beta} + \mathbb{E}\mathcal{V}_{t}^{1} + \left(1 + CEV\right)^{v_{z} - 1} \mathbb{E}\mathcal{V}_{t}^{2},$$

and the CEV is computed numerically and expressed as a percentage.

#### 3.3 Stabilising resource utilisation enhances welfare

We present our baseline line results in Figures 4.1 and 4.2. Figure 4.1 considers gradation in the CEV for each combination of policy response coefficients on inflation and resource utilisation. Each CEV surface is associated with a distinct measure of resource utilisation; the output gap, the employment gap and the unemployment rate gap. Let us first consider the case in which the central bank does not react to any measure of resource utilisation,  $r_x = 0$ , and instead pursues strict inflation targeting by increasing the inflation coefficent  $r_{\pi}$ . All three CEV surfaces decline, *i.e.* welfare increases, as the central bank targets only inflation. The welfare benefits of inflation targeting using simple and implementable monetary policy rules have been reported by Kollmann (2008) and Schmitt-Grohé and Uribe (2007) in closed-economy models for the United States, driven mainly by disturbances to aggregate demand and productivity. Kollmann (2002) finds similar results for a small open economy model augmented by shocks to productivity, uncovered interest rate parity, interest rates and inflation in the foreign economy.

The welfare implications of directly stabilising resource utilisation, over and above what can be achieved by merely stabilising inflation, are better understood by examining Figure 4.2. Each panel presents two-dimensional slices of the welfare surfaces presented earlier in Figure 4.1. We increase the response coefficients on the 3 measures of resource utilisation, when the inflation coefficient is set at distinct levels; firstly, at low levels at  $r_\pi=1.01$  which is high enough to guarantee model determinacy, then at  $r_\pi=1.52$  which is close to the means for prior distributions assigned to this parameter in estimated DSGE models (such as those in Footnote 8), and finally,  $r_\pi=2.5$ , the upper bound of the grid we consider.

Panel (a) distills the most unique insight that our framework delivers, relative to the rest of the literature. When monetary policy is nearly passive towards inflation at  $r_\pi=1.01$ , social welfare increases, and the CEV declines substantially as the central bank is more aggressive towards stabilising resource utilisation; fewer consumption units need to be compensated to agents for them to be as satisfied as in the more efficient counterfactual world where nominal price and wages rigidities and associated mark-up disturbances are absent. Between the 3 measures of resource utilisation, responding more strongly to the two labour market metrics - the employment gap and the unemployment rate gap - delivers welfare outcomes better than stabilising the most popular summary indicator of resource utilisation, the output gap. When the response coefficient on resource utilisation exceeds about 1.2 in the baseline model, the unemployment rate gap is the single most welfare-relevant measure of resource utilisation as far as monetary policy is concerned. The importance of the labour market variables for welfare is not a surprise; employment directly enters the agent's utility function,

and the stochastic mean and volatility of employment are crucial components of the social welfare metric. The relevance of the output gap, a combination of the capital stock gap and the employment gap in our model, is less direct.

Panels (b) and (c) demonstrate that the additional central bank response to resource utilisation continues to enhance social welfare, even when the inflation coefficient increases. However, the increases in welfare get progressively milder, and the differences *between* responding to different measures of resource utilisation start diminishing already in the more empirically-relevant case presented in Panel (b), when the inflation coefficient is set at about 1.5. The differences between the CEV curves are almost indistinguishable in Panel (c) when the inflation coefficient is set at the upper bound of the grid we consider.

The results in Figures 4.1 and 4.2 suggest that the dual mandate works to the benefit of society. Overall, conditional on all the endogenous frictions and shocks used for our baseline simulations, it also appears that the dual mandate gives the policy-maker a high degree of flexibility. When the response to inflation is low, responding more to resource utilisation yields better welfare outcomes. On the other hand, social welfare also improves even the central bank pursues strict inflation targeting, disregarding the stabilisation of resource utilisation. This latter result is reminiscent of the property of 'divine coincidence' in New Keynesian models highlighted by Blanchard and Galí (2007); that monetary policy can stabilise the welfare-relevant output gap by singularly focussing on stabilising the inflation gap. 10 The difference in our framework is that, within the set of rules we consider, welfare outcomes can at least improve mildly if the central bank additionally targets resource utilisation. The welfare gains become more substantial, as the central bank places a lower weight on inflation, and between the various measures of resource utilisation, responding to the labour market variables appear to be more welfare-enhancing than responding to the output gap. This is a novel result relative to the extant literature. In the following section, we examine how this result is strengthened or weakened as we alter the model configuration.

# 4 Cost-push shocks and the dual mandate

We will now validate the robustness of our baseline results. The first set of checks examine the implications of changing the strength of the endogenous frictions in the model: (a) full versus zero indexation in price and wage inflation

 $(\iota_d = \iota_m = \iota_x = \iota_w = 1, \ \iota_d = \iota_m = \iota_x = \iota_w = 0)$  (b) remove habit persistence  $(h_c = 0)$  (c) very low investment adjustment costs  $(\phi_{inv} = 0.01)$  (d) extremely low openness versus much higher openness  $(m_c = m_i = 0.01, \ m_c = m_i = 0.50)$  (e) lower price and wage adjustment costs  $(\chi_d = \chi_m = \chi_x = \chi_w = 25)$  and (f) extreme interest rate inertia  $(r_r = 0.85)$ . The qualitative insights from the baseline model are preserved in all these model variants; even though the welfare metrics change in quantitative terms, the analogous profiles of the welfare surfaces and curves are not dissimilar to those presented in Figures 4.1 and 4.2. The welfare enhancements implied by the dual mandate are apparent in all these reparameterisations.

<sup>&</sup>lt;sup>10</sup>The analogue of this property for the open economy has been noted in Corsetti, Dedola, and Leduc (2010).

Our model, besides being equipped with the standard set of endogenous frictions, is driven by a wide array of structural disturbances. Hence, our second set of robustness checks alter the shock composition. The domestic shocks in the small open economy can be categorised into three; demand-type disturbances such as those to consumption, investment and exogenous spending  $(\vartheta^c, \vartheta^{mei}, \vartheta^g)$ , efficient permanent as well as transitory disturbances on the supply side  $(\vartheta^a, \vartheta^{ist}, \vartheta^{neu}, \vartheta^l)$  and finally, inefficient cost-push disturbances to price and wage inflation  $(\vartheta^{pd}, \vartheta^{pm}, \vartheta^{px}, \vartheta^n)$ . The first two categories of shocks are embedded both in the economy with nominal rigidites and the counterfactual economy without nominal frictions, while the latter category of inefficient cost-push shocks to the price and wage markups do not appear in the counterfactual economy. We examine the welfare surfaces generated when we use the demand-type shocks and efficient supply-type shocks *one at a time*, keeping the baseline parameterisation of the endogenous frictions. Again, just as in the case of the first set of checks that strengthen or weaken the endogenous frictions, the qualitative insights remain quite similar to the baseline case. The quantitative implications for welfare, change depending on how strongly the relevant shock influences the business cycle.

The picture changes drastically when we consider the effects of powerful cost-push shocks. In this final check, we deactivate all other shocks, and empower the inefficient mark-up shocks to domestic price inflation, imported inflation and wage inflation  $(\vartheta^{pd}, \vartheta^{pm}, \vartheta^n)$  by assigning them 10 times the volatilities used in the baseline model. That is, the standard deviations of these shocks are set as  $\sigma_d = \sigma_m = \sigma_n = 0.1$  as opposed to  $\sigma_d = \sigma_m = \sigma_n = 0.01$  in the baseline. However, the persistence coefficients retain the same values as in the baseline:  $\rho_d = \rho_m = \rho_n = 0.75$ . We present the associate welfare (CEV) surfaces in Figure 4.3. Since we earlier saw that the welfare implications of responding to resource utilisation are most obvious when the inflation response coefficient is very low ( $r_\pi = 1.01$ ), we present this special case in Figure 4.4.

Figure 4.3 establishes that, in the presence of powerful cost-push shocks to price and wage inflation, welfare decreases (*CEV* increases) as the central bank reacts more to resource utilisation. This is the opposite pattern relative to that seen in our baseline specification, and is observed for all levels of inflation stabilisation. Furthermore, we also observe in Figure 4.4, that among the 3 measures of resource utilisation we experiment with, responding to the output gap delivers worse welfare outcomes than reacting to the labour market variables. A crucial determinant of the welfare implications of price and wage mark-up shocks, is the fact that they drive inflation and resource utilisation in opposite directions. When these shocks exert powerful influences on the business cycle, the central bank faces trade-offs between the dual objectives of stabilising inflation and stabilising resource utilisation. This is in contrast to the disturbances to aggregate demand or technical progress. Finally, reverting back to Figure 4.3, we find that, in the face of powerful cost-push shocks, society is better served when the central bank focusses solely on stabilising inflation, ignoring the resource utilisation objective.

<sup>&</sup>lt;sup>11</sup>Mankiw (2007) provides an intuitive account of the implications of demand shocks, productivity shocks, and cost-push shocks for the covariance between inflation and the output gap, and hence for monetary polcy tradeoffs.

Output gap
Unemployment rate gap
Employment gap 1.5 1 Response to resource utilisation Figure 4.1: The CEV under different combinations of monetary policy response coefficients 0.5 2 Response to inflation 1.5 ∨ 00.0 3.00 ~

2.5 2.0 (c)  $r_{\pi} = 2.50$ 1.0 0.5 0.00 3.00 2.50 2.00 1.50 1.00 0.50 2.5 2.0 (b)  $r_{\pi} = 1.52$ 1.0 0.5 0.00 0.0 3.00 [ 2.50 1.50 1.00 0.50 2.00 2.5 (a)  $r_{\pi} = 1.01$ 0.5 3.00 2.50 0.50 0.00

Figure 4.2: A closer examination of welfare as the central bank response stabilisation of inflation  $(r_{\pi})$  and resource utilisation  $(r_{x})$  increase

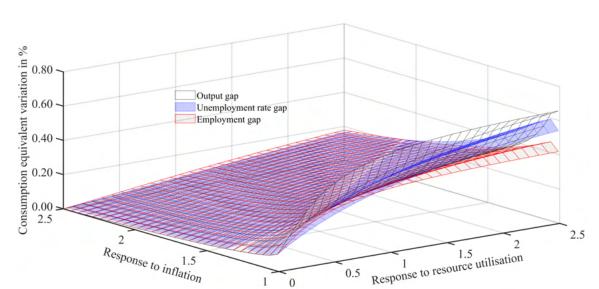
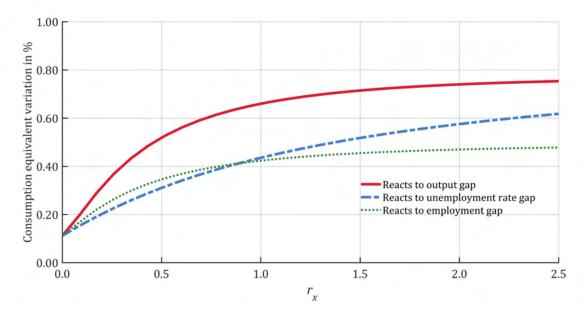


Figure 4.3: The CEV under extremely volatile cost-push shocks

Figure 4.4: The CEV under extremely volatile cost-push shocks when the inflation response is very weak  $(r_{\pi} = 1.01)$ 



#### 5 Conclusion

This paper demonstrates that the additional monetary policy response to resource utilisation, that is prescribed by the dual mandate for central banks, works to the benefit of society. Using a New Keynesian open-economy growth model calibrated to New Zealand data, we find that additionally stabilising resource utilisation improves social welfare at any given level of inflation stabilisation.

The welfare gains from stabilising resource utilisation are particularly striking when the central bank is nearly passive towards stabilising inflation response to inflation is weak. Interestingly, when the sensitivity to inflation is weak, monetary policy responses to the labour market variables such as the employment gap or the unemployment rate gap, that are more in line with the activity indicators stipulated in actual central bank legislation, yield better welfare outcomes than reacting to the most popular summary indicator of resource utilisation: the output gap. This is mainly because the levels and volatilities of employment and consumption are crucial components of the social welfare metric that underpins modern normative macroeconomics. The implications of stabilising the output gap, that is also influenced by several other elements in general equilibrium, are less direct as far as the welfare metric is concerned.

However, within the range of monetary policy settings we consider, the welfare gains from stabilising resource utilisation decline to mild levels when the central bank is already very sensitive to inflation. Not surprisingly, in this scenario, the differences between the welfare implications of stabilising the three different measures of resource utilisation also diminish.

The overall desirability of stabilising resource utilisation from the perspective of social welfare is robust across several variants of our theoretical framework. However, the challenge that the monetary policy-maker confronts is also an empirical one; the clouds of uncertainty that shroud assessments of the level of resource utilisation in the economy. Data series such as those for GDP and the unemployment rate are occasionally subject to revisions by statistical agencies, and the econometric measurement of the activity gap is also known to be complex (Orphanides and van Norden, 2002). From this practical viewpoint, the dual mandate is also the more difficult mandate for the policy-maker. Inflation targeting is a simpler framework, and since the policy response to inflation can deliver welfare outcomes almost as high as those generated by the response to resource utilisation in our model, we conclude that inflation targeting remains an appealing alternative monetary policy framework.

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# A Appendix

Figure A.1: Welfare (levels) in the baseline model as the central bank response stabilisation of inflation  $(r_{\pi})$  and resource utilisation  $(r_{x})$  in-2.5 2.0 (c)  $r_{\pi} = 2.50$ 1.0 0.5 48.0 L 0.0 51.0 F 50.5 49.5 48.5 50.0 49.0 2.5 2.0 (b)  $r_{\pi} = 1.52$ 1.5 1.0 0.5 48.0 L 0.0 51.0 F 50.5 49.5 50.0 49.0 48.5 2.5 (a)  $r_{\pi} = 1.01$ 0.5 51.0 [ 48.0 L 50.5 48.5

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