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Corporate asset pricing models and debt contracts

CAMA Working Paper 33/2015
August 2015

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Abstract

In the past decades financial markets rapidly gained on complexity due to an increased demand for risk diversification and hedging. A number of sophisticated instruments was developed that capture various aspects of price movements, correlations of assets, macro-economic developments, and other changes that might affect the future income generated by the considered securities. The pricing of these securities was not sufficiently accurate using the traditional asset pricing models. In the search for new methods two different approaches appeared. One stream of literature (called the reduced-form approach) focused on finding a purely mathematical way of asset pricing, without the effort of finding any economical intuition behind the models. In contrast, the other group of academics studied the firm and its evolution. These, so-called structural models have an intuitive connection to the underlying economics, and therefore they can be helpful in understanding the reasons of price movements. This work fits in the category of structural approaches.

Keywords

JEL Classification

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1 Introduction

In the past decades financial markets rapidly gained on complexity due to an increased demand for risk diversification and hedging. A number of sophisticated instruments was developed that capture various aspects of price movements, correlations of assets, macro-economical developments, and other changes that might affect the future income generated by the considered securities. The pricing of these securities was not sufficiently accurate using the traditional asset pricing models. In the search for new methods two different approaches appeared. One stream of literature (called the reduced-form approach) focused on finding a purely mathematical way of asset pricing, without the effort of finding any economical intuition behind the models. In contrast, the other group of academics studied the firm and its evolution. These, so-called structural models have an intuitive connection to the underlying economics, and therefore they can be helpful in understanding the reasons of price movements.

This work fits in the category of structural approaches. First it gives a brief overview to the development of these models, and proposes their extension to a stochastic interest rate environment. Second, it uses these models to examine the effects of parameter settings in debt contracts, and therefore gives a guidance for the design of an optimal credit contract that maximizes firm value. With the introduction of a stochastic interest rate environment, it is possible to consider the implications

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of the business cycle period on the optimal debt ratio, and—using stochastic default barrier—on the bankruptcy decision as well. Game theory is also invoked, therefore agency costs arising from asymmetric information are predicted and minimized with the help of safety covenants and properly chosen parameters.

2 Asset Pricing Models

Due to the risk-averse human nature the price of an asset is dependent on its riskiness (that is, on the volatility of its future returns): investors price assets below their expected payoff if they bear some risk. However, the idea of a risk-neutral probability measure deals with this issue: it is possible to adjust the probabilities of future states for risk in a way that assets are priced at their expected values.[1] To derive this risk-neutral probability measure we need the assumption that market prices include all available information, since known fair prices are needed in order to create a measure that produces expected values equal to these fair prices. Furthermore this risk-neutral probability measure is unique if markets are complete.

Models that require the assumption that market prices incorporate all available information are called market information based models. They can be further divided to structural and reduced-form models.

Models representing the first category are based on the Merton [42] framework that employs the option pricing theory presented by Black and Scholes [9]. In Merton's work a company defaults at the maturity of its debt if the value of its assets is below the sum of its liabilities. Default prior maturity is not possible. The subsequent models relaxed this assumption as well as others taken by Merton. The common attribute of these models is that they concentrate on the structural characteristics of a company, including asset volatility and financial leverage.

By contrast, reduced-form (aka hazard rate) models ignore structural characteristics, and treat bankruptcy as a possible exogenous event that is described as the first jump time of a point process, without trying to explain the reason of default. This approach was first proposed by Jarrow and Turnbull [30] and later extended in several works, for example [29], [37] or [16].

2.1 Merton's Structural Model

In his pathbreaking paper, Merton [42] paralleled the value of equity in a leveraged firm to a European call option on the firm's assets and used the option pricing theory developed by Black and Scholes [9] to value it. A corresponding debt is a zero-coupon bond with finite maturity with a promised terminal payoff B. This rather simplified description has many unrealistic restrictions, however, because of

[1] The probability measure that reflects the true probabilities is called the physical measure.

its simplicity and new perspective Merton built the basics of the framework used in structural models.

A large and growing body of literature has relaxed one or more assumptions posed by Merton. Some of the most important extensions are: more complex capital structure and safety covenants [8], interest paying debt [20], bankruptcy costs and tax benefits [34], short and long term debt types [53], or stochastic interest rate [36, 23, 11, 14].

The original framework's assumptions, mainly coming from the Black and Scholes [9] option pricing theory are[2]:

(A.1) There are no transactions costs, taxes, or problems with indivisibilities of assets.
(A.2) There is a sufficient number of investors with comparable wealth levels so that each investor believes that he can buy and sell as much of an asset as he wants at the market price.
(A.3) There exists an exchange market for borrowing and lending at the same rate of interest.
(A.4) Short-sales of all assets, with full use of the proceeds, are allowed.
(A.5) Trading in assets takes place continuously in time.
(A.6) The Modigliani-Miller theorem that the value of the firm is invariant to its capital structure obtains.
(A.7) The Term-Structure is "flat" and known with certainty. I.e., the price of a riskless discount bond which promises a payment of one dollar at time τ in the future is $P(\tau) = e^{-r\tau}$ where r is the (instantaneous) riskless rate of interest, the same for all time.
(A.8) The dynamics for the value of the firm, V, through time can be described by a diffusion-type stochastic process with Stochastic Differential Equation (SDE)[3]

$$dV = (\mu V - C)dt + \sigma V dW \tag{1}$$

where μ is the instantaneous expected rate of return on the firm per unit time, C is the total dollar payout by the firm per unit time to either its shareholders or liability-holders (e.g., dividends or interest payments) if positive, and it is the net dollars received by the firm from new financing if negative; σ^2 is the instantaneous variance of the return on the firm per unit time; dW is a standard Gauss-Wiener process.

Suppose a security with market value, Y dependent on the value of a firm. More specifically, its price can be written as a function of the firm value V, and time t: $Y = F(V,t)$. The dynamics of this security can be formally written using a SDE as

$$dY = [\mu_Y Y - C_Y]dt + \sigma_Y Y dW_Y, \tag{2}$$

where μ_Y, C_Y, σ_y and W_Y and defined similarly as in (1). Using the stochastic equivalent of chain-rule, the so-called Itō's Lemma we also have:

[2] The assumptions are written exactly in a way as Merton wrote them, except for the symbols used
[3] This process is called Geometric Brownian Motion.

$$\begin{aligned} dY &= F_V dV + \frac{1}{2} F_{VV} (dV)^2 + F_t \\ &= \left[\frac{1}{2}\sigma^2 V^2 F_{VV} + (\mu V - C) F_V + F_t\right] dt + \sigma V F_V dW, \end{aligned} \tag{3}$$

where subscripts denote partial derivatives, and the second equation comes from (1). Comparing terms in (2) and (3) we have

$$\mu_Y Y \equiv \frac{1}{2}\sigma^2 V^2 F_{VV} + (\mu V - C) F_V + F_t + C_Y \tag{4}$$

$$\sigma_Y Y \equiv \sigma V \tag{5}$$

$$dW_Y \equiv dW \tag{6}$$

The last equation indicates that Y_t and V_t are perfectly correlated, as they are driven by the same stochastic parameter. This implies the existence of such linear combination of these securities that the resulting payoff is non-stochastic. Using this fact Merton constructed a portfolio of three securities V, Y and riskless debt in a way that the initial investment was zero[4]. He showed that any security Y whose value can be written as a function of the firm value and time has to satisfy the following equation:

$$0 = \frac{1}{2}\sigma^2 V^2 F_{VV} + (\mu V - C) F_V - rF + F_t + C_Y \tag{7}$$

As we can see, F depends on the value of the firm, time, interest rate, the volatility of the firm's value, the payout policy of the firm and the payout policy to the holders of Y. It does not depend on the expected rate of return neither the risk preference of the investors. This is the result where the idea of risk-neutral valuation comes from. Also it should be noted, that the only thing that distinguishes one security from the other (debt vs. equity) is a pair of boundary conditions.

For pricing a simple corporate bond Merton took four further assumptions:

(A.9) The corporation has two classes of claims, a single homogeneous class of debt and the residual claim, called equity.
(A.10) The firm commits to pay $B to the bondholders at date T.
(A.11) If the payment is not met at T, the bondholders immediately take over the company, and so the shareholders receive nothing.
(A.12) The firm cannot issue any new claims that are not junior to the original one nor can pay dividends or do share repurchase before T.

As it can be seen this set-up ensures no default prior to maturity. Using equation (7) for the value of the debt, D, setting $C = C_Y = 0$ in line with the assumptions and defining $\tau = T - t$, so thus $D_t = -D_\tau$ we can write

[4] For the details about the construction of this portfolio, and for the complete derivation of equation (7) see [42] pp. 451–452.

$$0 = \frac{1}{2}\sigma^2 V^2 D_{VV} + rVD_V - rD - D_\tau \tag{8}$$

Denoting the value of equity as E and using (1), we have $V = D(V,\tau) + E(V,\tau)$. As E and D are non-negative, we know:

$$D(0,\tau) = E(0,\tau) = 0$$

and also $D(V,\tau) \leq V$, that is for $V > 0$ we have the other boundary condition

$$D(V,\tau)/V \leq 1$$

As the payment is made exactly when $V(T) > B$, the initial condition for the debt at $\tau = 0$ is

$$D(V,0) = \min[V,B]$$

The function $D(V,\tau)$ can be found using (8) and the above boundary conditions using standard methods as separation of variables. However, as Merton noticed, the problem can be transformed to another, already solved. For the value of equity holds $E(V,\tau) = V - D(V,\tau)$, so the solution for equity is given by (7):

$$0 = \frac{1}{2}\sigma^2 V^2 E_{VV} + rVE_V - rE - E_\tau \tag{9}$$

with a corresponding initial condition

$$E(V,0) = \max[0, V - B]$$

and the boundary conditions $E(0,\tau) = 0$ and $E(V,\tau)/V \leq 1$. This is identical to the equations for an European call option on a non-dividend-paying stock in the Black-Scholes option pricing model. The firm value corresponds to the stock price, the equity to the option value and B to the exercise price.

Therefore the equity price is

$$E(V,\tau) = V\Phi(d_1) - Be^{-r\tau}\Phi(d_2), \tag{10}$$

where

$$\begin{aligned} d_1 &= \frac{\ln(V/B) + \left(r + \frac{1}{2}\sigma^2\right)\tau}{\sigma\sqrt{\tau}} \\ d_2 &= d_1 - \sigma\sqrt{\tau} \end{aligned}$$

and $\Phi(\cdot)$ is the cumulative standard normal distribution.

As $D = V - E$, the debt value can be expressed as

$$D(V,\tau) = V\Phi(-d_1) + Be^{-r\tau}\Phi(d_2) \tag{11}$$

with the d_1 and d_2 as in (10).

2.2 First Passage Time Approach

The original Merton [42] model described in the previous section uses several assumptions that limit its practical implementability. One of the most unrealistic restriction is the impossibility of default before maturity. To solve this problem Black and Cox [8] came with a set-up where default occurs if the firm value touches a threshold level. This level is called the Default Barrier (DB), and generally can be constant [34, 36], deterministic [8, 35] or stochastic [11, 14] function of time. Models with a DB not only explain early default, but are also able to produce a large variety of Recovery Rates (RRs) and therefore reflect more precisely factors as bond covenants, bankruptcy costs or taxes.

The name of the First Passage Time (FPT) models corresponds to the method how the default is described mathematically: since the evolution of the firm value is represented using a Geometric Brownian Motion (GBM), it is possible to transform the probability distribution of the default to the FPT of a Wiener process. These models can be also divided to two groups in dependence on the determination of the DB: it can be set exogenously [8, 36], or be an endogenous result of an optimization process [34, 57]. The notation used throughout the section follows the one introduced in the description of Merton's model, unless it is explicitly defined otherwise.

2.2.1 Black and Cox Model

Black and Cox [8] extended the original Merton [42] framework to include several features of debt contracts, namely safety covenants, subordinated bonds, and restriction on asset sales. Since this section discusses basic asset pricing methods, only the introduction of a DB will be described.[5]

The evolution of the firm value is the same as in the Merton model [42], except a restriction that the continuous dividend payment received by the stockholders is a constant fraction of the firm value. Therefore equation (1) takes the form

$$dV = V(\mu - c)dt + \sigma V dW \tag{12}$$

with a constant $c = C/V$ representing the payout ratio received by the equity holders. Again, the short-term interest rate is assumed to be constant, and so the interest-rate risk is disregarded. The original case described in [8] also assumes zero bankruptcy costs.

A safety covenant, that provides a right for the bondholders to force bankruptcy if the firm is performing poorly, is introduced. This poor performance is signalled by the fall of the firm value under a time-dependent default barrier defined as $\bar{v}(t) = Ke^{-\gamma(T-t)}, t \in [0,T)$ for some constants $K > 0$ and γ. The creditors take over the firm as soon as the firm value hits this barrier. Consequently default could be triggered

[5] For pricing of more complex capital structures and the issue of contractural design see the original work of [8].

in two ways: prior to maturity (by reaching the threshold level) or at maturity, if the firm value was above the DB but is below the debt principal at T. To simplify the notation let us set the default barrier as one function:

$$v_t = \begin{cases} \bar{v}(t) & \text{for } t < T, \\ B & \text{for } t = T. \end{cases}$$

The default time τ is

$$\tau = \inf\{t \in [0,T] : V_t < v_t\}.$$

We also assume the following:

$$V_0 > \bar{v}(0)$$

$$Ke^{-\gamma(T-t)} \leq Be^{-r(T-t)}, \quad \forall t \in [0,T]$$

i.e. the firm is not in default initially and the default barrier (and hence the payment to the bondholder) is never higher than the present value of the principal amount. This holds also for $t = T$, therefore $K \leq L$.

Zero-Coupon Bond In Merton's model the debt pricing function solved equation 8. The analogous Partial Differential Equation (PDE) for zero-coupon debt value with default barrier is

$$0 = \frac{1}{2}\sigma^2 V^2 D_{VV} + (r-c)VD_V - rD + D_t \tag{13}$$

with the boundary condition

$$D(Ke^{-\gamma(T-t)}, t) = Ke^{-\gamma(T-t)}$$

and terminal condition

$$D(V,T) = \min(V,B).$$

Equation (13) can be solved using the classical methods used for PDEs or with a probabilistic approach.[6]

Note, that similarly as the equity value in [42] corresponds to a call option, it corresponds to a down-and-out barrier option here. Using the in-out parity (i.e. the plain vanilla option price equals to the sum of down-and-out and down-and-in barrier options price, all having the same strike price, underlining asset, maturity and the last two having the same barrier as well), the equity has a lower value by the price of a down-and-in barrier option in the presence of a DB. As there are no bankruptcy cots, this value is transferred to the bondholder.

Perpetual Coupon Bond A perpetual coupon bond has infinite maturity and continuous coupon payment at a constant rate c_D.[7] The net cost of the coupon is fi-

[6] The solution of (13) can be found in [8] p. 356

[7] Here we use the subscript D in order to distinguish this pay-out from c, which was the payout ratio to equity holders.

nanced by issuing additional equity. Its price $D_{c_D}(t)$ equals

$$D_{c_D}(t) = \lim_{T\to\infty} E\left(\int_t^T c_D e^{-r(s-t)} 1_{\{s<\bar{\tau}\}} ds\right) + \lim_{T\to\infty} E\left(Ke^{\gamma(\bar{\tau}-T)} e^{-r(\bar{\tau}-t)} 1_{\{t<\bar{\tau}<T\}}\right)$$

under risk-neutral probability measure with 1 used as a symbol for indicator function. Since the coupon payments are constant it is straightforward to define the default barrier constant as well, i.e. set $\gamma = 0$. With the assumption that dividends paid to equity holders are zero (that is $c = 0$) D_{c_D} can be written as[8]

$$D_{c_D} = \frac{c_D}{r}\left(1 - \left(\frac{\bar{v}}{V_t}\right)^{\alpha}\right) + \bar{v}\left(\frac{\bar{v}}{V_t}\right)^{\alpha}, \tag{14}$$

with $\alpha = 2r/\sigma^2$.

2.2.2 Leland's model

Leland [34] extended the perpetual coupon bond model described above by incorporating bankruptcy costs and tax benefits. Now V is a variable for the "asset value" of the firm; the total firm value is V less the expected costs of bankruptcy plus the value of the tax shield. V follows the same diffusion process as in (12) with no dividend payments ($c = 0$):

$$dV = V\mu dt + \sigma V dW,$$

hence V is not affected by the financial structure of the firm, thus the difference between coupon payments and tax benefits is financed by equity dilution.

When bankruptcy occurs at level $V_t = V_B$ a fraction $0 \le \omega \le 1$ is lost as costs due to bankruptcy, and the debt holders receive the remaining $(1-\omega)V_B$ leaving the equity holders with nothing. The value of the bond can be written as

$$D_{c_D}(V_t) = \frac{c_D}{r}\left(1 - \left(\frac{\bar{v}}{V_t}\right)^{\alpha}\right) + (1-\omega)\bar{v}\left(\frac{\bar{v}}{V_t}\right)^{\alpha}. \tag{15}$$

Note that with $\omega = 0$ this is identical to (14). If we denote $p_B = (\bar{v}/V_t)^{\alpha}$ (15) becomes

$$D_{c_D}(V_t) = \frac{c_D}{r}(1-p_B) + (1-\omega)\bar{v}p_B.$$

p_B represents the value of a contingent claim that pays \$1 when bankruptcy occurs, $\omega\bar{v}p_B$ is the present value of expected bankruptcy costs, and $c_D/r(1-p_B)$ is the present value of expected coupon payments. Consequently the value of the tax benefits is equal to:

$$TS = T_c\frac{c_D}{r}(1-p_B),$$

where T_c is the corporate tax rate.

[8] For the mathematical derivation see [7] p. 81 and the preceding calculations.

The total value of the firm, denoted by $G(V_t)$ is therefore equal to

$$G(V_t) = V_t - \omega \cdot \bar{v} \cdot p_B + T_c \frac{c_D}{r} (1 - p_B).$$

Since the total value of the firm is equal to the sum of its equity and debt value, the shareholders' claim can be found as

$$\begin{aligned} E(V_t) &= G(V_t) - D_{c_D}(V_t) \\ E(V_t) &= V_t - (1 - T_c)\frac{c_D}{r} (1 - p_B) - \bar{v} \cdot p_B. \end{aligned}$$

Intuitively the value of equity is equal to the value of firm's assets less the present value of expected coupon payments reduced by tax and the contingent claim on $\bar{v}$. Note that the value of equity is not dependent on the bankruptcy costs, and so that is paid in full by the bondholders.

2.3 Models with Stochastic Interest Rates

One of the shortcomings of the Black and Cox [8] model is the assumption of constant and known risk-free interest rate. This restriction is relaxed in models with stochastic interest rates. Because our work[9] assumes stochastic interest rate as well, we will make a review of the relevant literature at this point.

2.3.1 Longstaff and Schwartz

Longstaff and Schwartz [36] price corporate bonds reflecting both interest rate risk and credit risk using risk-neutral probability measure for both stochastic processes. The evolution of the short-term interest rate is inherited from the Vasicek [52] model:

$$dr_t = (a - br_t)dt + \sigma_r d\tilde{W}_t,$$

and the firm-s value is driven by the

$$dV_t = V_t(r_t dt + \sigma_V dW_t^*)$$

SDE. As we can see the constant drift from the Leland [34] model is replaced by the stochastically evolving short-term interest. Furthermore, following Longstaff and Schwartz [36] we have the following properties:

- Brownian motions $\tilde{W}$ and W^* are correlated with the instantaneous correlation $\rho_{V,r}$.
- DB is represented as a constant threshold level $\bar{v}$.

[9] See Sect. 4

- Recovery Rate (RR) is independent on the default time, proportional to the face value of the bond and paid out at maturity.
- $\bar{v} \geq B$, hence the debt is repaid in full if default does not occur prior maturity.[10]
- The firm has one or more debt classes with different recovery rates $(1-\omega_i)$, where ω_i is the writedown rate for the ith class. The seniority of the claims is already reflected in the writedown rates, and therefore does not play essential role.[11] It is natural to suppose the following relationship: $\bar{v} = \sum_{i=1}^{k}(1-\omega_i)B_i$ with B_i ($\sum_{i=1}^{k} B_i = B$) representing the total face value of debt from the ith class.

It we define τ, the time of default in the traditional way, that is

$$\tau - \inf\{t \in [0,T] : V_t < \bar{v}\},$$

then the bond's payoff at T can be written as

$$D_i(V_T,T) = B(1-\omega_i 1_{\{\tau \leq T\}}).$$

For finding an analytic solution of the bond value at time $t < T$ with given V_t there are basically two ways: by solving the fundamental PDE with the corresponding boundary and terminal conditions, or alternatively, by probabilistic approach. A closed-form solution however has not yet been produced using any of them. For this reason—even if some quasi-explicit results can be obtained analytically—numerical computations are required in order to obtain the results of the model. Such computations were made by the authors as well as others [33, 14]. A shortcoming of this model is, that it produces credit spreads close to zero for low debt maturities.

2.3.2 Briys and de Varenne

Briys and de Varenne [11] submitted a model that addressed some restrictive features and assumptions of the then available literature. For example, the previously analyzed Longstaff and Schwartz [36] model cannot work with a default barrier that would be lower than the present value of the debt principal. Their work also assumes stochastic default barrier, as it is derived from the instantaneous short-term interest.

The short-term rate dynamics follows the so-called generalized Vasicek model, which is a mean-reverting stochastic function:

$$dr_t = a(t)(b(t) - r_t)dt + \sigma(t)d\tilde{W}_t,$$

where $a, b, \sigma : [0,T] \to \mathbb{R}$ are known, deterministic functions. Consequently the price of a default-free zero-coupon bond, P follows the dynamics

$$dP(t,T) = P(t,T)(r_t dt + b(t,T)d\tilde{W}_t)$$

[10] In fact this inequality is not explicitly wrote down by [36], however it is implicitly assumed.

[11] Note that this set-up can easily catch Absolute Priority Rule (APR) violations.

for some deterministic $b(\cdot,T):[0,T]\to\mathbb{R}$. The firm value V is assumed to follow the process

$$\frac{dV_t}{V_t}=r_t dt+\sigma_V(\rho d\tilde{W}_t+\sqrt{1-\rho^2}d\hat{W}_t),$$

with constant $\sigma_V>0$, and mutually independent Brownian motions $\tilde{W}$ and $\hat{W}$. The local correlation coefficient between the risk-free rate and firm value is $\rho=\rho_{V,r}$. If we denote $W^*=\rho d\tilde{W}_t+\sqrt{1-\rho^2}d\hat{W}_t$, it is visible that the firm value process is defined in the same fashion as in [34].

The DB is defined as the price of a default-free bond with the same maturity and some face value $K\in(0;B]$ not greater than the defaultable bond principal:

$$v_t=\begin{cases}K\cdot P(t,T) & fort<T,\\ B & fort=T.\end{cases}$$

The default time is, as usually,

$$\tau=\inf\{t\in[0,T]:V_t<v_t\}.$$

The payoff at default is dependent on τ: for $\tau<T$ the bondholders receive a $(1-\omega_2)$ part of the remaining assets, whereas for $\tau=T$ this payoff ratio may be different, and is represented as $(1-\omega_1)$. The remaining ω_1 respectively ω_1 part is lost as bankruptcy cost and/or paid out to equity holders (APR). The bond's final cash flow at T is therefore

$$D(V_t,T)=(1-\omega_2)B1_{\{\tau<T\}}+(1-\omega_1)V_T1_{\{\tau=T\}}+B1_{\{\tau>T\}}$$

If the bond price volatility function $b(t,T)$ is known, than the price of a defaultable corporate bond can be derived as a closed-from solution:

$$D(t,T)=P(t,T)\cdot[B-D_1+D_2-\omega_2R_2-\omega_1R_1],\tag{16}$$

where $F_t=V_t/P(t,T)$

$$\begin{aligned}D_1&=B\Phi(d_1)-F_t\Phi(d_2),\\ D_2&=K\Phi(d_5)-(F_tL/K)\Phi(d_6),\\ R_2&=F_t\Phi(d_4)+K\Phi(d_3),\\ R_1&=F_t\big(\Phi(d_2)-\Phi(d_4)\big)+K\big(\Phi(d_5)-\Phi(d_3)\big),\end{aligned}$$

with

$$\begin{aligned}d_1&=\frac{\ln(B/F_t)+\frac{1}{2}\sigma^2(t,T)}{\sigma(t,T)}=d_2+\sigma(t,T),\\ d_3&=\frac{\ln(K/F_t)+\frac{1}{2}\sigma^2(t,T)}{\sigma(t,T)}=d_4+\sigma(t,T),\end{aligned}$$

$$d_5 = \frac{\ln(K^2/(F_t B)) + \frac{1}{2}\sigma^2(t,T)}{\sigma(t,T)} = d_6 + \sigma(t,T),$$

and

$$\sigma^2(t,T) = \int_t^T \left((\rho\sigma_V - b(u,T))^2 + (1-\rho^2)\sigma_V^2\right) du.$$

Let us analyze (16) here: $B - D_1$ corresponds to the Mertonian valuation (i.e. risk-free bond less put-to-default option), D_2 is associated with the value brought to the debt holders by the possibility of early default triggered by safety covenant. The last two terms, $\omega_2 R_2$ and $\omega_1 R_1$, are both positive,[12] and represent the costs of early default and default at maturity respectively. It is therefore clear that the bond's price is decreasing in ω_1 and ω_2.

3 Credit Contracts

This section explains the reasons for issuing debt, and gives an insight to the design of credit contracts that aims for the maximization of firm value and the prevention of unexpected losses in the contracting parties' claims. The answer to this problem is given using the tools described in the previous section, where we briefly introduced theoretical works that help us in pricing the two basic types of claims on the firm's assets: debt and equity.

3.1 Capital Structure

The capital structure of a firm refers to the proportion of securities that ensure the needed funds for financing the firm's projects. These securities have two basic types: a riskier asset called equity and a relatively safe one, the debt. Equity has two further sub-groups (preferred and common), debt has many flavours, and furthermore there exists a group called "hybrid securities" including, for example convertible bonds. In this work we will concentrate on the two basic types only, however the model presented in Sect. 4 can be easily extended to more complex capital structures as well.

The value of the firm is therefore the sum of the market value of its debts and its equity: $V = D + E$. Proposition I of the Modigliani-Miller (M-M) theorem [44] says that the market value of the firm is not dependent on its capital structure, if the following assumptions hold:

- There are no taxes
- The market is efficient (and consequently the bankruptcy costs are zero)
- Absence of asymmetric information

[12] See [7] pp. 105–106

Therefore under these assumptions capital structure does not matter. On the contrary, when capital structure matters, at least one of the M-M assumptions is violated. Consequently the M-M assumptions can guide us in finding the determinants of an optimal capital structure.

The M-M theorem can be extended to an environment with taxes, where interest payments are a tax deductible item. The amount saved on taxes due to leverage is called the Tax Shield (TS) and can be expressed as $TS = T_C \cdot D$, where T_C is the corporate tax rate and D is the value of a perpetual debt. The tax shield is therefore increasing in the debt/equity ratio.

It was showed[13] that the second assumption is violated as well: financial distress and bankruptcy have direct and indirect costs, such as loss of costumers, suppliers, and employees due to uncertain future, need of immediate sale of assets at lower prices, expenses on experts, and so on. As higher leverage means higher interest payments and thus higher probability of not meeting them and falling into financial distress, the expected distress costs are increasing with higher leverage. The effect on the overall firm value is therefore the opposite as for the tax shield.

Asymmetric information—i.e. the violation of the third assumption—implies agency costs, when the conflict of interest between different groups of stakeholders causes suboptimal investment decisions.[14] The typical examples of agency costs are over-investment, under-investment, and cashing-out problem, all of them gaining in significance in states of (or close to) financial distress. The negative effects of agency costs are increasing in leverage, and therefore shifting the optimal indebtedness to lower values.

3.2 Absolute Priority Rule

Absolute Priority Rule (APR) is a concept that describes how the assets should be divided among stakeholders after the event of bankruptcy. The basic order of the APR is, that a junior creditor receives some fraction of the remaining assets only in the case when senior creditors are paid in full. Similarly, equity holders receive nothing, unless all the creditors (both secured and unsecured) get the whole amount of their claim. Furthermore, when a class of stakeholders have the same seniority, they all receive the same ratio of their principal.

A considerable amount of literature[15] has been published on the violations of the APR: while under Chapter 7 liquidation absolute priority is generally enforced, in the case of Chapter 11 reorganizations[16] violation of APR is rather a rule than an exception. The reason is, that equity holders have the power to enforce APR deviation during workout negotiations due to the structure of Chapter 11 rules. The

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[13] See [47], or [10]

[14] More on this see, for example [2]

[15] See, for example, [39], [43], [54], [25], and [26]

[16] See [18] and [55]

management can put the firm in Chapter 11 at a moment when it is in the best interest of equity holders. As there is an automatic stay on payouts to claimants under Chapter 11, a renegotiation could enhance the situation of both equity and debt holders. In addition, the reorganization plan needs to be accepted by the shareholders as well, and therefore they can prolong the bargaining process, and therefore increase the costs of default. This is clearly not in the interest of the senior claimants, and so they rather distribute some value to equity holders and avoid long negotiations. For further discussion of optimality of negotiations during bankruptcy procedures see [27].

A large amount of empirical research have been done in the past two decades about the consequences of these absolute priority violations, and the result showed that APR deviations are beneficial ex ante. They decrease the severity of over-investment in assets requiring managers' special skills and under-investment in firm-specific human capital [5], might improve the timing of bankruptcy [48], hold back excessive risk taking [19] and help to resolve under-investment problem [56]. On the other hand, negative effects of absolute priority violation arise through the problem of moral hazard with respect to investment decisions [4].

3.3 Game Theory Analysis of Credit Contracts

As a typical company of our interest has complex capital structure with many parties of interest, it is reasonable to examine the problem of financing from the perspective of Game Theory. This section is therefore dedicated to this topic, and is particularly based on the work of Ziegler [57]. Our paper may be viewed as additonal, complementary, approach to the game theory analysis of the corporate bankruptcy provided by [28].

The method combines game theory and option pricing, so the maximized value of an option (note the parallel of options and credit contracts) can be calculated. The essence of the method is a three-step procedure:

1. The game between players is defined. The game tree is constructed.
2. The uncertain payoffs are valued using option pricing theory, where the parameters are the player's possible actions.
3. The game is solved using backward induction or subgame perfection.

The strengths of such a method are: taking into account the time value of money and the market price of risk, and separating the valuation problem from the analysis of strategic interaction.

3.3.1 Credit and Collateral

In financial contracting two forms of moral hazard occur: risk-shifting in the situation of hidden action, and observability problem in the situation of hidden information. In the following text these two basic problems are analyzed, whereas more complicated issues will be addressed in the upcoming parts of the section.

The Risk-Shifting Problem The origin of the risk-shifting problem is the borrower's incentive to influence the risk of the project, as he could increase his expected payoff on the expense of the lender. If he is able to change the risk of the project without the creditor's notice, we are talking about hidden action. The lender usually anticipates such behaviour, and requires higher interest rate that leads to adverse selection [50]. An alternative solution is to closely monitor the activities of the borrower, however this increases the costs of lending and therefore the interest rate. The best option would be a contract designed in a way that the borrower has no incentive for risk-shifting without the need of monitoring.

Ziegler [57] examined the situation when the borrower is able to set the riskiness of the project after the debt contract have been signed and the final payoff is observable to both parties with no cost. As it turned out, there exists an infinite number of contracts that preclude risk-shifting, however only contracts with proportional payout are renegotiation-proof (i.e. a situation, when a renegotiation is desirable for both the creditor and the debtor cannot occur). Renegotiation usually involves costs, and therefore both parties will have an incentive to agree on a contract that is not changed over its whole life. This means, that in the case of hidden action, only all-equity financing avoids risk-shifting.

The Observability Problem When the terminal value of the investment is not observable by both parties, a problem arises how the final transfer should be determined. In fact, it can be expected in many situations, that the borrower will have more accurate information about the terminal value, and therefore he can report distorted figures to minimize his payout to the lender.

According to Townsend's [51] costly state verification model—where the lender and the borrower agree in advance on situations when the verification should be taken—the optimal contract has the following properties (pure strategies allowed only):

- If verification does not take place, the payment to the lender is equal to some constant amount D.
- Verification should be taken when the terminal value is below some pre-defined threshold.

This contract is similar to a debt contract with fixed payment D and verification as a parallel to declaration of bankruptcy. Thus the observability problem can be addressed with constant promised payment in no-bankruptcy states. As risk-shifting can be solved only by proportional payment, there is no contract that could avoid both problems simultaneously.

Collateral is an asset, that can be—according to the credit contract—seized in the event of default to limit the lender's losses. A considerable amount of literature has been published on the role of collateral in providing motivation for the borrower to avoid default. For instance, in [3], the loan repayment decision is dependent entirely on the relative values of the collateral and the amount of outstanding debt, default occurring if the value of the collateral at maturity is below the amount due. An inverse relationship between agency costs and the amount of collateral available to borrowers has been shown by [6].

Chan and Kanatas [13] mentioned two types of collateral: it is an existing asset (for example the financed project) or it is an additional asset, normally not available to the lender. Ziegler's model examines the effects of the latter, and concludes that risk-shifting problem disappears only when the loan is fully collateralized, resulting riskless loan. However, collateral protects the lender in two ways: grants higher recovery after bankruptcy and reduces the borrowers incentives to risk-shifting behaviour.

3.3.2 Endogenous Bankruptcy and Capital Structure

In the previous section the credit was a finite maturity contract with a single payment to the lender at maturity. Although such approach is good to understand project financing, it is less useful to model corporate financing. In reality firms keep operating by issuing new debt to finance their new projects, or to repay the maturing debt and therefore keep the ongoing projects alive. Bankruptcy happens, when the entity is unable to meet its contractual payments. In fact equity holders can decide at any point in time whether they want the firm to make the agreed payments or default and trigger bankruptcy. Thereby bankruptcy is an endogenous decision made by equity holders, even if it might be initiated in principle by the creditor.

Ziegler [57] analyses endogenous bankruptcy building on the base of Leland's [34] infinite horizon model with the introduction of several modifications. First, interest on the loan is divided to two distinct types, a continuous effective payment and an increase in the face value of the loan. This division allows to investigate the role of these two components in finding market equilibrium. Second, endogenous bankruptcy is discussed as a principal-agent problem and the agency costs of the equity holder's socially suboptimal behaviour are quantified. Third, the effect of loan covenants and information asymmetry are considered. Fourth, the properties of optimal capital structure are studied, and finally, an incentive contract is developed that could influence equity holder's bankruptcy choice.

The Model A lender and a borrower signs the following contract: at initial time the lender transfers a loan of F_0,[17] and in exchange the borrower pays instantaneous interest of $\phi D(t)dt$, where $D(t) = D_0 e^{\kappa t}$ is the face value of the debt at time t and ϕ is the instantaneous interest rate to be effectively paid on the perpetual debt. Asset sales are prohibited, therefore net cash outflows on interest payments are financed by equity dilution. As κ is the rate of increase in the face value of debt (and therefore the rate of increase in interest payments as well), it is assumed, that $\kappa < r$, where r is the risk-free interest rate.[18] Sinking fund corresponds to the setting $\kappa < 0$.

[17] F_0 denotes the fair value of the loan at time 0, as it will be described in more details later.

[18] Otherwise the present value of the interest payments would converge to infinity.

If (and only if) the debtor defaults on his interest payments, the firm is liquidated with costs proportional to the asset value. The creditor therefore receives $(1-\omega)S_B$ in the event of default, where ω is the proportion lost due to liquidation and S_B is asset value at the time of bankruptcy.

The game has the following structure:

1. The amount of debt, D_0, and interest rates κ and ϕ are determined, the contract is signed. In exchange for its promised obligations the firm receives the fair value of the loan, F_0.
2. The firm makes its investment decision with the associated risk, represented by the volatility rate, σ. In the financing of additional (later) projects under-investment problem might occur.
3. Equity holders choose their default strategy S_B. In the event of bankruptcy ωS_B is lost, $(1-\omega)S_B$ is received by debt holders, and nothing remains to the equity holders.

The management is assumed to fully represent the equity holder's interest, hence there is no conflict of interest between these two parties. [57] assumes the asset value, S to follow the usual geometric Brownian motion, and estimates the firm, equity and debt value using the standard framework based on Merton.

In line with the principle of backward induction, the last stage of the game is examined at first. In this step the equity holders choose optimal asset level S_B for triggering bankruptcy. This level can be found using first-order condition, and is equal to

$$S_B = \frac{(1-\theta)\phi D(t)}{r-\kappa+\sigma^2/2},$$

where θ is the corporate tax rate.

As it can be noted, this optimal level is linear in $\phi D(t)$, and is independent on current asset value S. Furthermore, higher asset risk (σ) implies lower optimal bankruptcy boundary.

The Principal-Agent Problem and Agency Costs The principal-agent problem stems from the fact that the debtor (agent) adopts a different bankruptcy barrier than it would be optimal from the creditor's (principal's) view.[19] The creditor would choose a default boundary either to zero (to make his claim riskless) or as high as possible (to receive the firm's assets when they have a high value). The socially optimal bankruptcy strategy turns out to be the one with the lowest possible level of bankruptcy triggering, i.e. $S_B = 0$. This comes from the positive cost of bankruptcy for any asset value higher than zero.

In order to construct an incentive contract that would lead to socially optimal bankruptcy the effectively paid interest on debt, ϕ has to be zero, since for any other value the equity holders would trigger bankruptcy at a positive asset level. However, setting $\phi = 0$ means that the claim is worthless, as no interest is paid out. In other

[19] The optimal default levels from the debtor's and the creditor's points of view are derived in [57], pp. 48–49.

words, because of the borrower's limited liability, socially optimal default level can not be reached.

Armed with the above results the agency costs arising from endogenous bankruptcy can be expressed. The agency cost represents the expected deadweigth loss caused by the expected costs of bankruptcy. Intuitively, these costs are in direct relationship with the probability of bankruptcy (increasing in S_B and $\phi D(t)$), and with the proportional loss due to liquidation, ω.

The Investment Decision - Under-investment and Risk-shifting Once we have investigated the equity holder's optimal bankruptcy decision S_B, we should examine their investment choices. Two main issues are studied in the following paragraphs: under-investment and risk-shifting. Myers [45] highlighted that firms may abandon profitable projects in the existence of debt by refusing recapitalization of the firm. The reason of doing so is, that although equity holders would bear the full costs of the project, debt holders also benefit from this investment as the debt becomes less risky.

Ziegler [57] analyses the under-investment problem with a model that represents new investment as a scale up of the existing operations by some factor $w > 0$. The investment requires therefore additional wS of funding and increases the value of the firm's assets to $(1+w)S$. Since additional (equity funded) investment reduces expected bankruptcy costs and increases tax shield,[20] it always increases the overall firm value.

The model's calculated change in the value of the equity shows, that it is always lower than the costs of the investment, and therefore the overall return to equity holders is negative. Hence under-investment always arises. This problem can be addressed by renegotiation of the debt (reduction of D, ϕ, or κ) in order to ensure positive expected return on investment for the equity holders, or alternatively by sharing the costs of the new investment.

So far in the model of endogenous bankruptcy constant and known asset risk σ was considered, however in some cases this assumption might not hold. The question is, whether the agent has an incentive to increase the asset risk if the principal can not observe (and therefore control) his action. To answer this, Ziegler examined the partial derivative of the equity value with respect to σ^2. The result shows, that a leveraged firm has always incentives to increase asset risk. This has an implication for the optimal behaviour of the lender: he should focus on monitoring asset risk instead of asset value, as the risk is the relevant variable for the borrowers' bankruptcy decision.

Agency costs of risk-shifting can be expressed as a difference between the firm value at the social optimum less the firm value with the possibility of risk-shifting. Since firm value decreases with bankruptcy costs, it can be maximized by setting these costs to zero by approaching σ to zero. Agency cost is therefore equal to

$$C = \lim_{\sigma \to 0} W(S) - \lim_{\sigma \to \infty} W(S),$$

[20] Note that early bankruptcy means no tax deductibility in the future, and therefore it decreases the current value of the tax shield.

where, again firm value is W. As Ziegler [57] showed, the difference in the above limits is

$$C = \frac{\theta\phi D(t)}{r-\kappa},$$

i.e. to the value of the (safe) tax shields.

Effects of Loan Covenants It was shown in the previous sections, that under certain conditions, a "plain vanilla" debt contract[21] might imply deadweigth loss that moves the resulting firm value below its socially optimal level. To mitigate these losses, loan covenants might be introduced. A loan covenant is a condition agreed at debt issue that has to be fulfilled by the debtor. Covenants can take many forms, regulating operating activity, asset sale, cash payout and others[22]. Here, so-called safety covenants are analyzed which give the bondholder the right to force bankruptcy if certain conditions are met. More specifically, suppose a covenant that forces the firm into bankruptcy, if its asset value falls below some specified level $\overline{S_B}$. Reaching this level means transfer the ownership of the assets to the lender. As it turned out, the risk-shifting incentive depends on the level of this barrier: for low levels risk-shifting incentive is still present, however for higher values the situation changes and the debtor will have an incentive to decrease the risk of the investment. The breakpoint is naturally higher than S_B, the endogenous bankruptcy barrier set by the equity holders only.[23] Concluding the effects of such loan covenant, we should remark that they protect the lenders in two ways:

First, they reduce losses of the creditors by setting the default barrier higher, and
Second, they mitigate or even eliminate equity holder's risk-shifting incentives.

Hence, setting a safety covenant with an agreed level has similar effects as using collateral.

3.3.3 The Financing Decision

Using the results derived, we can investigate the way a firm should be financed. We will analyse—under endogenous bankruptcy—the optimal capital structure of a firm, and the effects of the way how the interest is divided between the interest effectively paid and growth rate in the face value of debt.

Optimal Capital Structure Assume that the asset risk is known to the lender and risk-shifting is not possible, or alternatively, it is possible only within certain bounds. In the latter case the lender would anticipate the borrower's risk-shifting behaviour, and therefore he will use the maximal available volatility value in his loan pricing calculations, $\bar{\sigma}$. We assume that the face value of the loan cannot be

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[21] Here "plain vanilla" refers to the absence of additional clauses defining loan covenants.

[22] A comprehensive analysis of covenants and their effect on debt pricing can be found in the work of Reisel [49].

[23] For the mathematical derivation of this statement see [57], pp. 58–59.

changed after the initial agreement, and that the borrower takes the offered interest rates κ and ϕ as given when selecting the initial face value of debt, D_0.

The financing decision is made with respect to the equity holders' effort to maximize the value of their holdings after the initial investment, I. Ziegler's calculations show, that there exists an interior maximum of the net equity value (that is the difference between the value of equity after the debt is taken and the equity holders' initial investment) in terms of optimal capital structure. As the rate of effective interest payments, ϕ rises—and consequently so does the cost of the debt service—the optimal face value of debt decreases. Similarly a higher growth rate in the face value of debt, κ, means lower optimal face value of debt. It also turns out, that changes in ϕ are perfectly offset by the endogenously chosen face value of the debt, and so the continuously paid coupon remains the same. Thus ϕ affects the nominal leverage (D_0/S_0), however it does not affect the leverage in market terms (F_0/S_0).

Interest Payments vs. Increase in the Face Value of Debt A natural question is, how the debt service should be divided between the interest payments ϕ, and the growth rate of face value of the debt κ. As the optimal leverage in market terms is not affected by ϕ, the borrower is indifferent to the interest rate effectively paid. On contrary, the rate κ does affect the optimal capital structure and the net equity value: with increasing κ the optimal leverage ratio and the net equity value decreases. Consequently equity holders prefer to pay higher effective interest instead of higher growth in the face value of debt.

Expected Life of Companies As the optimal capital structure and the conditions of the loan are given, it is possible to express the mean time of default. Using the analysis of Ingersoll [24], we know that the mean time of passing the origin for a standard geometric Brownian motion $dx = \mu dt + \sigma dW_t$ with initial value x_0 is given by

$$\bar{\tau} = \frac{x_0}{\mu}$$

With the help of this formula—after some computations—the mean time of default under endogenous bankruptcy can be revealed[24]. This value turns out to be independent on the parameter ϕ, in line with the finding that the borrower offsets the changes in the effective payout rate by changing the face value of debt. Again, the important parameter is κ, that influences mean time to bankruptcy.

An Incentive Contract It is worth to consider whether the lender can set the contract parameters ϕ and κ in a way that influences the borrower's bankruptcy strategy S_B. As it is in the lender's interest to have a higher default barrier, we will examine the possibilities of an incentive contract that induces the borrower to declare bankruptcy at a higher asset value. Early bankruptcy is interesting for the borrower for several reasons. First, the lender might be himself an agent and so he might have restrictions on the maximum risk he can take. Second, early liquidation may increase beliefs about the lender's solvency and therefore avoid some problems such

[24] See [57], pp. 67–68

as bank runs. Third, it enables the lender to save on monitoring costs as he can use early information provided by default on interest payments.

Since changes in the effective interest rate are perfectly offset by changes in D_0, ϕ does not affect the borrower's behavior. On the other hand the rate of growth in the face value of debt, κ does influence the borrower's optimal bankruptcy strategy. As a lower κ means faster debt repayment (through higher face value or equivalently higher ϕ), the resulting optimal bankruptcy triggering level is higher.

4 A proposed structural model

Section 3 gave an insight to the design of credit contracts, and showed the usability of game theory in pricing of corporate assets and predictions of rational actions taken by the parties concerned. Here, we extend the available literature of asset pricing models introduced in Sect. 2, and build up a framework with stochastic interest rate. This framework than serves as a valuation method for a similar game theory analysis as was introduced in Sect. 3.3. The starting-point of this work is the Goldstein et al. [21] EBIT-based model, that will be extended by the relaxation of the constant (or deterministic) interest rate requirement.

4.1 Assumptions

First of all we take the following assumptions:

(i) The management fully represents the equity holders' interest.
(ii) The APR is never violated.
(iii) Asset sales are prohibited, interest payments are financed by earnings and equity dilution.
(iv) When the earnings are above the paid interest, the difference is paid out as dividend.
(v) Paid interest is a tax deductible item, however no tax carry-back or carry-forward exists[25].
(vi) There is a sufficiently large number of investors, and only a limited amount of projects.

Assumptions (iii), (iv), and (v) imply the unimportance of the historical cash flow in the asset pricing. The current values of the two memoryless processes—the risk-free interest rate and the EBIT—are the only two stochastic variables that affect the debt, equity and firm value. Assumption (vi) has the consequence that the provided loan is always fairly priced, since the financial institutions perfectly compete with

[25] As Nejadmalayeri and Singh [46] showed, the US tax code's loss carry provisions affect the equity holders' bankruptcy decision.

each other. Next to these initial assumptions we will use further suppositions in the subsequent sections, particularly during the description of the stochastic evolution of the variables: the risk-free interest follows an Ornstein-Uhlenbeck process, the Earnings Before Interest and Taxes (EBIT) is supposed to follow a GBM, and so on.

4.2 Risk-free Interest Rate

Most of the models assume constant risk-free interest rate in order to simplify the calculation. However, in reality this interest rate does change in time, reflecting the situation of the overall economy. Modelling the interest rate stochastically allows us to include the possibility of a macro-level change and catch the correlation between the overall market and the modelled asset. Using this correlation the model could be extended to a risk averse measure, where higher return is expected just for the market risk—the one that can not be diversified (in line with modern portfolio theory, see [38]).

The risk-free interest rate $r(t)$ follows an Ornstein-Uhlenbeck process suggested by Vasicek [52], and used for example in the Longstaff and Schwartz [36] approach:

$$dr = \alpha(\gamma - r)dt + \sigma_r dW_t \tag{17}$$

where $\alpha > 0$ indicates the force pulling the interest rate back to its long-term mean γ at speed $\alpha(\gamma - r)$ per unit of time. The stochastic element is a standard Wiener process W_t times the volatility σ_r.

The expected value and variance at time s given $r(t)$ are

$$E_t[r(s)] = \gamma + (r(t) - \gamma)e^{-\alpha(s-t)}, \quad t \le s$$
$$Var_t[r(s)] = \frac{\sigma_r^2}{2\alpha}(1 - e^{-2\alpha(s-t)}), \quad t \le s$$

respectively. The distribution of $r(s)$ given $r(t)$, $t \le s$ can be written as

$$r(s) = r(t)e^{-\alpha(s-t)} + \gamma(1 - e^{-\alpha(s-t)}) + \frac{\sigma_r}{\sqrt{2\alpha}} W_t(e^{2\alpha(s-t)} - 1)e^{-\alpha(s-t)}$$

Having the assumption of risk-neutral measure (i.e. the yield to maturity is not dependent on the maturity date and thus there is no risk premium), the value of \$1 received at time $s \ge t$ has the value of

$$P(t,s) = E_t\left[\exp\left\{-\int_t^s r(\tau)d\tau\right\}\right] \tag{18}$$

received at t.

4.3 Earnings Before Interest and Taxes

Traditional models—building on the basis of Merton's [42] framework, including those introduced in Sect. 2—take unlevered equity as primitive variable with log-normal dynamics. However, for some models it seems to be more straightforward to use earnings instead of unlevered equity. Mella-Barral and Perraudin [40] consider a firm that produces output and sells it on the market, where the price of the sold product follows a geometric Brownian motion. Mello and Parsons [41] use a similar framework with a mining company and stochastic commodity price movements. Graham [22] models EBIT flow as a pseudo-random walk with drift, Goldstein et al. [21] and Broadie et al. [12] use geometric Brownian motion for the evolution of EBIT.

To see the advantages of such approach, we should review some of the main shortcomings of the traditional framework. **First**, unlevered equity ceases to exist as a traded asset when debt is issued. This problem is one of the motivating factors behind several subsequent frameworks [31, 32, 17]. **Second**, they treat tax payments in a different fashion as they deal with cash flows to debt and equity holders. In fact, they count tax benefit as capital inflow instead of using it for reduction of outflows. This implicitly assumes that it is always possible to deduce fully the interest costs from the tax payments, however, this is not the case when the cost of debt service is higher than the current EBIT. Another problem with the tax benefit approach is, that it implies higher firm value through higher tax shield as the tax rate increases. **Third,** as Goldstein et al. [21] noted, these models may significantly overestimate the risk-neutral drift, consequently underestimate the probability of bankruptcy and so the optimal leverage ratio.

Our model assumes an EBIT process with log-normal dynamics, and therefore is able to address the mentioned issues. The evolution of the firm's instantaneous EBIT, δ_t is modeled using geometric Brownian motion with risk-neutral measure $\mathbb{Q}$, similarly as Broadie et al. [12]:

$$\frac{d\delta_t}{\delta_t} = \mu dt + \sigma dX_t(\mathbb{Q}), \tag{19}$$

where

$$X_t = \rho W_t + \sqrt{(1-\rho^2)} Z_t.$$

W_t is the same process as in (17), Z_t is a standard Wiener process and ρ is the correlation coefficient between the risk-free interest rate and EBIT.

If the δ_t is known at $t = 0$, the differential equation (19) has the solution

$$\delta_t = \delta_0 \cdot \exp\left\{ \left(\mu - \frac{\sigma^2}{2} \right) t + \sigma X_t \right\} \tag{20}$$

Assuming no taxes and zero leverage, the value of the firm is the sum of discounted earnings. Using the notation V_t^0 for unlevered equity value at time t, we have

$$V_t^0 = \int_t^\infty \delta_t \cdot \exp\left\{ \left(\mu - \frac{\sigma^2}{2} \right)(s-t) + \sigma X_s - \int_t^s r(\tau) d\tau \right\} ds,$$

in line with (18).

4.4 Debt

The debt issuance and repayment is similar as in Ziegler's [57] model with endogenous bankruptcy, although several modifications are implemented. Most importantly, as the risk-free interest rate is considered to be stochastic, the interest payments are stochastic as well. Second, Ziegler considered a debt service divided between effective interest payments and growth in Face Value of debt (FV). As he proved that changes in effective interest rate are compensated by changes in face value of debt, its scalability will be left out from our model.

The debt is therefore set up it the following way:

1. The rate of growth in face value of debt, κ is chosen
2. The borrower (i.e. the firm) chooses the initial face value of debt, FV_0
3. The lender calculates the fair value of this debt, given the face value and κ, and provides a transfer to the borrower equal to this fair value.

After receiving the funds, the borrower starts to serve the interest payments. The FV at any point in time is given as:

$$FV_t = FV_0 \cdot e^{\kappa t}$$

The interest is continuously paid out at a rate $c_t = FV_t \cdot r(t)$ (coupon rate) with infinite horizon. We assume $\kappa < \gamma$, similarly as Ziegler, otherwise the discounted FV, and consequently the interest payments would growth to infinity.

The economic intuition behind this model is a floating coupon perpetual bond issue, where this corporate bond is (usually) sold below par. In order to catch constructions as a sinking fund, or alternatively a growth in debt principal, the parameter κ is introduced as well.

4.5 Default

The event of default corresponds to the situation, when the firm does not meet its obligation on interest payments. We assume, that creditors take over the firm immediately after its default and suffer the associated losses. Absolute priority rule is enforced, i.e. after bankruptcy equity holders receive nothing.

As the state variable is the instantaneous EBIT, it is convenient to define the recovery value as a multiple of the EBIT at the moment of default. Since a firm

Table 1 Notation

Symbol	Explanation	Base value
	Interest rate	
$r(t)$	Risk-free interest rate	$r(0) = \gamma$
γ	Long-term mean of risk-free interest rate	3%
α	Speed of expected risk-free interest rate convergence to γ	0.25
σ_r	The volatility of risk-free interest rate	0.5%
$P(t,s)$	The price of a \$1 face value riskless zero-coupon bond at time t, maturing at time s	
	Firm	
δ_t	EBIT	$\delta_0 = 100$
μ	Drift of EBIT under $\mathbb{Q}$	0.01
σ	Volatility of EBIT	20%
ρ	Correlation coefficient between $r(t)$ and δ_t	0.2
V^0	Firm value with no leverage and the assumption of zero taxes	
T_C	Corporate tax rate	35%
	Debt	
FV_t	Face value of debt	
κ	Growth rate of the face value of debt FV_t	1%
$D(\delta_t)$	Debt value	
c_t	Coupon rate, equals to $FV_t \cdot r(t)$	
	Default	
DB_t	Default Barrier	
τ	Time of default	
RR	Recovery rate defined as a multiple of yearly EBIT	10×

effectively becomes unlevered after bankruptcy (as its debt holders become the new equity holders), and we calculate the unlevered value during the iterations, this multiplier can be easily transformed to Loss Given Default (LGD)—a ratio that expresses the asset value lost due to bankruptcy.

4.5.1 Default Barrier

It is sensible to define the Default Barrier (DB) on the state (primitive) variable, since all the other values can be written as a function of this state variable. As we have an EBIT based model, DB will be defined on earnings. When the primitive variable is firm (or unlevered equity) value, DB is usually a function of the face value of debt. A straightforward modification for our model is to make the DB linearly dependent on the instantaneous coupon rate, c_t (as we show in Sect. 4.9, this setup proved to be consistent with the overall model).

Such modification would imply a lower barrier in recession (low risk-free rate), and thus work counter-cyclically. There are several facts that support this design: in recession the number of bankruptcies increases (see, for example [1]), thus banks experience losses in connection with other loans and might prefer immediate payments instead of triggering bankruptcy that yields uncertain income later. Furthermore as Altman et al. [1] also showed, the recovery rate is significantly lower in recession. Exactly the opposite holds for economic boom and high interest rates, therefore higher default barrier is reasonable.

4.5.2 The Bankruptcy Decision

The entity that does the bankruptcy decision is dependent on the transparency of the firm, on the credit contract, and possibly on other factors. When the state variable is not publicly observable, the firm's management is the only one who can trigger bankruptcy. On the contrary, when the state variable is observable, bankruptcy decision can be declared in the credit contract, and therefore support more favourable debt financing. This is in fact a safety covenant for the creditors, that ensures them the right to force bankruptcy if the firm performs poorly (that is crosses the DB).

4.6 Method and Calculations

Due to the high complexity of the model we use Monte Carlo simulations to uncover the model's sensitivity on its parameters (see Table 1 for parameter base values). The calculated results are used as payoff valuation for game trees analyzed in Sect. 4.7.

4.6.1 The Effects of Debt Face Value

The Face Value of debt (FV) is the most basic parameter of a corporate loan: it is the figure that appears on the firm's balance sheet and in other reports and statistics. It is also the exclusive right of the borrower to specify the loan's FV directly or through the amount of borrowed funds. The main questions addressed in the following lines are, whether it pays off to issue debt at all, whether there is a maximal firm value an if so, what level of FV corresponds to this maximum, and how this optimal value is dependent on the DB.[26]

Figure 1 illustrates the dependence of debt, equity and firm values on credit contracts with different face values. As it is visible, when the leverage is low, firm value can be enhanced if a debt with higher face value is issued due to increasing tax shield. At a certain point the rising bankruptcy costs exceed further tax savings, indicating an optimal face value of debt that maximizes firm value. With a low DB[27] equal to 0.3, for example the firm value can reach 35 times the yearly EBIT if a debt

[26] At this point we do not concentrate on the problem how the DB is chosen; that issue will be covered in Sect. 4.7.

[27] Recall that a default barrier of 0.3 means triggering default when the instantaneous earnings are at 30 percent of the coupon rate.

is issued with face value between 20 and 30 yearly earnings. This means an optimal debt ratio of circa $60-80\%$. As the DB rises, this optimal ratio declines due to higher Probability of Default (PD): with $DB = 0.7$ the maximal firm value declines below 3200 (i.e. 32 times the yearly EBIT) with debt ratio of 30% only. The effects of changes in the DB are described in details in Sect. 4.6.2.

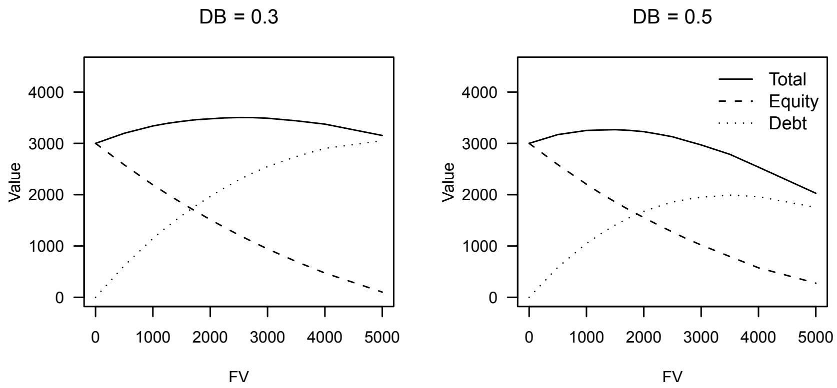

Fig. 1 Debt, equity and total value with different face values of debt

By observing the debt values it is apparent that at a certain FV the debt value reaches its maximum: this is the highest possible amount of money that could be reached with sole debt financing. The plotted equity values are not relevant as the equity holders are compensated for their decrease in equity value by receiving the funds obtained from the loan. Therefore the equity holders seek a loan agreement that ex-post maximizes firm value[28].

4.6.2 The Effects of Default Barrier Level

Next, we should explore how the output variables react on different levels of default barriers. To do so, we have plotted our basic calculation,[29] where no extreme values distort the picture. Figure 2 shows how the level of default barrier affects the equity, debt and overall firm value.

The overall firm value has the most unequivocal trend: it is declining as the barrier rises: the FV affects only the slope, not the tendency. Intuitively, setting the DB lower implies drop in the number of bankruptcies, later occurrence of the expected bankruptcy, and shrink of the LGD in absolute terms. Recall that the expected costs of bankruptcy equal to the product of these three factors: PD, LGD and the discount.

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[28] This holds only at the moment when the contract is signed. Later on both the debt and equity holders profit from an increase in the firm value.

[29] That is the one with parameters set to their base levels.

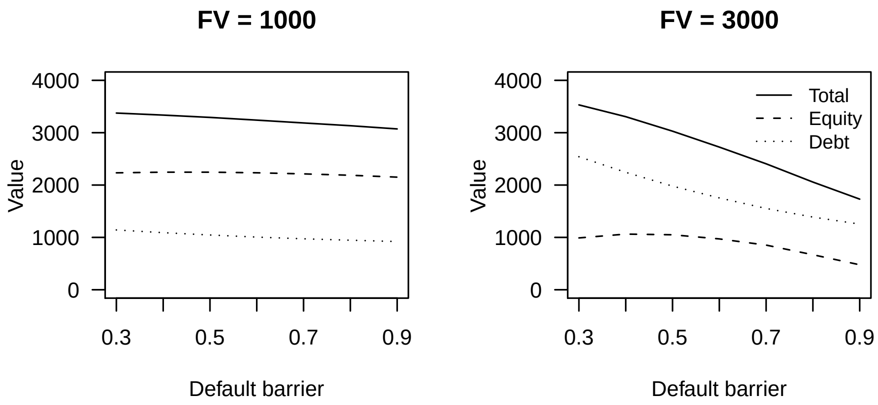

Fig. 2 Debt, equity and total value dependence on the DB with FV 1000 and 2750

The value of debt is rising with lower DB level. Again, this is intuitive, since default occurs later, therefore more money flows to creditors through equity dilution. If we examine the curves of the debt value on Fig. 2, a convergence in this value can be observed, as the DB rises. Because the initial EBIT is set to 100 and the base value of the RR multiple is 10, the debt value needs to be 1000 for sufficiently high DB that triggers default immediately. Consequently this needs to be the level where debt value converges to.

The third curve—the one that demonstrates the equity value sensitivity on shifts in the DB—is somewhat different: it has a "quadratic" shape with a maximum around 0.5. This means that, from the equity holders' point of view, there exists an optimal non-zero default decision. This result is highly important for our game theory analysis in Sect. 4.7, where we examine the rational behaviour of the involved parties. This conclusion, as well as the results related to the firm and debt values, is in line with Ziegler's [57] findings derived using closed form calculations in constant interest rate environment.

4.7 Agency Costs

4.7.1 Observable Actions

With observable actions, the creditor is able to control the parameters that affect the probability distribution of the EBIT flow, most importantly σ, which is determined by the riskiness of the firm's projects. This situation significantly simplifies the arrangement of the credit contract, since the lender does not need to study the set of possible actions that might be done by the debtor. In other words, the probability distribution of the payoffs is given, and therefore risk-shifting is not possible.[30]

Observable State Variable The simplest situation is, when the firm is completely transparent, and therefore the creditor can observe the management's actions and also the state of the firm. In this case a debt contract can be signed with such covenants that enforce both an agreed volatility and defines a default barrier at which bankruptcy will be triggered.

In this case such a combination of debt face value and default barrier will be chosen that maximizes firm value. This leads to a highly leveraged firm (to maximize the value of tax shield), and to low default barrier (to minimize the bankruptcy costs). Note, that it might be not always possible to specify an arbitrarily low DB: when the EBIT decreases so drastically, that the equity becomes worthless, it is not possible to finance the interest payments trough equity dilution. In a stock company the shareholders cannot be forced to transfer additional funds to the distressed firm. In contrast, when the considered firm is owned by a parent company, the interest payments can be guaranteed by the mother.

Not Observable State Variable Similarly as in the previous case, actions are observable, and therefore risk shifting is not possible. However, as the state variable is not followed by the creditor, a bankruptcy barrier as safety covenant can not be included in the credit contract, because it would be impossible to enforce it. Consequently the debtor will choose the default barrier in a way that maximizes its equity holders' value under the given circumstances. This decision is the bottom level of the game tree, and therefore it determines the expected payoffs under certain credit contract parameters. Table 2 shows an equity value matrix for several debt face values calculated using the base parameter setting.[31] As it can seen, the equity holders will choose to default on interest payments when the EBIT will be between 40 and 50% of the coupon rate (bold values in Table 2).

Table 2 Equity values - Basic parameters

	0.3	0.4	0.5	0.6	0.7	0.8	0.9
0	3037	3037	3037	3037	3037	3037	3037
500	2618	**2623**	**2623**	2619	2614	2604	2593
1000	2233	**2246**	**2246**	2234	2213	2187	2151
1500	1881	**1904**	**1903**	1881	1848	1792	1727
2000	1558	**1595**	**1595**	1562	1504	1404	1281
2500	1264	**1319**	**1316**	1257	1162	1037	865
3000	990	**1063**	**1050**	971	853	669	477
3500	742	**837**	**813**	725	563	354	121
4000	510	**623**	**610**	483	304	73	0

Default barrier on the X-axis and debt face value on the Y-axis

[30] More about risk shifting in the next section, where—in contrast with the present situation—it is possible.

[31] See Table 1

As the lender anticipates the borrower's behaviour in the bankruptcy triggering decision, he prices the loan according to this action. We have discussed in Sect. 4.6.1, that the equity holders want to maximize the overall firm value, and so they will choose FV that implies this highest possible value. Table 3 gives the valuation of this step in the game: the creditor offers loans priced according to the equity holders's default decision, therefore the equity holders' can choose total firm value only within the column specified by the planned (by shareholders) respectively assumed (by bondholders) DB. In this case the optimal face value of debt is 2000 for $DB = 0.4$ and 1500 for $DB = 0.5$. The corresponding firm values are 3400 and 3300 respectively.[32] The resulting total value, equal to 33–34 yearly EBITs is significantly higher than the unlevered value with 30 EBITs only. On the other hand, the maximally possible 3550 is not reached due to agency costs caused by asymmetric information.

Table 3 Total firm values - Basic parameters

	0.3	**0.4**	**0.5**	0.6	0.7	0.8	0.9
0	3037	3037	3037	3037	3037	3037	3037
500	3234	3222	3207	3191	3175	3156	3137
1000	3375	3336	3292	3240	3186	3133	3072
1500	3471	3393	**3306**	3211	3118	3010	2897
2000	3525	**3404**	3270	3122	2966	2778	2582
2500	3543	3375	3173	2952	2714	2463	2179
3000	3531	3306	3029	2725	2406	2057	1731
3500	3499	3203	2839	2454	2049	1632	1203
4000	3429	3045	2613	2123	1650	1147	1000

Default barrier on the X-axis and debt face value on the Y-axis

Paradoxically, the equity holders' ex post effort to increase the value of their claim decreases the total firm value (and so their total payoff) ex ante. This problem can be solved if they manage to ensure the lender, that they will default on their payments when the EBIT truly crosses the DB. Such contract requires monitoring with some associated costs, however if these costs are below the agency costs then monitoring should be introduced.

4.7.2 Hidden Actions

When the management's actions are not observable, the debtor is able to modify the parameters driving the EBIT flow, and so to change the expected payoffs of the involved parties. More specifically, he is able to shift the risk to the creditor,

[32] All these values are rounded: as we want to illustrate the decision process, the accurate numbers are not important. In real the DB is one number (between the mentioned 0.4 and 0.5) not an interval, and the FV that corresponds to the maximal firm value given this DB is determined unambiguously as well.

and consequently to enhance the value of his claim on the creditor's costs. Such behaviour is called risk-shifting or, in a wider sense, moral hazard.

To demonstrate this problem, recall section 2.1, where we described how Merton [42] proved that the value of equity in a leveraged firm can be expressed as European call option, and (using put-call parity) the value of debt is equal to a riskless bond with appropriate parameters less the value of a European put option. When the volatility of the asset's value rises, both options become more valuable, and therefore the equity value rises while the debt value declines. This model is valid only when there is no default prior debt maturity (and other assumptions made by Merton hold), however it illustrates the principle of risk-shifting.

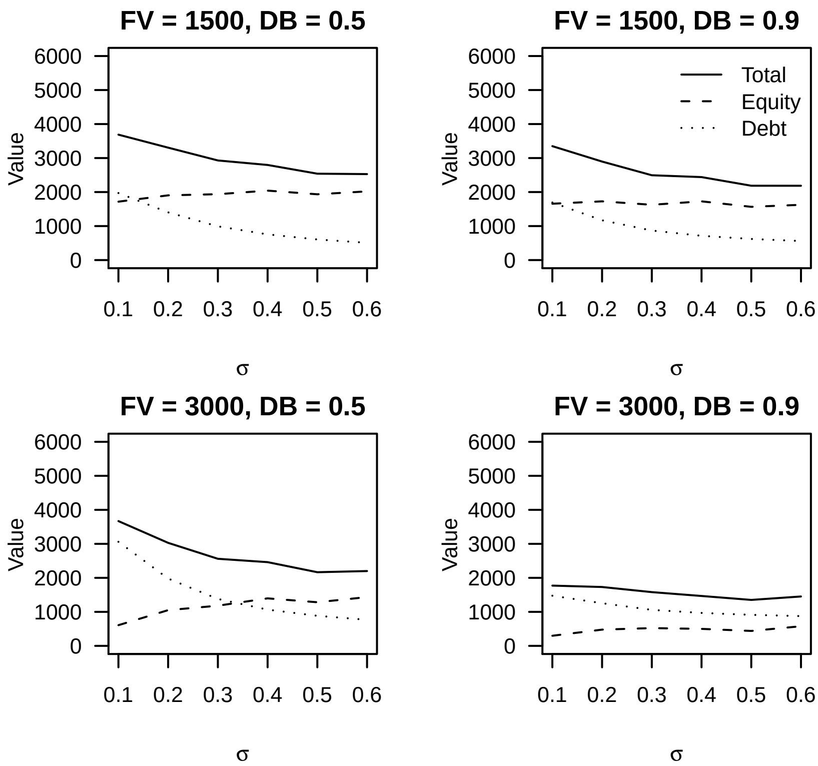

Fig. 3 Firm value dependence on σ

To find out whether risk-shifting appears in our model, and if so, what are its consequences, we have run simulations with several different EBIT volatility parameters. For the details of the simulation see [15]. With higher σ values we observed the following (see Fig. 3):

Equity value was rising, with steeper slopes for lower DB settings. In consequence the equity holders try to increase the EBIT volatility as much as they can, however they have a lower incentive to do so when the DB is higher. This means that if there are some additional costs of higher volatility paid by the equity holders[33], than they will not set the volatility to such high levels as they would so with lower DB.

Debt value was declining, however this decline was moderate for high DB settings. There are two reasons that support lower losses in debt value: First, and most importantly, default occurs at higher firm value, and therefore the firm has higher residual value after the bankruptcy that is transferred to the creditor. Second, default occurs earlier, therefore the asset value received has a smaller discount.

Probability of default rose.

Total firm value was decreasing due to increased PD.

Default barrier chosen by the equity holders was decreasing: their option on the firm's assets become more valuable with the increased volatility.

All of these observations are in line with the conclusions of Ziegler [57], who based his analysis on game theory and gave closed-form results for his model with constant risk-free interest rate. Next we examine how the observability of the instantaneous EBIT affects the credit contract's design and the behaviour of the involved parties.

Observable State Variable If the state variable is observable, it is feasible to mitigate the equity holders' risk-shifting incentive by setting a sufficiently high DB as a safety covenant. For a better understanding of the mechanism of this safety covenant we extend the Mertonian parallel of the equity value and a European call option. After the introduction of an exogenous default barrier the European call option is replaced by a down-and-out call barrier option.

Such an option has a similar price as a plain vanilla option if the DB is far below the spot price, and the volatility is not extremely high. However, as the spot price approaches the barrier, the option values begin to significantly differ. Fig. 4 shows[34] the prices of down-and-out barrier and plain vanilla call options as a function of the volatility, assuming a strike price 1000, barrier 900, constant risk-free interest 3% and time to maturity 1 year. As we can see, the equity holders' incentive to increase the volatility is mitigated when the firm value approaches the DB.

Our model shows a similar behaviour: when the DB is high (80–90% of the coupon rate), the equity value is not increasing significantly with higher volatility. A high DB can be used therefore as a safety covenant in order to avoid risk-shifting. This implies a loan with low FV (about 5 yearly EBITs in our basic setting; recall Fig. 1), and consequently results a total firm value of only circa 3150 (31.5 yearly EBITs). Comparing this number with the theoretical maximum of a fully transparent firm (3550), the losses caused by risk-shifting are equal to the firm's four yearly earnings. Similarly as in the case of not observable state variable, it might pay off to introduce monitoring on the management's actions, and therefore to avoid risk-shifting.

[33] This could be lower expected EBIT growth, or some risk of being exposed, for example.

[34] Source: author's calculations using Financial Derivatives Toolbox

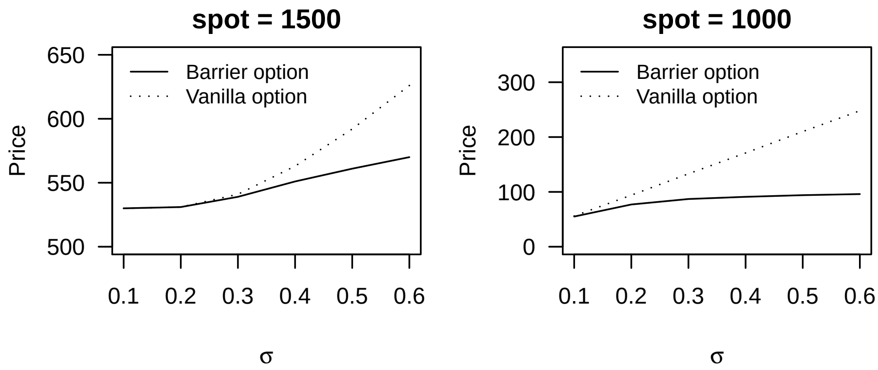

Fig. 4 Barrier option price dependency on volatility, barrier 90% of strike

Not Observable State Variable If the state variable is not observable, equity holders will increase the EBIT volatility and default on interest payments later. Since the creditor anticipates such behaviour, he prices the loan with respect to higher expected volatility. Consequently the resulting firm value (as it is depicted in Fig. 3) is lower than the value of the unlevered firm. The shareholders' ex-post behaviour therefore disables debt financing, and hence making the possible tax benefits unavailable.

4.8 Initial Interest Rate Level

An important advantage of the introduced mean-reverting interest rate environment is, that it can deal with a risk-free interest rate that is not on its long-term average (γ). In such case the interest rate is expected to return to γ, however, this takes some (random) time. In models with constant interest rate it is not possible to cover this situation. With a stochastic interest rate model though, it is just a question of different initial value $r(0)$ in the SDE (17). Furthermore, the effects of exogenous changes in this initial level can be examined. These exogenous changes in the risk-free interest rate correspond to the decisions of the central bank, and therefore we are able to predict the effects of the monetary policy on microeconomical level.

To see the effects of changes in the initial interest rate, we have run calculations with $r(0) = 1\%$, $r(0) = 3\%$, and $r(0) = 5\%$. Figure 5 demonstrates the obtained results for two different FVs. The tick lines show the total firm value dependence on the DB for three different initial interest rate levels. The gap between these lines represent the loss—ceteris paribus—when the interest rate suddenly increases to the next examined level. This drop in firm value is caused by two factors: higher

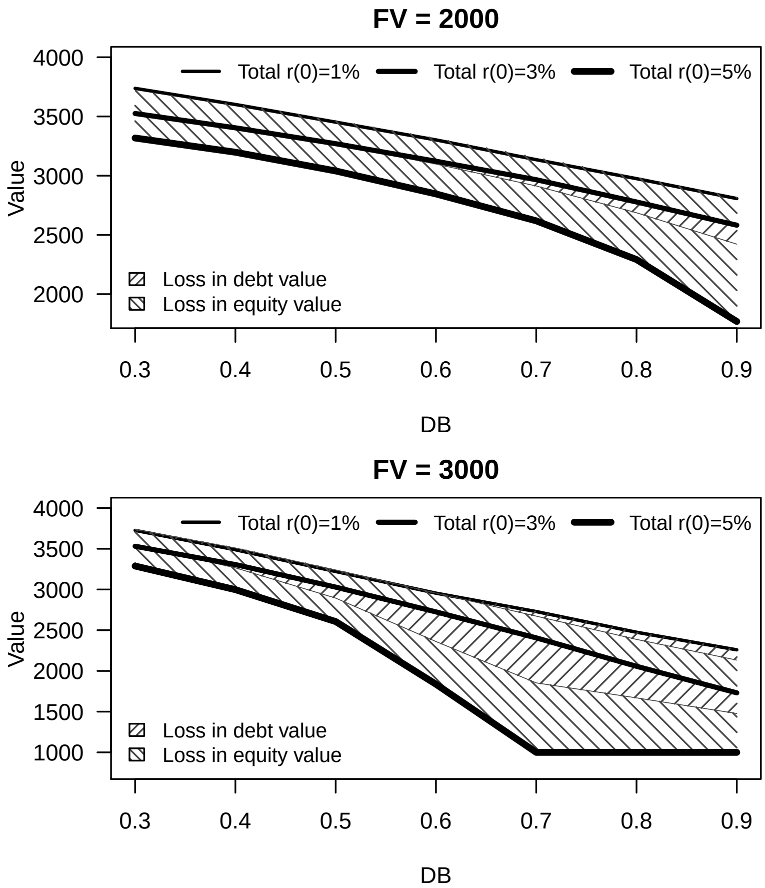

Fig. 5 Firm value dependence on initial interest rate

discount for all future earnings and increased PD due to higher interest payments.[35] The mentioned gap is a sum of declines in equity and debt value, and therefore we can divide this area to distinguish the losses of the two involved parties.

A larger fraction of the firm losses is booked by the equity holders (recall Fig. 5). Their claim is depreciated by the factors that affect the firm value (i.e. higher discount of future income and increased PD), and also by one additional: higher interest paid out to debt holders.

We can see that the debt value is insensitive to changes in initial interest rate, when the probability of early default is close to zero due to low FV and DB. Our conclusion is, that increased coupon payments perfectly offset higher discount on

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[35] Higher interest payments imply higher DB in absolute terms. The DB of the x axis on Fig. 5 is a ratio of the instantaneous interest payments.

future cash flows.[36] Consequently the only factor that decreases the bond's value is the increased default probability and its earlier expected occurrence.

Note, that this section explains how the central bank's interventions work. In economical downturn the monetary policy can support the companies by targeting a lower short-term rate. This increases the value of both traded and non-traded assets, reduces the number of defaults, and supports debt financing through the decrease of interest paid on the outstanding principal. The latter is favored by two factors: the risk-free interest is low, and the risk-premium drops due to lower PD. On the contrary, an overheated can be cooled down with higher risk-free interest.

4.9 Comparison of Stochastic and Deterministic Default Barrier

Stochastic risk-free interest rate and DB are the two features of our model that distinguish it from other EBIT-based works [21, 12]. The contribution of a stochastic interest rate is intuitive: a constant or deterministic risk-free rate is hardly acceptable. Its usefulness was presented also in Sect. 4.8, where our model have easily dealt with different initial interest rate levels and it was able to predict the implications of macro-level shocks. The benefits of a stochastic DB were however not proved. In the description of the DB for our model (see Sect. 4.5.1) we mentioned why banks might prefer a DB that is dependent on the interest rate. We saw however, that it is not the bank who sets the default triggering level: it is the debtor or it is specified in the debt contract, that is designed by both parties.

In order to examine whether it is correct to base our model on stochastic DB we simulated two firms with identical parameters[37] but different DB settings: one stochastic, driven by the instantaneous risk-free interest rate, and one deterministic DB, dependent only on FV_t.

The default triggering levels were therefore set to $FV_t \cdot r(t) \cdot DB$ in the stochastic case and to $FV_t \cdot \gamma \cdot DB$ in the deterministic case, where $DB > 0$ is the same variable in both cases. Figure 6 visualizes the comparison of results obtained by stochastic and deterministic DB setting. For the first sight it is apparent that the total firm value is higher when the DB is defined as a deterministic function. The reason is that a deterministic DB in fact softens the default triggering bound, and hence increases the firm value. The problem is however, that when the primitive variable is not ob-

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[36] For $\kappa = 0$ this is intuitive: the defaultable corporate bond can be represented as a risk-free bond with the same parameters minus the expected losses caused by default. Since the price of a riskless bond that pays continuous interest is always equal to its face value, it is not dependent on the current interest rate.

[37] These parameters were the same as in the basic setting, with the exception of lower recovery rate (5 yearly EBITs), and higher correlation between the EBIT and interest rate processes ($\rho = 0.5$). These modifications were made in order to make the results more sensible on the selection of the DB. Furthermore the number of iterations was doubled to increase the significance of small deviations between the two settings.

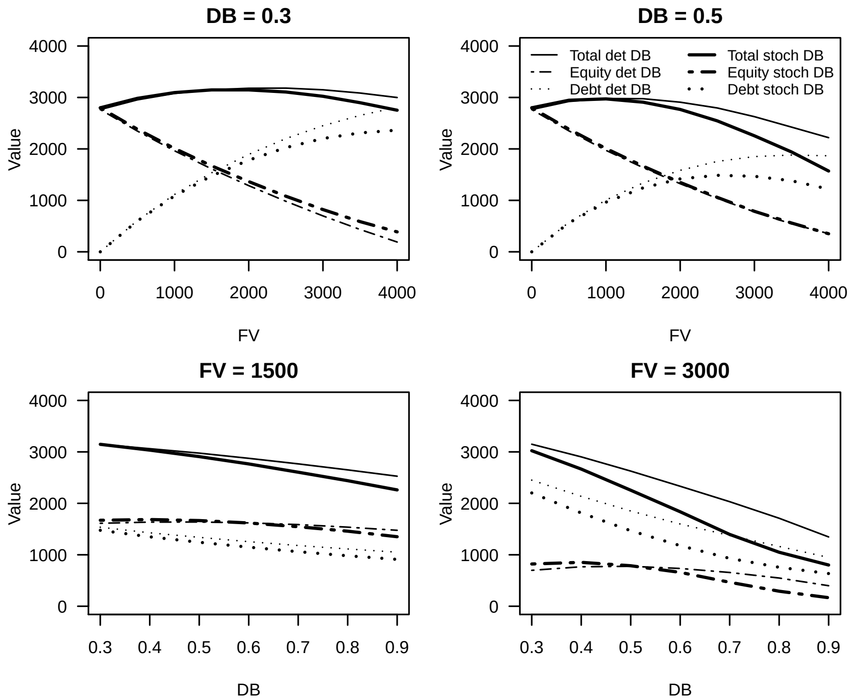

Fig. 6 Stochastic vs. Deterministic DB

servable[38], default is triggered by the equity holders in a way to maximize the value of their claim. Recall Fig. 6: a stochastic DB bears higher equity value for barrier ratios below 0.5. Since the equity-maximizing DB is below 0.5 (as we have seen in Sects. 4.6.2 and 4.7), the equity holders will prefer triggering default according to a stochastic barrier. In fact this is a logical conclusion: the situation of the overall economy, as well as the size of the interest payments is taken into account.

5 Conclusion

We first provide a brief overview of the relevant structural models of asset pricing. This is followed by a discussion of the design of incentive compatible credit contracts in connection with game theory approach to the pricing of corporate assets. Finally we apply these theoretical approaches to the construction of a new Earning Before Interest and Taxes (EBIT) based model of asset pricing.

[38] As it was discussed in Sect. 4.7, observable primitive variable implies low default triggering level. Consequently there is insignificant difference in the values produced by the two DB types.

The proposed structural model extends the available literature of asset pricing by an EBIT based model with stochastic interest rate. This framework is able to price equity and debt in a way consistent with the cash flow of the firm, and therefore to address some defects of the current frameworks. It solves the “delicate” issue of Leland [34], that the unlevered firm value might not be a traded asset, and deals with the problem of partial tax deductibility. The stochastic interest rate assumption contributes the possibility of analysing the effects of changes in the central bank’s monetary policy, and it is able to answer the question how the macroeconomical situation affects the optimal capital structure. The default is triggered using a stochastic default barrier, that is shown to be more accurate then its deterministic equivalent.

A weak point in our design is the assumption that the EBIT process is driven by a GBM, and therefore it cannot handle negative earnings. It might be argued that employing arithmetic Brownian motion would be a better choice for this reason, however it should be noted that our model has an infinite time horizon. As the prices of commodities grow exponentially, it is hard to accept a linear model for the EBIT evolution. Finding better alternatives for the EBIT process will be the subject of further research. A promising idea is to model the earnings as a difference of two correlated GBMs (representing revenues and expenses): it has a clear economic intuition, it is able to produce negative values, has an exponential expected evolution, and works with observable figures.

Acknowledgements The work on this paper was supported by the Grant Agency of the Czech Republic (grants 402/11/0948 and 15-00036S). The research leading to these results received funding from the People Programme (Marie Curie Actions) of the European Union’s Seventh Framework Programme FP7/2007-2013 under REA grant agreement number 609642. Karel Janda acknowledges research support provided during his long-term visits at Australian National University, Toulouse School of Economics, and University of California, Berkeley and the support he receives as an Affiliate Fellow at CERGE-EI, Prague. The views expressed here are those of the authors and not necessarily those of our institutions. All remaining errors are solely our responsibility.

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