CENTRE FOR APPLIED MACROECONOMIC ANALYSIS

The Australian National University

CAMA Working Paper Series **October, 2010**

BUSINESS CYCLE IMPLICATIONS OF INTERNAL CONSUMPTION HABIT FOR NEW KEYNESIAN MODELS

Takashi Kano
The University of Tokyo

James M. Nason
Federal Reserve Bank of Philadelphia
Centre for Applied Macroeconomic Analysis, ANU

CAMA Working Paper 31/2010
http://cama.anu.edu.au

Business Cycle Implications of Internal Consumption Habit for New Keynesian Models

Takashi Kano[†]
Faculty of Economics
Graduate School of Economics
The University of Tokyo
7–3–1 Hongo
Tokyo, JAPAN
email: tkano@e.u-tokyo.ac.jp

James M. Nason
Research Department
Federal Reserve Bank of Philadelphia
Ten Independence Mall
Philadelphia, PA
U.S.A. 19106–1574
email: jim.nason@phil.frb.org

Current Draft: August 12, 2010

Abstract

This paper studies the implications of internal consumption habit for new Keynesian dynamic stochastic general equilibrium (NKDSGE) models. Bayesian Monte Carlo methods are employed to evaluate NKDSGE model fit. Simulation experiments show that consumption habit often improves the ability of NKDSGE models to match output and consumption growth spectra. Nonetheless, the fit of NKDSGE models with consumption habit is susceptible to the source of the nominal rigidity, to spectra identified by permanent productivity shocks, to the frequencies used for evaluation, and to the choice of monetary policy rule. These vulnerabilities suggest that NKDSGE model specification is fragile.

Key Words: Habit; New Keynesian; Propagation; Monetary Transmission; Model Evaluation, Bayesian Monte Carlo.

JEL Classification Number: $E10$, $E20$, $E32$.

[†]We thank Evan Anderson, Heather Anderson, Jeannine Bailliu, Hafedh Bouakez, Toni Braun, Fabio Canova, Youngsung Chang, Richard Dennis, Mick Devereux, Mike Dotsey, Bill Dupor, Jon Faust, Martin Fukač, John Geweke, Pablo Guerron-Quintana, Tasos Karantounias, Keith Kuester, Thomas Lubik, Christian Matthes, Ricardo Nunes, Adrian Pagan, B. Ravikumar, Pedro Silos, Larry Schembri, Frank Schorfheide, Tan Wang, Tao Zha, and seminar and session participants at the Bank of Japan, Bank of Canada, 2004 Far Eastern Meetings of the Econometric Society, 10th International Conference on Computing in Economics and Finance (July 2004), 2007 Summer Meetings of the Econometric Society, HEC Montréal, Hitotsubashi University, University of Tokyo, Kobe University, Federal Reserve Bank of Atlanta's macro lunch workshop, Yonsei University, Workshop on Methods and Applications for DSGE Models at the Federal Reserve Bank of Cleveland (October 2008), 2008 Reserve Bank of Australia Research Workshop, "Monetary Policy in Open Economies", Federal Reserve Bank of Philadelphia, Northern Illinois University, System Macro Committee Meeting at the Federal Reserve Bank of San Francisco (May 2009), Hong Kong University of Science and Technology, Keio University, Tohoku University, Osaka University, and Johns Hopkins University for useful discussions. An early version of this paper circulated as, "Business Cycle Implications of Internal Habit Formation". The first author wishes to thank the Ministry of Education, Culture, Sports, Science and Technology of the Government of Japan for support from a grant-in-aid for scientific research (number 20730205). The views in this paper represent those of the authors and are not those of the Federal Reserve Bank of Philadelphia or the Federal Reserve System.

1. Introduction

It is a folk theorem of macroeconomics that dynamic stochastic general equilibrium (DSGE) models are refuted by a sufficiently rich description of aggregate fluctuations. This widely held belief stands in contrast to evaluation strategies that rely on the entire predictive density of a DSGE model. The tension between econometric evaluation of DSGE models and the folk theorem is that the latter implies the former is bound to fail. The issue remains that progress on DSGE models requires methods that evaluate fit on actual data.

This paper contributes to DSGE model research by evaluating the impact of consumption habit on propagation and monetary transmission in new Keynesian (NK)DSGE model using Bayesian Monte Carlo tools. Consumption habit is known to be successful at closing the distance between real business cycle models and aggregate quantity and asset price moments since Boldrin, Christiano, and Fisher (2001). Analysis by Eichenbaum and Hansen (1990) and Heaton (1995) suggest that habit achieves this success because it imposes costs on household utility that induces intertemporal complementarity in consumption. Given intertemporal complementarity and a positive consumption shock, households respond by substituting from current to future consumption. The stronger is habit the further consumption is pushed into the future and spread across more future dates given the shock.

We quantify this intuition by linearizing a one-period bond Euler equation in which consumption habit drives marginal utility. Solving the linearized Euler equation yields a first-order stochastic difference equation that generates a hump-shaped consumption growth response to a real rate shock that has a higher peak and is more persistent the stronger is habit.

A goal of this paper is to assess the extent to which this consumption habit mechanism affects propagation and monetary transmission in and the fit of NKDSGE models. The role of consumption habit in NKDSGE model propagation and monetary transmission is not settled. For example, Del Negro, Schorfheide, Smets, and Wouters (2007) find that consumption habit contributes to a NKDSGE model matching the hump-shaped output response to an interest rate rule shock, but Christiano, Eichenbaum, and Evans (2005) do not using a money growth shock. Christiano, Eichenbaum, and Evans (CEE) also report that their monetary policy shock is transmitted by sticky wages, but is not by sticky prices. In contrast, Del Negro and Schorfheide (2008) argue that Bayesian methods and aggregate data cannot discern whether sticky prices or sticky wages matter more for the fit of a NKDSGE model with consumption habit. However Dupor, Han, and Tsai (2009) obtain results that point to flexible prices and durability in consumption, instead of habit, by applying the CEE impulse response matching estimator to a NKDSGE model identified by productivity shocks in place of monetary policy shocks. Finally, little attention is paid to the disparate effects money growth and interest rate rules have on monetary transmission and the fit of NKDSGE models.

This paper reports that consumption habit matters for the fit of NKDSGE models. The evidence is garnered by evaluating 12 NKDSGE models with a Bayesian approach, advocated by Geweke (2010), that shuns estimation. He builds on methods pioneered by DeJong, Ingram, and Whiteman (1996) that employ Bayesian Monte Carlo simulations to gauge the fit of DSGE models to population moments.[1] This approach is labeled the minimal econometric interpretation (MEI) by Geweke to indicate neither an explicit dependence on likelihood-based estimators nor on estimators that focus on a subset of the potential universe of sample moments. A problem for these estimators is that confronting the predictive density of a DSGE model with a sufficiently large vector of sample moments, negates the model according to the folk theorem.[2] The MEI acknowledges that because a NKDSGE model is a partial depiction of economic behavior it has no predictive implications for sample moments.[3] Rather a NKDSGE model can only be judged on its population moments. The tie between prior distributions of population moments generated

[1] Kano (2009) and Nason and Rogers (2006) use Bayesian Monte Carlo simulation methods to examine the fit of small open economy-DSGE models on current account moments.

[2] The MEI differs from the limited information approach of CEE and the Smets and Wouters (2007) application of Bayesian likelihood methods. These estimators are proven useful, but do not guarantee problem free evaluation of NKDSGE models as noted by Del Negro and Schorfheide (2008), Schorfheide (2008), Canova and Sala (2009), Dupor, Han, and Tsai (2009), Iskrev (2010), and Guerron-Quintana (2010) among others.

[3] Geweke (2010) also distinguishes the MEI from prior predictive analysis. Prior predictive analysis relies on economic models having testable implications for sample moments.

by a NKDSGE model and observable data are econometric models that are also partial depictions of economic behavior and yield posterior distributions of population moments.[4]

We adapt the MEI and Bayesian Monte Carlo tools to gauge the fit of NKDSGE models with and without consumption habit on permanent and transitory output and consumption growth spectral densities. Our choice of these moments is guided by the permanent income hypothesis (PIH) and previous business cycle studies. The PIH predicts consumption growth has a flat spectral density, which Galí (1991) notes is at odds with U.S. data. Cogley and Nason (1995b) observe that DSGE models often cannot reproduce the spectral density of U.S. output growth because it peaks in the business cycle frequencies. Also they find, along with Nason and Cogley (1994), that many DSGE models fail to duplicate output's response to permanent and transitory shocks. Thus this paper confronts NKDSGE models with moments other DSGE models have problems replicating, but are necessary for NKDSGE models to match to be counted empirically relevant.

This paper addresses these issues by evaluating the fit of 12 NKDSGE models. We start with a baseline NKDSGE model that has sticky prices and wages similar to those studied by CEE and Smets and Wouters (2007). From this baseline, two NKDSGE models are created by stripping out one or the other nominal rigidity. Baseline, sticky price, and sticky wage NKDSGE models are endowed with household preferences that have either no consumption habit or internal consumption habit. These six NKDSGE models are doubled by defining monetary policy with either a money growth or an interest rate rule.

Judging fit on population permanent and transitory output and consumption growth spectral densities generates evidence about propagation and monetary transmission in NKDSGE models. We gather this evidence by studying the interplay of consumption habit with sticky prices, sticky wages, permanent total factor productivity (TFP) shocks, and transitory money

[4] Although a virtue of the MEI is that it avoids the problems Guerron-Quintana (2010) encounters about choosing the observables on which a NKDSGE model is estimated, intrinsic to Bayesian estimation is a step to update model parameters that is absent from and is a weakness of the MEI. The lack of parameter updating can bias MEI measures of model fit if priors are badly constructed, but poorly formed priors are also an issue when posteriors are used to compare the fit of models.

growth or interest rate rule shocks. These structural shocks meet the requirements of long-run monetary neutrality (LRMN) and the Blanchard and Quah (1989) decomposition. We invoke LRMN and the Blanchard and Quah (BQ) decomposition to map from a VAR of output growth-inflation or consumption growth-inflation to a structural vector moving average (SVMA) in actual and synthetic data. Under LRMN, an output growth-inflation (consumption growth-inflation) SVMA predicts a vertical long-run supply curve (PIH-consumption function). According to the BQ decomposition, these mappings also impose orthogonal shock innovations on the SVMAs that the NKDSGE models identify as TFP and monetary policy shocks. Thus, we assign to the SVMAs the task of computing permanent and transitory output and consumption growth spectral densities because the MEI recognizes that these econometric models connect observed data to population versions of these moments predicted by the NKDSGE models.

The Bayesian Monte Carlo experiments show that the fit of NKDSGE models to permanent and transitory output and consumption growth spectral densities is improved by including consumption habit. Thus, propagation and monetary transmission in NKDSGE models is more empirically relevant when consumption habit is combined with nominal rigidities. However, we find that NKDSGE model fit is sensitive to: (1) changes in the mix of nominal rigidities, (2) switching from a money growth rule to an interest rate rule, (3) identifying spectral densities on permanent TFP shocks instead of transitory monetary policy shocks, and (4) conducting evaluation on the entire spectrum rather than limiting it to the business cycle frequencies.

The rest of the paper is constructed as follows. Section 2 discusses internal consumption habit and NKDSGE models. Our application of the MEI to NKDSGE model evaluation is outlined in section 3. Results appear in section 4. Section 5 concludes.

2. Internal Consumption Habit and NKDSGE Models

This section describes household preferences with internal consumption habit, studies internal consumption habit propagation, connects it to intertemporal complementarity in future near-dated consumption, and sketches the baseline NKDSGE model.

2.1 *Internal consumption habit*

Consumption habit is often superinduced in DSGE models to improve fit.[5] This paper adopts additive internal consumption habit. Internal habit operates on lagged household consumption, unlike external habit which assume lags of aggregate consumption appear in utility, of which the (multiplicative) 'catching-up-with-the-Joneses' specification of Abel (1990) is typical. The model assumes that household preferences are intertemporally separable as well as separable across (net) consumption flow, labor disutility, and real balances

$$\mathcal{U}\left(c_t,\, c_{t-1},\, n_t,\, \frac{H_t}{P_t}\right) \;=\; \ln[c_t \,-\, hc_{t-1}] \,-\, \frac{\gamma}{1+\gamma} n_t^{1+\frac{1}{\gamma}} \,+\, \ln\left[\frac{H_t}{P_t}\right], \tag{1}$$

where c_t, n_t, γ, H_t, and P_t are household consumption, household labor supply, the strictly positive Frisch labor supply elasticity, household cash at the end of date $t-1$, and the aggregate price level, respectively. Since internal habit ties current consumption choice to date $t-1$ consumption for a household, the marginal utility of consumption is forward-looking,

$$\lambda_t = \frac{1}{c_t - hc_{t-1}} - \mathbf{E}_t\left\{\frac{\beta h}{c_{t+1} - hc_t}\right\},$$

assuming $0 < c_t - hc_{t-1}$ for all t, where the habit parameter $h \in (0,\, 1)$, the household discount factor $\beta \in (0,\, 1)$, and $\mathbf{E}_t\{\cdot\}$ is the mathematical expectation operator given date t information.[6]

2.2 *The internal consumption habit propagation mechanism*

Forward-looking marginal utility suggests internal habit acts as propagation mechanism for consumption. We study this mechanism with a log linear approximation of the Euler equa-

[5]Consumption habit is first grafted into a growth model by Ryder and Heal (1973). Nason (1988), Sundaresan (1989), and Constantinides (1990) are early attempts at solving risk-free rate and equity premium puzzles with consumption habit. Pollak (1976) shows that long-run utility with linear habit describes long-run behavior rather than long-run preferences. Rozen (2010) gives an axiomatic treatment of linear intrinsic habit. An excellent survey of habit in macro and finance is Schmitt-Grohé and Uribe (2007); also see Nason (1997).

[6]Eichenbaum and Hansen (1990) and Heaton (1995) estimate consumption-based asset pricing models with habit and local substitution through service flows. The adjustment cost hypothesis is rejected in favor of services flows according to their estimates. However, the data support habit if local substitutability operates at lower frequencies than the sampling frequency of consumption.

tion $\lambda_t = \beta\mathbf{E}_t\left\{\lambda_{t+1}R_{t+1}/(1+\pi_{t+1})\right\}$, where R_t is the nominal rate and $1+\pi_{t+1}$ $(= P_{t+1}/P_t)$ is date $t+1$ inflation. The log linear approximation gives a second order stochastic difference equation for demeaned consumption growth, $\widetilde{\Delta c}_t$, whose solution is

$$\widetilde{\Delta c}_t \;=\; \varphi_1\widetilde{\Delta c}_{t-1} \;+\; \frac{\Psi}{\varphi_2}\sum_{j=0}^{\infty}\varphi_2^{-j}\mathbf{E}_t\tilde{q}_{t+j}, \tag{2}$$

where the stable and unstable roots are $\varphi_1 = h\alpha^{*-1}$ and $\varphi_2 = \alpha^*(\beta h)^{-1}$, α^* is the steady state growth rate of the economy, the demeaned real rate is $\tilde{q}_t = \tilde{R}_t - \frac{\pi^*}{1+\pi^*}\tilde{\pi}_t$, π^* is mean inflation, and Ψ is a constant that is nonlinear in model parameters.[7]

We analyze internal consumption habit propagation using the solved linearized Euler equation (2). This is depicted in figure 1 with impulse response functions (IRFs) generated by equation (2) and a one percent shock to $\tilde{q}_t$. The calibration sets $[\beta\ \alpha^*]' = [0.993\ \ \exp(0.004)]'$ and $\tilde{q}_t$ to a quarterly first-order autoregression, AR(1), with a AR1 coefficient of 0.87.[8] We compute IRFs on the grid $h = [0.15\ 0.35\ 0.50\ 0.65\ 0.85]$. The IRFs drive $\widetilde{\Delta c}_t$ higher at impact as shown in Figure 1. However, its response falls from about one to 0.11 percent as h rises from 0.15 to 0.85. Figure 1 also displays IRFs that are shifted to the right with higher peaks and slower decay rates as h increases. Thus, as internal habit becomes stronger, it dictates greater intertemporal complimentarity that persuades the household to move in tandem long and longer sequences of future near-dated consumption.

The internal consumption habit propagation mechanism is also discussed by CEE. They note that in their NKDSGE model, in which h is estimated to be about 0.65, internal consumption habit generates a hump-shaped consumption response to a nominal shock. Figure 1 reveals a similar internal consumption habit propagation mechanism for equation (2). When $h \geq 0.5$, equation (2) produces a humped-shaped IRF with a peak at or beyond two quarters. This

[7] The appendix constructs equation (2), which assumes a unit root TFP shock drives trend consumption.

[8] The real demeaned federal funds rate $\tilde{q}_t$ equals the quarterly nominal federal funds rate net of implicit GDP deflator inflation multiplied by the ratio of its mean to one plus its mean. The SIC selects a AR(1) for $\tilde{q}_t$ over any lag length up to ten on a 1954Q1–2002Q4 sample. The appendix has details.

mechanism contrasts with $h \in (0, 0.5)$ or the non-habit model, $h = 0$, in which a linear approximation of the Euler equation sets $\mathbf{E}_t\left\{\widetilde{\Delta c}_{t+1} - \widetilde{q}_{t+1}\right\} = 0$. Given $h \leq 0.5$, figure 1 indicates that consumption growth dynamics are dominated by the time series properties of $\widetilde{q}_t$.

Greater risk aversion is often cited as the reason that consumption habit is a useful real rigidity to improve model fit. This explanation is bound up with consumption habit lowering the (local) elasticity of substitution. An equivalent notion is that consumption habit imposes costs on utility when consumption is substituted intertemporally. As h rises, a household views changes in current consumption as costly for future utility. These costs induce the household to treat near-dated consumption as complements rather than substitutes. According to figure 1, habit switches consumption from an intertemporal substitute to complement which creates an economically important propagation mechanism given $h \in (0.5, 1)$.

This paper studies the implications of internal consumption habit for NKDSGE models. Nonetheless, the results of this paper should extend beyond internal consumption habit to external habit. In the appendix, we show that internal and external habit produce equivalent consumption growth IRFs after impact given $\widetilde{q}_t$ is a AR(1).[9] This indicates little generality is lost by focusing on internal consumption habit.

2.3 A new Keynesian DSGE model

The baseline NKDSGE model contains (a) internal consumption habit, (b) capital adjustment costs, (c) variable capital utilization, (d) fully indexed Calvo-staggered price setting by monopolistic final goods firms, and (e) fully indexed Calvo-staggered wage setting by monopolistic households with heterogeneous labor supply. Households reside on the unit circle with addresses $\ell \in [0, 1]$. The budget constraint of household ℓ is

$$\frac{H_{t+1}}{P_t} + \frac{B_{t+1}}{P_t} + c_t + x_t + a(u_t)k_t + \tau_t \;=\; r_t u_t k_t + \frac{W_t(\ell)}{P_t} n_t(\ell) + \frac{H_t}{P_t} + R_t \frac{B_t}{P_t} + \frac{D_t}{P_t}, \qquad (3)$$

where B_{t+1} is the stock of government bonds the household carries from date t into date $t+1$,

[9]The observational equivalence extends to multiplicative internal and external consumption habit using the onto mapping from additive to multiplicative consumption habit parameters that Dennis (2009) constructs.

x_t is investment, k_t is capital owned by the household at the end of date $t-1$, τ_t is a lump sum government transfer, r_t is the real rental rate of k_t, $W_t(\ell)$ is the nominal wage paid to household ℓ, R_t is the nominal return on B_t, D_t is dividends received from firms, $u_t \in (0, 1)$ is the capital utilization rate, and $a(u_t)$ is its cost function. At the steady state, $u^* = 1$, $a(1) = 0$. To achieve determinate solutions, we set $\dfrac{a''(1)}{a'(1)} = 1.174$. Note that u_t forces household ℓ to forgo $a(\cdot)$ units of consumption per unit of capital. The adjustment costs specification is adapted from CEE, which places it into the law of motion of household capital

$$k_{t+1} = (1-\delta)k_t + \left[1 - S\left(\frac{1}{\alpha}\frac{x_t}{x_{t-1}}\right)\right]x_t, \quad \delta \in (0, 1), \quad 0 < \alpha, \tag{4}$$

where δ is the capital depreciation rate and α ($= \ln \alpha^*$) is deterministic TFP growth. The cost function $S(\cdot)$ is strictly convex, where $S(1) = S'(1) = 0$ and $S''(1) \equiv \varpi > 0$. In this case, the steady state is independent of the adjustment cost function $S(\cdot)$.

Given k_0, B_0, and c_{-1}, the expected discounted lifetime utility function of household ℓ

$$\mathbf{E}_t\left\{\sum_{i=0}^{\infty}\beta^i \mathcal{U}\left(c_{t+i}, c_{t+i-1}, n_{t+i}(\ell), \frac{H_t}{P_t}\right)\right\} \tag{5}$$

is maximized by choosing c_t, k_{t+1}, H_{t+1}, B_{t+1}, and $W_t(\ell)$ subject to period utility (1), budget constraint (3), the law of motion of capital (4), and downward sloping labor demand.

Households offer differentiated labor services to firms in a monopolistic market in which a Calvo staggered nominal wage mechanism operates. We assume the labor supply aggregator is $N_t = \left[\int_0^1 n_t(\ell)^{(\theta-1)/\theta}d\ell\right]^{\theta/(\theta-1)}$, where θ is the wage elasticity. Labor market monopoly imposes downward sloping labor demand schedules for differentiated labor services, $n_t(\ell) = \left[W_t/W_t(\ell)\right]^{\theta}N_t$, on firms, where the nominal wage index is $W_t = \left[\int_0^1 W_t(\ell)^{1-\theta}d\ell\right]^{1/(1-\theta)}$. The nominal wage aggregator is $W_t = \left[(1-\mu_W)W_{c,t}^{1-\theta} + \mu_W\left(\alpha^*\pi_{t-1}W_{t-1}\right)^{1-\theta}\right]^{1/(1-\theta)}$, which has households updating their desired nominal wage $W_{c,t}$ at probability $1-\mu_W$. With probability μ_W, households receive the date $t-1$ nominal wage indexed by steady state TFP growth, α^*, and π_{t-1}. In this case, the optimal nominal wage condition is

$$\left[\frac{W_{c,t}}{P_{t-1}}\right]^{1+\theta/\gamma} = \left(\frac{\theta}{\theta-1}\right) \frac{\mathbf{E}_t \sum_{i=0}^{\infty} \left[\beta\mu_W \alpha^{*-\theta(1+1/\gamma)}\right]^i \left[\left[\frac{W_{t+i}}{P_{t+i-1}}\right]^{\theta} N_{t+i}\right]^{1+1/\gamma}}{\mathbf{E}_t \sum_{i=0}^{\infty} \left[\beta\mu_W \alpha^{*(1-\theta)}\right]^i \lambda_{t+i} \left[\frac{W_{t+i}}{P_{t+i-1}}\right]^{\theta} \left[\frac{P_{t+i}}{P_{t+i-1}}\right]^{-1} N_{t+i}}, \tag{6}$$

because household ℓ solves a fully indexed Calvo-pricing problem. Equation (6) smooths nominal wage growth which forces labor supply to absorb TFP and monetary policy shocks conditional on the Frisch elasticity γ. Output and consumption respond because changes in labor supply alter production and the intra- and intertemporal margins of NKDSGE models.

Monopolistically competitive firms produce final goods that households consume. The consumption aggregator is $c_t = \left[\int_0^1 y_{D,t}(j)^{(\xi-1)/\xi} dj\right]^{\xi/(\xi-1)}$, where $y_{D,t}(j)$ is household final good demand for the output of a firm with address j on the unit interval. Final good firm j maximizes its profits by setting its price $P_t(j)$, subject to $y_{D,t}(j) = [P_t/P_t(j)]^{\xi} Y_{D,t}$, where ξ is the price elasticity, $Y_{D,t}$ is aggregate demand, and the price index is $P_t = \left[\int_0^1 P_t(j)^{1-\xi}\right]^{1/(1-\xi)}$.

The jth final good firm mixes capital, $K_t(j)$, rented and labor, $N_t(j)$, hired from households net of fixed cost N_0 given labor-augmenting TFP, A_t, in the constant returns to scale technology, $\left[u_t K_t(j)\right]^{\psi}\left[\left[N_t(j) - N_0\right] A_t\right]^{1-\psi}$, $\psi \in (0, 1)$, to create output, $y_t(j)$. Fixed labor cost N_0 is included to satisfy the needs of monopolistic competition in the final goods market. TFP is a random walk with drift, $A_t = A_{t-1}\exp\{\alpha + \varepsilon_t\}$, with its Gaussian innovation, $\varepsilon_t \sim \mathcal{N}(0, \sigma_\varepsilon^2)$, for the NKDSGE models to have a permanent shock.

Calvo-staggered price setting restricts a firm to update to optimal price $P_{c,t}$ at probability $1 - \mu_P$. Or with probability μ_P, firms are stuck with date $t-1$ prices scaled by inflation of the same date, π_{t-1}. This fixes the price aggregator $P_t = \left[(1-\mu_P)P_{c,t}^{1-\xi} + \mu_P\left(\pi_{t-1}P_{t-1}\right)^{1-\xi}\right]^{1/(1-\xi)}$. Under full price indexation, Calvo-pricing yields the optimal forward-looking price

$$\frac{P_{c,t}}{P_{t-1}} = \left(\frac{\xi}{\xi - 1}\right) \frac{\mathbf{E}_t \sum_{i=0}^{\infty} \left(\beta\mu_P\right)^i \lambda_{t+i}\phi_{t+i} Y_{D,t+i}\pi_{t+i}^{\xi}}{\mathbf{E}_t \sum_{i=0}^{\infty} \left(\beta\mu_P\right)^i \lambda_{t+i} Y_{D,t+i}\pi_{t+i}^{\xi-1}} \tag{7}$$

of a firm able to update its price. Under full price indexation, equation (7) implies restrictions that smooth inflation. Inflation smoothing forces the economy's response to shocks onto output and consumption, among other quantity variables. Along with habit inducing intertemporal complementarity in consumption, inflation and nominal wage growth smoothing are potential sources of propagation and monetary transmission in NKDSGE models.

We close the NKDSGE model with one of two monetary policy rules. CEE identify monetary policy with a money growth process that is a MA(∞). As they note, this MA(∞) is equivalent to the AR(1) money growth $\left(\ln M_{t+1}/M_t = m_{t+1}\right)$ supply rule

$$m_{t+1} = (1 - \rho_m)m^* + \rho_m m_t + \mu_t, \quad \left|\rho_m\right| < 1, \quad \mu_t \sim \mathcal{N}\left(0, \sigma_\mu^2\right), \tag{8}$$

where m^* is mean money growth and μ_t is the money growth innovation. We use NKDSGE-MG to label models with the money growth rule (8). The mnemonic NKDSGE-TR refers to models in which monetary policy is described with the Taylor rule

$$(1 - \rho_R\mathbf{L})R_t = (1 - \rho_R)\left(R^* + a_\pi \mathbf{E}_t\pi_{t+1} + a_{\widetilde{Y}}\widetilde{Y}_t\right) + \upsilon_t, \quad \left|\rho_R\right| < 1, \quad \upsilon_t \sim \mathcal{N}\left(0, \sigma_\upsilon^2\right), \tag{9}$$

where $R^* = \pi^*/\beta$ and $\pi^* = \exp(m^* - \alpha)$. Under the interest rate rule (9), the monetary authority obeys the 'Taylor' principle, $1 < a_\pi$, and sets $a_{\widetilde{Y}} \in (0, 1)$. This policy regime assumes the monetary authority computes private sector inflationary expectations, $\mathbf{E}_t\pi_{t+1}$, and mean zero transitory output, $\widetilde{Y}_t$, without inducing measurement errors.

The government finances B_t, interest on B_t, and a lump-sum transfer τ_t with new bond issuance $B_{t+1} - B_t$, lump-sum taxes τ_t, and money creation, $M_{t+1} - M_t$. Under either monetary policy rule, the government budget constraint is $P_t\tau_t = [M_{t+1} - M_t] + [B_{t+1} - (1 + R_t)B_t]$. We assume government debt is in zero net supply, $B_{t+1} = 0$ and the nominal lump-sum transfer equals the monetary transfer, $P_t\tau_t = M_{t+1} - M_t$, along the equilibrium path at all dates t.

Equilibrium requires goods, labor, and money markets to clear in the decentralized economy. This occurs when $K_t = k_t$ given $0 < r_t$, $N_t = n_t$ given $0 < W_t$, $M_t = H_t$, and also requires P_t, and R_t are strictly positive and finite. This leads to the aggregate resource constraint, $Y_t = C_t + I_t + a(u_t)K_t$, where aggregate consumption $C_t = c_t$ and aggregate investment $I_t = x_t$. A rational expectations equilibrium equates, on average, firm and household subjective forecasts of r_t and A_t to the objective outcomes generated by the decentralized economy. We add to this list μ_t and R_t, υ_t, P_t, or W_t under the money growth rule (8), the interest rate rule (9), a flexible price regime, or a competitive labor market, respectively.

3. Bayesian Monte Carlo Strategy

This section outlines the Bayesian Monte Carlo methods of DeJong, Ingram, and Whiteman (1996) and Geweke (2010). We adapt their procedures to assess the fit of 12 NKDGSE models on permanent and transitory output and consumption growth spectral densities. DeJong, Ingram, and Whiteman (DIW) and Geweke eschew standard calibration and estimation tools because, in their view, a DSGE model lacks predictions except for population moments. Geweke calls this the minimal econometric interpretation (MEI). We engage the MEI to evaluate NKDSGE models on population spectral densities generated from Bayesian Monte Carlo experiments. One set of experiments apply sample data, a structural vector moving average (SVMA), its priors, and a Markov chain Monte Carlo (MCMC) simulator to create posterior distributions of population spectral densities. Prior distributions of population spectral densities are approximated using a SVMA estimated on synthetic data that are simulated from a calibrated NKDSGE model whose parameters are drawn from independent priors.[10] The SVMAs are the econometric models that connect posterior and prior population moments to sample data. Posterior and prior population spectral densities are labeled empirical and theoretical spectral densities, $SD_{\mathcal{E}}$ and $SD_{\mathcal{T}}$, in the rest of the paper. The MEI gauges NKDSGE model fit on the overlap of distributions of permanent and transitory output and consumption growth $SD_{\mathcal{E}}$ and $SD_{\mathcal{T}}$.[11] Kolmogorov-Smirnov goodness of fit statistics give a concise measure of this overlap. Table 1 summarizes our implementation of the MEI to evaluate 12 NKDSGE models.

3.1 Output and consumption moments

We evaluate NKDSGE model fit with a vector of moments consisting of permanent and transitory output and consumption growth spectral densities. The spectral densities are calculated from SVMAs that are just-identified by the orthogonality of shock innovations along with a LRMN restriction embedded in the NKDSGE model of section 2. In this model, LRMN ties the TFP innovation ε_t to the permanent shock. The transitory shock is identified with the money growth innovation μ_t or Taylor rule innovation υ_t. Under LRMN, we recover SVMAs from unrestricted second-order VARs, VAR(2)s, of $[\Delta \ln Y_t \;\; \Delta \ln P_t]'$ and $[\Delta \ln C_t \;\; \Delta \ln P_t]'$ subsequent to applying the Blanchard and Quah (1989) decomposition.[12] A vertical long-run aggregate supply curve results from applying LRMN to the $\Delta \ln Y_t$–$\Delta \ln P_t$ system. The $\Delta \ln C_t$–$\Delta \ln P_t$ system represents a serially correlated demand-consumption function system giving rise to a vertical long-run PIH-consumption function assuming LRMN.

[10] Geweke (2010) develops the MEI by conditioning prior distributions of moments just on an economic model and its priors. Since LRMN and the assumptions of the BQ decomposition are built into the NKDSGE models, a SVMA(∞) can be recovered from the approximate linearized solution of a NKDSGE model. We choose instead to construct distributions of $SD_{\mathcal{T}}$s from SVMA(∞)s estimated on data simulated from linearized NKDSGE models. This approach is consistent with the MEI because population SVMA(∞)s are recovered with sufficiently long synthetic samples.

[11] The overlap of $SD_{\mathcal{E}}$ and $SD_{\mathcal{T}}$ distributions expresses the posterior odds, say, of a NKDSGE model with consumption habit against a NKDSGE model that lacks it. The favored NKDSGE model generates a prior distribution of a $SD_{\mathcal{T}}$ that better covers the posterior distribution of the relevant $SD_{\mathcal{E}}$.

[12] Blanchard and Quah (1989) include an appendix with a theorem that establishes necessary and sufficient conditions under which bivariate ARs identify the correct responses to a permanent shock and a transitory shock when truth is there are several permanent and transitory shocks. The theorem states that the BQ decomposition is satisfied when responses, say, of output growth and inflation to either permanent or transitory shocks are equivalent up to a scalar lag operator. Since the shocks that often appear in NKDSGE models are AR(1)s, adding these shocks to a NKDSGE model will not create spurious identification according to the theorem.

As an example consider the SVMA

$$\begin{bmatrix} \Delta \ln Y_t \\ \Delta \ln P_t \end{bmatrix} = \sum_{j=0}^{\infty} \mathbb{B}_j \begin{bmatrix} \varepsilon_{t-j} \\ \upsilon_{t-j} \end{bmatrix}, \quad \text{where} \quad \mathbb{B}_j = \begin{bmatrix} \mathbb{B}_{\Delta Y,\varepsilon,j} & \mathbb{B}_{\Delta Y,\upsilon,j} \\ \mathbb{B}_{\Delta P,\varepsilon,j} & \mathbb{B}_{\Delta P,\upsilon,j} \end{bmatrix}, \tag{10}$$

that equates the monetary policy shock with the Taylor rule innovation υ_t. Elements of $\mathbb{B}_j$ are just-identified (i) by the orthogonality of the TFP shock innovation ε_t and υ_t and (ii) by the LRMN restriction $\mathbb{B}_{\Delta Y,\upsilon}(\mathbf{1}) = 0$ (*i.e.*, output is independent of υ_t at the infinite horizon); see the appendix for details. These restrictions permit the SVMA (10) to be decomposed for output growth into univariate SMA(∞)s, $\mathbb{B}_{\Delta Y,\varepsilon}(\mathbf{L})\varepsilon_t$ and $\mathbb{B}_{\Delta Y,\upsilon}(\mathbf{L})\upsilon_t$. The former (latter) SMA(∞) is the IRF of output growth with respect to the permanent shock ε_t (transitory shock υ_t).

We grab the SMA(∞) of $\mathbb{B}_{\Delta Y,\varepsilon}(\mathbf{L})\varepsilon_t$ and $\mathbb{B}_{\Delta Y,\upsilon}(\mathbf{L})\upsilon_t$ from the SVMA (10) to calculate permanent and transitory output growth spectral densities. Since the SVMA (10) is also a Wold representation of $[\Delta \ln Y_t \;\; \Delta \ln P_t]'$, its spectrum (at frequency ω) is computed as $SD_{[\Delta Y \;\; \Delta P]}(\omega) = (2\pi)^{-1}\Gamma_{[\Delta Y \;\; \Delta P]} \exp(-i\omega)$, where $\Gamma_{[\Delta Y \;\; \Delta P]}(l) = \sum_{j=0}^{\infty} \mathbb{B}_j \mathbb{B}'_{j-l}$. The convolution $\Gamma_{[\Delta Y \;\; \Delta P]}(l)$ is expanded at horizon j to obtain

$$\mathbb{B}_j \mathbb{B}'_{j-l} = \begin{bmatrix} \mathbb{B}_{\Delta Y,\varepsilon,j}\mathbb{B}_{\Delta Y,\varepsilon,j-l} + \mathbb{B}_{\Delta Y,\upsilon,j}\mathbb{B}_{\Delta Y,\upsilon,j-l} & \mathbb{B}_{\Delta Y,\varepsilon,j}\mathbb{B}_{\Delta P,\varepsilon,j-l} + \mathbb{B}_{\Delta Y,\upsilon,j}\mathbb{B}_{\Delta P,\upsilon,j-l} \\ \mathbb{B}_{\Delta P,\varepsilon,j}\mathbb{B}_{\Delta Y,\varepsilon,j-l} + \mathbb{B}_{\Delta P,\upsilon,j}\mathbb{B}_{\Delta Y,\upsilon,j-l} & \mathbb{B}_{\Delta P,\varepsilon,j}\mathbb{B}_{\Delta P,\varepsilon,j-1} + \mathbb{B}_{\Delta P,\upsilon,j}\mathbb{B}_{\Delta P,\upsilon,j-l} \end{bmatrix},$$

whose off-diagonal elements imply output growth and employment cross-covariances and, therefore, co- and quad-spectra, while the upper left diagonal elements contain output growth autocovariances $\mathbb{B}_{\Delta Y,\varepsilon,j}\mathbb{B}_{\Delta Y,\varepsilon,j-l}$ ($\mathbb{B}_{\Delta Y,\upsilon,j}\mathbb{B}_{\Delta Y,\upsilon,j-l}$) with respect to ε_t (υ_t), the identified permanent (transitory) shock.[13] We exploit the SMAs $\mathbb{B}_{\Delta Y,\varepsilon}(\mathbf{L})\varepsilon_t$ and $\mathbb{B}_{\Delta Y,\upsilon}(\mathbf{L})\upsilon_t$, that are along the diagonal, to parameterize permanent and transitory output growth spectral densities, which extends ideas of Akaike (1969) and Parzen (1974). Given the BQ decomposition assumption

[13] The appendix constructs SVMAs from structural VARs and also reports that across all Bayesian Monte Carlo simulation the 12 NKDSGE models satisfy the invertibility condition of Fernández-Villaverde, Rubio-Ramírez, Sargent, and Watson (2007).

that the structural shock innovations have unit variances, the output growth spectral density at frequency ω is

$$SD_{\Delta Y,\iota}(\omega) \ = \ \frac{1}{2\pi}\left|\mathbb{B}_{\Delta Y,\iota,0} + \mathbb{B}_{\Delta Y,\iota,1}e^{-i\omega} + \mathbb{B}_{\Delta Y,\iota,2}e^{-i2\omega} + \ldots + \mathbb{B}_{\Delta Y,\iota,j}e^{-ij\omega} + \ldots\right|^{2}, \quad \iota \ = \ \varepsilon,\, \upsilon.$$

Before computing $SD_{\Delta Y,\iota}(\omega)$, we truncate its polynomial at $j = 40$, a ten year horizon.

3.2 Bayesian simulation methods I: Empirical Distributions

We engage MCMC software of Geweke (1999) and McCausland (2004) to create posterior distributions of SVMAs. These posterior distributions consist of $\mathcal{J} = 5000$ SVMA parameter vectors that are grounded on unrestricted VAR(2)s, LRMN, the BQ decomposition, priors, and a $1954Q1$–$2002Q4$ sample ($T = 196$) of U.S. output, consumption, and price growth.[14] These $\mathcal{J}$ vectors are used to calculate distributions of posterior or empirical permanent and transitory output and consumption growth spectral densities, $SD_{\mathcal{E},\Delta C}$ and $SD_{\mathcal{E},\Delta C}$.

3.3 Bayesian simulation methods II: Theoretical Distributions

Several steps are needed to solve and simulate NKDSGE models. The models have a permanent TFP shock, which requires stochastic detrending of optimality and equilibrium conditions before log-linearizing around deterministic steady states that is described in the appendix. We engage an algorithm of Sims (2002), sketched in the appendix, to solve for linear approximate equilibrium laws of motion of a NKDSGE model. Synthetic samples result from feeding sequences of TFP and monetary policy shock innovations into these equilibrium laws of motion given initial conditions and draws from priors of NKDSGE model parameters.

Priors embed our uncertainty about NKDSGE model parameters that is reflected in distributions of prior or theoretical population permanent and transitory output and consumption growth spectral densities, $SD_{\mathcal{T},\Delta Y}$ and $SD_{\mathcal{T},\Delta C}$. Table 2 lists these priors. For example, h has an uninformative prior that is drawn from an uniform distribution with end points 0.05 and 0.95 in table 2. The uninformative prior reflects a belief that any $h \in \left[0.05,\, 0.95\right]$ is as likely

[14]The software is found at `http://www2.cirano.qc.ac/~bacc`, while the appendix describes the data.

as another. Non-habit NKDSGE models are defined by the degenerate prior $h = 0$.

Priors are also taken from earlier DSGE model studies.[15] We place degenerate priors on $\left[\beta\ \delta\ \alpha\ \psi\right]' = \left[0.9930\ \ 0.0200\ \ 0.0040\ \ 0.3500\right]'$ that are consistent with the Cogley and Nason (1995b) calibration. However, the micro estimates of Kimmel and Kniesner (1998) supply the mean of the prior of the Frisch labor supply elasticity, $\gamma = 1.55$. Uncertainty about $\left[\beta\ \gamma\ \delta\ \alpha\ \psi\right]'$ is captured by 95 percent coverage intervals, which include values in Nason and Cogley (1994), Hall (1996), and Chang, Gomes, and Shorfheide (2002). We set the prior of the investment cost of adjustment parameter ϖ to estimates reported by Bouakez, Cardia, and Ruge–Murcia (2005). The standard deviation of TFP shock innovations, σ_ϵ, is given an uniform prior because the DSGE literature suggests that any draw of σ_ϵ from [0.0070, 0.0140] is equally likely.

There are four sticky price and wage parameters to calibrate. The relevant prior means are $[\xi\ \ \mu_P\ \ \theta\ \ \mu_W]' = \left[12.0\ 0.55\ 15.0\ 0.7\right]'$. The mean of ξ implies a steady state price markup, $\xi/(\xi-1)$, of nine percent with a 95 percent coverage interval that runs from five to 33 percent. This coverage interval blankets estimates found in Basu and Fernald (1997) and CEE. More uncertainty surrounds the priors of μ_P, θ, and μ_W. Sbordone (2002), Nason and Slotsve (2004), Lindé (2005), and CEE suggest a 95 percent coverage interval for μ_P of $[0.45,\ 0.65]$. Likewise, a 95 percent coverage interval of $[0.04,\ 0.25]$ suggests substantial uncertainty around the seven percent prior mean household wage markup, $\theta/(\theta-1)$. The degenerate mean of μ_W and its 95 percent coverage interval reveals stickier nominal wages than prices, as found by CEE and Rabanal and Rubio-Ramírez (2005), but we imbue it with greater uncertainty.

The money growth rule (8) is calibrated to estimates from a $1954Q1$–$2002Q4$ sample of the monetary base. The point estimates are degenerate priors for $\left[m^*\ \ \rho_m\ \ \sigma_\mu\right]' = \left[0.011\ 0.628\ 0.006\right]'$. We give these prior means less precision than found in sample. For example, the lower end of the 95 percent coverage interval of ρ_m is near 0.46. CEE note that $\rho_m \approx 0.5$ implies the money growth rule (8) mimics the persistence of their MA(∞) monetary policy shock process.

[15]The means of several priors match sample means of the consumption-output ratio, labor input, federal funds rate, and inflation on a $1955Q1$–$2002Q4$ sample. We also fix $N_0 = 0.1678$ and $r^* = 1.0050$.

The calibration of the interest rate rule (9) obeys the Taylor principle and $a_{\mathcal{Y}} \in (0, 1)$. The degenerate prior of a_π is 1.80. We assign a small role to movements in transitory output, $\widetilde{Y}$, with a prior mean of 0.05 for $a_{\mathcal{Y}}$. The 95 percent coverage intervals of a_π and $a_{\mathcal{Y}}$ rely on estimates reported by Smets and Wouters (2007). The interest rate rule (9) is also calibrated to smooth R_t given a prior mean of 0.65 and a 95 percent coverage interval of [0.55, 0.74] that incorporates estimates found in Guerron-Quintana (2010). Ireland (2001) is the source of the prior mean of the standard deviation of the monetary policy shock, $\sigma_\upsilon = 0.0051$, and its 95 percent coverage interval, [0.0031, 0.0072]. We assume all shock innovations are uncorrelated at leads and lags (*i.e.*, $\mathbf{E}\{\varepsilon_{t+i}\,\upsilon_{t+q}\} = 0$, for all i, q).

Draws from the priors of the parameters of a NKDSGE model are applied to its linearized approximation to generate a synthetic sample of length $\mathcal{M} = \mathcal{W} \times T$. On the $\mathcal{J}$ synthetic samples of length $\mathcal{M}$, SVMAs are estimated subsequent to estimating unrestricted VAR(2)s, invoking LRMN and the assumptions of the BQ decomposition. We set $\mathcal{W} = 5$ to compute prior population permanent and transitory $SD_{\mathcal{T},\Delta Y}$ and $SD_{\mathcal{T},\Delta C}$.

3.4 Measures of fit

The fit of NKDSGE models is gauged with a tool that updates one Cogley and Nason (1995a) exploit. They measure the fit of DSGE models to the spectral density of U.S. output growth with the Kolmogorov-Smirnov (*KS*) goodness of fit statistic. The *KS* statistic is useful because it maps a multidimensional *SD* into a scalar statistic that summarizes model fit.

This paper employs the *KS* statistic to gauge NKDSGE model fit, but in the context of Bayesian Monte Carlo experiments. The experiments produce (posterior) empirical and (prior) theoretical distributions of *KS* statistics, $KS_{\mathcal{E}}$ and $KS_{\mathcal{T}}$, that are normalized on sample output or consumption growth spectral densities, $\widehat{SD}_T$, which is constructed from a SVMA estimated on actual data of length T. Define $\mathcal{R}_{\mathcal{D},j}(\omega) = \widehat{SD}_T(\omega)/SD_{\mathcal{D},j}(\omega)$ at replication j and frequency ω, where $\mathcal{D} = \mathcal{E}$, $\mathcal{T}$. Next, compute the partial sum $\mathcal{V}_{\mathcal{D},j}\left(2\pi q/\mathcal{H}\right) = 2\pi\mathcal{H}^{-1}\sum_{\ell=1}^{q}\mathcal{R}_{\mathcal{D},j}\left(2\pi\ell/\mathcal{H}\right)$, where $\mathcal{H} = T$ when $\mathcal{D} = \mathcal{E}$ and $\mathcal{H} = \mathcal{M}$ otherwise. The partial sums are used to construct the partial difference $\mathcal{B}_{\mathcal{D},j}(\kappa) = 0.5\pi^{-1}\sqrt{2\mathcal{H}}\Big[\mathcal{V}_{\mathcal{D},j}(\kappa\pi) - \kappa\mathcal{V}_{\mathcal{D},j}(\pi)\Big],\ \kappa \in [0, 1]$. The restriction placing κ on $[0, 1]$ requires that the partial difference $\mathcal{B}_{\mathcal{D},j}(\kappa)$ is evaluated on the entire spectrum. The $KS_{\mathcal{D}}$ statistic at replication j is calculated as the maximal absolute value of $\mathcal{B}_{\mathcal{D},j}(\kappa)$, $KS_{\mathcal{D},j} = Max_{\kappa\in[0,1]}\Big|\mathcal{B}_{\mathcal{D},j}(\kappa)\Big|$. The $KS_{\mathcal{E},j}$s and $KS_{\mathcal{T},j}$s statistics are collected into vectors of length $\mathcal{J}$ to form distributions of $KS_{\mathcal{E}}$ and $KS_{\mathcal{T}}$ statistics. Substantial overlap of these distributions indicate a good fit for a NKDSGE model. This constitutes a 'joint test' of NKDSGE model fit because the distribution of $SD_{\mathcal{T}}$ must match the distribution of $SD_{\mathcal{E}}$ at several frequencies for distributions of $KS_{\mathcal{E}}$ and $KS_{\mathcal{T}}$ statistics to display significant area in common.

DIW advocate using the confidence interval criterion (*CIC*) to quantify the intersection of $KS_{\mathcal{E}}$ and $KS_{\mathcal{T}}$ distributions. The *CIC* measures the fraction of $\mathcal{J}$ elements of a $KS_{\mathcal{T}}$ distribution that occupies an interval defined by lower and upper quantiles of the associated $KS_{\mathcal{E}}$ distribution given a $1 - p$ percent confidence level.[16] We set $p = 0.05$. If a habit NKDSGE model yields a $CIC > 0.3$ (as DIW imply in their analysis of RBC models), say, for the transitory output growth spectral density and the non-habit model's $CIC \leq 0.3$ on this moment, the former model is viewed as providing a more plausible match in this case.

We calculate $KS_{\mathcal{E},j}$ and $KS_{\mathcal{T},j}$ statistics on the entire spectrum and on business cycle horizons from eight to two years per cycle. By isolating the business cycle fluctuations, we build on an insight of Diebold, Ohanian, and Berkowitz (1998). Their insight is that a focus on the business cycle frequencies matters for NKDSGE model evaluation when model misspecification corrupts measurement of short- and long-run fluctuations. We address these problems by compiling $KS_{\mathcal{E}}$ and $KS_{\mathcal{T}}$ distributions in which κ is limited to frequencies between eight and two years per cycle, or $KS_{\mathcal{D},j} = Max_{\kappa\in[0.064,\,0.25]}\Big|\mathcal{B}_{T,\mathcal{D},j}(\kappa)\Big|$. This mitigates problems of discounting NKDSGE models that perform well at business cycle horizons, but poorly on the lower growth and higher short-run frequencies.

[16] Following DIW, the *CIC* of a $KS_{\mathcal{T}}$ statistic distribution is set to $\frac{1}{1-p}\int_a^b KS_{\mathcal{T},j}dj$ for a $1-p$ percent confidence level, where a (b) is the lower $0.5p$ (upper $1-0.5p$) quantile. The *CIC* is normalized by $1-p$ to equal $\int_a^b KS_{\mathcal{E},j}dj$.

4. Habit and Non-Habit NKDSGE Model Evaluation

This section judges the fit of 12 NKDGSE models to distributions of permanent and transitory $SD_{\mathcal{E},\Delta Y}$ and $SD_{\mathcal{E},\Delta C}$. The evaluation is based on the overlap of $KS_{\mathcal{E}}$ and $KS_{\mathcal{T}}$ statistic densities that are plotted in the second and third columns of figures 3–8 and quantified by *CIC* reported in table 3. Along with mean permanent and transitory $SD_{\mathcal{E},\Delta Y}$ and $SD_{\mathcal{E},\Delta C}$, mean permanent and transitory $SD_{\mathcal{T},\Delta Y}$ and $SD_{\mathcal{T},\Delta C}$ are presented in the first column of figures 3–8 to give information about propagation and monetary transmission in NKDSGE models.

4.1 *Summary of moments to match: Mean $SD_{\mathcal{E},\Delta Y}$ and $SD_{\mathcal{E},\Delta C}$*

Figure 2 plots mean permanent and transitory $SD_{\mathcal{E},\Delta Y}$ and $SD_{\mathcal{E},\Delta C}$. These *SD*s decompose the average variation frequency by frequency of the response of output and consumption growth to permanent and transitory shocks.[17] The top (bottom) panel of figure 2 contains mean permanent (transitory) $SD_{\mathcal{E},\Delta Y}$ and $SD_{\mathcal{E},\Delta C}$. Mean $SD_{\mathcal{E},\Delta Y}$ appear as solid (blue) lines in figure 2, while mean $SD_{\mathcal{E},\Delta C}$ plots are thicker with (blue) '♦' symbols.

Mean permanent $SD_{\mathcal{E},\Delta Y}$ and $SD_{\mathcal{E},\Delta C}$ display greatest variation or power at frequency zero (*i.e.*, long-run) in the top panel of figure 2. This is followed by immediate decay across the remaining frequencies. However, the mean permanent $SD_{\mathcal{E},\Delta Y}$ has about five times the amplitude (*i.e.*, volatility) at the long-run that is found in the mean permanent $SD_{\mathcal{E},\Delta C}$.

The lower panel of figure 2 presents mean transitory $SD_{\mathcal{E},\Delta Y}$ and $SD_{\mathcal{E},\Delta C}$ with disparate shapes. The latter *SD* peaks around six years per cycle. Rather than a peak, mean transitory $SD_{\mathcal{E},\Delta C}$ plateaus from the growth frequencies (*i.e.*, more than eight years per cycle) to four years per cycle before decaying in the high frequencies. At the business cycle frequencies, this plateau exhibits about 20 percent of the volatility of mean transitory $SD_{\mathcal{E},\Delta Y}$.

Mean permanent and transitory $SD_{\mathcal{E},\Delta Y}$ and $SD_{\mathcal{E},\Delta C}$ suggest the underlying empirical distributions pose challenges for NKDSGE models. Sufficient periodicity is displayed by mean $SD_{\mathcal{E},\Delta C}$s in figure 2 at low and business cycle frequencies to reject the PIH. Thus, NKDSGE

[17] A mean *SD* is computed across an ensemble of SD_j, $j = 1, \ldots, \mathcal{J}$ pointwise or frequency by frequency.

models must violate the PIH to generate distributions of permanent and transitory $SD_{\mathcal{T},\Delta C}$ that match distributions of $SD_{\mathcal{E},\Delta C}$s. Economically meaningful propagation and monetary transmission mechanisms are also needed by NKDSGE models to produce distributions of permanent and transitory $SD_{\mathcal{T},\Delta Y}$ that achieve a good fit to distributions of $SD_{\mathcal{E},\Delta Y}$.

4.2 Quantify NKDSGE model fit: CIC

Table 3 presents *CIC* that measure the overlap of $KS_{\mathcal{E}}$ and $KS_{\mathcal{T}}$ statistic distributions. The extent of the overlap of these distributions is a gauge of the fit of NKDSGE models to permanent and transitory $SD_{\mathcal{E},\Delta Y}$ and $SD_{\mathcal{E},\Delta C}$ distributions. The top panel of table 3 lists *CIC* of sticky price and wage (baseline), sticky price only (SPrice), and sticky wage only (SWage) habit and non-habit NKDSGE-MG models in which the money growth rule (8) defines monetary policy.[18] The lower panel contains *CIC* of NKDSGE-TR models that replace equation (8) with the Taylor rule (9). Columns titled $\infty : 0$ ($8 : 2$) include *CIC* quantifying the overlap of $KS_{\mathcal{E}}$ and $KS_{\mathcal{T}}$ statistic distributions on the entire frequency domain (business cycle frequencies that run from eight to two years per cycle).

Table 3 shows that placing consumption habit in NKDSGE models produces a superior fit. Of the 27 $CIC \geq 0.3$ listed in table 3, 18 are tied to habit NKDSGE models. This is twice as many as non-habit NKDSGE models produce. The SPrice habit NKDSGE-MG model generates four of the five $CIC \geq 0.3$ in the top panel of table 3. In the bottom panel of table 3, 14 of the 22 $CIC \geq 0.3$ are produced by baseline, SPrice, and SWage habit NKDSGE-TR models. SPrice habit NKDSGE-TR models enjoy six of these 14 *CIC*. The remaining eight $CIC \geq 0.3$ are divided evenly between baseline and SWage habit NKDSGE-TR models in the bottom panel of table 3.

A striking feature of table 3 is the disparate effects habit, sticky prices, and sticky wages have on the fit of NKDSGE models to distributions of permanent $SD_{\mathcal{E},\Delta Y}$ and $SD_{\mathcal{E},\Delta C}$. The Bayesian Monte Carlo experiments reveal that on these distributions only SPrice NKDSGE models yield $CIC \geq 0.3$. In the middle of the top panel of table 3, these matches occur for the

[18] The SWage NKDSGE model requires the degenerate prior $\mu_P = 0$ with fixed markup $\phi = (\xi - 1)/\xi$. When the nominal wage is flexible, households set their optimal wage period by period in SPrice NKDSGE models. In this case, the markup in the labor market is fixed at $(\theta - 1)/\theta$, which equals $n^{-1/\gamma}$, given $\mu_W = 0$.

SPrice habit NKDSGE-MG model on permanent $SD_{\mathcal{E},\Delta Y}$ and $SD_{\mathcal{E},\Delta C}$ distributions exclusively at the business cycle frequencies. The SPrice habit NKDSGE-TR model obtains similar results in the middle of the bottom panel of table 3 with four $CIC \geq 0.3$ in the columns labeled 8 : 2 (years per cycle). An additional $CIC \geq 0.3$ is generated by this model for the permanent $SD_{\mathcal{E},\Delta Y}$ distribution when the evaluation is conducted on the entire spectrum. In comparison, SPrice non-habit NKDSGE-MG and -TR models are responsible for three $CIC \geq 0.3$ that measure the overlap of permanent $SD_{\mathcal{E}}$ and $SD_{\mathcal{T}}$ distributions. Thus, the most empirically relevant propagation mechanisms are attributed to SPrice habit NKDSGE-MG and -TR models.

Several NKDSGE models have empirically credible monetary transmission mechanisms. According to table 3, transitory $SD_{\mathcal{E},\Delta Y}$ and $SD_{\mathcal{E},\Delta C}$ distributions are replicated by habit NKDSGE models whether sticky prices and wages are combined or used one at a time. Nonetheless, it is evident from table 3 that NKDSGE-TR models fit these distributions better than do NKDSGE-MG models. Baseline and SWage habit NKDSGE-TR models realize transitory $SD_{\mathcal{T},\Delta Y}$ and $SD_{\mathcal{T},\Delta C}$ distributions that match transitory $SD_{\mathcal{E},\Delta Y}$ and $SD_{\mathcal{E},\Delta C}$ distributions with $CIC \geq 0.44$ on the entire spectrum and at the business cycle frequencies in the bottom panel of table 3. SPrice habit NKDSGE-MG and -TR models are adept at fitting transitory $SD_{\mathcal{E},\Delta Y}$ and $SD_{\mathcal{E},\Delta C}$ distributions, but only on the business cycle frequencies. These successes are not duplicated by baseline and SWage habit NKDSGE-MG models given *CIC* in the top panel of table 3.

In summary, the *CIC* of table 3 show that consumption habit confers a superior fit to NKDSGE models.[19] These results echo the support Del Negro, Schorfheide, Smets, and Wouters (2007) obtain for consumption habit in their NKDSGE model. However, we find that the fit of habit NKDSGE models to $SD_{\mathcal{E},\Delta Y}$ and $SD_{\mathcal{E},\Delta C}$ distributions is not robust to the mix of nominal rigidities or choice of monetary policy rule. The frequencies on which the habit NKDSGE models are evaluated also matter for judgments about fit.

[19]The appendix presents Bayesian Monte Carlo experiments that estimate VAR(4)s instead of VAR(2)s, substitute the Cramer-von Mises (*CvM*) goodness of fit statistic for the *KS* statistic to quantify NKDSGE model fit, and replace the prior $h \sim U(0.05, 0.95)$ with either the prior $h \sim U(0.05, 0.499)$, $h \sim U(0.50, 0.95)$, or $h \sim \beta(0.65, 0.15)$. The latter prior implies a 95 percent coverage interval for h of $[0.38, 0.88]$. These experiments are reported in the appendix and reinforce the message table 3 has for the impact of consumption habit on NKDSGE model fit.

4.3 Visualize NKDSGE model dynamics and fit: Figures 3–8

The content of figures 3–8 is described in this section. Evidence to evaluate the fit of baseline NKDSGE-MG and -TR models is reported in figures 3 and 4, respectively. Figures 5 and 6 contain results for SPrice NKDSGE-MG and -TR models. The final two figures present plots generated by SWage NKDSGE-MG and -TR models.

Figures 3–8 summarize evidence about propagation, monetary transmission, and NKDSGE model fit in 12 windows spread across four rows and three columns. From top to bottom, the four rows report results for permanent $SD_{\Delta Y}$, transitory $SD_{\Delta Y}$, permanent $SD_{\Delta C}$, and transitory $SD_{\Delta Y}$, respectively. The first column of figures 3–8 plots mean *SD*s, while the second and third columns display densities of *KS* statistics.

Visual testimony about NKDSGE model propagation and monetary transmission appears in the first column of figures 3–8. This column consists of four windows containing plots of mean permanent $SD_{\mathcal{T},\Delta Y}$, mean transitory $SD_{\mathcal{T},\Delta Y}$, mean permanent $SD_{\mathcal{T},\Delta C}$, and mean transitory $SD_{\mathcal{T},\Delta C}$. Mean $SD_{\mathcal{T},\Delta Y}$ and $SD_{\mathcal{T},\Delta C}$ are denoted by (red) dot-dash plots for NKDSGE models with consumption habit and (green) dashed plots for NKDSGE models without consumption habit. The solid (blue) plots of these windows are the mean $SD_{\mathcal{E},\Delta Y}$ and $SD_{\mathcal{E},\Delta C}$ displayed in figure 2 and are included in figures 3–8 for comparison.

The second and third columns of figures 3–8 furnish densities that map from distributions of permanent and transitory $SD_{\mathcal{E}}$s and $SD_{\mathcal{T}}$s to densities of $KS_{\mathcal{E}}$ and $KS_{\mathcal{T}}$ statistics. The overlap of $KS_{\mathcal{E}}$ and $KS_{\mathcal{T}}$ statistic densities are a visual depiction of *CIC* and thus of NKDSGE model fit. Figures 3–8 display this overlap in columns two and three with a scheme similar to that described for the first column. Solid (blue) lines, (green) dashed lines, and (red) dot-dash lines denote $KS_{\mathcal{E}}$ statistic densities, $KS_{\mathcal{T}}$ statistic densities generated by non-habit NKDSGE models, and $KS_{\mathcal{T}}$ statistic densities produced by habit NKDSGE models, respectively. The second (third) column of figures 3–8 evaluates NKDSGE model fit with $KS_{\mathcal{E}}$ and $KS_{\mathcal{T}}$ statistics computed on the entire spectrum (restricted to the business cycle frequencies).

4.4 NKDSGE model propagation: Habit and nominal rigidities

This section explores the impact of different combinations of consumption habit, sticky prices, and sticky wages on the propagation of TFP shocks in NKDSGE models. For example, the first and third rows of the second and third columns of figures 3, 4, 7, and 8 present $KS_{\mathcal{T}}$ densities whose mass are to the right of $KS_{\mathcal{E}}$ densities. The lack of overlap explains the associated $CIC < 0.3$ for baseline and SWage NKDSGE models in table 3. The inability to match distributions of permanent $SD_{\mathcal{E},\Delta Y}$ and $SD_{\mathcal{E},\Delta C}$ extends to whether consumption habit is included in or is excluded from baseline and SWage NKDSGE models. Switching from the money growth rule (8) to the Taylor rule (9) also cannot repair the poor fit of these NKDSGE models to distributions of permanent $SD_{\mathcal{E}}$s.

The odd numbered rows of the first column of figures 3, 4, 7, and 8 display mean permanent $SD_{\mathcal{E},\Delta Y}$, $SD_{\mathcal{E},\Delta C}$, $SD_{\mathcal{T},\Delta Y}$, and $SD_{\mathcal{T},\Delta C}$ consistent with $KS_{\mathcal{E}}$ and $KS_{\mathcal{T}}$ statistic densities exhibiting little overlap. These mean permanent $SD_{\mathcal{E}}$s decay slowly from the infinite horizon into the business cycle frequencies. The same charts include mean permanent $SD_{\mathcal{T}}$s that often peak between eight and four years per cycle besides possessing substantial amplitude at the growth frequencies. Thus, the poor fit of baseline and SWage NKDSGE models to distributions of permanent $SD_{\mathcal{E},\Delta Y}$ and $SD_{\mathcal{E},\Delta C}$ cannot be attributed to weak propagation of TFP shocks.

Nontheless, baseline and SWage NKDSGE models have powerful propagation mechanisms. Fully indexed sticky wages induce these propagation mechanisms, in part, by smoothing nominal wages which forces households to adjust labor supply in response to permanent TFP shocks. This response contributes to an empirically unreasonable propagation mechanism.

Stripping out sticky wages conveys an empirically credible propagation mechanism to habit NKDSGE-MG and -TR models when fit to distributions of permanent $SD_{\mathcal{E},\Delta Y}$ and $SD_{\mathcal{E},\Delta C}$ is limited to the business cycle frequencies. The first and third rows of the third column of figures 5 and 6 provide this evidence with $KS_{\mathcal{E}}$ and $KS_{\mathcal{T}}$ statistic densities that display considerable overlap. This shows SPrice habit NKDSGE models possess economically meaningful propagation mechanisms at the business cycle frequencies marked by combining intertemporal complementarity in consumption with inflation smoothing.

There are two successes for SPrice habit and non-habit NKDSGE-TR models when asked to replicate the distribution of permanent $SD_{\mathcal{E},\Delta Y}$ on the entire spectrum. In the middle window of the first row of figure 6, these NKDSGE models yield $KS_{\mathcal{T}}$ statistic densities that overlap the $KS_{\mathcal{E}}$ statistic density. This is further evidence of the difficulties NKDSGE models have at matching distributions of permanent $SD_{\mathcal{E},\Delta Y}$ and $SD_{\mathcal{E},\Delta C}$, especially on the entire distributions.

SPrice habit NKDSGE-MG and -TR models are responsible for mean permanent $SD_{\mathcal{T},\Delta Y}$ and $SD_{\mathcal{T},\Delta C}$ that possess less volatility and periodicity compared to those produced by baseline and SWage NKDSGE models. The odd numbered rows of figures 5 and 6 show that removing sticky wages reduces volatility and periodicity in mean permanent $SD_{\mathcal{T}}$s. This moves these SDs closer to mean permanent $SD_{\mathcal{E}}$s on the business cycle frequencies. The most striking example of a SPrice habit NKDSGE model generating empirically relevant mean dynamics is found in the (red) dot-dash plots in the first column of the first and third rows of figure 6. In this figure, plots of mean permanent $SD_{\mathcal{T},\Delta Y}$ and $SD_{\mathcal{T},\Delta C}$ decay smoothly from the long-run into the business cycle frequencies. This mimics the behavior of mean permanent $SD_{\mathcal{E},\Delta Y}$ and $SD_{\mathcal{E},\Delta C}$.

This section reports that consumption habit combines with fully indexed sticky prices to create empirically relevant and economically meaningful propagation of TFP shocks in NKDSGE models at the business cycle frequencies. With this mix of real and nominal rigidities, SPrice habit NKDSGE models tie propagation to intertemporal consumption complementarity and inflation smoothing. Thus, the match between NKDSGE models and distributions of $SD_{\mathcal{E},\Delta Y}$ and $SD_{\mathcal{E},\Delta C}$ identified by a permanent TFP shock is sensitive to the mix of nominal rigidities, a result in line with Dupor, Han, and Tsai (2009), and to the frequencies used to judge fit.

4.5 NKDSGE model monetary transmission: Habit and monetary policy rules

Erceg, Henderson, and Levin (2000) recognize that implementing optimal monetary policy can be problematic when faced with sticky prices and wages. Nonetheless, their conclusions about monetary policy analysis rests on sticky prices and wages transmitting monetary policy shocks to the real economy in ways that match empirical observation. This section assesses the empirical relevance of different combinations of sticky prices and wages for monetary transmission in NKDSGE models with and without consumption habit. The specification of monetary policy matters for evaluating NKDSGE model fit because monetary transmission is described with transitory $SD_{\mathcal{T},\Delta Y}$s and $SD_{\mathcal{T},\Delta C}$s that are identified with respect to either the money growth rule (8) or Taylor rule (9) shock innovations.

We find that baseline, SPrice, and SWage NKDSGE models achieve greater success in matching distributions of transitory $SD_{\mathcal{E},\Delta Y}$ and $SD_{\mathcal{E},\Delta C}$ given the Taylor rule (9) defines monetary policy. The Taylor rule contributes to a superior fit, especially when the entire spectrum is used for evaluation, by dampening output and consumption growth fluctuations. These results are anticipated by Poole (1970). In a sticky price Keynesian macro model, he shows that an interest rate rule minimizes the variance of output relative to a money growth rule when real shocks are more volatile than nominal shocks. Since we apply priors to the NKDSGE models that respect this ordering of the relative volatilities of TFP, money growth rule, and Taylor rule shocks, our Bayesian Monte Carlo experiments underline the sensitivity of NKDSGE model fit to the specification of monetary policy rules.

Money growth rule (8) shock innovations are transmitted by baseline and SWage NKDSGE models into fluctuations in output and consumption growth. However, these monetary transmission mechanisms are unable to produce distributions of transitory $SD_{\mathcal{T},\Delta Y}$ and $SD_{\mathcal{T},\Delta Y}$ that cover distributions of transitory $SD_{\mathcal{E},\Delta C}$ and $SD_{\mathcal{E},\Delta C}$. The even numbered rows of the second and third columns of figures 3 and 7 depict the poor fit of baseline and SWage NKDSGE-MG models with distributions of transitory $SD_{\mathcal{E}}$ and $SD_{\mathcal{T}}$ that do not overlap. This lack of fit is translated into excessive amplitude and periodicity in mean transitory $SD_{\mathcal{T},\Delta Y}$ and $SD_{\mathcal{T},\Delta C}$ compared to mean transitory $SD_{\mathcal{E},\Delta Y}$ and $SD_{\mathcal{E},\Delta C}$ that are found in the even numbered rows of the first column of figures 3 and 7.

An exception to this poor fit is obtained by the SPrice habit NKDSGE-MG model. This NKDSGE model generates distributions of transitory $SD_{\mathcal{T},\Delta Y}$ and $SD_{\mathcal{T},\Delta C}$ that intersect distributions of transitory $SD_{\mathcal{I},\Delta Y}$ and $SD_{\mathcal{I},\Delta C}$ on the business cycle frequencies in the second and third rows of the third column of figure 5. The good fit helps explain mean transitory $SD_{\mathcal{T}}$s of the SPrice habit NKDSGE-MG model that cross mean transitory $SD_{\mathcal{I}}$s (from below) at between eight to two years per cycle in the even numbered rows of the first column of figure 5. When the entire spectrum serves to judge fit, the second and fourth rows of the second column of figure 5 display $KS_{\mathcal{T}}$ statistic densities far to the right of the associated $KS_{\mathcal{I}}$ statistic densities.

Baseline, SPrice, and SWage habit NKDGSE-TR models produce mean transitory $SD_{\mathcal{T},\Delta Y}$ and $SD_{\mathcal{T},\Delta C}$ that are more similar to mean transitory $SD_{\mathcal{I},\Delta Y}$ and $SD_{\mathcal{I},\Delta C}$ as shown in the second and fourth rows of the first column of figures 4, 6, and 8. Thus, Taylor rule shock innovations are transmitted into output and consumption growth fluctuations on average in economically relevant ways by baseline, SPrice, and SWage habit NKDGSE-TR models using intertemporal complementarity created by consumption habit and nominal wage growth smoothing engendered by fully indexed sticky wages. Although these NKDSGE-TR models yield mean transitory $SD_{\mathcal{T}}$ that are close to the mean transitory $SD_{\mathcal{I}}$, note that mean transitory $SD_{\mathcal{I}}$s are most nearly realized by the SWage habit NKDSGE-TR model.

Monetary transmission differs across baseline and SPrice habit NKDSGE-TR models. The baseline habit NKDSGE-TR model produces periodicity in its mean transitory $SD_{\mathcal{T},\Delta Y}$ that is close to displayed by the mean transitory $SD_{\mathcal{I},\Delta Y}$ in the second row of the first column of figure 4, but the former mean spectral density lacks the volatility of the latter. The lack of volatility is reversed by the SPrice habit NKDSGE-TR model. The second row of the first column of figure 6 displays mean transitory $SD_{\mathcal{I},\Delta Y}$ and $SD_{\mathcal{T},\Delta Y}$ that have about the same amplitude. However, the latter *SD* has much of its power in the high frequency rather than at the business cycle frequencies.

Mean transitory $SD_{\mathcal{I},\Delta C}$ and $SD_{\mathcal{T},\Delta C}$ expose more disparities in the monetary transmission mechanisms of baseline and SPrice habit NKDSGE-TR models. The baseline NKDSGE-TR model yields a mean transitory $SD_{\mathcal{T},\Delta C}$ in the bottom panel of the first column of figure 4 that is out of phase in the lower frequencies with the mean transitory consumption growth $SD_{\mathcal{E},\Delta C}$, but captures its amplitude. When the lone nominal rigidity is sticky prices, the fourth row of the first column of figure 6 shows that the SPrice habit NKDSGE-TR model produces a mean transitory $SD_{\mathcal{T},\Delta C}$ that is out of phase in the high frequencies, but mimics the amplitude of the mean transitory consumption growth $SD_{\mathcal{E},\Delta C}$.

The Bayesian Monte Carlo experiments reveal that the fit of baseline, SPrice, and SWage NKDSGE-TR models to distributions of transitory $SD_{\mathcal{E},\Delta Y}$ and $SD_{\mathcal{E},\Delta C}$ is vulnerable to the frequencies used for evaluation. A good fit for these models is affirmed on the business cycle frequencies by the overlap of $KS_{\mathcal{E}}$ and $KS_{\mathcal{T}}$ statistic densities in the far right columns of figures 4, 6, and 8. The NKDSGE-TR models match the transitory $SD_{\mathcal{E}}$s on the business cycle frequencies whether or not household preferences include consumption habit. However, only baseline and SWage habit NKDSGE-TR models replicate transitory $SD_{\mathcal{E}}$s on the entire spectrum given the overlap of $KS_{\mathcal{E}}$ and $KS_{\mathcal{T}}$ statistic densities in the even numbered rows of the middle columns of figures 4 and 8.

This section shows there are several combinations of consumption habit, sticky prices, sticky wages, and the money growth rule (8) or Taylor rule (9) that create empirically significant and economically meaningful monetary transmission in NKDSGE models. When fit is measured on the business cycle frequencies, the baseline, SPrice, and SWage NKDGSE-TR and SPrice habit NKDSGE-MG models match transitory output and consumption growth $\text{SD}_{\mathcal{E}}$s. These models face problems when evaluated on these posterior moments using the entire spectrum. This metric limits a satisfactory fit just to the baseline and SWage habit NKDGSE-TR models. Common to these models is fully indexed Calvo nominal wage setting. This nominal rigidity contributes to empirically relevant monetary transmission by trading smoother nominal wage growth for greater variation in labor supply. Nonetheless, our evidence lends support to the contention of Del Negro and Schorfheide (2008) that it is difficult to choose among competing nominal rigidities when evaluating NKDSGE model fit, especially to monetary policy shocks.

5. Conclusion

This paper studies the business cycle implications of internal consumption habit for new Keynesian dynamic stochastic general equilibrium (NKDSGE) models. We examine the fit of 12 NKDSGE models that have different combinations of internal consumption habit, Calvo staggered prices and nominal wages, along with several other real rigidities. The NKDSGE models are confronted with population output and consumption growth spectral densities (*SD*s) identified by permanent productivity and transitory monetary shocks.

The fit of NKDSGE models with and without consumption habit is explored using Bayesian Monte Carlo methods that avoid estimation. We view this approach as a low cost way to explore the fit of competing NKDSGE model specifications that complement results obtained from estimation. The evidence produced using these techniques favors retaining consumption habit in NKDSGE models. Nonetheless, the Bayesian Monte Carlo experiments show that the fit of NKDSGE models with consumption habit is susceptible to (1) changing the mix of nominal rigidities, (2) identifying *SD*s on permanent productivity shocks instead of transitory monetary policy shocks, (3) evaluating *SD*s on the entire spectrum rather than the business cycle frequencies, and (4) tying monetary policy to a money growth rule instead of a Taylor rule. These results suggest that there remain ambiguities about the specification of real and nominal rigidities in NKDSGE models. The resolution of these ambiguities should inspire further research into the role real and nominal rigidities play in propagation and monetary transmission.

References

Abel, Andrew B. (1990) "Asset Prices under Habit Formation and Catching up with the Joneses." *American Economic Review Paper and Proceedings*, 80, 38–42.

Akaike, Hirotugu. (1969) "Power Spectrum Estimation through Autoregressive Model Fitting." *Annals of the Institute of Statistical Mathematics*, 21, 407–419.

Basu, Susanto, and John G. Fernald. (1997) “Returns to Scale in U.S. Production: Estimates and Implications.” *Journal of Political Economy*, 105, 249–283.

Blanchard, Olivier J., and Danny Quah. (1989). “The Dynamic Effects of Aggregate Demand and Supply Disturbances.” *American Economic Review*, 79, 655–673.

Boldrin, Michele, Lawrence Christiano, Jonas D.M. Fisher (2001). “Habit Persistence, Asset Returns, and the Business Cycle.” *American Economic Review* 91, 149–166.

Bouakez, Hafedh, Emanuela Cardia, and Francisco Ruge–Murcia. (2005) “Habit Formation and the Persistence of Monetary Policy Shocks.” *Journal of Monetary Economics*, 52, 1073–1088.

Canova, Fabio, and Luca Sala. (2009) “Back to Square One: Identification Issues in DSGE Models.” *Journal of Monetary Economics*, 56, 431–449.

Chang, Yongsung, Joao F. Gomes, and Frank Schorfheide. (2002) “Leaning-by-Doing as a Propagation Mechanism.” *American Economic Review*, 92, 1498–1520.

Christiano, Lawerence, Martin Eichenbaum, Charles L. Evans. (2005) “Nominal Rigidities and the Dynamic Effects of a Shock to Monetary Policy.” *Journal of Political Economy*, 113, 1–45.

Cogley, Tim, and James M. Nason. (1995a) “Effects of the Hodrick-Prescott Filter on Trend and Difference Stationary Time Series: Implications for Business Cycle Research.” *Journal of Economic Dynamics and Control*, 19, 253–278.

Cogley, Tim, and James M. Nason. (1995b) “Output Dynamics in Real-Business-Cycle Models.” *American Economic Review*, 85, 492–511.

Constantinides, George M. (1990) “Habit Formation: a Resolution of the Equity Premium Puzzle”, *Journal of Political Economy*, 98, 519–543.

DeJong, David N., Beth F. Ingram, Charles H. Whiteman. (1996) “A Bayesian Approach to Calibration.” *Journal of Business and Economic Statistics*, 14, 1–9.

Del Negro, Marco, and Frank Schorfheide. (2008) “Forming Priors for DSGE Models (and how it affects the assessment of nominal rigidities).” *Journal of Monetary Economics*, 55, 1191–1208.

Del Negro, Marco, Frank Schorfheide, Frank Smets, and Raf Wouters. (2007) “On the Fit of New Keynesian Models.” *Journal of Business and Economic Statistics*, 25, 123–143.

Dennis, Richard. (2009) “Consumption-Habits in a New Keynesian Business Cycle Model.” *Journal of Money, Credit, and Banking*, 41, 1015–1030.

Diebold, Francis X., Lee E. Ohanian, and Jeremy Berkowitz. (1998) “Dynamic Equilbrium Economies: A Framework for Comparing Models and Data.” *Review of Economic Studies*m 65, 433–451.

Dupor, Bill, Jing Han, and Yi-Chan Y.C. (2009) “What Do Technology Shocks Tell Us about the New Keynesian Paradigm?” *Journal of Monetary Economics*, 56, 560–569.

Eichenbaum, Martin, and Lars P. Hansen. (1990) "Estimating Models with Intertemporal Substitution Using Aggregate Time Series Data." *Journal of Business and Economic Statistics*, 8, 53–69.

Erceg, Christopher J., Dale W. Henderson, and Andrew T. Levin. (2000) "Optimal Monetary Policy with Staggered Wage and Price Contracts." *Journal of Monetary Economics*, 46, 281–313.

Fernández-Villaverde, Jesus, Juan F. Rubio-Ramírez, Thomas J. Sargent, and Mark W. Watson. (2007) "ABC's (and D)'s for Understanding VARS." *American Economic Review*, 97, 1021–1026.

Galí, Jordi. (1991) "Budget Constraints and Time-Series Evidence on Consumption." *American Economic Review*, 81, 1238–1253.

Geweke, John. (2010) *Complete and Incomplete Econometric Models*, Princeton, NJ: Princeton University Press.

Geweke, John. (1999) "Using Simulation Methods for Bayesian Econometric Models: Inference, Development, Communication." *Econometric Reviews*, 18, 1–73.

Guerron-Quintana, Pablo A. (2010) "What You Match Does Matter: The Effects of Data on DSGE Estimation." *Journal of Applied Econometrics*, 25, 774–804.

Hall, George J. (1996) "Overtime, Effort, and the Propagation of Business Cycle Shocks." *Journal of Monetary Economics*, 38, 139–160.

Heaton, John. (1995) "An Empirical Investigation of Asset Pricing with Temporary Dependent Preference Specifications." *Econometrica*, 63, 681–717.

Ireland, Peter N. (2001) "Sticky-price Models of the Business Cycle: Specification and Stability." *Journal of Monetary Economics*, 47, 3–18.

Iskrev, Nikolay. (2010) "Local Identification in DSGE Models." *Journal of Monetary Economics*, 57, 189–202.

Kano, Takashi. (2009) "Habit Formation, and the Present-value Model of the Current Account: Yet Another Suspect." *Journal of International Economics*, 78, 72–85.

Kimmel, Jean, and Thomas J. Kniesner. (1998) "New Evidence on Labor Supply: Employment versus Hours Elasticities by Sex and Marital Status." *Journal of Monetary Economics* 42, 289–301.

Lindé, Jesper. (2005) "Estimating New-Keynesian Phillips Curves: A Full Information Maximum Likelihood Approach." *Journal of Monetary Economics*, 52, 1135–1149.

McCausland, William. (2004) "Using the BACC Software for Bayesian Inference." *Journal of Computational Economics*, 23, 201–218.

Nason, James M. (1988) "The Equity Premium and Time-varying Risk Behavior." FEDS Paper no. 11, Board of Governors of the Federal Reserve System, Washington, DC.

Nason, James M. (1997) "Taste Shocks." In *Business Cycles and Depressions: An Encyclopedia*, edited by David Glaser, pp. 673–676. New York, NY: Garland Publishing.

Nason, James M., and Cogley, Tim. (1994) "Testing the Implications of Long-run Neutrality for Monetary Business Cycle Models." *Journal of Applied Econometrics*, 9, S37–S70.

Nason, James M., and Rogers, John H. (2006) "The Present-Value Model of the Current Account Has Been Rejected: Round up the Usual Suspects." *Journal of International Economics*, 68, 159–187.

Nason, James M., and Slotsve, George A. (2004) "Along the New Keynesian Phillips Curve with Nominal and Real Rigidities." Working Paper 2004–09, Federal Reserve Bank of Atlanta.

Parzen, Emanuel. (1974) "Some Recent Advances in Time Series Modelling." *I.E.E.E. Transactions on Automatic Control*, AC-19, 723–729.

Pollak, Robert A. (1976) "Habit Formation and Long-run Utility Functions." *Journal of Economic Theory*, 13, 272–297.

Poole, William. (1970) "Optimal Choice of Monetary Policy Instruments in a Simple Stochastic Macro Model." *Quarterly Journal of Economics*, 84, 197–218.

Rabanal, Pau, and Juan F. Rubio-Ramírez. (2005) "Comparing New Keynesian Models of the Business Cycle: A Bayesian Approach." *Journal of Monetary Economics*, 52, 1151–1166.

Rozen, Kareen. (2010) "Foundations of Intrinsic Habit Formation." *Econometrica*, forthcoming.

Ryder, Harl E., Jr., and Geoffrey M. Heal. (1973) "Optimal Growth with Intertemporally Dependent Preferences." *Review of Economic Studies*, 40, 1–31.

Sbordone, Argia M. (2002) "Prices and Unit Costs: A New Test of Price Stickiness." *Journal of Monetary Economics*, 49, 235–456.

Schorfheide, Frank. (2008) "DSGE Model-Based Estimation of the New Keynesian Phillips Curve." *Economic Quarterly*, Federal Reserve Bank of Richmond 94, 397–433.

Sims, Christopher. (2002) "Solving Linear Rational Expectations Models." *Computational Economics*, 20, 1–20.

Schmitt-Grohé, Stephanie, and Martin Uribe. (2007) "Habit Persistence." In *The New Palgrave Dictionary of Economics, second edition*, edited by Steven N. Durlauf, and Lawerence Blume, New York, NY: Palgrave Macmillan.

Smets, Frank, and Raf Wouters. (2007) "Shocks and Frictions in US Business Cycles: A Bayesian DSGE approach." *American Economic Review*, 97, 1123–1175.

Sundaresan, Suresh. (1989) "Intertemporally Dependent Preferences and the Volatility of Consumption and Wealth." *Review of Financial Studies*, 2, 73–89.

TABLE 1: SUMMARY OF MEI AND BAYESIAN MONTE CARLO METHODS

	Empirical (Posterior)	Theoretical (Prior)
Sample	Actual	Synthetic
Sample Length	$T = 196$	$\mathcal{M} = T \times \mathcal{W}$, $\mathcal{W} = 5$
Number of Replications	$\mathcal{J} = 5000$	$\mathcal{J} = 5000$
Priors	VAR coefficients	NKDSGE model parameters
Simulator	MCMC produce BVAR coefficients	Estimate VARs on synthetic data generated by NKDSGE Models
BQ Decomposition under LRMN	Invert BVAR to obtain SVMA(∞)	Invert estimated VAR to produce SVMA(∞)
Distributions	$SD_{\mathcal{E},\Delta Y}$ and $SD_{\mathcal{E},\Delta C}$ mapped into $KS_{\mathcal{E}}$ statistics	$SD_{\mathcal{T},\Delta Y}$ and $SD_{\mathcal{T},\Delta C}$ mapped into $KS_{\mathcal{T}}$ statistics

Table 2: Bayesian Calibration of NKDSGE Models

		Prior Distribution	Mean	Standard Deviation	95 Percent Cover Interval
h	Internal Consumption Habit	Uniform	—	—	[0.0500, 0.9500]
β	H'hold Subjective Discount	Beta	0.9930	0.0020	[0.9886, 0.9964]
γ	Labor Supply Elasticity	Normal	1.5500	0.5360	[0.4995, 2.6005]
δ	Depreciation Rate	Beta	0.0200	0.0045	[0.0122, 0.0297]
α	Deterministic Growth Rate	Normal	0.0040	0.0015	[0.0011, 0.0064]
ϖ	Capital Adjustment Costs	Normal	4.7710	1.0260	[2.7601, 6.7819]
ψ	Capital's Share of Output	Beta	0.3500	0.0500	[0.2554, 0.4509]
σ_ϵ	TFP Growth Shock Std.	Uniform	—	—	[0.0070, 0.0140]
ξ	Final Good Dmd Elasticity	Normal	12.0000	4.0820	[3.9994, 20.0006]
μ_P	No Price Change Probability	Beta	0.5500	0.0500	[0.4513, 0.6468]
θ	Labor Demand Elasticity	Normal	15.0000	3.0800	[8.9633, 21.0367]
μ_W	No Wage Change Probability	Beta	0.7000	0.0500	[0.5978, 0.7931]
m^*	$\Delta \ln M$ Mean	Beta	0.0114	0.0030	[0.0063, 0.0180]
ρ_m	$\Delta \ln M$ AR1 Coef.	Beta	0.6278	0.0800	[0.4653, 0.7767]
σ_μ	$\Delta \ln M$ Shock Std.	Beta	0.0064	0.0012	[0.0043, 0.0090]
a_π	Taylor Rule $\mathbf{E}_t \pi_{t+1}$ Coef.	Normal	1.8250	0.2300	[1.3742, 2.2758]
$a_{\hat{Y}}$	Taylor Rule $\hat{Y}_t$ Coef.	Normal	0.1000	0.0243	[0.0524, 0.1476]
ρ_R	Taylor Rule AR1 Coef.	Beta	0.6490	0.0579	[0.5317, 0.7578]
σ_υ	Taylor Rule Shock Std.	Beta	0.0051	0.0016	[0.0025, 0.0087]

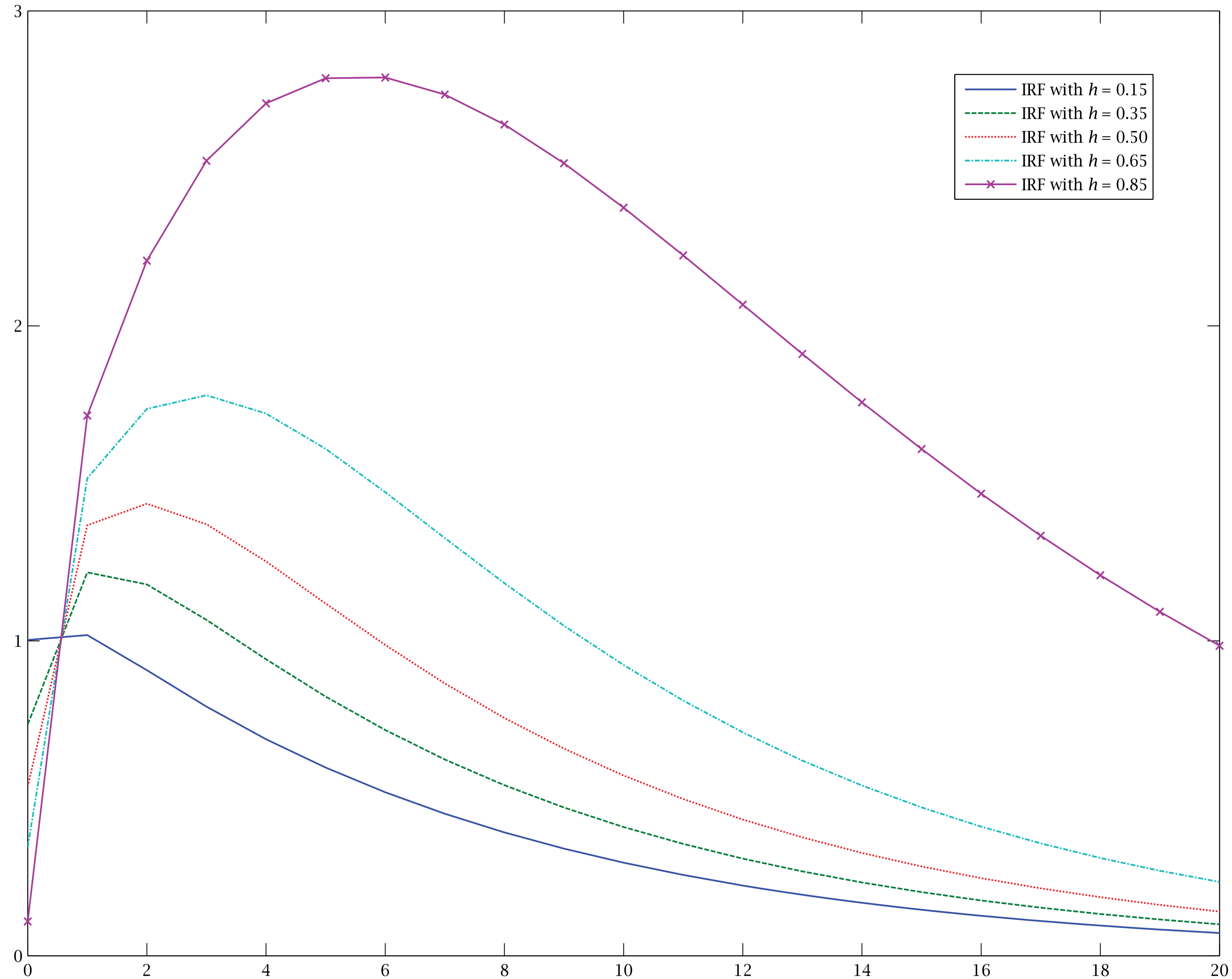
The calibration relies on existing DSGE model literature; see the text for details. For a non-informative prior, the right most column contains the lower and upper end points of the uniform distribution. When the prior is based on the beta distribution, its two parameters are $a = \overline{\Gamma}_{i,n}\left[(1-\overline{\Gamma}_{i,n})\overline{\Gamma}_{i,n}/STD(\Gamma_{i,n})^2 - 1\right]$ and $b = a(1-\overline{\Gamma}_{i,n})/\overline{\Gamma}_{i,n}$, where $\overline{\Gamma}_{i,n}$ is the degenerate prior of the ith element of the parameter vector of model $n = 1, \ldots, 4$, and its standard deviation is $STD(\Gamma_{i,n})$.

Table 3: *CIC* of Kolmogorov-Smirnov Statistics

Model	ΔY w/r/t Trend Sh'k		ΔY w/r/t Transitory Sh'k		ΔC w/r/t Trend Sh'k		ΔC w/r/t Transitory Sh'k	
	$\infty : 0$	$8 : 2$	$\infty : 0$	$8 : 2$	$\infty : 0$	$8 : 2$	$\infty : 0$	$8 : 2$
NKDSGE-MG								
Baseline								
Non-Habit	0.02	0.03	0.00	0.01	0.00	0.00	0.00	0.00
Habit	0.00	0.04	0.16	0.18	0.02	0.16	0.13	0.18
SPrice								
Non-Habit	0.03	0.47	0.00	0.23	0.01	0.17	0.00	0.04
Habit	0.14	0.64	0.11	0.59	0.09	0.44	0.29	0.49
SWage								
Non-Habit	0.00	0.00	0.00	0.06	0.00	0.00	0.00	0.06
Habit	0.00	0.01	0.18	0.23	0.01	0.10	0.13	0.23
NKDSGE-TR								
Baseline								
Non-Habit	0.01	0.00	0.12	0.71	0.00	0.00	0.08	0.68
Habit	0.00	0.03	0.64	0.52	0.03	0.14	0.53	0.85
SPrice								
Non-Habit	0.40	0.57	0.00	0.76	0.01	0.16	0.00	0.49
Habit	0.43	0.74	0.29	0.65	0.15	0.46	0.33	0.76
SWage								
Non-Habit	0.00	0.00	0.21	0.37	0.00	0.00	0.02	0.81
Habit	0.00	0.05	0.55	0.45	0.03	0.13	0.44	0.77

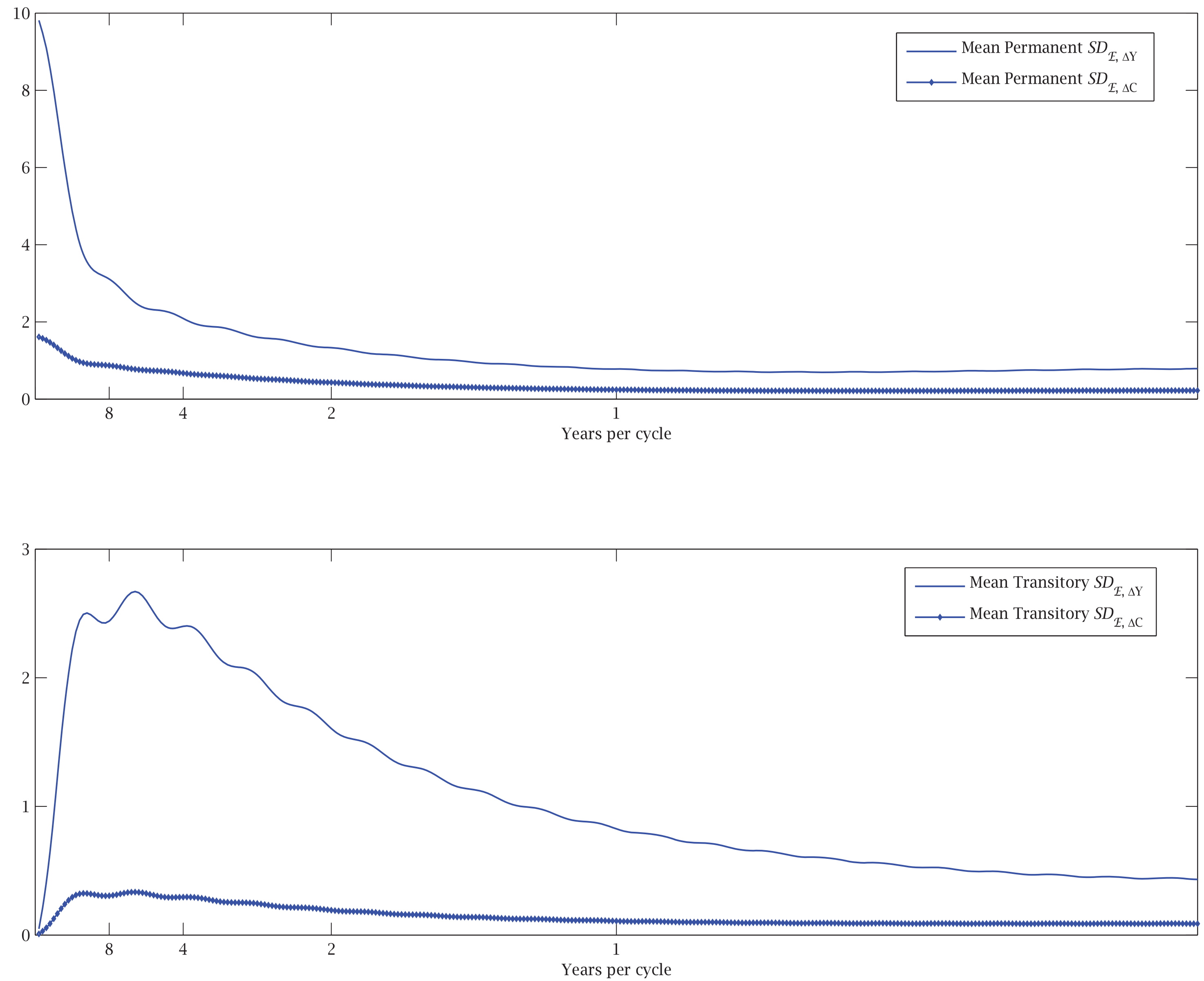
NKDSGE-MG and NKDSGE-TR denote the NKDSGE model with the AR(1) money supply rule (8) and the Taylor rule (9), respectively. Baseline NKDSGE models include sticky prices and sticky wages. The acronyms SPrice and SWage represent NKDSGE models with only sticky prices or sticky nominal wages, respectively. The column heading $\infty : 0$ ($8 : 2$) indicates that *CIC* quantify the intersection of $\mathcal{E}$ and $\mathcal{T}$ *KS* statistic distributions computed from permanent and transitory output and consumption growth SDs with domains on the entire spectrum (from eight to two years per cycle).

Figure 1: ΔC Response to Real Interest Rate Shock



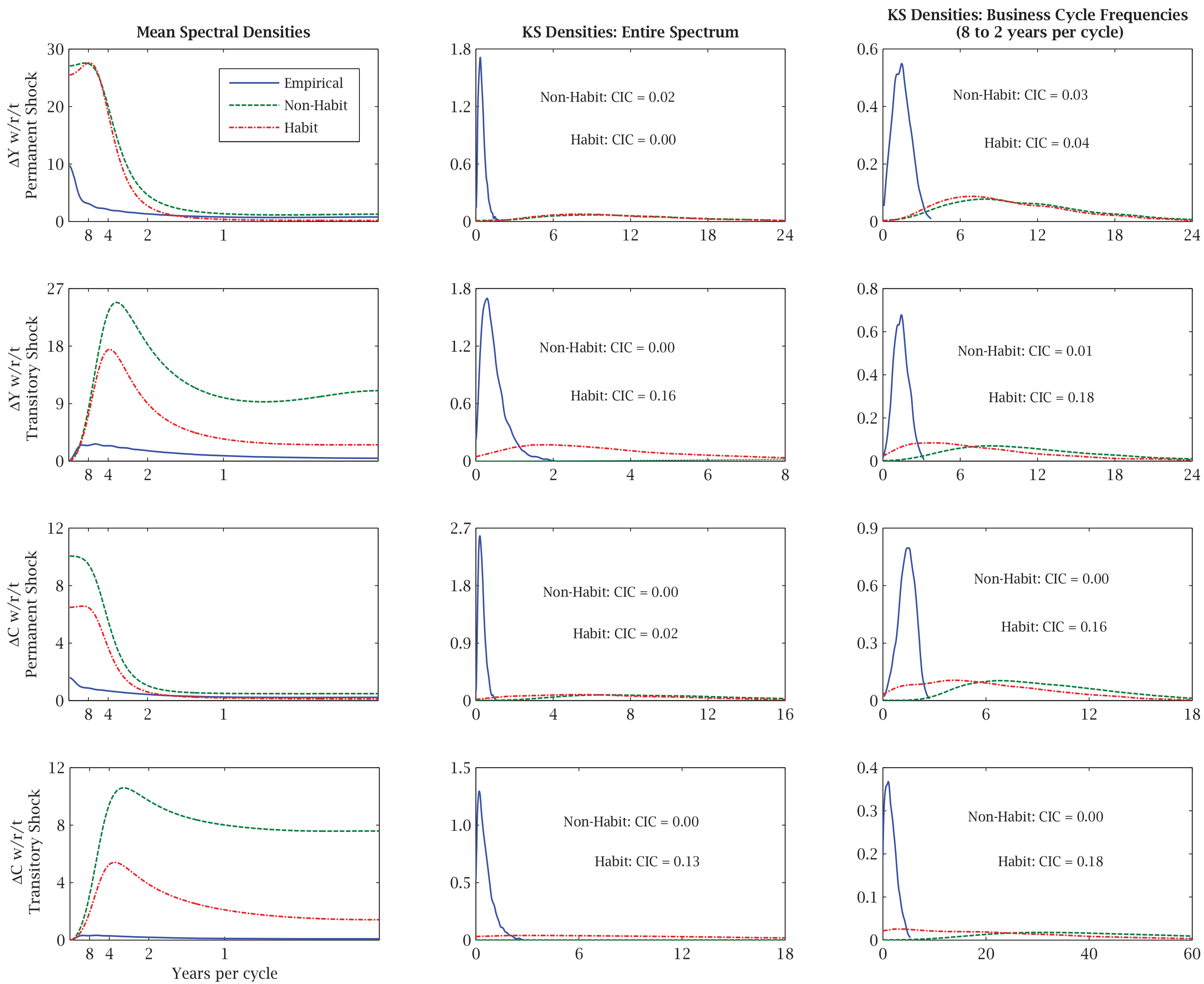
The plots are the impulse response functions (IRFs) of consumption growth (ΔC) generated from the solved linearized Euler (2) given a one percent shock to the forecast innovation of the AR(1) of the real rate, q_t.

FIGURE 2: MEAN STRUCTURAL $\mathcal{E}$ SPECTRA OF ΔY AND ΔC



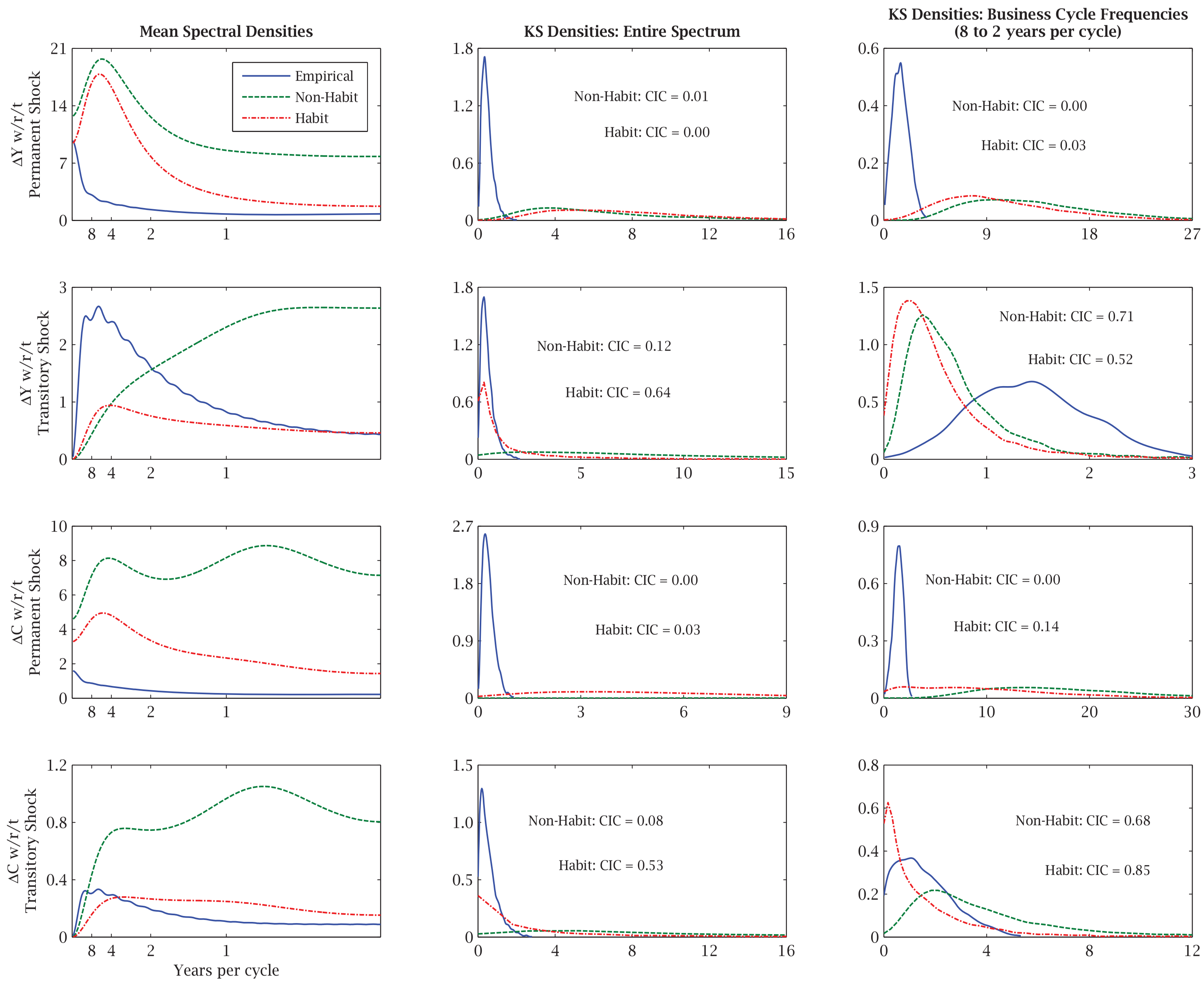
Mean permanent and transitory $SD_{\mathcal{E},\Delta Y}$ and $SD_{\mathcal{E},\Delta C}$ are averaged frequency by frequency across ensembles that consist of $\mathcal{J}$ of these SDs. The SDs are constructed using SVMA(∞)s that rely on LRMN, the BQ decomposition, unrestricted VAR(2)s.

FIGURE 3: MEAN STRUCTURAL $\mathcal{E}$ AND $\mathcal{T}$ SDs AND KS DENSITIES FOR BASELINE NKDSGE MODELS WITH AR(1) MONEY GROWTH RULE



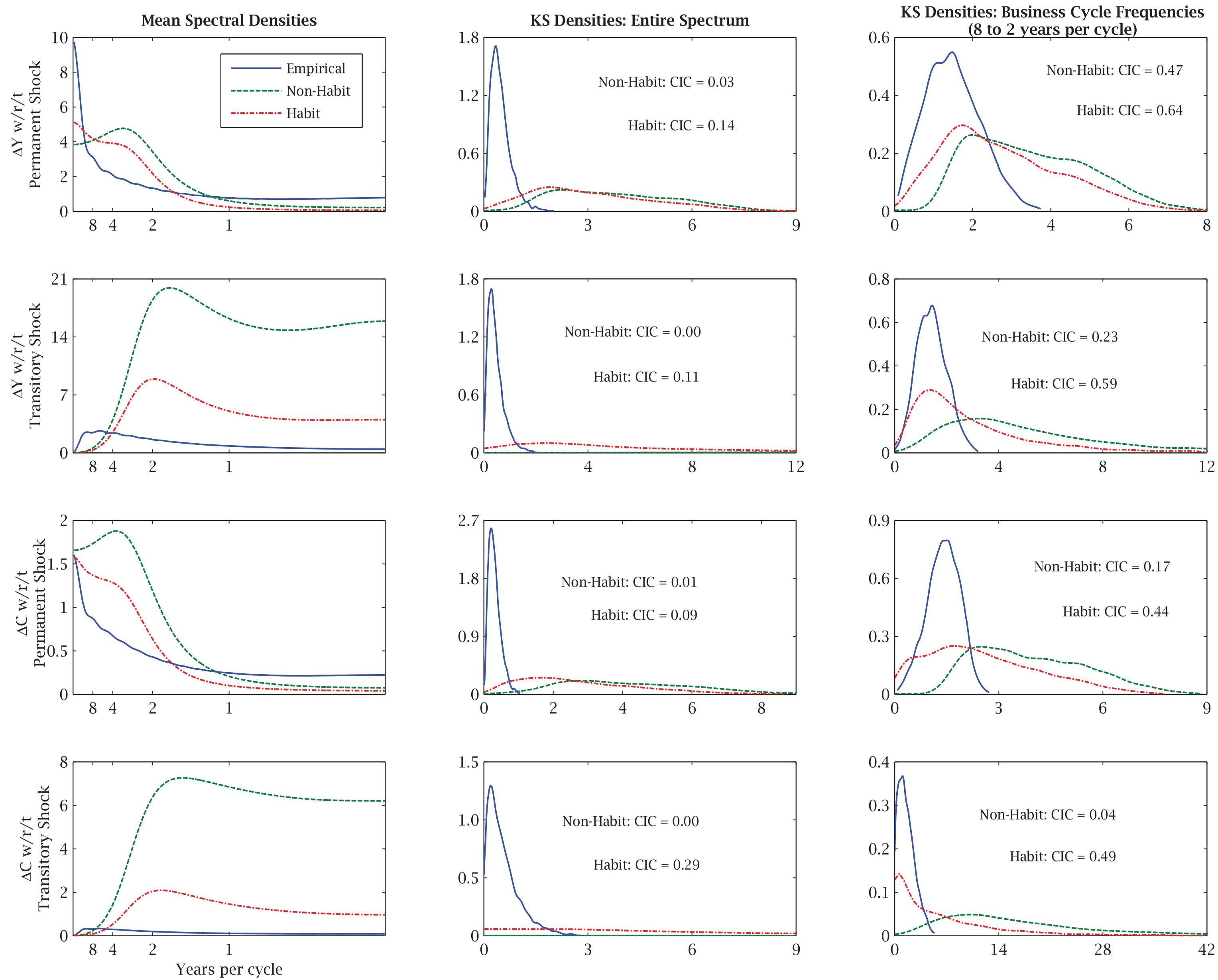
See section 4.3 for details.

Figure 4: Mean Structural $\mathcal{E}$ and $\mathcal{T}$ SDs and KS Densities for Baseline NKDSGE Models with Taylor Rule



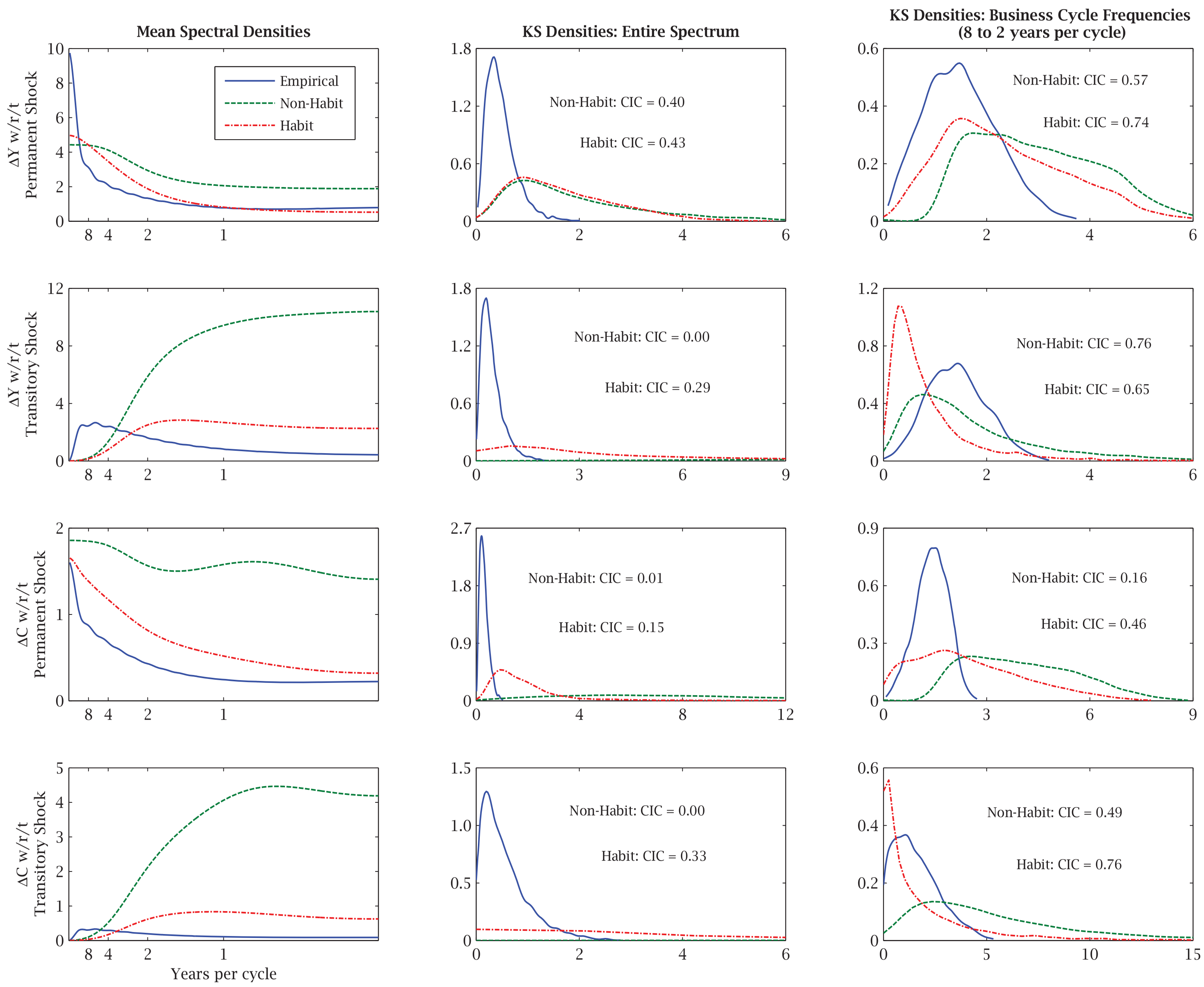
See section 4.3 for details.

Figure 5: Mean Structural $\mathcal{E}$ and $\mathcal{T}$ SDs and KS Densities for NKDSGE Models with AR(1) Money Growth Rule and Only Sticky Prices



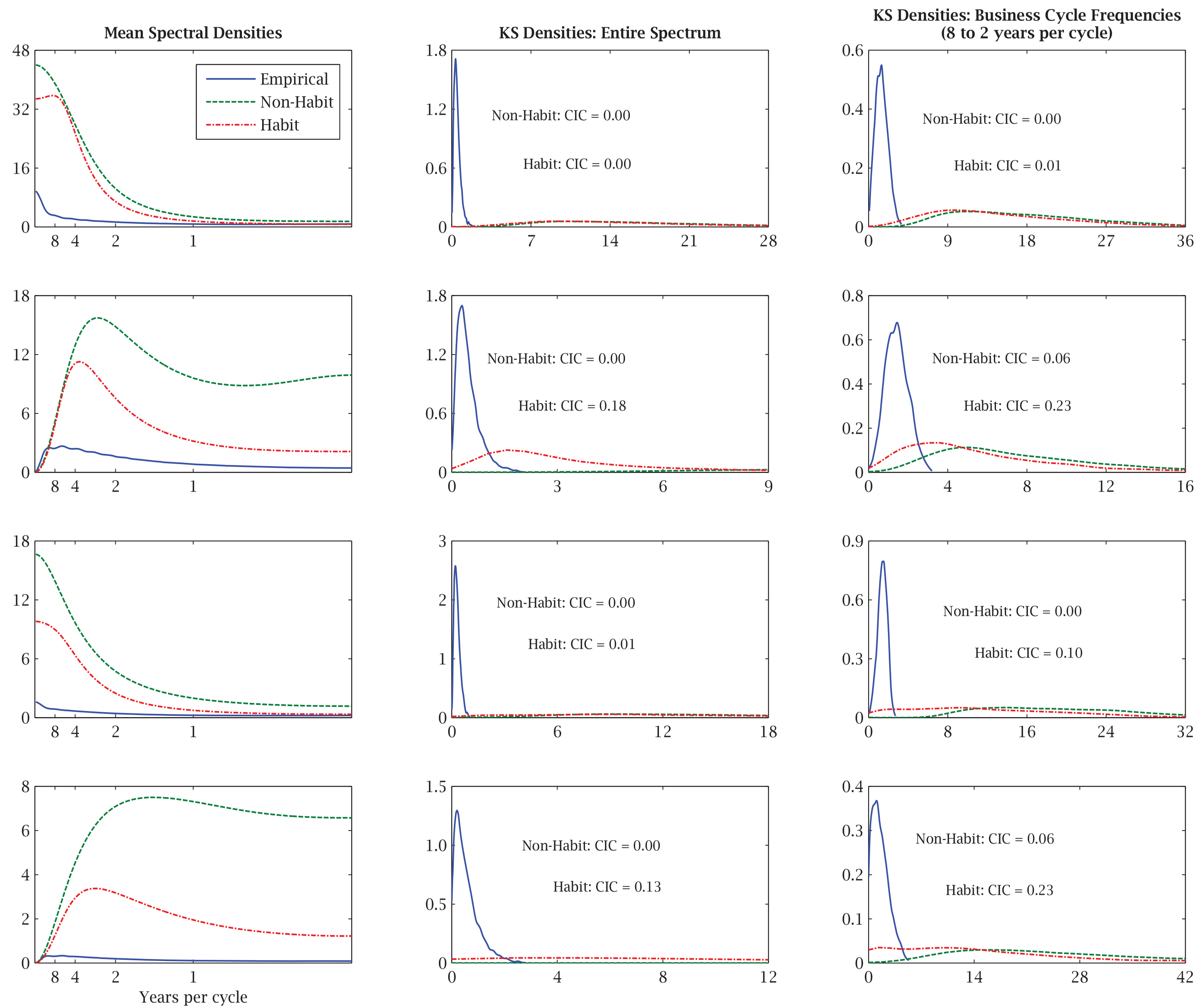
See section 4.3 for details.

Figure 6: Mean Structural $\mathcal{E}$ and $\mathcal{T}$ SDs and KS Densities for NKDSGE Models with Taylor Rule and only Sticky Prices



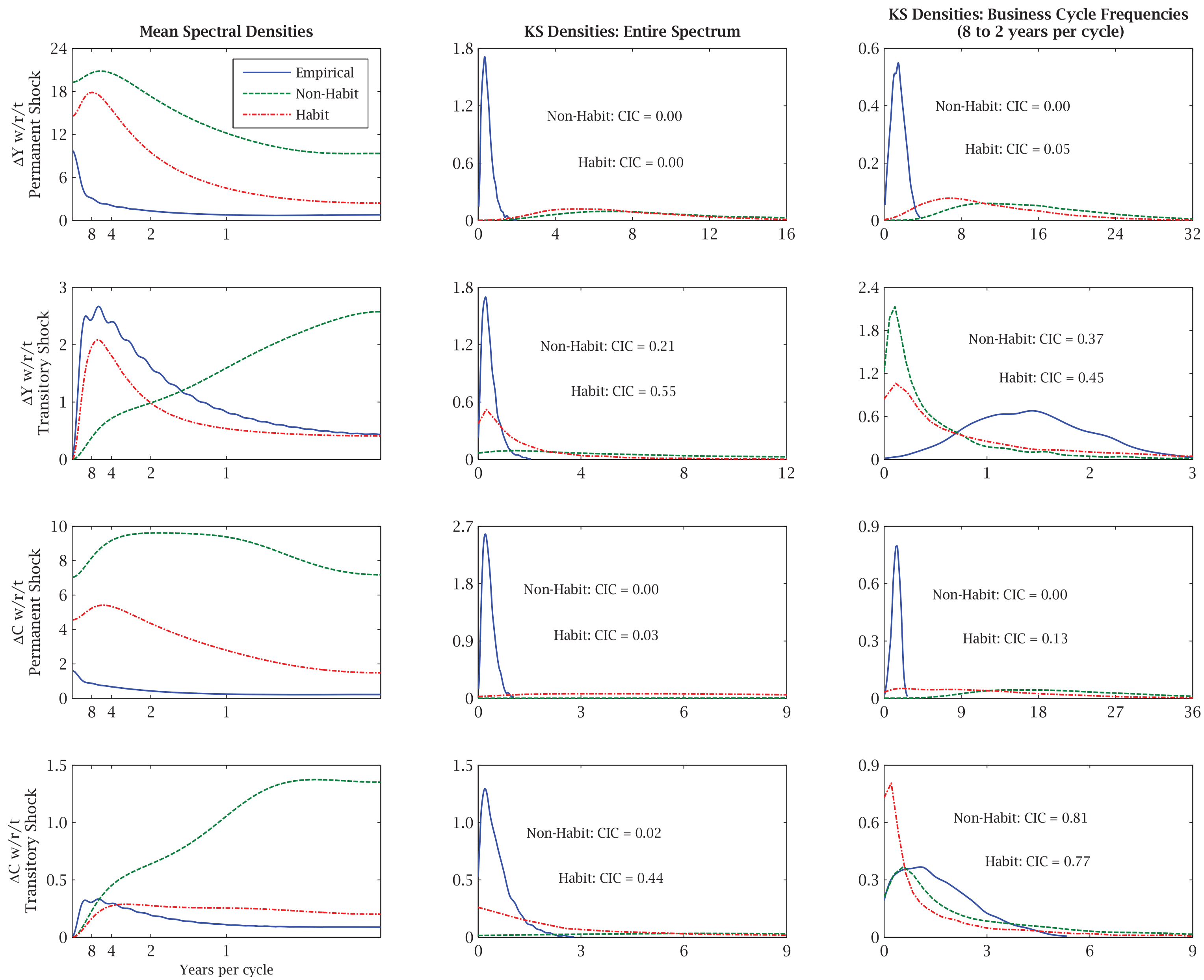
See section 4.3 for details.

Figure 7: Mean Structural $\mathcal{E}$ and $\mathcal{T}$ SDs and KS Densities for NKDSGE Models with AR(1) Money Growth Rule and Only Sticky Wages



See section 4.3 for details.

Figure 8: Mean Structural $\mathcal{E}$ and $\mathcal{T}$ SDs and KS Densities for NKDSGE Models with Taylor Rule and only Sticky Wages



See section 4.3 for details.

Appendix

The appendix consists of five sections. The sample data is described in section **A0**. Section **A1** enlarges on our discussion in the paper about the internal consumption habit propagation mechanism. We present optimality and equilibrium conditions of the baseline habit new Keynesian dynamic stochastic general equilibrium (NKDSGE) model in section **A2**, along with stochastically detrended, steady state, and linearized versions of these equations. This section also outlines the algorithm applied to solve the linearized NKDSGE models. The next section gives instructions to identify and estimate infinite order structural vector moving averages, SVMA(∞)s. Next, we engage existing literature to show that the SVMA(∞)s retrieve the economic shocks of the NKDSGE models. This is followed by formulas to compute the identified permanent and transitory output and consumption growth spectral densities, $SD_{\Delta Y}$ and $SD_{\Delta C}$. Section **A4** completes the appendix with summaries of NKDSGE model fit given the habit parameter h is endowed with a β prior, two different uniform priors, SVMA(∞)s are estimated with VAR(4)s instead of VAR(2)s, and the Kolmogorov-Smirnov goodness of fit statistic is replaced with the Cramer-von Mises goodness of fit statistics.

A0. Data Sources and Construction

This section sketches the 1954Q1–2002Q4 sample data. The source of the data is FRED-II maintained by the Federal Reserve Bank of St. Louis at *http://research.stlouisfed.org/fred2/*. Mnemonics appear in parentheses. The NIPA data are real chained 1996 billion dollars and seasonally adjusted at annual rates. The consumption series equals *Real Personal Consumption Expenditures on Nondurables* (PCNDGC96) plus *Real Personal Consumption Expenditures on Services* (PCESVC96). Investment is constructed by adding together *Real Personal Consumption Expenditures on Durables* (PCDGCC96), *Real Gross Private Domestic Investment* (GPDIC1), *Real National Defense Gross Investment* (DGIC96), and *Real Federal Nondefense Gross Investment* (NDGIC96). Government spending subtracts *Real National Defense Gross Investment* plus *Real*

Federal Nondefense Gross Investment from *Real Government Consumption Expenditures and Gross Investment* (GCEC1). Output equals the sum of consumption, investment and government spending. Aggregate quantities are divided by *Civilian Labor Force* (CLF16OV) to create per capita series. Since the *Civilian Labor Force* is monthly, temporal aggregation produces quarterly observations. Finally, the money stock is equated with the seasonally adjusted, *St. Louis Adjusted Monetary Base* (AMBSL). This monthly series is temporally aggregated to obtain a quarterly series and also made per capita.

A1. Consumption Dynamics under Internal and External Consumption Habits

This sections studies the propagation mechanism of additive internal consumption habit. We also show that subsequent to log linearization additive internal and external consumption habit produce observationally equivalent consumption growth dynamics up to a normalization on the impact shock of the AR(1) real rate.

A1.1 The Internal Consumption Habit Propagation Mechanism

Section 2.2 of the paper presents a calibration exercise that discusses the additive internal consumption habit propagation mechanism. The discussion begins with the Euler equation

$$\lambda_t = \beta \mathbf{E}_t \left\{ \frac{\lambda_{t+1} R_{t+1}}{1 + \pi_{t+1}} \right\}, \tag{$\mathcal{A}$1.1}$$

where the forward-looking marginal utility of consumption is $\lambda_t = \dfrac{1}{c_t - hc_{t-1}} - \mathbf{E}_t \left\{ \dfrac{\beta h}{c_{t+1} - hc_t} \right\}$, h is the habit parameter, c_t is household consumption, β is the household discount factor, the mathematical expectations operator conditional on date t information is $\mathbf{E}_t\{\cdot\}$, R_t is the nominal rate, and $1 + \pi_{t+1}$ $(= P_{t+1}/P_t)$ is date $t+1$ inflation. Given a random walk (with drift) drives total factor productivity (TFP) A_t, the Euler equation ($\mathcal{A}$1.1) and λ_t are stochastically detrended according to

$$\hat{\lambda}_t = \beta \mathbf{E}_t \left\{ \frac{\hat{\lambda}_{t+1} R_{t+1}}{\alpha_{t+1}(1 + \pi_{t+1})} \right\}, \tag{$\mathcal{A}$1.2}$$

and

$$\hat{\lambda}_t = \frac{\alpha_t}{\alpha_t \hat{c}_t - h\hat{c}_{t-1}} - \mathbf{E}_t \left\{ \frac{\beta h}{\alpha_{t+1}\hat{c}_{t+1} - h\hat{c}_t} \right\}, \tag{$\mathcal{A}$1.3}$$

where $\hat{\lambda}_t \equiv A_t \lambda_t$ and $\alpha_t = A_t / A_{t-1} = \exp(\alpha + \varepsilon_t)$, $\alpha > 0$, and ε_t is the mean zero, homoskedastic TFP shock innovation. Since household consumption and the marginal utility of consumption are stationary, the Euler equation ($\mathcal{A}$1.2) and marginal utility function ($\mathcal{A}$1.3) can be log linearized around the means (*i.e.* steady state) of $\hat{\lambda}_t$, $\hat{c}_t$, R_t, and π_t. The results are

$$\tilde{\lambda}_t = \mathbf{E}_t \left\{ \tilde{\lambda}_{t+1} - \varepsilon_{t+1} + \tilde{R}_{t+1} - \frac{\pi^*}{1 + \pi^*}\tilde{\pi}_{t+1} \right\}, \tag{$\mathcal{A}$1.4}$$

and

$$(\alpha^* - \beta h)(\alpha^* - h)\tilde{\lambda}_t = \alpha^* \beta h \mathbf{E}_t \tilde{c}_{t+1} - (\beta h^2 + \alpha^{*2})\tilde{c}_t + \alpha^* h \tilde{c}_{t-1} - \alpha^* \beta h \mathbf{E}_t \varepsilon_{t+1} + \alpha^* h \varepsilon_t, \tag{$\mathcal{A}$1.5}$$

where, for example, $\tilde{c}_t = \ln \hat{c}_t - \ln c^*$ or $\tilde{R}_t = \ln R_t - \ln R^*$, and $\alpha^* = \exp(\alpha)$ is the deterministic TFP growth rate. We combine equations ($\mathcal{A}$1.4) and ($\mathcal{A}$1.5) to obtain

$$\begin{aligned} &\alpha^* \beta h \mathbf{E}_t \left\{ \Delta\tilde{c}_{t+2} + \varepsilon_{t+2} \right\} - (\beta h^2 + \alpha^{*2})\mathbf{E}_t \left\{ \Delta\tilde{c}_{t+1} + \varepsilon_{t+1} \right\} + \alpha^* h(\Delta\tilde{c}_t + \varepsilon_t) \\ &\qquad = -(\alpha^* - \beta h)(\alpha^* - h)\mathbf{E}_t \tilde{q}_{t+1}, \end{aligned} \tag{$\mathcal{A}$1.6}$$

where the demeaned real rate is $\tilde{q}_t = \tilde{R}_t - \frac{\pi^*}{1 + \pi^*}\tilde{\pi}_t$. By exploiting stochastic detrending, the linearized Euler equation ($\mathcal{A}1.6$) can be written as a second-order expectational stochastic difference equation in demeaned household consumption growth

$$\alpha^* \beta h \mathbf{E}_t \widetilde{\Delta c}_{t+2} - (\beta h^2 + \alpha^{*2}) \mathbf{E}_t \widetilde{\Delta c}_{t+1} + \alpha^* h \widetilde{\Delta c}_t = -(\alpha^* - \beta h)(\alpha^* - h) \mathbf{E}_t \widetilde{q}_{t+1}, \qquad (\mathcal{A}1.7)$$

where $\widetilde{\Delta c}_t = \Delta \ln \widehat{c}_t + \varepsilon_t$ denotes demeaned household consumption growth.

We solve equation ($\mathcal{A}1.7$) to obtain the backward-looking stable root $\varphi_1 = h\alpha^{*-1}$ and forward-looking unstable root $\varphi_2 = \alpha^*/(\beta h)$. These roots are exploited by the lag polynomial $-\mathbf{L}^{-1}(1 - \varphi_1 \mathbf{L})(1 - \varphi_2^{-1}\mathbf{L}^{-1})\varphi_2 \alpha^* \beta h \widetilde{\Delta c}_t$, which is an alternative to the left side of equation ($\mathcal{A}1.7$). After applying the lag polynomial, we have

$$\left(1 - \frac{h}{\alpha^*}\mathbf{L}\right) \widetilde{\Delta c}_t = \frac{(\alpha^* - \beta h)(\alpha^* - h)}{\alpha^{*2}} \sum_{j=0}^{\infty} \left(\frac{\beta h}{\alpha^*}\right)^j \mathbf{E}_t \widetilde{q}_{t+j}, \qquad (\mathcal{A}1.8)$$

which is the unique (*i.e.*, sunspot free) solution of the second-order stochastic difference equation ($\mathcal{A}1.7$). This solution is equation (2) of the paper, where $\Psi = \dfrac{(\alpha^* - \beta h)(\alpha^* - h)}{\alpha^* \beta h}$. Equation ($\mathcal{A}1.8$) is forward-looking in the expected discounted present value of $\widetilde{q}_t$ and backward-looking in the lag of demeaned consumption growth. Assume $\widetilde{q}_t$ is a AR(1) with persistence parameter ρ_q. In this case, the Wiener-Kolmogorov formulas alter equation ($\mathcal{A}1.8$) to

$$\left(1 - \frac{h}{\alpha^*}\mathbf{L}\right) \widetilde{\Delta c}_t = \frac{(\alpha^* - \beta h)(\alpha^* - h)}{\alpha^*(\alpha^* - \rho_q \beta h)} \widetilde{q}_t. \qquad (\mathcal{A}1.9)$$

We employ equation ($\mathcal{A}1.9$), put h on the grid $[0.15\ 0.35\ 0.50\ 0.65\ 0.85]$, calibrate $[\beta\ \ \alpha^*]' = [0.993\ \ \exp(0.004)]'$, and estimate a AR(1) for $\widetilde{q}_t$ to generate the impulse response functions plotted in figure 1.

The real federal funds rate $\widetilde{q}_t$ is measured with the demeaned quarterly nominal federal funds rate and demeaned implicit GDP deflator inflation. The latter is multiplied by the ratio of its mean to one plus its mean and subtracted from the former to create the real federal funds rate $\tilde{q}_t$ on a 1954Q1–2002Q4 sample. Although likelihood ratio tests and the Hannan-Quinn criterion suggest a AR(3), we settle on a AR(1) using the SIC against AR(2) to AR(10) specifications. On the 1954Q1–2002Q4 sample, OLS estimates of the AR(1) of $\tilde{q}_t$ are ρ_q = 0.8687 and the standard error of the regression is 1.2059.

A1.2 An Observational Equivalence Result for Internal and External Consumption Habits

We show in this section that under internal and external additive consumption habit the preferences $\ln[c_t - hc_{t-1}]$ yield log linearized Euler equations that are observationally equivalent up to a normalization of the AR(1) real rate, $\tilde{q}_t$. The consumption habit specification $\ln[c_t - hc_{t-1}]$ is found in the NKDSGE models that Christiano, Eichenbaum, and Evans (2005) and Smets and Wouter (2007) estimate. The latter (former) paper uses internal (external) consumption habit. According to Dennis (2009), the economic content of estimates of linearized NKDSGE models appears to be unaffected by the choice of consumption habit specification. He also shows that the mapping from additive to multiplicative (*i.e.* 'keeping up with the Jones') consumption habit parameters is onto (only in this direction).

Additive external consumption habit restricts the marginal utility of consumption to be purely backward-looking. After stochastic detrending, $\hat{\lambda}_{t,ECH} = \dfrac{\alpha_t}{\alpha_t\hat{c}_t - h\hat{c}_{t-1}}$, where *ECH* denotes external consumption habit. The log linearized Euler equation ($\mathcal{A}$1.4) becomes

$$\alpha^*\tilde{c}_t - h\tilde{c}_{t-1} + h\varepsilon_t = \mathbf{E}_t\Big\{\alpha^*\tilde{c}_{t+1} - h\tilde{c}_t + \alpha^*\varepsilon_{t+1} + (\alpha^* - h)\tilde{q}_{t+1}\Big\}, \qquad (\mathcal{A}1.10)$$

with $(\alpha^* - h)\tilde{\lambda}_{t,ECH} = -\alpha^*\tilde{c}_t + \alpha^*\tilde{c}_{t-1} - h\varepsilon_t$. A bit of rearranging transforms the linearized Euler equation ($\mathcal{A}$1.10) into the first-order expectational stochastic difference equation

$$\mathbf{E}_t\Big\{\Delta\tilde{c}_{t+1} + \varepsilon_{t+1}\Big\} = \frac{h}{\alpha^*}(\Delta\tilde{c}_t + \varepsilon_t) - \frac{\alpha^* - h}{\alpha^*}\mathbf{E}_t\tilde{q}_{t+1}. \qquad (\mathcal{A}1.11)$$

Given a mean zero, homoskedastic expectation error $\vartheta_{\tilde{c},t} = \tilde{c} - \mathbf{E}_{t-1}\tilde{c}$, the first-order stochastic difference equation ($\mathcal{A}$1.11) can be written

$$\left(1 - \frac{h}{\alpha^*}\mathbf{L}\right)\widetilde{\Delta c}_t \;=\; \frac{\alpha^* - h}{\alpha^*}\tilde{q}_t + \vartheta_{\tilde{c},t}, \qquad (\mathcal{A}1.12)$$

which represents reduced-form consumption growth dynamics under additive ECH.

Equations ($\mathcal{A}$1.9) and ($\mathcal{A}$1.12) produce observationally equivalent dynamics in $\widetilde{\Delta c}_t$ up to the impact coefficient on $\tilde{q}_t$ given it is a AR(1). The dynamics are equivalent because equations ($\mathcal{A}$1.9) and ($\mathcal{A}$1.12) share the same leading autoregressive root, which equals h/α^*. Thus across additive internal and external consumption habit, a shock to $\tilde{q}_t$ generates identical responses in $\widetilde{\Delta c}_t$ beyond impact. Only at impact can internal and external consumption habit yield disparate responses in $\widetilde{\Delta c}_t$ to an innovation in $\tilde{q}_t$. As $h \longrightarrow 1$ the impact responses of $\widetilde{\Delta c}_t$ differ by a factor of 12 for internal and external consumption habit at the calibration of section 2.2, but the impact responses converge as $h \longrightarrow 0$.

A2. Solving the Habit NKDSGE Models

This section presents the optimality and equilibrium conditions of the baseline habit NKDSGE models, the stochastically detrended versions of these conditions, the steady state of this economy, the log linearized optimality and equilibrium conditions, and solution method invoked to compute a multivariate linear approximate equilibrium law of motion.

A2.1 Optimality and equilibrium conditions

The baseline habit NKDSGE models have first-order necessary conditions (FONCs) that are restricted by the primitives of preferences, technology, market structure, and monetary policy regime. The FONCs imply optimality and equilibrium conditions that must be satisfied by any candidate equilibrium time series. The optimality and equilibrium conditions are

$$\lambda_t = \frac{1}{C_t - hC_{t-1}} - \beta h \mathbf{E}_t \left\{ \frac{1}{C_{t+1} - hC_t} \right\}, \tag{$\mathcal{A}$2.1}$$

$$\frac{1-q_t}{q_t} + S\left(\frac{X_t}{\alpha^* X_{t-1}}\right) + S'\left(\frac{X_t}{\alpha^* X_{t-1}}\right)\frac{X_t}{\alpha^* X_{t-1}} = \frac{\beta}{\alpha^*}\mathbf{E}_t\left\{\frac{\lambda_{t+1} q_{t+1}}{\lambda_t q_t} S'\left(\frac{X_{t+1}}{\alpha^* X_t}\right)\left[\frac{X_{t+1}}{X_t}\right]^2\right\}, \tag{$\mathcal{A}$2.2}$$

$$q_t = \beta \mathbf{E}_t \left\{ \frac{\lambda_{t+1}}{\lambda_t} \left[\psi u_{t+1} \phi_{t+1} \frac{Y_{A,t+1}}{K_{t+1}} - a(u_{t+1}) + q_{t+1}(1-\delta) \right] \right\}, \tag{$\mathcal{A}$2.3}$$

$$\frac{\lambda_t}{P_t} = \beta \mathbf{E}_t \frac{\lambda_{t+1}}{P_{t+1}} R_{t+1}, \tag{$\mathcal{A}$2.4}$$

$$\frac{\lambda_t}{P_t} = \beta \mathbf{E}_t \left\{ \frac{\lambda_{t+1}}{P_{t+1}} + \frac{1}{M_{t+1}} \right\}, \tag{$\mathcal{A}$2.5}$$

$$a'(u_t) = \psi \phi_t \frac{Y_{A,t}}{K_t}, \tag{$\mathcal{A}$2.6}$$

$$\frac{W_t}{P_t} = \phi_t (1-\psi) \frac{Y_{A,t}}{N_t - N_0}, \tag{$\mathcal{A}$2.7}$$

$$\frac{Y_{A,t}}{Y_{D,t}} = \left(\frac{P_{A,t}}{P_t}\right)^{-\xi}, \qquad (\mathcal{A}2.8)$$

$$\frac{P_{c,t}}{P_{t-1}} = \left(\frac{\xi}{\xi-1}\right) \frac{\mathbf{E}_t \sum_{i=0}^{\infty} \left[\beta\mu_P\right]^i \lambda_{t+i}\phi_{t+i} Y_{D,t+i} \left[\frac{P_{t+i}}{P_{t+i-1}}\right]^{\xi}}{\mathbf{E}_t \sum_{i=0}^{\infty} \left[\beta\mu_P\right]^i \lambda_{t+i} Y_{D,t+i} \left[\frac{P_{t+i}}{P_{t+i-1}}\right]^{\xi-1}}, \qquad (\mathcal{A}2.9)$$

$$\frac{N_t}{n_t} = \left(\frac{W_{D,t}}{W_t}\right)^{-\xi}, \qquad (\mathcal{A}2.10)$$

$$\left[\frac{W_{c,t}}{P_{t-1}}\right]^{1+\theta/\gamma} = \left(\frac{\theta}{\theta-1}\right) \frac{\mathbf{E}_t \sum_{i=0}^{\infty} \left[\beta\mu_W \alpha^{*-\theta(1+1/\gamma)}\right]^i \left[\left[\frac{W_{t+i}}{P_{t+i-1}}\right]^{\theta} N_{t+i}\right]^{1+1/\gamma}}{\mathbf{E}_t \sum_{i=0}^{\infty} \left[\beta\mu_W \alpha^{*(1-\theta)}\right]^i \lambda_{t+i} \left[\frac{W_{t+i}}{P_{t+i-1}}\right]^{\theta} \left[\frac{P_{t+i}}{P_{t+i-1}}\right]^{-1} N_{t+i}}, \qquad (\mathcal{A}2.11)$$

$$K_{t+1} = (1-\delta)K_t + \left[1 - S\left(\frac{X_t}{\alpha^* X_{t-1}}\right)\right] X_t, \qquad (\mathcal{A}2.12)$$

$$Y_{D,t} = C_t + X_t + a(u_t)K_t, \qquad (\mathcal{A}2.13)$$

$$Y_{A,t} = \left[u_t K_t\right]^{\psi} \left[(N_t - N_0)A_t\right]^{1-\psi}, \qquad (\mathcal{A}2.14)$$

$$P_t^{1-\xi} = \mu_P \left[\frac{P_{t-1}}{P_{t-2}} P_{t-1} \right]^{1-\xi} + (1-\mu_P) P_{c,t}^{1-\xi}, \tag{$\mathcal{A}$2.15}$$

$$P_{A,t}^{-\xi} = \mu_P \left[\frac{P_{A,t-1}}{P_{A,t-2}} P_{A,t-1} \right]^{-\xi} + (1-\mu_P) P_{c,t}^{-\xi}, \tag{$\mathcal{A}$2.16}$$

$$W_t^{1-\theta} = \mu_W \left(\alpha * \frac{P_{t-1}}{P_{t-2}} W_{t-1} \right)^{1-\theta} + (1-\mu_W) W_{c,t}^{1-\theta}, \tag{$\mathcal{A}$2.17}$$

and

$$W_{D,t}^{-\theta} = \mu_W \left(\alpha * \frac{P_{t-1}}{P_{t-2}} W_{D,t-1} \right)^{-\theta} + (1-\mu_W) W_{c,t}^{-\theta}, \tag{$\mathcal{A}$2.18}$$

where λ_t, C_t, q_t, $S(\cdot)$, X_t, u_t, r_t, δ, $a(u_t)$, P_t, M_t, $Y_{A,t}$, $P_{A,t}$, ξ, $Y_{D,t}$, ϕ_t, ψ, K_t, W_t, $P_{c,t}$, μ_P, θ, μ_W, $W_{c,t}$, $W_{D,t}$, and γ denote the marginal utility of consumption, aggregate consumption, the shadow price of capital (per unit of consumption), the investment growth cost function, the deterministic TFP growth rate, aggregate investment, capital utilization rate, the rental rate of capital, the depreciation rate of capital, the household cost of capital utilization, the aggregate (demand) price level, the aggregate money stock at the end of date $t-1$, aggregate output, the aggregate supply price level, the price elasticity, aggregate demand, real marginal cost, capital's share of output, the aggregate capital stock at the end of date $t-1$, the aggregate nominal wage, the firm's optimal date t price, the fraction of firms forced to update their price at the previous period's inflation rate, the wage elasticity, the fraction of households forced to update their nominal wage at the previous period's inflation rate, the household's optimal date t nominal wage, the aggregate demand nominal wage, and the inverse of the Frisch labor supply elasticity, respectively.

A symmetric equilibrium is imposed on the markets in which final good firms and households have monopolistic power. Along the symmetric equilibrium path, firms i and j choose the same commitment price $P_{c,t} = P_{i,t} = P_{j,t}$. The same restriction is placed on the nominal wages $W_{c,t} = W_{\ell,t} = W_{\wp,t}$ of households ℓ and $\wp$. The optimality conditions ($\mathcal{A}$2.9) and ($\mathcal{A}$2.11) reflect the impact of the symmetric equilibrium assumptions. Rather than $P_{i,t}$ and $W_{\ell,t}$, the symmetric equilibrium impose the final good price $P_{c,t}$ and nominal wage $W_{c,t}$ on the optimality conditions ($\mathcal{A}$2.9) and ($\mathcal{A}$2.11).

The impulse vector consists of TFP and monetary policy shocks. We assume TFP, $\ln A_t$, is a random walk with drift

$$\ln A_t = \alpha + \ln A_{t-1} + \varepsilon_t, \quad \varepsilon_t \sim \mathcal{N}\left(0, \sigma_\varepsilon^2\right). \qquad (\mathcal{A}2.19)$$

The monetary policy shock is either the innovation μ_t of the first order autoregression, AR(1), money growth (MG) supply rule

$$m_{t+1} = (1-\rho_m)m^* + \rho_m m_t + \mu_t, \quad \left|\rho_m\right| < 1, \quad \mu_t \sim \mathcal{N}\left(0, \sigma_\mu^2\right), \qquad (\mathcal{A}2.20)$$

of the NKDSGE-MG model, where m^* is the steady state money growth rate, or the innovation υ_t to the interest rate smoothing Taylor rule (TR)

$$R_t = \rho_R R_{t-1} + (1-\rho_R)\left(R^* + a_\pi \mathbf{E}_t\left\{\frac{P_{t+1}}{P_t}\right\} + a_{\tilde{Y}}\tilde{Y}_t\right) + \upsilon_t, \ \left|\rho_R\right| < 1, \ \upsilon_t \sim \mathcal{N}\left(0, \sigma_\upsilon^2\right), \quad (\mathcal{A}2.21)$$

of the NKDSGE-TR model, where the steady state nominal rate R^* is the ratio of steady state inflation to the household discount factor, π^*/β, π^* equals the differential of steady state money growth and deterministic TFP growth, $\exp(m^* - \alpha)$, and $\widetilde{Y}_t$ is the output gap (*i.e.*, deviations of output from its trend). The TFP and money growth (or Taylor rule) innovations are assumed to be uncorrelated at leads and lags, $\mathbf{E}\{\varepsilon_{t+i}\,\mu_{t+j}\} = 0$, (or $\mathbf{E}\{\varepsilon_{t+i}\,\upsilon_{t+j}\} = 0$) for all i, j.

Equations ($\mathcal{A}$2.1)–($\mathcal{A}$2.11) are the optimality conditions of the baseline habit NKDSGE model. Internal consumption habit creates the forward-looking marginal utility of current consumption, which is restated by equation ($\mathcal{A}$2.1). Equation ($\mathcal{A}$2.2) sets the cost of adding one unit of aggregate investment, X_t, to its discounted expected benefit. The cost is represented by the ratio of the cost of installing a unit of investment to the market value of extant capital (*i.e.*, the inverse of Tobin's q), $(1 - q_t)/q_t$, plus the total cost of installing a unit of investment and the marginal cost of adding a unit of capital at the investment growth rate, X_t/X_{t-1}, net of steady state growth α^*. The expected benefit equals the foregone marginal cost of future investment valued at the pricing kernel, $\beta\lambda_{t+1}/\lambda_t$, which is weighted by the change in the price of capital. The Euler equation of capital ($\mathcal{A}$2.3) equates the price of increasing the capital stock by one unit to the discounted expected return on the service flow of that unit of capital net of the cost of capital services (or utilization) plus the net value of the unit of capital after production evaluated at the pricing kernel. The riskless bond is priced in the Euler equation ($\mathcal{A}$2.4). The dynamics of the purchasing power of money is described by the Euler equation ($\mathcal{A}$2.5), where money is valued at the marginal utility of consumption. Equations ($\mathcal{A}$2.4) and ($\mathcal{A}$2.5) yield the money demand function of the baseline habit NKDGSE model. Equation ($\mathcal{A}$2.6) is an intratemporal optimality condition that forces the marginal capital utilization rate to match the marginal product of capital, which equals the rental rate of capital. Final good firm labor demand is tied down by the intratemporal optimality condition ($\mathcal{A}$2.7). The ratio of aggregate supply to aggregate demand is connected to the ratio of the alternative aggregate price level to the aggregate price level raised to the negative of the price elasticity by equation

($\mathcal{A}$2.8). Equation ($\mathcal{A}$2.9) specifies optimal pricing of a monopolistically competitive final good firm. This decision is restricted by the Calvo staggered price technology, the firm's discount factor, real marginal cost, aggregate demand, and full indexation to lagged inflation of those firms unable to obtain their optimal price at date t. Aggregate labor demand is equated to aggregate labor supply in equation ($\mathcal{A}$2.10) up to the ratio of the aggregate nominal wage indices raised to the negative of the nominal wage elasticity. The optimal nominal wage decision is characterized by equation ($\mathcal{A}$2.11). The household settles on its optimal nominal wage by balancing the discounted expected disutility of labor supply to the benefits of greater real labor income in marginal utility of consumption units (*i.e.*, the marginal rate of substitution between the expected discounted lifetime disutility of work to the expected discounted value of permanent income). Note that these costs and benefits are affected by the wage and labor supply elasticities, and that those households unable to update their date t nominal wage reset using lagged inflation.

Equilibrium conditions are given by equations ($\mathcal{A}$2.12)–($\mathcal{A}$2.18) for the baseline habit NKDSGE model. Equation ($\mathcal{A}$2.12) is the law of motion of capital with capital adjustment costs. Aggregate demand equals its constituent parts according to equation ($\mathcal{A}$2.13). The constant returns to scale aggregate technology is found in equation ($\mathcal{A}$2.14). Equations ($\mathcal{A}$2.15), ($\mathcal{A}$2.16), ($\mathcal{A}$2.17), and ($\mathcal{A}$2.18) are the laws of motion of the aggregate price levels and nominal wages under Calvo price and nominal wage setting with full indexation.

The laws of motion ($\mathcal{A}$2.16) and ($\mathcal{A}$2.18) are added to avoid the curse of dimensionality. Under Calvo staggered price and nominal wage setting, Yun (1996) points out that the price and nominal wage aggregators ($\mathcal{A}$2.15) and ($\mathcal{A}$2.17) place the histories P_t and W_t (from date t = 0) into the state vector of the baseline habit NKDSGE model. The reason is the histories of P_t and W_t drive the process that restrict $P_{C,t}$ and $W_{C,t}$ along any candidate equilibrium path. The aggregate supply price and aggregate demand nominal wage laws of motion ($\mathcal{A}$2.16) and ($\mathcal{A}$2.18) are used to replace $P_{C,t}$ and $W_{C,t}$ with $P_{A,t}$ and $W_{D,t}$ in the state vector. This leaves the state vector with P_t, $P_{A,t}$, W_t, and $W_{D,t}$ rather than their histories.

A2.2 Stochastically detrended optimality and equilibrium conditions

The NKDSGE models contain a permanent technology shock A_t. Since this shock is a random walk (with drift), stochastic detrending renders the equilibrium path of state and other endogenous variables stationary. Stochastic detrending consists of $\hat{C}_t \equiv C_t/A_t$, $\hat{X}_t \equiv X_t/A_t$, $\hat{Y}_{j,t} \equiv Y_{j,t}/A_t$, $j = A, D$, $\hat{K}_{t+1} \equiv K_{t+1}/A_t$, $\hat{P}_t \equiv P_t A_t/M_t$, $\hat{P}_{i,t} \equiv P_{i,t} A_t/M_t$, $i = A, c$, $\widehat{W}_t \equiv W_t/M_t$, and $\widehat{W}_{c,t} \equiv W_{\wp,t}/M_t$, $\wp = D, c$. Applying these definitions to equations ($\mathcal{A}$2.1)–($\mathcal{A}$2.18) yields the stochastically detrended optimality and equilibrium conditions

$$\hat{\lambda}_t = \frac{\alpha_t}{\alpha_t \hat{C}_t - h\hat{C}_{t-1}} - \beta h \mathbf{E}_t \left\{ \frac{1}{\alpha_{t+1}\hat{C}_{t+1} - h\hat{C}_t} \right\}, \tag{$\mathcal{A}$2.22}$$

$$\frac{1-q_t}{q_t} + S\left(\frac{\alpha_t \hat{X}_t}{\alpha^* \hat{X}_{t-1}}\right) + S'\left(\frac{\alpha_t \hat{X}_t}{\alpha^* \hat{X}_{t-1}}\right)\frac{\alpha_t \hat{X}_t}{\alpha^* \hat{X}_{t-1}}$$

$$= \frac{\beta}{\alpha^*}\mathbf{E}_t \left\{ \alpha_{t+1} \frac{q_{t+1}\hat{\lambda}_{t+1}}{q_t \hat{\lambda}_t} S'\left(\frac{\alpha_{t+1}\hat{X}_{t+1}}{\alpha^* \hat{X}_t}\right)\left[\frac{\hat{X}_{t+1}}{\hat{X}_t}\right]^2 \right\}, \tag{$\mathcal{A}$2.23}$$

$$q_t = \beta \mathbf{E}_t \left\{ \frac{\hat{\lambda}_{t+1}}{\hat{\lambda}_t} \left[\psi u_{t+1} \phi_{t+1} \frac{\hat{Y}_{t+1}}{\hat{K}_{t+1}} + \frac{q_{t+1}[1-\delta] - a(u_{t+1})}{\alpha_{t+1}} \right] \right\}, \tag{$\mathcal{A}$2.24}$$

$$\frac{\hat{\lambda}_t}{\hat{P}_t} = \beta \mathbf{E}_t \left\{ \frac{\hat{\lambda}_{t+1}}{\hat{P}_{t+1}} \frac{R_{t+1}}{\exp(m_{t+1})} \right\}, \tag{$\mathcal{A}$2.25}$$

$$\frac{\widehat{\lambda}_t}{\widehat{P}_t} = \beta \mathbf{E}_t \left\{ \left[\frac{\widehat{\lambda}_{t+1}}{\widehat{P}_{t+1}} + 1 \right] \exp(-m_{t+1}) \right\}, \tag{$\mathcal{A}$2.26}$$

$$a'(u_t) = \psi \phi_t \alpha_t \frac{\widehat{Y}_t}{\widehat{K}_t}, \tag{$\mathcal{A}$2.27}$$

$$\frac{\widehat{W}_t}{\widehat{P}_t} = (1 - \psi) \phi_t \frac{\widehat{Y}_t}{N_t - N_0}, \tag{$\mathcal{A}$2.28}$$

$$\frac{\widehat{Y}_{A,t}}{\widehat{Y}_{D,t}} = \left(\frac{\widehat{P}_{A,t}}{\widehat{P}_t} \right)^{-\xi}, \tag{$\mathcal{A}$2.29}$$

$$\exp(m_t - \varepsilon_t) \frac{\widehat{P}_{c,t}}{\widehat{P}_{t-1}} =$$

$$\left(\frac{\xi}{\xi - 1} \right) \frac{\mathbf{E}_t \sum_{i=0}^{\infty} (\beta \mu_P)^i \, \widehat{\lambda}_{t+i} \phi_{t+i} \widehat{Y}_{D,t+i} \left[\exp(m_{t+i} - \varepsilon_{t+i}) \frac{\widehat{P}_{t+i}}{\widehat{P}_{t+i-1}} \right]^{\xi}}{\mathbf{E}_t \sum_{i=0}^{\infty} (\beta \mu_P)^i \, \widehat{\lambda}_{t+i} \widehat{Y}_{D,t+i} \left[\exp(m_{t+i} - \varepsilon_{t+i}) \frac{\widehat{P}_{t+i}}{\widehat{P}_{t+i-1}} \right]^{\xi - 1}}, \tag{$\mathcal{A}$2.30}$$

$$\frac{N_t}{n_t} = \left(\frac{\widehat{W}_{D,t}}{\widehat{W}_t} \right)^{-\xi}, \tag{$\mathcal{A}$2.31}$$

$$\left[\exp(m_t)\frac{\widehat{W}_{c,t}}{\widehat{P}_{t-1}}\right]^{1+\theta/\gamma} = \frac{\theta}{\theta-1}$$

$$\times\frac{\mathbf{E}_t\sum_{i=0}^{\infty}(\beta\mu_W)^i\exp(\theta(1+1/\gamma)(m_{t+i}+\sum_{j=1}^{i}\varepsilon_{t+j-1})\left[\left[\frac{\widehat{W}_{t+i}}{\widehat{P}_{t+i-1}}\right]^{\theta}N_{t+i}\right]^{1+1/\gamma}}{\mathbf{E}_t\sum_{i=0}^{\infty}(\beta\mu_W)^i\lambda_{t+i}\exp(-(1-\theta)(m_{t+i}+\sum_{j=1}^{i}\varepsilon_{t+j-1}))\left[\frac{\widehat{W}_{t+i}}{\widehat{P}_{t+i-1}}\right]^{\theta}\left[\frac{\widehat{P}_{t+i}}{\widehat{P}_{t+i-1}}\right]^{-1}N_{t+i}}, \quad (\mathcal{A}2.32)$$

$$\hat{K}_{t+1} = \frac{(1-\delta)\hat{K}_t}{\alpha_t} + \left[1 - S\left(\frac{\alpha_t\hat{X}_t}{\alpha^*\hat{X}_{t-1}}\right)\right]\hat{X}_t, \quad (\mathcal{A}2.33)$$

$$\hat{Y}_t = \hat{C}_t + \hat{X}_t + \frac{a(u_t)\hat{K}_t}{\alpha_t}, \quad (\mathcal{A}2.34)$$

$$\hat{Y}_t = \left[u_t\frac{\hat{K}_t}{\alpha_t}\right]^{\psi}\left[N_t - N_O\right]^{1-\psi}, \quad (\mathcal{A}2.35)$$

$$\hat{P}_t^{1-\xi} = \mu_P\left[\exp(-m_t + m_{t-1} + \varepsilon_t - \varepsilon_{t-1})\frac{\hat{P}_{t-1}}{\hat{P}_{t-2}}\hat{P}_{t-1}\right]^{1-\xi} + (1-\mu_P)\hat{P}_{c,t}^{1-\xi}, \quad (\mathcal{A}2.36)$$

$$\hat{P}_{A,t}^{-\xi} = \mu_P\left[\exp(-m_t + m_{t-1} + \varepsilon_t - \varepsilon_{t-1})\frac{\hat{P}_{A,t-1}}{\hat{P}_{A,t-2}}\hat{P}_{A,t-1}\right]^{-\xi} + (1-\mu_P)\hat{P}_{c,t}^{-\xi}, \quad (\mathcal{A}2.37)$$

$$\widehat{W}_t^{1-\theta} = \mu_W \left[\exp(-m_t + m_{t-1} - \varepsilon_{t-1}) \frac{\widehat{P}_{t-1}}{\widehat{P}_{t-2}} \widehat{W}_{t-1} \right]^{1-\theta} + (1-\mu_W)\widehat{W}_{c,t}^{1-\theta}, \qquad (\mathcal{A}2.38)$$

and

$$\widehat{W}_{D,t}^{-\theta} = \mu_W \left[\exp(-m_t + m_{t-1} - \varepsilon_{t-1}) \frac{\widehat{P}_{t-1}}{\widehat{P}_{t-2}} \widehat{W}_{D,t-1} \right]^{-\theta} + (1-\mu_W)\widehat{W}_{c,t}^{-\theta}, \qquad (\mathcal{A}2.39)$$

where it is understood in equation ($\mathcal{A}$2.32) that at $i = 0$ the sum $\sum_{j=1}^{i} \varepsilon_{t+j-1}$ equals one. Equations ($\mathcal{A}$2.22)–($\mathcal{A}$2.39) constitute the basis of the steady state equilibrium and the first-order linear approximation of the baseline habit NKDSGE models.

A2.3 Deterministic steady state

Let λ^*, C^*, Y^*, X^*, N^*, K^*, q^*, W^*, r^*, P^*, u^*, ϕ^*, and R^* denote deterministic steady state values of the corresponding endogenous variables. The steady state equilibrium rests on $u^* = 1$, $a(1) = 0$, and $S(1) = S'(1) = 0$, which is consistent with Christiano, Eichenbaum, and Evans (2005). Given these assumptions, the following equations characterize the deterministic steady state of the stochastically detrended system ($\mathcal{A}$2.22)–($\mathcal{A}$2.39)

$$C^*\lambda^* = \frac{\alpha^* - \beta h}{\alpha^* - h}, \qquad (\mathcal{A}2.40)$$

$$q^* = 1, \qquad (\mathcal{A}2.41)$$

$$\frac{K^*}{Y^*} = \frac{\beta\alpha^*\psi\phi^*}{\alpha^* - \beta(1-\delta)}, \qquad (\mathcal{A}2.42)$$

$$R^* = \frac{\exp(m^*)}{\beta}, \qquad (\mathcal{A}2.43)$$

$$\frac{\lambda^*}{P^*} = \frac{\beta}{\exp(m^*) - \beta}, \qquad (\mathcal{A}2.44)$$

$$a'(1) = \psi\phi^*\alpha^*\frac{Y^*}{K^*}, \qquad (\mathcal{A}2.45)$$

$$\frac{W^*}{P^*} = (1-\psi)\phi^*\frac{Y^*}{N^* - N_0}, \qquad (\mathcal{A}2.46)$$

$$\phi^* = \frac{\xi - 1}{\xi}, \qquad (\mathcal{A}2.47)$$

$$\frac{W^*}{P^*} = \left(\frac{\theta}{\theta - 1}\right)\frac{N^{*1/\gamma}}{\lambda^*}, \qquad (\mathcal{A}2.48)$$

$$\frac{X^*}{K^*} = 1 - \frac{(1-\delta)}{\alpha^*}, \tag{$\mathcal{A}$2.49}$$

$$Y^* = C^* + X^*, \tag{$\mathcal{A}$2.50}$$

and

$$Y^* = \left[\frac{K^*}{\alpha^*}\right]^{\psi} \left[N^* - N_O\right]^{1-\psi}. \tag{$\mathcal{A}$2.51}$$

Note that equations ($\mathcal{A}$2.46), ($\mathcal{A}$2.48), and ($\mathcal{A}$2.51) imply that the solution for N^* is nonlinear. Also, at the steady state equilibrium, $P^* = P_A^* = P_c^*$ and $W^* = W_D^* = W_c^*$.

A2.4 Log-linearized baseline habit NKDSGE models

We log linearize the optimality and equilibrium conditions of the baseline NKDSGE models in this section. The log linear approximations (*i.e.*, first-order Taylor expansions) of the stochastically detrended system ($\mathcal{A}$2.22)–($\mathcal{A}$2.39) are around the deterministic steady state given by equations ($\mathcal{A}$2.40)–($\mathcal{A}$2.51). The approximations exploit, for example, the definitions $\tilde{C}_t = \ln \hat{C}_t - \ln C^*$ or $\tilde{N}_t = \ln N_t - \ln N^*$.

A symmetric equilibrium has several implications for the log linear approximation of the baseline habit NKDSGE models. Subsequent to log linearizing around the steady state, the aggregate price indices are equated $\tilde{P}_t = \tilde{P}_{A,t}$, as are the aggregate nominal wages $\widetilde{W}_t = \widetilde{W}_{D,t}$, given $P_0 = P_{A,0}$ and $W_0 = W_{D,0}$. This further reduces the dimension of the state vector.

Log linearizing the stochastically detrended system ($\mathcal{A}$2.22)–($\mathcal{A}$2.39) yields the linear approximate optimality and equilibrium conditions of the baseline habit NKDSGE model. The relevant conditions are

$$(\alpha^* - h)(\alpha^* - \beta h)\widetilde{\lambda}_t = \beta\alpha^* h\mathbf{E}_t\widetilde{C}_{t+1} - (\beta h^2 + \alpha^{*2})\widetilde{C}_t + \alpha^* h(\widetilde{C}_{t-1} - \varepsilon_t), \qquad (\mathcal{A}2.52)$$

$$\beta\varpi\mathbf{E}_t\widetilde{X}_{t+1} - (1+\beta)\varpi\widetilde{X}_t + \varpi\widetilde{X}_{t-1} + \widetilde{q}_t = \varpi\varepsilon_t, \qquad (\mathcal{A}2.53)$$

$$\widetilde{q}_t + \widetilde{\lambda}_t = \mathbf{E}_t\left\{\widetilde{\lambda}_{t+1} + \beta\psi\phi^*\frac{Y^*}{K^*}\left[\widetilde{\phi}_{t+1} + \widetilde{Y}_{t+1} - \widetilde{K}_{t+1}\right] + \beta\frac{1-\delta}{\alpha^*}\widetilde{q}_{t+1}\right\}, \qquad (\mathcal{A}2.54)$$

$$\widetilde{\lambda}_t - \widetilde{P}_t = \mathbf{E}_t\left\{\widetilde{\lambda}_{t+1} - \widetilde{P}_{t+1} + \widetilde{R}_{t+1}\right\} - \widetilde{m}_{t+1}, \qquad (\mathcal{A}2.55)$$

$$\widetilde{\lambda}_t - \widetilde{P}_t = \frac{\lambda^*}{\lambda^* + P^*}\mathbf{E}_t\left\{\widetilde{\lambda}_{t+1} - \widetilde{P}_{t+1}\right\} - \widetilde{m}_{t+1}, \qquad (\mathcal{A}2.56)$$

$$\varrho\widetilde{u}_t = \widetilde{\phi}_t + \widetilde{Y}_t - \widetilde{K}_t + \varepsilon_t, \qquad (\mathcal{A}2.57)$$

$$\widetilde{W}_t - \widetilde{P}_t = \widetilde{\phi}_t + \widetilde{Y}_t - \frac{N^*}{N^* - N_0}\widetilde{N}_t, \qquad (\mathcal{A}2.58)$$

$$\mu_P(1+\beta)\tilde{\pi}_t \;=\; \beta\mu_P\mathbf{E}_t\tilde{\pi}_{t+1}+\mu_P\tilde{\pi}_{t-1}$$

$$+\;\; (1-\mu_P)(1-\beta\mu_P)\tilde{\phi}_t+\beta\mu_P\widetilde{m}_{t+1}-\mu_P(1+\beta)(\widetilde{m}_t-\varepsilon_t)+\mu_P(\widetilde{m}_{t-1}-\varepsilon_{t-1}), \quad (\mathcal{A}2.59)$$

$$\left[1+\beta\mu_W^2-\frac{\theta(1-\mu_W)(1-\beta\mu_W)}{\theta+\gamma}\right]\widetilde{W}_t \;=\; \beta\mu_W\mathbf{E}_t\widetilde{W}_{t+1}+\mu_W\widetilde{W}_{t-1}+\left[\frac{(1-\mu_W)(1-\beta\mu_W)}{\theta+\gamma}\right]\tilde{N}_t$$

$$-\left[\frac{\gamma(1-\mu_W)(1-\beta\mu_W)}{\theta+\gamma}\right](\tilde{\lambda}_t-\tilde{P}_t)-\beta\mu_W\tilde{\pi}_t+\mu_W\tilde{\pi}_{t-1}+\beta\mu_W\widetilde{m}_{t+1}-(1+\beta)\mu_W\widetilde{m}_t+\mu_W\widetilde{m}_{t-1}$$

$$+\;\; \beta\mu_W\varepsilon_t-\mu_W\varepsilon_{t-1}, \quad (\mathcal{A}2.60)$$

$$\tilde{K}_{t+1}=\frac{(1-\delta)}{\alpha^*}\left(\tilde{K}_t-\varepsilon_t\right)+\frac{X^*}{K^*}\tilde{X}_t, \quad (\mathcal{A}2.61)$$

$$\tilde{Y}_t=\frac{C^*}{Y^*}\tilde{C}_t+\frac{X^*}{Y^*}\tilde{X}_t+\psi\phi^*\tilde{u}_t, \quad (\mathcal{A}2.62)$$

$$\tilde{Y}_t = \psi\left(\tilde{u}_t + \tilde{K}_t\right) + (1-\psi)\frac{N^*}{N^* - N_0}\tilde{N}_t - \psi\varepsilon_t, \qquad (\mathcal{A}2.63)$$

and

$$\widetilde{m}_{t+1} \;=\; \rho_m \widetilde{m}_t + \mu_t, \qquad (\mathcal{A}2.64)$$

for NKDSGE-MG rule models, or for NKDSGE-TR models the interest rate rule

$$(1-\rho_R\mathbf{L})\tilde{R}_t \;=\; (1-\rho_R)\left(a_\pi \mathbf{E}_t\tilde{\pi}_{t+1} + a_\pi \widetilde{m}_{t+1} + a_y\tilde{Y}_t\right) \;+\; \upsilon_t, \qquad (\mathcal{A}2.65)$$

where $\varrho \equiv \dfrac{a''(1)}{a'(1)}$ (= 1.174) and $\tilde{\pi}_t \equiv \tilde{P}_t - \tilde{P}_{t-1}$. The linear approximate habit NKDSGE-TR model consists of the linear stochastic difference equations ($\mathcal{A}$2.52)–($\mathcal{A}$2.63) and ($\mathcal{A}$2.65) with the unknowns $\tilde{\lambda}_t$, $\tilde{C}_t$, $\tilde{X}_t$, $\tilde{q}_t$, $\tilde{Y}_t$, $\tilde{K}_{t+1}$, $\tilde{R}_t$, $\tilde{u}_t$, $\tilde{\phi}_t$, $\tilde{N}_t$, $\tilde{P}_t$, and $\widetilde{W}_t$. When the AR(1) money growth rule ($\mathcal{A}$2.64) replaces the Taylor rule ($\mathcal{A}$2.65) in the system of linear stochastic difference equations that approximate the baseline habit NKDSGE-MG model, the linearized detrended bond Euler equation ($\mathcal{A}$2.55) can be dropped along with the demeaned nominal rate $\tilde{R}_t$.

A2.5 Solving the baseline habit NKDSGE models

This section describes the solution method we apply to solve the linear stochastic difference equations that approximate the NKDSGE models. Consider the baseline habit NKDSGE-TR model that employs the monetary policy rule ($\mathcal{A}$2.65). For this model, the vector of endogenous variables is

$$H_t = \left[\tilde{\lambda}_t \;\; \tilde{C}_t \;\; \tilde{X}_t \;\; \tilde{q}_t \;\; \tilde{Y}_t \;\; \tilde{K}_{t+1} \;\; \tilde{R}_t \;\; \tilde{u}_t \;\; \tilde{\phi}_t \;\; \tilde{N}_t \;\; \tilde{P}_t \;\; \widetilde{W}_t \;\; \widetilde{m}_{t+1}\right]'.$$

Next define the expectational forecast errors $\vartheta_{\tilde{\lambda},t+1} = \tilde{\lambda}_{t+1} - E_t\tilde{\lambda}_{t+1}$, $\vartheta_{\tilde{C},t+1} = \tilde{C}_{t+1} - E_t\tilde{C}_{t+1}$, $\vartheta_{\tilde{X},t} = \tilde{X}_{t+1} - E_t\tilde{X}_{t+1}$, $\vartheta_{\tilde{q},t+1} = \tilde{q}_{t+1} - E_t\tilde{q}_{t+1}$, $\vartheta_{\tilde{Y},t+1} = \tilde{Y}_{t+1} - E_t\tilde{Y}_{t+1}$, $\vartheta_{\tilde{u},t+1} = \tilde{u}_{t+1} - E_t\tilde{u}_{t+1}$, $\vartheta_{\tilde{\phi},t+1} = \tilde{\phi}_{t+1} - E_t\tilde{\phi}_{t+1}$, $\vartheta_{\tilde{P},t} = \tilde{P}_{t+1} - E_t\tilde{P}_{t+1}$, and $\vartheta_{\widetilde{W},t+1} = \widetilde{W}_{t+1} - E_t\widetilde{W}_{t+1}$. Collect these forecast errors into the vector ϑ_{t+1}. We use H_t and ϑ_t, the linear approximate optimality and equilibrium conditions ($\mathcal{A}$2.52)–($\mathcal{A}$2.63) and the Taylor rule ($\mathcal{A}$2.65) to form the multivariate first-order stochastic difference equation system of the baseline habit NKDSGE-TR model

$$\mathbb{G}_0\mathbb{H}_t = \mathbb{G}_1\mathbb{H}_{t-1} + \mathbb{V}\zeta_t + \mathbb{K}\vartheta_t, \qquad (\mathcal{A}2.66)$$

where $\mathbb{H}_t = [H_t \;\; \mathbf{E}_t H_{t+1}]'$ and $\zeta_t = [\varepsilon_t \;\; \upsilon_t]'$ (or when monetary policy is defined by the AR(1) money growth rule ($\mathcal{A}$2.64), $\zeta_t = [\varepsilon_t \;\; \mu_t]'$). It is understood that $\mathbf{E}_t H_{t+1}$ contains only those elements of H_t that enter equations ($\mathcal{A}$2.52)–($\mathcal{A}$2.63) as one-step ahead expectations. The matrices $\mathbb{G}_0$, $\mathbb{G}_1$, and $\mathbb{V}$ contain cross-equation restriction embedded in the optimality and equilibrium conditions ($\mathcal{A}$2.52)–($\mathcal{A}$2.63), and the Taylor rule ($\mathcal{A}$2.65).

Sims (2002) studies and solves multivariate linear rational expectations models that match ($\mathcal{A}$2.66). His solution algorithm taps the QZ (or generalized complex Schur) decomposition of matrices $\mathbb{G}_0$ and $\mathbb{G}_1$. The QZ decomposition employs $\mathcal{Q}'\mathcal{F}\mathcal{Z}' = \mathbb{G}_0$ and $\mathcal{Q}'\mathcal{O}\mathcal{Z}' = \mathbb{G}_1$, where $\mathcal{Q}'\mathcal{Q} = \mathcal{Z}'\mathcal{Z} = \mathbf{I}$ and matrices $\mathcal{F}$ and $\mathcal{O}$ are upper triangular. Matrices $\mathcal{Q}$, $\mathcal{Z}$, $\mathcal{F}$ and $\mathcal{O}$ are possibly complex. Let $\mathcal{D}_t = \mathcal{Z}'\mathbb{H}_t$ and premultiply equation ($\mathcal{A}$2.66) by $\mathcal{Q}$ to obtain

$$\begin{bmatrix} \mathcal{F}_{11} & \mathcal{F}_{12} \\ \mathbf{0} & \mathcal{F}_{22} \end{bmatrix} \begin{bmatrix} \mathcal{D}_{1,t} \\ \mathcal{D}_{2,t} \end{bmatrix} = \begin{bmatrix} \mathcal{O}_{11} & \mathcal{O}_{12} \\ \mathbf{0} & \mathcal{O}_{22} \end{bmatrix} \begin{bmatrix} \mathcal{D}_{1,t-1} \\ \mathcal{D}_{2,t-1} \end{bmatrix} + \begin{bmatrix} \mathcal{Q}_{1\cdot} \\ \mathcal{Q}_{2\cdot} \end{bmatrix} \left(\mathbb{V}\zeta_t + \mathbb{K}\vartheta_t\right), \qquad (\mathcal{A}2.67)$$

where $\mathcal{Q}_{j\cdot}$ denotes the jth block of rows of $\mathcal{Q}$. Although the QZ decomposition of $\mathbb{G}_0$ and $\mathbb{G}_1$ never fails to exist, these decompositions are not unique. Nonetheless, generalized eigenvalues of $\mathcal{F}$ and $\mathcal{O}$ can be unique if infinite values are allowed and zero eigenvalues for $\mathbb{G}_0$ and $\mathbb{G}_1$ are ruled out. Denote the generalized eigenvalues of $\mathcal{F}$ and $\mathcal{O}$ as $f_{ii}^{-1}o_{ii}$. These eigenvalues are ordered to partition the system ($\mathcal{A}$2.67) in such a way to place only explosive elements in $\mathcal{D}_{2,t}$. The 'reduced form' process of $\mathcal{D}_{2,t}$ is the second row of the system ($\mathcal{A}$2.67), which is written

$$\mathcal{D}_{2,t} = \mathcal{M}\mathcal{D}_{2,t-1} + \mathcal{M}\mathcal{O}_{22}^{-1}\mathcal{Q}_{2\cdot}\left(\mathbb{V}\zeta_t + \mathbb{K}\vartheta_t\right), \tag{$\mathcal{A}$2.68}$$

where $\mathcal{M} \equiv \mathcal{F}_{22}^{-1}\mathcal{O}_{22}$. Forward iteration of equation ($\mathcal{A}$2.68) gives

$$\mathcal{D}_{2,t} = -\sum_{i=0}^{\infty}\mathcal{M}^{-i}\mathcal{O}_{22}^{-1}\mathcal{Q}_{2\cdot}\left(\mathbb{V}\zeta_{t+i+1} + \mathbb{K}\vartheta_{t+i+1}\right), \tag{$\mathcal{A}$2.69}$$

where the transversality condition $\mathcal{M}^{-i}\mathcal{D}_{2,t+i}$, $i \longrightarrow \infty$ holds.

Extrinsic or sunspot equilibria are excluded from the solution of the present value ($\mathcal{A}$2.69) of $\mathcal{D}_{2,t}$. The present value invokes a no sunspot result because the expectation error vector ϑ_t has no impact on $\mathcal{D}_{1,t}$ and $\mathcal{D}_{2,t}$. The implications is that $\mathcal{D}_{2,t}$ belongs only to the date t information set (*i.e.*, it includes only the intrinsic shocks of ζ_t), which mean that

$$\mathbf{E}_t\sum_{i=0}^{\infty}\mathcal{M}^{-i}\mathcal{O}_{22}^{-1}\mathcal{Q}_{2\cdot}\mathbb{V}\zeta_{t+i+1} = \sum_{i=0}^{\infty}\mathcal{M}^{-i}\mathcal{O}_{22}^{-1}\mathcal{Q}_{2\cdot}\left(\mathbb{V}\zeta_{t+i+1} + \mathbb{K}\vartheta_{t+i+1}\right),$$

For an intrinsic equilibrium to exist, Sims (2002) shows that the necessary and sufficient conditions are that the set of equations $\mathcal{Q}_{2\cdot}\mathbb{V}\zeta_{t+1} + \mathcal{Q}_{2\cdot}\mathbb{K}\vartheta_{t+1}$ equal a column vector of zeros. A solution is available for the multivariate first order system ($\mathcal{A}$2.66) if (and only if) the column space of $Q_{2\cdot}\mathbb{V}$ is contained in that of $Q_{2\cdot}\mathbb{K}$. Given ζ_t is uncorrelated, the solution follows immediately. This is not true for the NKDSGE-MG models. When the intrinsic shocks are serially correlated, $\mathcal{Q}_{2\cdot}\mathbb{K}\vartheta_t$ is calculated from information in $\mathcal{Q}_{2\cdot}\mathbb{V}\zeta_t$.

Suppose that an intrinsic solution exists. When there is no sunspot equilibria, the row space of $\mathcal{Q}_{1\cdot}\mathbb{K}$ is contained in that of $\mathcal{Q}_{2\cdot}\mathbb{K}$. This is a necessary and sufficient condition for uniqueness of the solution of the linear approximate system ($\mathcal{A}$2.66), as Sims (2002) shows. He suggests working with a matrix Φ that yields $\mathcal{Q}_{1\cdot}\mathbb{K} = \Phi Q_{2\cdot}\mathbb{K}$. By premultiplying equation ($\mathcal{A}$2.67) with $[\mathbf{I} \quad -\Phi]$, combining this with equation ($\mathcal{A}$2.68), and noting that this wipes out the expectational forecast errors ϑ_t, we have

$$\mathcal{F}_{11}\mathcal{D}_{1,t} + (\mathcal{F}_{12} - \Phi\mathcal{F}_{22})\mathcal{D}_{2,t} = \mathcal{O}_{11}\mathcal{D}_{1,t-1} + (\mathcal{O}_{12} - \Phi\mathcal{O}_{22})\mathcal{D}_{2,t-1} + (Q_{1\cdot} - \Phi Q_{2\cdot})\mathbb{V}\zeta_t.$$

Stacking these equations on top of the equations of ($\mathcal{A}$2.69) produces

$$\begin{bmatrix} \mathcal{F}_{11} & \mathcal{F}_{12} - \Phi\mathcal{F}_{22} \\ \mathbf{0} & \mathbf{I} \end{bmatrix} \begin{bmatrix} \mathcal{D}_{1,t} \\ \mathcal{D}_{2,t} \end{bmatrix} = \begin{bmatrix} \mathcal{O}_{11} & \mathcal{O}_{12} - \Phi\mathcal{O}_{22} \\ \mathbf{0} & \mathbf{0} \end{bmatrix} \begin{bmatrix} \mathcal{D}_{1,t-1} \\ \mathcal{D}_{2,t-1} \end{bmatrix}$$

$$+ \begin{bmatrix} Q_{1\cdot} - \Phi Q_{2\cdot} \\ \mathbf{0} \end{bmatrix} \mathbb{V}\zeta_t + \begin{bmatrix} \mathbf{0} \\ \mathbf{E}_t \sum_{i=0}^{\infty} \mathcal{M}^{-i}\mathcal{O}_{22}^{-1}\mathcal{Q}_{2\cdot}\mathbb{V}\zeta_{t+i+1} \end{bmatrix}.$$

This matrix system maps into the unique intrinsic solution for $\mathbb{H}_t$

$$\mathbb{H}_t = \Theta_{\mathbb{H}}\mathbb{H}_{t-1} + \Theta_\zeta\zeta_t, \qquad (\mathcal{A}2.70)$$

where

$$\Theta_{\mathbb{H}} = \mathcal{Z} \begin{bmatrix} \mathcal{F}_{11} & \mathcal{F}_{12} - \Phi\mathcal{F}_{22} \\ \mathbf{0} & \mathbf{I} \end{bmatrix}^{-1} \begin{bmatrix} \mathcal{O}_{11} & \mathcal{O}_{12} - \Phi\mathcal{O}_{22} \\ \mathbf{0} & \mathbf{0} \end{bmatrix} \mathcal{Z}'$$

and

$$\Theta_{\zeta} = \mathcal{Z} \begin{bmatrix} \mathcal{F}_{11} & \mathcal{F}_{12} - \Phi\mathcal{F}_{22} \\ \mathbf{0} & \mathbf{I} \end{bmatrix}^{-1} \begin{bmatrix} Q_{1\cdot} - \Phi Q_{2\cdot} \\ \mathbf{0} \end{bmatrix}.$$

We engage the system of first-order stochastic difference equations ($\mathcal{A}$2.70) to produce linear approximate solutions for the NKDSGE models. These solutions generate synthetic data sets that are inputs into our Bayesian simulation experiments.

A3. Estimating SVMAs, Checking Their ABC and Ds, and Spectral Density Computation

This section fills in a few gaps about the methods used to evaluate the NKDSGE models. We review the Blanchard and Quah (1989) decomposition and apply it to vector autoregressions (VARs) of output growth (or consumption growth) and inflation. These VARs are identified with a long-run monetary neutrality (LRMN) restriction that the level of output or consumption is independent of monetary shocks at $t \longrightarrow \infty$. The LRMN restriction yields SVMA(∞), processes of output (or consumption growth) and inflation. We show that it retrieves the TFP and monetary policy shock innovations of the NKDSGE models as in the ABCs and Ds of Fernández-Villaverde, Rubio-Ramírez, Sargent, and Watson (2007). The SVMA(∞) also provides a map to permanent and transitory output and consumption growth spectral densities, $SD_{\Delta Y}$ and $SD_{\Delta C}$. This section ends with a review of several methods available to compute these $SD_{\Delta Y}$ and $SD_{\Delta C}$.

A3.1 VARs and SVMAs

The SVMAs are constructed from a VAR of $\mathcal{X}_t = [\Delta \ln Y_t \;\; \Delta \ln P_t]'$ or $[\Delta \ln C_t \;\; \Delta \ln P_t]'$ and the LRMN restriction using the Blanchard and Quah (1989) decomposition. The unrestricted joint probability distribution of $\mathcal{X}_t$ is approximated by the finite-order VAR

$$\mathcal{X}_t = \mathbb{A}(\mathbf{L})\mathcal{X}_{t-1} + e_t, \quad \mathbb{A}(\mathbf{L}) = \sum_{j=1}^{p} \mathbb{A}_j \mathbf{L}^j, \tag{$\mathcal{A}$3.1}$$

where constants are ignored, the forecast errors $e_t = \mathcal{X}_t - \mathbf{E}\left\{\mathcal{X}_t \,\middle|\, \mathcal{X}_{t-1}, \mathcal{X}_{t-2}, \ldots, \mathcal{X}_{t-p}\right\}$ are Gaussian, and its covariance matrix is Σ. We set $p = 2$ in sample estimation and for the Bayesian Monte Carlo experiments, but below we report results for $p = 4$.

The unrestricted VAR of ($\mathcal{A}$3.1) is invertible whether estimated or under the NKDSGE models. Inverting this VAR yields the reduced form VMA(∞), $\mathcal{X}_t = \left[\mathbf{I} - \mathbb{A}(\mathbf{L})\right]^{-1} e_t$, or Wold representation of $\mathcal{X}_t$, $\mathbb{C}(\mathbf{L})e_t$, where $\mathbb{C}(\mathbf{L}) = \sum_{i=0}^{\infty} \mathbb{C}_i \mathbf{L}^i$ and the reduced form impact matrix $\mathbb{C}_0 = \mathbf{I}$. The corresponding SVMA(∞) is

$$\mathcal{X}_t = \mathbb{B}(\mathbf{L})\varsigma_t, \quad \varsigma_t \sim \mathcal{N}\left(\mathbf{0}, \mathbf{I}\right), \tag{$\mathcal{A}$3.2}$$

which summarizes equation (10) of the paper. The NKDSGE models predict that in the long run the levels of output and consumption are independent of monetary policy innovations (*i.e.*, the money growth rule innovation μ_t or Taylor rule innovation υ_t). This is the LRMN restriction, which forces the upper right element of $\mathbb{B}(\mathbf{1})$ to be zero, or $\sum_{j=0}^{\infty} \mathbb{B}_{j,1,2} = \mathbb{B}(\mathbf{1})_{1,2} = 0$. The SVMA ($\mathcal{A}$3.2) and the reduced form VMA(∞) also force $e_t = \mathbb{B}_0\varsigma_t$ and $\mathbb{B}_j = \mathbb{A}_j\mathbb{B}_0$. Note that once estimates of the four unknown elements of the structural impact response matrix $\mathbb{B}_0$ are available, we can compute the SVMA of ($\mathcal{A}$3.2) from the reduced form VMA(∞).

Our goal is to recover the four unknown coefficients of $\mathbb{B}_0$. The map from the structural shocks to the reduced form errors, $e_t = \mathbb{B}_0\varsigma_t$, and the covariances matrices of e_t and ς_t place three restrictions on the four unknowns of $\mathbb{B}_0$. These three restrictions present us with three nonlinear equations that follow from expanding $\Sigma = \mathbb{B}_0\mathbb{B}_0'$ to

$$\begin{aligned} \Sigma_{1,1} &= \mathbb{B}^2_{0,1,1} + \mathbb{B}^2_{0,1,2}, \\ \Sigma_{1,2} &= \mathbb{B}_{0,1,1}\mathbb{B}_{0,2,1} + \mathbb{B}_{0,1,2}\mathbb{B}_{0,2,2}, \\ \Sigma_{2,2} &= \mathbb{B}^2_{0,2,1} + \mathbb{B}^2_{0,2,2}. \end{aligned} \tag{$\mathcal{A}$3.3}$$

The remaining restriction is found by summing both sides of $\mathbb{B}_j = \mathbb{A}_j\mathbb{B}_0$ from $j \geq 0$, which leads to $\mathbb{B}(\mathbf{1}) = \mathbb{C}(\mathbf{1})\mathbb{B}_0$. The LRMN restriction imposes

$$\mathbb{C}(\mathbf{1})_{1,1}\mathbb{B}_{0,1,2} + \mathbb{C}(\mathbf{1})_{1,2}\mathbb{B}_{0,2,2} = 0, \tag{$\mathcal{A}$3.4}$$

which is a fourth nonlinear equation. We solve the four nonlinear equations ($\mathcal{A}$3.3) and ($\mathcal{A}$3.4) to calculate estimates of the four unknown coefficients of $\mathbb{B}_0$.

Markov chain Monte Carlo (MCMC) simulations of the SVMA(∞) of equation ($\mathcal{A}$3.2) engage the BACC software of Geweke (1999) and McCausland (2004). The MCMC simulators need priors that are obtained, only in part, from ordinary least squares (OLS) estimates of the reduced form VAR(2) of equation ($\mathcal{A}$3.1). These estimates and related covariance matrices are the prior information used to generate $\mathcal{J}$ (= 5,000) posterior draws of the reduced form VAR(2) coefficients. Next we calculate the reduced form VMA(∞) and apply the BQ decomposition by imposing the LRMN restriction to recover the SVMA(∞) of equation ($\mathcal{A}$3.2). The $\mathcal{J}$ samples of the $\mathbb{B}(\mathbf{L})$s are the basis of the empirical distributions of the permanent and transitory $SD_{\mathcal{E},\Delta Y}$ and $SD_{\mathcal{E},\Delta C}$. The theoretical, $\mathcal{T}$, distributions of these moments are estimated in the same manner, but on synthetic samples generated by NKDSGE models.

We treat synthetic samples generated by MCMC simulations of the unrestricted VAR ($\mathcal{A}3.1$) and the NKDSGE models in the same way, with one caveat. The exception is that although the off diagonal elements of the NKDSGE model structural shock covariance matrix $\Xi = \mathbf{E}\left\{\zeta_t\zeta_t'\right\}$ are zero, its diagonal elements are not unity. The NKDSGE model SVMAs are normalized for the Blanchard and Quah (BQ) decomposition with a correction that relies on the Choleski decomposition of Ξ, $\Xi^{1/2}$. Given $\mathbb{D}(\mathbf{L})$ is the infinite-order lag polynomial matrix of the theoretical SVMA(∞), the normalization is $\mathbb{D}(\mathbf{L})\ \Xi^{1/2}$. This normalization is imposed by the BQ decomposition on the ensemble of $\mathcal{J}$ theoretical SVMAs that are created from synthetic time series of length $\mathcal{M} \times T$ obtained from Bayesian simulations of the NKDSGE models.

A3.2 The ABCs and Ds of the NKDSGE models, LRMN, and SVARs

This section shows how the SVMA(∞) of equation ($\mathcal{A}3.2$) retrieves the economic shocks of a NKDSGE model. This involves restating a result from Fernández-Villaverde, Rubio-Ramírez, Sargent, and Watson (2007). They study a condition that equates the shocks identified by an econometric model to those of a DSGE model. We exploit their condition to tie the shocks of a structural VAR (SVAR) identified by LRMN to the NKDSGE model shocks ζ_t.

Fernández-Villaverde, Rubio-Ramírez, Sargent, and Watson (FVRRSW) construct a VAR(∞) driven by DSGE model shocks to expose the condition that links these shocks to those identified by a SVAR(∞). The baseline habit NKDSGE Taylor rule model yields the VAR(∞)

$$\mathcal{X}_t = \Gamma_{\mathbb{H}} \sum_{j=0}^{\infty} \left[\Theta_{\mathbb{H}} - \Theta_\zeta \Gamma_\zeta^{-1} \Gamma_{\mathbb{H}}\right]^j \Theta_\zeta \Gamma_\zeta^{-1} \mathcal{X}_{t-j-1} + \Gamma_\zeta \zeta_t, \qquad (\mathcal{A}3.5)$$

which combines the equilibrium law of motion ($\mathcal{A}2.70$), the system

$$\mathcal{X}_t = \Gamma_{\mathbb{H}} \mathbb{H}_{t-1} + \Gamma_\zeta \zeta_t, \qquad (\mathcal{A}3.6)$$

that relates the observables of $\mathcal{X}_t$ to $\mathbb{H}_t$ and ζ_t, and several steps described by FVRRSW. Note that Γ_ζ is square and its inverse is taken to exist. FVRRSW also examine

$$\mathbb{H}_t = \sum_{j=0}^{\infty} \left[\Theta_{\mathbb{H}} - \Theta_\zeta \Gamma_\zeta^{-1} \Gamma_{\mathbb{H}}\right]^j \Theta_\zeta \Gamma_\zeta^{-1} \mathcal{X}_{t-j}, \qquad (\mathcal{A}3.7)$$

which results from passing Γ_ζ^{-1} through equation ($\mathcal{A}$3.6), substituting it into the equilibrium law of motion ($\mathcal{A}$2.70), and rearranging terms. Equation ($\mathcal{A}$3.7) recovers the state vector $\mathbb{H}_t$ from the history of $\mathcal{X}_{t-j}$, which consists of observed variables (*i.e.*, there are no latent state variables), if (and only if) the eigenvalues of $\Theta_{\mathbb{H}} - \Theta_\zeta \Gamma_\zeta^{-1} \Gamma_{\mathbb{H}}$ are strictly less than one in modulus. This is the condition FVRRSW require to equate shocks identified by a SVAR to the NKDSGE model shocks ζ_t. Given the FVRRSW condition is satisfied by $\Theta_{\mathbb{H}} - \Theta_\zeta \Gamma_\zeta^{-1} \Gamma_{\mathbb{H}}$, the coefficients of the lag polynomial implied by $\left[\mathbf{I} - \left(\Theta_{\mathbb{H}} - \Theta_\zeta \Gamma_\zeta^{-1} \Gamma_{\mathbb{H}}\right) \mathbf{L}\right]$ also fulfill the needs of square summability. By also assuming that $\Gamma_\zeta \zeta_t$ is orthogonal to $\mathcal{X}_{t-j-1}$ ($j = 0, 1, \ldots, \infty$), equation ($\mathcal{A}$3.5) can be interpreted as the theoretical VAR(∞) of $\mathcal{X}_t$.

We rely on LRMN for identification of the SVMA(∞) of equation ($\mathcal{A}$3.2). This complicates the problem of using the FVRRSW condition to connect NKDSGE model shock innovations ζ_t to innovations identified by an econometric model. A solution is to exploit an approach of King and Watson (1997) that imposes the LRMN restriction on the SVAR(∞)

$$\begin{bmatrix} 1 & -\Lambda_{\Delta Y, \Delta P, 0} \\ -\Lambda_{\Delta P, \Delta Y, 0} & 1 \end{bmatrix} \mathcal{X}_t = \begin{bmatrix} \Lambda_{\Delta Y, \Delta Y}(\mathbf{L}) & \Lambda_{\Delta Y, \Delta P}(\mathbf{L}) \\ \Lambda_{\Delta P, \Delta Y}(\mathbf{L}) & \Lambda_{\Delta P, \Delta P}(\mathbf{L}) \end{bmatrix} \mathcal{X}_{t-1} + \begin{bmatrix} \eta_{1,t} \\ \eta_{2,t} \end{bmatrix}, \qquad (\mathcal{A}3.8)$$

where the impact matrix Λ_0 is nonsingular, $\Lambda(\mathbf{L})$ summarizes the lag polynomial attached to $\mathcal{X}_{t-1}$, $\eta_t = \left[\eta_{1,t} \;\; \eta_{2,t}\right]'$, Ω is the diagonal covariance matrix of $\mathbf{E}\left\{\eta_t \eta_t'\right\}$, $\mathbf{E}\eta_t = \mathbf{0}$, and $\mathbf{E}\left\{\eta_t \eta_{t-i}'\right\}$ $= \mathbf{0}$, for all non-zero i.

King and Watson (1997) are interested in identifying and estimating SVARs with impact and long run restrictions. We focus on the latter type of restriction to identify the SVAR of ($\mathcal{A}$3.8) with LRMN. The identification relies on the response of the level of output to a permanent change in the nominal shock $\eta_{2,t}$, which is

$$\mathcal{L}_{\Delta Y,\Delta P} = \frac{\Lambda_{\Delta Y,\Delta P,0} + \Lambda_{\Delta Y,\Delta P}(\mathbf{1})}{1 - \Lambda_{\Delta Y,\Delta Y}(\mathbf{1})}.$$

This ratio is zero when LRMN holds because it measures the long run response of output to a monetary shock. Following King and Watson, the LRMN restriction is imposed on the structural VAR of ($\mathcal{A}$3.8) by rewriting its top equation as

$$\Delta \ln Y_t = [\Lambda_{\Delta Y,\Delta P,0} + \Lambda_{\Delta Y,\Delta P}(\mathbf{1})]\Delta \ln P_t + \Lambda_{\Delta Y,\Delta Y}(\mathbf{1})\Delta \ln Y_{t-1}$$

$$+ \Psi_{\Delta Y,\Delta Y}(\mathbf{L})\Delta^2 \ln Y_{t-1} + \Psi_{\Delta Y,\Delta P}(\mathbf{L})\Delta^2 \ln P_{t-1} + \eta_{1,t},$$

where, for example, $\Psi_{\Delta Y,\Delta P,i} = -\sum_{s=i+1}^{\infty} \Lambda_{\Delta Y,\Delta P,s}$. Next, multiply and divide the first term after the equality by $\mathcal{L}_{\Delta Y,\Delta P}$ to produce

$$\Delta \ln Y_t - \mathcal{L}_{\Delta Y,\Delta P}\Delta \ln P_t = \Lambda_{\Delta Y,\Delta Y}(\mathbf{1})[\Delta \ln Y_{t-1} - \mathcal{L}_{\Delta Y,\Delta P}\Delta \ln P_t]$$

$$+ \Psi_{\Delta Y,\Delta Y}(\mathbf{L})\Delta^2 \ln Y_{t-1} + \Psi_{\Delta Y,\Delta P}(\mathbf{L})\Delta^2 \ln P_{t-1} + \eta_{1,t},$$

or under LRMN

$$\Delta \ln Y_t = \Lambda_{\Delta Y,\Delta Y}(\mathbf{1})\Delta \ln Y_{t-1} + \Psi_{\Delta Y,\Delta Y}(\mathbf{L})\Delta^2 \ln Y_{t-1} + \Psi_{\Delta Y,\Delta P}(\mathbf{L})\Delta^2 \ln P_{t-1} + \eta_{1,t}.$$

The previous equation and the bottom equation of ($\mathcal{A}$3.8) form a just-identified SVAR from which $\eta_{1,t}$ and $\eta_{2,t}$ can be computed. An estimator of these shocks does not rely on identifying either impact coefficient $\Lambda_{\Delta Y,\Delta P,0}$ or $\Lambda_{\Delta P,\Delta Y,0}$. Rather the former coefficient is obtained from $\mathcal{L}_{\Delta Y,\Delta P,0} = 0$ given $\Lambda_{\Delta Y,\Delta Y}(\mathbf{1})$, $\Psi_{\Delta Y,\Delta Y}(\mathbf{L})$ and $\Psi_{\Delta Y,\Delta P}(\mathbf{L})$, while the latter coefficient is obtained from the bottom equation of ($\mathcal{A}$3.8). King and Watson (1997) use an instrumental variable (IV) estimator with $\eta_{1,t}$ serving as the additional instrument. Instead of the IV estimator, we apply the BQ decomposition, equations ($\mathcal{A}$3.3) and ($\mathcal{A}$3.4), to synthetic samples of $\mathcal{X}_t$, rather than estimate SVAR(∞)s.

The FVRRSW condition enables us to match the shocks of the SVAR of ($\mathcal{A}$3.8) with the NKDSGE shocks ζ_t. This SVAR implies the reduced form VAR(∞)

$$\mathcal{X}_t = \mathbb{S}(\mathbf{L})\mathcal{X}_{t-1} + \nu_t, \quad \mathbb{S}(\mathbf{L}) = \sum_{j=1}^{\infty} \mathbb{S}_j \mathbf{L}^j, \qquad (\mathcal{A}3.9)$$

is associated with the SVAR of ($\mathcal{A}$3.8), where $\nu_t = \mathcal{X}_t - \mathbf{E}\left\{\mathcal{X}_t \,\middle|\, \mathcal{X}_{t-1}, \mathcal{X}_{t-2}, \ldots, \mathcal{X}_{t-p}, \ldots\right\}$, $\mathbb{S}(\mathbf{L}) = \Lambda_0^{-1}\,\Lambda(\mathbf{L})$, and $\nu_t = \Lambda_0^{-1}\,\eta_t$. Equation ($\mathcal{A}$3.9) serves to represent the VAR(∞) of $\mathcal{X}_t$ when the sum from $i = 1, \ldots, \infty$ of $\mathbb{S}^2_{\Delta Y,\Delta Y,i} + \mathbb{S}^2_{\Delta Y,\Delta P,i} + \mathbb{S}^2_{\Delta P,\Delta Y,i} + \mathbb{S}^2_{\Delta P,\Delta Y,i}$ is finite and the orthogonality condition $\mathbf{E}\left\{\nu_t \mathcal{X}'_{t-j}\right\} = \mathbf{0}$, holds for all $j \geq 1$. We can acquire shocks from this reduced form VAR that match those of the baseline habit NKDSGE-TR model when $\Lambda_0\nu_t = \Gamma_\zeta\zeta_t$. FVRRSW show that the equality links the econometric and NKDSGE model shocks if (and only if) the eigenvalues of $\Theta_{\mathbb{H}} - \Theta_\zeta\Gamma_\zeta^{-1}\Gamma_{\mathbb{H}}$ are strictly less than one in modulus.

The FVRRSW restriction is checked at each of the $\mathcal{J} = 5000$ replications of the Bayesian simulations of the 12 NKDSGE models. The simulations reveal that the NKDSGE models satisfy the FVRRSW restriction on the eigenvalues $\Theta_{\mathbb{H}} - \Theta_\zeta\Gamma_\zeta^{-1}\Gamma_{\mathbb{H}}$ at all $\mathcal{J}$ replications. Thus, the theoretical SVMA(∞)s estimated on synthetic data always recover the economic shocks of the 12 NKDSGE models.

A3.3 Computing permanent and transitory spectral densities

In Section 3.2, the paper presents the map from the SVMA(∞) of equation ($\mathcal{A}$3.2) to permanent and transitory $SD_{\Delta Y}$ and $SD_{\Delta C}$. We reproduce the SD (at frequency ω) that appears at the end of section 3.2 here as

$$SD_{\Delta Y,\iota}(\omega) = \frac{1}{2\pi}\sum_{j=0}^{40}\left|\mathbb{B}_{\Delta Y,\iota,j}e^{-ij\omega}\right|^2, \quad \iota = \varepsilon,\, \upsilon, \tag{$\mathcal{A}$3.10}$$

where it is understood that the Bayesian simulations of the NKDSGE models account for the non-unit diagonal elements of Ξ. Although we calculate the permanent and transitory $SD_{\Delta Y}$ and $SD_{\Delta C}$ using ($\mathcal{A}$3.10), there are (at least) two other methods available to compute these moments. First, the $SD(\omega)$ can be represented as

$$SD_{\Delta Y,\iota}(\omega) = \frac{\mathbb{B}^2_{\Delta Y,\iota,0}}{2\pi}\sum_{j=0}^{40}\left|\frac{\mathbb{B}_{\Delta Y,\iota,j}}{\mathbb{B}_{\Delta Y,\iota,0}}e^{-ij\omega}\right|^2,$$

which leads to the factorization

$$1 + \frac{\mathbb{B}_{\Delta Y,\iota,1}}{\mathbb{B}_{\Delta Y,\iota,0}}z + \frac{\mathbb{B}_{\Delta Y,\iota,2}}{\mathbb{B}_{\Delta Y,\iota,0}}z^2 + \ldots + \frac{\mathbb{B}_{\Delta Y,\iota,40}}{\mathbb{B}_{\Delta Y,\iota,0}}z^{40} = (1-\chi_{\iota,1}z)(1-\chi_{\iota,2}z)\cdots(1-\chi_{\iota,40}z),$$

in terms of the eigenvalues, the $\chi_{\iota,h}$s, of the MA(40) process of output growth with respect to the NKDSGE shocks ε, μ, or υ. The eigenvalue factorization gives

$$SD_{\Delta Y,\iota}(\omega) = \frac{\mathbb{B}^2_{\Delta Y,\iota,0}}{2\pi}\prod_{j=1}^{40}\left[1 + \chi^2_{\iota,j} - 2\chi_{\iota,j}\cos(\omega)\right],$$

which provides a third method to compute $SD(\omega)$s.

A4. Additional NKDSGE model evaluation

Our paper grounds its evaluation of 12 NKDSGE models on the minimal econometric interpretation (MEI) of Geweke (2010). The MEI is useful to judge the fit of a NKDSGE model because no pretense is made that it provides a complete description of economic behavior. Thus, the folk theorem that all models are false is not violated by the MEI.

This section discusses five Bayesian Monte Carlo experiments grounded on the MEI. To review, we engage the MEI to evaluate 12 NKDSGE models on prior and posterior population moments, permanent and transitory $SD_{\Delta Y}$ and $SD_{\Delta C}$, that are functions of actual observable data. By drawing from priors of parameters of a NKDSGE model, its linearized version produces prior population *SD*s from SVMA(∞)s estimated on synthetic samples of length $\mathcal{M}$ (= $T \times \mathcal{W}$). We label these posterior moments theoretical *SD*s, or $SD_{\mathcal{T}}$s. The same SVMA(∞)s are used to build posterior *SD*s, tagged as empirical *SD*s or $SD_{\mathcal{E}}$s, on synthetic samples of length T generated by MCMC simulators. Actual data, unrestricted VAR(2)s, and priors of these models are the conditioning information on which the $SD_{\mathcal{E}}$s are built.

We offer the five Bayesian Monte Carlo experiments to check the robustness of the evaluation of the 12 NKDSGE models conducted by the paper. The first of these experiments replaces the prior of the habit parameter, $h \sim U(0.05,\ 0.95)$, with a prior drawn from the β distribution, $h \sim \beta(0.65, 0.15)$. Next, we break the prior $h \sim U(0.05,\ 0.95)$ in half to conduct two experiments. One set of simulations condition on the prior $h \sim U(0.50,\ 0.95)$, while the other set relies on the prior $h \sim U(0.05,\ 0.499)$. The fourth experiment retains the original priors, including $h \sim U(0.05,\ 0.95)$, but uses VAR(4)s, rather than VAR(2)s, to construct the SVMA(∞)s. In the final experiment, we return to the structure of the Bayesian Monte Carlo experiments presented in the paper except that the *CIC* are calculated using distributions of Cramer-von Mises (*CvM*) goodness of fit statistics instead of Kolmogorov-Smirnov (*KS*) statistics.

Results of the five NKDSGE model evaluation exercises appear in tables A1–A5 and figures A1–A30. Tables A1–A4 contain *CIC* that measure the overlap of $KS_{\mathcal{E}}$ and $KS_{\mathcal{T}}$ distributions generated by Bayesian Monte Carlo experiments employing the prior $h \sim \beta(0.65, 0.15)$, the prior $h \sim U(0.50,\ 0.95)$, the prior $h \sim U(0.050,\ 0.499)$, and switching from VAR(2)s to VAR(4)s, respectively. The source of *CIC* reported in table A5 are densities of *CvM* statistics constructed from distributions of permanent and transitory $SD_{\mathcal{E},\Delta Y}$, $SD_{\mathcal{E},\Delta C}$, $SD_{\mathcal{T},\Delta Y}$, and $SD_{\mathcal{T},\Delta C}$.

Figures A1–A30 are laid out in the fashion as figures 3–8 of the paper. From top to bottom, the rows of figures A1-A30 list results for permanent $SD_{\Delta Y}$, transitory $SD_{\Delta Y}$, permanent $SD_{\Delta C}$, and transitory $SD_{\Delta C}$. Mean permanent and transitory $SD_{\mathcal{E},\Delta Y}$, $SD_{\mathcal{E},\Delta C}$, $SD_{\mathcal{T},\Delta Y}$, and $SD_{\mathcal{T},\Delta C}$ appear in the first column of figures A1–A30. The second (third) column of figures A1–A24 contain densities of *KS* statistics computed using the entire spectrum (constrained to eight to two years per cycle). We denote mean $SD_{\mathcal{E}}$s and $KS_{\mathcal{E}}$ statistic densities with (blue) solid lines, mean $SD_{\mathcal{T}}$s and $KS_{\mathcal{T}}$ statistic densities generated by non-habit NKDSGE models with (green) dashed lines, and mean $SD_{\mathcal{T}}$s and $KS_{\mathcal{T}}$ statistic densities created by habit NKDSGE models with (red) dot-dash lines in figures A1–A24. Densities of *CvM* statistics are displayed in the second and third columns of figures A25-A30 using the same scheme. Goodness of fit statistic densities appear with associated *CIC* in figures A1–A30.

A4.0 Preliminary NKDSGE model fit: Prior predictive analysis

Before reviewing the five Bayesian Monte Carlo experiments, this section presents a prior predictive analysis of the 12 NKDSGE models. Our prior predictive analysis asks if a NKDSGE model can account for sample innovation variances of VAR(2)s estimated on output growth and inflation and consumption growth and inflation data that starts in $1955Q1$ and ends with $2002Q4$, $T = 196$. The data is described in section A0. Draws from priors of NKDSGE model parameters and linearized NKDSGE models generate synthetic samples of output growth, consumption growth, and inflation of length T on which equivalent VARs are estimated to extract $\mathcal{J} = 5000$ pairs of output growth and consumption growth regression forecast innovation variances. These VAR innovation variances form the prior distributions of interest.

Scatter plots of the prior distributions are reported in the three rows and four columns of figure A0. The baseline, SPrice, and SWage versions of these models appear in the rows of figure A0 from top to bottom. From left to right, the columns contain results for non-habit NKDSGE-MG, habit NKDSGE-MG, non-habit NKDSGE-TR, and habit NKDSGE-TR models. The 12 scatter plots of figure A0 place the innovation variance of the consumption (output) growth regression on the horizontal (vertical) axes. The plus symbols, "+", in these plots denote the combination of sample innovation variance estimates obtained from output and consumption growth regressions. In each scatter plot of figure A0, clouds of points represent prior distributions of artificial innovation variances.

Figure A0 shows that non-habit NKDSGE models fail to explain sample innovation variances of the output and consumption growth regressions. Neither baseline, SPrice, nor SWage non-habit NKDSGE models produce prior distributions of synthetic innovation variances in the first and third columns of figure A0 that cover the plus sign, "+", that symbolizes the intersection of the sample shock innovations. This is preliminary evidence that non-habit NKDSGE models cannot describe fluctuations in U.S. output and consumption growth data on the $1955Q1$–$2002Q4$ sample.

The habit NKDSGE model are better able to explain the sample innovation variances of the output and consumption growth regression. The second and fourth columns of figure A0 present clouds of prior distributions of shock innovation variances that blanket the sample innovation variances. This result holds for baseline, SPrice, and SWage habit NKDSGE models. Thus, NKDSGE models find it useful to include consumption habit to explain sample output and consumption growth innovation variances.

A4.1 NKDSGE model fit under the prior $h \sim \beta(0.65,\ 0.15)$

The uniform prior for the consumption habit parameter, $h \sim U(0.05,\ 0.95)$, only utilizes information about the theoretical restriction that h takes values on the open interval between zero and one. We replace this uninformative prior for h with a β prior informed by evidence from previous DSGE model studies, $h \sim \beta(0.65, 0.15)$. The β prior gives h a mean of 0.65, a standard deviation of 0.15, and a 95 percent coverage interval of [0.3842, 0.8765]. This calibration focuses on estimates of Christiano, Eichenbaum, and Evans (2005) and also covers values of h found in Boldrin, Christiano, and Fisher (2001) and Francis and Ramey (2005), among others. The non-habit NKDSGE models remain defined by the degenerate prior $h = 0$.

Table A1 reports *CIC* generated from Bayesian Monte Carlo experiments of the NKDSGE models given the β prior for h. We include density plots of *KS* statistic distributions based on distributions of permanent and transitory $SD_{\mathcal{E},\Delta Y}$, $SD_{\mathcal{E},\Delta C}$, $SD_{\mathcal{E},\Delta Y}$, and $SD_{\mathcal{E},\Delta C}$ in figures A1–A6. The *CIC* and *KS* statistic densities indicate that the β prior for h produces only minimal changes in NKDSGE model fit compared to *CIC* found in table 3. Figures A1–A6 reinforce this conclusion.

A4.2 NKDSGE model fit under the prior $h \sim U(0.50, 0.95)$

Bounding the prior of h from below at 0.5 yields one important change in the evaluation of the 12 NKDSGE models discussed in the paper. Although the prior $h \sim U(0.50, 0.95)$ is uninformative, it eliminates values of h that suggest weaker consumption habit induced propagation and monetary transmission. With only the prior $h \sim U(0.50, 0.95)$ different, the top half of table A3 shows that habit NKDSGE-MG models achieve six more $CIC \geq 0.3$ compared to results found in the top half of table 3. These additional matches are mostly made by baseline and SWage habit NKDSGE-MG models to distributions of transitory $SD_{\mathcal{E},\Delta Y}$ and $SD_{\mathcal{E},\Delta C}$ when fit is restricted to eight to two years per cycle. The SPrice habit NKDSGE-TR model generates an additional $CIC \geq 0.3$ in the bottom half of table A3 when drawing from the prior $h \sim U(0.50, 0.95)$ instead of $h \sim U(0.05, 0.95)$. This match occurs on the transitory $SD_{\mathcal{E},\Delta Y}$ distribution when the evaluation is conducted using the entire spectrum. Visual support for these results are $KS_{\mathcal{E}}$ and $KS_{\mathcal{T}}$ densities displayed in figures A7–A12. The first column of these figures present mean permanent and transitory $SD_{\mathcal{T},\Delta Y}$ and $SD_{\mathcal{T},\Delta C}$ that are qualitatively similar to those found in the first column of figures 3–8 of the paper.

A4.3 NKDSGE model fit under the prior $h \sim U(0.050, 0.499)$

The reason for replacing the prior $h \sim U(0.50, 0.95)$ with $h \sim U(0.050, 0.499)$ is to generate evidence about the impact of a weaker consumption habit process on NKDSGE model propagation and monetary transmission. Moving to the prior $h \sim U(0.050, 0.499)$ has the unsurprising effect of reducing the number of successful matches, $CIC \geq 0.3$, by five. Compared to the top half of table 3, the top half of table A3 reveals that the SPrice habit NKDSGE-MG model achieves two fewer matches. SPrice and SWage habit NKDSGE-TR models exhibit three fewer $CIC \geq 0.3$ in the bottom half of table A3 when set next to the bottom half of table 3. These failed matches are to distributions of permanent and transitory $SD_{\mathcal{T},\Delta C}$. The deterioration in the fit of SPrice habit NKDSGE-MG, SPrice habit NKDSGE-TR, and SWage habit NKDSGE-TR models is reflected in in figures A13–A18 by mean permanent and transitory $SD_{\mathcal{T},\Delta Y}$ and $SD_{\mathcal{T},\Delta C}$ that are farther from mean permanent and transitory $SD_{\mathcal{E},\Delta Y}$ and $SD_{\mathcal{E},\Delta C}$ as well as $KS_{\mathcal{E}}$ and $KS_{\mathcal{T}}$ densities that display less overlap.

A4.4 NKDSGE model fit using VAR(4)s to estimate SVMA(∞)s

We estimate unrestricted VARs with longer lags to examine the impact on the construction of SVMA(∞)s, permanent and transitory $SD_{\Delta Y}$ and $SD_{\Delta C}$, and our Bayesian evaluation to NKDSGE model. Table A4 includes CIC that indicate switching to VAR(4)s from VAR(2)s has little impact on judging the fit of the 12 NKDSGE models to distributions of $SD_{\mathcal{E},\Delta Y}$ and $SD_{\mathcal{E},\Delta C}$. For the experiments relying on VAR(4)s, there are in net two additional $CIC \geq 0.3$ compared to those found in table 3. The bottom half of table A4 reveals that the SPrice non-habit NKDSGE-TR model fails to duplicate the distribution of permanent $SD_{\mathcal{E},\Delta Y}$. This model succeeds at the task of matching this distribution according to the bottom panel of figure 3. The SWage habit NKDSGE-MG model is responsible for two $CIC \geq 0.3$ as shown in the top half of table A4 given VAR(4)s, instead of VAR(2)s, are engaged by the Bayesian Monte Carlo experiments. These matches occur on distributions of transitory $SD_{\mathcal{E},\Delta Y}$ and $SD_{\mathcal{E},\Delta C}$ when evaluation is constrained to the business cycle frequencies. Another $CIC \geq 0.3$ is provided by the SWage habit NKDSGE-TR model in the bottom half of table A4 that is not observed in the table 3. This

NKDSGE model replicates the distribution of the transitory $SD_{\mathcal{E},\Delta Y}$ on the entire spectrum.

Figures A19-A24 indicate that the impact of estimating VAR(4)s, rather than VAR(2)s, falls on the mean transitory $SD_{\mathcal{E},\Delta C}$. This *SD* displays a peak in the business cycle frequencies. Note that figure 2, which is constructed on VAR(2)s, contains a mean transitory $\mathrm{SD}_{\mathcal{E},\Delta C}$ that has a plateau from the growth into the business cycle frequencies. Otherwise, the VAR(4)s have few qualitative implications for mean $SD_{\mathcal{E}}$, mean $SD_{\mathcal{T}}$, $KS_{\mathcal{E}}$ densities, and $KS_{\mathcal{T}}$ densities comparing those in figures A19–A24 to those in figures 3–8.

A4.5 Gauging NKDSGE model fit with the Cramer-von Mises statistic

Table A5 contains *CIC* for 12 NKDSGE models based on the *CvM* statistic. We ground *CIC* on densities of *CvM* statistics to check the robustness of measures of NKDSGE model fit presented in the paper. The *CvM* statistic is

$$CvM_{\mathcal{D},j} = \int_0^1 \mathcal{B}^2_{\mathcal{D},j}(\kappa)d\kappa,$$

for $\mathcal{D} = \mathcal{E}, \mathcal{T}$ and replication j of the ensemble of $\mathcal{J}$ (= 5000) $\mathcal{E}$ and $\mathcal{T}$ synthetic samples. Section 3.4 provides details about computing $\mathcal{B}_{\mathcal{D},j}(\cdot)$, but to review

$$\mathcal{B}_{\mathcal{D},j}(\kappa) \;=\; \frac{\sqrt{2\mathcal{H}}}{2\pi}\left[\mathcal{V}_{\mathcal{D},j}(\kappa\pi) - \kappa\mathcal{V}_{\mathcal{D},j}(\pi)\right],$$

where $\kappa \in [0, 1]$ ($[0.064, 0.25]$) when evaluation is conducted on the entire spectrum (on the business cycle frequencies of eight to two years per cycle) and $\mathcal{H} = T$ if $\mathcal{D} = \mathcal{E}$, otherwise $\mathcal{H} = \mathcal{M}$. Also, the partial sum $\mathcal{V}_{\mathcal{D},j}\left(2\pi q/\mathcal{H}\right) = 2\pi \sum_{\ell=1}^{q} \mathcal{R}_{\mathcal{D},j}\left(2\pi\ell/\mathcal{H}\right)/\mathcal{H}$ and the ratio $\mathcal{R}_{\mathcal{D},j}(\omega) = \widehat{I_T}(\omega) \,/\, I_{\mathcal{D},j}(\omega)$, where the numerator (denominator) is the sample (jth $\mathcal{E}$ or $\mathcal{T}$) output or consumption growth SD at frequency ω. Distributions of $CvM_{\mathcal{E}}$ and $CvM_{\mathcal{T}}$ statistics are the basis of *CIC* that quantify the overlap of the ensemble of distributions of permanent and transitory $SD_{\mathcal{E},\Delta Y}$, $SD_{\mathcal{E},\Delta C}$, $SD_{\mathcal{T},\Delta Y}$, and $SD_{\mathcal{T},\Delta C}$.

The fit of the NKDSGE models is qualitatively similar across table A5 and table 3 with two exceptions. First, table 3 shows that the SPrice non-habit NKDSGE-MG model produces one *CIC* > 0.30. Using the *CvM* statistic allows this NKDSGE model to produce an additional *CIC* ≥ 0.30 in the third row of table A5. This row of *CIC* shows that the SPrice non-habit NKDSGE-MG model replicates distributions of permanent and transitory $SD_{\mathcal{E},\Delta Y}$s when evaluation is grounded on frequencies between eight to two years per cycle. However, distributions of $CvM_{\mathcal{E}}$ and $CvM_{\mathcal{T}}$ statistics generate *CIC* limited on the business cycle frequencies that indicate the fit of the SPrice habit NKDSGE-MG model dominates the fit of the SPrice non-habit NKDSGE model-MG. The former model also duplicates the distribution of transitory $SD_{\mathcal{E},\Delta C}$ on the entire spectrum using the *CvM* statistic, which is the other difference between table A5 and table 3.

Figures A25–A30 plot mean $SD_{\mathcal{E}}$s and $SD_{\mathcal{T}}$s and densities of $CvM_{\mathcal{E}}$ and $CvM_{\mathcal{T}}$ statistics. A striking feature of these figures is that for *CIC* > 0.3 measured on the entire spectrum, the $CvM_{\mathcal{T}}$ statistic densities often decay smoothly from left to right instead of showing well-defined peaks. For example the middle panel of the bottom row of figure A26 shows that the baseline habit NKDSGE-TR model yields a *CIC* = 0.56 on the distribution of transitory $SD_{\mathcal{E},\Delta C}$, but the relevant *CvM* statistic density is relatively flat with a long right-hand tail. The explanation is that the quadratic form of the *CvM* statistic can place weight on large deviations between sample *SD*, $\widehat{SD_T}(\omega)$, and say, the jth draw from the distribution of $SD_{\mathcal{T}}(\omega)$, $SD_{\mathcal{T},j}(\omega)$. In this case, the density of the $CvM_{\mathcal{T}}$ statistic will be disperse with a long thin right tail. The supremum of the *KS* statistic, $KS_{\mathcal{T},j} = Max_{\kappa\in[0,1]}\left|\mathcal{B}_{\mathcal{T},j}(\kappa)\right|$, is immune from this dispersion, especially in cases when *CIC* > 0.3. Nonetheless, using the *CvM* statistic does not alter our evaluation of habit and non-habit NKDSGE models.

References

Blanchard, O.J., Quah, D., 1989. The dynamic effects of aggregate demand and supply disturbances. American Economic Review 79, 655–673.

Boldrin, M., Christiano, L.J., Fisher, J.D.M. 2001. Habit persistence, asset returns, and the business cycle. American Economic Review 91, 149–166.

Christiano, L. J., Eichenbaum, M., Evans, C.L., 2005. Nominal rigidities and the dynamic effects of a shock to monetary policy. Journal of Political Economy 113, 1–45.

Dennis, R., 2009. Consumption-habits in a new Keynesian business cycle model. Journal of Money, Credit, and Banking, 41, 1015–1030.

Fernández-Villaverde, J., Rubio-Ramírez, J.F., Sargent T.J., Watson M.W., 2007. ABCs (and D)s for understanding VARS. American Economic Review 97, 1021–1026.

Francis, N., Ramey, V.A. 2005. Is the technology-driven real business cycle hypothesis dead? Shocks and aggregate fluctuations revisited. Journal of Monetary Economics 52, 1379-1399.

Geweke, John. (2010) *Complete and Incomplete Econometric Models*, Princeton, NJ: Princeton University Press.

Geweke, J., 1999. Using simulation methods for Bayesian econometric models: Inference, development, communication. Econometric Reviews 18, 1–73.

King, R.G., Watson, M.W., 1997. Testing long run neutrality, Economic Quarterly, Federal Reserve Bank of Richmond, 83(Summer), 69–101.

McCausland, W., 2004. Using the BACC Software for Bayesian Inference. Journal of Computational Economics 23, 201–218.

Sims, C.A., 2002. Solving linear rational expectations models. Computational Economics 20, 1–20.

Smets, F., Wouters, R., 2007. Shocks and frictions in US business cycles: A Bayesian DSGE approach. American Economic Review 97, 1123–1175.

Yun, T. 1996. Nominal price rigidity, money supply endogeneity, and business cycle. Journal of Monetary Economics 37, 345–370.

Table A1: *CIC*s of Kolmogorov-Smirnov Statistics

Replace the prior $h \sim U(0.05, 0.95)$ with $h \sim \beta(0.65, 0.15)$

Model	ΔY w/r/t Trend Sh'k $\infty:0$	ΔY w/r/t Trend Sh'k $8:2$	ΔY w/r/t Transitory Sh'k $\infty:0$	ΔY w/r/t Transitory Sh'k $8:2$	ΔC w/r/t Trend Sh'k $\infty:0$	ΔC w/r/t Trend Sh'k $8:2$	ΔC w/r/t Transitory Sh'k $\infty:0$	ΔC w/r/t Transitory Sh'k $8:2$
NKDSGE-MG								
Baseline								
Non-Habit	0.02	0.03	0.00	0.01	0.00	0.00	0.00	0.00
Habit	0.00	0.03	0.20	0.22	0.01	0.20	0.12	0.22
SPrice								
Non-Habit	0.03	0.47	0.00	0.23	0.01	0.17	0.00	0.04
Habit	0.17	0.73	0.15	0.72	0.07	0.62	0.54	0.79
SWage								
Non-Habit	0.00	0.00	0.00	0.06	0.00	0.00	0.00	0.06
Habit	0.00	0.01	0.23	0.29	0.01	0.09	0.14	0.29
NKDSGE-TR								
Baseline								
Non-Habit	0.01	0.00	0.12	0.71	0.00	0.00	0.08	0.68
Habit	0.00	0.03	0.80	0.46	0.02	0.15	0.53	0.85
SPrice								
Non-Habit	0.40	0.57	0.00	0.76	0.01	0.16	0.00	0.49
Habit	0.37	0.83	0.45	0.59	0.14	0.65	0.33	0.76
SWage								
Non-Habit	0.00	0.00	0.21	0.37	0.00	0.00	0.02	0.81
Habit	0.00	0.05	0.62	0.47	0.02	0.14	0.44	0.77

The prior $h \sim \beta(0.65, 0.15)$ implies a 95 percent coverage interval of $[0.3842, 0.8765]$. NKDSGE-MG and NKDSGE-TR denote the NKDSGE model with the AR(1) money supply rule (8) and the Taylor rule (9), respectively. Baseline NKDSGE models include sticky prices and sticky wages. The acronyms SPrice and SWage represent NKDSGE models with only sticky prices or sticky nominal wages, respectively. The column heading $\infty : 0$ ($8 : 2$) indicates that *CIC*s measure the intersection of distributions of $KS_{\mathcal{I}}$ and $KS_{\mathcal{T}}$ statistics computed over the entire spectrum (from eight to two years per cycle).

Table A2: *CIC*s of Kolmogorov-Smirnov Statistics Replace the prior $h \sim U(0.05, 0.95)$ with $h \sim U(0.50, 0.95)$

Model	ΔY w/r/t Trend Sh'k $\infty:0$	ΔY w/r/t Trend Sh'k $8:2$	ΔY w/r/t Transitory Sh'k $\infty:0$	ΔY w/r/t Transitory Sh'k $8:2$	ΔC w/r/t Trend Sh'k $\infty:0$	ΔC w/r/t Trend Sh'k $8:2$	ΔC w/r/t Transitory Sh'k $\infty:0$	ΔC w/r/t Transitory Sh'k $8:2$
NKDSGE-MG								
Baseline								
Non-Habit	0.02	0.03	0.00	0.01	0.00	0.00	0.00	0.00
Habit	0.00	0.06	0.27	0.32	0.03	0.32	0.26	0.36
SPrice								
Non-Habit	0.03	0.47	0.00	0.23	0.01	0.17	0.00	0.04
Habit	0.24	0.80	0.21	0.78	0.17	0.63	0.57	0.77
SWage								
Non-Habit	0.00	0.00	0.00	0.06	0.00	0.00	0.00	0.06
Habit	0.00	0.03	0.29	0.39	0.03	0.20	0.26	0.39
NKDSGE-TR								
Baseline								
Non-Habit	0.01	0.00	0.12	0.71	0.00	0.00	0.08	0.68
Habit	0.00	0.06	0.78	0.39	0.05	0.28	0.76	0.83
SPrice								
Non-Habit	0.40	0.57	0.00	0.76	0.01	0.16	0.00	0.49
Habit	0.44	0.88	0.55	0.56	0.23	0.66	0.61	0.73
SWage								
Non-Habit	0.00	0.00	0.21	0.37	0.00	0.00	0.02	0.81
Habit	0.01	0.10	0.60	0.47	0.05	0.26	0.67	0.70

See the notes to table A1 except that the results of this table rely on $h \sim U(0.50, 0.95)$.

Table A3: CICs of Kolmogorov-Smirnov Statistics
Replace the prior $h \sim U(0.05, 0.95)$ with $h \sim U(0.050, 0.499)$

Model	ΔY w/r/t Trend Sh'k $\infty:0$	ΔY w/r/t Trend Sh'k $8:2$	ΔY w/r/t Transitory Sh'k $\infty:0$	ΔY w/r/t Transitory Sh'k $8:2$	ΔC w/r/t Trend Sh'k $\infty:0$	ΔC w/r/t Trend Sh'k $8:2$	ΔC w/r/t Transitory Sh'k $\infty:0$	ΔC w/r/t Transitory Sh'k $8:2$
NKDSGE-MG								
Baseline								
Non-Habit	0.02	0.03	0.00	0.01	0.00	0.00	0.00	0.00
Habit	0.01	0.02	0.06	0.02	0.00	0.00	0.00	0.00
SPrice								
Non-Habit	0.03	0.47	0.00	0.23	0.01	0.17	0.00	0.04
Habit	0.03	0.48	0.00	0.39	0.00	0.25	0.00	0.20
SWage								
Non-Habit	0.00	0.00	0.00	0.06	0.00	0.00	0.00	0.06
Habit	0.00	0.00	0.09	0.08	0.00	0.00	0.00	0.08
NKDSGE-TR								
Baseline								
Non-Habit	0.01	0.00	0.12	0.71	0.00	0.00	0.08	0.68
Habit	0.00	0.00	0.50	0.66	0.00	0.00	0.30	0.87
SPrice								
Non-Habit	0.40	0.57	0.00	0.76	0.01	0.16	0.00	0.49
Habit	0.41	0.60	0.04	0.74	0.08	0.26	0.04	0.80
SWage								
Non-Habit	0.00	0.00	0.21	0.37	0.00	0.00	0.02	0.81
Habit	0.00	0.00	0.52	0.42	0.00	0.00	0.20	0.84

See the notes to table A1 except that the results of this table rely on $h \sim U(0.050, 0.499)$.

TABLE A4: *CIC*s OF KOLMOGOROV-SMIRNOV STATISTICS USING VAR(4)S TO CONSTRUCT SVMA(∞)S

Model	ΔY w/r/t Trend Sh'k $\infty:0$	ΔY w/r/t Trend Sh'k $8:2$	ΔY w/r/t Transitory Sh'k $\infty:0$	ΔY w/r/t Transitory Sh'k $8:2$	ΔC w/r/t Trend Sh'k $\infty:0$	ΔC w/r/t Trend Sh'k $8:2$	ΔC w/r/t Transitory Sh'k $\infty:0$	ΔC w/r/t Transitory Sh'k $8:2$
NKDSGE-MG								
Baseline								
Non-Habit	0.01	0.04	0.00	0.03	0.00	0.00	0.00	0.00
Habit	0.02	0.09	0.14	0.19	0.04	0.18	0.14	0.21
SPrice								
Non-Habit	0.14	0.53	0.00	0.36	0.01	0.12	0.00	0.04
Habit	0.29	0.67	0.23	0.75	0.14	0.46	0.28	0.52
SWage								
Non-Habit	0.00	0.00	0.00	0.09	0.00	0.00	0.00	0.09
Habit	0.01	0.03	0.23	0.30	0.03	0.11	0.14	0.30
NKDSGE-TR								
Baseline								
Non-Habit	0.00	0.01	0.24	1.00	0.00	0.00	0.03	0.56
Habit	0.02	0.07	0.70	0.98	0.05	0.15	0.44	0.83
SPrice								
Non-Habit	0.13	0.78	0.00	0.92	0.00	0.13	0.00	0.52
Habit	0.40	0.88	0.33	0.89	0.16	0.48	0.34	0.81
SWage								
Non-Habit	0.00	0.00	0.31	1.00	0.00	0.00	0.01	0.65
Habit	0.02	0.06	0.60	0.92	0.04	0.14	0.40	0.84

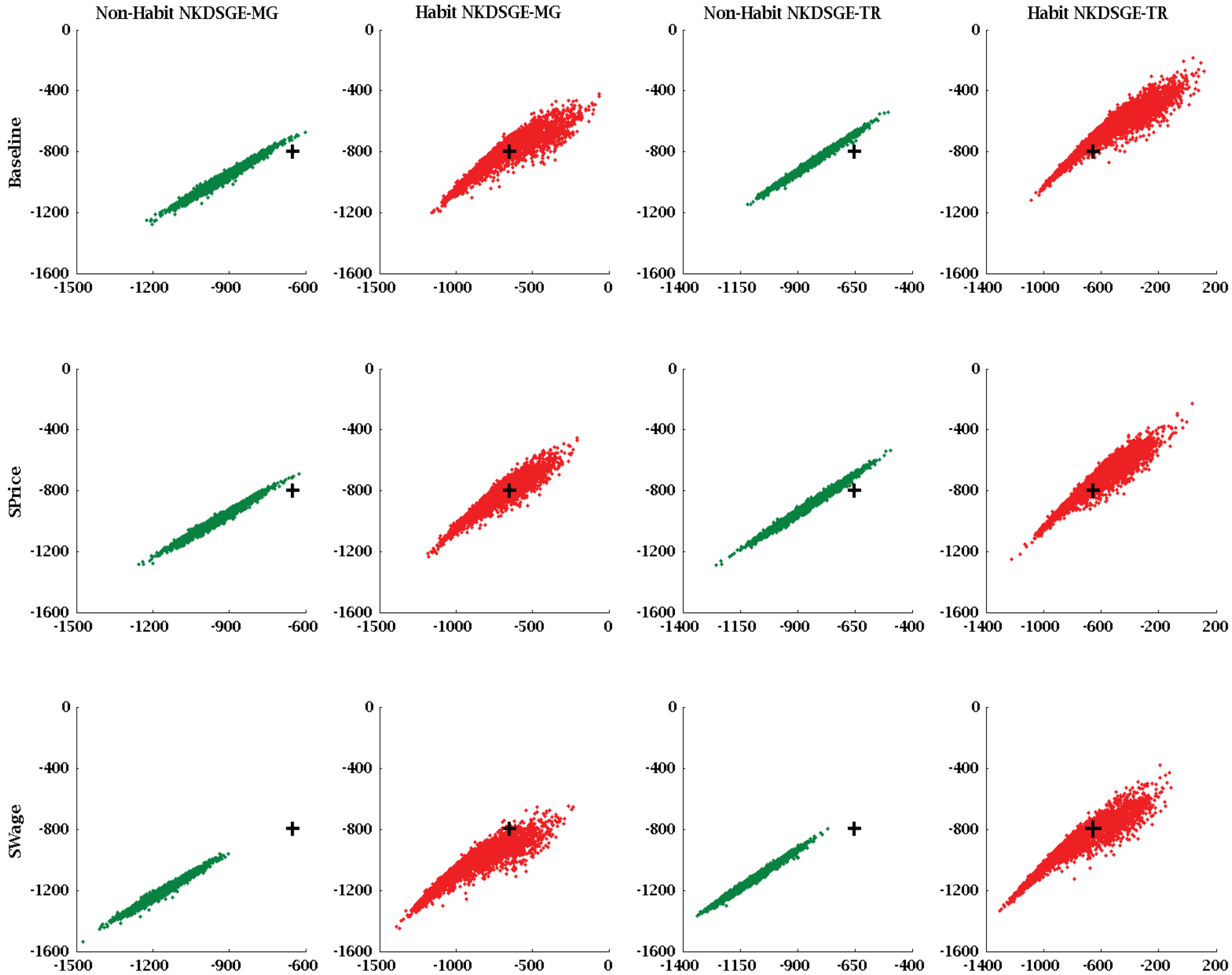
The SVMA(∞) are constructed from unrestricted VAR(4)s estimated on actual and synthetic data. Otherwise, see the notes at the bottom of table A1.

Table A5: *CIC*s of Cramer-von Mises Statistics

Model	ΔY w/r/t Trend Sh'k		ΔY w/r/t Transitory Sh'k		ΔC w/r/t Trend Sh'k		ΔC w/r/t Transitory Sh'k	
	$\infty : 0$	$8 : 2$	$\infty : 0$	$8 : 2$	$\infty : 0$	$8 : 2$	$\infty : 0$	$8 : 2$
NKDSGE-MG								
Baseline								
Non-Habit	0.02	0.02	0.00	0.02	0.00	0.00	0.00	0.00
Habit	0.00	0.02	0.20	0.18	0.02	0.12	0.14	0.18
SPrice								
Non-Habit	0.04	0.52	0.00	0.49	0.02	0.21	0.00	0.14
Habit	0.14	0.65	0.09	0.75	0.09	0.46	0.32	0.60
SWage								
Non-Habit	0.00	0.00	0.00	0.11	0.00	0.00	0.00	0.01
Habit	0.00	0.01	0.23	0.26	0.01	0.01	0.14	0.19
NKDSGE-TR								
Baseline								
Non-Habit	0.03	0.00	0.12	0.62	0.00	0.00	0.10	0.75
Habit	0.00	0.02	0.63	0.50	0.03	0.14	0.56	0.87
SPrice								
Non-Habit	0.42	0.61	0.00	0.80	0.01	0.21	0.00	0.71
Habit	0.42	0.73	0.27	0.56	0.16	0.50	0.35	0.81
SWage								
Non-Habit	0.00	0.00	0.21	0.33	0.00	0.00	0.03	0.87
Habit	0.00	0.04	0.54	0.43	0.02	0.13	0.46	0.78

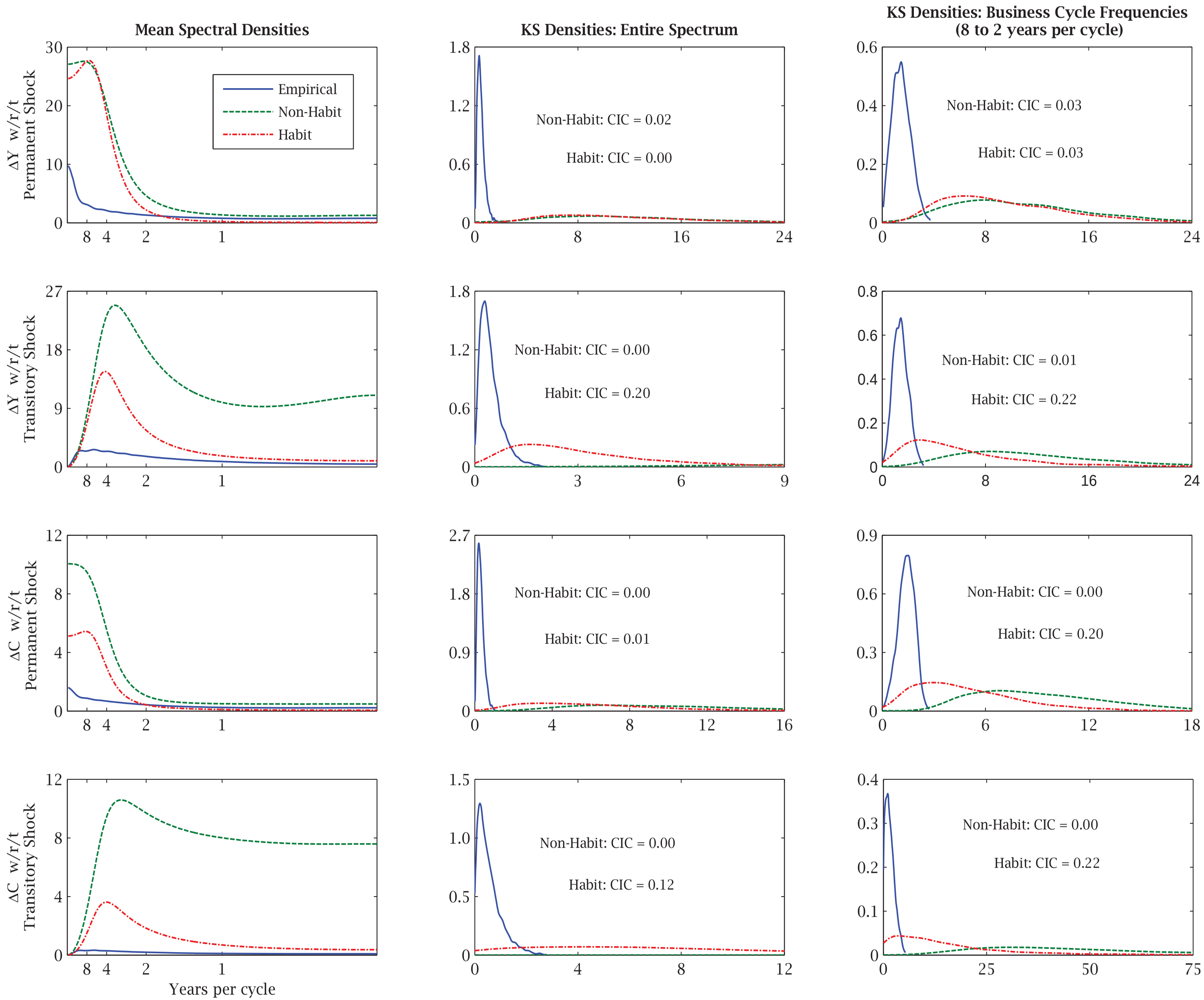
The column heading $\infty : 0$ ($8 : 2$) indicates that *CIC*s measure the intersection of distributions of $CvM_{\mathcal{I}}$ and $CvM_{\mathcal{T}}$ statistics computed over the entire spectrum (from eight to two years per cycle). Otherwise, see the notes at the bottom of table A1.

Figure A0: Prior Predictive Analysis: Sample and NKDSDE Prior Estimates of VAR(2) Innovation Variances



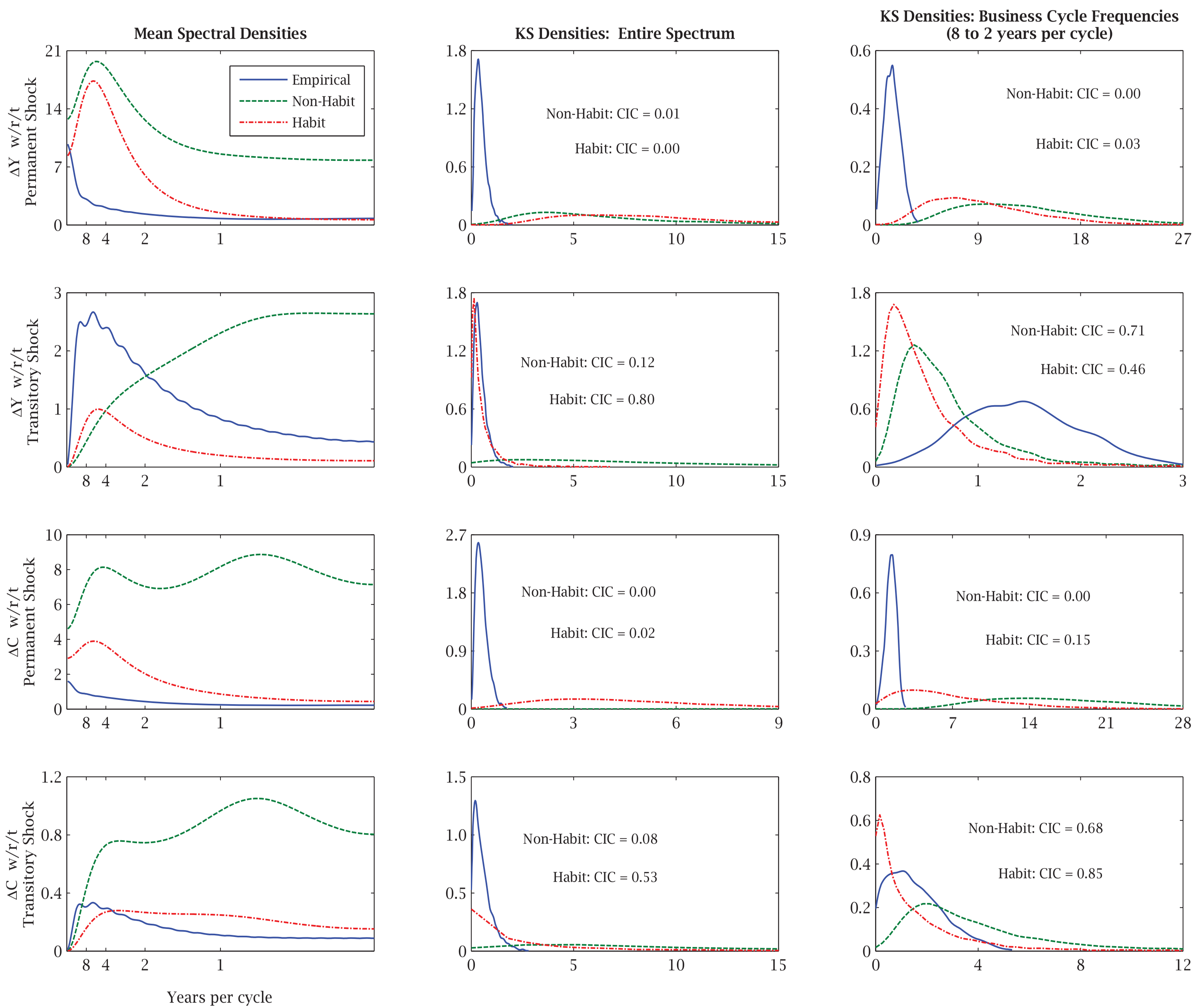
The horizontal (vertical) axes contain the innovation variance of the consumption (output) growth regression. The plus symbols denote the intersection of sample innovation variance estimates of the consumption and output growth regressions. The clouds of points are the prior distributions of estimated synthetic innovation variances from the consumption and output growth regressions generated by the NKDSGE models.

Figure A1: Mean Structural $\mathcal{E}$ and $\mathcal{T}$ SDs and KS Densities for Baseline NKDSGE Models with Money Growth Rule and $h \sim \beta$



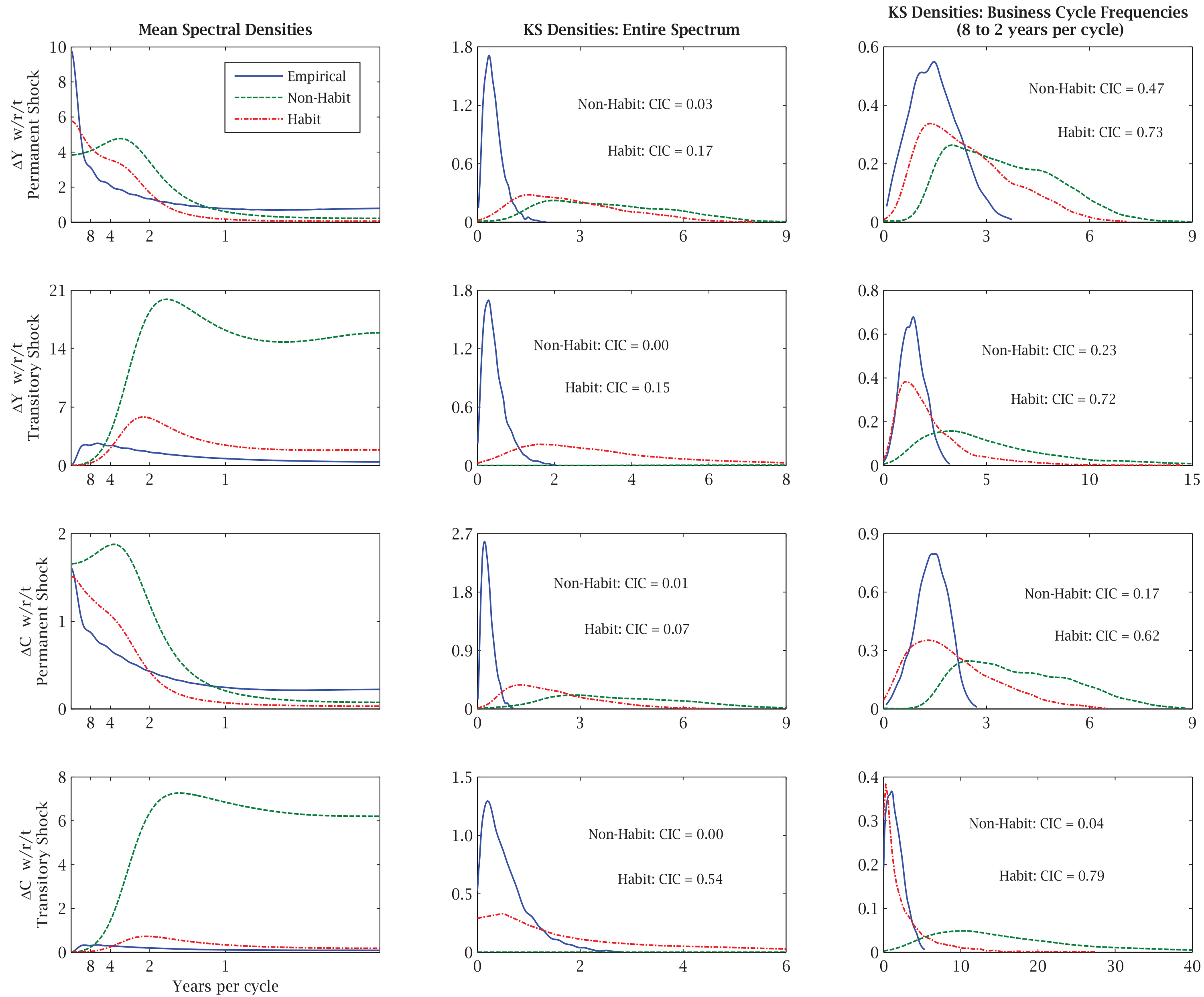
See section $A4.1$ and section 4.3 of the paper for details.

Figure A2: Mean Structural $\mathcal{E}$ and $\mathcal{T}$ SDs and KS Densities for Baseline NKDSGE Models with Taylor Rule and $h \sim \beta$



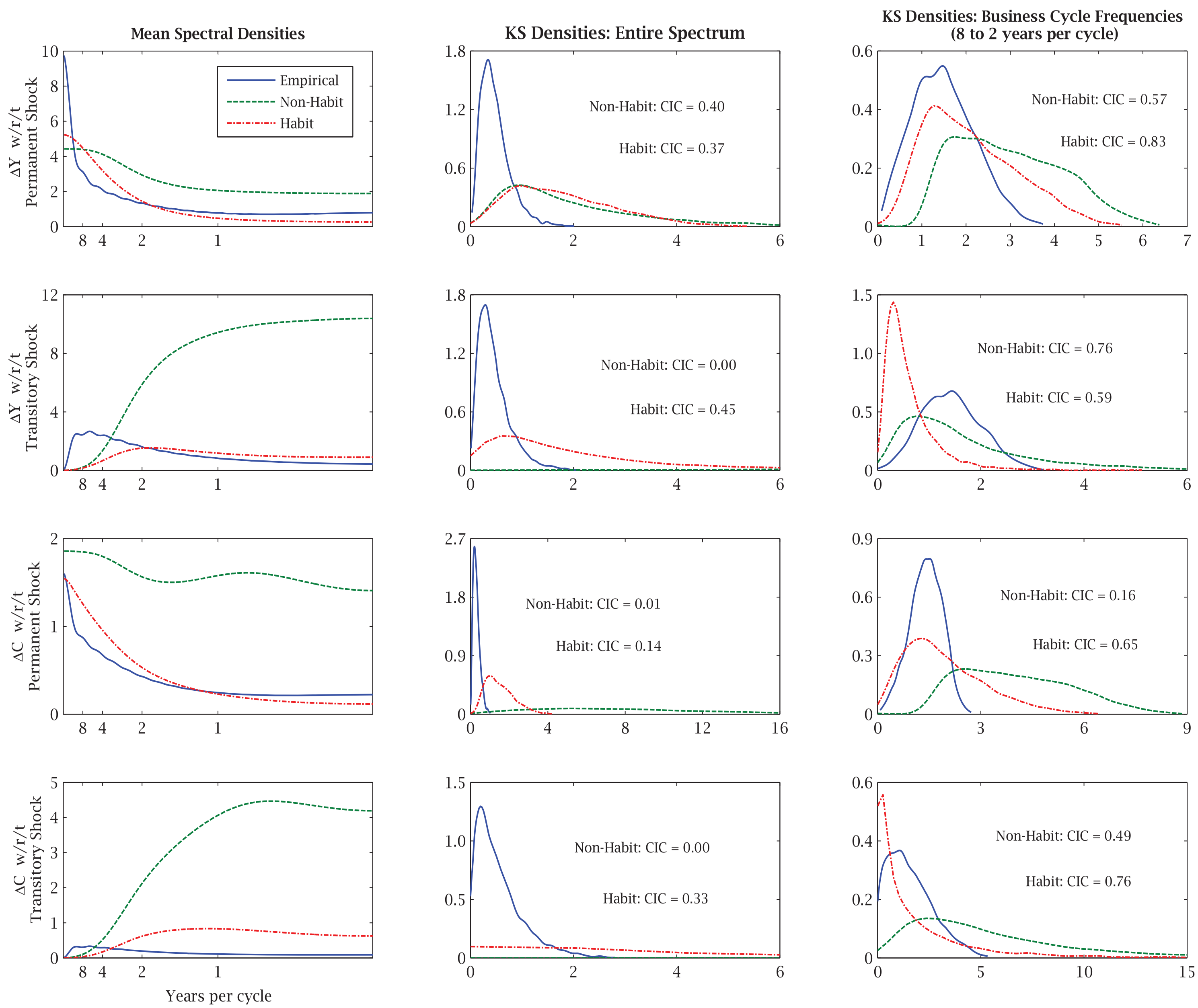
See section $A4.1$ and section 4.3 of the paper for details.

Figure A3: Mean Structural $\mathcal{E}$ and $\mathcal{T}$ SDs and KS Densities for NKDSGE Models with Money Growth Rule, Only Sticky Prices, and $h \sim \beta$



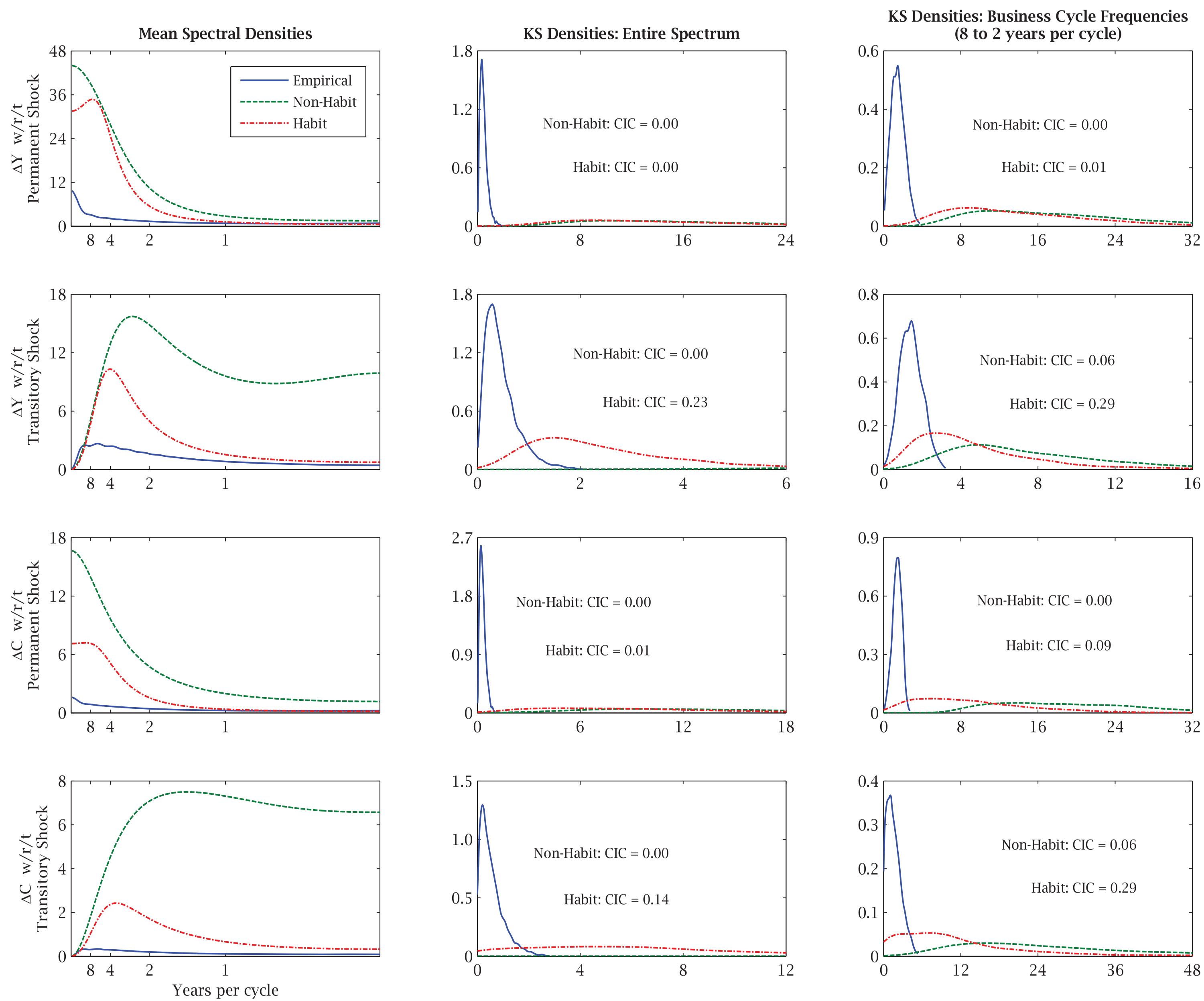
See section *A*4.1 and section 4.3 of the paper for details.

Figure A4: Mean Structural $\mathcal{E}$ and $\mathcal{T}$ SDs and KS Densities for NKDSGE Models with Taylor Rule, only Sticky Prices, and $h \sim \beta$



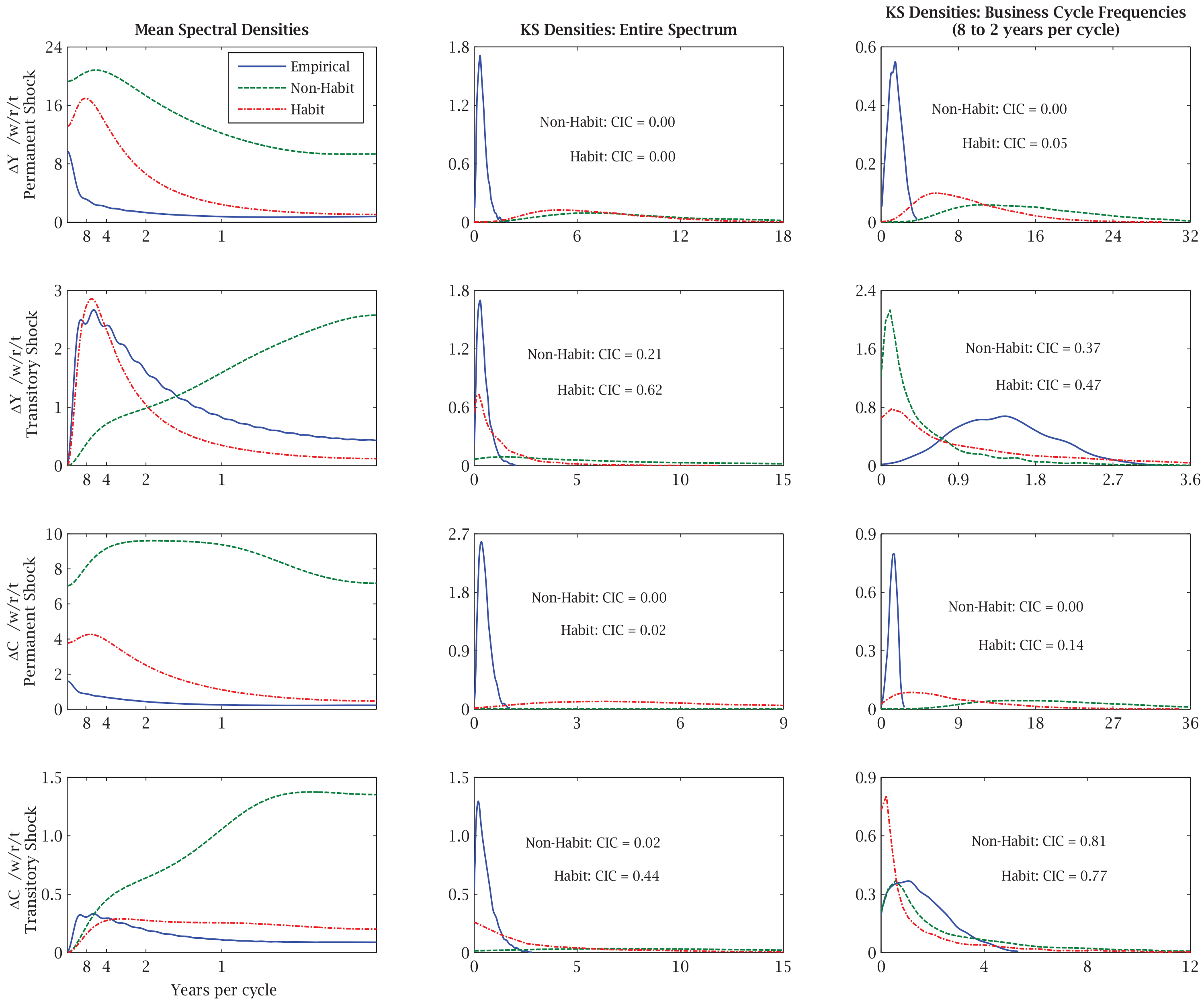
See section $A4.1$ and section 4.3 of the paper for details.

Figure A5: Mean Structural $\mathcal{E}$ and $\mathcal{T}$ SDs and KS Densities for NKDSGE Models with Money Growth Rule, Only Sticky Wages, and $h \sim \beta$



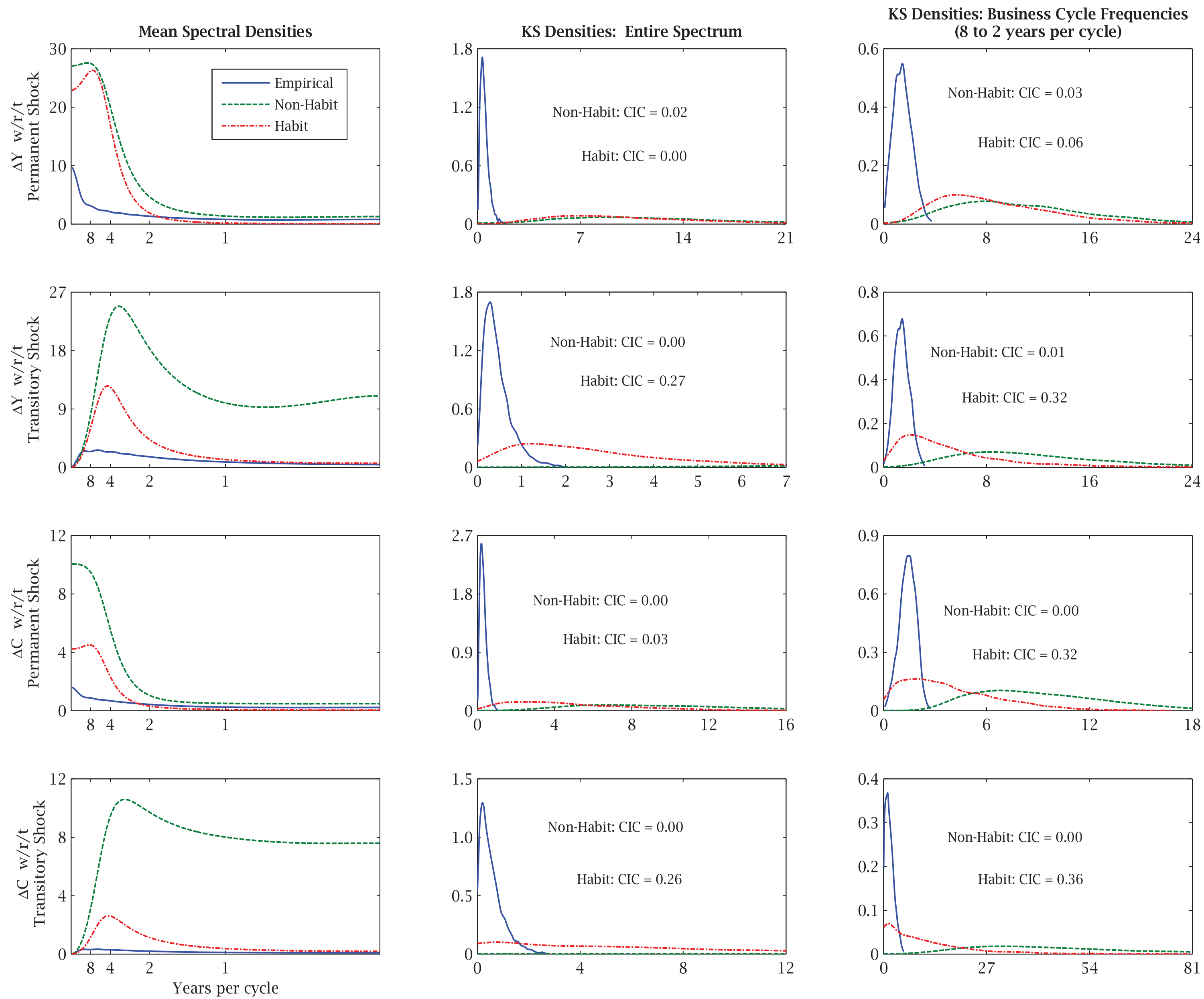
See section *A*4.1 and section 4.3 of the paper for details.

Figure A6: Mean Structural $\mathcal{E}$ and $\mathcal{T}$ SDs and KS Densities for NKDSGE Models with Taylor Rule, only Sticky Wages, and $h \sim \beta$



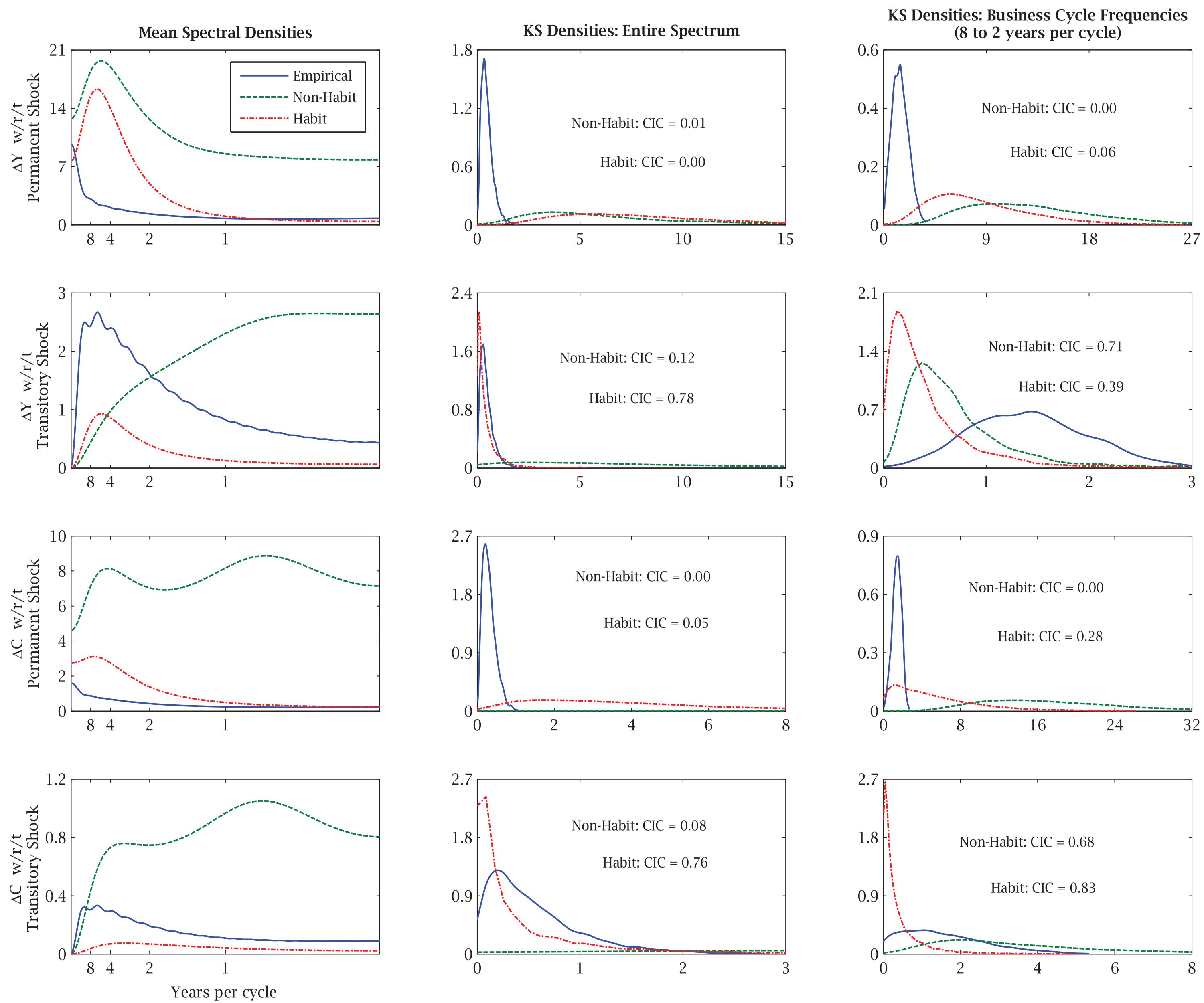
See section *A*4.1 and section 4.3 of the paper for details.

Figure A7: Mean Structural $\mathcal{E}$ and $\mathcal{T}$ SDs and KS Densities for Baseline NKDSGE Models with Money Growth Rule and $h \sim U(0.50, 0.95)$



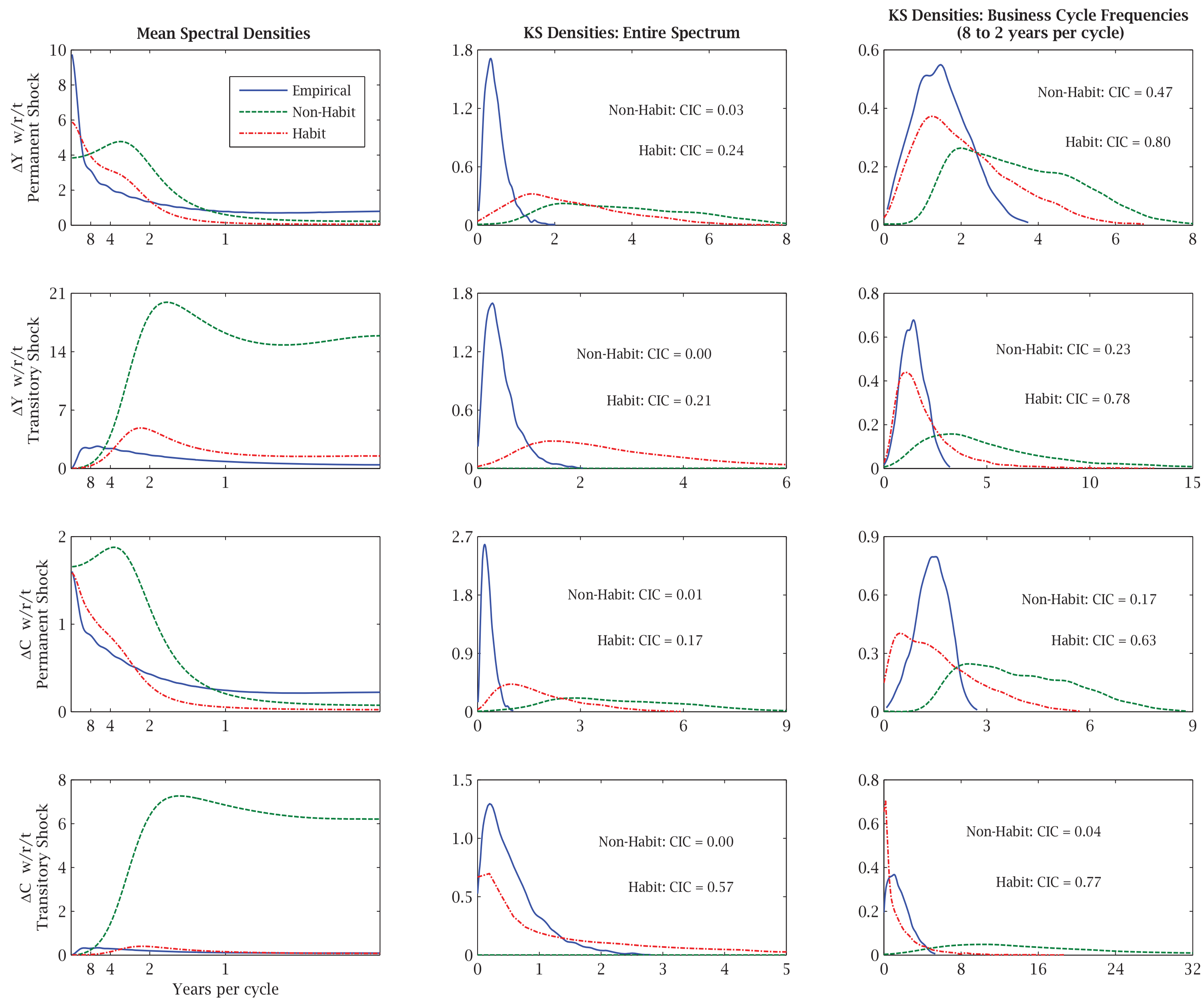
See section $A4.2$ and section 4.3 of the paper for details.

Figure A8: Mean Structural $\mathcal{E}$ and $\mathcal{T}$ SDs and KS Densities for Baseline NKDSGE Model with Taylor Rule and $h \sim U(0.50, 0.95)$



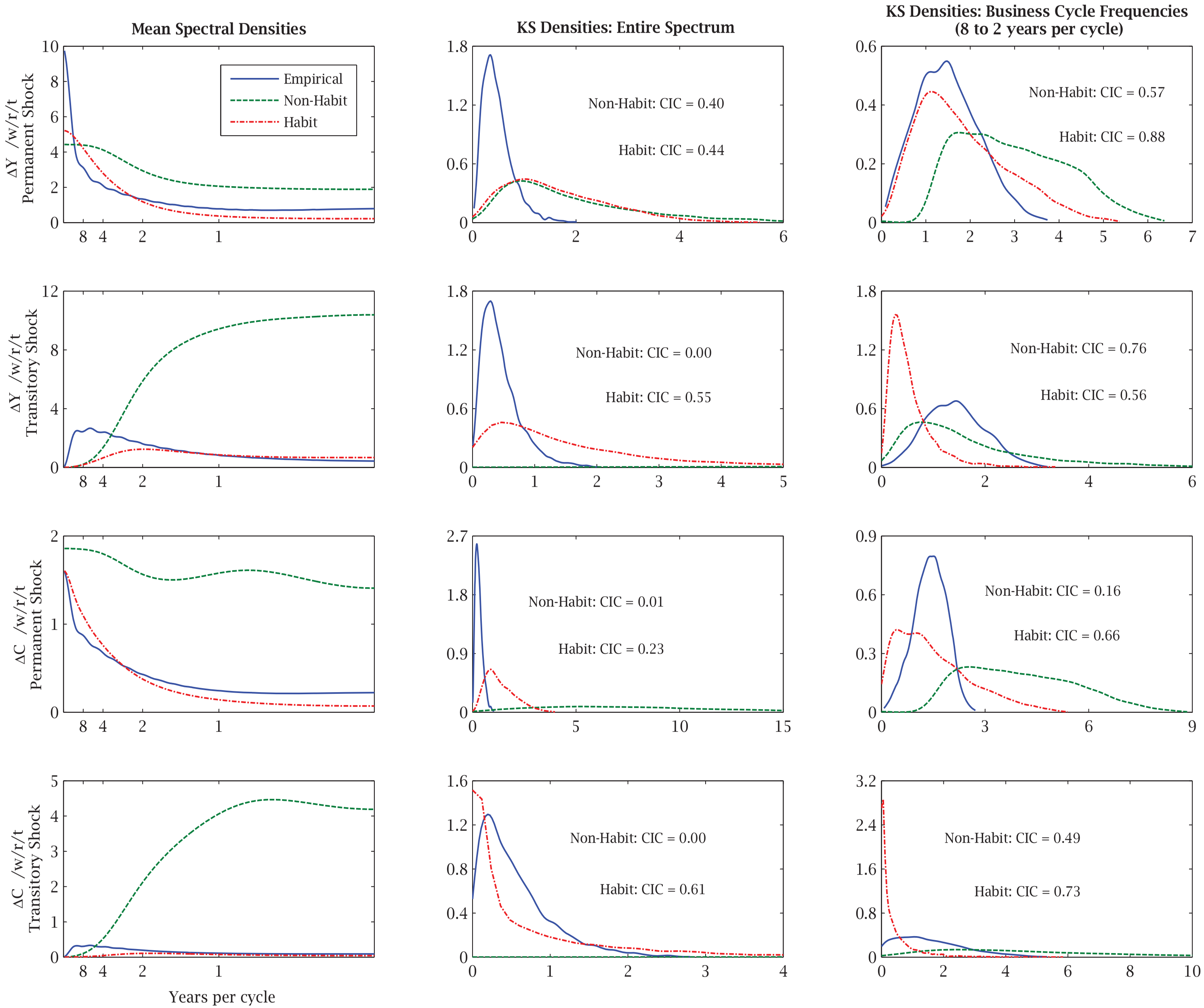
See section *A*4.2 and section 4.3 of the paper for details.

Figure A9: Mean Structural $\mathcal{E}$ and $\mathcal{T}$ SDs and KS Densities for NKDSGE Models with Money Growth Rule, Only Sticky Prices, and $h \sim U(0.50, 0.95)$



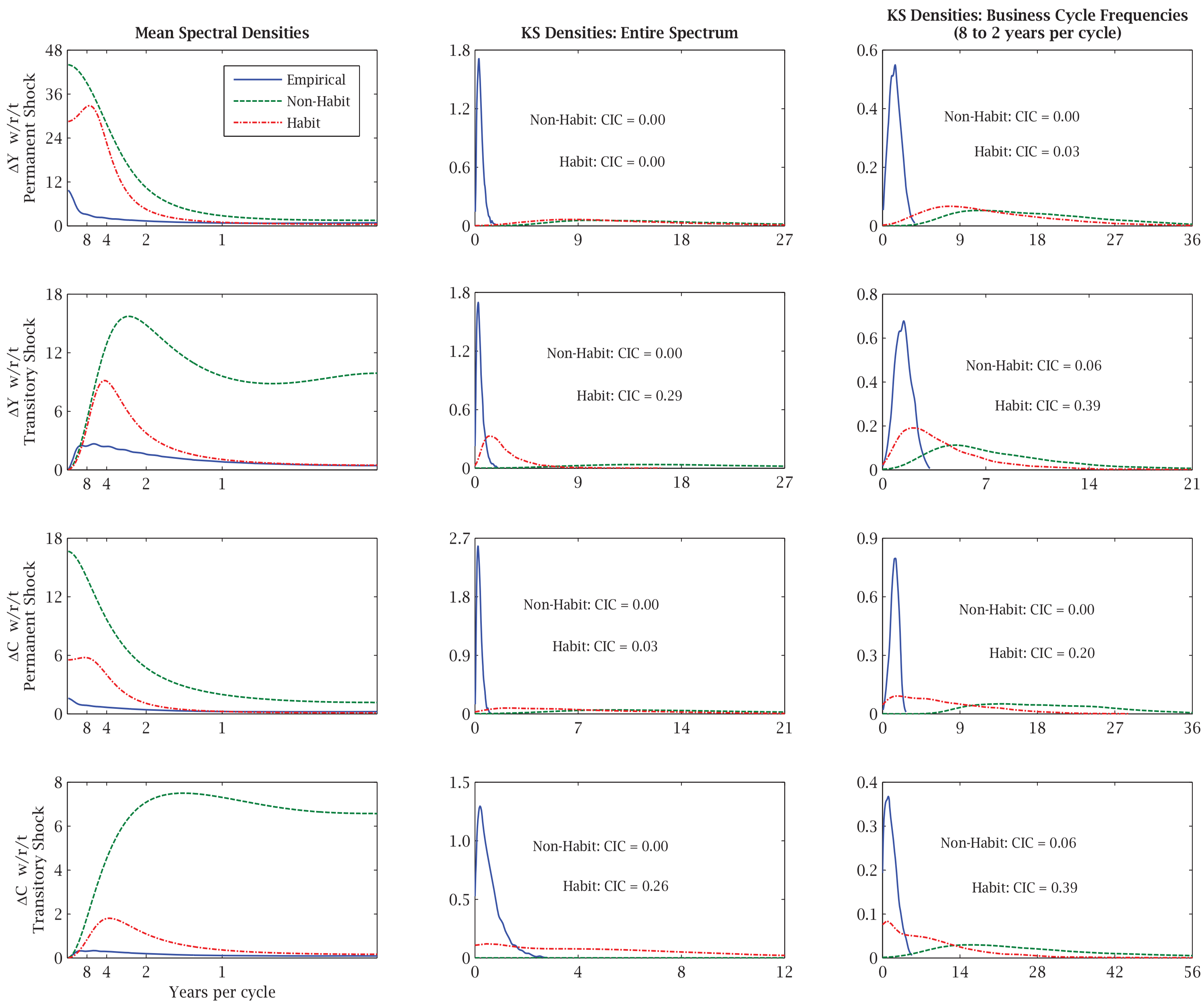
See section *A*4.2 and section 4.3 of the paper for details.

Figure A10: Mean Structural $\mathcal{E}$ and $\mathcal{T}$ SDs and KS Densities for NKDSGE Models with Taylor Rule, only Sticky Prices, and $h \sim U(0.50, 0.95)$



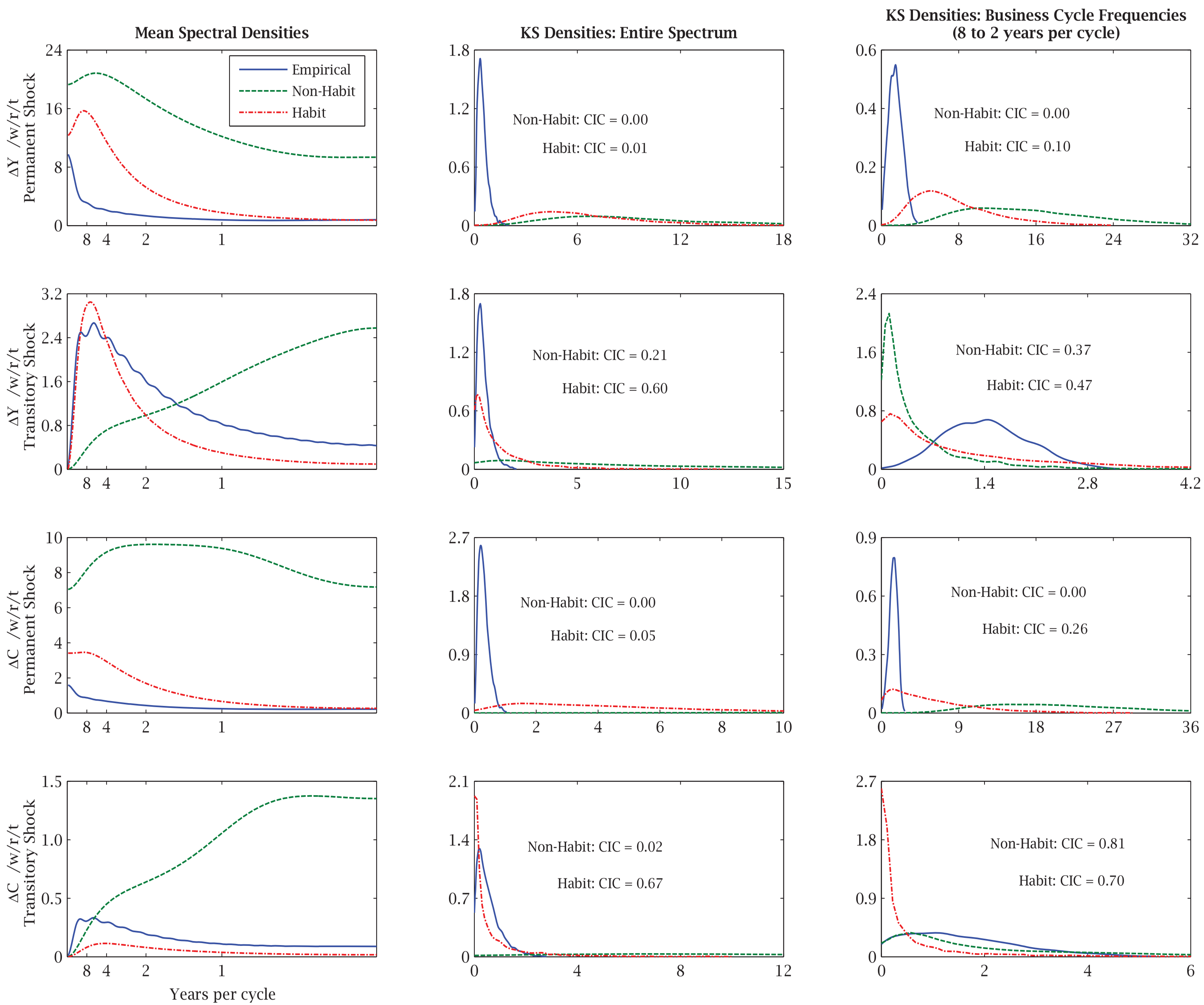
See section $A4.2$ and section 4.3 of the paper for details.

Figure A11: Mean Structural $\mathcal{E}$ and $\mathcal{T}$ SDs and KS Densities for NKDSGE Models with Money Growth Rule, Only Sticky Wages, and $h \sim U(0.50, 0.95)$



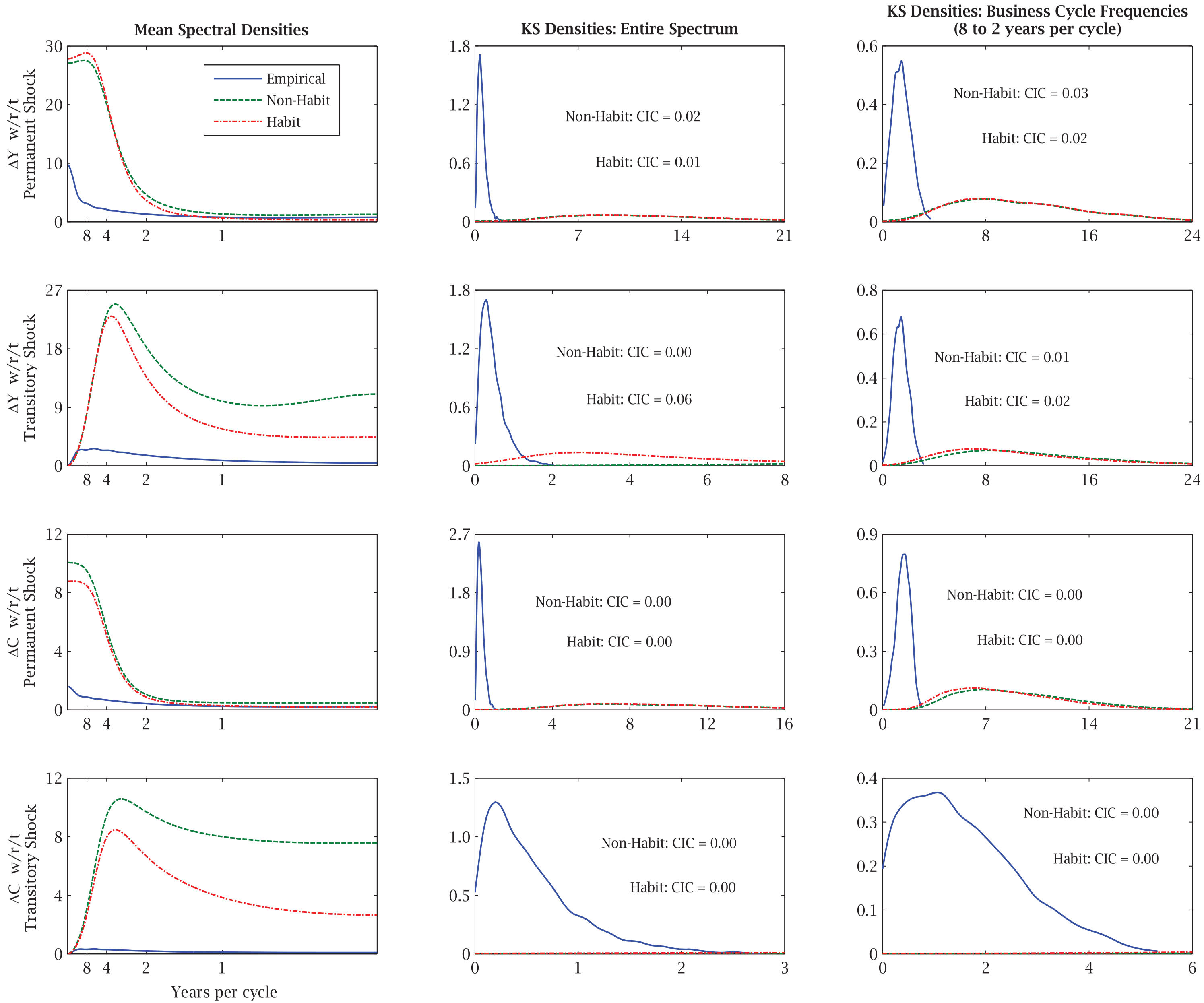
See section *A*4.2 and section 4.3 of the paper for details.

Figure A12: Mean Structural $\mathcal{E}$ and $\mathcal{T}$ SDs and KS Densities for NKDSGE Models with Taylor Rule, only Sticky Wages, and $h \sim U(0.50, 0.95)$



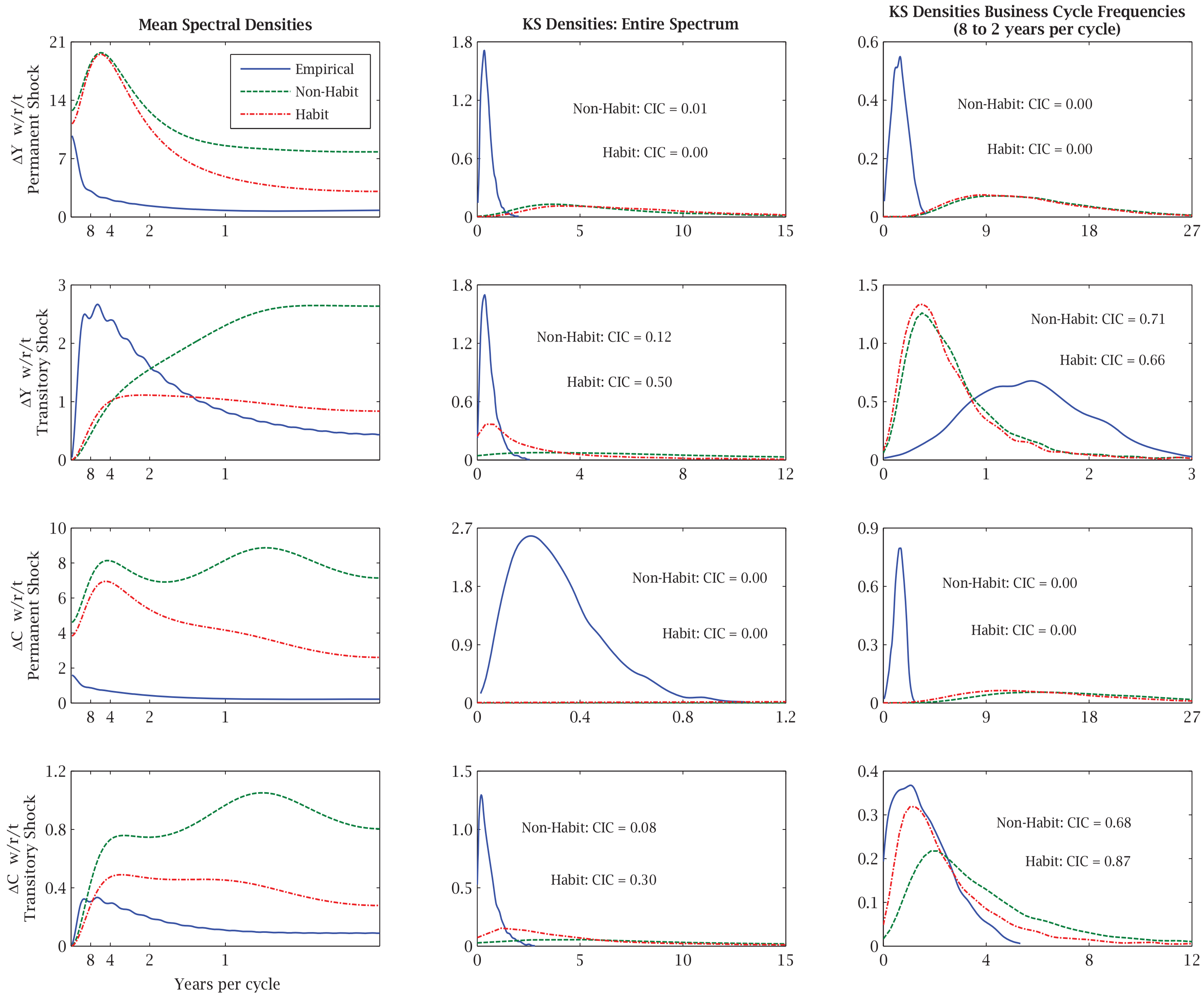
See section $A4.2$ and section 4.3 of the paper for details.

Figure A13: Mean Structural $\mathcal{E}$ and $\mathcal{T}$ SDs and KS Densities for Baseline NKDSGE Models with Money Growth Rule and $h \sim U(0.050, 0.499)$



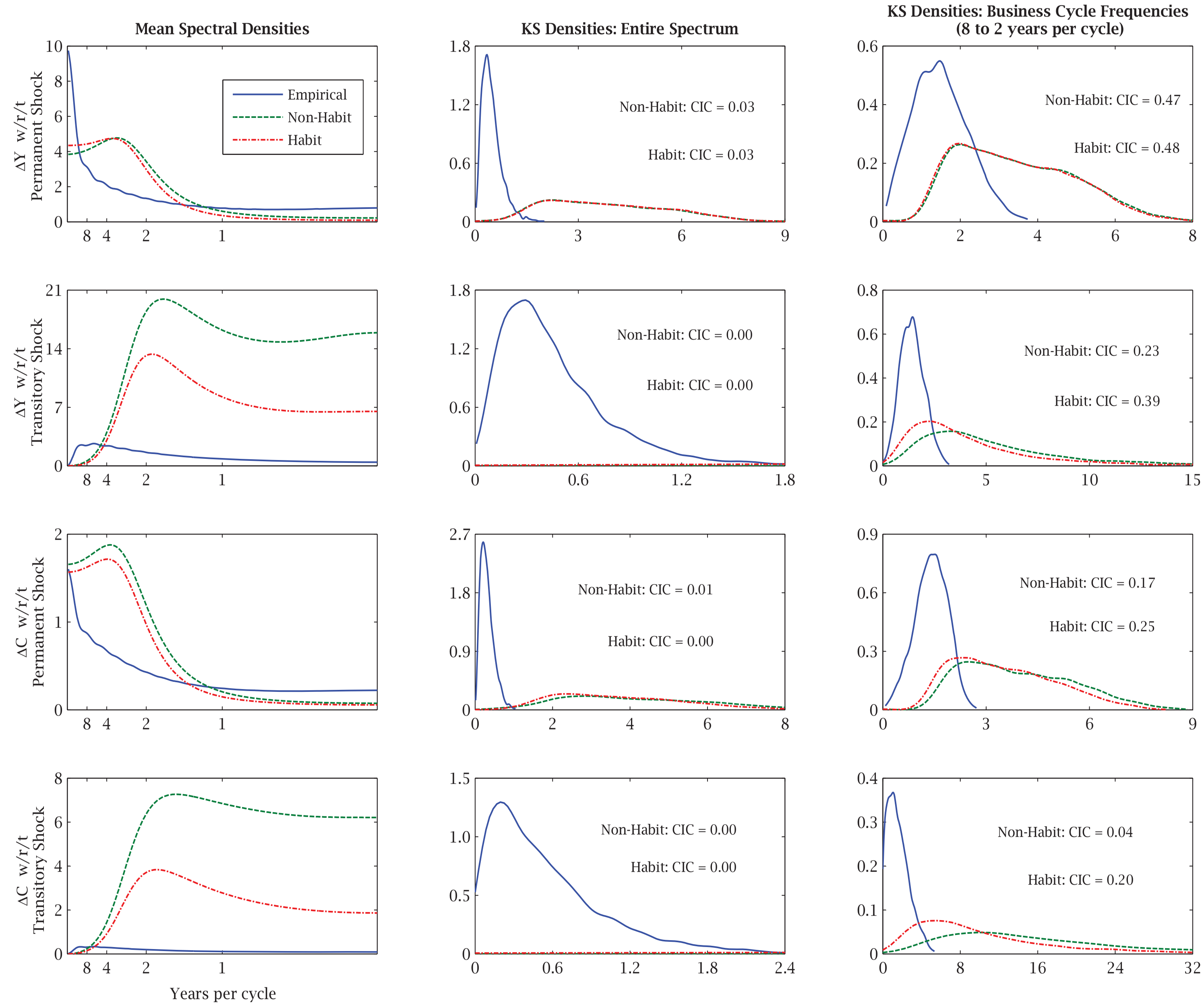
See section *A*4.3 and section 4.3 of the paper for details.

Figure A14: Mean Structural $\mathcal{E}$ and $\mathcal{T}$ SDs and KS Densities for Baseline NKDSGE Models with Taylor Rule and $h \sim U(0.050, 0.499)$



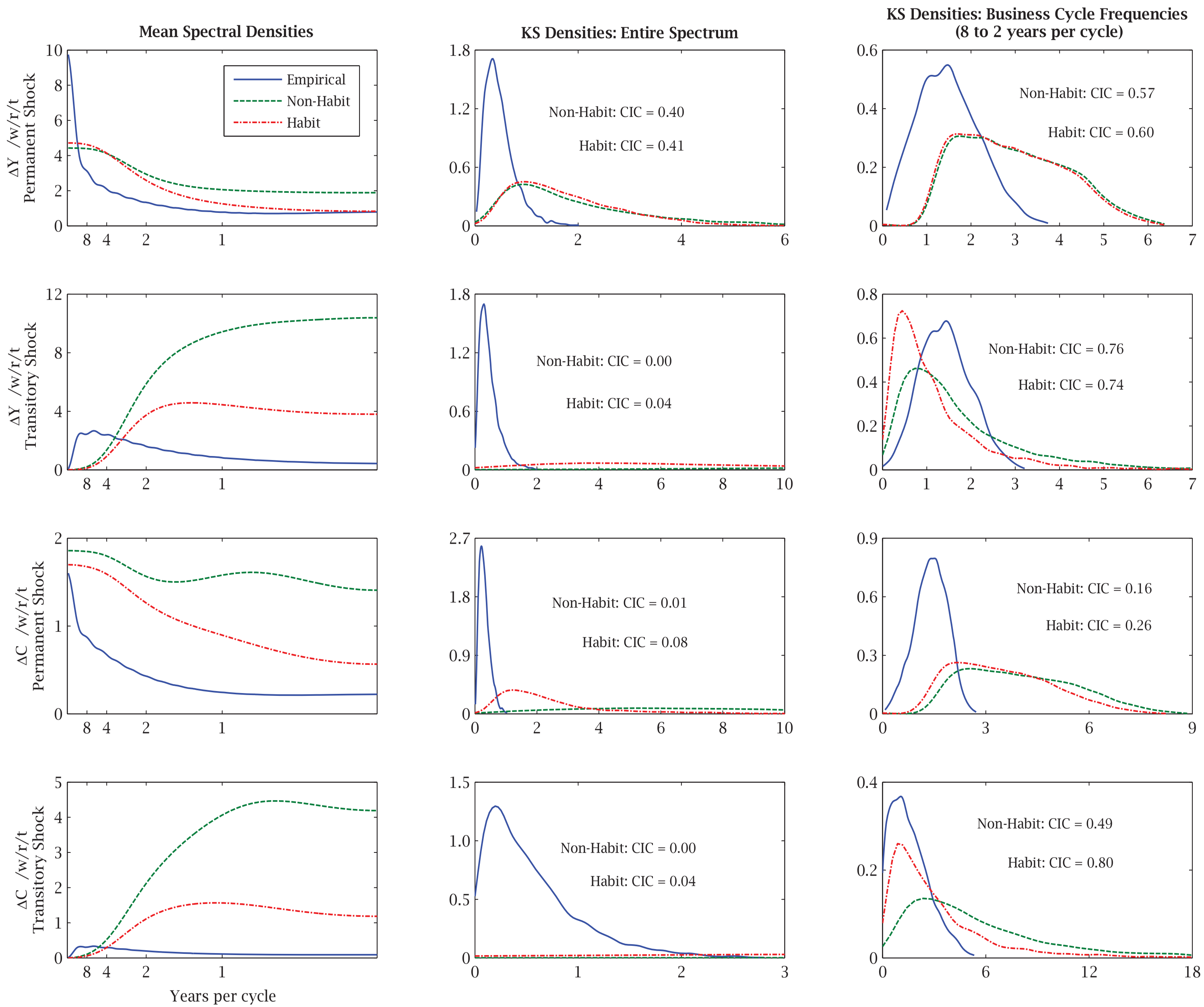
See section $A4.3$ and section 4.3 of the paper for details.

FIGURE A15: MEAN STRUCTURAL $\mathcal{E}$ AND $\mathcal{T}$ SDs AND KS DENSITIES FOR NKDSGE MODELS WITH MONEY GROWTH RULE, ONLY STICKY PRICES, AND $h \sim U(0.050, 0.499)$



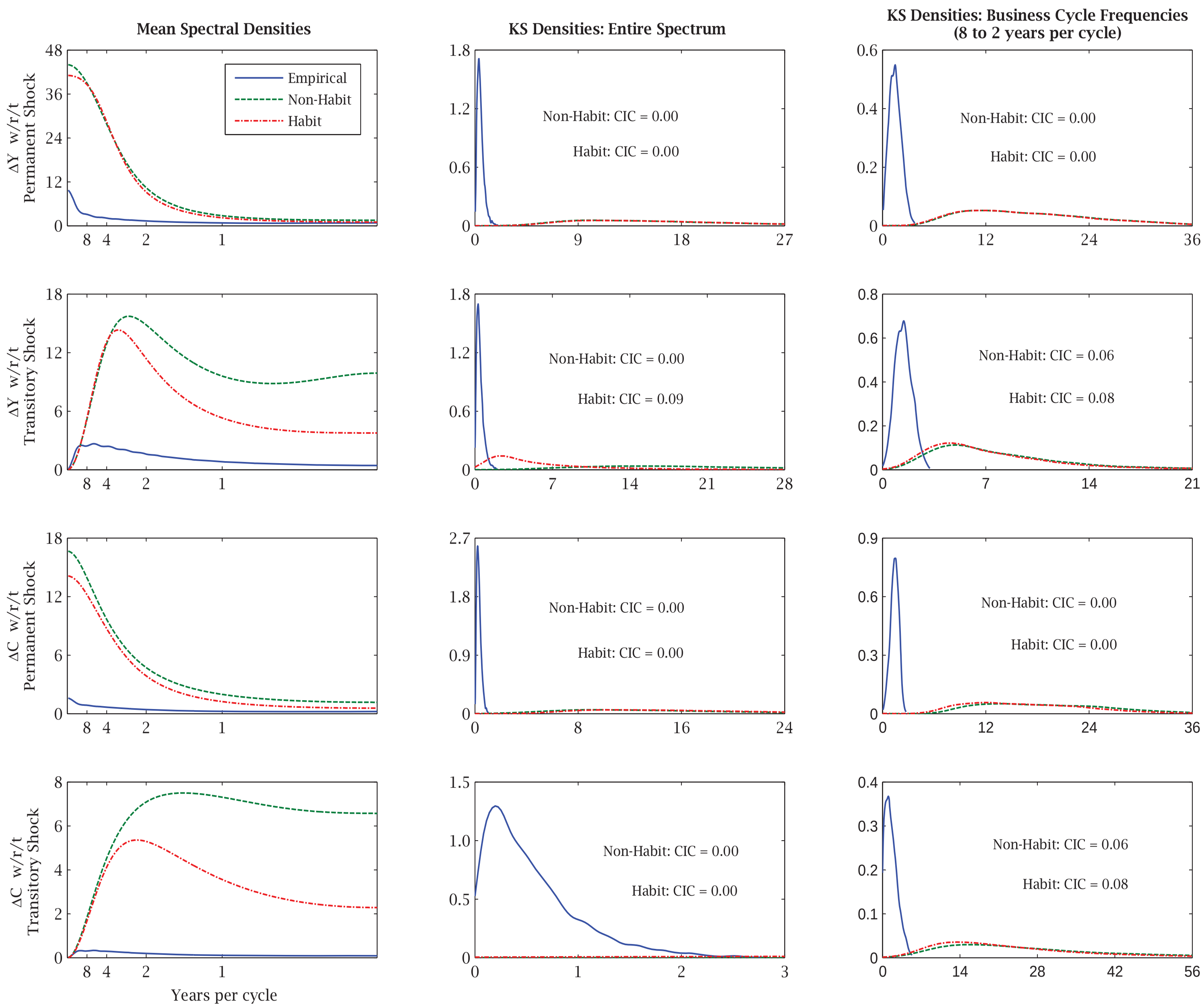
See section *A*4.3 and section 4.3 of the paper for details.

Figure A16: Mean Structural $\mathcal{E}$ and $\mathcal{T}$ SDs and KS Densities for NKDSGE Models with Taylor Rule, only Sticky Prices, and $h \sim U(0.050, 0.499)$



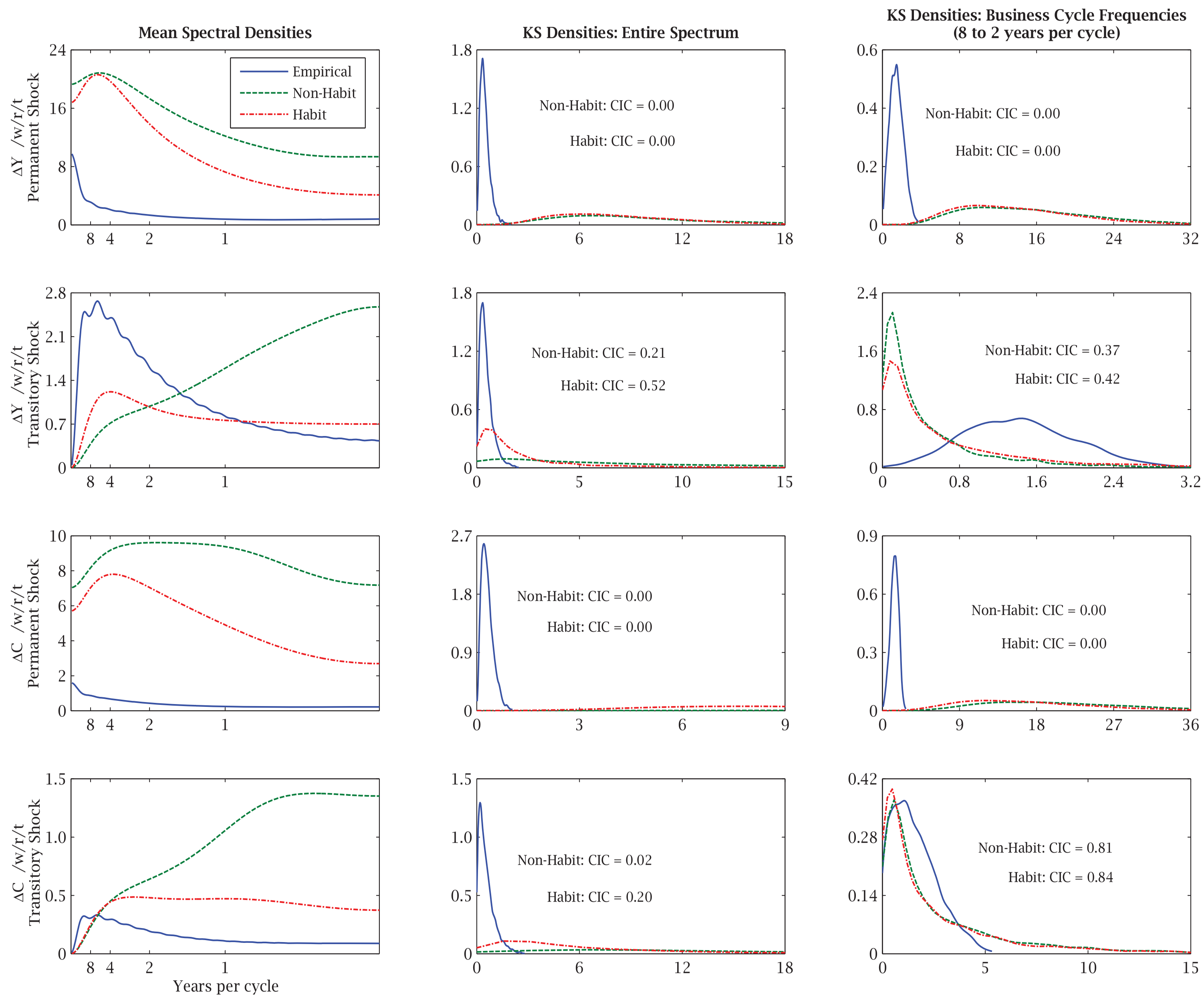
See section *A*4.3 and section 4.3 of the paper for details.

Figure A17: Mean Structural $\mathcal{E}$ and $\mathcal{T}$ SDs and KS Densities for NKDSGE Models with Money Growth Rule, Only Sticky Wages, and $h \sim U(0.050, 0.499)$



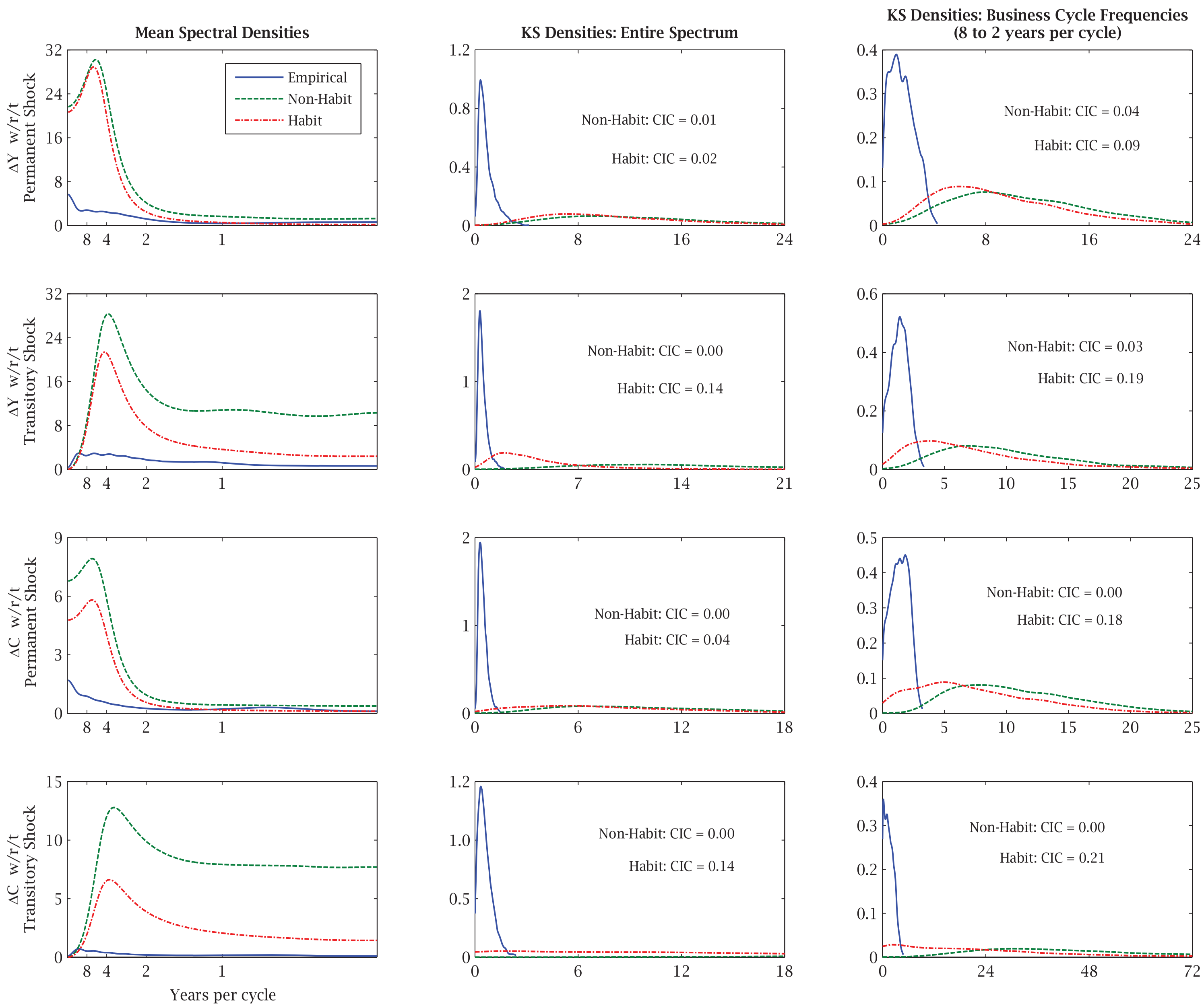
See section $A4.3$ and section 4.3 of the paper for details.

Figure A18: Mean Structural $\mathcal{E}$ and $\mathcal{T}$ SDs and KS Densities for NKDSGE Models with Taylor Rule, only Sticky Wages, and $h \sim U(0.050, 0.499)$



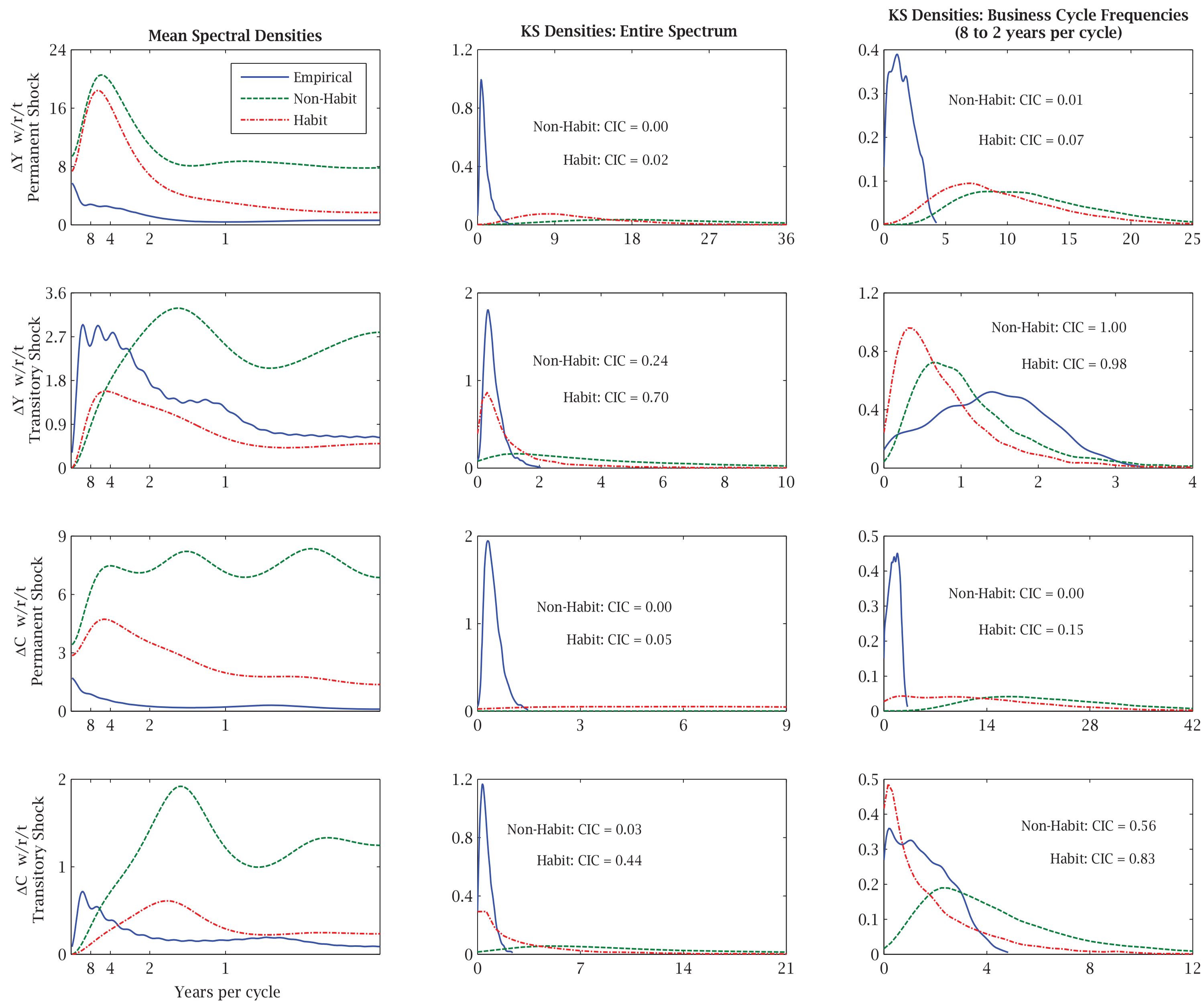
See section *A*4.3 and section 4.3 of the paper for details.

Figure A19: Mean Structural $\mathcal{E}$ and $\mathcal{T}$ SDs and KS Densities for Baseline NKDSGE Models with Money Growth Rule Using VAR(4)



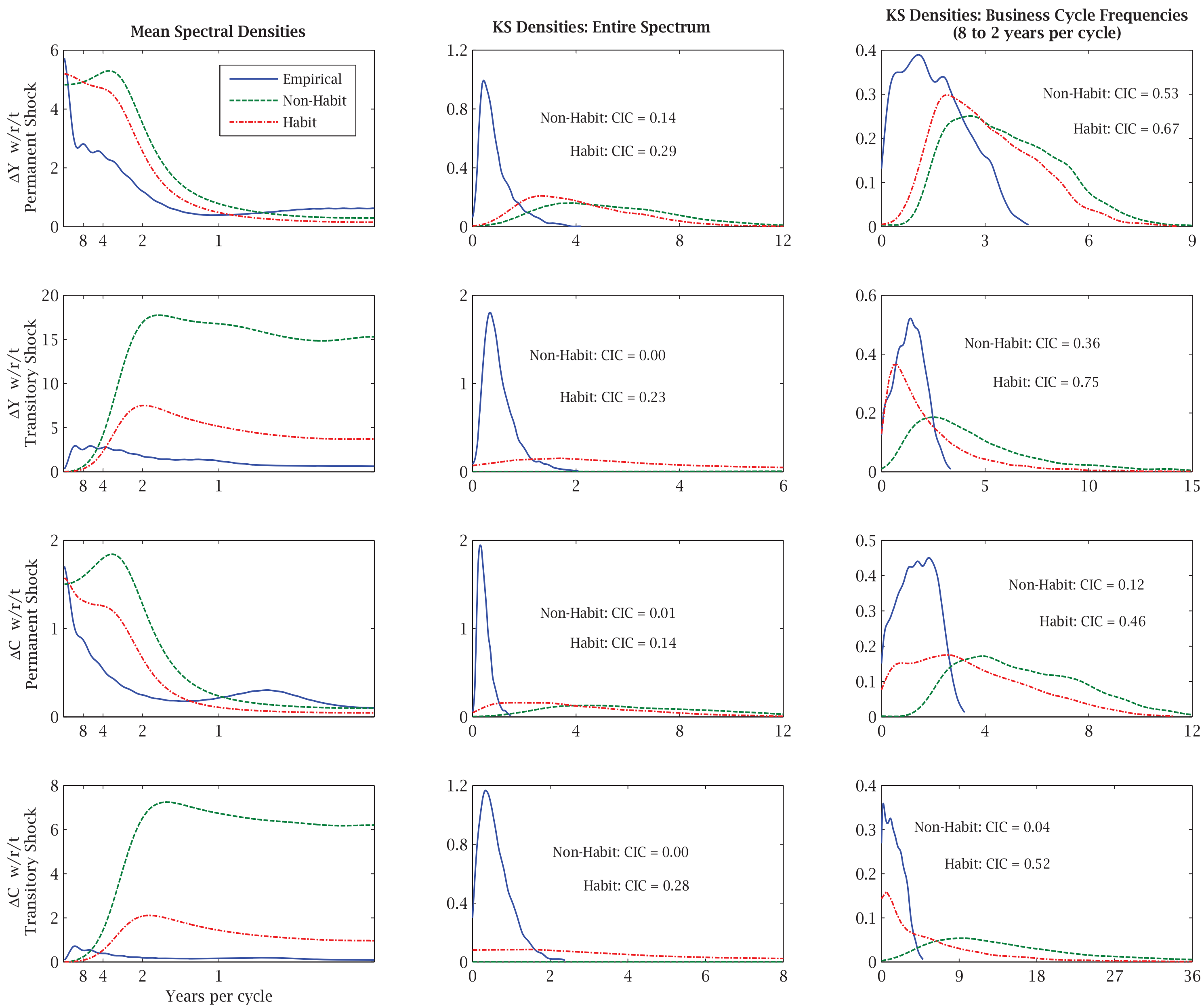
See section *A*4.4 and section 4.3 of the paper for details.

Figure A20: Mean Structural $\mathcal{E}$ and $\mathcal{T}$ SDs and KS Densities for Baseline NKDSGE Models with Taylor Rule Using VAR(4)



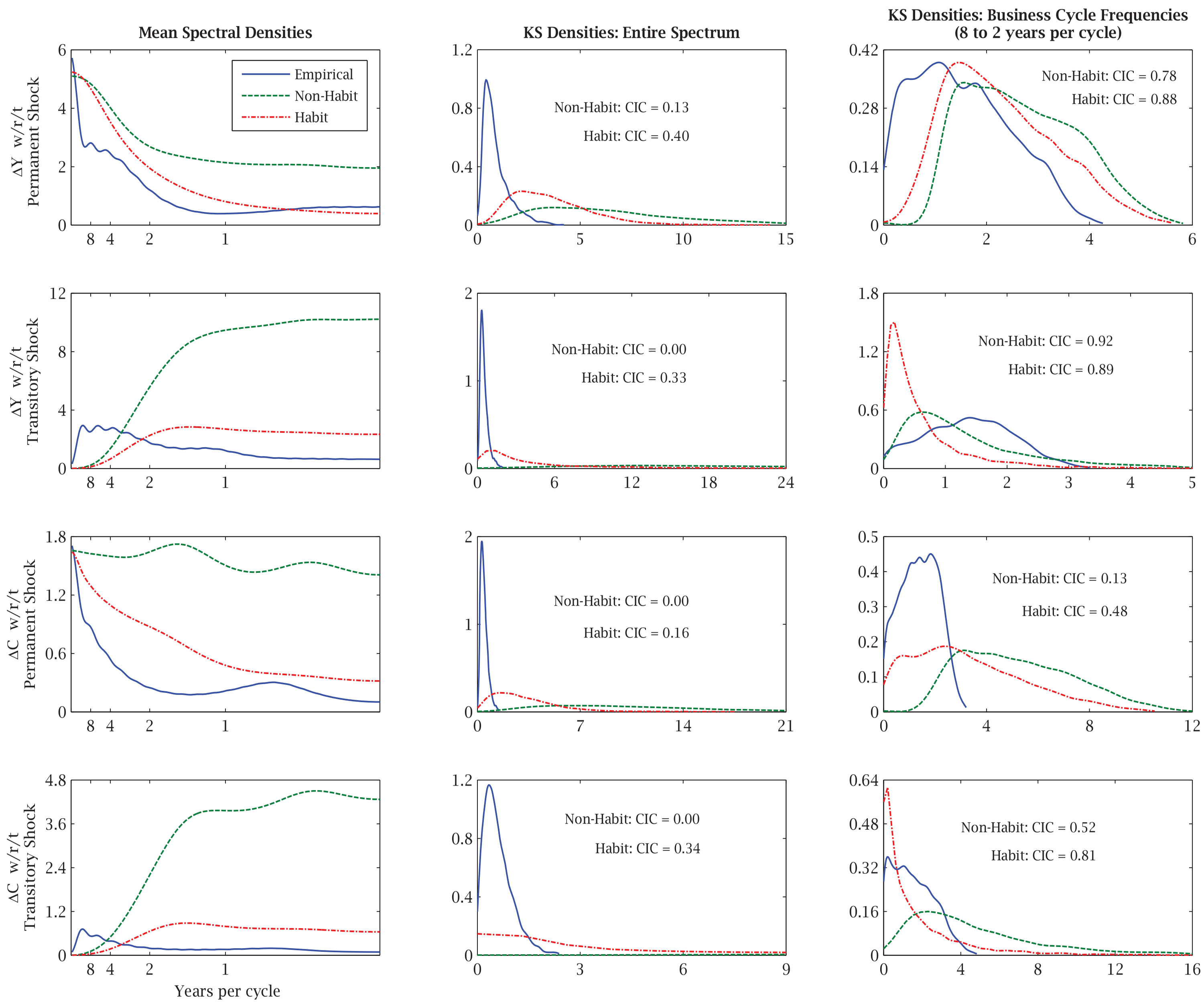
See section A4.4 and section 4.3 of the paper for details.

Figure A21: Mean Structural $\mathcal{E}$ and $\mathcal{T}$ SDs and KS Densities for NKDSGE Models with Money Growth Rule and Only Sticky Prices Using VAR(4)



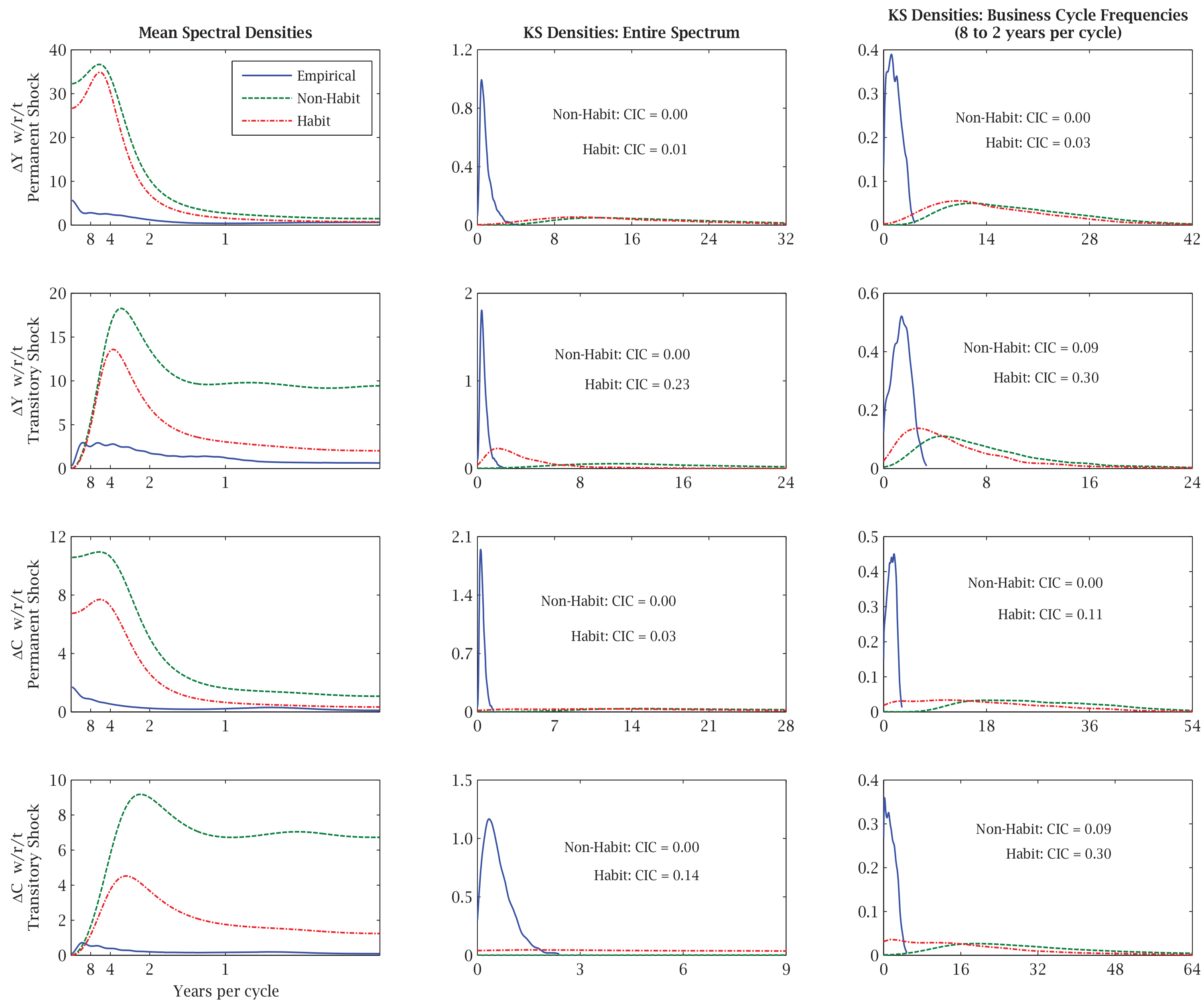
See section *A*4.4 and section 4.3 of the paper for details.

Figure A22: Mean Structural $\mathcal{E}$ and $\mathcal{T}$ SDs and KS Densities for NKDSGE Models with Taylor Rule and only Sticky Prices Using VAR(4)



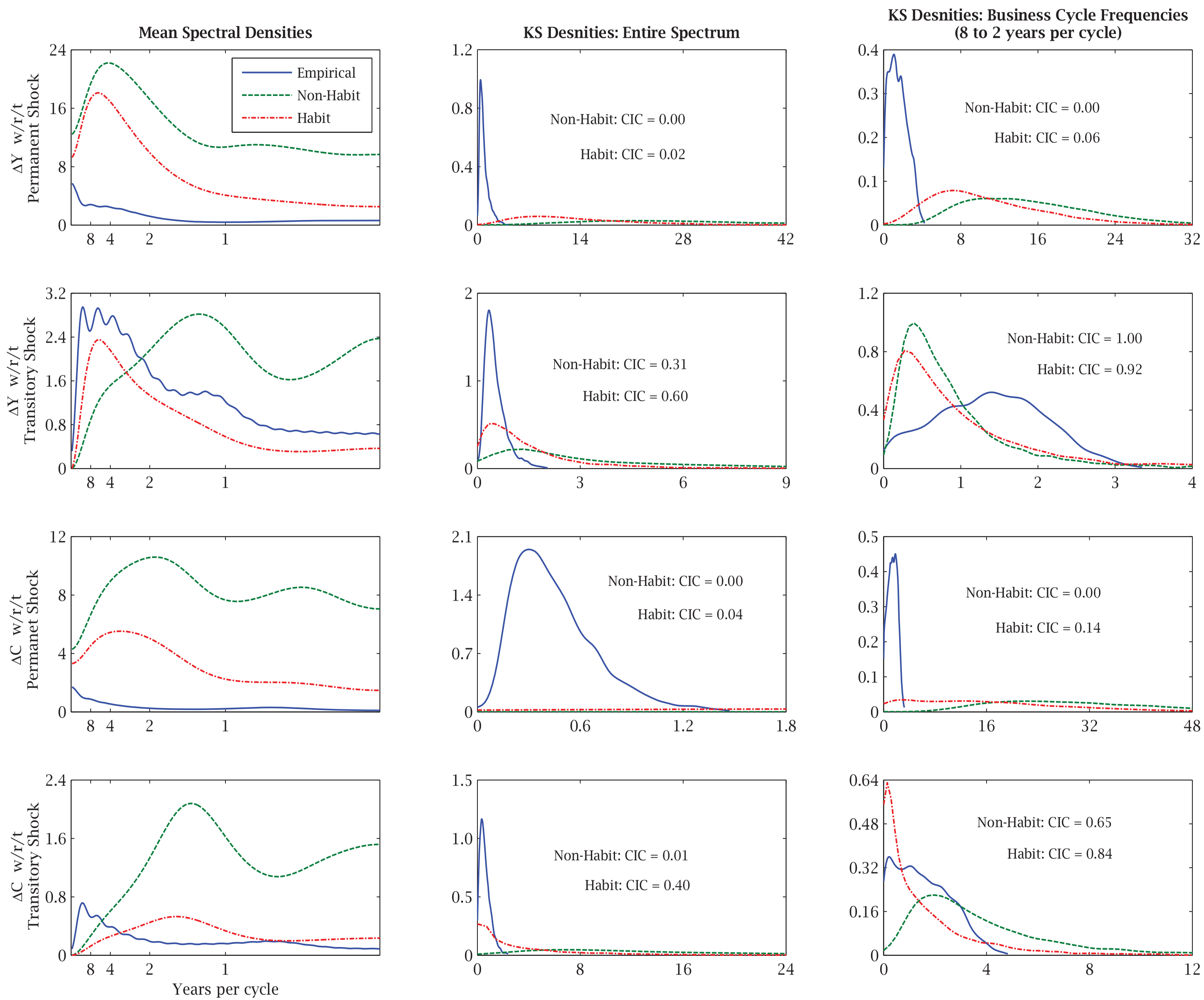
See section *A*4.4 and section 4.3 of the paper for details.

Figure A23: Mean Structural $\mathcal{E}$ and $\mathcal{T}$ SDs and KS Densities for NKDSGE Models with Money Growth Rule and Only Sticky Wages Using VAR(4)



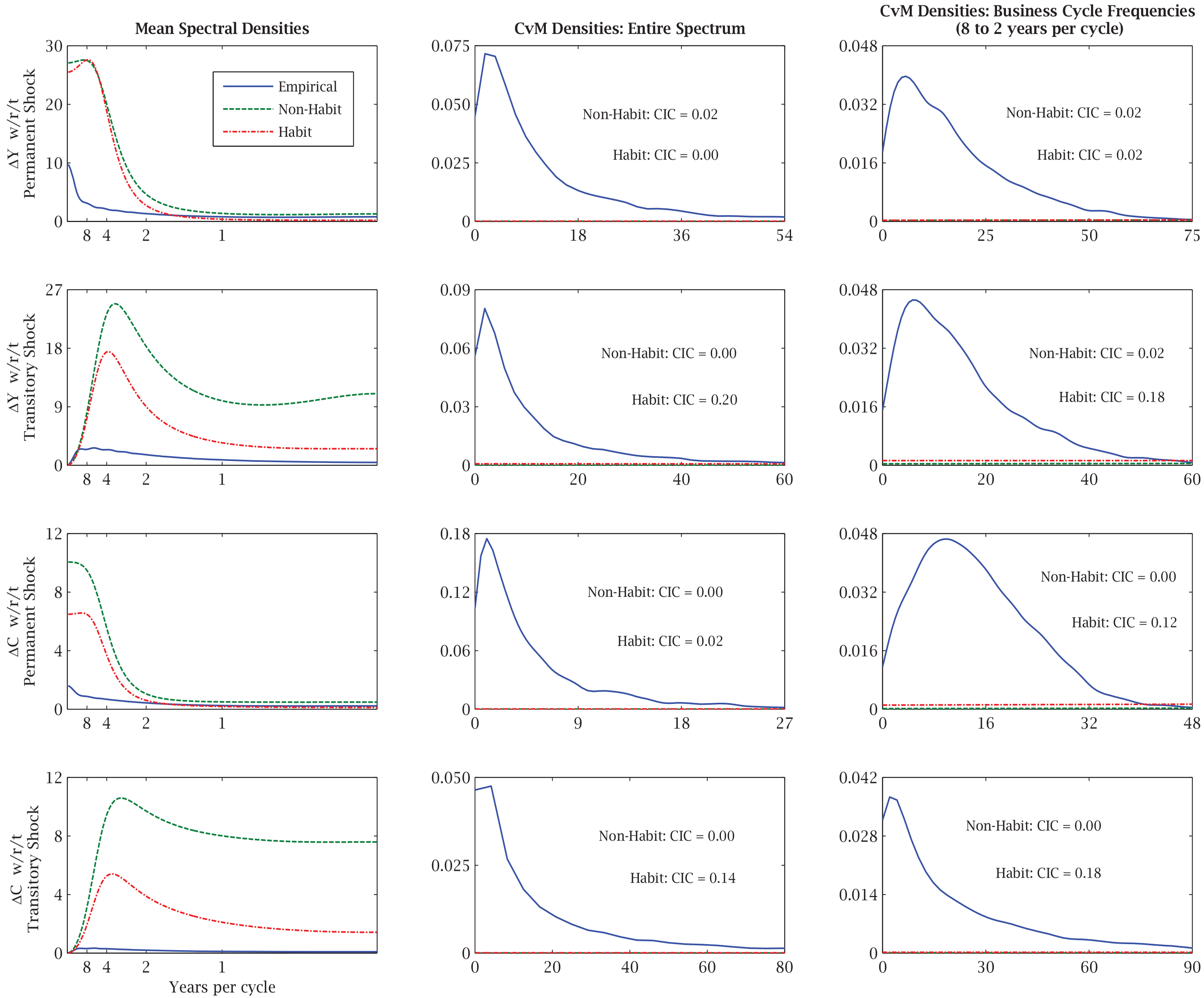
See section *A*4.4 and section 4.3 of the paper for details.

Figure A24: Mean Structural $\mathcal{E}$ and $\mathcal{T}$ SDs and KS Densities for NKDSGE Models with Taylor Rule and only Sticky Wages Using VAR(4)



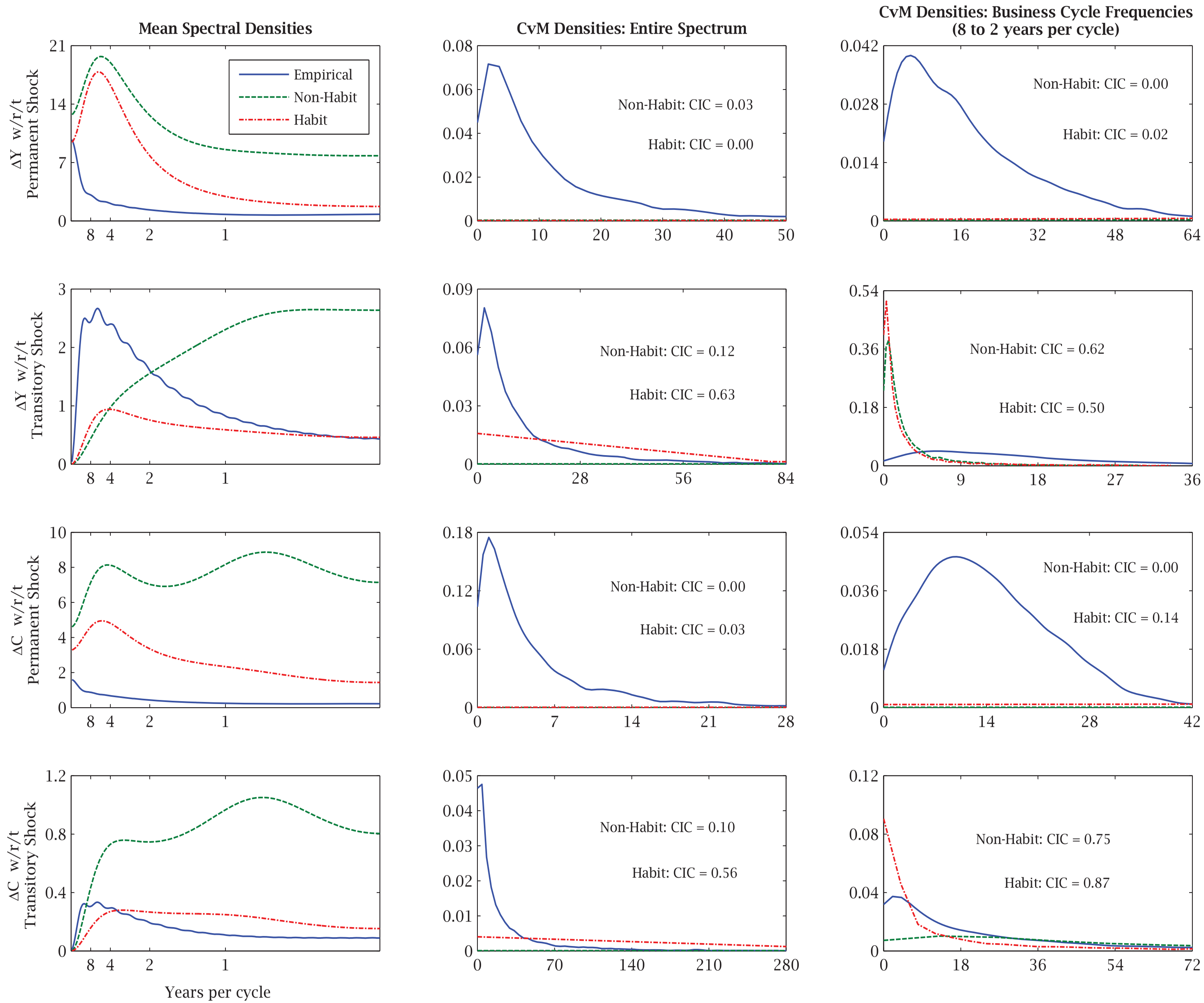
See section *A*4.4 and section 4.3 of the paper for details.

Figure A25: Mean Structural $\mathcal{E}$ and $\mathcal{T}$ SDs and CvM Densities for Baseline NKDSGE Models with Money Growth Rule



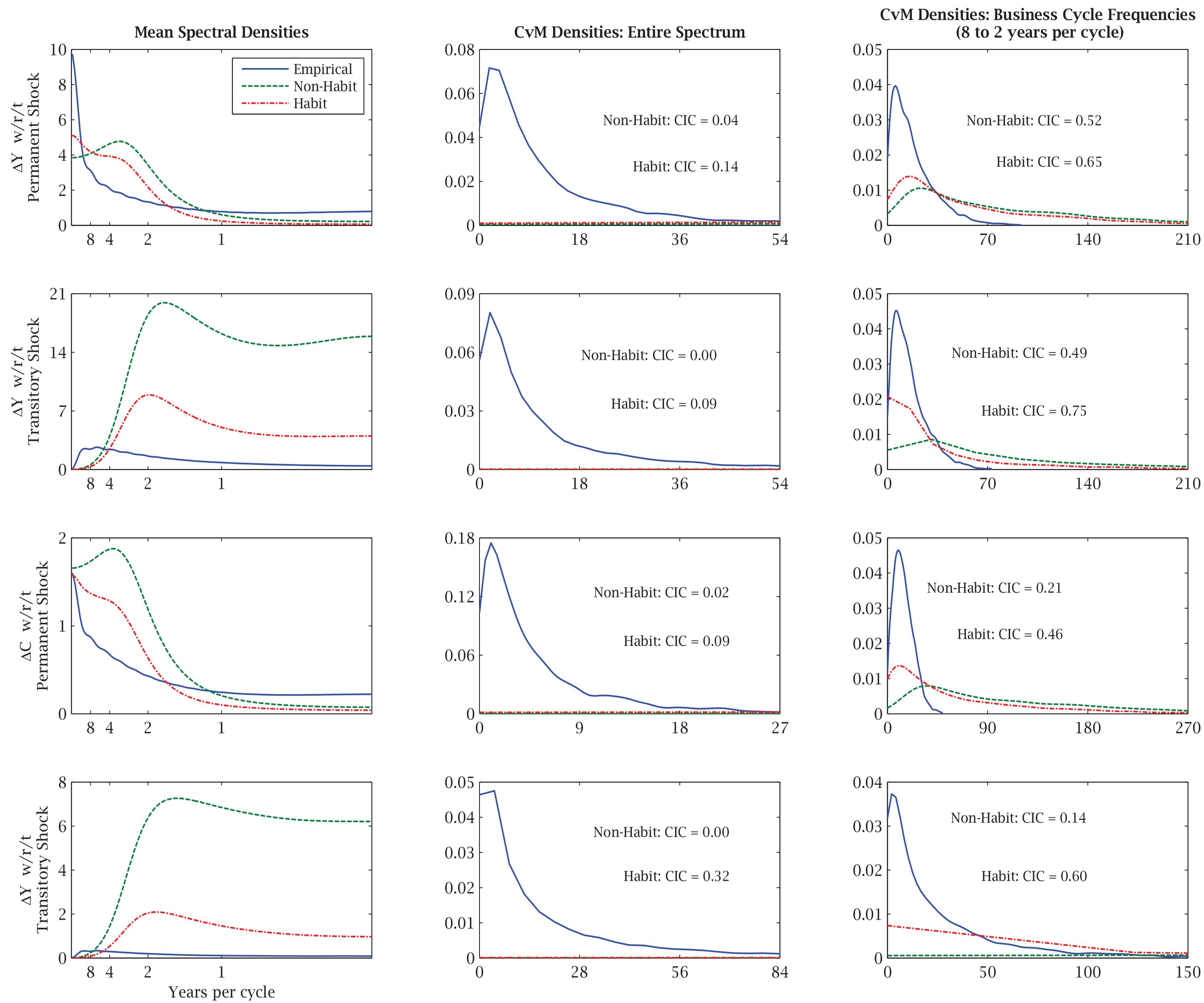
See section $A4.5$ and section 4.3 of the paper for details.

Figure A26: Mean Structural $\mathcal{E}$ and $\mathcal{T}$ SDs and CvM Densities for Baseline NKDSGE Models with Taylor Rule



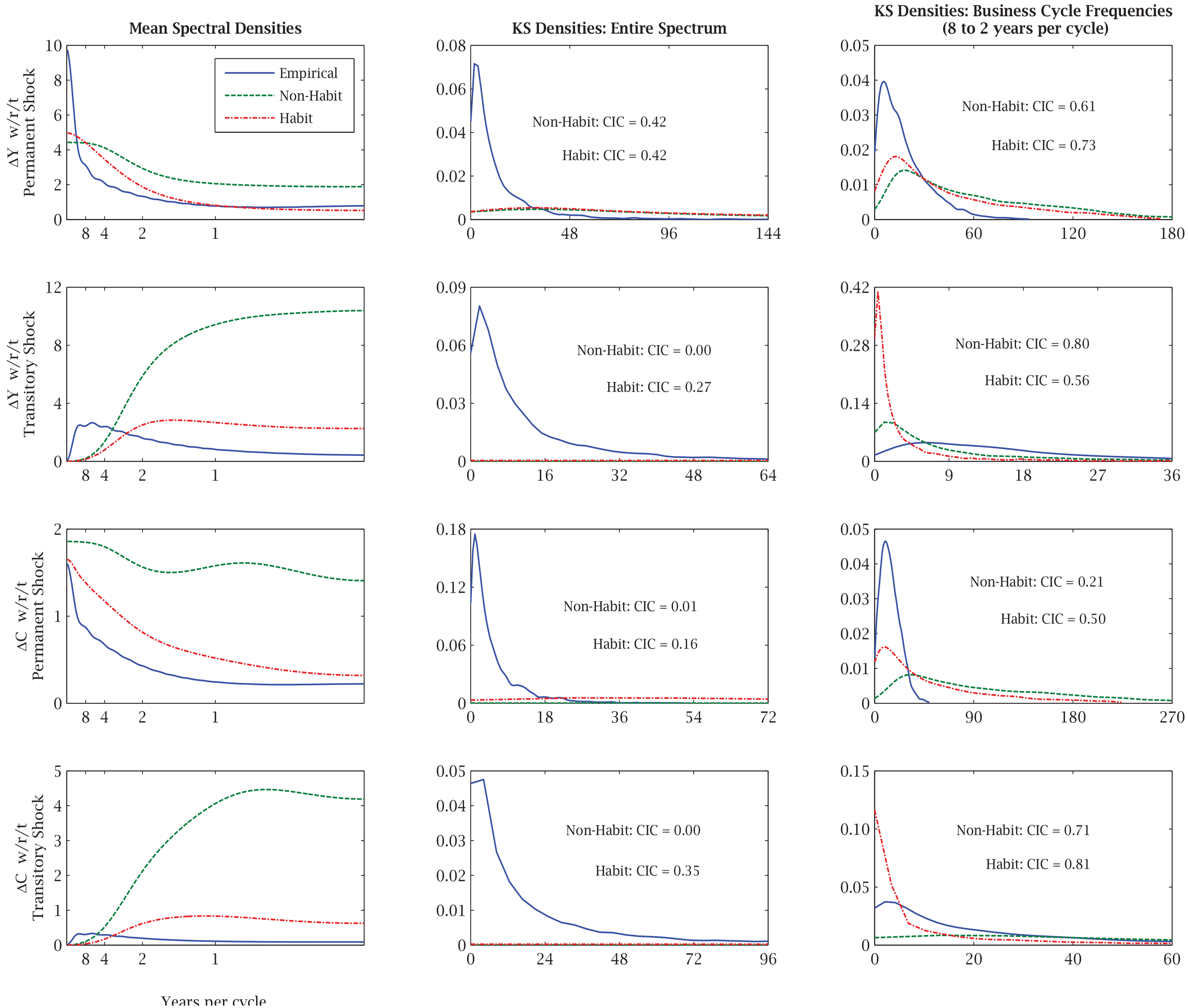
See section *A*4.5 and section 4.3 of the paper for details.

Figure A27: Mean Structural $\mathcal{E}$ and $\mathcal{T}$ SDs and CvM Densities for NKDSGE Models with Money Growth Rule and Only Sticky Prices



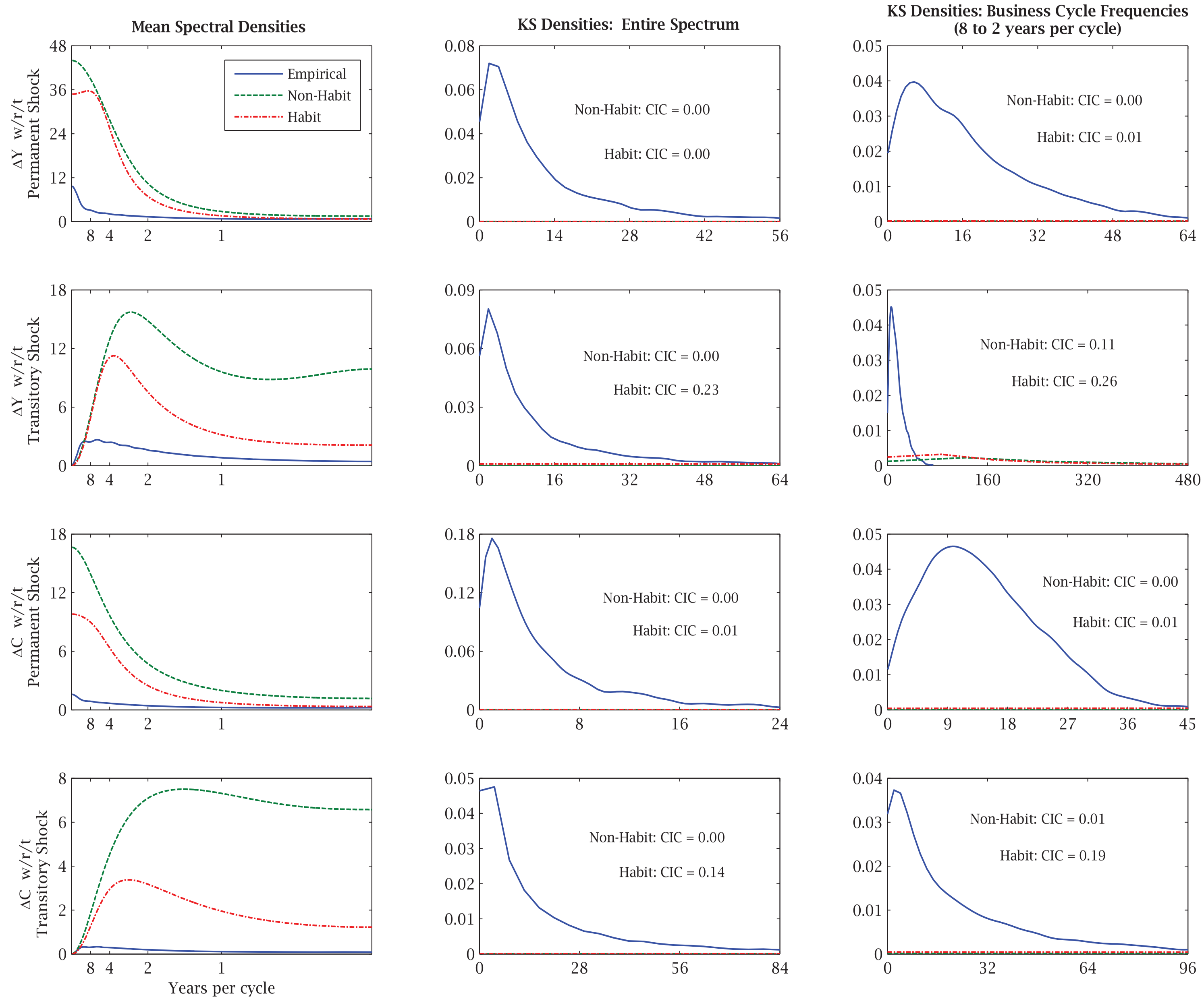
See section A4.5 and section 4.3 of the paper for details.

Figure A28: Mean Structural $\mathcal{E}$ and $\mathcal{T}$ SDs and CvM Densities for NKDSGE Models with Taylor Rule and only Sticky Prices



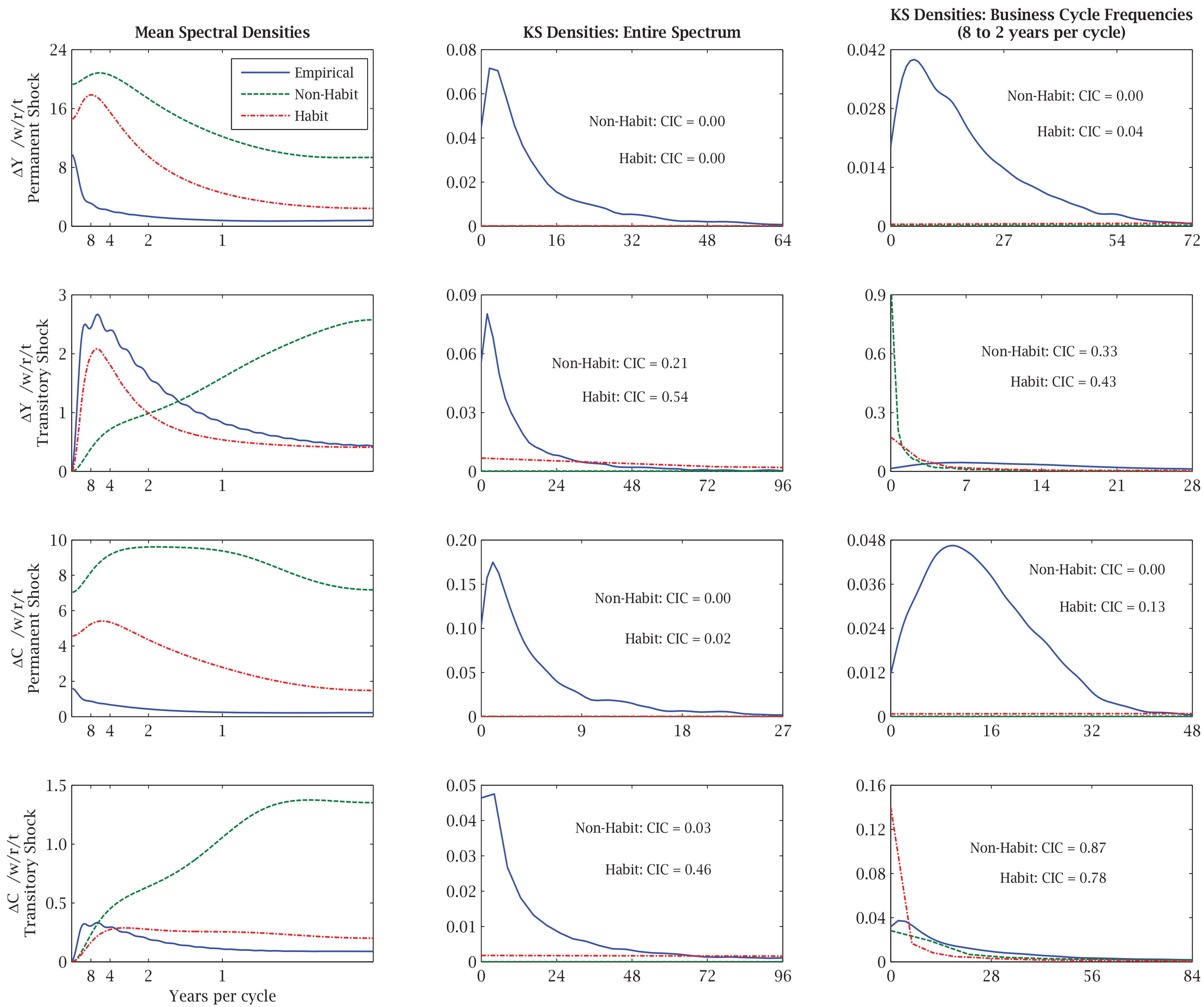
See section *A*4.5 and section 4.3 of the paper for details.

Figure A29: Mean Structural $\mathcal{E}$ and $\mathcal{T}$ SDs and CvM Densities for NKDSGE Models with Money Growth Rule and Only Sticky Wages



See section *A*4.5 and section 4.3 of the paper for details.

Figure A30: Mean Structural $\mathcal{E}$ and $\mathcal{T}$ SDs and CvM Densities for NKDSGE Models with Taylor Rule and only Sticky Wages



See section *A*4.5 and section 4.3 of the paper for details.