Australian National University

Crawford School of Public Policy

CAMA

Centre for Applied Macroeconomic Analysis

Exchange rates, expected returns and risk: UIP unbound

CAMA Working Paper 73/2014
December 2014

Anella Munro
Reserve Bank of New Zealand and
Centre for Applied Macroeconomic Analysis (CAMA), ANU

Abstract

No-arbitrage implies a close link between exchange rates and interest returns, but evidence of that link has been elusive. This paper derives an exchange rate asset price model with consumption-risk adjustments. Interest rates and exchange rates reflect common risks which bias their reduced-form relationship. As markets become more complete, the model predicts increasing disconnect between exchange rates and observed interest rates, and between premia that price bonds and premia that price currency returns. When accounting for risk, the estimated interest rate - exchange rate relationship is considerably closer to theory for eight USD currency pairs. Exchange rates, risk and returns need to be jointly modeled.

Keywords

Exchange rate, asset price, risk adjustment, uncovered interest parity, bond premium, currency premium

JEL Classification

F31, G12

Address for correspondence:

(E) cama.admin@anu.edu.au

Exchange rates, expected returns and risk: UIP unbound

Anella Munro*

November 27, 2014

Abstract

No-arbitrage implies a close link between exchange rates and interest returns, but evidence of that link has been elusive. This paper derives an exchange rate asset price model with consumption-risk adjustments. Interest rates and exchange rates reflect common risks which bias their reduced-form relationship. As markets become more complete, the model predicts increasing disconnect between exchange rates and observed interest rates, and between premia that price bonds and premia that price currency returns. When accounting for risk, the estimated interest rate - exchange rate relationship is considerably closer to theory for eight USD currency pairs. Exchange rates, risk and returns need to be jointly modeled.

JEL codes: F31, G12

Keywords: Exchange rate, asset price, risk adjustment, uncovered interest parity, bond premium, currency premium

*Senior Adviser, Economic Department, Reserve Bank of New Zealand, 2 The Terrace, PO Box 2498, Wellington, New Zealand. Tel: (64 4) 471 3663. anella.munro@rbnz.govt.nz.
This paper has benefitted from comments from Yu Chin Chen, Charles Engel, Ippei Fujiwara, Ryan Greenaway-McGrevy, Punnoose Jacob, Leo Krippner, Richard Levich, Chris McDonald, Maurice Obstfeld, Weshah Razzak, and Konstantinos Theodoris, Hugo Vega and anonymous referees. I thank participants at the Australian Macro Workshop, Canberra, 4-5 April 2013, and the RBNZ-BIS Conference on Cross Border Financial Linkages, Wellington, October 2014 for their comments.

1 Introduction

A large literature[1] has argued that risk premia are at teh heart of the weak empirical relationship between exchange rates and interest rates.[2] A no-arbitrage condition - uncovered interest parity (UIP) - implies a close link between exchange rates and relative interest returns, but evidence of that link has proved elusive. Empirical tests of UIP fail systematically across currency pairs and across time periods. Fama (1984) argues that the exchange rate premium is time varying and systematically correlated with expected exchange rate depreciation. Similarly, Engel and West (2010) argue that covariance between fundamentals and unobserved variables, such as risk, may bias the estimated relationship between exchange rates and fundamentals. (Sarno et al, 2012) show that, when risk premia are accounted for, predicted currency returns are unbiased. The potential estimation bias problem associated with correlation between observed interest rates and risk premia is the subject of this paper.

Observed interest rates include a risk-free return and premia that compensate for risk.[3] Even government bill rates or central bank rates reflect risk. Those rates reflect low credit default and liquidity risk, but can include substantial 'specialness' premia associated with their collateral value (Krishnamurthy and Vissing-Jorgensen, 2012). Any interest rate with a maturity greater than zero reflects interest rate risk, a term premium and, from the foreign investor's perspective, currency revaluation risk (Lustig and Verdelhan 2007, Duarte and Stockman 2005). If the risk adjustments embodied in observed interest rates - the 'bond premium' - are correlated with the exchange rate risk premium, then reduced-form estimates of the exchange rate-interest rate relationship will be biased.

This paper derives a two-equation, risk-augmented exchange rate asset price model. The model extends the exchange rate asset price model used by Engel and West (2005, 2010) by including explicit consumption-risk adjustments (Backus et al 2001 and Lustig and Verdelhan 2007). With explicit risk adjustments, we see that, the exchange rate and observed interest rates reflect a common bond premium. If not accounted for, that common bond premium biases the estimated relationship between interest rates and exchange rates. Intuitively, higher home risk-free returns and a higher home bond premium both raise home yields, but the former appreciates the home currency while the latter does not. Only risk-adjusted relative returns affect the currency. As markets become more complete, the model predicts increasing disconnect in the reduced-form relationship between exchange rates and observed interest returns, and disconnect between risk factors that price bond markets and risk factors that price currencies.

[1] For example, Sarno et al (2012), Lustig and Verdelhan (2007), Brennan and Xia (2006), (Duarte and Stockman, 2005), (Obstfeld et al, 2002), Backus et al (2001) and Fama (1984)

[2] See Bilson (1981), Fama (1984) for early contributions and, for reviews of the literature, see Engel (2013), Engel (2012), Engel (1996), and Flood and Rose (1996).

[3] Here risk-free rates are defined by investors' consumption discount factors which are unobserved.

The common bond premium motivates a two-equation, structural asset price model to inform on the degree of estimation bias and to identify the bond premium empirically. The empirical model employs the sign restrictions implied by the theoretical model and forecasts of future interest returns constructed from interest rate swaps.[4] For a set of eight advanced country USD currency pairs, I estimate the common bond premium to be large enough to severely bias single-equation estimates. When risk is accounted for, the estimated exchange rate response to expected returns averages 0.75, compared to 0.34 in the reduced-form model, and a theoretical value of one (Dornbusch, 1976). Innovations in the idiosyncratic currency premium are correlated with 'speculative' positioning in foreign exchange futures markets and, for non-reserve currencies, with changes in the VIX index.[5] When risk is accounted for, I estimate expected risk-free returns to account for about 20% of exchange rate variance, on average, compared to about 6% in the reduced-form model. These results suggest that exchange rates, expected returns and risk need to be modeled jointly.

This paper relates to several strands of the literature. The model used in this paper is an extension of the asset price model of the exchange rate employed by Engel and West (2010)[6] that accounts for movements in the expected future expected path of interest rates. The single equation results here are comparable to those in Engel and West (2005), despite different approaches to forecasting future returns and different time periods. Here, I extend that single-equation model to a 2-equation model with explicit consumption-risk adjustments and show that the weak correlation between exchange rates and expected returns can be partly understood in terms of a bond premium that is priced into both observed interest returns and exchange rates.

This paper relates excess returns on assets to the consumption risk associated with holding those assets (Duffie 1992, Cochrane 2001). Lustig and Verdelhan (2007) relate excess returns on bonds denominated in different currencies to consumption risk premia, and particularly to currency revaluation risk. Their approach is extended to incorporate other types of risk and motivates the second equation in the structural model.

Risk premia have been widely examined as an explanation of excess short-term returns, or 'carry trade returns'.[7] Empirically, traditional risk factors that help to explain domestic stock returns (Burnside, 2012) or bond returns (Sarno et al, 2012), do not explain currency returns; and less traditional currency risk factors[8] that help

[4] The approach here is conceptually different to Chinn and Quayyum (2012) and Chinn and Meredith (2004) who relate multi-period bond differentials to multi-period exchange rate changes. Here, long-term swaps are used as forecasts of expected future short-term rates and forecast innovations are related to short-term exchange rate movements.

[5] The VIX index is the implied volatility of S&P500 equities, inferred from options prices.

[6] See also Nason and Rogers (2008) and Kano (2014).

[7] See Burnside (2012), Chen and Tsang (2013), Sarno et al (2012), Farhi et al (2009), Lustig and Verdelhan (2007), Froot and Frankel (1989) and references therein.

[8] These include a dollar or global volatility factor (Menkhoff et al, 2012), a high-to-low carry factor

to explain currency returns, do not explain domestic asset returns. The exchange rate asset model derived in this paper, predicts disconnect between measures of risk that price domestic assets and measures of risk that price the currency. That theoretical prediction is consistent with the empirical findings of Burnside (2012) and Sarno et al (2012). Intuitively, the bond premium has little effect on currency returns because exchange rates are priced according to relative *risk-adjusted* returns.

Papers that relate empirical affine yield curve factors to exchange rates[9] model expected short-term interest rates, bond premia, and the currency premium. This paper is perhaps closest to (Sarno et al, 2012). Both model a bond premia and currency premium explicitly. The model derived here is built on consumption-risk premia. Sarno et al's is built on empirical yield curve factors. In this paper, the yield curve is decomposed into risk-free returns and a bond premium by exploiting their opposite-signed effects on the exchange rate and on expected returns. In yield curve models, interest rates are decomposed into the future path of short-term rates and factors informed by the cross-sectional shape of the yield curve. The predictions of the model derived here are consistent with empirical results from the yield curve literature in two respects. First, yield curve factors should help to resolve the forward premium puzzle (Brennan and Xia 2006, Sarno et al 2012). Second, yield curve factors should have little predictive power for currency depreciation rates (Sarno et al, 2012).

Event studies provide a potential means of addressing the estimation bias problem by focusing on periods dominated by changes in monetary policy. Kearns and Manners (2006) and Zettelmeyer (2006) estimate a relatively strong exchange rate response to monetary policy and Coleman and Karagedikli (2008) find a strong exchange rate response to yield curve shocks, using swap rates as a measure of expected returns. Through the lens of the asset price model used here, the identification problem should be less severe during periods dominated by monetary policy, if movements in monetary policy rates are more correlated with risk-free rates than with bond premia.

Many papers in the vector auto-regression (VAR) literature seek to identify the exchange rate response to changes in interest rates. The timing of the exchange rate response to monetary policy is sensitive to identifying assumptions (Faust and Rogers, 2003). If risk-free returns and the currency premium affect both the exchange rate and expected returns, then identification based on a Cholesky factorisation cannot identify the exchange rate response to a monetary policy shock. It precludes the contemporaneous and correlated relationship between exchange rates, expected returns and risk. Some VAR studies identify a strong immediate exchange rate response to monetary policy shocks using sign restrictions (Scholl and Uhlig, 2008) or long-run restrictions (Bjørnland, 2009) that allow for a contemporaneous

(Lustig et al, 2011), and skewness/crash risk (Brunnermeier et al 2009, Rafferty 2012).

[9]For example, Chen and Tsang 2013, Sarno et al 2012, De Los Rios 2009, Brennan and Xia 2006 and Backus et al 2001.

relationship between interest rates and the exchange rate. Bjørnland identifies a Dornbusch 'jump' response for a range of advanced small open economy currencies. Neither of those papers explicitly addresses the role of risk.

The next section derives an asset price model of the exchange rate, incorporating consumption risk premia. Section 3 describes the empirical identification strategy and the data, and explains how forecasts of expected real interest returns are constructed. Section 4 presents the estimation results, and relates the unobserved risk-free factor and risk factors to measures of monetary policy and to observed measures of risk. Section 5 considers changes to the empirical model and section 6 concludes.

2 An asset price model of the exchange rate with consumption-risk adjustments

Observed interest rates reflect risk-free rates plus risk premia. Government bills and central bank rates are often assumed to be risk free because their credit default risk and liquidity risk are relatively low. However, they are not strictly risk-free: they can reflect 'specialness' premia associated with investment mandates and collateral value, interest rate risk, a term premium, and from the perspective of a global portfolio, currency revaluation risk.

Risk-free interest rates, defined by investors' consumption discount factors, are not observed. The home real, risk-free rate, r_t^f, is defined by the home investor's willingness to give up a unit of consumption today to consume $(1 + r_t^f)$ units of consumption next period:

$$M_{t+1} = E_t \beta U'_{C,t+1}/U'_{C,t} = \tfrac{1}{1+r_t^f}$$

where M_t is the stochastic discount factor (or pricing kernel), E_t indicates expectations at time t, β is the home investor's subjective discount factor, and $U'_{C,t}$ is the marginal utility of consumption.[10]

The home investor's Euler equation for home bonds is:

$$1 \;=\; E_t[M_{t+1}(1 + r_t)] \tag{1}$$

Equation (1) is a no-arbitrage condition that equates the cost of buying a unit of home bond this period to the expected, discounted return on the bond at time $t+1$. The short-term real interest rate, $\tilde{r}_t \equiv log E_t(1 + r_t)$ can be written as the risk-free rate plus a risk adjustment (See Lustig and Verdelhan (2007) and Appendix A):

$$\tilde{r}_t \;=\; r_t^f - E_t cov_t(m_{t+1}, r_t), \tag{2}$$

[10] The risk-free rate is lower when people save more because they are patient (β), are averse to varying consumption across time (inter-temporal substitution), are averse to varying consumption across states (risk aversion), or expect consumption growth to be volatile (precautionary savings). See Cochrane (2001).

where m_t is the log of the stochastic discount factor. Similarly, the foreign short-term interest rate, $\tilde{r}_t^* \equiv logE_t(1 + r_t^*)$, can be written as the foreign investor's risk-free rate, r_t^{*f}, plus a risk adjustment that is priced according to the foreign investor's stochastic discount factor, m_t^*:

$$\tilde{r}_t^* \quad = \quad r_t^{*f} - E_t cov_t(m_{t+1}^*, r_t^*) \tag{3}$$

Combining (2) and (3), the observed short-term home-foreign interest differential, $r_t^d \equiv \tilde{r}_t - \tilde{r}_t^*$, is relative risk-free returns plus a relative bond premium:

$$r_t^d = (r_t^f - r_t^{*f}) - E_t[cov_t(m_{t+1}, r_t) - cov_t(m_{t+1}^*, r_t^*)] \tag{4}$$

From the perspective of the home investor, UIP is derived from the Euler equation for home short-term bonds (equation 1) and the following Euler equation for foreign short-term bonds:

$$Q_t \quad = \quad E_t M_{t+1}(1 + r_t^*)Q_{t+1}, \tag{5}$$

where Q_t is the real exchange rate (value of the foreign currency). Equation (5) is a no-arbitrage condition that equates the cost of buying a unit of the foreign bond this period, Q_t, to the expected, discounted return of the foreign bond at time, $t+1$, in domestic currency terms.

Taking logs, UIP can be written as:

$$E_t(q_{t+1}) - q_t = r_t^d + \lambda_t, \tag{6}$$

where $q_t = log(Q_t)$, and $\lambda_t \equiv E_t(q_{t+1}) - q_t - r_t^d$ is the expected 'excess return' to holding foreign currency or the foreign exchange rate premium. Abstracting from risk, if the home real interest rate is expected to rise relative to the foreign real rate, the no-arbitrage condition requires an immediate appreciation of the home currency (Dornbusch, 1976) so that it can depreciate over the period of high home returns. The initial appreciation eliminates all future excess returns, the subsequent depreciation offsets the higher interest payoffs so there is no excess return to holding either the home or foreign asset.

Interpreting excess returns in terms of consumption risk premia,

$$\begin{aligned} \lambda_t &= E_t\Big[cov_t(m_{t+1}, r_t) - cov_t(m_{t+1}, r_t^*) - cov_t(m_{t+1}, q_{t+1})\Big] \\ &= E_t\Big[cov_t(m_{t+1}, r_t) - cov_t(m_{t+1}, r_t^*) - cov_t(m_{t+1}, \Delta q_{t+1}) - cov_t(m_{t+1}, q_t)\Big] \end{aligned} \tag{7}$$

The first two terms increase yields on home and foreign bonds that perform poorly in bad times, when consumption is expected to fall (the marginal utility of consumption is expected to rise). The final two terms in equation (7) increase the value of currencies that are expected to appreciate (safe-haven currencies) or are strong when the marginal utility of consumption is expected to rise.[11] Interpreting the 'excess return' as a risk premium, equation (6) says that the home currency should depreciate to offset relative *risk-adjusted* returns $(r_t^d + \lambda_t)$.

Comparing (4) and (7), we see that the interest differential and the exchange rate reflect a common risk premium, $-cov_t(m_{t+1}, r_t)$.[12] That common premium is a potential source of estimation bias in the standard interest parity test that regresses ex-post exchange rate changes on ex-ante interest differentials:

$$\Delta q_{t+1} = c + \beta r_t^d + \epsilon_t \tag{8}$$

where the coefficient β has a theoretical value of one,[13] but empirical estimates of β tend to be small and are often negative.

Adding $q_{t+1} = E_{t+1}q_{t+1}$ to equation (6) and rearranging, we can see that ϵ_t includes the exchange rate premium, λ_t, plus the change in expectations, from time t to $t+1$ about q_{t+1}. The latter should be unanticipated if we assume rational expectations.

$$\Delta q_{t+1} = r_t^d + \underbrace{\lambda_t + [E_{t+1}(q_{t+1}) - E_t(q_{t+1})]}_{\epsilon_t} \tag{9}$$

If there is a common premium in the residual, ϵ_t, and the explanatory variable, r_t^d, then the estimate of β in equation (8) will be biased:

$$\hat{\beta} = \beta + \frac{cov_t(r_t^d, \epsilon_t)}{var(r_t^d)}$$

As markets become more complete, the the model predicts increasingly severe estimation bias. When all investors hold a global portfolio and consumption is perfectly correlated across countries, $m_t = m_t^*$, home and foreign risk-free rates are equal and the interest rate differential (equation 4) reflects only the relative bond premium, λ_t^R:

$$r_t^d \;\; = \;\; -\lambda_t^R = E_t[-cov_t(m_{t+1}, r_t) + cov_t(m_{t+1}, r_t^*) + cov(m_{t+1}, \Delta q_{t+1})] \tag{10}$$

The first three terms of the risk premium, λ_t, in the exchange rate equation (7) reflects the bond premium (equation 10) plus an idiosyncratic currency premium: $\lambda_t = \lambda_t^R - cov_t(m_{t+1}, q_t)$. In that case, the estimation bias is -1: we expect to estimate $\beta = 0$, ie, there is no reduced-form empirical relationship between Δq_{t+1} and r_t^d.

[11] Lustig and Verdelhan (2007) show that investors earn excess returns on portfolios of high interest currencies that depreciate when US consumption growth is low; and negative excess returns on low interest currencies that provide a hedge against US consumption growth risk.

[12] Although r_t and r_t^* are known ex ante, the payoff is also subject to credit default risk and liquidity risk. For now, we abstract from term premia and interest rate risk.

[13] The test assumes that the assets denominated in home and foreign currency have similar risk characteristics. In practice, that can only be true if currency revaluation risk is small.

In the complete markets case, the model also predicts disconnect between the bond premium and risks that price the currency. Rearranging (6) and substituting in (7) and (10):

$$\begin{aligned} q_t &= -r_t^d - \lambda_t + E_t(q_{t+1}) \\ &= \lambda_t^R - (\lambda_t^R - cov_t(m_{t+1}, q_t)) + E_t(q_{t+1}) \\ &= cov_t(m_{t+1}, q_t) + E_t(q_{t+1}), \end{aligned} \tag{11}$$

we see that the premium that prices the bond market, λ_t^R, has no role in pricing currency returns. That theoretical result provides an interpretation of Burnside (2012) and Sarno et al (2012)'s empirical finding that risk factors that help to price domestic assets do not help to price currency returns. Intuitively, only *risk-adjusted* returns are important in pricing the currency.

In a multi-period setting, consider the following investment: borrow one unit of home currency at the short-term rate, r_t, invest it abroad at the foreign short-term rate, r_t^*, and keep rolling over the bonds indefinitely. Using the notation of Engel and West (2010), the real exchange rate can be written as:[14]

$$q_t = -R_t - \Lambda_t + E_t\bar{q}_t, \tag{12}$$

where q_{t+N} is the real exchange rate, the 'level' excess return, Λ_t, is the expected forward sum of future short-term excess returns, $\Lambda_t = E_t \sum_{k=0}^{\infty} \lambda_{t+k}$, and the sum of expected future relative interest payoffs, R_t, is $E_t \sum_{k=0}^{\infty} r_{t+k}^d$. The expected long-run equilibrium exchange rate, $E_t\bar{q}_t$, reflects factors such as the terms of trade and relative productivity (Benigno and Thoenissen, 2003). Defining m_{t+j} as the j-step ahead log stochastic discount factor, the level exchange rate risk premium, Λ_t, can be written in terms of risk adjustments:

$$\Lambda_t = E_t \sum_{j=1}^{\infty} \left[cov_t(m_{t+j}, r_{t+j-1}) - cov_t(m_{t+j}, r_{t+j-1}^*) - cov_t(m_{t+j}, q_{t+j}) \right] \tag{13}$$

For simplicity of exposition, for now, I abstract from terms in $cov_t(m_{t+j}, m_{t+j+1})$ and $cov_t(m_{t+j}^*, m_{t+j+1}^*)$ (term premia), and terms in $cov_t(r_{t+j-1}, r_{t+j})$ and $cov_t(r_{t+j-1}^*, r_{t+j}^*)$ and $cov_t(q_{t+j}, q_{t+j+1})$. Those terms are reflected in multi-period returns. Their contribution to the common bond premium is discussed below.

Summing (4) forward, and abstracting from the same multi-period covariance terms, the expected sum of future short-term interest returns, R_t is the sum of

[14] This relationship can be expressed in real terms (Engel and West 2005 and Nason and Rogers 2008) or nominal terms (Engel and West, 2010).

expected 'risk-free' relative returns plus risk adjustments:

$$
\begin{aligned}
R_t &= R_t^f - \Lambda_t^R \qquad (14)\\
\text{where, } R_t^f &= E_t \sum_{j=1}^{\infty} (r_{t+j-1}^{f} - r_{t+j-1}^{f*}), \text{ and}\\
\Lambda_t^R &= E_t \sum_{j=1}^{\infty} \left[cov_t(m_{t+j}, r_{t+j-1}) - cov_t(m_{t+j}^*, r_{t+j-1}^*) \right] \qquad (15)
\end{aligned}
$$

Equations (12) and (14) form a two-equation partial equilibrium asset price model that incorporates consumption risk adjustments:

$$
\begin{aligned}
q_t &= -R_t - \Lambda_t + E_t \bar{q}_t \qquad (16)\\
R_t &= R_t^f - \Lambda_t^R \qquad (17)
\end{aligned}
$$

Equation 16 is Engel and West (2010)'s exchange rate asset price equation. Equation 17 extends Lustig and Verdelhan (2007)'s 'currency premium', that expresses relative returns as risk-free returns plus risk-adjustments, to an infinite horizon. Here I refer to those risk adjustments as the 'bond premium', Λ_t^R.

Comparing equations (13) and (15), we can see that there is a common component, $E_t \sum_{j=1}^{\infty} cov_t(m_{t+j}, r_{t+j-1})$ in the two premia. In addition, when we consider multi-period investments, the common premium will include the home term premium (terms in $cov(m_{t+j}, m_{t+j+1})$), and terms in $cov(r_{t+j-1}, r_{t+j})$ and $cov(r_{t+j-1}^*, r_{t+j}^*)$). When R_t and Λ_t reflect a common premium, reduced-formestimates of (16) will be biased.

Engel and West (2005, 2010) find correlations between changes in exchange rates and changes expected returns, $-\Delta R_t$ to be generally positive, but weak compared to the UIP-implied level. One potential interpretation of those weak correlations, as Engel and West suggest, is covariance between fundamentals (R_t) and unobserved variables (Λ_t).

If we estimate the exchange rate asset price equation in reduced form:

$$
\Delta q_t = -\alpha \Delta R_t \underbrace{- \Delta \Lambda_t + [E_{t+1}\bar{q}_t - E_t \bar{q}_t]}_{\text{unobservables, } \varepsilon_t} \qquad (18)
$$

and there is a common premium in R_t and Λ_t, then our estimate of α in equation (18) will be biased:

$$
\begin{aligned}
\hat{\alpha} &= \alpha + cov_t(-\Delta R_t, \varepsilon_t)/var(\Delta R_t)\\
&= \alpha - cov_t(\Delta \Lambda^R, \Delta \Lambda_t)/var(\Delta R_t)
\end{aligned}
$$

As in the one-period case, the estimation bias is increasingly severe as markets become more complete. With complete markets, $R_t^f = 0$ so relative interest returns reflect only the bond premium ($R_t = -\Lambda_t^R$). The bond premium, Λ_t^R, is fully reflected in the exchange rate premium, Λ_t, so the estimation bias converges on -1.

There is no reduced-form relationship between the exchange rate and relative interest returns. The exchange rate reflects only an idiosyncratic 'currency premium', $E_t \sum_{j=1}^{\infty} cov_t(m_{t+j}, q_{t+j-1})$, and the equilibrium real exchange rate $E_t\bar{q}_t$:

$$q_t \;=\; \underbrace{\Lambda_t^R}_{-R_t} - \Lambda_t^R - E_t \sum_{j=1}^{\infty} cov_t(m_{t+j}, q_{t+j-1}) + E_t\bar{q}_t \qquad (19)$$

The bond premium does not help to explain currency returns because it enters with opposite signs in R_t and Λ_t.

3 Empirical strategy

How can we assess the estimation bias problem empirically? There are at least two potential approaches. One is the method of instrumental variables.[15] If we can find an instrument for risk-free returns, or for the bond premium, then we can estimate a single-equation model without bias in the estimation of β in equation (8) or α in equation (18). Yield curve factors may serve as instruments, if those factors reflect the bond premium and are uncorrelated with risk-free returns, or vice versa. A potential problem with this approach is that many types of risk are included in the bond premium - consumption risk, credit default risk, term premium, interest rate risk, currency revaluation risk - and those premia may be correlated. Moreover, the stochastic discount factor both defines the risk-free rate and prices risk, so risk-free rates and bond premia may be correlated. Kiley (2013) finds results closer to UIP using surprises in Eurodollar futures four quarters ahead as instruments for innovations in the future path of short-term rates during monetary policy event windows.[16]

A second approach to assessing the degree of estimation bias is to estimate a structural model. The identification problem is similar to a simple supply-demand identification problem: we can't estimate a demand curve without accounting for changes in supply because demand and supply have opposite-signed effects on price and quantity. Similarly, we can't estimate the relationship between exchange rate movements and interest rates without accounting for the different effect of the risk-free rate and the bond premium on the two observed variables. Here, I estimate the structural two-equation model (16) and (17), using Bayesian techniques.

[15] Thanks to Hugo Vega for suggesting this.

[16] Through the lens of the model derived here, that may work because Eurodollar futures, like the Treasury bonds used as an explanatory variable, reflect common movements in risk-free rates, but have a different risk profile. Treasury bonds reflect government risk (or 'specialness') while Eurodollar futures reflect the underlying interbank Libor curve. There are also differences in term premia and collateralisation.

3.1 Identification

To enable identification of the bond premium, we can rewrite equations (16) and (17) as follows:

$$q_t = -R_t - \Lambda_t^R - \Lambda_t^{FX} \tag{20}$$
$$R_t = R_t^f - \Lambda_t^R \tag{21}$$

The unobserved component Λ_t^{FX}, is the exchange rate premium, Λ_t, net of the bond premium and the equilibrium exchange rate, $\bar{q}_t$. For convenience, I will call Λ_t^{FX} the idiosyncratic 'currency premium', while noting that it also includes non-risk fundamentals, $\bar{q}_t$, such as the terms of trade and relative productivity.

$$\Lambda_t^{FX} \equiv \Lambda_t - \Lambda_t^R - E_t\bar{q}_t$$
$$\Lambda_t^{FX} = E_t \sum_{j=1}^{N} \Big[(cov_t((m_{t+j} - m_{t+j}^*), r_{t+j-1}^*) + cov_t(m_{t+j}, q_{t+j})\Big] - E_t\bar{q}_t \tag{22}$$

In the case of incomplete markets (22), the risk component of Λ_t^{FX}, reflects the difference in home and foreign discount factors and a currency premium. In the case of complete markets ($m_t = m_t^*$), Λ_t^{FX} includes the premium $E_t \sum_{j=1}^{N} cov_t(m_{t+j}, q_{t+j-1})$ and long-run fundamentals.

There are two main issues associated with estimating this model. First, unit root tests (Table 2) show that q_t, R_t and $\Lambda_t = -(q_t + R_t)$ test as integrated for most currency pairs. Therefore, following Engel and West (2005), the relationship is estimated in differences:[17]

$$\Delta q_t = -\alpha \Delta R_t - \Delta\Lambda_t^R - \Delta\Lambda^{FX} \tag{23}$$
$$\Delta R = \Delta R_t^f - \Delta\Lambda_t^R \tag{24}$$

The parameter α is added because we want to estimate the exchange rate response to expected returns, not impose UIP.

The second potential problem is that the model (23) and (24) is under-identified. We want to estimate three shock variances and one parameter but, with q_t and R_t as observed variables, there are only three distinct elements of the reduced-form variance covariance matrix. The sign restriction $\alpha > 0$ from the theoretical model provides the identifying restriction for the structural interpretation.[18] Intuitively, variation in q_t and R_t is put into three buckets: negative co-movement between q_t and R_t is attributed to R_t^f; positive co-movement to Λ_t^R; and exchange rate

[17] Estimating the model in differences is more demanding because the unconditional correlations between the exchange rate and expected returns are weaker in differences than in levels (Table 1). Estimating in differences also means that estimates should be less affected by any structural shifts. Estimation in levels with AR(1) innovation produces qualitatively similar results.

[18] For consistency, the same sign restriction ($\alpha > 0$) is imposed in the reduced-form model. That restriction potentially affects the results for the CAD for which the unconditional correlation between Δq_t and ΔR_t is positive (see Table 1).

fluctuations that don't fit well in those buckets are attributed to the idiosyncratic currency premium Λ_t^{FX}.

The model nests Engel and West's asset price model. If we assume that interest returns are risk-free, then there is no common bond premium, and the model reduces to:

$$\begin{aligned} \Delta q_t &= -\alpha \Delta R_t - \Delta \Lambda_t^{FX} &(25)\\ \Delta R_t &= \Delta R_t^f &(26) \end{aligned}$$

In this 'reduced-form' model, risk is an exogenous process that affects q_t but not R_t. We can estimate the parameter, α, and the standard deviations of the innovations in R_t^f and Λ_t^{FX}, respectively σ_{Rf} and σ_{FX} from three distinct elements of the covariance matrix.

If the true model is (23) and (24), but we estimate the reduced-form model (25) and (26), then the estimated parameter, $\hat{\alpha}$ is biased:

$$\begin{aligned} \hat{\alpha} &= \alpha + cov_t(-\Delta R_t, -\Delta \Lambda_t^R)/var(\Delta R_t) \\ &= \alpha - var(\Delta \Lambda_t^R)/var(\Delta R_t) &(27) \end{aligned}$$

Since variances must be positive, the parameter α will be biased downwards from its theoretical value of one, consistent with the weak unconditional correlation between Δq_t and $-\Delta R_t$ in Engel and West (2005) and Table 1.

Estimates of the standard deviations of relative risk-free returns and the bond premium from the full model (equations 23 and 24) inform on potential estimation bias in the reduced-form model. If the variance of Λ_t^R is estimated to be small compared to the variance of R_t^f, then the bias in the reduced-form estimate of α should be small.

3.2 Forecasts of real returns

To estimate the model (23) and (24) we need measures of q_t and R_t. The real exchange rate is constructed from the nominal exchange rate adjusted for relative CPI inflation. To measure R_t, we need a forecast of future relative interest returns. I employ the market-based measure of expected future short-term (Libor or equivalent) nominal returns provided by the interest rate swap market. The swap rate is the rate the market is willing to pay (receive) in exchange for floating-rate interest payments (receipts). When participants agree on a fixed rate, it should be a good forecast of future floating rate payments.

A zero-coupon interest rate swap[19] equates the value of a single fixed payment at maturity to the expected compounded returns on floating interest rates up to that

[19] Zero-coupon swap rates are derived from ordinary interest swap rates (see Hull (2000), p90-92). Both the exchange rate and the swap are priced under 'risk-neutral' probabilities, that is, they are arbitrage-free prices.

maturity. Abstracting from risk, the N-period zero-coupon swap rate, i_t^z, multiplied by N periods provides a forecast of the sum of future short-term nominal floating Libor interest rates, i_{t+k}, over the N-period horizon:

$$\begin{aligned} (1+i_t^Z)^N &= E_t \prod_{k=1}^{N}(1+i_{t+k-1}) \\ \text{taking logs,} \qquad N\, i_t^Z &\simeq E_t \sum_{k=1}^{N} i_{t+k} \end{aligned}$$

That is not the infinite un-discounted sum we would like, but it is a forecast of short-term interest returns over a long horizon, based on transacted prices.[20]

Expected relative real returns, R_t, are defined as expected relative nominal returns net of expected relative inflation:

$$\begin{aligned} R_t &= \sum_{k=0}^{119}(i_{t+k} - i^*_{t+k}) - E_t \sum_{k=1}^{120}(\pi_{t+k} - \pi^*_{t+k}) \\ &\approx 120\,(i_t^{sw10} - i_t^{*sw10}) - \frac{(\rho_\pi)^2(1-\rho_\pi^{120})}{1-\rho_\pi}(\pi_{t-1} - \pi^*_{t-1}) \end{aligned} \tag{28}$$

where home and foreign ten-year nominal swap rates i_t^{sw10} and i_t^{*sw10} (% per month) are multiplied by 120 months to proxy a 120 month sum of returns. The expected 10-year sum of future relative inflation is proxied by an N-period AR1 forecast, based on observed $t-1$ inflation.[21] The AR(1) coefficient for inflation is estimated jointly with other parameters.

Swap contracts, compared to bonds of the same maturity, have less credit default risk,[22] but still reflect other premia such as term premia, interest rate risk and currency revaluation risk. Feldhütter and Lando (2008) argue that the risk-free rate is better proxied by the swap rate than the Treasury rate for all maturities. However, they show that, even the swap rate has a substantial risk component. Using an observed interest rate that is relatively close to the risk-free rate should bias our results towards a small role for the common premium.

3.3 Data

The data set covers eight US dollar (USD) currency pairs: the Australian dollar (AUD), Canadian dollar (CAD), Swiss franc (CHF), euro (EUR), British pound

[20] Those forecasts are the basis for a vast volume of transactions: the Bank for International Settlements (2013) reports that the notional amount of interest rate swaps outstanding globally in December 2012 was $490 trillion.

[21] Break-even inflation rates, derived from inflation-indexed bonds, might provide a better measure of expected inflation. In practice, inflation-indexed bonds are only systematically issued in a few jurisdictions, markets are often not very liquid and data samples are short.

[22] Credit default risk is low because no principal is exchanged and collateral is posted against out-of-the-money positions. Relative credit default risk is also low because the counter-parties are often similarly-rated banks. See Duffie and Singleton (1997).

(GBP), Japanese yen (JPY), New Zealand dollar (NZD) and Swedish krona (SEK). The home currency is the USD. The data used are the nominal exchange rate, relative CPI prices and the nominal ten-year zero coupon swap rate differential. The sample period begins on the month that zero-coupon interest rate swap data for the currency pair is available on Bloomberg, and ends in March 2014. Exchange rate and interest rate data are end-month. Data sources are shown in Appendix B.

The real exchange rates and forecasts of un-discounted relative real interest rate returns are shown in Figure 1 for the eight USD currency pairs. Forecast revisions and exchange rate changes are shown in Figure 2.

3.4 Estimation method and prior distributions

The model is estimated using Bayesian techniques.[23] First, the mode of the posterior distribution is estimated by maximizing the log posterior function, which combines the prior information on the parameters with the likelihood of the data. Second, the Metropolis-Hastings algorithm is used to sample the posterior space and build the posterior distributions. The posterior distributions are from a Metropolis Hastings chain of 400,000 draws, of which the first third is discarded. Acceptance rates are about 20-40%. Convergence is established using chi-squared statistics comparing the means of the beginning and end of the retained Markov chain (Geweke, 1992). Priors restrict α and shock variances to be positive (top of Table 3). The restriction on α provides the identifying sign restriction for the structural interpretation.

The observed, demeaned, data are the CPI-based real exchange rate, the ten-year zero-coupon swap differential, and relative annual CPI inflation. For estimation, the full model also includes an expression for the forecast of expected real returns R_t (equation 28), accounting identities that relate levels and differences, and an AR(1) process for the evolution of annual inflation ($\pi_t^{12} - \pi_t^{12*}$).

4 Results

4.1 Estimation bias

Prior and posterior distributions, for the reduced-form model (left column) and the model with risk (right column), are shown in Figure 3. The posterior distributions are identified in the sense that they are distinct from the prior distribution. The posterior mode parameter estimates are summarised in Table 3.

For the reduced-form model, the results are qualitatively similar to those in Engel and West (2005), despite the different approach to forecasting future returns (Engel and West construct forecasts of future fundamentals (R_t) using AR(1) and VAR(2) forecasts; here we use interest rate swaps) and the different time periods.

[23]See An and Schorfheide (2007) for a description of this methodology. The estimation is implemented in Dynare (see Adjemian et al 2011).

Engel and West find the relationship between exchange rate movements and changes in expectations of future returns to be mostly of the correct sign, but weak relative to theory. Here the estimated value of α is positive by construction because of the sign restriction. With the exception of the CAD/USD, the sign restriction should have no effect on the reduced-form estimates because the unconditional correlation between Δq_t and $-\Delta R_t$ is otherwise positive (Table 1). As in Engel and West (2005), the estimated exchange rate response to changes in expected returns is well below one. The modes of the posterior distributions for α in the reduced-form model average 0.34 and range from 0.13 (CAD) to 0.51 (EUR). One is outside the 90% confidence bounds in all cases.

To assess estimation bias, we want to know whether the variance of the common bond premium, in the model with risk, is large enough relative to the variance of R_t, to materially bias estimates of α in the reduced-form model? As shown in Figure 3, the bond premium variance, σ^R, is well identified in the sense that it is distinct from the prior distribution. The implied estimation bias in the reduced-form model is the ratio of the variance of changes in the common bond premium to the variance of changes in expected gross returns. As shown in the final column of Table 3 (bottom panel), the implied reduced-form estimation bias is, on average, -0.50, so α is severely biased downwards from a theoretical value of one.

The estimation bias is most severe for the CAD/USD model. In that case, the large bias is the result of a relatively small variance of relative risk-free returns, so a small variance of ΔR_t relative to the variance of $\Delta\Lambda_t^R$ in equation (27). The low variance of Canadian risk-free returns, relative to US risk-free returns, is interesting. As markets become more complete, relative risk-free rates converge. So the smaller variance of changes in US-Canadian risk-free rates suggests a greater degree of risk sharing between Canada and the US. Economic integration may mean that consumption growth, and so stochastic discount factors, and in turn monetary policy are more correlated.

When accounting for a common risk factor, the estimated exchange rate response to expected relative interest returns is considerably stronger. The average estimate of α rises from 0.34 in the reduced-form model to 0.75 in the model with risk. The theoretical value of one is only outside the 90% confidence bounds for one (CAD) of the eight currency pairs. For seven of the eight currency pairs, we cannot reject the no-arbitrage UIP condition.

4.2 Variance decomposition

The results described in the previous section show that, when risk is accounted for, exchange rates and expected returns are estimated to be closely linked, in the way predicted by asset price models. What does that tell us about the drivers of interest rates and exchange rates? The variance decomposition is shown in Table 4.

In the reduced-form model (top panel), the variance of expected returns, R_t is attributed only to innovations in the risk-free rate, by construction. In contrast, in the model with risk, on average 52% of the variance in expected returns is attributed to the bond premium. That is large relative to the assumption that swap rates are near risk-free. However, even though Feldhütter and Lando (2008) argue that the swap rate is a good proxy for the the risk-free rate, they estimate a sizeable premium component. Moreover, even if swap rates are near risk-free for the domestic investor, exchange rate revaluation risk may be large for the foreign investor.

In the model with risk, risk-free returns account for 20% of exchange rate variance, on average, compared to 6% in the reduced-form model. In the model with risk, risk-free returns play a larger role for all currency pairs. Nevertheless, interest rates still account for a minor share of exchange rate variance. Several factors contribute to that result. First, only risk-free returns matter for the exchange rate. For the CAD, in particular, a small variance of relative risk-free returns translated into a small contribution of relative returns to exchange rate variance (9%, up from 1% in the reduced-form model). If markets become more complete, we can expect the contribution of relative returns to exchange rate variance to decline, as risk-free rates converge.

Second, the bond premium component of observed interest rates contributes little to exchange rate variance. The bond premium accounts for about 52% of changes in relative asset returns R_t, on average, but only about 3% of changes in the exchange rate. That decomposition is consistent with Burnside (2012) and Sarno et al (2012)'s result that risk factors that price domestic asset markets do not explain currency returns, and that risk factors that price currency returns do not price domestic asset markets. When $\hat{\alpha}$ is near one, the bond premium component of interest returns, R_t, and the bond premium component the exchange rate premium cancel out in the exchange rate equation (23). When $\hat{\alpha}$ is near one, the real exchange rate reflects only risk-adjusted interest returns, an idiosyncratic currency premium, and fundamentals that affect the equilibrium exchange rate.

In the model with risk, the idiosyncratic currency premium, Λ_t^{FX}, still dominates exchange rate variance for all currency pairs. On average, it accounts for almost 80% of exchange rate variance. That large share reflects the relatively large variance of innovations in the currency premium (average standard deviation of 2.9% per month) compared to innovations in relative risk-free returns (average standard deviation of 1.8%). The 'currency premium' includes not only risk, but also time-varying drivers of the equilibrium exchange rate such as the relative terms of trade and relative productivity (Benigno and Thoenissen, 2003). Those effects may be large, particularly for countries with volatile terms of trade (Chen and Rogoff, 2002).

4.3 Unobserved components and 'monetary policy'

In the theoretical model, the risk-free rate is defined by the investor's consumption discount factor. While the investor's discount rate is often equated with the monetary policy rate in macroeconomic models, the monetary policy rate is not necessarily equal to the investor's risk-free rate. As discussed in Broadbent (2014), if the risk-free rate is low[24] then inflation pressures are likely to be weak, leading to an easing of monetary policy.

Correlations between the unobserved components and changes in 'monetary policy' measured by changes in the relative nominal 30-day interest differential, are shown in Table 5. Changes in short-term interest rates are significantly positively correlated with changes in relative risk-free rates for all currency pairs, supporting the idea that monetary policy is, in some way, related to the unobserved 'risk-free rate'. The correlation between changes in relative short-term nominal rates and changes in expected risk-free rates ΔR_t^f averages 0.18, which is about the same as the correlation with changes in the expected forward interest path, ΔR_t.

A rise in the relative home short-term rate is also negatively correlated with changes in the foreign bond premium, but only weakly correlated with changes in the currency premium. The correlation with the bond premium could reflect the measure of monetary policy: the change in 30-day Libor, or equivalent, is really a measure of expected changes in policy rates over the next month plus the Libor-OIS premium. It could also reflect a more general correlation between risk-free returns and bond premia (6). When expected US relative risk-free returns rise, the relative foreign bond premium falls. That correlation is consistent with the counter-cyclical relationship between foreign currency premia and the US economy described in Sarno et al (2012), Lustig et al (2011) and Lustig and Verdelhan (2007). In a general equilibrium model, time-varying risk premia are correlated with changes in economic variables through the optimising behaviour of consumers (Obstfeld et al, 2002). The stochastic discount factor both defines the risk-free rate and prices risk.

4.4 Unobserved components and risk

In the theoretical model, Λ_t^R is a bond premium and Λ_t^{FX} is a currency premium plus fundamentals. Can we relate the unobserved components from the empirical model to observed measures of risk? There are many potential measures of bond and currency premia (Sarno et al 2012, Burnside 2012). Here I consider two currency-related measures: the VIX index, which is often referred to as a measure of 'risk aversion', and non-commercial positioning in foreign exchange futures markets on the Chicago Mercantile Exchange International Money Market (IMM), that is often referred to as 'speculative positioning'.

Correlations between the changes in the unobserved components and changes in the VIX index are shown in Table 7. With the exception of the GBP and JPY, 'reserve currencies', a rise in the VIX index is associated with a rise in the foreign currency premium. That result is consistent with studies of currency excess returns

[24]Risk free rate may be low because people are uncertain about the future, so save more, or because expected consumption growth is low.

(for example, Sarno et al 2012 and Lustig et al 2011), and the well-known flight to quality that tends to accompany a rise in the VIX index. Correlations with changes in the bond premium are weaker and less systematic.

Correlations between changes in the unobserved components with IMM 'speculative positioning' in foreign exchange markets is shown in Table 8. Changes in IMM positioning are strongly correlated with changes in the idiosyncratic 'currency premium' for all currencies. When speculative positioning in a currency rises relative to the USD, that currency's premium falls. The correlation coefficients range from -0.43 to -0.53, and average -0.49. In contrast, the correlation between IMM positioning and the common bond premium is weak, averaging 0.01, and is only weakly significant for the CAD.

Changes in IMM positioning are also correlated with innovations in relative risk-free rates. Through the lens of the structural model with risk, when positioning in a foreign currency increases, that currency strengthens for two reasons: (i) foreign risk-free rates rise relative to home risk-free rates, and (ii) the foreign currency premium falls. There is a variety of potential causal interpretations behind those correlations. For example, the fall in the foreign currency premium (or an improvement in fundamentals such as the terms of trade) that appreciates the foreign currency, may stimulate the foreign economy, increasing expected consumption growth and the foreign risk-free rate.

In the empirical model, Λ_t^R and Λ_t^{FX} are interpreted as risk premia and fundamentals. However, we cannot rule out a role for the supply and demand effects of cross-border capital flows. Empirically, cross-currency flows are large and volatile.[25] Evans and Lyons (2002) and Evans and Lyons (2006) show that flows through foreign currency markets have strong explanatory power for exchange rate movements. A capital outflow could be reflected in either the common bond premium[26] or, if the outflow has a larger effect on the foreign exchange market than on the fixed income market, in the 'currency premium'. In principle, assets can be repriced without actual flows (Fama, 1965). Therefore a lasting role for flows would imply some sort of limit to capital free arbitrage (Shleifer and Vishny, 1997). Innovations in the unobserved components in our model premia have lasting effects because they are near-random walk processes.

[25] For advanced countries, gross current account credits and debits typically account for less than 1% of foreign exchange market turnover reported in the BIS Triennial Central Bank Survey of Foreign Exchange and Derivatives Market Activity.

[26] A capital flow has the same sign properties as the bond premium. A capital outflow from the foreign country implies a fall in demand for the foreign currency, so a depreciation of the foreign currency; a fall in the supply of funding in the foreign fixed-income market, so a rise in foreign yields; and an increase in the supply of funding in the home currency fixed income market, so a fall in home yields.

5 Robustness

Several changes to the empirical specification were considered. Those changes are briefly described below and discussed in more detail in Appendix C.

When R_t is constructed from 10-year **plain vanilla swaps** rather than from 10-year zero-coupon swaps, R_t is a smaller, discounted sum of expected returns. In the model with risk, the estimated exchange rate response to that smaller sum is predictably larger: α averages 0.97 compared to 0.75 when R_t is an un-discounted sum. The posterior distributions in the benchmark model are less normal in shape, a result that is invariant to a longer Metropolis Hastings chain. It is possible that constructing zero-coupon swaps from plain vanilla swaps distorts the data. Constructing zero-coupon swaps, to get an un-discounted sum of future returns, requires assumptions to be made about the risk-free rate, that may be at odds with the decomposition used here. Whether an un-discounted sum - ie. a discount rate of one - is the correct for the UIP condition is examined empirically in Engel and West (2005), Nason and Rogers (2008) and Kano (2014).

In the model with risk, the estimated exchange rate response to the **nominal component of** R_t (equation 28) is a bit stronger (α_i=0.83) and the response to the **inflation component** is a bit weaker (α_π=0.72). While a rise in home inflation implies a depreciation of the long-run level of the exchange rate, a Taylor rule monetary policy response implies a rise in the home real rate in the short term. Clarida and Waldman (2007) show that, empirically, the home exchange rate initially appreciates in response to a rise in home inflation, consistent with the weaker response to the inflation component of returns estimated here.

The estimated exchange rate response to a 5-year sum of expected returns, is 1.21, on average, and more variable across currency pairs. The estimated response to a longer 15-year sum of expected returns, averages 0.98, and the posterior mode estimates are more consistently centered near the theoretical value of one. Those results favour using a longer maturity forecast.

When changes in the unobserved components R_t^f, Λ_t^R and Λ_t^{FX} are modeled as **AR(1) processes**, rather than iid innovations, or include a **lagged level term**, the results are qualitatively similar: $\hat{\alpha}$ averages 0.76 and 0.78 respectively. The AR(1) coefficients and error correction coefficients are estimated to be small, consistent with the near-random walk behaviour of the level variables. Finally, when the model is **estimated in levels** rather than in differences, the average posterior estimate for α was qualitatively similar: 0.34 for the reduced-form model and 0.88 for the model with risk.

An **alternative identification scheme** is estimated. In the alternative scheme, only one risk premium is estimated and the bond premium is an estimated multiple of the exchange rate premium. This identification scheme serves as a check that we haven't biased the results by including Λ_t^R in the exchange rate equation with a coefficient of one. The alternative scheme produces more severe reduced-form estimation bias, averaging -0.68, compared to -0.50 in the benchmark model. Accordingly the posterior mode estimates for α are higher, averaging 0.97, up from 0.75. Perhaps the greatest difference is that 42.5% of exchange rate variation is attributed to risk-free returns, compared to 19% in the benchmark model.

The empirical approach used here is parsimonious, using two observable variables to inform on estimation bias. The empirical model can be disaggregated in a number of other ways and extended to a linear general equilibrium framework where it would provide measures of expected risk-free rates and bond premia.

6 Conclusion

Exchange rates, risk and return need to be jointly modeled. This paper derives an asset price model with explicit risk adjustments, and shows that relative interest returns and the exchange rate reflect a common bond premium. That bond premium is estimated to severely bias the reduced-form relationship between exchange rates and expected returns.

The model derived here predicts two types of disconnect that become more pronounced as markets become more complete. First, it predicts disconnect in the reduced-form relationship between exchange rates and interest rates. That result helps to explain the empirical failure of UIP (Fama, 1984) and the weak exchange rate response to relative returns in Engel and West (2005, 2010). Second, the model predicts disconnect between risk factors that price bond markets and risk factors that price exchange rates. That result is consistent with the empirical findings of Burnside (2012) and Sarno et al (2012). Only risk-adjusted asset returns matter for exchange rates.

When the common premium is accounted for, the estimated exchange rate response to expected returns is considerably closer to the theoretical value of one for a set of eight advanced country USD currency pairs. In the model with risk, risk-free returns account for an average 19% of exchange rate variance, compared to an 6% when observed interest rates are assumed to be risk-free. The unobserved currency premium and fundamentals, such as the terms of trade and productivity, account for 79% of exchange rate variance, on average. Overall, the results support the idea that a time-varying risk premium is at the heart of the failure of UIP. When accounting for risk, exchange rates and interest rates are estimated to be linked in the way that is predicted by asset price models (Dornbusch, 1976).

References

Adjemian, S, H Bastani, F Karamé, M Juillard, J Maih, F Mihoubi, G Perendia, M Ratto, and S Villemot (2011), "Dynare: Reference manual version 4," *CEPREMAP*, *Dynare Working Papers*, 1.

An, S and F Schorfheide (2007), “Bayesian Analysis of DSGE Models,” *Econometric Reviews*, 26(2-4), 113–172.

Backus, D, S Foresi, and C Telmer (2001), “Affine term structure models and the forward premium anomaly,” *The Journal of Finance*, 56(1), pp. 279–304.

Bank for International Settlements (2013), *Quarterly Bulletin*, Bank for International Settlements, June.

Benigno, G and C Thoenissen (2003), “Equilibrium exchange rates and supply-side performance,” *Economic Journal*, 113(486), C103–C124.

Bilson, J (1981), “The ‘speculative efficiency’ hypothesis,” *Journal of Business*, 54(3), 433–451.

Bjørnland, H C (2009), “Monetary policy and exchange rate overshooting: Dornbusch was right after all,” *Journal of International Economics*, 79(1), 64–77.

Brennan, M J and Y Xia (2006), “International Capital Markets and Foreign Exchange Risk,” *Review of Financial Studies*, 19(3), 753–795.

Broadbent, B (2014), “Monetary policy, asset prices and distribution,” Speech given by B. Broadbent, Deputy Governor for Monetary Policy, Bank of England, at the Society of Business Economists Annual Conference 23 October 2014.

Brunnermeier, M K, S Nagel, and L H Pedersen (2009), “Carry Trades and Currency Crashes,” in *NBER Macroeconomics Annual 2008*, vol 23 of *NBER Chapters*, 313–347, National Bureau of Economic Research, Inc.

Burnside, C (2012), “Carry trades and risk,” in *Handbook of Exchange Rates*, eds I W M Jessica James and L Sarno, Handbook of Monetary Economics, 283–312, John Wiley & Sons.

Chen, Y and K Rogoff (2002), “Commodity Currencies and Empirical Exchange Rate Puzzles,” *International Monetary Fund*, *IMF Working Papers*.

Chen, Y and K Tsang (2013), “What does the yield curve tell us about exchange rate predictability?” *The Review of Economics and Statistics*, 95(1), 185–205.

Chinn, M and G Meredith (2004), “Monetary policy and long-horizon uncovered interest parity,” *Staff Papers*, 513.

Chinn, M and S Quayyum (2012), “Long horizon uncovered interest parity re-assessed,” *Working Paper*, 18482.

Clarida, R and D Waldman (2007), “Is bad news about inflation good news for the exchange rate?” *National Bureau of Economic Research, Inc*, *NBER Working Papers*, 13010.

Cochrane, J (2001), *Asset Pricing*, Princeton University Press.

Coleman, A and O Karagedikli (2008), "The relative size of new zealand exchange rate and interest rate responses to news," *Reserve Bank of New Zealand*, *Reserve Bank of New Zealand Discussion Paper Series*, DP2008/12.

De Los Rios, A D (2009), "Can affine term structure models help us predict exchange rates?" *Journal of Money, Credit and Banking*, 41(4), 755–766.

Dornbusch, R (1976), "Expectations and exchange rate dynamics," *Journal of Political Economy*, 84(6), 1161–76.

Duarte, M and A C Stockman (2005), "Rational speculation and exchange rates," *Journal of Monetary Economics*, 52(1), 3–29.

Duffie, D (1992), *Dynamic Asset Pricing Theory*, Princeton University Press, Princeton, NJ.

Duffie, D and K J Singleton (1997), "An econometric model of the term structure of interest-rate swap yields," *Journal of Finance*, 52(4), 1287–1321.

Engel, C (1996), "The forward discount anomaly and the risk premium: A survey of recent evidence," *Journal of Empirical Finance*, (3), 123–192.

Engel, C (2012), "The real exchange rate, real interest rates, and the risk premium," *Paper presented at Victoria University of Wellington macroworkshop October 29 2012*.

Engel, C (2013), "Exchange rates and interest parity," *Handbook of International Economics, vol. 4 (Elsevier)*.

Engel, C and K West (2005), "Exchange rates and fundamentals," *Journal of Political Economy*, 113(3), 485–517.

Engel, C and K West (2010), "Global interest rates, currency returns, and the real value of the dollar," *American Economic Review*, 100(2), 562–567.

Evans, M D D and R K Lyons (2002), "Order flow and exchange rate dynamics," *Journal of Political Economy*, 110(1), 170–180.

Evans, M D D and R K Lyons (2006), "Understanding order flow," *International Journal of Finance and Economics*, 11(1), 3–23.

Fama, E F (1965), "The behavior of stock market prices," *Journal of Business*, 38, 34–105.

Fama, E F (1984), "Forward and spot exchange rates," *Journal of Monetary Economics*, 14(3), 319–338.

Farhi, E, S Fraiberger, X Gabaix, R Ranciere, and A Verdelhan (2009), "Crash risk in currency markets," *National Bureau of Economic Research, Inc*, *NBER Working Papers*, 15062.

Faust, J and J H Rogers (2003), "Monetary policy's role in exchange rate behavior," *Journal of Monetary Economics*, 50(7), 1403–1424.

Feldhütter, P and D Lando (2008), "Decomposing swap spreads," *Journal of Financial Economics*, 88(2), 375–405.

Flood, R P and A K Rose (1996), "Fixes: Of the Forward Discount Puzzle," *The Review of Economics and Statistics*, 78(4), 748–52.

Froot, K and J Frankel (1989), "Forward discount bias: Is it an exchange risk premium?" *Quarterly Journal of Economics*, 104.

Geweke, J (1992), "Evaluating the accuracy of sampling-based approaches to the calculation of posterior moments," 169–194, Oxford University Press.

Hull, J (2000), *Options, Futures and Other Derivatives*, Prentice Hall.

Kano, T (2014), "Exchange Rates and Fundamentals: Closing a Two-country Model," *Graduate School of Economics, Hitotsubashi University*, *Discussion Papers*.

Kearns, J and P Manners (2006), "The impact of monetary policy on the exchange rate: a study using intraday data," *International Journal of Central Banking*, 2, 157–183.

Kiley, M T (2013), "Exchange rates, monetary policy statements, and uncovered interest parity: before and after the zero lower bound," *Board of Governors of the Federal Reserve System (U.S.)*, *Finance and Economics Discussion Series*, 2013-17.

Krishnamurthy, A and A Vissing-Jorgensen (2012), "The aggregate demand for treasury debt," *Journal of Political Economy*, (2), 233–267.

Lustig, H, N Roussanov, and A Verdelhan (2011), "Common risk factors in currency markets," *Review of Financial Studies*, 24(11), 3731–3777.

Lustig, H and A Verdelhan (2007), "The cross section of foreign currency risk premia and consumption growth risk," *American Economic Review*, 97(1), 89–117.

Menkhoff, L, L Sarno, M Schmeling, and A Schrimpf (2012), "Carry Trades and Global Foreign Exchange Volatility," *Journal of Finance*, 67(2), 681–718.

Nason, J M and J H Rogers (2008), "Exchange rates and fundamentals: a generalization," *Board of Governors of the Federal Reserve System (U.S.), International Finance Discussion Papers*, 948.

Obstfeld, M, K Rogoff, E Helpman, and E Sadka (2002), *Risk and Exchange Rates*, Cambridge University Press, Cambridge.

Rafferty, B (2012), "Currency returns, skewness and crash risk," *Duke University, Working Paper*.

Sarno, L, P Schneider, and C Wagner (2012), "Properties of foreign exchange risk premiums," *Journal of Financial Economics*, 105(2), 279–310.

Scholl, A and H Uhlig (2008), "New evidence on the puzzles: Results from agnostic identification on monetary policy and exchange rates," *Journal of International Economics*, 76(1), 1–13.

Shleifer, A and R W Vishny (1997), "The limits of arbitrage," *Journal of Finance*, 52(1), 35–55.

Zettelmeyer, J (2006), "The impact of monetary policy on the exchange rate: evidence from three small open economies," *Journal of Monetary Economics*, 51, 635 – 652.

Table 1: Estimated standard deviations and correlations of q, R, and Λ

		Levels						Differences				
		q	R		Λ			Δq	ΔR		$\Delta\Lambda$	
AUD	q	22.11	-0.73	***	-0.92	***	Δq	3.61	-0.25	***	-0.84	***
	R		9.43		0.40	***	ΔR		2.06		-0.32	***
	Λ				16.59		$\Delta\Lambda$				3.69	
CAD	q	14.23	-0.01		-0.91	***	Δq	2.44	0.04		-0.82	***
	R		6.40		-0.40	***	ΔR		1.77		-0.61	***
	Λ				15.52		$\Delta\Lambda$				3.07	
CHF	q	14.24	-0.60	***	-0.86	***	Δq	3.22	-0.22	***	-0.76	***
	R		7.31		0.10		ΔR		2.37		-0.47	***
	Λ				11.48		$\Delta\Lambda$				3.55	
EUR	q	15.24	-0.42	***	-0.91	***	Δq	3.16	-0.32	***	-0.77	***
	R		6.16		0.02		ΔR		2.14		-0.35	***
	Λ				13.81		$\Delta\Lambda$				3.19	
GBP	q	8.02	-0.23	***	-0.73	***	Δq	2.41	-0.21	***	-0.68	***
	R		6.37		-0.50	***	ΔR		2.18		-0.58	***
	Λ				9.05		$\Delta\Lambda$				2.89	
JPY	q	13.97	-0.17	***	-0.74	***	Δq	3.22	-0.19	***	-0.70	***
	R		11.07		-0.54	***	ΔR		2.83		-0.58	***
	Λ				16.31		$\Delta\Lambda$				3.87	
NZD	q	20.54	-0.58	***	-0.93	***	Δq	3.67	-0.13	*	-0.84	***
	R		7.55		0.25	***	ΔR		2.20		-0.43	***
	Λ				17.30		$\Delta\Lambda$				4.03	
SEK	q	13.41	-0.53	***	-0.62	***	Δq	3.29	-0.22	***	-0.75	***
	R		11.20		-0.34	***	ΔR		2.50		-0.48	***
	Λ				12.13		$\Delta\Lambda$				3.66	
averages	q	15.22	-0.41		-0.83		Δq	3.13	-0.19		-0.77	
	R		8.19		-0.13		ΔR		2.26		-0.48	
	Λ				14.02		$\Delta\Lambda$				3.49	

Following Table 1 in Engel and West (2010), diagonal elements are standard deviations; off-diagonal elements are correlations; and $\Lambda_t \equiv -(q_t + R_t)$. Expected relative returns, R_t, is the 10-year interest rate swap differential, net of an AR(1) forecast of relative inflation. *** indicates significance to the 1% level; ** indicates significance to the 5% level, * indicates significance to the 10% level.

Table 2: Unit root tests

	Real exchange rate, q_t			Forecast returns, R_t			$\Lambda_t = -(q_t + R_t)$		
	statistic	lag		statistic	lag		statistic	lag	
				Levels					
AUD	-2.00	0		-2.73	0	*	-2.26	0	
CAD	-2.17	0		-2.60	0		-2.15	0	
CHF	-2.31	0		-3.60	0	***	-2.46	0	
EUR	-2.24	0		-2.41	0		-2.17	0	
GBP	-2.28	0		-2.80	0	*	-2.44	0	
JPY	-2.61	0		-2.21	0		-2.79	0	*
NZD	-2.32	0		-3.46	0	**	-2.00	0	
SEK	-2.04	0		-2.87	0	*	-2.47	0	
				Differences					
AUD	-14.79	0	***	-16.47	0	***	-15.75	0	***
CAD	-16.71	0	***	-16.01	0	***	-15.55	0	***
CHF	-15.93	0	***	-13.36	1	***	-15.92	0	***
EUR	-13.99	0	***	-14.86	0	***	-13.36	0	***
GBP	-14.99	0	***	-13.69	1	***	-15.05	0	***
JPY	-15.44	0	***	-14.76	1	***	-17.07	0	***
NZD	-6.52	2	***	-12.77	1	***	-15.52	0	***
SEK	-15.26	0	***	-16.48	0	***	-15.91	0	***

Notes: Dickey-Fuller (1979) test using the Schwarz/Bayesian Information Criterion to select lag length. Maximum lag of 2.

Dickey, D and W Fuller (1979), "Distribution of the estimators for autoregressive time series with a unit root," Journal of the American Statistical Association, 74, (427-431).

*** indicates significance at the 1% level; ** indicates significance at the 5% level; * indicates significance at the 10% level. Significance levels based on sample of 200.

Table 3: Prior and posterior estimates: reduced-form model and model with risk

	α	σ_{R^f}	σ_R	σ_{FX}	ρ^{π}	σ_{π}	$\hat{\alpha}$ bias[a]
Prior							
Distribution	γ	γ^{-1}	γ^{-1}	γ^{-1}	β	γ^{-1}	
Prior mean	1	0.020	0.020	0.020	0.8	0.0003	
Prior mode	0.76	0.009	0.009	0.009	0.85	0.0001	
Prior stdev	0.5	0.500	0.500	0.500	0.1	0.0050	
Reduced-form model: interest rates are risk-free, $\Lambda^R = 0$)							
AUD	0.47	0.020	-	0.034	0.898	0.00048	-
CAD	0.13	0.018	-	0.024	0.908	0.00028	-
CHF	0.33	0.023	-	0.031	0.897	0.00028	-
EUR	0.51	0.021	-	0.030	0.894	0.00029	-
GBP	0.28	0.021	-	0.023	0.933	0.00033	-
JPY	0.25	0.028	-	0.031	0.922	0.00037	-
NZD	0.38	0.021	-	0.037	0.884	0.00045	-
SEK	0.34	0.025	-	0.032	0.914	0.00035	-
Avg.	0.34	0.022	-	0.030	0.906	0.00036	-
Model with risk							
AUD	0.79	0.017	0.015	0.033	0.898	0.00048	-0.32
CAD	0.65	0.011	0.015	0.023	0.906	0.00028	-0.64
CHF	0.68	0.019	0.020	0.030	0.899	0.00029	-0.54
EUR	0.77	0.018	0.016	0.028	0.897	0.00029	-0.29
GBP	0.75	0.017	0.018	0.021	0.933	0.00033	-0.58
JPY	1.02	0.023	0.025	0.029	0.920	0.00038	-0.65
NZD	0.71	0.018	0.018	0.036	0.882	0.00045	-0.43
SEK	0.62	0.021	0.021	0.030	0.916	0.00035	-0.51
Avg.	0.75	0.018	0.019	0.029	0.906	0.00036	-0.50

This table reports the posterior mode, which is the maximum of posterior distribution. The standard asset price model is subject to the restriction $\alpha > 0$ and Bayesian priors.
α is the exchange rate response to expected interest returns.
ρ_{π} is the AR(1) coefficients of relative inflation.
$\sigma_{\mathbf{R^f}}$, $\sigma_{\mathbf{R}}$, $\sigma_{\mathbf{FX}}$ and σ_{π} are the standard deviations of risk-free relative returns, the bond premium, the currency premium and relative inflation respectively.
[a] This column shows the implied bias in the reduced-form estimate of $\hat{\alpha}$.

Table 4: Variance decomposition

variable →	Exchange rate changes Δq_t			Expected returns ΔR_t	
shock →	risk-free factor ΔR^f_t	common risk factor $\Delta \Lambda^R_t$	currency risk factor $\Delta \Lambda^{FX}_t$	risk-free factor ΔR^f_t	common risk factor $\Delta \Lambda^R_t$
Reduced-form model: interest rates are risk-free, $\Lambda^R = 0$)					
AUD	7.4	-	92.6	100	-
CAD	0.9	-	99.1	100	-
CHF	5.6	-	94.4	100	-
EUR	11.0	-	89.0	100	-
GBP	6.0	-	94.0	100	-
JPY	4.9	-	95.1	100	-
NZD	4.4	-	95.6	100	-
SEK	6.4	-	93.6	100	-
	5.8		94.2	100	
Model with risk					
AUD	14.6	0.8	84.6	56.7	43.3
CAD	8.7	4.5	86.8	35.6	64.4
CHF	15.8	3.9	80.3	47.7	52.3
EUR	18.6	1.5	79.9	54.1	45.9
GBP	27.0	3.1	69.9	48.6	51.4
JPY	39.9	0.1	60.0	45.0	55.0
NZD	10.5	1.8	87.7	49.4	50.6
SEK	14.7	5.9	79.4	49.5	50.5
	18.7	2.7	78.6	48.3	51.7

Note: for this random walk model, the unconditional variance decomposition and forecast error variance decomposition are identical.

Table 5: Correlation of unobserved components with changes in 'monetary policy'

	$corr(\Delta R_t, \Delta\hat{i}_t)$		Unobserved components $corr(\Delta R^f_t, \Delta\hat{i}_t)$		$corr(\Delta\Lambda^R_t, \Delta\hat{i}_t)$		$corr(\Delta\Lambda^{FX}_t, \Delta\hat{i}_t)$	
AUD	0.21	***	0.25	***	-0.17	***	0.15	***
CAD	0.22	***	0.16	***	-0.22	**	-0.05	***
CHF	0.15	**	0.14	**	-0.15	**	0.01	**
EUR	0.11		0.14		-0.08	*	0.08	
GBP	0.12	*	0.11	*	-0.11		0.02	
JPY	0.21	***	0.20	***	-0.20	***	0.06	***
NZD	0.20	***	0.20	***	-0.19	***	0.03	***
SEK	0.24	***	0.24	***	-0.22	***	0.06	***
average	0.18		0.18		-0.17		0.05	

Unobserved components are from the model with risk. 'Monetary policy' is measured as changes in the nominal short-term 90-day interbank interest rate. *** indicates significance to the 1% level; ** indicates significance to the 5% level, * indicates significance to the 10% level.

Table 6: Correlation among unobserved components

	$corr(\Delta R^f_t, \Delta\Lambda^P_t)$		$corr(\Delta R^f_t, \Delta\Lambda^{FX}_t)$		$corr(\Delta\Lambda^R_t, \Delta\Lambda^{FX}_t)$	
AUD	-0.84	***	0.31	***	0.25	***
CAD	-0.78	***	0.49	***	0.18	***
CHF	-0.77	***	0.47	***	0.20	***
EUR	-0.78	***	0.34	***	0.32	***
GBP	-0.67	***	0.57	***	0.23	***
JPY	-0.69	***	0.58	***	0.18	***
NZD	-0.85	***	0.35	***	0.20	***
SEK	-0.75	***	0.47	***	0.22	***
average	-0.77		0.45		0.22	

Unobserved components are from the model with risk. *** indicates significance to the 1% level; ** indicates significance to the 5% level, * indicates significance to the 10% level.

Table 7: Correlations of unobserved components with changes in the VIX index

	Δq_t		Unobserved components ΔR^f_t		$\Delta \Lambda^R_t$		$\Delta \Lambda^{FX}_t$	
AUD	-0.50	***	0.20	***	0.08		0.50	***
CAD	-0.43	***	0.10		0.21	***	0.44	***
CHF	-0.13	**	0.03		0.07		0.14	**
EUR	-0.36	***	0.12		0.13	*	0.38	***
GBP	-0.06		-0.04		0.11	*	0.08	
JPY	0.06		-0.06		0.03		-0.05	
NZD	-0.39	***	0.07		0.15	**	0.41	***
SEK	-0.32	***	0.17	***	0.05		0.32	***
average	-0.27		0.07		0.11		0.27	

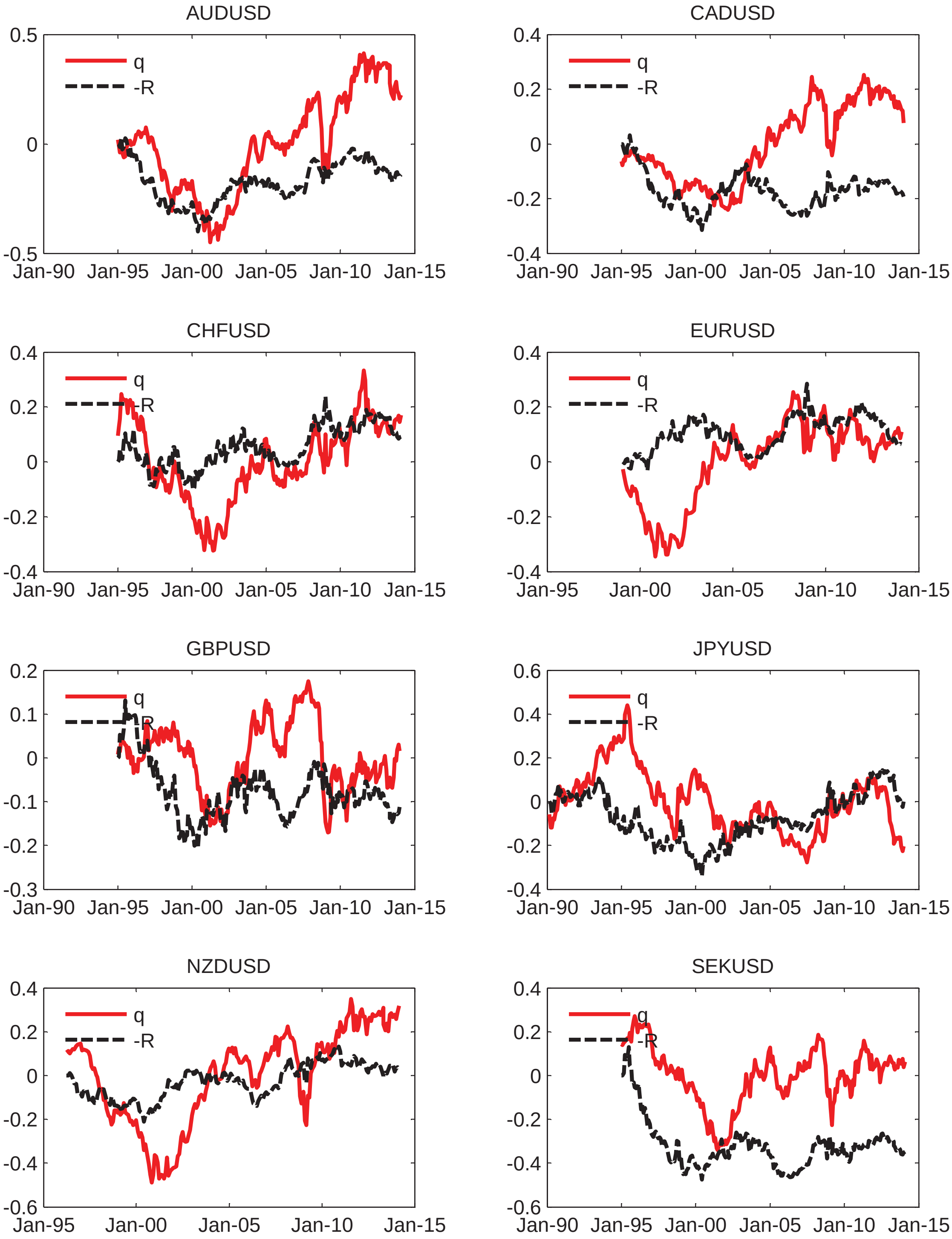
Unobserved components are from the model with risk. The VIX index is the implied volatility of the S&P500 equity index, calculated from options prices, and is commonly referred to as a measure of 'risk aversion'. A rise in the exchange rate is a depreciation of the USD. Relative risk-free returns are US minus foreign. A rise in VIX is correlated with appreciation of the USD relative to non-reserve currencies. Through the lens of the model, the USD appreciates because the non-reserve currency premium rises. For some currencies, the bond premium also rises significantly. *** indicates significance to the 1% level; ** indicates significance to the 5% level, * indicates significance to the 10% level.

Table 8: Correlation of unobserved components with changes in IMM positioning

	Δq_t		Unobserved components ΔR^f_t		$\Delta \Lambda^R_t$		$\Delta \Lambda^{FX}_t$	
AUD	0.53	***	-0.32	***	0.04		-0.50	***
CAD	0.46	***	-0.19	***	-0.12	*	-0.46	***
CHF	0.51	***	-0.37	***	0.05		-0.49	***
EUR	0.46	***	-0.25	***	-0.05		-0.46	***
GBP	0.54	***	-0.43	***	0.03		-0.52	***
JPY	0.46	***	-0.39	***	0.10		-0.43	***
NZD	0.54	***	-0.31	***	0.02		-0.53	***
SEK	n.a.		n.a.		n.a.		n.a.	
average	0.50		-0.32		0.01		-0.49	

Unobserved components are from the model with risk. IMM positioning is non-commercial positioning in FX futures in the Chicago Mercantile Exchange International Money Market (IMM). A rise in the exchange rate is a depreciation of the USD. Relative interest returns are US minus foreign. A rise in non-commercial positioning in a foreign currency relative to the USD is correlated with appreciation of the foreign currency for two reasons: the foreign currency premium falls and foreign risk-free returns rise relative to USD returns.

Figure 1: Real exchange rates and forecast sums of future relative returns



Real exchange rates (red lines) are % deviation from sample mean so are subject to a level shift if the sample mean differs from the long-run real equilibrium. Dashed grey lines show UIP-consistent exchange rates ($-R_t$), assuming R_t is risk-free. R_t is constructed as 120 times the ten-year nominal zero-coupon swap differential (monthly rate), net of an AR(1) forecast of the relative inflation paths.

Figure 2: Exchange rate changes (Δq_t) and forecast revisions ($-\Delta R_t$) and

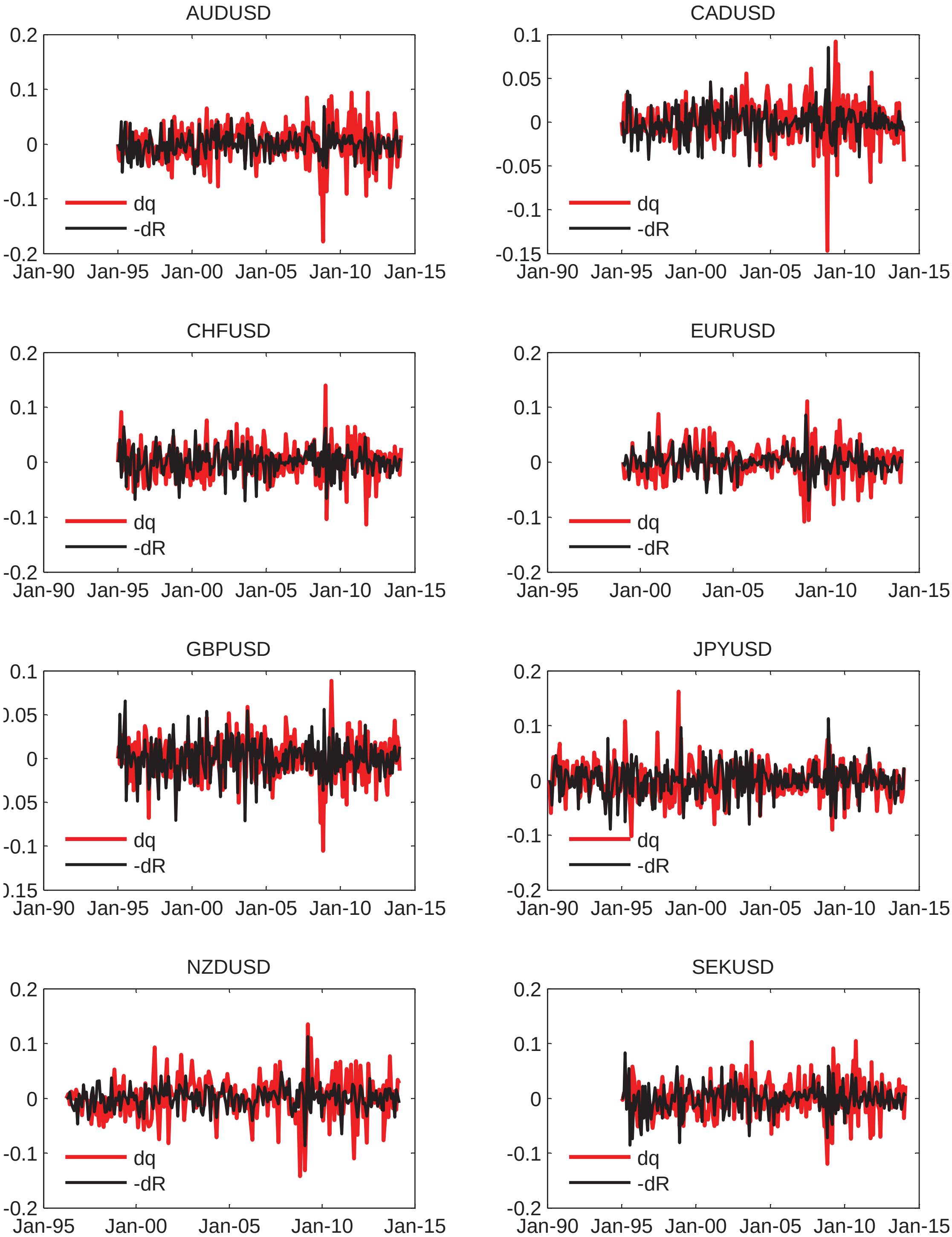
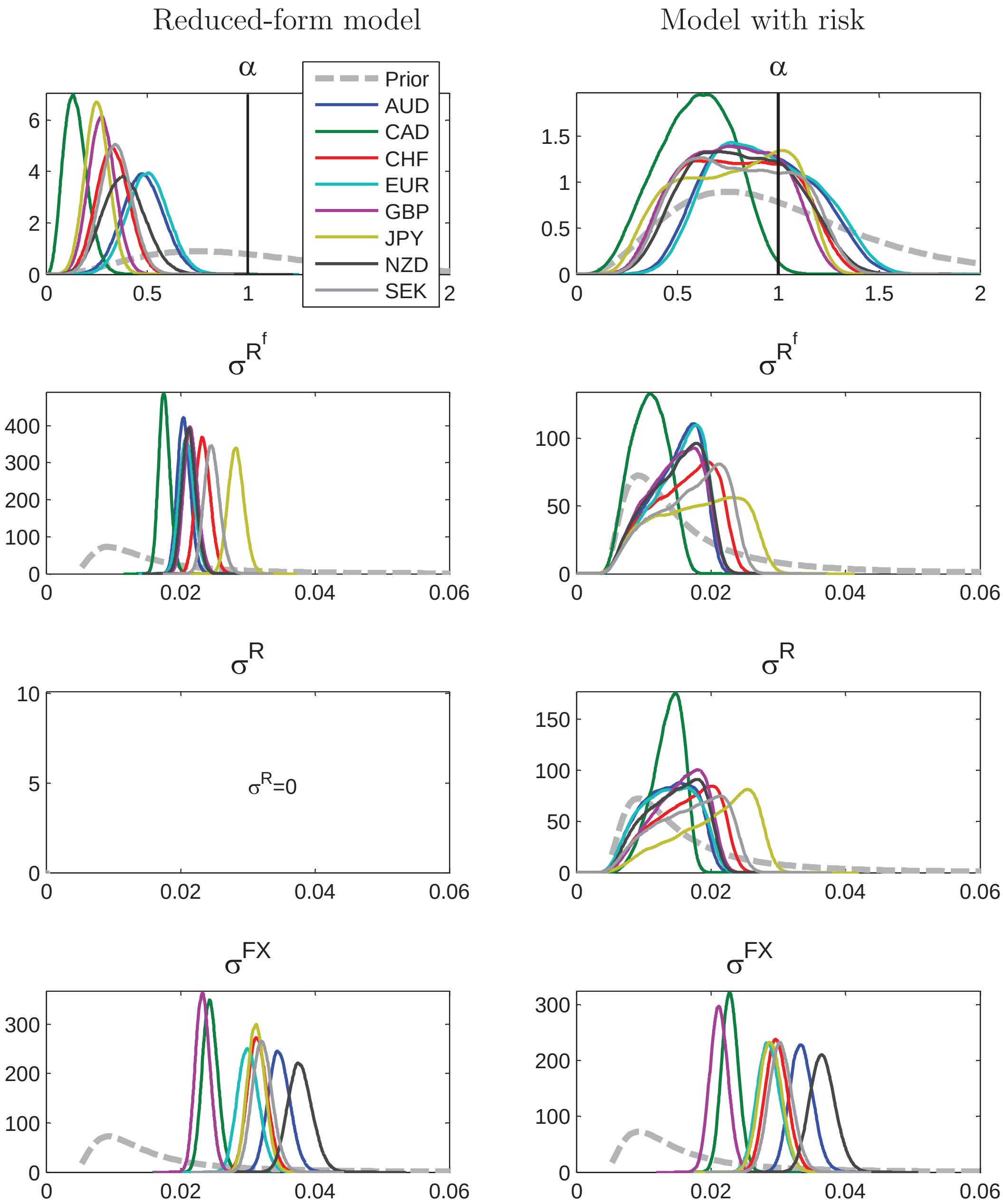


Figure 3: Posterior Densities



A Derivations

A.1 One-period bonds

The investor's stochastic discount factor, or pricing kernel, is defined in terms of the marginal utility of consumption $U'_{C,t}$:

$$E(M_{t+1}) \;=\; \beta E_t \frac{U'_{C,t+1}}{U'_{C,t}}$$

The unobserved risk-free rate, r_t^f, is defined by the investor's stochastic discount factor (SDF), M_{t+1}:

$$1 \;=\; (1 + r_t^f)E[M_{t+1}] \tag{A.1}$$

Define $x_{t+1} = log(X_{t+1})$. If x_{t+1} is normally distributed, then $X_{t+1} = \exp(x_{t+1})$ is log-normally distributed and $E(X_{t+1}) = \exp(\bar{x} + \frac{1}{2}\sigma_x^2)$. Taking logs, equation (A.1) can be written:

$$\begin{aligned} 0 &= log((1 + r_t^f)E_t[M_{t+1}]) \\ &= log[\,\exp(r_t^f + \bar{m} + \tfrac{1}{2}var(m_{t+1}))\,] \end{aligned} \tag{A.2}$$

The home investor's Euler equation for domestic short-term bonds is:

$$1 \;=\; E_t[M_{t+1}(1 + r_t)\,]$$

where r_t, which is known at time t, is the payoff on the domestic bond at time $t+1$. Taking logs,

$$\begin{aligned} 0 &= log(E_t[M_{t+1}(1 + r_t)]) \\ &= \exp(\bar{m} + \bar{r} + \tfrac{1}{2}var(m_{t+1} + r_t)) \\ &= \exp(\bar{m} + \bar{r} + \tfrac{1}{2}var(m_{t+1}) + \tfrac{1}{2}var(r_t) + cov_t(m_{t+1}, r_t)) \end{aligned} \tag{A.3}$$

Combining A.2 and A.3,

$$\begin{aligned} \bar{r} + \tfrac{1}{2}var(r_t) &= r_t^f - cov_t(m_{t+1}, r_t) \\ log(E_t(1 + r_t)) &= r_t^f - cov_t(m_{t+1}, r_t) \end{aligned}$$

Defining, $\tilde{r}_t \equiv logE_t(1 + r_t)$,

$$\tilde{r}_t = r_t^f - E_t cov_t(m_{t+1}, r_t) \tag{A.4}$$

The expected payoff on the home bond is equal to the risk-free rate plus a risk-adjustment. If the payoff co-varies negatively with the discount factor, m_{t+1}, then it co-varies positively with expected consumption. Holding assets with payoffs that co-vary positively with expected consumption makes consumption more volatile, so those assets pay higher returns to induce investors to hold them (Cochrane, 2001).

Similarly, the foreign investor's Euler equation for foreign bonds can be written in terms of the foreign risk-free rate, r_t^{f*} and a risk adjustment that depends on the foreign investor's stochastic discount factor m_{t+1}^*:

$$\tilde{r}_t^* = r_t^{f*} - E_t cov_t(m_{t+1}^*, r_t^*) \tag{A.5}$$

Combining equations (A.4) and (A.5), the short-term interest differential, $r_t^d = \tilde{r}_t - \tilde{r}_t^*$ can be written in terms of the relative risk-free rate and home and foreign bond market risk-adjustments:

$$r_t^d = (r_t^f - r_t^{f*}) - E_t cov_t(m_{t+1}, r_t) + E_t cov_t(m_{t+1}^*, r_t^*) \tag{A.6}$$

The home investor's Euler equation for foreign short-term bonds is:

$$Q_t \;=\; E_t[M_{t+1}(1 + r_t^*)Q_{t+1}] \tag{A.7}$$

where Q_t is the real exchange rate (defined as the value of the foreign currency, so a rise is a depreciation of the home currency). Taking logs,

$$\begin{aligned}
log(Q_t) &= log(E_t[M_{t+1}(1 + r_t^*)Q_{t+1}] \\
q_t &= log[\exp(\bar{m} + \bar{r^*} + \bar{q} + \tfrac{1}{2}var(r_t^* + q_{t+1} + m_{t+1}))] \\
q_t &= \bar{m} + \bar{r^*} + \bar{q} + \tfrac{1}{2}var(r_t^*) + \tfrac{1}{2}var(q_{t+1}) + \tfrac{1}{2}var(m_{t+1}) \\
&\quad + cov_t(m_{t+1}, r_t^*) + cov_t(m_{t+1}, q_{t+1}) \\
q_t &= -r_t^f + r_t^* + E_t(q_{t+1}) + E_t cov_t(m_{t+1}, r_t^*) + E_t cov_t(m_{t+1}, q_{t+1}) \qquad (A.8)
\end{aligned}$$

Combining, (A.4) and (A.8), we get the UIP condition:

$$q_t = -(r_t - r_t^*) + E_t(q_{t+1}) - \lambda_t \tag{A.9}$$

where, $\lambda_t = E_t\ [\ cov_t(m_{t+1}, r_t) - cov_t(m_{t+1}, r_t^*) - cov_t(m_{t+1}, q_{t+1})\ ]$

A.2 Multi-period bonds

Now consider rolling over short term bonds for N-periods. Define,

$$R_t = E_t \sum_{j=1}^{N} (r_{t+j-1} - r_{t+j-1}^*), \text{ and } \Lambda_t^q = E_t \sum_{j=1}^{N} \lambda_{t+j-1} - E_t q_{t+N}$$

Substituting equation (A.9) forward, the exchange rate can be written as an asset price, ie. the long-run level plus a forward-looking sum of expected relative returns and risk:

$$q_t \;=\; -R_t - \Lambda_t^q \text{ where,} \tag{A.10}$$

$$\Lambda_t^q = E_t \sum_{j=1}^{N} [cov_t(m_{t+j}, r_{t+j-1}) - cov_t(m_{t+j}, r_{t+j-1}^*) - cov_t(m_{t+j}, q_{t+j})] - E_t q_{t+N}$$

For simplicity of exposition, we abstract from the terms in $cov_t(m_{t+j}, m_{t+j+1})$ and $cov_t(m_{t+j}^*, m_{t+j+1}^*)$ (home and foreign term premia respectively), and in $cov_t(r_{t+j-1}, r_{t+j})$, $cov_t(r_{t+j-1}^*, r_{t+j}^*)$ and $cov_t(q_{t+j}, q_{t+j+1})$. The home term premium, and terms in r_t and r_t^* also contribute to the common bond premium that is reflected in both q_t and R_t, so are part of the bond premium identified here. Since exchange rate require pricing returns into the future, these terms may be large.

Summing equation (A.6) forward, and defining $R_t^f \equiv E_t \sum_{j=1}^{N} E_t(r_{t+j-1}^f - r_{t+j-1}^{f*})$,

$$\begin{aligned}
R_t &= R_t^f - E_t \sum_{j=1}^{N} [\ cov_t(m_{t+j}, r_{t+j-1}) - cov_t(m_{t+j}^*, r_{t+j-1}^*)] \\
&= R_t^f - \Lambda_t^R
\end{aligned}$$

A.3 Complete markets

In the case of complete markets, the common component is larger. When consumption is correlated across countries, risk-free rates are equal and $m_t = m_t^*$. In that case, the interest differential reflects only risk which is priced according to the global pricing kernel, m_t. Replacing equation (A.5) with $r_t^* = r_t^f - E_t cov_t(m_{t+1}, r_t^*) - E_t cov_t(m_{t+1}, \Delta q_{t+1})$ from equation (A.7),

$$\begin{aligned} r_t^d &= E_t\big[-cov_t(m_{t+1}, r_t) + cov_t(m_{t+1}, r_t^*) + cov_t(m_{t+1}, \Delta q_{t+1})\big] \\ &= -\lambda_t^R \end{aligned}$$

Using, $q_{t+1} = q_t + \Delta q_{t+1}$, the currency excess return is:

$$\begin{aligned} \lambda_t &= E_t\big[-cov_t(m_{t+1}, r_t) + cov_t(m_{t+1}, r_t^*) + cov_t(m_{t+1}, \Delta q_{t+1}) + cov_t(m_{t+1}, q_t)\big] \\ &= -\lambda_t^R + cov_t(m_{t+1}, q_t) \end{aligned}$$

For the multi-period case,

$$\begin{aligned} R_t &= -\Lambda_t^R \\ q_t &= -R_t - \Lambda_t^q \\ &= -(-\Lambda_t^R) - \Lambda_t^R + E_t \sum_{j=1}^{N} cov_t(m_{t+j}, q_{t+j-1}) + E_t q_{t+N} \end{aligned}$$

When UIP holds, the reduced-form estimate of α is zero and the estimation bias is -1.

B Data

Exchange rates and nominal interest rates are end-month rates. Real exchange rates are measured ex-post. The inflation component of real interest rates is forecast on the basis of distributed lag equations. CPI data is assumed to be released with in a month. Nominal 30-day interest rates, zero-coupon swap rates and spot exchange rates are end-month rates from Bloomberg:

Table B.1: Bloomberg codes

	30-day interest rate	10-year interest rate swap	10-year zero-coupon swap	exchange rate
AUD	ADBB1M Curncy	ADSW10 Curncy	I00110y index	AUD Curncy
CAD	CD001M Curncy	CDSW10 Curncy	I00710Y Index	CAD Curncy
CHF	SF001M Curncy	SFSW10 Curncy	I05710y index	CHF Curncy
EUR	EU001M Curncy	EUSa10 Curncy	I05310Y Index	EUR Curncy
GBP	BP001M Curncy	BPSW10 Curncy	I05510Y Index	GBP Curncy
JPY	JY001M Curncy	JYSW10 Curncy	I05610Y Index	JPY Curncy
NZD	NDBB1M Curncy	NDSW10 Curncy	I04910y index	NZD Curncy
SEK	STIBOR01M Index	SKSW10 Curncy	I08710y index	SEK Curncy
USD	US0001M Index	USSW10 Curncy	I05210Y Index	1

For 5-year and 15-year swaps used in the robustness section, the codes are as above except that “10” is replaced with “5” or “15”.

Nominal 30-day interest rates are Libor rates or a local equivalent rate where the local benchmark rate is more heavily traded (e.g., Australia and New Zealand bank bill rates). Ten year zero-coupon swap rates are available from December 1989 for the JPY/USD pair, from December 1994 for all other currency pairs except the euro which is available from January 1999. The sample ends in March 2014. Consumer price indices and import and export price indices are from the IMF International Financial Statistics. For Australia and New Zealand, quarterly price indices are interpolated so that observed inflation is the same for three months between quarterly inflation data.

C Robustness

Figure C.1 compares the posterior distributions for the model with risk when R_t is constructed from 10-year zero-coupon swaps (left panel) and 10-year **plain vanilla swaps** (right panel). The UIP condition is derived from Euler equations for home and foreign bonds that relate the price of a bond today to expected discounted returns on that bond. In combination, we have an un-discounted sum. Whether an un-discounted sum or a discounted sum is the correct measure is examined empirically in Engel and West (2005), Nason and Rogers (2008) and Kano (2014).

There are two main differences in the empirical results using plain vanilla swaps. First, when R_t is a smaller, discounted sum, the average estimate of $\alpha = 0.97$ is predictably higher than the average benchmark estimate of 0.75. Second, the posterior distributions for the zero-coupon swaps are less normal in shape, a result that is invariant to a longer Metropolis Hastings chain, raising the question of whether the construction of zero-coupon swaps from plain vanilla swaps distorts the data. Constructing zero-coupon swaps from plain vanilla swaps requires estimation of discount factors. There are both conceptual and practical issues related to that transformation. Conceptually, because the transformation makes assumptions about the discount factor, it makes assumptions about the risk-free rate (the inverse of the discount factor). Those assumptions may distort the risk-free rate backed out here using sign restrictions. If assumptions about the discount rate introduce errors at the short-end of the curve, on which the bootstrapping is based, then longer-term zero-coupon rates may be significantly distorted. Because the short-end of the yield curve is dominated by monetary policy, which not necessarily equal the risk-free rate defined by the investor's stochastic discount factor, some distortion might be expected.

Figure C.2 shows the results for shorter and longer forecast horizons. The posterior estimate of α is more consistent across currencies for longer horizon forecasts of returns, and more closely clustered around the theoretical value of one. Those results favour use of a longer horizon forecast, with the caveat that longer-term markets tend to be less liquid. As forecast horizon increases, the variability of both risk-free returns and the bond premium rise. In contrast, the currency premium variance is stable over different forecast horizons.

Figure Figure C.3 shows the posterior distributions for models with innovations in the unobserved components modeled as **AR(1) processes** (left panel) or innovations that include a lagged level term - an **error correction term** (right panel). Figure C.4 shows the posterior distributions for models **estimated in levels**.

Alternative identification scheme

In equations (23) and (24), the bond premium is fully reflected in the exchange rate equation. That is true in the complete markets case, but evidence does not favour complete markets. In an alternative identification scheme, we identify only one risk component and estimate a parameter, γ, to capture the common component:

$$\Delta q_t = -\alpha \Delta R_t - \Delta \Lambda_t^R \quad \text{(C.1)}$$

$$\Delta R = \Delta R_t^f - \gamma \Delta \Lambda_t^R \quad \text{(C.2)}$$

The parameter, γ, captures the degree of covariance between the exchange rate premium and the bond premium. The sign restrictions α, $\gamma > 0$ implied by the theoretical model provide the identifying restrictions. The parameter, γ, is well-identified, as shown in Figure C.5, and posterior estimates average 0.35. In this model, the estimation bias, $-\gamma var(\Delta\Lambda_t^R)/var(\Delta R_t)$, is more severe, averaging -0.68, compared to -0.50 in the benchmark model. Accordingly, the average posterior mode estimate for α in the model with risk is higher, averaging 0.97. In this specification, a larger share of exchange rate variation variation is attributed to innovations in risk-free rates: 42.5% on average, compared to 19% in the benchmark model. The higher contribution to exchange rate variance arises from the higher estimate of α and the lower variance of the bond premium, $\gamma\Lambda_t^R$.

Table C.1: Prior and posterior estimates: alternative identification scheme

	α	γ	σ_{R^f}	σ_R	σ_{FX}	ρ^π	σ_π	Bias in $\hat{\alpha}$
Prior Distribution	γ	γ	γ^{-1}	γ^{-1}	γ^{-1}	β	γ^{-1}	
Prior mean	1	1	0.020	0.020	0.020	0.8	0.0003	
Prior mode	0.76	0.76	0.009	0.009	0.009	0.85	0.0001	
Prior stdev	0.5	0.5	0.500	0.500	0.500	0.1	0.0050	
AUD	1.16	0.28	0.019	0.037	-	0.900	0.00048	-0.68
CAD	0.82	0.35	0.015	0.028	-	0.906	0.00028	-0.88
CHF	0.97	0.36	0.021	0.035	-	0.896	0.00028	-0.68
EUR	1.13	0.33	0.019	0.033	-	0.894	0.00029	-0.57
GBP	0.81	0.44	0.019	0.026	-	0.934	0.00033	-0.65
JPY	0.84	0.44	0.025	0.035	-	0.920	0.00037	-0.63
NZD	1.07	0.26	0.019	0.041	-	0.884	0.00045	-0.72
SEK	0.96	0.37	0.022	0.036	-	0.915	0.00035	-0.66
Avg.	0.97	0.35	0.020	0.034	-	0.906	0.00035	-0.68

This table reports the posterior mode, which is the maximum of posterior distribution. The standard asset price model is subject to the restriction $\alpha > 0$ and Bayesian priors.
α is the exchange rate response to expected interest returns.
ρ_π is the AR(1) coefficients of relative inflation.
$\sigma_{\mathbf{R^f}}$, $\sigma_{\mathbf{R}}$, $\sigma_{\mathbf{FX}}$ and σ_π are the standard deviations of risk-free relative returns, the bond premium, the currency premium and relative inflation respectively.

Table C.2: Unconditional variance decomposition

variable $\rightarrow$	Exchange rate changes Δq_t			Expected returns ΔR_t		
shock $\rightarrow$	risk-free factor ΔR_t^f	common risk factor $\Delta\Lambda_t^R$	currency risk factor $\Delta\Lambda_t^{FX}$	risk-free factor ΔR_t^f	common risk factor $\Delta\Lambda_t^R$	currency risk factor $\Delta\Lambda_t^{FX}$
AUD	42.5	57.5	-	76.9	23.1	-
CAD	28.4	71.6	-	70.2	29.8	-
CHF	44.3	55.7	-	73.9	26.1	-
EUR	52.5	47.5	-	76.5	23.5	-
GBP	46.3	53.7	-	73.1	26.9	-
JPY	46.5	53.5	-	72.3	27.7	-
NZD	33.2	66.8	-	76.7	23.3	-
SEK	46.3	53.7	-	73.6	26.4	-
	42.5	57.5		74.2	25.9	

Note: for this random walk model, the unconditional variance decomposition and forecast error variance decomposition are identical.

Figure C.1: Posterior Densities: zero-coupon swaps vs plain vanilla swaps

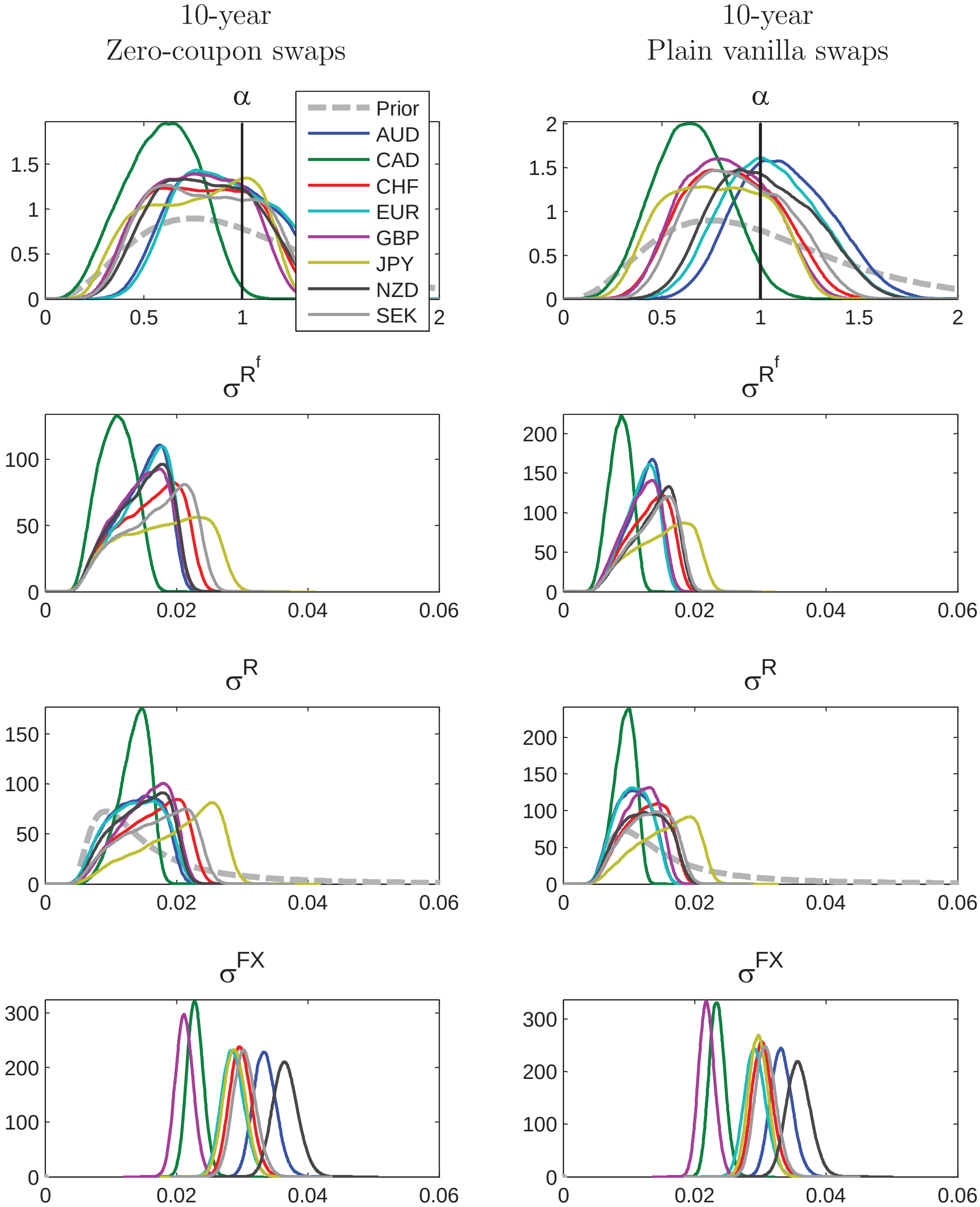


Figure C.2: Posterior Densities: different forecast horizons for R_t

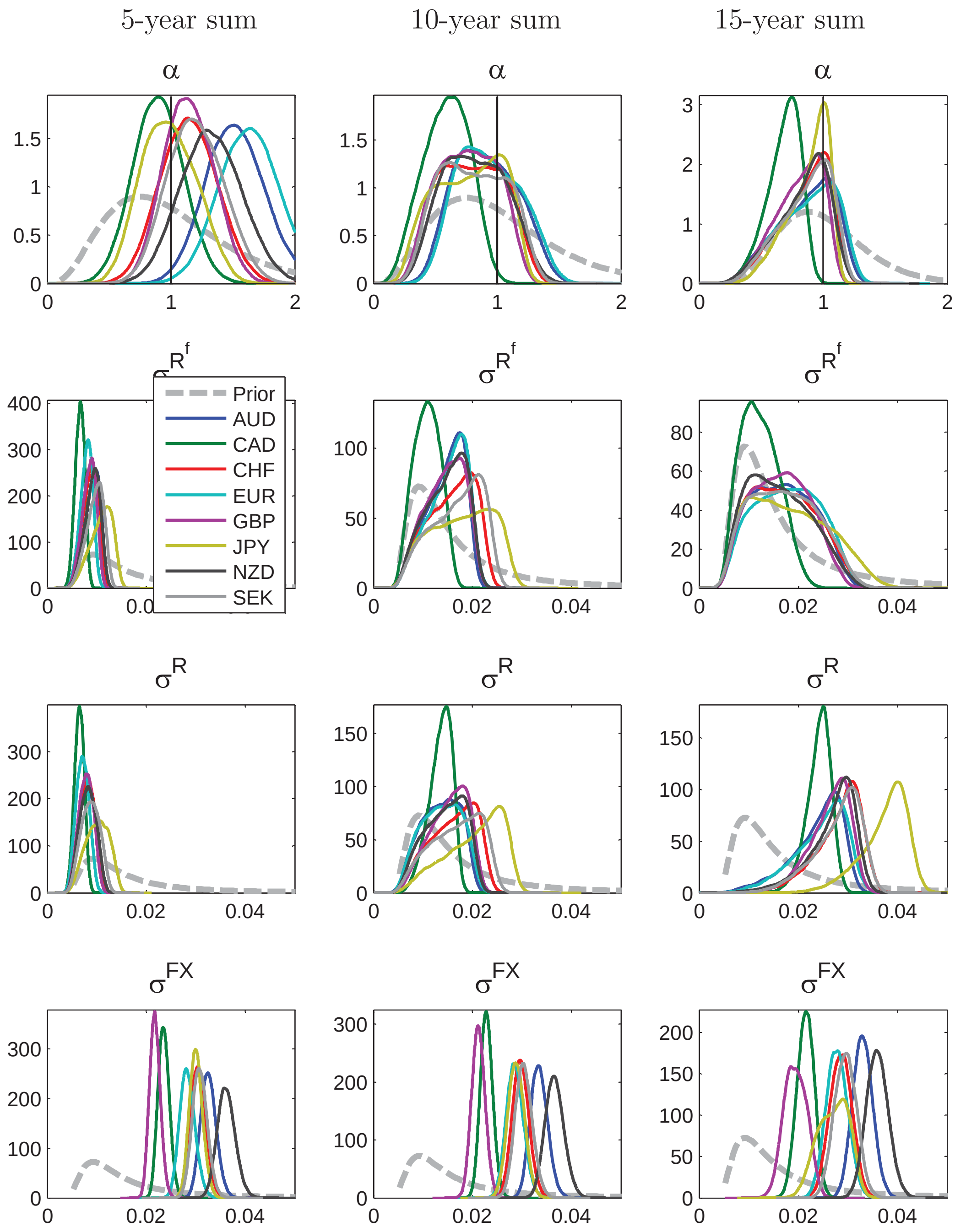


Figure C.3: Posterior Densities: innovations modeled with AR(1) or error correction component

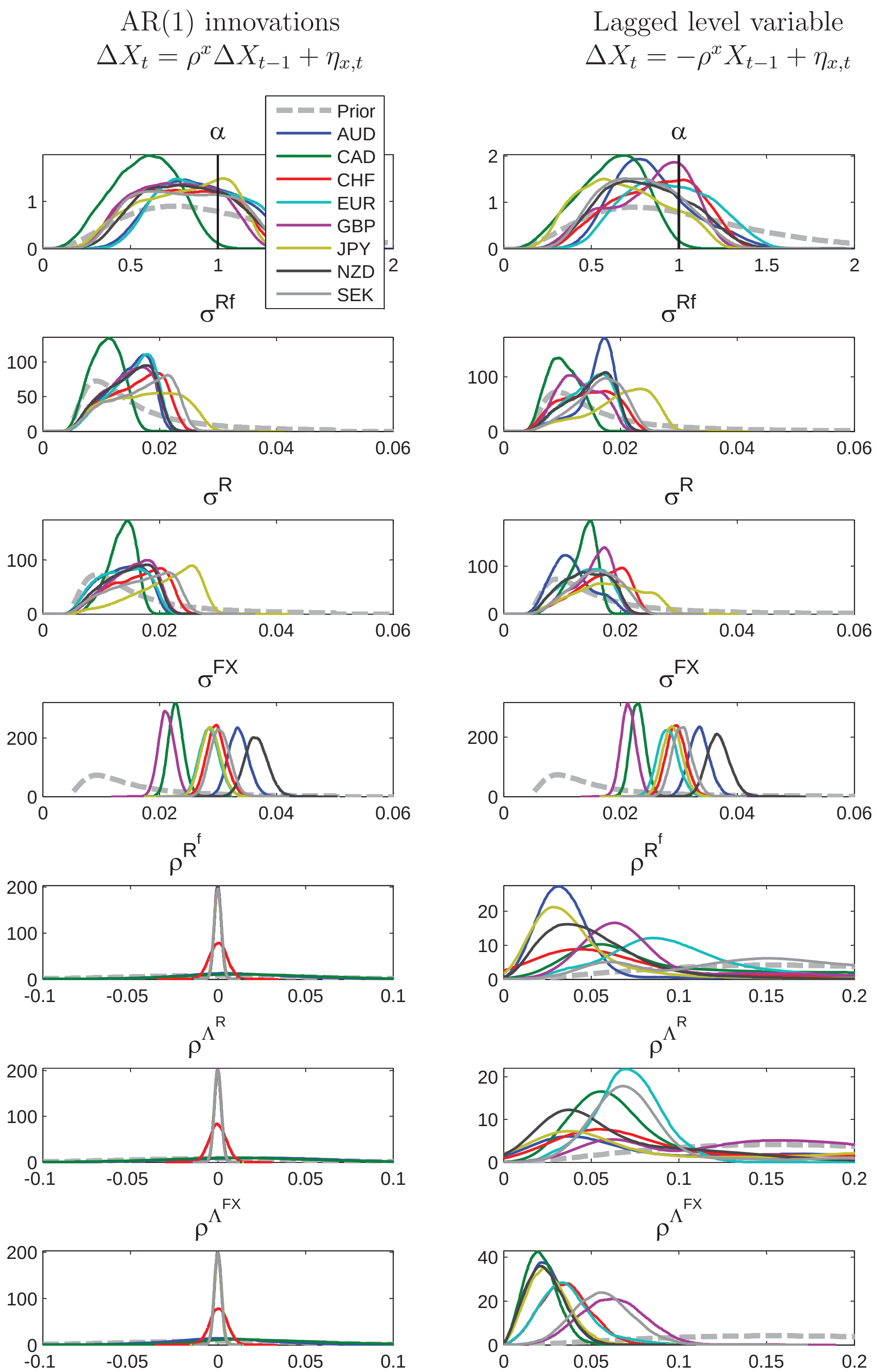


Figure C.4: Posterior Densities: model estimated in levels, rather than differences

Reduced-form model | Model with risk

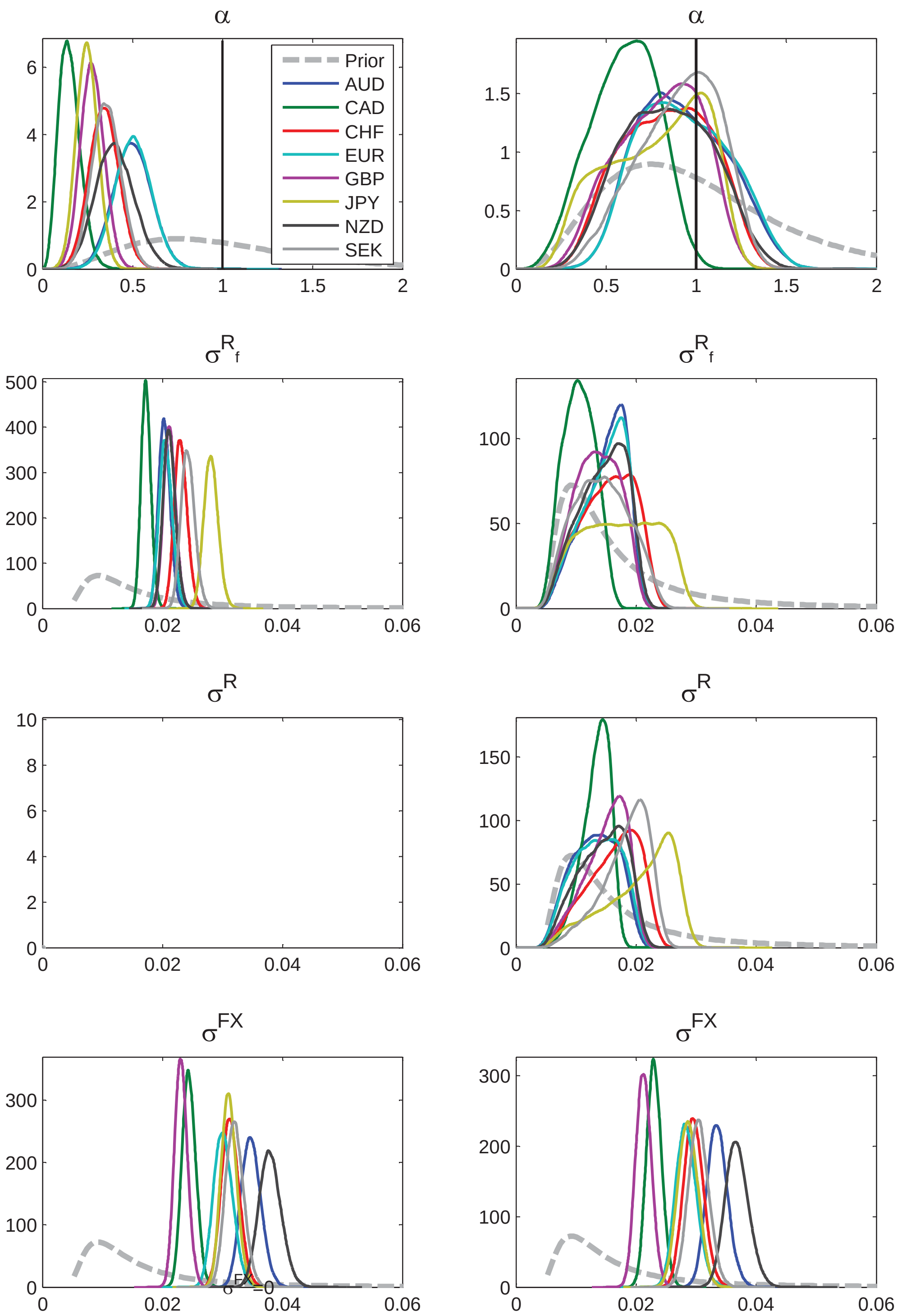


Figure C.5: Posterior Densities: Alternate identification scheme