CENTRE FOR APPLIED MACROECONOMIC ANALYSIS

The Australian National University

CAMA Working Paper Series **April, 2010**

FORECAST DENSITIES FOR ECONOMIC AGGREGATES FROM DISAGGREGATE ENSEMBLES

Francesco Ravazzolo
Norges Bank

Shaun P. Vahey
The Australian National University

CAMA Working Paper 10/2010
http://cama.anu.edu.au

Forecast Densities for Economic Aggregates from Disaggregate Ensembles*

Francesco Ravazzolo† (Norges Bank)
Shaun P. Vahey‡ (ANU)

March 12, 2010

Abstract

We propose a methodology for producing forecast densities for economic aggregates based on disaggregate evidence. Our ensemble predictive methodology utilizes a linear mixture of experts framework to combine the forecast densities from potentially many component models. Each component represents the univariate dynamic process followed by a single disaggregate variable. The ensemble produced from these components approximates the many unknown relationships between the disaggregates and the aggregate by using time-varying weights on the component forecast densities. In our application, we use the disaggregate ensemble approach to forecast US Personal Consumption Expenditure inflation from 1997Q2 to 2008Q1. Our ensemble combining the evidence from 11 disaggregate series outperforms an aggregate autoregressive benchmark, and an aggregate time-varying parameter specification in density forecasting.

Keywords: Ensemble forecasting, disaggregates
JEL codes: C11; C32; C53; E37; E52

*We benefited greatly from discussions with Todd Clark, Anthony Garratt, Kirstin Hubrich, Christian Kascha, James Mitchell, Tara Sinclair and Michael Smith. We thank conference and seminar participants at Oslo University, the European Central Bank, the Veissmann European Research Centre, and the Society for Nonlinear Dynamics and Econometrics 17th Annual Symposium. The views expressed in this paper are our own and do not necessarily reflect those of Norges Bank. We thank the ARC (LP 0991098) for support.

†Norges Bank, Research Department. francesco.ravazzolo@norges-bank.no
‡*Corresponding author*: Shaun Vahey, CAMA, ANU. spvahey@gmail.com

1 Introduction

Policymakers regularly combine the leading evidence in disaggregate series to carry out probabilistic assessments of aggregate behavior; see, for example, Greenspan (2004), and the discussions by Feinstein, King, and Yellen (2004). To our knowledge, economists have not explored formally the scope for producing density forecasts for economic aggregates based on disaggregate information. This is surprising given the widespread recognition that evaluations of point forecast accuracy are only relevant for highly restricted loss functions. More generally, complete probability distributions over outcomes provide information which is helpful for making economic decisions; see, for example, Granger and Pesaran (2000) and Timmermann (2006). Accordingly, several central banks, including the US Federal Reserve, have committed to density or interval forecasts in recent years.

In this paper, we propose an ensemble methodology for combining the evidence in disaggregate series to make probabilistic forecasts for an economic aggregate. We formulate the forecasting problem as one in which a forecaster (recursively) selects a linear combination of component forecast densities to produce an ensemble forecast density for the aggregate. Each component forecast is produced from a univariate time series model for a single disaggregate series. The resulting ensemble approximates the many unknown relationships between the disaggregates and the aggregate using time-varying weights across the disaggregate forecast densities. Construction of the disaggregate ensemble forecast for the economic aggregate uses out of sample density combination methods; see, for example, Jore, Mitchell and Vahey (2010).

In our application based on US Personal Consumption Expenditure deflator data, we assess the forecast performance of the disaggregate ensemble approach over the out of sample period 1997Q2 to 2008Q1. An ensemble combining the evidence from 11 disaggregate series outperforms an aggregate autoregressive benchmark, and also an aggregate time-varying parameter specification in density forecasting. Our applied macroeconomic work extends the case for forecast combinations, made by (among others) Stock and Watson (2003) and Clark and McCracken (2010), to forecast densities for economic aggregates

from disaggregate information.

The remainder of this paper is structured as follows. In Section 2, we describe our methods for ensemble modeling of the relationship between the economic aggregate and the disaggregates. In Section 3, we apply our methodology to US data to produce aggregate inflation forecast densities from an ensemble system utilizing disaggregate information. We compare and contrast the ensemble predictive densities with those resulting from our alternative specifications which ignore disaggregate information. In the final section, we conclude.

2 Disaggregate Ensemble Forecast Methodology

The theoretical insights of Bates and Granger (1969) and the macroeconomic forecast evaluation studies by (among others) Stock and Watson (2003) and Clark and McCracken (2010) suggest that forecast combination can be an effective tool for point forecasting. Jore, Mitchell and Vahey (2010) and Garratt, Mitchell and Vahey (2009) establish the performance credentials of forecast combinations for macroeconomic aggregates using ensembles of vector autoregression (VAR) components.

Outside of the economics literature, meteorologists commonly construct ensemble densities to deal with model and/or measurement uncertainty. For an early description of a weather ensemble forecasting see Molteni et al (1996), and more recent contributions in this field by Raftery et al (2005), and Bao et al (2010). Murphy et al (2004) discuss ensembles for modeling climate change; Lopez et al (2009) examine the impact of climate change on resources.

The methodology proposed in this paper extends the scope of the ensemble macroeconomic forecasting framework, developed by Jore, Mitchell and Vahey (2010) and Garratt, Mitchell and Vahey (2009), to disaggregate systems. These existing macroeconomic forecasting exercises consider combinations of forecast densities from models with a small number of (three, or less) candidate variables. With a large number of (lagged) disaggregate variables which could be used to forecast an economic aggregate, the applied re-

searcher faces a severe computational difficulty. For example, given 10 disaggregates, and restricting attention to a single lag of each disaggregate, the researcher faces $2^{10} = 1024$ feasible forecasting specifications for the aggregate. Allowing for anything beyond first-order dynamics is prohibitively burdensome computationally. For example, just one or two lags of each disaggregate variable would give $2^{20} = 1,048,576$ variants. And of course, whatever model selection methodology is applied by the researcher, there will be considerable model uncertainty about which specification is 'best' in practice.

In this paper, we overcome the curse of dimensionality resulting from forecasting with disaggregates by approximating the interactions between the many disaggregates and the aggregate. Each component in the ensemble represents the dynamic univariate time series process for a single disaggregate variable. Then we take time-varying weighted combinations of the forecast densities produced from the individual components to construct the ensemble predictive density for the aggregate. The calibration properties of the ensemble forecast densities provides guidance on the appropriateness of the approximation. Bache et al (2009) and Geweke (2009) discuss the interpretation of forecast density combinations in the presence of an incomplete model space.

2.1 Disaggregate Ensemble Construction

We consider a forecaster combining out of sample forecast densities provided by component models. Timmermann (2006, p177) discusses out of sample density combination. Recent applications include Wallis (2005), aggregating survey information, and Mitchell and Hall (2005), combining forecasts from two institutions.

We assume that the forecaster has uninformative priors over the forecast densities produced by the component models. In principle, off-model information—such as assigning prior mass to the expenditure shares used to define the aggregate index—could be helpful in forecasting applications. However, a prior elicitation problem arises with dynamic interrelationships between (potentially, a large number of) disaggregates. Hence, we leave an investigation of the scope for informative component priors to subsequent research.

Given $i = 1, \ldots, N$ disaggregates (where N could be a large number), we define the disaggregate ensemble (DE) by the convex combination sometimes referred to as a linear opinion pool. The disaggregate ensemble is defined as:

$$DE \;=\; g(Y_\tau) \;=\; \sum_{i=1}^{N} w_{i,\tau}\, h(Y_\tau \mid I_{i,\tau}), \qquad \tau = \underline{\tau}, \ldots, \overline{\tau}, \tag{1}$$

where $h(Y_\tau \mid I_{i,\tau})$ are the one step ahead forecast densities from component model i, $i = 1, \ldots, N$ of the economic aggregate Y_τ, conditional on the information set $I_{i,\tau}$.

Each component produces one step ahead forecasts for the aggregate. Hence, the variables used to produce a one step ahead forecast density for τ are dated $\tau - 1$ or earlier. Although we do not explore this issue here, a density combination framework can easily be extended to forecast horizons greater than one; see, for example, Jore, Mitchell and Vahey (2010). The non-negative weights, $w_{i,\tau}$, in this finite mixture sum to unity, are positive, and vary by recursion in the evaluation period $\tau = \underline{\tau}, \ldots, \overline{\tau}$.

Notice that our ensemble framework does not restrict the way in which the component forecasts are produced. The component models could have time-varying or constant parameters. The members of the ensemble could be estimated by frequentist or Bayesian methods, with or without the aid of conventional regression diagnostics. And the component models need not utilize the same in-sample observations for parameter estimation—rolling regression variants can be accommodated in the out of sample density combination exercise. Notice also that the disaggregate ensemble will be a mixture of the forecast densities produced by the components. Hence, the ensemble given by equation (1) can accommodate non-Gaussian predictive densities. This flexibility can be very useful in adapting the methodology to applied economic issues. Kascha and Ravazzolo (2010) discuss the methods to restrict the ensemble densities to be both unimodal and symmetric if required.

2.2 Component Model Space

Macroeconomic disaggregate time series variables commonly exhibit parameter change, and applied researchers often utilize Bayesian methods to accommodate this feature. With this in mind, consider a mixture innovation model for a single disaggregate variable, π:

$$\pi_t = \beta_{0t} + \sum_{p=1}^{k} \beta_{pt}\pi_{t-p} + \sigma_t\varepsilon_t$$

$$\beta_{jt} = \beta_{j,t-1} + \kappa_{jt}\eta_{jt}, \quad j = 0, \ldots, k \tag{2}$$

$$ln\sigma_t^2 = ln\sigma_{t-1}^2 + \kappa_{k+1,t}\eta_{k+1,t}$$

where $t = 1, ..., \tau - 1$, $\varepsilon_t \sim N(0, 1)$, $\eta_t = (\eta_{0t}, ..., \eta_{k+1,t})' \sim N(0, Q)$ with Q a diagonal matrix and elements $q_0^2, \ldots, q_{k+1}^2$, and $\kappa_t = (\kappa_{0t}, \ldots, \kappa_{k+1,t})'$ is a $((k + 2) \times 1)$ vector of unobserved uncorrelated 0/1 processes with $\Pr[\kappa_{jt} = 1] = p_j$ for $j = 0, \ldots, k + 1$.

Hence, each of the regression parameters β_{jt} and the residual variance σ_t^2 remain the same as their previous values $\beta_{j,t-1}$ and σ_{t-1}^2 unless $\kappa_{jt} = 1$ and $\kappa_{k+1,t} = 1$ in which case β_{jt} changes with η_{jt} and $ln(\sigma_t)^2$ changes with $\eta_{k+1,t}$ respectively. See, for example, Koop and Potter (2007) and Giordani, Kohn, van Dijk (2007) for similar approaches. As the changes in the variance parameters $\ln\sigma_t^2$ are stochastic we allow for a form of stochastic volatility; see Giordani and Kohn (2008). The flexibility of the specification in (2) stems from the fact that the parameters $\beta_t = (\beta_{0t}, \ldots, \beta_{kt})'$ and σ_t^2 are allowed to change every time period, but they need not change. The occurrence of a change is described by the latent binary random variable κ_{jt}, while the magnitude of the change is determined by η_{jt}, which is assumed to be normally distributed with mean zero. An attractive property of (2) is that the changes in the individual regression parameters are not restricted to coincide but rather are allowed to occur at different points in time. Given the popularity of this specification for modeling time variation in autoregressions, we relegate our discussion of the computational steps to Appendix A.1. We describe the disaggregate forecast densities from equation (2) in Appendix A.2.

We emphasize that the component specification described by equation (2) repre-

sents an autoregressive forecasting relationship (with parameter change) for a single disaggregate—the aggregate variable of interest does not enter equation (2). In applied ensemble work, the component (in our case, disaggregate) model forecasts might be badly behaved. The forecast densities from a given component could be too diffuse, or too narrow, and/or the forecasts might exhibit individual bias. It is common in the ensemble literature to consider adjusting the spread and/or the central location of each component density prior to combination; see, the discussions in (among others) Atger (2003), Stensrud and Yussouff (2007) and Bao et al (2010). In our disaggregate forecasting exercise, the disaggregate forecast, π_τ, may not be an efficient forecast of the aggregate, Y_τ. Although more flexible approaches are feasible, a simple bias-correction step to the component forecasts has often been found to be sufficient to ensure well-calibrated ensemble densities in practice; see, for example, Stensrud and Youssoff (2007). To implement this post-processing step, estimate with (recursive) Ordinary Least Squares (OLS):

$$Y_s \;=\; a + p^e(\pi_s \mid I_{i,s}) + \varepsilon_s, \qquad s = \underline{s}, \ldots, \tau - 1 \tag{3}$$

where $p^e(\pi_s|I_{i,s})$ is the expected value (for example, the median) of the predictive density $p(\pi_s \mid I_{i,s})$ from the i^{th} disaggregate component. Then, define the bias-corrected disaggregate forecast density for the aggregate:

$$h(Y_\tau \mid I_{i,\tau}) \;=\; p(\pi_\tau \mid I_{i,\tau}) + \widehat{a} \tag{4}$$

where $\widehat{a}$ is the OLS estimate of a in (3). The bias-corrected disaggregate forecast density $h(Y_\tau \mid I_{i,\tau})$ is used to construct the ensemble forecast density for the aggregate, $g(Y_\tau)$.

We note that although we consider a time-varying parameter model for the disaggregate time series, this is not a necessary feature of the ensemble approach. For example, Ravazzolo and Vahey (2009) utilize (recursively-estimated) constant parameter autoregressive components to forecast inflation in Australia.

2.3 Disaggregate Ensemble Weights

We complete our description of the disaggregate ensemble prediction system by specifying the construction of the time-varying weights. A number of studies in the economics literature have used density scoring rules. Mitchell and Wallis (2010) provide a recent discussion of scoring rules and the justification for testing relative density forecasting performance from the perspective of the Kullback-Leibler Information Criterion (KLIC). Gneiting and Raftery (2007) analyze the relationships between scoring rules and Bayes factors. Corradi and Swanson (2006) provide an extensive review of measures of density forecast performance.

Outside the econometrics literature, Hersbach (2000), Gneiting and Raftery (2007) and Panagiotelis and Smith (2008) have argued that the Continuous Ranked Probability Score (CRPS), which rewards predictive densities with high probabilities near (and at) the outturn, provides a robust metric of density forecast performance. Gneiting and Raftery (2007) refer to the concentration of a forecast density about its central location as 'sharpness', and the location as 'distance'. The CRPS metric favors densities with small distance and high sharpness.

The CRPS is measured as the difference between the predicted and actual cumulative distribution. Figure 1 provides an illustrative example for a particular observation: the CRPS measures the area between the predictive (for this example, assumed to be Gaussian) and the actual cumulative distribution (marked by shading). The (positive) score approaches zero as the predictive density converges on the true (but unobserved) density.

More formally, following Panagiotelis and Smith (2008), the CRPS of a component density for a particular observation can be defined as:

$$CRPS = E_h|y - Y| - 0.5E_h|y - y'| \tag{5}$$

where E_h is the expectation for the predictive $h(Y_\tau)$, y and y' are independent random draws from the predictive, and Y is the observed outturn. The expectation terms can be approximated using the Monte Carlo draws from the component forecast density; Panagiotelis and Smith (2008, equation 4.5) provide the computational steps required.

For each bias-corrected disaggregate forecast density, we construct the mean CRPS averaged over the evaluation period. The weight on an individual component density i in each observation of the evaluation period is then calculated by:

$$w_{i,\tau} = \frac{\left[\sum_{\underline{s}}^{\tau-1} X(h(Y_\tau \mid I_{i,\tau}))\right]}{\sum_{i=1}^{N}\left[\sum_{\underline{s}}^{\tau-1} X(h(Y_\tau \mid I_{i,\tau}))\right]}, \qquad \tau = \underline{s}, \ldots, \underline{\tau}, \ldots, \overline{\tau}. \tag{6}$$

with X is the inverse of the mean CRPS, $0 \leq X \leq \infty$, and higher scores are preferred.

2.4 Methodological Summary

Our disaggregate ensemble methodology can be summarized as follows. For each observation in the forecaster's evaluation period, we estimate N univariate time series representations, one for each disaggregate. The 'fit' of each bias-corrected component forecast density is assessed with the CRPS, and used to construct weights for the ensemble forecast density. These weights vary through the evaluation period. In this manner, we approximate the forecast densities for the true, but unknown, relationships between the disaggregates and the aggregate. The appropriateness of the approximation can be assessed by examining the calibration properties of the ensemble forecast densities. (We shall utilize a number of well-known calibration tests in the subsequent application.)

3 Application: forecasting inflation for the US

In this forecasting US inflation application, we consider US Personal Consumption Expenditure deflator (PCE) data. We construct a disaggregate ensemble using an evaluation period from 1997Q2 to 2008Q1, and then examine the calibration of the ensemble aggregate inflation forecast densities using probability integral transforms, *PITS*, at the end of the evaluation. We also examine forecast performance relative to a number of aggregate benchmarks. We stress that our focus in this example is the predictive performance of the ensemble. We do not aim to select a preferred single disaggregate predictor of aggregate inflation from the (likely) misspecified disaggregate components.

We begin our analysis by describing the US data. Then we describe our disaggregates ensemble, aggregate benchmarks, density evaluation methods, and results.

3.1 Data

The dataset contains time series for the disaggregate components of the PCE. The data are available on the Bureau of Economic analysis http://www.bea.gov/national/nipaweb. To our knowledge, the disaggregate data used in this study are not available on a real-time basis, although Croushore (2009) discusses the revisions in aggregate PCE. The PCE data permit breakdowns at various levels of disaggregation. Tables AI-AIII in Clark (2006) provide further details on levels of disaggregation in the US PCE data.

We emphasize that, in principle, our methodology could be applied to any level of disaggregation. In our application, we illustrate our technique with 11 disaggregates. These are Motor Vehicles, Household Equipment, Other Durables, Clothing, Other Nondurables, Housing, Household Operation, Transport, Medical Care, Recreation and Other Services. For all inflation series, PCE and its disaggregates, we work with the quarterly growth rates (calculated as 100 time the log difference in the price levels) plotted in figure 2. The volatility and the mean of PCE measured inflation vary through the sample as figure 2 makes clear, providing some motivation for time-varying parameter specifications. The 11 disaggregate series display varying degrees of shifting levels and changes in volatility: Motor Vehicles, Household Equipment, Other Durables, and Other Nondurables have marked changes in levels; Household Operation, Transport, Recreation and Other Services show signs of volatility changes; and Clothing, Housing, and Medical Care exhibit both characteristics relatively strongly. Overall, the data display considerable time variation and heterogeneity across disaggregates.

Using conventional assumptions about the timing of Great Moderation, we start our sample for component estimation with 1984Q1 and end with 2008Q1. Hence, we restrict our analysis to the period in which conventional wisdom has it that inflation is difficult to predict in terms of point forecast accuracy; see, for example, Stock and Watson

(2007). With our evaluation period (τ) from 1997Q2 ($\underline{\tau}$) to 2008Q1 ($\overline{\tau}$), the period 1993Q2 to 1997Q1 comprises a 'training period' to initialize the ensemble weights. The bias-correction step is based on a rolling window of 20 quarters, denoted $s = \tau - 20, \ldots, \tau - 1$, for the results reported below. (Using the training period plus the evaluation period for bias-correction gave some degradation in relative performance but the disaggregate ensemble always outperformed the aggregate benchmark.)

3.2 Disaggregate Ensemble and Aggregate Benchmarks

The ensemble forecast densities for aggregate inflation use equations (1)-(6) described above. In addition to our disaggregate ensemble, DE11, we also evaluate the predictive densities from two time series models of aggregate inflation. The first uses a linear model to forecast measured inflation without disaggregate information. That is, using a linear autoregressive model for aggregate measured inflation, with two lags, AR(2). We use uninformative priors for the AR(2) parameters with an expanding window. The predictive densities follow the t-distribution, with mean and variance equal to OLS estimates; see, for example, Koop (2003) for details. We use this AR model as our benchmark in tests of relative forecast performance.

The second aggregate variant uses a single time-varying parameter autoregressive specification similar to equation (2), but for aggregate inflation, Y_τ, with no disaggregate information. For both the aggregate and the disaggregate time-varying specifications we use four autoregressive terms (that is, we set $k = 4$).

3.3 Density Evaluation

Following (among others) Jore, Mitchell and Vahey (2010), we evaluate the ensemble predictive densities using a battery of (one-shot) tests of absolute forecast accuracy, relative to the 'true' but unobserved density. Like Rosenblatt (1952) and Diebold, Gunther and Tay (1998), we utilize the probability integral transforms, *PITS*, of the realization of the variable with respect to the forecast densities. A forecast density is preferred if the density

is correctly calibrated, regardless of the forecasters loss function. The *PITS* are:

$$z_\tau = \int_{-\infty}^{\pi_\tau} p(u)du.$$

The *PITS* should be both uniformly distributed, and independently and identically distributed if the forecast densities are correctly calibrated. Hence, calibration evaluation requires the application of tests for goodness-of-fit and independence. Given the large number of bias-corrected component forecast densities under consideration in the ensemble, we do not allow for estimation uncertainty in the components when evaluating the *PITS*. Corradi and Swanson (2006) review tests computationally feasible for small N.

The goodness-of-fit tests employed include the Likelihood Ratio (LR) test proposed by Berkowitz (2001), the Anderson-Darling test, and the Pearson (χ^2) test used by Wallis (2003). Our Berkowitz test is a three degrees of freedom variant, with a test for independence, where under the alternative z_τ follows an AR(1) process. The Anderson-Darling (AD) test for uniformity, a modification of the Kolmogorov-Smirnov test, gives more weight to the tails of the forecast density. The Pearson (χ^2) tests divides the range of the z_τ into eight equiprobable classes and tests for uniformity in the histogram. We also test directly for independence of the *PITS* using a Ljung-Box (LB) test, based on autocorrelation coefficients up to four. A well-calibrated ensemble should give high probability values for all four of these tests—implying the null hypothesis of no calibration failure cannot be rejected.

Turning to our analysis of relative predictive accuracy, we consider a Kullback-Leibler Information Criterion (KLIC) based test, utilizing the expected difference in the Logarithmic Scores of the candidate forecast densities; see, for example, Bao, Lee and Saltoglu (2007), Mitchell and Hall (2005) and Amisano and Giacomini (2007). Suppose there are two density forecasts, $g(Y_\tau \mid I_{1,\tau})$ and $g'(Y_\tau \mid I_{2,\tau})$, and consider the loss differential $d_\tau = \ln g(Y_\tau \mid I_{1,\tau}) - \ln g'(Y_\tau \mid I_{2,\tau})$. The null hypothesis of equal accuracy is $\mathcal{H}_0 : E(d_\tau) = 0$. The sample mean, $\overline{d}_\tau$, has under appropriate assumptions the limiting distribution: $\sqrt{T}(\overline{d}_\tau - d_\tau) \to N(0, \Omega)$. The Logarithmic Score of the i^{th} density forecast, $\ln g(Y_\tau \mid I_{i,\tau})$, is the logarithm of the probability density function $g(. \mid I_{i,\tau})$, evaluated

at the outturn Y_τ. In our LS test of relative forecast performance, we abstract from the estimation procedure used to generate the forecast densities. Mitchell and Wallis (2010) discuss the value of information-based methods for evaluating forecast densities that are well-calibrated on the basis of *PITS* tests.

3.4 Results

Before considering the density evaluations for our disaggregate ensemble, we summarize the point forecast performance. Both the disaggregate ensemble (DE11) and the time-varying parameter aggregate autoregressive model (TVPAR) are considerably outperformed by the aggregate AR(2) model in terms of root mean squared prediction error (RMSPE). For the AR(2) benchmark, the raw RMSPE is 0.163. The other specifications give figures approximately 60 percent higher. Stock and Watson (2007) discuss the difficulty of outperforming simple benchmarks in terms of RMSPE with Great Moderation data; see also Groen, Paap and Ravazzolo (2009) for similar results.

The evaluation of the forecast densities are presented in table 1. The three rows refer to the disaggregate ensemble, DE11, the aggregate autoregressive benchmark, AR(2), and the aggregate time-varying parameter model, TVPAR, respectively. The five columns of table 1 report the p-values for the Berkowitz LR test, the Anderson-Darling AD test, the χ^2, the LB test, and the Logarithmic Scores (averaged over the evaluation period).

Looking at the DE11 results shown in the top row, we see that the null hypothesis of no calibration failure cannot be rejected at the 1 percent significance level for all of the four individual diagnostic tests, marked in bold. (Using a 5 percent significance level, the χ^2 test is (just) failed with a 4.8 percent probability value.) We note that each of these diagnostic tests for calibration is conducted on an individual basis. A 5 percent significance level on each individual test would imply a Bonferroni-corrected p-value of 5/4=1.25 percent (reported as 0.0125 in the table).

The aggregate specifications, shown in the remaining two rows of table 1, display a number of instances of calibration failure. The AR(2) benchmark, first row, fails all of

the diagnostic tests, with three p-values below 1 percent. The more flexible aggregate specification, TVPAR, fails two of the four tests at the 1 percent level. Namely, the LR and the χ^2.

Figure 3 plots the *PITS* histograms for the three candidates, the DE11, the AR(2) and the TVPAR. The histogram for the AR(2) displays severe departures from uniformity. The TVPAR and DE11 are more evenly spread across the decile counts, although visual inspection suggests calibration could be improved in both cases.

Turning to the Logarithmic Scores of the forecast densities, shown in the fifth column of table 1, we see that the disaggregate ensemble DE11 records the best relative performance, roughly 26 percent of the AR(2). The LS test p-value (marked in bold) indicates that the null hypothesis of equal forecast performance can be rejected at the 1 percent significance level. The time-varying parameter aggregate specification also improves on the the AR(2) benchmark, at roughly 60 percent. An LS test of the DE11 relative to the TVPAR confirms the superiority of the DE11 at the 1 percent significance level.

To shed further light on the contribution of disaggregate information, figure 4 plots the weights in the disaggregate ensemble DE11. As we might expect, given the univariate nature of the components, there is uncertainty about the relative importance of disaggregate components through the evaluation. The weights lie in the (approximate) interval [0.04, 0.18] at the beginning of the evaluation. But the dispersion in the weights drops as the top three disaggregates decline in importance through the evaluation.

In figure 5, we plot the median from our disaggregate ensemble, together with aggregate PCE inflation. The 25^{th} and 75^{th} percentiles from the ensemble density are also shown. The plot shows that the median of the DE11 is considerably less volatile than the actual aggregate inflation series. The central mass of the predictive density is around 0.3 percent prior to 2004, and slightly higher thereafter. The difference between the two percentiles shown varies very little through the evaluation, typically remaining close to 0.4 percentage points. We note that the number of inflation outturns above the 75^{th} percentile is somewhat larger than the number below the 25^{th}percentile. Although the *PITS* tests indicate that the ensemble forecast densities are correctly calibrated, clearly there

is scope for further improvement.

We draw the following conclusions from our forecast density evaluations. First, the disaggregate ensemble DE11 performs well in both tests of absolute and relative density forecasting performance. Second, as Jore, Mitchell and Vahey (2010) and Clark (2009) emphasize, although simple autoregressive models of aggregate inflation produce accurate point forecasts, the benchmark can be bettered in terms of forecast densities.

4 Conclusions

In this paper, we have proposed a methodology for constructing forecast densities for economic aggregates based on disaggregate evidence using an ensemble predictive system. In our application, we have shown that the disaggregate ensemble approach delivers well-calibrated forecast densities for US PCE aggregate inflation from 1997Q2 to 2008Q1. Alternative forecasting specifications for the aggregate based on time-varying models or simple autoregressive benchmarks failed to match the density forecasting performance of our disaggregate ensemble.

Our applied work indicates that including disaggregate information via an ensemble system improves probabilistic forecasts for US aggregate inflation. This result mirrors similar findings in other fields where ensemble methods have been widely adopted by practitioners to provide a pragmatic framework for probabilistic assessment. Our results also confirm formally the view endorsed by many economic policymakers that disaggregate information can be helpful for forecasting.

5 References

Altger, F. (2003) "Spatial and interannual variability of the reliability of ensemble-based probabilistic forecasts: Consequences for calibration", *Monthly Weather Review*, 131, 1509-1523.

Amisano, G. and R. Giacomini (2007), "Comparing Density Forecasts via Likelihood Ratio Tests", *Journal of Business and Economic Statistics*, 25, 2, 177-190.

Bache, I.W., J. Mitchell, F. Ravazzolo and S.P. Vahey (2009) "Macro modeling with many models", Norges Bank Working Paper, 2009/15.

Bao, Y., T-H. Lee and B. Saltoglu (2007), "Comparing Density Forecast Models", *Journal of Forecasting*, 26, 203-225.

Bao, L., T. Gneiting, E.P. Grimit, P. Guttop, and A.E. Raftery (2010), "Bias Correction and Bayesian Model Averaging for Ensemble Forecasts of Surface Wind Direction", *Monthly Weather Review*, forthcoming.

Bates, J.M. and C.W.J. Granger (1969), "Combination of Forecasts", *Operational Research Quarterly*, 20, 451-468.

Berkowitz, J. (2001) "Testing density forecasts, with applications to risk management", *Journal of Business and Economic Statistics*, 19, 465-474.

Carter, C. and R. Kohn (1994) "On Gibbs sampling for state-space models", *Biometrika*, 81, 541-553.

Carter, C. and R. Kohn (1997) "Semiparametric Bayesian inference for time series with mixed spectra", *Journal of the Royal Statistical Society*, Series B, 255-268.

Clark, T.E. (2006) "Disaggregate evidence on the persistence of consumer price inflation, *Journal of Applied Econometrics*, 21, 563-587.

Clark, T.E. (2009) "Real-time density forecasts from VARs with stochastic volatility", FRB Kansas City Working Paper, RWP 09-08.

Clark T.E. and M.W. McCracken (2010) "Averaging forecasts from VARs with uncertain instabilities", *Journal of Applied Econometrics*, 25, 5-29.

Corradi, V., and N.R. Swanson (2006) "Predictive density evaluation", in G. Elliot, C. W. J. Granger, and A. Timmermann(eds.) Handbook of Economic Forecasting, Elsevier, 197 - 284.

Croushore, D. (2009) "Revisions to PCE inflation measures: implications for monetary policy", FRB Philadelphia Working Paper 08-8, revised July 2009.

Diebold, F.X., T.A. Gunther, and A.S. Tay (1998) "Evaluating density forecasts; with applications to financial risk management", *International Economic Review*, 39, 863-83.

Feinstein, M., M.A. King, and J. Yellen (2004) "Innovations and issues in monetary policy: panel discussion", *American Economic Review*, Papers and Proceedings, May, 41-48.

Garratt, A., J. Mitchell and S.P. Vahey (2009) "Measuring output gap uncertainty", Reserve Bank of New Zealand Discussion Paper, DP 2009/15.

Geman, S. and D. Geman (1984) "Stochastic relaxation, Gibbs distributions and the Bayesian restoration of images", IEEE Transaction on Pattern Analysis and Machine Intelligence, 6, 721-741.

Gerlach, R., C. Carter, and R. Kohn (2000) "Efficient Bayesian inference for dynamic mixture models", *Journal of the American Statistical Association*, 95, 819-828.

Geweke, J. (2009) "Complete and Incomplete Econometric Models", Princeton University Press.

Giordani, P. and R. Kohn (2008) "Efficient Bayesian inference for multiple change-point and mixture innovation models", *Journal of Business and Economic Statistics*, 26, 66-77.

Giordani, P., R. Kohn, and D. van Dijk (2007) "A united approach to nonlinearity, outliers and structural breaks", *Journal of Econometrics*, 137, 112-137.

Gneiting, T. and A.E. Raftery (2007) "Strictly proper scoring rules, prediction and estimation", *Journal of the American Statistical Society*, 102, 477, 359-378.

Granger, C. and M.H. Pesaran (2000) Economic and statistical measures of forecast accuracy, *Journal of Forecasting*, 19, 537-560.

Greenspan, A. (2004) "Risk and uncertainty in monetary policy", *American Economic*

Review, Papers and Proceedings, May, 33-40.

Groen, J.J.J., R. Paap and F. Ravazzolo (2009) "Real-time inflation forecasting in a changing world", Norges Bank Working Paper, 2009/16.

Hersbach, H. (2000) "Decomposition of the continuous ranked probability score for ensemble prediction systems", *Weather and Forecasting*, 15, 559-570.

Jore, A.S., J. Mitchell and S.P. Vahey (2010)"Combining forecast densities from VARs with uncertain instabilities", *Journal of Applied Econometrics*, forthcoming.

Kascha, C. and F. Ravazzolo (2010) "Combining inflation density forecasts", *Journal of Forecasting*, 29, 231-250.

Kim, S., S. Shephard, and S. Chib (1998) "Stochastic volatility: Likelihood inference and comparison with ARCH models, *Review of Economic Studies*, 65, 361-393.

Koop, G. (2003) *Bayesian Econometrics*, Wiley.

Koop, G. and S. Potter (2007) "Estimation and forecasting in models with multiple breaks", *Review of Economic Studies*, 2007, 74, 763-789.

Lopez, A., F. Fung, M. New, G. Watts, A. Weston, R.L. Wilby (2009) "From climate model ensembles to climate change impacts and adaptation: A case study of water resource management in the southwest of England", *Water Resources Research*, 45, W08419, 1-21.

Mitchell, J. and S.G. Hall (2005) "Evaluating, comparing and combining density forecasts using the KLIC with an application to the Bank of England and NIESR fan charts of inflation", *Oxford Bulletin of Economics and Statistics*, 67, 995-1033.

Mitchell, J. and K.F. Wallis (2010) "Evaluating density forecasts: Forecast combinations, model mixtures, calibration and sharpness", *Journal of Applied Econometrics*, forthcoming.

Molteni, F., R. Buizza, T.N. Palmer and T. Petroliagis (1996) "The new ECMWF ensemble prediction system: methodology and validation", *Quarterly Journal of the Royal Meteorological Society*, 122, 73-119.

Murphy, J.M., D.M.H. Sexton, D.N. Barnett, G.S. Jones, M.J. Webb, M. Collins and D.A. Stainforth (2004) "Quantification of modelling uncertainties in large ensembles of

climate change simulations", *Nature*, 430, 768-772.

Panagiotelis, A. and M. Smith (2008) "Bayesian density forecasting if intraday electricity prices using multivariate skew t distribution", *International Journal of Forecasting*, 24, 710-727.

Raftery, A.E., T. Gneiting, F. Balabdaoui and M. Polakowski, (2005) "Using Bayesian model averaging to calibrate forecast ensembles", *Monthly Weather Review*, 133, 1155-1174.

Ravazzolo, F. and S.P. Vahey (2009) "Measuring core inflation in Australia with disaggregate ensembles", available from http://www.rba.gov.au/publications/confs/2009/index.html.

Rosenblatt, M. (1952) "Remarks on a multivariate transformation", *The Annals of Mathematical Statistics*, 23, 470-472.

Shephard, N. (1994), "Partial non-Gaussian state-space models", *Biometrika*, 81, 115-131.

Stensrud, D.J. and N. Yussouf (2007) "Bias-corrected short-range ensemble forecasts of near surface variables", *Meteorological Applications*, 12, 217-230.

Stock, J.H. and M.W. Watson (2003) "Forecasting output and inflation: The role of asset prices", *Journal of Economic Literature*, 41, 788-829.

Stock, J.H. and M.W. Watson (2007) "Why has US inflation become harder to forecast?", *Journal of Money, Credit and Banking*, 39, 3-34.

Tanner, M. and W. Wong (1987) "The calculation of posterior distributions by data augmentation, *Journal of the American Statistical Association*, 82, 528-550.

Timmermann, A. (2006) "Forecast combination", G. Elliot, C. Granger, C. and A. Timmermann (eds.) *Handbook of Economic Forecasting,* North-Holland, 197-284.

Wallis, K.F. (2003) "Chi-squared tests of interval and density forecasts, and the Bank of England's fan charts", *International Journal of Forecasting*, 19, 165-175.

Wallis, K.F. (2005) "Combining density and interval forecasts: a modest proposal", *Oxford Bulletin of Economics and Statistics*, 67, 983-994.

A Time-Varying parameter model

A.1 Prior Specification and Posterior Simulation

We specify the following mixture innovation model for a given time series $\pi = \{\pi_t\}_{t=1}^{\tau-1}$:

$$\pi_t = \beta_{0t} + \sum_{p=1}^{k} \beta_{pt}\pi_{t-p} + \sigma_t\varepsilon_t$$

$$\beta_{jt} = \beta_{j,t-1} + \kappa_{jt}\eta_{jt}, \quad j = 0, ..., k \tag{A-1}$$

$$ln\sigma_t^2 = ln\sigma_{t-1}^2 + \kappa_{k+1,t}\eta_{k+1,t}$$

where $\varepsilon_t \sim N(0,1)$, $\eta_t = (\eta_{0t}, ..., \eta_{k+1,t})' \sim N(0, Q)$ with Q a diagonal matrix and elements $q_0^2, ..., q_{k+1}^2$, and $\kappa_t = (\kappa_{0t}, ..., \kappa_{k+1,t})'$ is a $((k+2)\times 1)$ vector of unobserved uncorrelated 0/1 processes with $\Pr[\kappa_{jt} = 1] = p_j$ for $j = 0, ..., k+1$. The model parameters are the structural break probabilities $p = (p_0, ..., p_{k+1})'$ and the vector of variances of the size of the breaks $q = (q_0, ..., q_{k+1})'$. We collect the model parameters in a $(2(k+1)\times 1)$ vector $\theta = (p_0, ..., p_{k+1}, q_0, ..., q_{k+1})'$.

To facilitate the posterior simulation we make use of independent conjugate priors. For the structural break probability parameters we take Beta distributions

$$p_j \sim Beta(a_j, b_j) \tag{A-2}$$

The parameters a_j and b_j can be set according to our prior belief about the occurrence of structural breaks. For the variance parameters we take the inverted Gamma-2 prior

$$q_j^2 \sim \text{IG-2}(\nu_j, \delta_j) \tag{A-3}$$

where ν_j, δ_j are parameters which can be chosen to reflect the prior beliefs about the variances. Realistic values of the parameters in the different prior distributions depend on the problem at hand. In general, we suggest to assign to ν_j high values. This means to have strong believes that the magnitude of a break at time t for parameter β_{jt} (σ_t^2) associated to $\Pr[\kappa_{jt} = 1] = 1$ is equal to δ_j. The prior on (A-2) can consequently be chosen

to limit the number of these breaks. As the posterior probability $\Pr[\kappa_{jt} = 1]$ is lower than 1, prior information is weak on breaks with magnitude lower than δ_j or situations of not changes.

Posterior results are obtained using the Gibbs sampler of Geman and Geman (1984) combined with the technique of data augmentation of Tanner and Wong (1987). The latent variables $B = \{\beta_t\}_{t=1}^{\tau-1}$, $R = \{\sigma_t^2\}_{t=1}^{\tau-1}$ and $K = \{\kappa_t\}_{t=1}^{\tau-1}$ are simulated alongside the model parameters θ.

The complete data likelihood function is given by

$$\begin{aligned} p(\pi, B, K, R|\theta) = \textstyle\prod_{t=1}^{\tau-1} p(\pi_t|\beta_t, \sigma_t^2) \prod_{j=0}^{k} p(\beta_{jt}|\beta_{j,t-1}, \kappa_{jt}, q_j^2) \\ p(\sigma_t^2|\sigma_{t-1}^2, \kappa_{k+1,t}, q_{k+1}^2) \prod_{j=0}^{k+1} p_j^{\kappa_{jt}} (1-p_j)^{1-\kappa_{jt}}, \end{aligned} \tag{A-4}$$

where $\pi = (\pi_1, \ldots, \pi_{\tau-1})$. The terms $p(\pi_t|\beta_{jt}, \sigma_t^2)$ and $p(\beta_{jt}|\beta_{j,t-1}, \kappa_{jt}, q_j^2)$ are normal density functions which follow directly from (A-1) and $p(\sigma_t^2|\sigma_{t-1}^2, \kappa_{k+1,t}, q_{k+1}^2)$ is an exponential normal density function. If we combine (A-4) together with the prior density $p(\theta)$, which follows from (A-2)-(A-3), we obtain the posterior density

$$p(B, K, R, \theta|\pi) \propto p(\theta) p(\pi, B, K, R|\theta). \tag{A-5}$$

For the Gibbs sampling procedure we employ the efficient sampling algorithm of Gerlach, Carter and Kohn (2000) to handle the (occasional) structural breaks. If we define $K_\beta = \{\kappa_{0t}, \ldots, \kappa_{kt}\}_{t=1}^{\tau-1}$ and $K_\sigma = \{\kappa_{k+1,t}\}_{t=1}^{\tau-1}$, the sampling scheme can be summarized as follows:

1. Draw K_β conditional on R, K_σ, θ and π.
2. Draw B conditional on R, K, θ, and π.
3. Draw K_σ conditional on B, K_β, θ, and π.
4. Draw R conditional on B, K, θ, and π.
5. Draw θ conditional on B, K, and π.

The (occasional) structural breaks, measured by the latent variable κ_{jt}, are drawn using the algorithm of Gerlach, Carter and Kohn (2000), which derives its efficiency from generating κ_{jt} without conditioning on the states β_{jt} (σ_t^2). The conditional posterior density for $\kappa_{*,t}$, $t = 1, \ldots, \tau - 1$ unconditional on B is

$$\begin{aligned} p(\kappa_{*,t}|K_{*,-t}, K_{k+1}, R, \theta, \pi) &\propto p(\pi|K_*, K_{k+1}, R, \theta) p(\kappa_{*,t}|K_{*,-t}, \theta) \\ &\propto p(\pi_{t+1}, \ldots, \pi_{\tau-1}|\pi_1, \ldots, \pi_t, K, R, \theta) \\ &\quad p(\pi_t|\pi_1, \ldots, \pi_{t-1}, \kappa_1, \ldots, \kappa_t, R, \theta) p(\kappa_{*,t}|K_{*,-t}, \theta), \end{aligned} \tag{A-6}$$

where $K_{*,-t} = \{\kappa_{*,s}\}_{s=1,s\neq t}^{\tau-1}$. Note that the term $p(\kappa_{*,t}|K_{*,-t}, \theta)$ is simply given by $\prod_{j=0}^{k} p_j^{\kappa_{jt}}(1-p_j)^{1-\kappa_{jt}}$. The two remaining densities $p(\pi_{t+1}, \ldots, \pi_{\tau-1}|\pi_1, \ldots, \pi_t, K, R, \theta)$ and $p(\pi_t|\pi_1, \ldots, \pi_{t-1}, \kappa_1, \ldots, \kappa_t, R, \theta)$ can be evaluated as shown in Gerlach, Carter and Kohn (2000). Because $\kappa_{*,t}$ can take a finite number of values, the integrating constant can easily be computed by normalization.

The full conditional posterior density for the latent regression parameters B is computed using the simulation smoother as in Carter and Kohn (1994). The Kalman smoother is applied to derive the conditional mean and variance of the latent factors.

To draw K_σ and R in steps 3 and 4 we want to follow the same approach. As the model for $\ln \sigma_t^2$ does not result in a linear state space model the Kalman filter cannot be applied. Therefore, we apply the approach of Giordani and Kohn (2008) and rewrite the model (A-1) as

$$\begin{aligned} \ln(\pi_t - \beta_{0t} - \sum_{p=1}^{k} \beta_{pt}\pi_{it-p})^2 &= \ln \sigma_t^2 + u_t \\ \beta_{jt} &= \beta_{j,t-1} + \kappa_{jt}\eta_{jt}, \qquad j = 0, \ldots, k, \\ \ln \sigma_t^2 &= \ln \sigma_{t-1}^2 + \kappa_{k+1,t}\eta_{k+1,t} \end{aligned} \tag{A-7}$$

where $u_t = \ln \varepsilon_t^2$ has a log χ^2 distribution with 1 degree of freedom. We follow Carter and Kohn (1994), Carter and Kohn (1997), Shephard (1994) and Kim, Shephard, Chib (1998) who show that the $\ln \chi^2(1)$ distribution can be approximated very accurately by a finite mixture of normal distributions. We consider a mixture of five normal distributions such

that the density of u_t is given by

$$f(u_t) = \sum_{s=1}^{5} \varphi_s \frac{1}{\omega_s} \phi((u_t - \mu_s)/\omega_s). \tag{A-8}$$

with $\sum_{s=1}^{5} \varphi_s = 1$. The appropriate values for μ_s, ω_s^2 and φ_s can be found in Carter and Kohn (1997, Table 1). In each step of the Gibbs sampler we simulate a component of the mixture distribution from the distribution of the mixing distribution. Given the value of the mixture component we can apply standard Kalman filter techniques. Hence, the variables K_σ and R can be sampled in a similar way as K_β and B in step 1 and 2.

To sample the parameters θ we can use standard results in Bayesian inference. Hence, the probabilities π_j are sampled from Beta distributions, and the variance parameters q_j^2 are sampled from inverted Gamma-2 distributions.

A.2 Forecast density

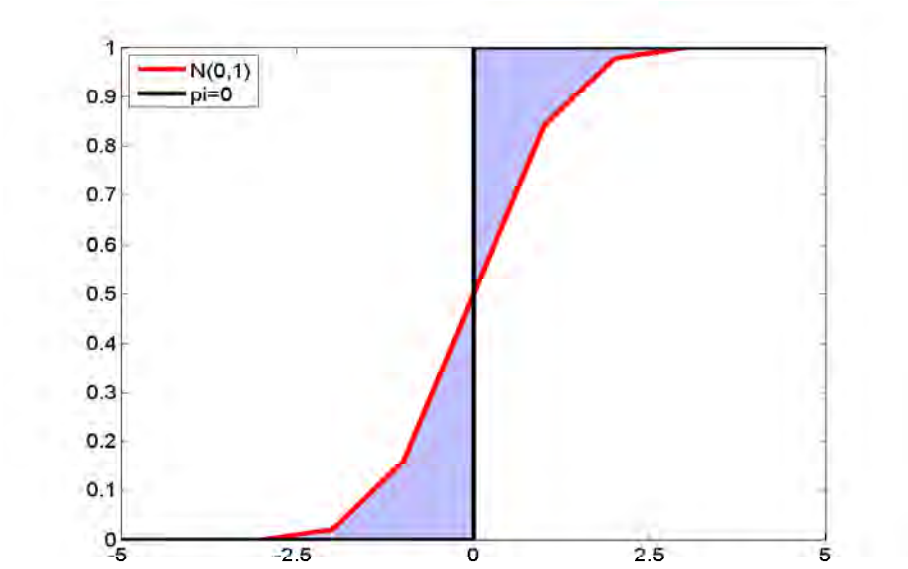
The one-step ahead forecast density of π_τ at time τ conditional on I_τ is given by

$$\begin{aligned} p(\pi_\tau|I_\tau) = \iint \sum_K \sum_{\kappa_\tau} p(\pi_\tau|S, \beta_\tau, \sigma_\tau^2) \prod_{j=0}^{k} p(\beta_{j,\tau}|\beta_{j,\tau-1}, \kappa_{j,\tau}, q_j^2) \\ p(\sigma_\tau^2|\sigma_{\tau-1}^2, \kappa_{k+1,\tau}, q_{k+1}^2) \prod_{j=0}^{k+1} p_j^{\kappa_{j,\tau}} (1-p_j)^{1-\kappa_{j,\tau}} p(B, K, R, S, \theta|\pi) dB dR d\theta, \end{aligned} \tag{A-9}$$

where $p(\pi_\tau|S, \beta_\tau, \sigma_\tau^2)$ and $p(\beta_{j,\tau}|\beta_{j,\tau-1}, \kappa_{j,\tau}, q_j^2)$ and $p(\sigma_\tau^2|\sigma_{\tau-1}^2, \kappa_\tau, q_{k+1}^2)$ follow directly from (A-1) and where $p(B, K, R, S, \theta|\pi)$ is the posterior density in (A-5) using information I_τ. Computation of this predictive density is straightforward using the Gibbs draws. In each Gibbs step, we simulate the π_τ using (A-1) as data generating process, where we replace the parameters and the latent variables by the draw from the posterior distribution. As point estimate we use the posterior median.

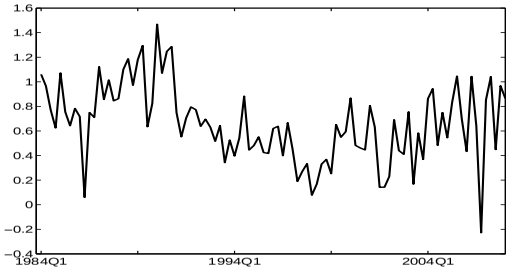
The procedure can be applied to derive the predictive density $p(\pi_{\tau,h} \mid I_{i,\tau})$ for each disaggregate i, $i = 1, \ldots, N$.

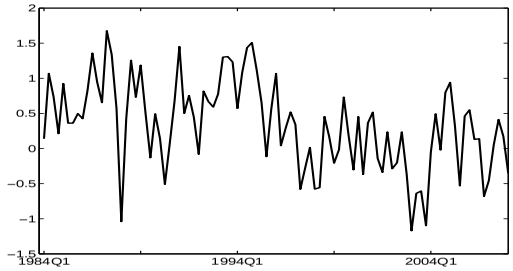
Figure 1: CRPS

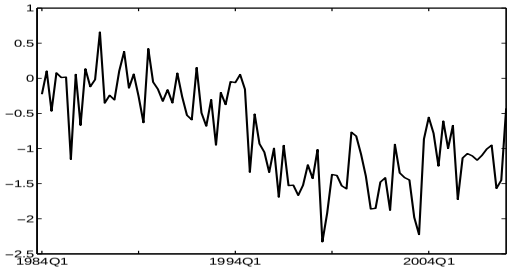


Note: The figure shows the cumulative distribution of a normal density with zero mean and unit variance, N(0,1), and the cumulative distribution of the realized value 0. The colored area measures the CRPS.

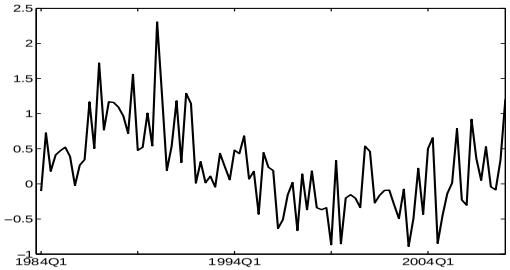
Figure 2: Data, 1984Q1-2008Q1

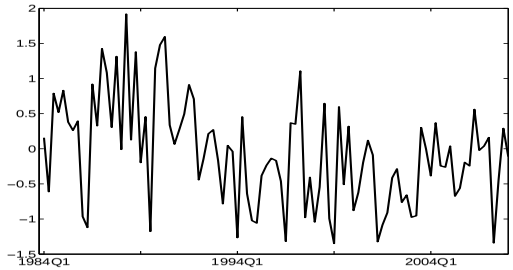


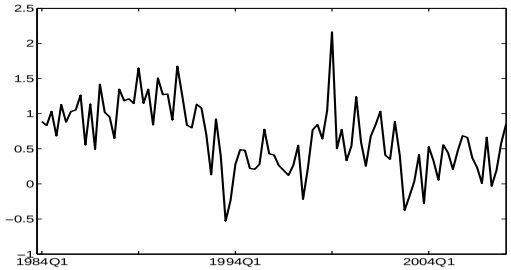




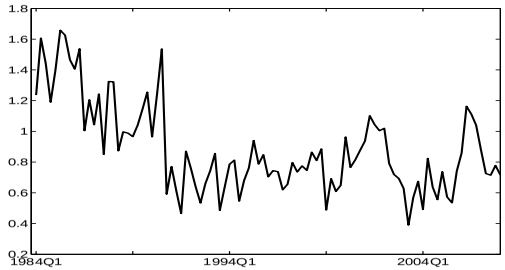
(a) PCE - Motor - Household Equipment

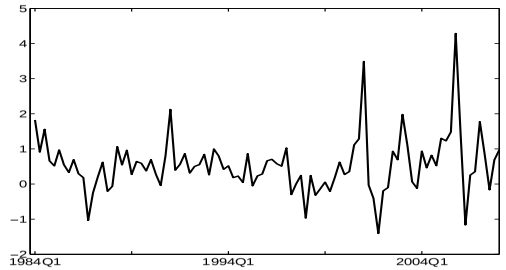


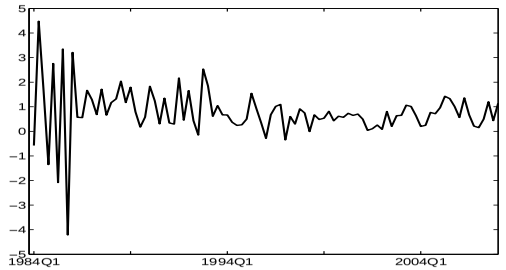




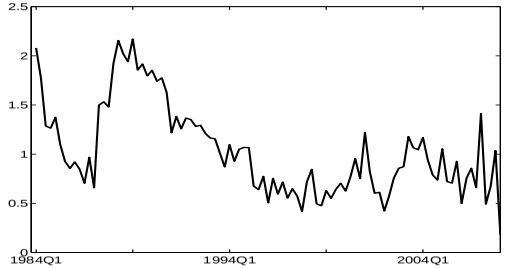
(b) Oth. Durables - Clothing - Oth. Non-dururables

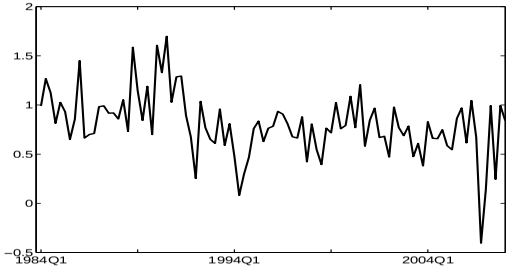


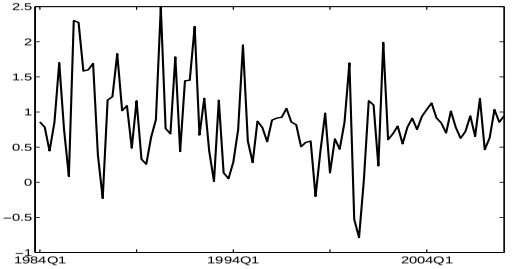




(c) Housing - Household Operation - Transport

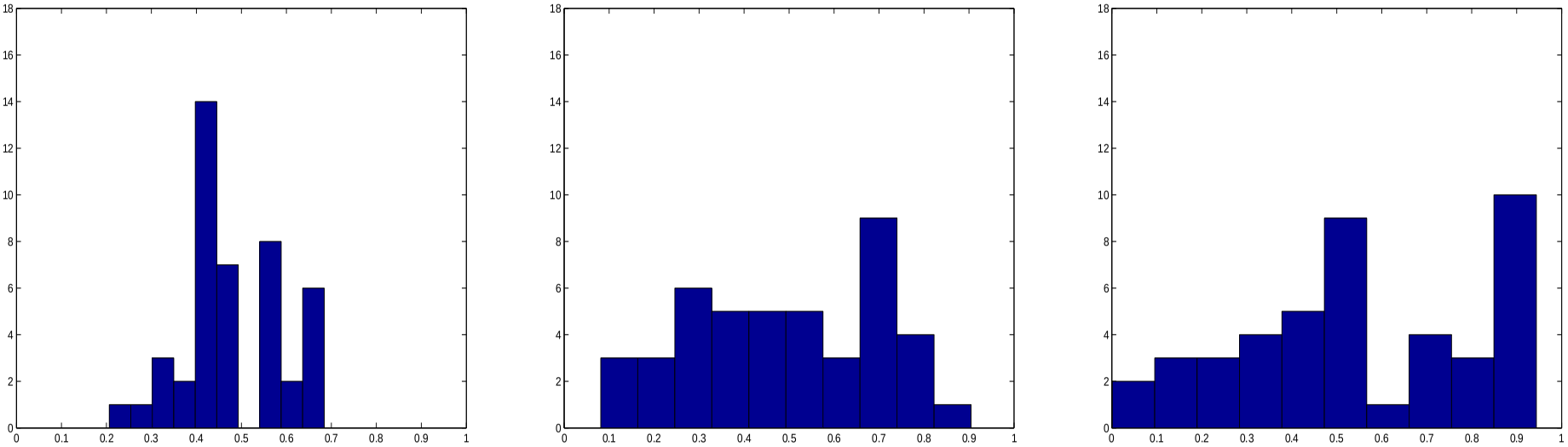






(d) Medical Care - Recreation - Oth. Service

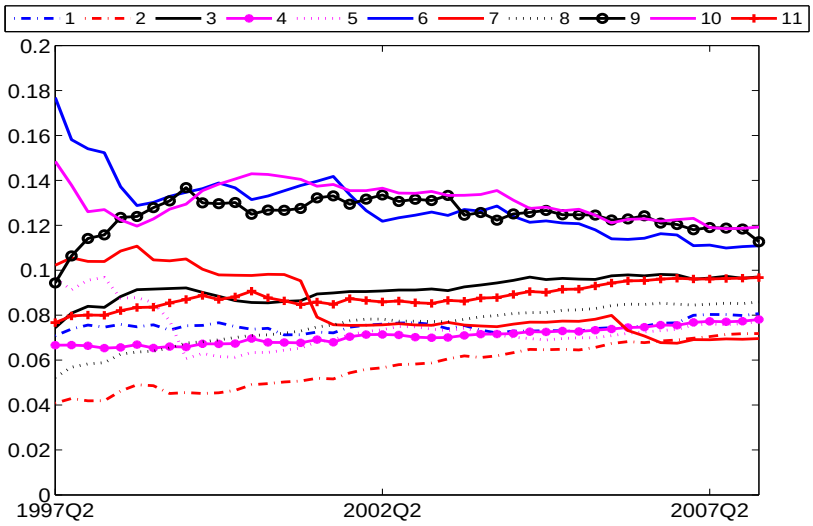
Figure 3: *PITS* histogram



(a) AR - TVPAR - DE11

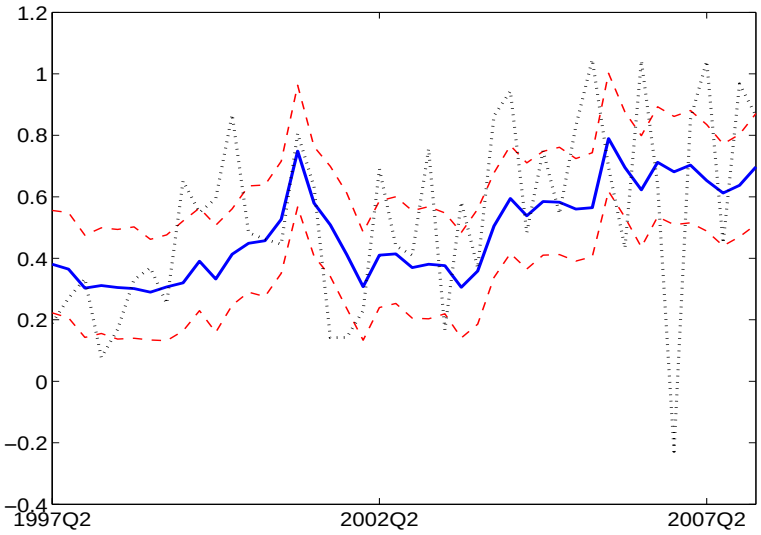
Note: The histogram shown are the decile counts of the *PITS* transforms.

Figure 4: DE11 weights



Note: The figures plot the weights given by disaggregate ensemble DE11. The disaggregate order 1-11 for DE11 corresponds to the order in figure 2.

Figure 5: PCE inflation forecasts



Note: The figure shows the posterior median (blue solid line) of the predictive density given by disaggregate ensemble DE11 and the actual inflation (black dashed line), together with the 25^{th} and 75^{th} percentiles of the predictive density (red dashed line).

Table 1: Forecast density performance, 1997Q2 - 2008Q1

	LR	AD	χ^2	LB	LS
DE11	**0.603**	**0.086**	**0.048**	**0.677**	**0.262**
	Individual models				
AR	0.000	0.000	0.000	**0.014**	*-0.427*
TVPAR	0.000	**0.057**	0.001	**0.122**	**0.602**

Note: The column LR is the Likelihood Ratio p-value of the test of zero mean, unit variance and independence of the inverse normal cumulative distribution function transformed *PITS*, with a maintained assumption of normality for transformed *PITS*. AD is the p-value for the Anderson-Darling test for uniformity of the pits. The small-sample (simulated) 1% p-values computed assuming independence of the *PITS* for the Anderson-Darling test is 3.905. χ^2 is the p-value for the Pearson chi-squared test of uniformity of the *PITS* histogram in eight equiprobable classes. LB is the p-value from a Ljung-Box test for independence of the *PITS*. A bold number indicates that the null hypothesis of a correctly specified model cannot be rejected at 1% significance level for LR, AD, χ^2 and LB. LS is the average Logarithmic Score over the evaluation period. The value using an AR(2) for PCE is reported in *italics*; all the other numbers report statistics relative to those of the AR(2). Numbers below one indicate that the competitor provides a mean LS lower than the AR(2), i.e. the benchmark is outperformed. Bold numbers indicate that the null of the LS test of equal density predictive accuracy relative to the AR(2) benchmark is rejected at 1% significance level.