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Keywords

fertility, housing price, gender wages, working age population

JEL Classification

J10, J13, J16

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Fertility and housing

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Abstract

Young households in Hong Kong face particularly steep increases in house prices and low fertility despite low gender wage gaps. The model of fertility and housing in this paper explains why fertility decline need not reverse as female wages rise relative to male wages where housing land is scarce. For given house prices, demand for children may rise with female relative wages if housing comprises a sufficiently large share of childrearing. If the user cost of housing falls with rising house prices then fertility also rises. For endogenous house prices, however, growth in wages and a burgeoning working age population raises the market price of housing. In turn, fertility no longer rises with female relative wages. The analysis provides a novel mechanism whereby high population support ratios depress fertility and the results fit recent evidence that house prices affect fertility.

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1. Introduction

This paper explains recent evidence that house prices affect fertility and are affected by population age structure. In doing so, we provide novel insights into the persistence of low fertility in Hong Kong, while fertility decline reverses in most high income economies. Referring to Figure 1, Hong Kong, SAR, has experienced three decades of low fertility that is below the replacement rate of 2.1 and even lower than the low total fertility rates of Japan and Singapore. Fertility in these high income Asian economies ranks among the lowest in the world. Even if immigration continues at its current pace, Hong Kong's bustling population of seven million is expected to fall by more than one million this century should current fertility trends persist (United Nations, 2013).

[Figure 1 about here]

Recent evidence suggests a possible reversal in the well established negative relationship between fertility and economic growth across high income OECD countries (Luci-Greulich and Thevenon, 2014). Figure 2 depicts the once strictly negative cross-country association between fertility and per capita Gross Domestic Product (GDP). More than a convex relationship, Figure 3 shows an emerging divide between countries with high levels of per capita GDP, but with either high or low

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fertility (Luci and Thevenon, 2011; Luci-Greulich and Thevenon, 2014).[1] Japan and South Korea drag the bottom of a convex relationship across OECD countries. Hong Kong and Singapore, which are not included in the OECD sample, are clear outliers with high per capita GDP and very low fertility.

[Figures 2 and 3 about here]

Recent research identifies gender inequality as a key factor determining on which side of the divide in Figure 3 an economy falls (Luci and Thevenon, 2011; Luci-Greulich and Thevenon, 2014). Highly developed economies ranking low in gender equity continue to experience low fertility (Myrskyla et al, 2011). OECD countries with the highest gender wage gaps, Japan and South Korea, also have the lowest fertility. However, reducing the gender wage gap is necessary but not sufficient for fertility rebound. In Hong Kong and Singapore low fertility has persisted, even though gender wage gaps are commensurate with other developed economies where fertility decline has reversed.

Referring to Figures 4 and 5, real house prices have increased in several high income OECD countries over the past three decades, with the exception of Japan. Hong Kong has witnessed a steep rise in real house prices over the last decade, with strong appreciations since the depth of the global financial crisis in late 2008.

[Figures 4 and 5 about here]

Seminal endogenous fertility models explain that economic growth induces fertility decline by raising female wages relative to male wages, thereby raising the opportunity cost of maternal time (Barro and Becker, 1988; Galor and Weil, 1996). However, Apps and Rees (2004) and Martinez and Iza (2004) show that fertility may rise with female relative wages if households can substitute child care for maternal time. These models assume that the cost of child rearing comprises the opportunity cost of maternal time or purchased child care. However, to rear children families also need housing space. We may expect that higher house prices raise the cost of child rearing and reduce household disposable income available for other spending, thereby reducing demand for children.

A growing body of empirical work suggests that rising house prices affect fertility, especially in China. Yi and Zhang (2010) find that high house prices account for approximately 65 per cent of the fertility decline in Hong Kong from 1971 to 2005. Pan and Xu (2012) recognize that since fertility is negatively correlated with urban disposable income per capita, which in turn is positively correlated with house prices, estimating the effect of the ratio of house prices to income on fertility more accurately measures the effect of house prices on fertility. They find that after controlling for explanatory variables suggested by existing theoretical models, such as female labor force participation and female education, the average ratio of house prices to income from 2001 to 2010 remains significant in explaining lower fertility in China.

Moreover, Yi and Zhang (2010) show that female and male wages can be treated as exogenous variables that affect house prices as well as fertility. They find that rising male wages have a strong positive effect on fertility, which is attributed to an income effect on the demand for children. However, rising female wages have a weak negative effect because rising female wages have both a negative substitution effect and positive income effect on the demand for children when female labor is used to rear children.[2] As female wages rise relative to male wages, the overall effect on fertility is ambiguous. This empirical evidence motivates the focus of theoretical analysis in this paper on the effect on fertility of both rising house prices and female wages relative to male wages.

These findings are consistent with international evidence. Dettling and Kearney (2014) find that a 10 per cent increase in house prices leads to a 4 per cent increase in births among home owners and a 1 per cent decrease among non-owners. Countries where home ownership is most difficult also have the lowest fertility (McDonald, 2008). Despite this evidence and calls for theoretical research into low fertility and the housing market (McDonald, 2008; Malmberg, 2012), the theoretical literature overlooks the effect of house prices on fertility.

Evidence also suggests that population age structure affects house prices (Malmberg, 2012). However, the theoretical link between demography and house prices has been largely overlooked since Mankiw and Weil (1989) and Poterba (1991). Existing models explain the effect of population change on house prices, but ignore the effect of house prices on population via household fertility choice. This paper explains both directions of causality.

This paper fills these gaps in the literature by providing a theoretical analysis of the simultaneous events of homeownership and parenthood. In this paper, we analyze whether a positive impact of female relative wages on fertility can be sustained as rising demand for housing puts pressure on prices to rise. Population age structure affects house prices, because aggregate demand for housing is increasing in the working age population, whereas aggregate supply is increasing in the elderly population and rate of government land release.

This is the first paper to model the effect of rising female relative wages on fertility with endogenous increases in house prices and to link rising house prices to an increase in the working age to elderly population ratio. Of the theoretical models explaining rising house prices, the two period, constant population growth and three period, fixed population models of Deaton and Laroque (2001) and Garino and Sarno (2004), respectively, provide a useful framework on which this paper builds. We find their approach to be useful for the analysis in this paper which focuses on the interrelationship between fertility, gender wages and house prices.

Two particularly interesting results of the four period model presented in this paper are, firstly, that demand for children may rise with female relative wages, for given house prices, if housing comprises a sufficiently large share of child rearing. Secondly, wages growth and a burgeoning working age population raises the market price of housing, which in turn implies that, in equilibrium, fertility no longer rises with female relative wages. These results suggest that gender equity may hold the key to fertility rebound in Japan and South Korea and that the market for housing provides a novel mechanism whereby past demographic change may impact current fertility in Hong Kong and Singapore.

Section 2 of this paper provides the theoretical framework for household choice of number of children and consumption, and child rearing inputs of maternal time and housing. Within this framework, we analyze the effect of rising female relative wages and house prices on fertility. Section 4 examines the robustness of fertility decline reversal to endogenous house prices, which are determined by the market for housing in Section 3.

2. The model

Consider an economy where the basic unit of analysis is the couple. They decide on how many children to have, how much maternal time to spend rearing children and how much to save for the purchase of housing at an endogenously determined market price.

The household lives for four periods: childhood, young working age, middle working age and old age. A young working age household consumes and saves for a deposit to purchase housing, with which they rear children in middle age. A middle aged household consumes, rears children, repays a mortgage and saves for retirement. On retiring, the household funds old age consumption through the sale of the house and interest on retirement savings.

We proceed by drawing on the three period models presented in Apps and Rees (2004) and Garino and Sarino (2004). Apps and Rees (2004) could be thought of as performing comparative statics on an extended version of Galor and Weil (1996) because it does not model physical capital accumulation and the dynamics of female relative wages. Similarly, Garino and Sarno (2004) abstracts from the accumulation of physical capital and housing and the dynamics of wages. The four period model in this paper treats wages as exogenous, as in Apps and Rees (2004) and Garino and Sarno (2004), in order to focus the analysis on the effect of an increase in female relative wages and house prices on fertility.[3]

The periods of young, middle aged and retired adulthood are denoted $0, 1, 2$, respectively. For a person who is young in period 0, lifetime utility to be maximized is

$$U = u(c_0) + \delta \left[u(c_1, n)\right] + \delta^2 u(c_2) \tag{1}$$

where n denotes number of children and c_0, c_1, c_2 denote household consumption when young, middle aged and retired, respectively, and $\delta = 1/(1+\rho)$ is the discount rate with a constant time preference parameter, ρ. Individuals do not own land in the first period of adulthood. As in Deaton and Laroque (2001), we may think of young working adults as living with their parents. Similarly, as in Garino and Sarno (2004), there is no utility from children or land in old age.

Each agent is endowed with a unit of time. Men allocate their unit time endowment to the paid labor force. To raise children, women employ a fraction of their time endowment, $\hat{z} \in (0, 1)$, in which case their paid labor supply is $(1 - \hat{z}n)$. Women and men receive real wages of w_t^m and w_t^f, respectively, when young and middle aged, $t = 0, 1$.

Land yields housing services. Working age households buy $\hat{h}$ units of housing per pair of children, so that each household demands $\hat{h}n$ units (square metres) of housing. Working age households purchase housing from newly retired households at the market price, p_1^h. Newly retired households sell housing to the next generation of working age households at the market price, p_2^h. The market price of housing is endogenously determined so that aggregate housing demand coincides with aggregate housing supply, which in turn is affected by the ratio of working age households to retired households.

2.1 *Utility Maximization*

A young working age household faces the budget constraints

$$c_0 + D_0 = w_0^m + w_0^f \tag{2a}$$

$$c_1 + (1 + v_1)(p_1^h \hat{h} n - D_0) + w_1^f \hat{z} n + s_1 = w_1^m + w_1^f \tag{2b}$$

$$c_2 = (1 + r_2)\, s_1 + p_2^h \hat{h} n \tag{2c}$$

when young, middle aged and retired, respectively.[4] The mortgage size is $(p_1^h \hat{h} n - D_0)$ where the deposit D_0 is optimally chosen by the young working age household. The mortgage repayment in middle age is $(1 + v_1)(p_1^h \hat{h} n - D_0)$ where v_1 is the real mortgage rate. On retiring, the household sells their house and receives the prevailing real interest rate, r_2, on savings in middle age, s_1. We allow v_1 and r_2 to differ because financial institutions set different mortgage rates and interest rates on savings. We herein specify lifetime utility and derive the lifetime budget constraint.

The household derives direct utility from children. Housing does not yield direct utility to the household, but is necessary to rear children. The utility functions are of log linear form,

$$\begin{aligned} u(c_0) &= \ln c_0 && \text{(3a)} \\ u(c_1, n) &= \beta \ln c_1 + (1-\beta)\ln n && \text{(3b)} \\ u(c_2) &= \ln c_2 && \text{(3c)} \end{aligned}$$

where $(1-\beta) \in (0,1)$ captures the relative preference for children, and satisfy strict concavity of $U(.)$, so that the optimally chosen quantities of fertility and consumption are expected to be strictly positive and female relative wages will have a strictly negative effect on fertility when maternal time is used to rear children (Galor and Weil, 1996, Apps and Rees, 2004).

From (1) and (3a) - (3c), the household's lifetime utility

$$U = \ln c_0 + \delta\left[\beta \ln c_1 + (1-\beta)\ln n\right] + \delta^2 \ln c_2$$

which is maximized subject to an intertemporal budget constraint. Substituting for s_1 from (2c) in (2b) and for D_0 from (2b) in (2a) gives the lifetime household budget constraint

$$\left(w_0^m + w_0^f\right) + \frac{\left(w_1^m + w_1^f\right)}{(1+v_1)} = c_0 + \frac{1}{(1+v_1)}\left[c_1 + p^n n + \frac{c_2}{(1+r_2)}\right] \tag{4}$$

where $p^n = \left[\left((1+v_1)p_1^h - \frac{p_2^h}{(1+r_2)}\right)\hat{h} + w_1^f \hat{z}\right] > 0$ is the per unit cost of rearing children. The lifetime budget constraint clarifies that we abstract housing from its investment role and consider it as an input of child rearing. The household plays two roles: consumer and producer of children. As a producer, the household, given n, chooses the optimal inputs of $\hat{h}$ and $\hat{z}$. Thus, as a consumer, the household can treat $\hat{h}$ and $\hat{z}$ as given in the utility maximization problem.

We assume a general child rearing production function

$$n = f(z, h) \tag{5}$$

which is linear homogeneous, continuously differentiable and strictly quasi-concave and where z and h denote total time input and units of housing, respectively. Because the child rearing production function is linear homogeneous, the household optimization problem can be solved in two stages. The household chooses c_0, c_1, c_2 and n for a given input mix, so as to maximize lifetime utility subject to the lifetime budget constraint and then, for a given n, the cost minimizing input mix of h and z.

Referring to the appendix, for a given child rearing input mix, $\hat{h}$ and $\hat{z}$, the household chooses fertility

$$n = \frac{\delta(1-\beta)\left[(w_1^f + w_1^m)/(1+v_1) + (w_0^m + w_0^f)\right]}{\left[1+\delta(1+\delta)\right]p^n/(1+v_1)} \tag{6}$$

which is increasing in the present value of lifetime labor income, $(w_1^f + w_1^m)/(1+v_1) + (w_0^m + w_0^f)$, and decreasing in the per unit child rearing cost, p^n, which in turn depends on the optimal child rearing input mix.

2.2 *Cost minimization*

The total cost of rearing children is

$$\left[\pi(p_1^h, p_2^h, v_1, r_2)\hat{h} + w_1^f \hat{z}\right] n \tag{7}$$

where $\pi(.) = \left((1+v_1)p_1^h - p_2^h/(1+r_2)\right)$ is the user cost of land, which is increasing in p_1^h and decreasing in p_2^h. We herein explain that p_1^h is determined by equilibrium in the market for housing land. In equilibrium, the user cost is strictly positive $\pi(.) > 0$, otherwise the demand for housing would be unbounded. However, as house prices rise, the user cost may fall if the increase in p_2^h relative to p_1^h is sufficiently large, $dp_2^h/dp_1^h > (1+v_1)(1+r_2)$.

The household chooses the input mix, for a given n, to minimize (7) subject to the child rearing production function,

$$n = z^{\alpha} h^{1-\alpha} \tag{8}$$

where z and h denote total time input and units of housing, respectively, and $\alpha \in (0,1)$ captures the relative importance of housing and time in child rearing. The production function is Cobb-Douglas where maternal time and housing are essential child rearing inputs.

The Cobb-Douglas functional form implies that $n = 0$ if either $h = 0$ or $z = 0$, which is a realistic assumption when we consider that at least either some maternal time and some housing is needed to rear children. If we relax the assumption that the elasticity of substitution between h and z equals one ($\epsilon = 1$) and consider a CES functional form, we could allow for varying degrees of complementarity ($\epsilon < 1$) or substitutability ($\epsilon > 1$). However, this would also allow for corner solutions, meaning $n > 0$ if either $h = 0$ or $z = 0$, which is unrealistic. It is important to note that the results in this paper are robust to a range of functional forms, such as CES production, encompassed by the general child rearing production function, (6).[5]

The input demands for housing and time are, respectively

$$h = \hat{h}n = \left(\frac{1-\alpha}{\alpha}\right)^{\alpha} \left(\frac{w_1^f}{\pi}\right)^{\alpha} n \tag{9a}$$

$$z = \hat{z}n = \left(\frac{\alpha}{1-\alpha}\right)^{1-\alpha} \left(\frac{\pi}{w_1^f}\right)^{1-\alpha} n \tag{9b}$$

and the optimal per unit cost of child rearing is

$$p^n = a\left(w_1^f\right)^{\alpha} \left((1+v_1)p_1^h - \frac{p_2^h}{(1+r_2)}\right)^{1-\alpha} \tag{10}$$

where $a = \left[(\alpha/(1-\alpha))^{1-\alpha} + ((1-\alpha)/\alpha)^{\alpha}\right]$. $p^n(.)$ is strictly increasing and concave, which means that the input demands are positive, $\partial p^n/\partial w_1^f = \hat{z} > 0$ and $\partial p^n/\partial \pi = \hat{h} > 0$, and downward sloping, $\partial \hat{z}/\partial w_1^f < 0$ and $\partial \hat{h}/\partial \pi < 0$.

2.3 *Optimal fertility for exogenous price of housing*

From (6) and (10), household demand for children is

$$n = \frac{\delta\,(1-\beta)\left[\frac{(w_1^f + w_1^m)}{(1+v_1)} + (w_0^m + w_0^f)\right]}{\left[1+\delta(1+\delta)\right] a \left(\frac{w_1^f}{(1+v_1)}\right)^{\alpha} \left(p_1^h - \frac{p_2^h}{(1+r_2)(1+v_1)}\right)^{1-\alpha}} \tag{11}$$

the key properties of which are summarized in the following proposition and lemma.

Proposition 1 *For given house prices, demand for children may rise with female relative wages if housing comprises a sufficiently large share of child rearing.*

Proof. Refer to appendix. ■

The intuitive explanation for a positive response of fertility demand to female relative wages lies in a comparison of substitution and income effects. To see this, consider the special case where maternal time is the only child rearing input ($\alpha = 1$). Fertility unambiguously declines with rising female relative wages when maternal time is the only child rearing input. Intuitively, when both husband and wife work, female wages constitute a portion of household income. When maternal time is the only child rearing input, the cost of child rearing rises proportionate to female wages. This means that an x per cent rise in female wages leads to an x per cent rise in the cost of child rearing, because female wages is the only cost of child rearing when maternal time is the only child rearing input. However, an x per cent rise in female wages leads to a less than x per cent rise in household income because female wages contribute a portion of household income. As female relative wages rise, the substitution effect dominates the income effect and fertility declines.

However, when both maternal time and housing are child rearing inputs, the cost of child rearing rises less than proportionate to female wages. This means that an x per cent rise in female wages leads to a less than x per cent rise in the cost of child rearing, because female wages contribute a portion of the cost of child rearing when housing is also an input.[6] A rise in female relative wages has competing effects on the demand for children. On the one hand, the increase in household wages makes children more affordable. On the other hand, children are more costly to raise using maternal time. The larger the intensity with which housing is used in child rearing, the smaller the effect of rising female wages on the marginal cost of child rearing. The income effect of rising female wages now dominates the substitution effect and fertility rises.

We have assumed that the real mortgage rate, v_1, and real interest rate on retirement savings, r_2, differ. If the two interest rates are the same, Proposition 1 is unchanged. From equation (11), if $v_1 = r_2$ then

$$n = \frac{\delta\,(1-\beta)\left[\frac{(w_1^f + w_1^m)}{(1+v_1)} + (w_0^m + w_0^f)\right]}{\left[1+\delta(1+\delta)\right] a \left(\frac{w_1^f}{(1+v_1)}\right)^{\alpha} \left(p_1^h - \frac{p_2^h}{(1+v_1)^2}\right)^{1-\alpha}} \tag{12}$$

where w_1^f and w_1^m are discounted at the rate $1/(1+v_1)$ and p_2^h is discounted at the rate $1/(1+v_1)^2$. Intuitively, p_2^h is received in the final period and is therefore discounted at a rate compounded over two periods. Referring to the appendix, the rate at which p_2^h is discounted does not affect the proof of Proposition 1, which summarizes the response of demand for children to rising female relative wages, holding $p_1^h - p_2^h/(1+v_1)^2$ constant.

Lemma 1 *If the user cost of housing falls with rising house prices, then demand for children increases, all else equal.*

From (11), fertility is decreasing in p_1^h, and increasing in $p_2^h/(1+r_2)(1+v_1)$.[7] Intuitively, children are a normal good. Housing is used to rear children. On the one hand, an increase in current house prices reduces the fertility of a young household that is saving for a deposit. On the other hand, a young household also realizes that the family home is a means of storing and transferring wealth from working age to retirement. Thus, an increase in the discounted value of future house prices has a positive wealth effect on the demand for children. This is consistent with evidence that rising house prices reduces the fertility of non-home owners and increases the fertility of home owners via a wealth effect (Yi and Zhang, 2010; Dettling and Kearney, 2014). If future house prices increase relative to current house prices such that the user cost of land falls, then the wealth effect dominates and fertility rises.

Thus far the effect of rising female relative wages on fertility has been analyzed for given house prices, p_1^h and p_2^h. By (9a), rising female wages also raises demand for housing per child. At the individual household level, the household may take house prices as given. However, at the aggregate level, house prices are endogenous. In the following section, house prices are endogenized in a straightforward manner by assuming that housing land is bought at a price determined by market demand and supply.

3. Market for housing land

From (9a) and (11), the final demand for housing by a working age household is

$$h_1 = \frac{\delta\left(1-\beta\right)\left(1-\alpha\right)\left[\frac{\left(w_1^f+w_1^m\right)}{\left(1+v_1\right)}+\left(w_0^m+w_0^f\right)\right]}{\left[1+\delta(1+\delta)\right]\left[p_1^h-\frac{p_2^h}{(1+r_2)(1+v_1)}\right]} \tag{13}$$

which is increasing in household preference for children, $(1-\beta)$, the share of housing in child rearing, $(1-\alpha)$, the present value of lifetime labor income and decreasing in the user cost of land.

Land has intrinsic value through the provision of housing services in child rearing. There are L^w working age households purchasing housing land and L^o retired households that are selling housing land to the new young working age households.

Aggregate demand for housing is

$$h^d = \frac{\delta\left(1-\beta\right)\left(1-\alpha\right)\left[\frac{\left(w_1^f+w_1^m\right)}{\left(1+v_1\right)}+\left(w_0^m+w_0^f\right)\right]}{\left[1+\delta(1+\delta)\right]\left[p_1^h-\frac{p_2^h}{(1+r_2)(1+v_1)}\right]}L^w \tag{14}$$

Let $\bar{h}$ denote housing land available for sale from the recently retired. We can allow for the stock of land available for purchase by young working age households to grow at a rate of government land release, g. Under perfect competition, this land sells for the same market price at which the older generation is selling housing land.

The aggregate supply of housing is

$$\begin{aligned} h^s &= \bar{h}(1+g)L^o \\ &= \bar{h}\frac{(1+g)L^w}{(1+N)} \end{aligned} \tag{15}$$

where, since there are $L^w/L^o = (1+N)$ as many working age households buying as there are old age households selling, actual land purchased by each young household is $\bar{h}(1+g)/(1+N)$.

The market price of housing land is determined so that aggregate demand coincides with aggregate supply, yielding

$$p_1^h = \frac{p_2^h}{(1+r_2)(1+v_1)} + \frac{\delta(1-\beta)(1-\alpha)}{[1+\delta(1+\delta)]}\frac{(1+N)}{(1+g)\bar{h}}y_1 \tag{16}$$

where $y_1 = (w_0^m + w_0^f) + (w_1^f + w_1^m)/(1+v_1)$ denotes the present value of lifetime labor income.

Lemma 2 *The market price of housing is increasing in the future price and the fundamentals of the present value of lifetime labor income and the working age to elderly population ratio.*

The market demand for housing land is downward sloping or decreasing in the user cost of land, which in turn is increasing in the current price and decreasing in the discounted future price. Demand for housing land is derived from demand for children, a normal good. Accordingly, increases in the present value of lifetime labor income shift out the market demand for housing land. The market supply of housing land is inelastic. The stock of land is depleted by an increase in the ratio of working age households who demand housing land to old age households who sell housing land. Thus, past demographic shocks shift the supply of land inward, raising the market price. Government land release replenishes the stock of housing land, shifting the supply of land outward, relieving upward pressure on the market price.[8]

Despite ranking among countries with the lowest fertility in the world, Hong Kong and Singapore have experienced above average population growth for high income economies. International migrant stocks now approximate 39 percent and 38 percent of the total population in Hong Kong and Singapore, respectively, which is more than three times the average for high income economies (World Bank, 2013).[9] There are approximately 8.3 and 5.9 working age persons per elderly persons in Hong Kong and Singapore. Conversely, in Japan, international migrants comprise a mere 1.7 percent of the population and there are around 2.9 working age persons per elderly. Immigration of working age adults ameliorates the pace of aging. However, massive immigration places pressure on physical and infrastructure constraints (Yap, 2011). In the presence of such constraints, the above analysis suggests that, all else equal, house prices would be highest (lowest) in Hong Kong and Singapore (Japan) due to the high (low) working age to old age population support ratios. We herein analyze how fertility responds to female relative wages under endogenous house prices.

4. Equilibrium

From (11) and (16), fertility is

$$n = \left[\frac{\alpha(1-\beta)\left[\frac{(w_1^f + w_1^m)}{(1+v_1)} + (w_0^m + w_0^f)\right]\delta}{\left(\frac{w_1^f}{(1+v_1)}\right)[1+\delta(1+\delta)]}\right]^{\alpha}\left[\frac{\bar{h}(1+g)}{(1+N)}\right]^{1-\alpha} \tag{17}$$

where $(1+N)$ is the ratio of the working age population to the current retired population: $L^w = (1+N)L^o$. The following remark summarizes the key properties of fertility with endogenous house prices.

Proposition 2 *For endogenous house prices, fertility is non-increasing in female relative wages. An increase in the working age to old age population support ratio reduces fertility. An increase in the rate of land release raises fertility.*

Intuitively, household income rises less than proportionate to female wages as female wages rise because female wages contribute a portion of household income. This means that an x per cent rise in female wages leads to a less than x per cent rise in household income. The increase in derived demand for housing raises the user cost of housing, which in turn raises the per unit cost of child rearing. The cost of child rearing now rises proportionate to female wages. This means that an x per cent rise in female wages leads to an x per cent rise in the cost of child rearing. Accordingly, the substitution effect of rising female wages dominates the income effect and fertility declines when there is any degree of substitutability between maternal time and housing.

In Hong Kong, female and male median hourly wages currently differ by around 17 percent, which is the OECD average. However, in Japan, the gender gap in median hourly wages remains much higher at 28 per cent.[10] The above analysis explains why low fertility persists in Hong Kong despite rising female relative wages. When housing land is scarce, as it is in Hong Kong, a large working age to retired population ratio raises the market price of housing. This in turn depresses fertility since housing is used to rear children.

If the real mortgage rate and real interest rate on retirement savings are the same, $v_1 = r_2$, then p_2^h is discounted at the rate $1/(1+v_1)^2$. However, Proposition 2 is unchanged. Referring to equation (17), the present value of p_2^h does not affect fertility, because the user cost of housing, $p_1^h - p_2^h/(1+v_1)^2$ is endogenously determined by the market for housing.

5. Conclusion

This paper analyzes the effect of female relative wages on fertility in a model that incorporates a housing market. Land yields housing services in the rearing of children. In contrast to the existing literature, endogenous increases in house prices play an important role in the effect of rising female relative wages and past demographic shocks on fertility.

The analysis predicts the following. Firstly, for given house prices, fertility increases with female relative wages if housing comprises a sufficiently large share of child rearing. Secondly, if the user cost of housing land falls with rising house prices, then fertility rises. Thirdly, for endogenous house prices, fertility unambiguously declines as female relative wages and working age populations grow.

The first result fits the observation that the once negative association between fertility and per capita income has broken down across most high income countries and suggests that closing the gender wage gap may hold the key to reversing fertility decline in Japan and South Korea. The second is consistent with recent evidence that suggests increasing house prices could raise fertility. The third fits evidence that rising house prices decrease fertility in Hong Kong (Yi and Zheng, 2010).

The analysis provides a novel mechanism whereby past demographic change can impact present fertility choice. High ratios of working age persons per elderly may explain high house prices and low fertility in Hong Kong. Some interesting implications for policy arise. The results in this paper explain why the desired fertility of households wishing to enter the market may rise with female relative wages, for given house prices, whilst actual fertility falls further behind other high income countries as high population support ratios raise house prices when land is scarce. This suggests that housing in Hong Kong may hold the key to reversing fertility decline and unlocking future population growth.

Notes

[1] The total fertility rate is a cross sectional measure that indicates the average births per woman in her lifetime if she were to experience the current age-specific fertility rates throughout her lifetime. Hence, Luci and Thevenon (2011) examine a possible convex impact of per capita GDP on fertility, after controlling for birth postponement and country-specific effects across OECD countries.

[2] According to the substitution effect, higher female wages raise the opportunity cost of maternal time spent rearing children, which encourages the household to choose less children. According to the income effect, higher female wages make children more affordable, which encourages the household to choose more children.

[3] In our model, housing accumulates at an exogenous rate of land release and physical capital accumulates from savings in middle age, which referring to appendix, depends on house prices and the present value of lifetime wages. Analyzing a dynamic system for the model presented in this paper is an interesting direction for further research.

[4] All variables are real, with the price of aggregate consumption normalized to 1.

[5] Developing the model with the general production function, (5), would not overturn the results in this paper since, in a market equilibrium, the user cost of housing increases with the cost of unpaid time. The appendix solves the model for a perfect complements production function which is not linear homogeneous and continuously differentiable as required by (5). This functional form implies that child rearing must use h and z in fixed proportions, as if they were left shoes and right shoes. Perfect complements is unrealistic as we would reasonably expect a household to change the combination of housing and maternal time as house prices and female wages change. Nonetheless, this provides a robustness check.

[6] The same results hold if we relax the assumption that child rearing production has a Cobb-Douglas functional form and replace it with a general child rearing production function (5). Intuitively, the cost of maternal time contributes a portion of the cost of childrearing when housing is a child rearing input.

[7] If $v_1 = r_2$ then the present value of the future house price is $p_2^h/(1+v_1)^2$. However, Lemma 1 still holds.

[8] If the real mortgage rate and real interest rate on retirement savings are the same, $v_1 = r_2$, then equation (16) implies $p_1^h = \frac{p_2^h}{(1+v_1)^2} + \frac{\delta(1-\beta)(1-\alpha)}{[1+\delta(1+\delta)]}\frac{(1+N)}{(1+g)h}\left[(w_0^m + w_0^f) + \frac{(w_1^f+w_1^m)}{(1+v_1)}\right]$, for which Lemma 2 still holds. However, it is worth noting that the current house price depends on the future house price, p_2^h, discounted at a rate compounded over two periods because it is received in the final period, whereas future wages are discounted at a single period rate because they are received in the second period.

[9] International migrant stocks approximate 10 percent and 14 percent of the total population in the United Kingdom and the United States, respectively.

[10] Authors' calculations using 2013 hourly wage data from http://www.censtad.gov.hk/ and http://www.e-stat.go.jp/.

Appendix

Utility maximization problem

The household's problem can be written as

$$\begin{aligned}\max\Omega \quad = \quad & \ln c_0 + \delta\left[\beta \ln c_1 + (1-\beta)\ln n\right] + \delta^2 \ln c_2 \\ & +\lambda\left[\left(w_0^m + w_0^f\right) + \frac{\left(w_1^m + w_1^f\right)}{(1+v_1)} - c_0 - \frac{1}{(1+v_1)}\left[c_1 + p^n n + \frac{c_2}{(1+r_2)}\right]\right]\end{aligned}$$

The first order conditions are

$$\begin{aligned}
\frac{1}{c_0} &= \lambda \\
\frac{\delta\beta}{c_1} &= \frac{\lambda}{(1+v_1)} \\
\frac{\delta\,(1-\beta)}{n} &= \frac{\lambda p^n}{(1+v_1)} \\
\frac{\delta^2}{c_2} &= \frac{\lambda}{(1+v_1)\,(1+r_2)} \\
\left(w_0^m + w_0^f\right) + \frac{\left(w_1^m + w_1^f\right)}{(1+v_1)} &= c_0 + \frac{1}{(1+v_1)}\left[c_1 + p^n n + \frac{c_2}{(1+r_2)}\right]
\end{aligned}$$

which yield the optimal young consumption, middle age consumption, fertility and old age consumption

$$c_0^* = \frac{\left(w_0^m + w_0^f\right) + \frac{\left(w_1^m + w_1^f\right)}{(1+v_1)}}{[1+\delta(1+\delta)]} \tag{A1.1}$$

$$c_1^* = \frac{\delta\beta\left[\left(w_0^m + w_0^f\right) + \frac{\left(w_1^m + w_1^f\right)}{(1+v_1)}\right]}{[1+\delta(1+\delta)]\,/(1+v_1)} \tag{A1.2}$$

$$n^* = \frac{\delta\,(1-\beta)\left[(w_1^f + w_1^m)/(1+v_1) + (w_0^m + w_0^f)\right]}{[1+\delta(1+\delta)]\,p^n/(1+v_1)} \tag{A1.3}$$

$$c_2^* = \frac{\delta^2\left[\left(w_0^m + w_0^f\right) + \frac{\left(w_1^m + w_1^f\right)}{(1+v_1)}\right]}{[1+\delta(1+\delta)]\,/(1+v_1)\,(1+r_2)} \tag{A1.4}$$

where substituting for (A1.1) in (2a) gives the optimal deposit for a housing loan

$$D_0^* = \frac{\delta(1+\delta)(1+v_1)(w_0^m + w_0^f) - (w_1^m + w_1^f)}{[1+\delta(1+\delta)]\,(1+v_1)} \tag{A1.5}$$

which is increasing in the mortgage interest rate, $\partial D_0/\partial(1+v_1) > 0$. Substituting for (A1.4) in (2c) gives the optimal savings in middle age

$$s_1^* = \frac{\delta\left[\beta - \frac{p_2^h \hat{h}}{p^n}\,(1-\beta)\right]\left[\left(w_0^m + w_0^f\right) + \frac{\left(w_1^m + w_1^f\right)}{(1+v_1)}\right](1+v_1)}{[1+\delta(1+\delta)]\,(1+r_2)} \tag{A1.6}$$

which is positive if $p^n/\,(1-\beta) > p_2^h/\beta$, where $(1-\beta)$ is the utility weight for children and β is the utility weight for consumption.

Proof of Proposition 1
From (11),

$$d\ln n = \frac{\partial \ln n}{\partial \ln w_1^f} d\ln w_1^f + \frac{\partial \ln n}{\partial \ln w_1^m} d\ln w_1^m - (1-\alpha)\, d\ln \pi \tag{A2.1}$$

where $(d\ln w_1^f - d\ln w_1^m) > 0$ if female wages rise relative to male wages and

$$\frac{\partial \ln n}{\partial \ln w_1^f} = \frac{w_1^f}{(1+v_1)(w_0^m + w_0^f) + (w_1^f + w_1^m)} - \alpha \gtreqless 0 \tag{A2.2}$$

$$\frac{\partial \ln n}{\partial \ln w_1^m} = \frac{w_1^m}{(1+v_1)(w_0^m + w_0^f) + (w_1^f + w_1^m)} > 0 \tag{A2.3}$$

By (A2.3), fertility rises unambiguously in male wages. If fertility also rises with female wages, then fertility rises as female wages rise relative to male wages. From (A2.2), fertility rises with female wages when the following condition is met

$$\frac{\partial \ln n}{\partial \ln w_1^f} > 0 \Leftrightarrow \frac{(1-\alpha)}{\alpha} > \frac{(w_0^m + w_0^f)}{w_1^f/(1+v_1)} + \frac{w_1^m}{w_1^f} \tag{A2.4}$$

where $w_1^m/w_1^f < 1$ if a gender wage gap exists and $(1-\alpha)/\alpha > 1$ if housing is a greater portion of child rearing than maternal time ($\alpha > 0.5$). According to (A2.4), fertility is more likely to increase with female wages, the higher: the share of housing in child rearing, $(1-\alpha)$; female relative wages, (w_1^f/w_1^m); and present value of middle age female wages relative to young household income, $(w_1^f/(1+v_1))/(w_0^m + w_0^f)$.

Referring to (A2.2), if $\alpha = 1$, then $\partial \ln n/\partial \ln w_1^f < 0$, since middle age female wages are a fraction of present value lifetime labor income, $w_1^f < (1+v_1)(w_0^m + w_0^f) + (w_1^f + w_1^m)$. Substituting for $\partial \ln n/\partial \ln w_1^f < 0$, $\partial \ln n/\partial \ln w_1^f > 0$ and $d\ln \pi = 0$ in (A2.1), $d\ln n < 0$ since $-w_1^m(d\ln w_1^f - d\ln w_1^m) - (1+v_1)(w_0^m + w_0^f)d\ln w_1^f < 0$.

If $v_1 = r_2$, then $\pi = (1+v_1)p_1^h - \frac{p_2^h}{(1+v_1)}$ and $\frac{\pi}{(1+v_1)} = p_1^h - \frac{p_2^h}{(1+v_1)^2}$. However, the rate at which p_2^h is discounted does not alter the above proof that $d\ln n|_{d\ln \pi = 0} > 0 \Rightarrow \frac{(1-\alpha)}{\alpha} > \frac{(w_0^m + w_0^f)}{w_1^f/(1+v_1)} + \frac{w_1^m}{w_1^f}$ since $d\ln \pi = 0$.

Perfect complements child rearing production
Consider that the household minimizes (7) subject to

$$n = \min(az, bh) \tag{A3.1}$$

which implies the household demands per unit inputs,

$$\hat{h} = 1/b \tag{A3.2}$$

$$\hat{z} = 1/a \tag{A3.3}$$

regardless of inputs prices. Hence, the optimal per unit cost of child rearing is

$$p\left(w_1^f, \pi\right) = \frac{w_1^f}{a} + \frac{\pi}{b} \tag{A3.4}$$

and household demand for children, for exogenous house prices, is

$$n^* = \frac{\delta\,(1-\beta)\left[(w_1^f + w_1^m)/(1+v_1) + (w_0^m + w_0^f)\right]}{[1+\delta(1+\delta)]\left[\frac{w_1^f}{a} + \frac{\pi}{b}\right]/(1+v_1)} \tag{A3.5}$$

which is unambiguously increasing in w_1^m. However,

$$\begin{aligned}
\frac{\partial \ln n}{\partial \ln w_1^f} &= w_1^f \left\{ \frac{1/(1+v_1)}{\left[(w_1^f + w_1^m)/(1+v_1) + (w_0^m + w_0^f)\right]} - \frac{1/a}{\left[\frac{w_1^f}{a} + \frac{\pi}{b}\right]} \right\} \gtreqless 0 \\
\frac{\partial \ln n}{\partial \ln w_1^f} &> 0 \Leftrightarrow \frac{\pi}{b} > \frac{w_1^m + (1+v_1)(w_0^m + w_0^f)}{a}
\end{aligned}$$

which is more likely to hold the lower the value of b. A lower value for b means more housing, h, is needed to rear a given number of children, n. Thus, Proposition 1 is robust.
If $v_1 = r_2$ then the second inequality simplifies to

$$\begin{aligned}
\frac{\partial \ln n}{\partial \ln w_1^f} &> 0 \Leftrightarrow \frac{(1+v_1)p_1^h - \frac{p_2^h}{(1+v_1)}}{b} > \frac{w_1^m + (1+v_1)(w_0^m + w_0^f)}{a} \\
&\Leftrightarrow \frac{p_1^h - \frac{p_2^h}{(1+v_1)^2}}{b} > \frac{\frac{w_1^m}{(1+v_1)} + (w_0^m + w_0^f)}{a}
\end{aligned}$$

which is still more likely to hold the lower the value of b, implying that Proposition 1 is robust. Moreover, the higher the real interest rate $v_1 = r_2$, the more likely the inequality will hold.
From (A3.2) and (A3.5), demand for housing is

$$h_1 L^w = \frac{\delta\,(1-\beta)\left[(w_1^f + w_1^m)/(1+v_1) + (w_0^m + w_0^f)\right]}{b\,[1+\delta(1+\delta)]\left[\frac{w_1^f}{a} + \frac{\pi}{b}\right]/(1+v_1)} L^w \tag{A3.6}$$

which equated with (15) gives the market price of housing

$$p_1^h = \frac{p_2^h}{(1+r_2)\,(1+v_1)} + \left\{ \frac{\delta\,(1-\beta)}{[1+\delta(1+\delta)]} \frac{(1+N)}{(1+g)\bar{h}} y_1 - \frac{w_1^f}{(1+v_1)} \right\} \tag{A3.7}$$

From (A3.5) and (A3.7), fertility is

$$n = \frac{b\bar{h}\,(1+g)}{(1+N)} \tag{A3.8}$$

which is non-increasing in w_1^f, decreasing in $(1+N)$ and increasing in $(1+g)$. Thus, Proposition 4 is robust to the extreme case where h and z are perfect complements in child rearing.

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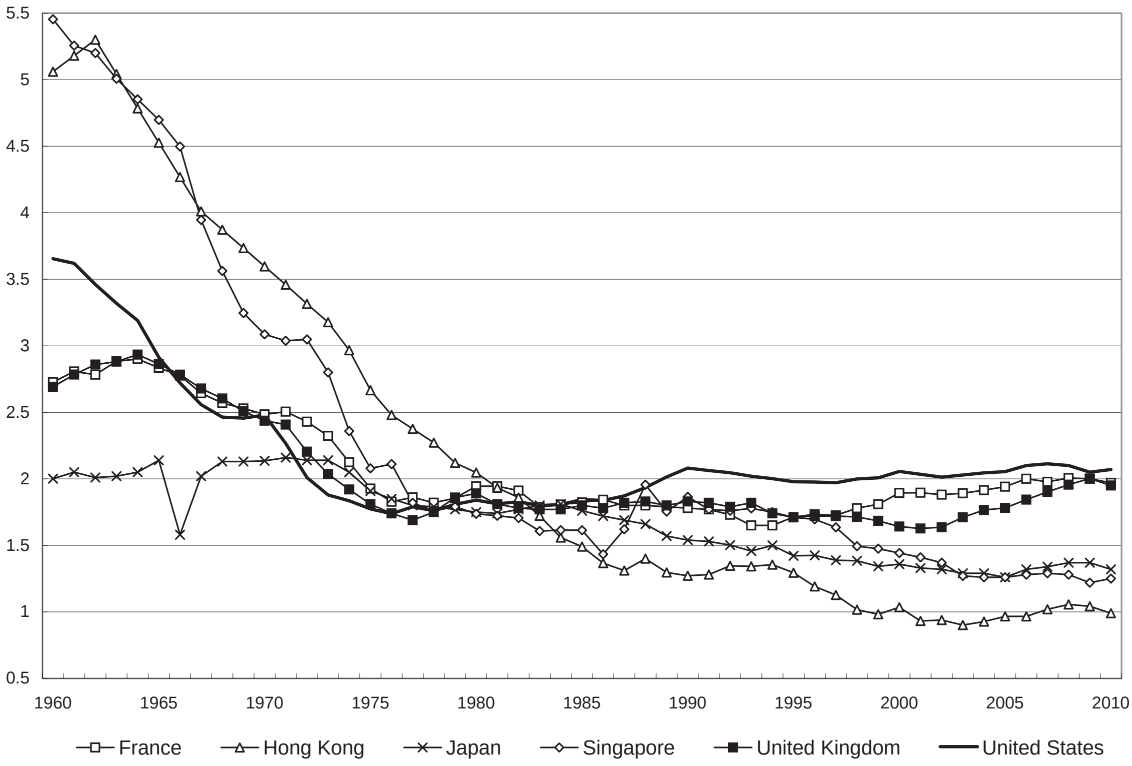

Figure 1. Total Fertility Rate (births per woman), 1960-2010

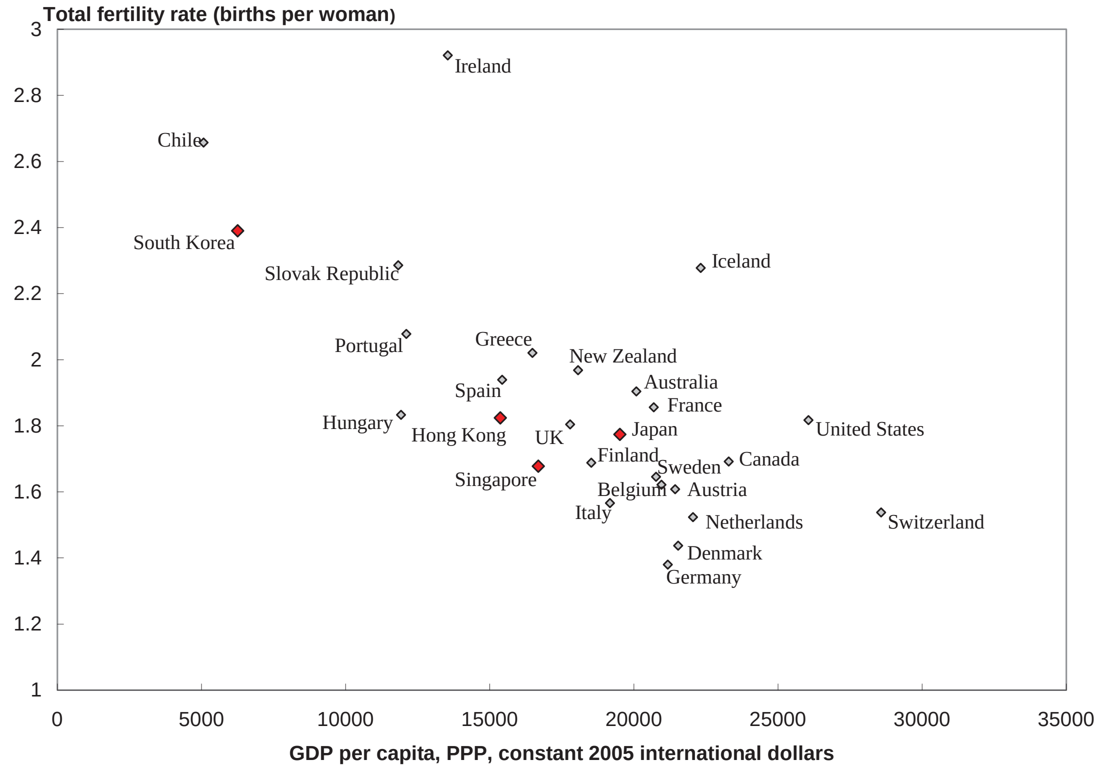

Figure 2. TFR vs. GDP per capita, PPP (constant 2005$), 1980-84

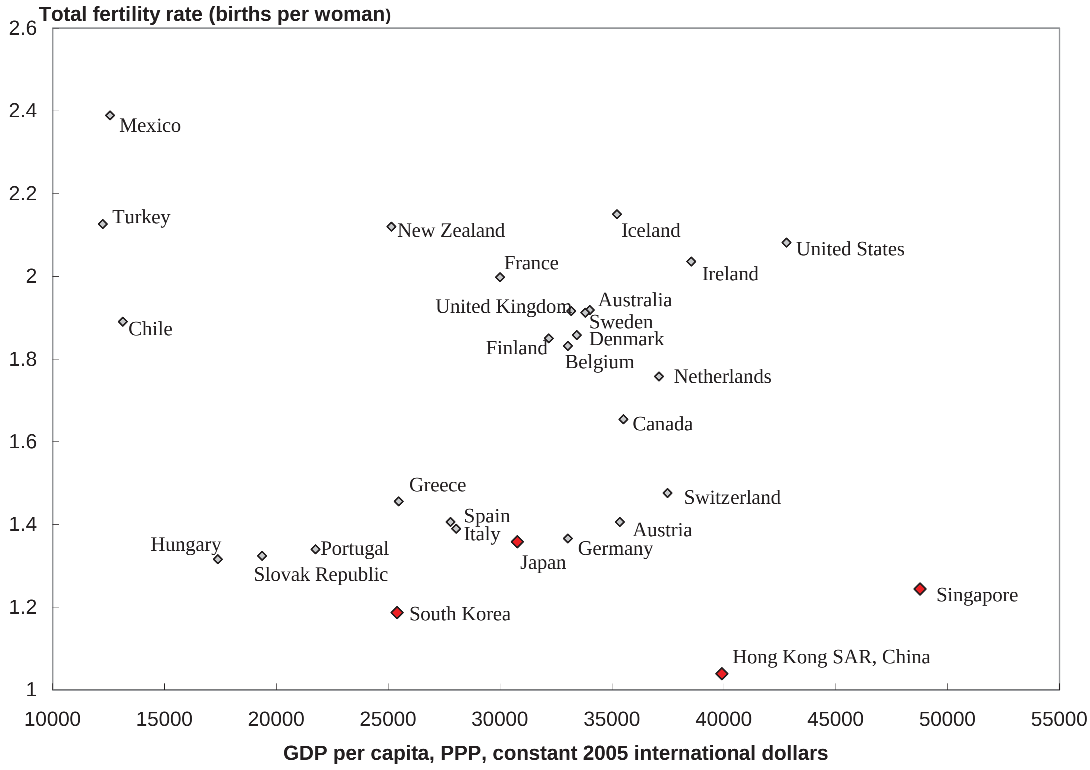

Figure 3. TFR vs. GDP per capita, PPP (constant 2005$), 2006-10