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Fertility in an Unequal, Innovative World

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Abstract

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Keywords

fertility, growth, technological change, inequality

JEL Classification

D1, J1, O10

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Fertility in an Unequal, Innovative World*

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Keywords: Fertility; Growth; Technological change; Inequality
JEL codes: D1, J1, O10

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1 Introduction

The relationship between income and fertility is one of the most studied empirical regularities in economics, yet no single theory accounts for all its observed forms. In early, Malthusian economies fertility rises with income as relaxed subsistence constraints allow larger families. During the demographic transition fertility falls sharply with income as rising returns to education shift household preferences toward child quality over quantity. And in several advanced economies today the relationship has flattened or turned mildly positive again, a pattern recently documented by Doepke et al. (2023) across OECD countries. Understanding what drives these regime changes is central to predicting long-run population dynamics and their consequences for growth.

This paper offers a unified explanation. We develop a theory in which technological progress shapes fertility through two interconnected but opposing channels. The first is an *education channel*: technological progress raises the return to education and therefore the opportunity cost of raising children, pushing households toward quality over quantity. The second is an *inequality channel*: because high-human-capital households benefit disproportionately from technological progress, it widens the distribution of relative human capital across households, which depending on the development stage, can raise aggregate fertility. In Proposition 1, we present the interaction between these two channels - one reducing and the other raising fertility - which generates the full non-monotonic historical fertility path within a single framework.

The central message is that inequality is not a side outcome of the growth process; it is part of the mechanism through which technological progress reshapes fertility. This matters for aggregate dynamics because inequality does not merely shift the *level* of fertility - it changes the *composition* of fertility across households. This composition effect, which we formalize in Proposition 2, is regime-dependent and is absent from representative-agent models of the demographic transition.

The theory generates three fertility regimes. In the **Malthusian (ML) regime**, inequality raises both fertility and education; combined with a positive income effect, technological progress raises fertility. In the **Modern Growth (MG) regime**, inequality reduces both, and the substitution effect dominates, so fertility falls unambiguously. In the **More-Modern-Growth (MMG) regime** - corresponding to the contemporary partial fertility recovery observed in high-income OECD countries - inequality again raises fertility and education, and the income effect re-emerges. These predictions are formalised in Propositions 1–2 and confirmed through calibrated simulations.

Our paper contributes to a rich literature on fertility and growth.[1] Galor and Weil

[1]The long-run process by which economies transition from Malthusian stagnation to sustained modern growth through the interaction between technological progress, population dynamics, and the rising importance of education has been systematically explored in a body of influential work; see Galor (2011) for a comprehensive treatment.

(2000) explain the demographic transition through rising returns to education in a representative-household framework. Galor and Moav (2002) introduce a rich form of household heterogeneity through genetically determined preferences for child quality rather than endowments, and their focus is on the evolutionary origins of growth rather than on how technological progress widens the distribution of human capital across income groups and reshapes fertility differentially — the mechanism we develop here. de la Croix and Doepke (2003), building on Glomm and Ravikumar (1992) and Becker and Barro (1998), introduce differential fertility and inequality within the MG regime, finding that greater inequality unambiguously reduces growth.[2] We show this conclusion is regime-dependent: in the Malthusian and MMG regimes, rising inequality *accelerates* growth through the composition effect. Jones (2022) studies the macroeconomic consequences of an Empty Planet by modelling technological progress as a function of total population while treating education as exogenous.[3] We endogenise it by modelling technological progress as a function of total population and average education and show that the EP outcome is one of four possible long-run equilibria — alongside the ML trap, sustained growth, and a renewed fertility rise — with inequality playing a necessary role in determining which one prevails (Proposition 3).[4]

We provide supporting empirical evidence using US state-level data for 1880–1940, spanning the Modern Growth transition. Using state-level total factor productivity as an instrument for land-concentration inequality, the 2SLS estimates show that higher inequality is associated with higher fertility and lower educational investment. Once inequality and education are jointly controlled for, TFP has no residual explanatory power for fertility, consistent with the model's prediction that technology affects fertil-

[2]Several other studies also contribute important perspectives. Demographic transition, mortality, longevity, human capital, and growth have also been studied together by Boucekkine et. al. (2002, 2003), de la Croix and Licandro (2013) among others. Cordoba and Ripoll (2014) observes that *income-fertility* relationships vanish if parents could legally impose debt on children to recover upbringing costs, but emerge when such constraints exist, turning negative when intergenerational elasticity of substitution exceeds unity (also see altruistic agents setups in Cordoba and Ripoll (2019) and Cordoba et al. (2016)). Bhattacharya and Chakraborty (2007) links fertility behavior to child mortality and contraception, Aksan and Chakraborty (2014) presents demographic, disease and economic transition in a general equilibrium.

[3]Kogel and Prskawetz (2001), Hansen and Prescott (2002), Tamura (2002), and Doepke (2004) - analyze the transition from pre-industrial stagnation to sustained growth, often highlighting demographic shifts. Moav (2005) develops a theory where families jointly choose fertility and child education, with higher human capital enhancing individuals' effectiveness as teachers. It explains persistent poverty across and within countries and shows that inequality hampers growth both through its impact on fertility decisions and by reducing the relative returns to human and physical capital. Aiyar, Dalgaard and Moav (2008) captures positive feedback between the evolution of population and the adoption of technology in the pre-industrial world.

[4]The relationship between intergenerational transfers and endogenous fertility — including the education–pension nexus (e.g. Amol et al. (2023a), Bishnu et al. (2023b)) — as well as the broader literature on intergenerational transfers without endogenous fertility (e.g. Andersen and Bhattacharya (2017), Bishnu (2013), Bishnu et al. (2021), Bishnu and Wang (2013), Boldrin and Montes (2005), Kaganovich and Zilcha (1999), Lancia and Russo (2016)) — is rich and well-developed, though it is not the focus of this paper.

ity through these channels alone.[5]

The rest of the paper is organised as follows. Section 2 presents the model. Section 3 characterises how technological progress and inequality shape fertility, education, and growth across regimes. Section 4 analyses the global dynamics and the conditions for EP, ML, and sustained-growth outcomes. Section 5 presents the quantitative analysis, and Section 6 concludes.

2 Model

2.1 Production of final output

Production follows a constant-returns-to-scale technology that is subject to endogenous technological progress. Precisely, the final output produced at time t, Y_t, is given by

$$Y_t = K_t^{\alpha}(A_t L_t)^{1-\alpha},$$

where K_t is aggregate capital, L_t is aggregate labor supply, $A_t > 0$ represents the endogenously determined technological progress at time t, and the parameter $\alpha \in (0,1)$. Physical capital completely depreciates in one period. The firm chooses inputs by maximizing profits $\Pi_t = Y_t - w_t(A_t L_t) - R_t K_t$ where w_t and R_t represent wage rate and gross rate of interest on capital respectively. Factor prices follow from the assumed competitive setup of firms, which leads to equalization of marginal costs and productivities:

$$w_t = (1-\alpha)\left(\frac{K_t}{A_t L_t}\right)^{\alpha} \equiv (1-\alpha)\kappa_t^{\alpha},$$

$$R_t = \alpha\left(\frac{K_t}{A_t L_t}\right)^{\alpha-1} \equiv \alpha\kappa_t^{\alpha-1},$$

where $\kappa_t = \frac{K_t}{A_t L_t}$ is defined as capital per effective labor at time t.[6]

2.2 Preferences and budget constraints

Member of generation t live for three periods: childhood, adulthood and old age. Time is discrete and goes from 0 to ∞. In the first period of life in period $t-1$, individual consumes a fraction of their parent's time. All the decisions taken by the individuals are in period t when they are adult in the second period of life. While adult, households are endowed with one unit of time which they allocate between

[5]See Appendix C for details.

[6]Krueger and Ludwig (2007) finds the effect of demographic change including rate of fertility for rates of returns to capital, and the distribution of wealth and welfare in a multi-country large-scale overlapping generations model.

child-rearing and labor force participation. The preferences of members of generation t are defined over consumption as well as over the potential aggregate income of their children. Precisely, the utility of a generation t agent i is given by

$$u_t = ln(c_t) + \beta ln(d_{t+1}) + \gamma ln(w_{t+1}h_{t+1}n_t) \tag{1}$$

where c_t and d_{t+1} are the adulthood and old age consumption respectively[7], n_t is the number of children chosen by individuals of generation t, h_{t+1} represents the level of human capital of the children, and w_{t+1} is the wage per efficiency unit of labor at time $t+1$. The parameter $\beta > 0$ is the psychological discount factor and $\gamma > 0$ represents the altruism factor. The role of old-age consumption is to provide a motive for savings and therefore generate an endogenous supply of capital.

For a member of generation t, let $\phi \in (0,1)$ and $\delta \in (0,1)$ be the time cost of raising and educating a child respectively and ψ be the monetary cost of raising a child. While middle aged in period t (parenthood), individual faces the budget constraint:

$$c_t + s_t + \psi n_t + e_{t+1}n_t w_t \bar{h}_t = w_t h_t \left(1 - \left(\phi + \delta e_{t+1}\right) n_t\right). \tag{2}$$

We have both time cost and monetary cost of educating and raising a child in order to determine the complete effect of income on fertility. A member of generation t is endowed with different unit of human capital, h_t, i.e., we have intra-generational inequality present in our model and w_t is wage rate per unit of human capital. Therefore, potential income of member of generation t is defined as $y_t \equiv w_t h_t$. Average human capital in the population is equal to teachers' average human capital and is represented by $\bar{h}_t$ and therefore, monetary cost of education per child is given by $e_{t+1}w_t\bar{h}_t$ where e_{t+1} is the schooling time per child. The budget constraint for the old-age is as follows:

$$d_{t+1} = R_{t+1}s_t. \tag{3}$$

An agent in generation t maximizes her utility as defined in (1) subject to period-wise budget constraints (2) and (3) by choosing saving s_t, number of children n_t and schooling time per child e_{t+1}.

2.3 Production of human capital

The level of human capital of children of members of generation t, h_{t+1}, is constructed in the following way so that it combines the features of Galor and Weil (2000), Galor

[7]The quantitative outcome will remain unchanged if we multiply the part of utility which represents the adulthood and old age consumption by $1-\gamma$ as in Galor and Weil (2000).

and Moav (2002) and de la Croix and Doepke (2003):

$$h_{t+1} = \frac{B_t(\theta + e_{t+1})^{\eta(g_{t+1})}(h_t)^{\tau}(\bar{h}_t)^{\vartheta}}{1 + (g_{t+1})^{\xi}}. \tag{4}$$

This depicts the inter-generational inequality in our model. In the above specification, h_{t+1} is an increasing function of education e_{t+1} and a decreasing function of progress in the state of technology from period t to $t+1$, $g_{t+1} \equiv \frac{A_{t+1}-A_t}{A_t}$. The parameter $\tau \in [0, 1]$ captures the inter generational transmission of human capital within the family, whereas $\vartheta \in [0, 1-\tau]$ represents externalities at the societal level. Crucial to our analysis, $\eta \geq 0$ captures the returns to education investment as mentioned above and, $\theta > 0$ ensures that children have some human capital even without parental investment in education. The efficiency parameter B_t is defined similar to de la Croix and Doepke (2003):

$$B_t = B(1 + \rho)^{(1-\tau-\vartheta)t}.$$

We now define the relative human capital of household as $a_t = \frac{h_t}{\bar{h}_t}$. With this representation, h_{t+1} now depends on relative human capital a_t which parents are endowed with and, the average human capital of the society $\bar{h}_t$. Further, we are interested in endogenous growth, therefore, as in Rangazas (2000) and de la Croix and Doepke (2003), equation (4) is compatible with endogenous growth for $\vartheta = 1 - \tau$, that is, finally

$$h_{t+1} = \frac{B(\theta + e_{t+1})^{\eta(g_{t+1})}(a_t)^{\tau}(\bar{h}_t)}{1 + (g_{t+1})^{\xi}}, \tag{5}$$

where $h(e_{t+1}, g_{t+1}, a_t, \bar{h}_t) > 0$, with $h_e > 0$, $h_{ee} < 0$, $h_g < 0$, $h_{gg} > 0$, and $h_{eg} > 0$. These assumptions on human capital production function with respect to education and technological progress is taken from Galor and Weil (2002). Our assumption that technological progress compliments rate of human capital due to an increase in education, that is $h_{eg} > 0$, implies that η increases with the level of technological progress. Thus, crucially, η in our setup is a function of technological progress, that is, $\eta(g_{t+1})$ with $\eta'(g_{t+1}) > 0$. Our assumption of $h_{eg} > 0$ will not be satisfied if η is assumed to be exogenous (constant), as for example, in de la Croix and Doepke (2003). Thus, individual human capital is an increasing and strictly concave function of education, and a decreasing and strictly convex function of the rate of technological progress. Moreover, technological progress increases the return to education in terms of human capital. We have empirically tested this assumption using Penn World Tables 10.1 and Barro and Lee (2018) dataset and find that there exists a significant and positive relationship between technological progress and returns to education.[8]

[8]See Appendix B.5 for empirical evidence supporting this assumed relationship.

2.4 Technological progress

The rate of technological progress g_{t+1} effectively depends on the aggregate education E_t and the population size P_t of the working generation in period t, precisely

$$g_{t+1} = \frac{A_{t+1} - A_t}{A_t} = g(E_t, P_t), \tag{6}$$

where for $E_t > 0$ and $P_t > 0$, $g(0, P_t) > 0$, along with $g_i(E_t, P_t) > 0$ and $g_{ii}(E_t, P_t) < 0$ for $i = E_t, P_t$ as assumed in Galor and Weil (2000). Hence, the rate of technological progress between time t and $t+1$ is a positive, increasing and strictly concave with respect to both the population and the level of education of the working generation at time t. Note that g is positive even if labor quality is zero. This assumption ensures a positive rate of technological progress during the Malthusian regime when people were investing nothing in quality at the extreme but the population size was sufficiently large.

2.5 Quantity-Quality trade off: Initial income or technology

As mentioned above, members of generation t chooses the number of their children, n_t, quality of their children, e_{t+1} and therefore their own consumption, so as to maximize their inter temporal utility function. Further, we assume that life-time consumption may be constrained by $\bar{c}$, precisely,

$$c_t + \frac{d_{t+1}}{R_{t+1}} \geq \bar{c}, \tag{7}$$

which, using (2) and (3), essentially gives us the following:

$$w_t \bar{h}_t \left(a_t(1 - (\phi + \delta e_{t+1})n_t) - e_{t+1}n_t\right) - \psi n_t \geq \bar{c}. \tag{8}$$

The optimization problem of a member of generation t is given by

$$\{n_t, e_{t+1}, s_t\} = argmax\{ln(c_t) + \beta ln(d_{t+1}) + \gamma ln(w_{t+1}h_{t+1}n_t)\}$$

subject to (8) along with

$$(n_t, e_{t+1}) \geq 0,$$

given (5) and (6).

It is easy to verify that the first order condition with respect to n_t implies that, as long as relative human capital a_t is sufficiently high so as to ensure that $c_t + s_t > \bar{c}$, the optimal number of children chosen by individual t satisfies $n_t = \frac{\gamma c_t}{a_t(\phi+\delta e_{t+1})w_t\bar{h}_t + e_{t+1}w_t\bar{h}_t + \psi}$. However, for individuals with low levels of a_t, the subsistence constraint (7) binds.

They save a constant amount of income, consume the subsistence level $\bar{c}$, and use rest of the time for child rearing. Precisely, the optimal amount of savings by members of generation t is given by,

$$s_t = \begin{cases} \frac{\beta\bar{c}}{(1+\beta)}, & \text{if } \frac{\bar{c}}{w_t\bar{h}_t} \leq a_t < \frac{\bar{c}(1+\beta+\gamma)}{(1+\beta)w_t\bar{h}_t}, \\ \frac{\beta w_t h_t}{(1+\beta+\gamma)}, & a_t \geq \frac{\bar{c}(1+\beta+\gamma)}{(1+\beta)w_t\bar{h}_t}. \end{cases}$$

We now focus on two important decisions by members of generation t - optimal n_t and e_{t+1} - both of which depend on the different levels of initial $w_t\bar{h}_t$.[9] We provide the base case with the situation where initial level of potential income is very low (under a certain cutoff ω as defined below).[10]

2.5.1 Countries too poor to start with

We consider the case where countries are initially too poor. In our framework, this is represented by $w_t\bar{h}_t < \omega \equiv \frac{\phi\eta(g_{t+1})-\theta\delta}{\theta}\left[\frac{\bar{c}(1+\beta+\gamma)}{(1+\beta)} + \frac{\eta(g_{t+1})\psi}{\phi\eta(g_{t+1})-\theta\delta}\right]$, meaning the country's initial level of potential income is very low — specifically, less than ω. Under this scenario, the optimal level of fertility n_t with respect to a_t is as follows:

$$n_t = \begin{cases} \frac{(a_t w_t\bar{h}_t-\bar{c})}{(a_t\phi w_t\bar{h}_t+\psi)} & \text{if } \frac{\bar{c}}{w_t\bar{h}_t} \leq a_t < \frac{\theta}{(\phi\eta(g_{t+1})-\theta\delta)} - \frac{\eta\psi}{(\phi\eta(g_{t+1})-\theta\delta)w_t\bar{h}_t}, \\ \frac{(a_t w_t\bar{h}_t-\bar{c})(1-\eta(g_{t+1}))}{(a_t\phi w_t\bar{h}_t+\psi-w_t\bar{h}_t\theta(1+\delta a_t))} & \text{if } \frac{\theta}{(\phi\eta(g_{t+1})-\theta\delta)} - \frac{\eta\psi}{(\phi\eta(g_{t+1})-\theta\delta)w_t\bar{h}_t} \leq a_t < \frac{\bar{c}(1+\beta+\gamma)}{(1+\beta)w_t\bar{h}_t}, \\ \frac{\gamma(1-\eta(g_{t+1}))a_t w_t\bar{h}_t}{(1+\beta+\gamma)\left[a_t\phi w_t\bar{h}_t+\psi-w_t\bar{h}_t\theta(1+\delta a_t)\right]} & \text{if } \frac{\bar{c}(1+\beta+\gamma)}{(1+\beta)w_t\bar{h}_t} \leq a_t. \end{cases} \tag{9}$$

Further, investment in education of their children e_{t+1} is given by

$$e_{t+1} = \begin{cases} 0, & \text{if } \frac{\bar{c}}{w_t\bar{h}_t} \leq a_t < \frac{\theta}{(\phi\eta(g_{t+1})-\theta\delta)} - \frac{\eta\psi}{(\phi\eta(g_{t+1})-\theta\delta)w_t\bar{h}_t}, \\ \frac{\eta(g_{t+1})(a_t\phi w_t\bar{h}_t+\psi)-w_t\bar{h}_t\theta(1+\delta a_t)}{w_t\bar{h}_t(1-\eta(g_{t+1}))(1+\delta a_t)}, & \text{if } \frac{\theta}{(\phi\eta(g_{t+1})-\theta\delta)} - \frac{\eta\psi}{(\phi\eta(g_{t+1})-\theta\delta)w_t\bar{h}_t} \leq a_t. \end{cases} \tag{10}$$

Potential income in period t is determined by the combination of fixed income, $w_t\bar{h}_t$, which remains constant across all households, and relative human capital, a_t, which varies from one household to another. The income-fertility trajectory of households in period $t+1$ is influenced by the dominance of either fixed income or relative human capital. Figure (1a) and (1b) below represent the paths of fertility and parental investment in education with respect to relative human capital. In Figure (2a) and (2b) we present the effect of fixed income on the paths of fertility and parental investment

[9] We ignore the trivial case $0 \leq a_t < \frac{\bar{c}}{w_t\bar{h}_t}$ for which the optimal choice of $\{n_t, e_{t+1}, s_t\}$= {0,0,0,0}.

[10] The knife-edge case and the case when countries are not very poor to start with that is, $w_t\bar{h}_t$ is greater than or equal to the cutoff has been presented in the **Appendix B.1**.

in education, keeping relative human capital fixed. With all these, finally Figure (3) shows the paths of fertility and parental investment in education with respect to potential income.

We can see from Figure (2a) and (2b), with an increase in the level of fixed income, the optimal choice of fertility rises while education falls, as the monetary cost of educating a child increases. Figure (3) shows that households with a lifetime subsistence consumption constraint, $c_t + s_t = \bar{c}$ - those countries operating under a Malthusian regime - will exhibit a positive income-fertility relationship. This implies that when the initial level of relative human capital satisfies $\frac{\bar{c}}{w_t \bar{h}_t} \leq a_t \leq \frac{\bar{c}(1+\beta+\gamma)}{(1+\beta) w_t \bar{h}_t}$, an increase in fixed income and relative human capital will lead to a rise in the optimal fertility choice, since:

$$\frac{dn_t}{dy_t} = \frac{2(\psi + \phi \bar{c})}{\left(a_t \phi w_t \bar{h}_t + \psi\right)^2} > 0.$$

After the Malthusian regime, that is, given the initial level of relative human capital satisfies $\frac{\bar{c}(1+\beta+\gamma)}{(1+\beta) w_t \bar{h}_t} \leq a_t$, with a rise in relative human capital the optimal choice of fertility decreases but the level of education increases, since the opportunity cost of raising children at the margin rises. During the **MG** regime - when $\frac{\bar{c}(1+\beta+\gamma)}{(1+\beta) w_t \bar{h}_t} \leq a_t \leq \frac{\bar{c}\theta}{2\psi - w_t \bar{h}_t \theta + 2\bar{c}(\phi - \theta\delta)}$ - a household's optimal choice of fertility decreases while education increases with a rise in potential income, since:

$$\frac{dn_t}{dy_t} = \frac{(1-\eta)\left[2\psi - w_t \bar{h}_t \theta + 2\bar{c}(\phi - \theta\delta) - \frac{\bar{c}\theta}{a_t}\right]}{\left(a_t w_t \bar{h}_t (\phi - \theta\delta) + \psi - w_t \bar{h}_t \theta\right)^2} < 0,$$

as the net returns from raising more children are negative, whereas the returns from educating them are positive.[11] Further, during the **MMG** regime - when $a_t \geq \frac{\bar{c}\theta}{2\psi - w_t \bar{h}_t \theta + 2\bar{c}(\phi - \theta\delta)}$ - a household's optimal choice of fertility increases, since:

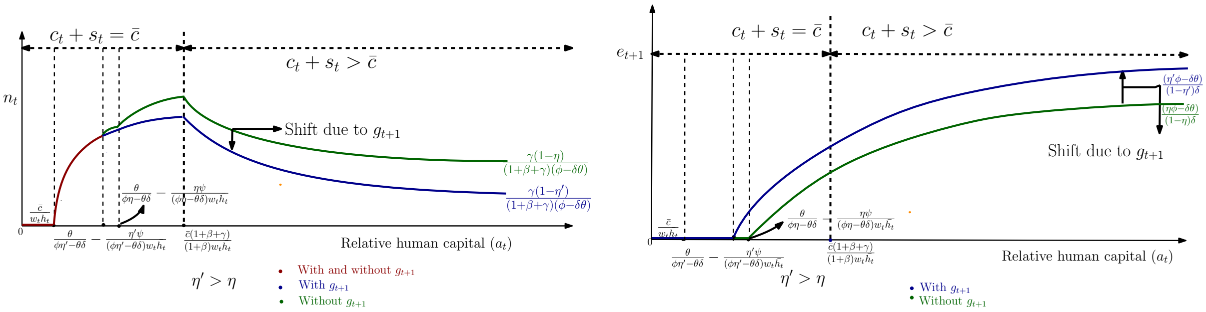
$$\frac{dn_t}{dy_t} = \frac{\gamma(1-\eta)\left(2\psi - w_t \bar{h}_t \theta\right)}{(1+\beta+\gamma)\left(a_t w_t \bar{h}_t (\phi - \theta\delta) + \psi - w_t \bar{h}_t \theta\right)^2} > 0,$$

when $\psi > \frac{w_t \bar{h}_t \theta}{2}$, while education rises with potential income.[12] This occurs because the net returns from raising more educated kids are positive. It might be the case that

[11]We refer to the second regime as the Modern Growth (MG) regime, in which the substitution effect of rising education costs dominates and fertility declines with income — consistent with the standard demographic transition. The third regime we call the More-Modern-Growth (MMG) regime: at very high income levels, the income effect reasserts itself and fertility may rise again.

[12]The threshold $a_t = \frac{\bar{c}\theta}{2\psi - w_t \bar{h}_t \theta + 2\bar{c}(\phi - \theta\delta)}$ separating the MG and MMG regimes is derived from the sign change of $\frac{dn_t}{dy_t}$ along a dynamic path where both a_t and $w_t \bar{h}_t$ change together as potential income $y_t = w_t \bar{h}_t \cdot a_t$ rises. Specifically, since $y_t = w_t \bar{h}_t \cdot a_t$, we have $\frac{da_t}{dy_t} = \frac{1}{w_t \bar{h}_t}$ and $\frac{d(w_t \bar{h}_t)}{dy_t} = \frac{1}{a_t}$, so the total

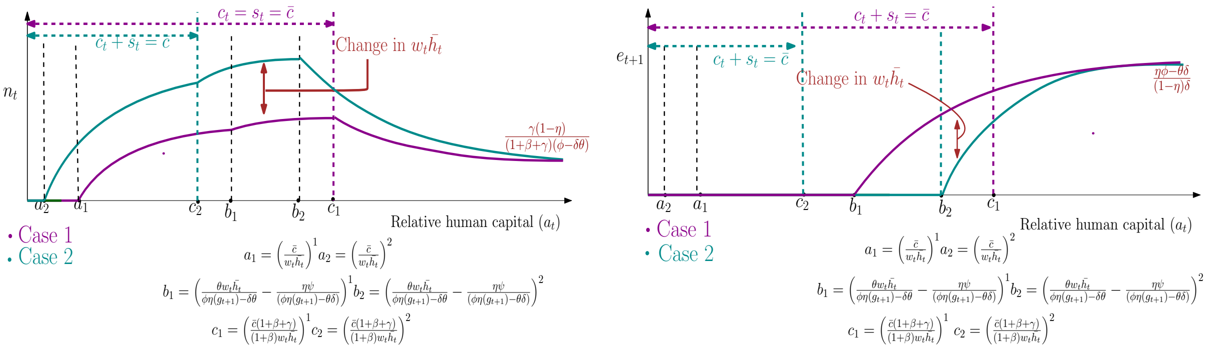
during **MMG** the net returns from raising more children is zero or negative. Hence, during this regime the income-fertility path may fall or stay constant depending on country specific parameter values. Note that Cordoba and Ripoll (2014), Cordoba and Ripoll (2019) and Cordoba et.al. (2016) find that in a purely altruistic setup where no consumption constraint is imposed, the negative income-fertility relationship demands a restriction that the intergenerational elasticity of substitution to be greater than unity.



(a) Relative human capital - Fertility relationship

(b) Relative human capital - Education relationship

Figure 1: Effect of an increase in relative human capital, a_t.



(a) Relative human capital - fertility curve.

(b) Relative human capital - education curve.

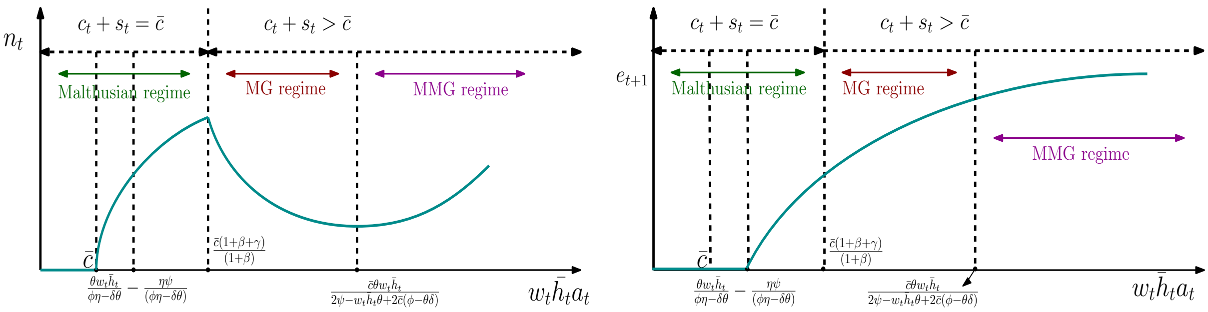
Figure 2: Effect of an increase in fixed income, $w_t\bar{h}_t$.

According to Galor and Weil (2000) and Galor and Moav (2002), if the potential income of parents exceeds $\tilde{z} = w_t h_t = \frac{\bar{c}}{1-\gamma}$, an increase in income neither raises the number of children nor improves their quality, as parents only incur the time cost of raising children. In their framework, time spent on raising children remains constant

derivative is:

$$\frac{dn_t}{dy_t} = \frac{1}{w_t\bar{h}_t}\frac{\partial n_t}{\partial a_t} + \frac{1}{a_t}\frac{\partial n_t}{\partial(w_t\bar{h}_t)},$$

where both scale factors carry units of inverse goods, ensuring dimensional consistency. The threshold is obtained by setting $\frac{dn_t}{dy_t} = 0$ and solving for a_t, which yields $a_t = \frac{\bar{c}\theta}{2\psi - w_t\bar{h}_t\theta + 2\bar{c}(\phi-\theta\delta)}$. This threshold is determined by the condition that the negative opportunity-cost effect (captured by $\frac{1}{w_t\bar{h}_t}\frac{\partial n_t}{\partial a_t} < 0$) is exactly offset by the positive income effect (captured by $\frac{1}{a_t}\frac{\partial n_t}{\partial(w_t\bar{h}_t)} > 0$).



(a) Potential income - fertility curve.

(b) Potential income - education curve.

Figure 3: Effect of an increase in potential income, $w_t\bar{h}_t a_t$.

at γ, regardless of the increase in potential income. Consequently, the division of child-rearing time between quantity and quality remains unaffected. In contrast, we conclude that if potential income of parents exceeds $\tilde{z} = w_t h_t = \frac{\bar{c}(1+\beta+\gamma)}{(1+\beta)}$, an increase in potential income reduces the number of children while improving their quality when the opportunity cost of raising a child outweighs the monetary cost of educating them, that is, when:

$$\frac{dn_t}{dy_t} = \frac{(1-\eta)\left[2\psi - w_t\bar{h}_t\theta + 2\bar{c}(\phi - \theta\delta) - \frac{\bar{c}\theta}{a_t}\right]}{\left(a_t w_t \bar{h}_t(\phi - \theta\delta) + \psi - w_t\bar{h}_t\theta\right)^2} < 0,$$

which holds in the **MG** regime when $\frac{\bar{c}(1+\beta+\gamma)}{(1+\beta)w_t\bar{h}_t} \leq a_t \leq \frac{\bar{c}\theta}{2\psi - w_t\bar{h}_t\theta + 2\bar{c}(\phi-\theta\delta)}$. At higher levels of potential income — specifically in the **MMG** regime when $a_t \geq \frac{\bar{c}\theta}{2\psi - w_t\bar{h}_t\theta + 2\bar{c}(\phi-\theta\delta)}$ — an increase in income may lead to a rise in both the quantity and quality of children, since:

$$\frac{dn_t}{dy_t} = \frac{\gamma(1-\eta)\left(2\psi - w_t\bar{h}_t\theta\right)}{(1+\beta+\gamma)\left(a_t w_t\bar{h}_t(\phi - \theta\delta) + \psi - w_t\bar{h}_t\theta\right)^2} > 0,$$

when $\psi > \frac{w_t\bar{h}_t\theta}{2}$. The outcome depends on the relative strength of opportunity costs versus monetary costs, which in turn is determined by country-specific parameter values. The simultaneous increase in quality and quantity, observed at very high income levels, has been documented recently by Doepke et al. (2023).

In our analysis, η is positively linked to technological progress (g_{t+1}). With constant income, technological progress increases educational investment (e_{t+1}) and reduces fertility (n_t), as higher η raises the return on education. This shift begins early with technological advancements. Keeping relative human capital (a_t) fixed, an increase in g_{t+1} boosts e_{t+1} and lowers n_t, as shown in Figures (1a) and (1b). In Figure (1b), technological growth shifts the income-education curve upward and moves the threshold for positive education investment leftward. In Figure (1a), technological progress shifts the *income-fertility* curve downward. Additionally, the impact of g_{t+1} on e_{t+1} is stronger for households with higher a_t, evidenced by $\frac{de_{t+1}}{dg_{t+1}} > 0$ and $\frac{de_{t+1}}{da_t dg_{t+1}} > 0$. This intensifies the trade-off for those prioritizing quality over quantity.

Note that income plays a crucial role in determining whether households prioritize quantity or quality, as captured by the total derivative:

$$\frac{dn_t}{dy_t} = \frac{1}{w_t \bar{h}_t} \frac{\partial n_t}{\partial a_t} + \frac{1}{a_t} \frac{\partial n_t}{\partial (w_t \bar{h}_t)},$$

where the first term reflects the opportunity-cost effect of rising relative human capital and the second term reflects the income effect of rising fixed income. Specifically:

- In the **Malthusian** regime, both effects are positive, so $\frac{dn_t}{dy_t} > 0$ and households prioritize quantity.
- In the **MG** regime, the opportunity-cost effect dominates, so $\frac{dn_t}{dy_t} < 0$ and households shift toward quality over quantity.
- In the **MMG** regime, when $\psi > \frac{w_t \bar{h}_t \theta}{2}$, the income effect reasserts itself, so $\frac{dn_t}{dy_t} > 0$ and households may increase both quantity and quality simultaneously.

Technological progress, on the other hand, influences the strength of this trade-off. Furthermore, with technological advancements, the *income-fertility* curve shifts downward, and the education-fertility curve shifts upward. This is because technological progress increases the incentive to invest in a child's education by enhancing the returns to education, captured by $\eta(g_{t+1})$. A rise in g_{t+1} increases η, which raises the threshold separating the MG and MMG regimes:

$$a_t = \frac{\bar{c}\theta}{2\psi - w_t \bar{h}_t \theta + 2\bar{c}(\phi - \theta\delta)},$$

thereby expanding the range of relative human capital over which households substitute quality for quantity. With the above results, we now present our proposition below.

Proposition 1. *Let $y_t \equiv w_t h_t = w_t \bar{h}_t a_t$ denote potential income, with a_t the relative human capital and $w_t \bar{h}_t$ fixed income. The following hold:*

(i) $\frac{\partial n_t}{\partial a_t} > 0$ *if* $\frac{\bar{c}}{w_t \bar{h}_t} \leq a_t \leq \frac{\bar{c}(1+\beta+\gamma)}{(1+\beta) w_t \bar{h}_t}$, *and* $\frac{\partial n_t}{\partial a_t} < 0$ *if* $a_t \geq \frac{\bar{c}(1+\beta+\gamma)}{(1+\beta) w_t \bar{h}_t}$.

(ii) $\frac{\partial n_t}{\partial (w_t \bar{h}_t)} > 0$ *for all* $a_t \geq \frac{\bar{c}}{w_t \bar{h}_t}$.

(iii) Along any development path where both a_t and $w_t \bar{h}_t$ grow with y_t, the total derivative

$$\frac{dn_t}{dy_t} = \frac{1}{w_t \bar{h}_t} \frac{\partial n_t}{\partial a_t} + \frac{1}{a_t} \frac{\partial n_t}{\partial (w_t \bar{h}_t)}$$

satisfies:

$$\frac{dn_t}{dy_t} = \begin{cases} \dfrac{2(\psi + \phi\bar{c})}{\left(a_t\phi w_t\bar{h}_t + \psi\right)^2} > 0, & \text{(Malthusian regime)}, \\ \dfrac{(1-\eta)\left[2\psi - w_t\bar{h}_t\theta + 2\bar{c}(\phi - \theta\delta) - \frac{\bar{c}\theta}{a_t}\right]}{\left(a_t w_t\bar{h}_t(\phi - \theta\delta) + \psi - w_t\bar{h}_t\theta\right)^2} < 0, & \text{(MG regime)}, \\ \dfrac{\gamma(1-\eta)\left(2\psi - w_t\bar{h}_t\theta\right)}{(1+\beta+\gamma)\left(a_t w_t\bar{h}_t(\phi - \theta\delta) + \psi - w_t\bar{h}_t\theta\right)^2} > 0, & \text{(MMG regime)}, \end{cases}$$

where the MMG regime requires $\psi > \dfrac{w_t\bar{h}_t\theta}{2}$ *and the MMG threshold is:*

$$a_t \geq \frac{\bar{c}\theta}{2\psi - w_t\bar{h}_t\theta + 2\bar{c}(\phi - \theta\delta)}.$$

Hence the income–fertility locus is non-monotonic: ascending in the Malthusian regime, declining in the MG regime, and ascending again in the MMG regime.

Proof 1. *Parts (i) and (ii) follow by direct differentiation of the optimal fertility expressions in equation (A.24); the explicit partial derivatives are computed in Appendix* ***A.1****.*

For Part (iii), since $y_t = w_t\bar{h}_t \cdot a_t$*, we have* $\frac{da_t}{dy_t} = \frac{1}{w_t\bar{h}_t}$ *and* $\frac{d(w_t\bar{h}_t)}{dy_t} = \frac{1}{a_t}$*, so the total derivative along any path where both arguments increase with* y_t *is:*

$$\frac{dn_t}{dy_t} = \underbrace{\frac{\partial n_t}{\partial a_t}}_{\text{sign changes}} \cdot \frac{1}{w_t\bar{h}_t} + \underbrace{\frac{\partial n_t}{\partial(w_t\bar{h}_t)}}_{>0} \cdot \frac{1}{a_t}.$$

Both scale factors $\frac{1}{w_t\bar{h}_t}$ *and* $\frac{1}{a_t}$ *carry units of inverse goods, ensuring dimensional consistency of the sum.*

In the Malthusian regime (Case 1), both partial derivatives are positive, so $dn_t/dy_t > 0$ *unambiguously.*

In the MG regime (Case 2 and the relevant part of Case 3), $\frac{\partial n_t}{\partial a_t} < 0$ *and the negative opportunity-cost effect dominates, giving* $dn_t/dy_t < 0$ *when:*

$$2a_t\psi - a_t w_t\bar{h}_t\theta + 2a_t\bar{c}(\phi - \theta\delta) < \bar{c}\theta.$$

In the MMG regime (Case 3), the positive income effect $\frac{1}{a_t}\frac{\partial n_t}{\partial(w_t\bar{h}_t)} > 0$ *dominates when* $\psi > \frac{w_t\bar{h}_t\theta}{2}$*, giving* $dn_t/dy_t > 0$*. The MMG threshold is obtained by setting* $dn_t/dy_t = 0$ *in Case 2 and solving for* a_t*:*

$$a_t = \frac{\bar{c}\theta}{2\psi - w_t\bar{h}_t\theta + 2\bar{c}(\phi - \theta\delta)}.$$

*See Appendix **A.1** for the explicit calculations.*

We now explain how we connect this model that generates the empirically observed income-fertility path with the studies we have mentioned above. We confirm that we can generate the desired path at least from the two following modeling choices: (i) when we bring inequality and monetary cost of having children in Galor and Weil (2000) and, (ii) when we bring consumption constraint, time cost of educating and monetary cost of raising a child in de la Croix and Doepke (2003). Incorporating inequality through education costs, represented by $e_{t+1}w_t\bar{h}_t$, highlights that education is relatively more expensive for poorer parents. Galor and Weil (2000) predicts a declining *income-fertility* curve for skilled parents because, while education costs are fixed, the time or opportunity cost of raising additional children increases with income. Consequently, skilled parents with high relative human capital experience a declining relationship between human capital and fertility, consistent with the **MG** regime in our model where:

$$\frac{dn_t}{dy_t} = \frac{(1-\eta)\left[2\psi - w_t\bar{h}_t\theta + 2\bar{c}(\phi-\theta\delta) - \frac{\bar{c}\theta}{a_t}\right]}{\left(a_t w_t\bar{h}_t(\phi-\theta\delta) + \psi - w_t\bar{h}_t\theta\right)^2} < 0,$$

for $\frac{\bar{c}(1+\beta+\gamma)}{(1+\beta)w_t\bar{h}_t} \leq a_t \leq \frac{\bar{c}\theta}{2\psi - w_t\bar{h}_t\theta + 2\bar{c}(\phi-\theta\delta)}$.

On the other hand, the model by de la Croix and Doepke (2003) does not address how inequality impacts fertility during the Malthusian regime. By integrating factors like consumption constraints, the monetary cost of raising a child, and the time cost of education into their framework, we observe a positive effect of higher relative human capital on fertility during the Malthusian period, consistent with:

$$\frac{dn_t}{dy_t} = \frac{2(\psi + \phi\bar{c})}{\left(a_t\phi w_t\bar{h}_t + \psi\right)^2} > 0,$$

for $\frac{\bar{c}}{w_t\bar{h}_t} \leq a_t \leq \frac{\bar{c}(1+\beta+\gamma)}{(1+\beta)w_t\bar{h}_t}$. Furthermore, these additions enable an analysis of how monetary and opportunity costs influence the trade-off between having more children and investing in their quality, captured by the total derivative:

$$\frac{dn_t}{dy_t} = \frac{1}{w_t\bar{h}_t}\frac{\partial n_t}{\partial a_t} + \frac{1}{a_t}\frac{\partial n_t}{\partial (w_t\bar{h}_t)},$$

where the first term captures the opportunity-cost effect and the second term captures the monetary income effect.

Further, as mentioned earlier, unlike these studies, our findings are also compatible with the recent observation made by Doepke et al. (2023) that the *income-fertility* relationship has recently shown a slight upward trend. In our model, this upward trend emerges in the **MMG** regime when $a_t \geq \frac{\bar{c}\theta}{2\psi - w_t\bar{h}_t\theta + 2\bar{c}(\phi - \theta\delta)}$ and $\psi > \frac{w_t\bar{h}_t\theta}{2}$, where:

$$\frac{dn_t}{dy_t} = \frac{\gamma(1-\eta)\left(2\psi - w_t\bar{h}_t\theta\right)}{(1+\beta+\gamma)\left(a_t w_t\bar{h}_t(\phi - \theta\delta) + \psi - w_t\bar{h}_t\theta\right)^2} > 0.$$

The emerging trend is driven by inequality dynamics and technological growth, captured through the fixed monetary cost of education, with or without childcare services. Childcare is closely linked to inequality, as only the wealthy, who face relatively higher education costs, can afford it and benefit by converting opportunity costs into monetary costs. This weakens the impact of the opportunity cost effect $\frac{1}{w_t\bar{h}_t}\frac{\partial n_t}{\partial a_t}$, and fertility starts rising as the income effect $\frac{1}{a_t}\frac{\partial n_t}{\partial(w_t\bar{h}_t)}$ gains dominance over the opportunity cost effect. Therefore any such services can well be explained through inequality dynamics only.

3 Linking Inequality with Growth via Technological progress

We now explicitly determine how inequality via technological progress affect aggregate fertility, education and therefore, growth. With the evolution of aggregate level of education, E_t, and technological progress, $g_{t+1}(E_t, P_t)$, inequality - defined as an increase in the spread of income distribution - also increases. From (5),

$$a_{t+1} = \frac{h_{t+1}}{\bar{h}_{t+1}} = \frac{B(\theta + e_{t+1})^{\eta(g_{t+1})} a_t^{\tau}\bar{h}_t}{(1 + g_{t+1}^{\xi})\bar{h}_{t+1}}.$$

For a given dispersion of relative human capital, a_t, an increase in aggregate education, E_t, leads to a rise in technological progress denoted by $g_{t+1}(E_t, P_t)$. This implies that parents' decide to invest more in their children's education, e_{t+1}. With the rise in technological progress, the investment in children's education in relation to relative human capital remains positive, i.e., $\frac{de_{t+1}}{da_t dg_{t+1}} > 0$. Consequently, a_{t+1} increases at different rates for different households, depending on the initial level of a_t.
We observe that, with the evolution of aggregate education, E_t, $\bar{h}_t$ remains unchanged, while the average human capital in period $t+1$, $\bar{h}_{t+1}$, changes in a similar magnitude for all households. However, as parents' education evolves, the relative human capital of children with wealthier parents rises more significantly because their initial level of relative human capital, a_t, is higher. Therefore, with technological progress, the positive effect of increased education is much greater for wealthy households. The overall effect of technological progress leads to a rise in a_{t+1}. In contrast, for poorer households, the positive effect of increased education is much smaller, and the overall effect of technological progress may result in a slight rise or fall in a_{t+1}. Hence, inequality increases as the disparity in relative human capital widens. Thus, with

the evolution of aggregate education, E_t, both technological progress, $g_{t+1}(E_t, P)$, and therefore, inequality increases for a given population level. Formally, since $\frac{de_{t+1}}{da_t\, dg_{t+1}} > 0$, technological progress raises educational investment disproportionately for high-a_t households, causing a_{t+1} to increase faster for wealthier households than for poorer ones, thereby widening the dispersion of G_{t+1} relative to G_t.

3.1 Evolution of the Human Capital Distribution

For analytical transparency, we assume that the initial distribution $F_0(h)$ is uniform on a bounded support. Thereafter, the distribution evolves endogenously according to the law of motion derived below. Let $F_t(h)$ denote the cumulative distribution function (CDF) of human capital among adults at time t, and P_t the adult population size. The distribution F_t is endogenous and evolves over time according to individual human capital dynamics and fertility choices. Each adult with human capital h_t gives birth to $n_t = n_t(h_t)$ children, and the human capital of each child is given by

$$h_{t+1} = H_t(h_t).$$

The distribution of human capital in the next period is therefore given by the population-weighted pushforward of F_t:

$$F_{t+1}(h) = \frac{1}{P_{t+1}} \int \mathbb{I}\{H_t(h_t) \leq h\}\, n_t(h_t)\, dF_t(h_t), \tag{11}$$

where total population evolves according to

$$P_{t+1} = P_t N_t. \tag{12}$$

Equation (11) provides the general law of motion for the cross-sectional distribution of human capital. It states that the mass of individuals with human capital below h in period $t+1$ is obtained by aggregating all parents whose offspring attain human capital less than h, weighted by their fertility. Note that if the mapping H_t is strictly monotone and invertible, the above expression admits an equivalent density representation via a change of variables. Let f_t denote the density associated with F_t. Then, for $h' = H_t(h)$,

$$f_{t+1}(h') = \frac{N_t(H_t^{-1}(h'))\, f_t(H_t^{-1}(h'))}{\int N_t(z)\, dF_t(z)} \left| \frac{d}{dh'} H_t^{-1}(h') \right|. \tag{13}$$

We present (11) as it remains valid without requiring invertibility or differentiability of H_t. Average human capital $\bar{h}_t$ is given by

$$\bar{h}_t = \int_0^\infty h_t \, dF_t(h_t). \tag{14}$$

Market clearing conditions for capital and labor are

$$K_{t+1} = P_t \int_0^\infty s_t \, dF_t(h_t), \tag{15}$$

and

$$L_t = P_t \left[\int_0^\infty h_t(1 - (\phi + \delta e_{t+1})n_t) \, dF_t(h_t) - \int_0^\infty e_{t+1} n_t \bar{h}_t \, dF_t(h_t) \right]. \tag{16}$$

Equation (16) reflects the fact that time available for teaching is not available for goods production. The state of the technology at time $t+1$, A_{t+1}, is defined as,

$$A_{t+1} = (1 + g_{t+1})A_t.$$

The key decision variable is the human capital of a family relative to average human capital, $\bar{h}_t$ of the population. Therefore, relative human capital is defined as $a_t = \frac{h_t}{\bar{h}_t}$. We define the distribution of the relative human capital level as,

$$G_t(a_t) \equiv F_t(a_t \bar{h}_t).$$

Therefore, equation (11), (12), and (14) can be written as,

$$N_t = \int_0^\infty n_t \, dG_t(a_t),$$

$$G_{t+1}(a) = \frac{1}{N_t} \int_0^\infty n_t \mathbb{I}(a_{t+1} \le a) \, dG_t(a_t),$$

and

$$1 = \int_0^\infty a_t \, dG_t(a_t).$$

If h_t is assumed to have a support $[\underline{h_t}, \bar{h}_t]$, we can measure differential fertility as $\frac{n_t^{\text{poor}(\underline{h_t})}}{n_t^{\text{rich}(\bar{h}_t)}}$.[13] With the evolution of education and technological progress, inequality rises as the spread of the distribution increases. Assuming an economy lies within Malthusian regime and households are optimally choosing positive level of investment in child's education. Due to the shape of the fertility and education curve, rising inequal-

[13]In calibration, following de la Croix and Doepke (2003), differential fertility is measured by the difference in top and bottom income quintiles.

ity leads to an increase in differential fertility, total fertility, and average education.[14] In the **MG** regime, where fertility declines with rising potential income, increasing inequality raises differential fertility while reducing total fertility and weighted average education.[15] In the **MMG** regime, an increase in inequality raises differential fertility, total fertility, and weighted average education. With a mean-preserving spread, fertility increases more for those who initially invest more in education, resulting in a rise in weighted average education (as $\frac{n_2'}{n_2} > \frac{n_1'}{n_1}$ following the notational expressions in the above two footnotes). Lower fertility among individuals with less human capital during the Malthusian and **MMG** regimes amplifies inequality's positive effect on human capital accumulation. While de la Croix and Doepke (2003) focus on inequality's effect on growth via differential fertility during the **MG** regime, incorporating the Malthusian and **MMG** regimes suggests that rising inequality may actually enhance growth.[16] Thus, we are in a position to present our next proposition as follows.

Proposition 2. *Let G_t (fertility-weighted) denote the distribution of relative human capital a_t, and let $N_t \equiv \int n_t(a_t)\, dG_t(a_t)$ and $E_{t+1} \equiv \int e_{t+1}(a_t)\, dG_t(a_t)$ denote aggregate fertility and fertility-weighted average education respectively. Consider a mean-preserving spread of G_t (a rise in inequality). Then:*

(i) ***Malthusian regime*** $\left(\frac{\partial n_t}{\partial a_t} > 0,\ \frac{\partial e_{t+1}}{\partial a_t} > 0\right)$*:* $\dfrac{dN_t}{d\sigma} > 0$ *and* $\dfrac{dE_{t+1}}{d\sigma} > 0$*, where σ parameterises the spread of G_t.*

(ii) ***MG regime*** $\left(\frac{\partial n_t}{\partial a_t} < 0,\ \frac{\partial e_{t+1}}{\partial a_t} > 0\right)$*:* $\dfrac{dN_t}{d\sigma} < 0$ *and* $\dfrac{dE_{t+1}}{d\sigma} < 0$.

(iii) ***MMG regime*** $\left(\frac{\partial n_t}{\partial a_t} > 0,\ \frac{\partial e_{t+1}}{\partial a_t} > 0\right)$*:* $\dfrac{dN_t}{d\sigma} > 0$ *and* $\dfrac{dE_{t+1}}{d\sigma} > 0$.

Proof. We analyse each part in turn.

Let σ parameterise a mean-preserving spread of G_t, so that $\int a_t\, dG_t(a_t) = 1$ is preserved for all σ. A mean-preserving spread redistributes mass away from the centre toward

[14] For instance, if two types of parents initially invest n_1 and n_2 in fertility and e_1 and e_2 in education, where $n_2 > n_1$ and $e_2 > e_1$, total fertility is $n_1 + n_2$, total education is $e_1 + e_2$, and weighted average education is $\frac{n_1 e_1}{n_1+n_2} + \frac{n_2 e_2}{n_1+n_2}$. As inequality grows, n_1 and e_1 increase to n_1' and e_1', while n_2 and e_2 increase more substantially to n_2' and e_2', resulting in higher total fertility, total education, and weighted average education. Notably, the disproportionate rise in n_2 and e_2 compared to n_1 and e_1 implies $\frac{n_2'}{n_1'+n_2'} > \frac{n_2}{n_1+n_2}$, driving the increase in weighted average education. Thus, rising inequality during this regime boosts differential fertility, total fertility, and weighted average education.

[15] Consider two types of parents specified exactly as in the previous footnote. After an increase in inequality, n_1 and n_2 fall to n_1' and n_2', reducing total fertility to $(n_1' + n_2')$, while e_1 and e_2 rise to e_1' and e_2', increasing total education to $(e_1' + e_2')$. As inequality grows, the fall in n_2 and rise in e_2 outweigh the changes in n_1 and e_1, leading to $\frac{n_2'}{n_1'+n_2'} < \frac{n_2}{n_1+n_2}$ and $\frac{n_1'}{n_1'+n_2'} > \frac{n_1}{n_1+n_2}$, i.e., $\frac{n_2'}{n_2} < \frac{n_1'}{n_1}$. Consequently, weighted average education declines with rising inequality. Thus, in the **MG** regime, inequality increases differential fertility while reducing total fertility and weighted average education.

[16] In Appendix C, we provide empirical evidence consistent with the model's mechanism in the **MG** regime, using US state-level data for 1880–1940.

both tails: formally, $\partial a_t/\partial\sigma > 0$ for households above the mean and $\partial a_t/\partial\sigma < 0$ for households below the mean. Since $N_t = \int n_t(a_t)\, dG_t(a_t)$ and $E_{t+1} = \int e_{t+1}(a_t)\, dG_t(a_t)$, differentiating with respect to σ gives

$$\frac{dN_t}{d\sigma} = \int \frac{\partial n_t}{\partial a_t} \cdot \frac{\partial a_t}{\partial \sigma}\, dG_t(a_t), \qquad \frac{dE_{t+1}}{d\sigma} = \int \frac{\partial e_{t+1}}{\partial a_t} \cdot \frac{\partial a_t}{\partial \sigma}\, dG_t(a_t). \tag{17}$$

The sign of each integral is determined by the product of (i) the slope of the choice variable with respect to a_t, and (ii) the sign of $\partial a_t/\partial\sigma$, which is positive at the top and negative at the bottom of the distribution.

Part (i): Malthusian regime ($\partial n_t/\partial a_t > 0,\ \partial e_{t+1}/\partial a_t > 0$).

Both fertility and education are increasing in relative human capital a_t. Under a mean-preserving spread, high-a_t households see their a_t rise and low-a_t households see theirs fall. Since $\partial n_t/\partial a_t > 0$, the increase in n_t among richer households more than offsets the decrease among poorer households: the product $(\partial n_t/\partial a_t) \cdot (\partial a_t/\partial\sigma)$ is positive throughout the support of G_t, so

$$\frac{dN_t}{d\sigma} > 0.$$

The same logic applies to education. Because $\partial e_{t+1}/\partial a_t > 0$, the spread disproportionately raises education among high-a_t households, who also attract greater fertility weight, giving

$$\frac{dE_{t+1}}{d\sigma} > 0.$$

Part (ii): MG regime ($\partial n_t/\partial a_t < 0,\ \partial e_{t+1}/\partial a_t > 0$).

In this regime fertility declines with relative income while education rises. A mean-preserving spread raises a_t for high-income households, who respond by having *fewer* children, and lowers a_t for low-income households, who respond by having *more* children. The product $(\partial n_t/\partial a_t) \cdot (\partial a_t/\partial\sigma)$ is therefore negative throughout the support of G_t, so

$$\frac{dN_t}{d\sigma} < 0.$$

For education, although $\partial e_{t+1}/\partial a_t > 0$, the fertility-weighted average E_{t+1} declines because the composition of births shifts toward low-a_t parents who invest less in each child's education. Formally, the fall in n_t among high-education parents and the rise among low-education parents reduces the fertility weight on high e_{t+1}, so

$$\frac{dE_{t+1}}{d\sigma} < 0.$$

Part (iii): MMG regime ($\partial n_t/\partial a_t > 0,\ \partial e_{t+1}/\partial a_t > 0$).

Both fertility and education are again increasing in a_t, as in the Malthusian regime. The identical argument therefore applies: a mean-preserving spread concentrates births among high-a_t, high-education parents. The products $(\partial n_t/\partial a_t) \cdot (\partial a_t/\partial \sigma)$ and $(\partial e_{t+1}/\partial a_t) \cdot (\partial a_t/\partial \sigma)$ are positive across the support of G_t, giving

$$\frac{dN_t}{d\sigma} > 0 \qquad \text{and} \qquad \frac{dE_{t+1}}{d\sigma} > 0.$$

□

There is no consistent negative relationship between differential fertility and growth during the Malthusian and **MMG** regimes. Economies can move from the MG regime to the more prosperous **MMG** regime as fertility begins to rise with potential income. Interestingly, economies may also regress to the Malthusian regime under certain conditions, leading history to repeat itself. As we show, importantly, inequality plays a key role in preventing such regression. These dynamics, including the resemblance of the possibility outlined in Jones (2022) within our framework, are detailed in the next section.

4 Global dynamics

The development of an economy is shaped by various factors, including changes in output per worker, population growth, technological progress, education per worker, relative human capital, capital per effective worker, and levels of inequality. Since inequality is the central force in our framework, we analyse the global dynamics directly in the presence of heterogeneous households.

4.1 Dynamical system with inequality

Remark on the conditional phase diagram. The full dynamical system has three state variables (κ_t, e_t, g_t) together with population P_t evolving according to equation (12). A complete characterisation would therefore require a four-dimensional analysis. Following the approach standard in this literature (see Galor and Weil (2000)), we present two-dimensional cross-sections of this system by conditioning on a given population size P and, within each cross-section, on a given level of technological progress $g_t = \bar{g}$. The resulting *conditional* phase diagrams in the (e_t, κ_t) plane are therefore projections of the full dynamics onto a lower-dimensional space; they are explicitly labelled "conditional" to reflect this. We discuss in Section 4 how the diagrams shift as P_t changes and, in particular, how a sustained population decline feeds back into $g(E_t, P_t)$ to produce the Empty Planet outcome.

Remark on regime boundaries. The Malthusian, MG, and MMG regimes are defined by the position of (κ_t, e_t) relative to the *Conditional Malthusian Frontier* $MM|g_t$, which is itself a curve in the state space (e_t, κ_t) at fixed g_t. Crucially, this frontier is determined entirely by predetermined state variables: $w_t(\kappa_t)$ depends only on κ_t (a state variable), and $h_t(e_t, \bar{g})$ depends only on e_t and the given g_t. The cutoff ω introduced in Section 2 characterises households whose initial potential income places them in the Malthusian region; it is equivalently interpreted as the level of $w_t\bar{h}_t$ at which the Conditional Malthusian Frontier crosses the $(e_t = 0)$ axis. Hence the regime classification is not circular: given the state (κ_t, e_t, g_t), one can determine which regime the economy is in without first solving for equilibrium fertility or education. The household optimization then delivers the equilibrium objects (n_t, e_{t+1}) *within* that regime.

The dynamical system. The XX *locus* is the set of (κ_t, e_t) satisfying $\Xi^i(\kappa_t, e_t, g_t; P) = 1$, i.e. capital per effective worker is stationary. The EE *locus* is the set satisfying $\Phi^i(\kappa_t, e_t, g_t; P) = 0$, i.e. education per worker is stationary. The *Conditional Malthusian Frontier* $MM|g_t$ is the curve along which $w_t(\kappa_t)h_t(e_t, \bar{g}) = \bar{c}(1 + \beta + \gamma)/(1 + \beta)$; below it the lifetime subsistence constraint binds. We now introduce heterogeneous households $(a_t \sim G_t)$ and examine how inequality shifts these loci and determines the long-run outcome.

If the income distribution positions the economy within the Malthusian regime, that is, the relative human capital, a_t, is uniformly distributed within the range, $\frac{\bar{c}}{w_t\bar{h}_t} \leq a_t \leq \frac{\bar{c}(1+\beta+\gamma)}{(1+\beta)w_t\bar{h}_t}$, then we have,

$$K_{t+1} = P_t * A, \tag{18}$$

$$L_t = \frac{P_t(1 + \beta)}{w_t\gamma\bar{c}} \left[f(w_t\bar{h}_t = z_t, \eta(g(e_t; P))) \right], \tag{19}$$

$$\kappa_{t+1} = \Xi^a(e_t, g_t, \kappa_t; P)\kappa_t, \tag{20}$$

$$e_{t+1} = \Phi^a(e_t, g_t, \kappa_t; P). \tag{21}$$

During this regime, technological progress increases the returns to education (η) and investment in education (e_{t+1}), leading to a rise in overall fertility (n_t) as the income effect outweighs the substitution effect. This relationship is characterized by $\Xi^a_\kappa(.) < 0$, $\Xi^a_e(.) > 0$, and $\frac{d\kappa_t}{de_t} > 0$. Simultaneously, with technological progress, $\Phi^a_\kappa(.) < 0$ and $\Phi^a_e(.) < 0$ as the income effect continues to dominate, implying $\frac{d\kappa}{de_t} < 0$. Technological progress during this period also exacerbates inequality in the subsequent period $(t+1)$. Consequently, fertility (n_{t+1}) and investment in children's education (e_{t+2}) further rise during the Malthusian regime, as highlighted in Proposition 2. This is reflected in $\Xi^a_e(.) > 0$, $\frac{d\kappa_t}{de_t} > 0$, and $\Phi^a_e(.) = 0$, causing a rightward shift in the e curve. As education evolves, the XX locus is upward-sloping due to the influence of technological progress and inequality, while the EE locus is downward-sloping with technological progress but shifts upward in response to increased inequality.

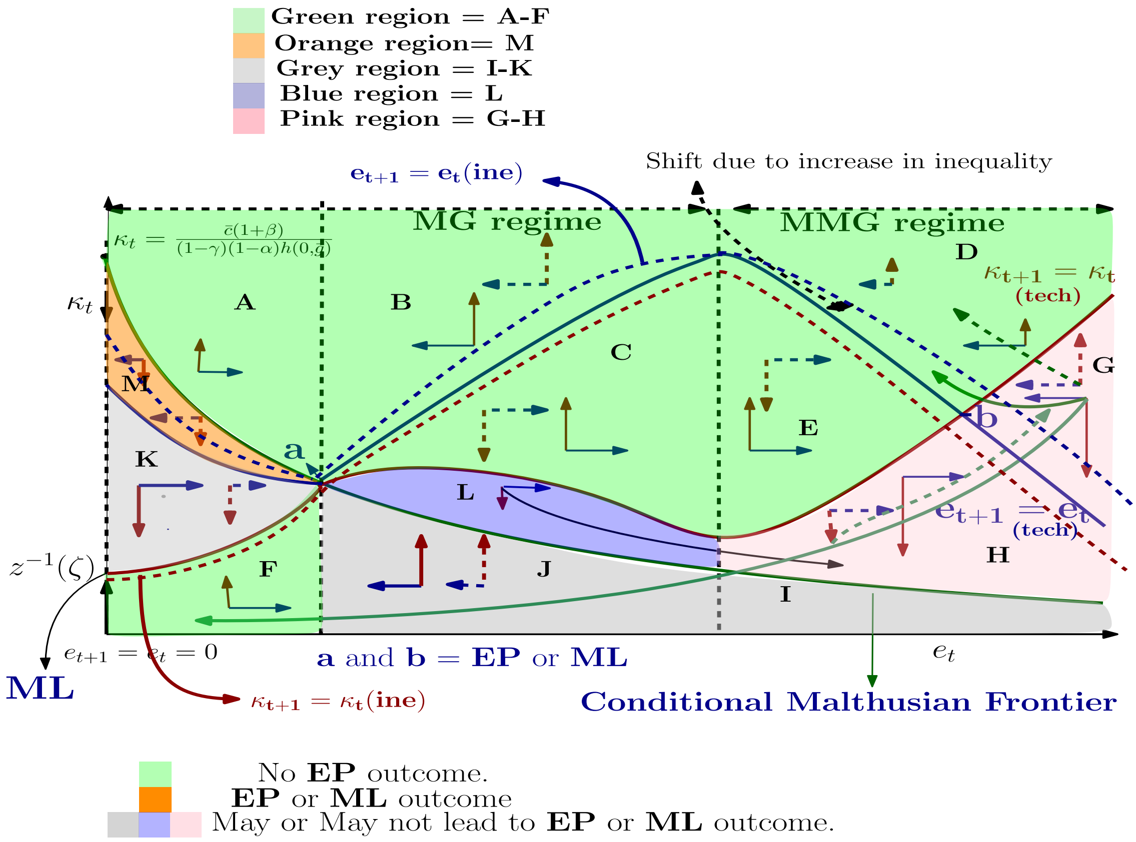


Figure 4: The conditional dynamic system with inequality.

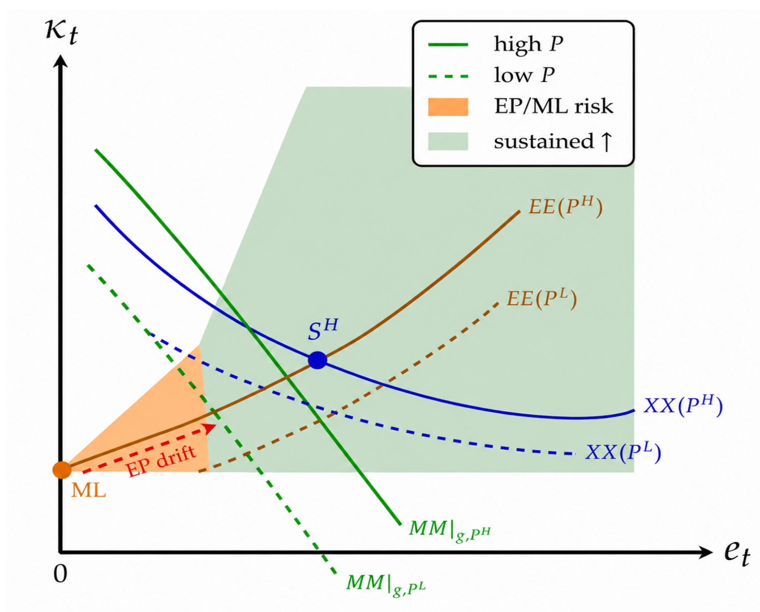


Figure 5: Locus shifts as P declines: the EE locus shifts down and the XX locus shifts left as population falls from P^H to P^L. The interior steady state S^H disappears and the economy drifts toward the EP/ML outcome.

If the income distribution places the economy within the **MG** regime, where relative human capital a_t is uniformly distributed as $\frac{\bar{c}(1+\beta+\gamma)}{(1+\beta)w_t\bar{h}_t} \leq a_t \leq (w_t\bar{h}_t)\left(\frac{w_t\bar{h}_t\theta-\psi}{\psi}\right)$, then we have,

$$K_{t+1} = P_t * B(w_t\bar{h}_t), \tag{22}$$
$$\kappa_{t+1} = \Xi^c(e_t, g_t, \kappa_t; P),$$
$$e_{t+1} = \Phi^b(z_t, \kappa_t, g_t; P).$$

During the **MG** regime, technological progress leads to rising returns to education (η) and investment in education (e_{t+1}), while overall fertility (n_t) declines as the substitution effect outweighs the income effect. This is reflected in $\Xi^c_\kappa(.) > 0$, $\Xi^c_e(.) > 0$, and $\frac{d\kappa_t}{de_t} < 0$. Simultaneously, with technological progress, $\Phi^b_\kappa(.) < 0$ and $\Phi^b_e(.) > 0$, indicating the dominance of the substitution effect, which implies $\frac{d\kappa}{de_t} > 0$. As previously noted, technological progress also contributes to greater inequality in period $t+1$. Consequently, fertility (n_{t+1}) and average investment in children's education (e_{t+2}) decline during the **MG** regime, as observed in Proposition 2. This results in $\Xi^c_e(.) < 0$, $\frac{d\kappa_t}{de_t} > 0$, and $\Phi^b_e(.) = 0$, causing a leftward shift in the e curve. As education evolves, the XX locus becomes downward sloping due to technological progress and upward sloping due to inequality. Meanwhile, the EE locus is upward sloping with technological progress but shifts leftward in response to increasing inequality.
If the income distribution places the economy within the **MMG** regime, where relative human capital, a_t, is uniformly distributed as $(w_t\bar{h}_t)\left(\frac{w_t\bar{h}_t\theta-\psi}{\psi}\right) \leq a_t$, then we have,

$$K_{t+1} = P_t * C(w_t\bar{h}_t), \tag{23}$$
$$\kappa_{t+1} = \Xi^d(e_t, g_t, \kappa_t; P),$$
$$e_{t+1} = \Phi^c(e_t, g_t, \kappa_t; P).$$

Technological progress results in higher returns to education (η) and greater investment in education (e_{t+1}), accompanied by a rise in overall fertility (n_t) due to the income effect dominating the substitution effect. This relationship is expressed as $\Xi^d_\kappa(.) > 0$, $\Xi^d_e(.) < 0$, and $\frac{d\kappa_t}{de_t} > 0$. At the same time, technological progress results in $\Phi^c_\kappa(.) < 0$ and $\Phi^c_e(.) < 0$, as the income effect continues to dominate, leading to $\frac{d\kappa}{de_t} < 0$. With rising inequality during the **MMG** regime, both fertility (n_{t+1}) and investment in children's education (e_{t+2}) increase, as highlighted in Proposition 2. This outcome is characterized by $\Xi^d_e(.) > 0$, $\frac{d\kappa_t}{de_t} < 0$, and $\Phi^c_e(.) = 0$, resulting in an upward shift in the e curve. These dynamics reflect that the XX locus slopes upward with technological progress but slopes downward with rising inequality. On the other hand, the EE locus is downward-sloping with technological progress but shifts upward with increased inequality.

Figure 4 illustrates the pivotal role of technological progress and inequality in driving the economic transition from the Malthusian regime to the **MG** regime, and subsequently to the **MMG** regime. As education improves, inequality and technological progress increase, altering economic outcomes. The dotted lines and arrows indicate the effects of rising inequality, while the bolder lines and arrows represent the impact of increased technological progress on capital per effective labor (κ) and education investment (e). Below the Conditional Malthusian Frontier (green curve), households are limited to subsistence-level consumption. Technological progress alone is insufficient to drive the transition from the Malthusian to the **MG** regime. As education evolves and technological progress increases, an economy could converge to a Conditional Malthusian steady state, that is, **ML** trap (orange region). Introducing inequality shows that when its effects surpass those of technological progress in the absence of inequality, economies in the grey region show transition from the Malthusian to the **MG** regime. This shift eliminates the orange region, moves the grey region upward, and fosters economic growth. Inequality thus plays a more crucial role in this transition. In the dark blue region, both inequality and technological progress together lead to either an *EP* outcome or an economy may move to **MMG** regime. Upon entering the **MMG** regime (pink region), technological progress may return the economy to the **Malthusian** region, causing another **ML** trap. Despite this, rising inequality plays a crucial role in preserving a positive growth rate and averting this regression to **ML** trap, thereby reinforcing its dominant influence in achieving long-term economic growth and development. Figure 5 illustrates how a decline in population from P^H to P^L changes the economy's dynamic structure. As population falls, the *EE* locus shifts downward and the *XX* locus shifts leftward, reducing the set of feasible high-growth equilibrium. Consequently, the interior steady state S^H disappears, and the economy begins to drift toward the low-development EP/ML region. The shaded orange area represents the zone of endogenous Malthusian trap, whereas the green region denotes trajectories consistent with sustained growth and development. We now present the following proposition.

Proposition 3. *(i)* ***Empty Planet (EP).*** *In the* **MG** *regime, suppose $\dot{e}/e \geq 0$ and $\dot{n}/n < 0$. The economy converges to an EP outcome — characterised by $g_{t+1}(E_t, P_t) < 0$ and $\dot{A}/A < 0$ — if and only if the negative effect of population decline dominates the positive effect of rising education:*

$$\left|\frac{\partial g}{\partial P_t}\dot{P}_t\right| > \left|\frac{\partial g}{\partial E_t}\dot{E}_t\right|.$$

(ii) ***Malthusian-like (ML) trap.*** *In the* ***MMG*** *regime, the economy converges to the Conditional Malthusian steady state $(\kappa^*, e^*) = (z^{-1}(\zeta), 0)$ with $\dot{y}/y = 0$ if capital per effective worker declines due to rising population: $\Phi^c(\kappa_t, e_t, g_t; P) < 0$ and $\kappa_t < \kappa(e_t)$.*

(iii) ***Sustained growth.*** *Starting from the* ***MMG*** *regime, the economy achieves a positive long-run output growth rate if inequality rises sufficiently to shift the EE locus rightward and the XX locus upward, so that their intersection* (κ^{**}, e^{**}) *lies strictly above the Conditional Malthusian Frontier* $MM|g_t$*.*

(iv) *In all cases, technological progress alone is insufficient to determine the long-run outcome; the interaction between* g_{t+1} *and the endogenous evolution of inequality is necessary and sufficient for the transition between regimes.*

Proof 2. *Part (i) — EP outcome. From the technological-progress function* $g_{t+1} = g(E_t, P_t)$ *with* $g_E > 0$ *and* $g_P > 0$*, the total time derivative along a trajectory with* $\dot{E}_t \geq 0$ *and* $\dot{P}_t < 0$ *is* $\dot{g}_{t+1} = g_E\dot{E}_t + g_P\dot{P}_t$*. This is negative — giving* $g_{t+1} < 0$ *and* $\dot{A}/A < 0$ *— if and only if* $|g_P\dot{P}_t| > |g_E\dot{E}_t|$*, i.e. the condition in part (i). See Appendix A.2 for the explicit derivation of the threshold on* κ_t *below which* $\dot{P}_t < 0$ *in the MG regime.*

Part (ii) — ML trap. In the MMG regime the capital dynamics satisfy $\kappa_{t+1} = \Xi^d(\kappa_t, e_t, g_t; P)\kappa_t$ *with* $\Xi^d_\kappa > 0$*. When* $\kappa_t < \kappa(e_t)$ *(below the XX locus) and* $\Phi^c < 0$ *(education declining), the state* (κ_t, e_t) *converges to the unique Conditional Malthusian steady state* $(z^{-1}(\zeta), 0)$ *with* $\dot{y}/y = 0$*.*

Parts (iii) and (iv). The sustained-growth outcome requires the XX–EE intersection to lie above the Conditional Malthusian Frontier. From the phase diagram analysis in Section 4.1, inequality shifts the XX locus upward and the EE locus rightward in the MMG regime; without this shift (i.e. under zero inequality), the system converges to the ML trap. Hence technological progress alone is insufficient, establishing part (iv).

Negative population growth leads to an **EP** outcome. With constant investment in education and a declining population growth rate, the stock of ideas decreases, i.e., $\frac{\dot{A}_t}{A_t} < 0$. This aligns with negative technological progress represented by $g_{t+1} < 0$. Such a scenario arises when the negative impact of population decline outweighs the positive influence of rising education, resulting in the persistence of the **EP** outcome.

During the **MG** regime, where $\frac{\dot{e}}{e} \geq 0$ and $\frac{\dot{n}}{n} < 0$, if $\frac{\dot{e}}{e} = 0$ and $\frac{\dot{n}}{n} < 0$, the economy consistently converges to the **EP** outcome. This occurs because education contributes nothing to the stock of ideas while fertility decline has a negative effect, resulting in $g_{t+1}(e_t, P_t) < 0$. In cases of a quantity-quality trade-off, where $\frac{\dot{e}}{e} > 0$ and $\frac{\dot{n}}{n} < 0$, the economy reaches the **EP** outcome if the negative population growth ($\frac{\dot{n}}{n}$) surpasses the positive educational effect ($\frac{\dot{e}}{e}$). This leads to declining technological progress, $g_{t+1}(e_t, P_t) < 0$, reinforcing the **EP** outcome. In summary, we show that despite advancements in education and technological progress, as well as a decline in fertility and population growth, the dominance of negative population effects over rising education drives the economy toward the **EP** outcome during the **MG** regime. When an economy moves from the **MG** regime to the **MMG** regime, fertility begins to rise alongside potential income. Technological progress, coupled with the effects of inequality, can then propel the economy onto a path of sustained growth.

5 Computation

The theoretical findings in the previous section highlight several crucial aspects. First, we examine how inequality in human capital drives disparities in fertility and education, subsequently influencing economic growth. Second, historical fertility trends can jointly be explained by technological progress and its effect on inequality. Third, we analyze whether a declining population growth rate results in stagnant economic growth, represented as the **EP** outcome. The model assumes a generational length of 30 years for calibration. The calibration process is divided into two parts: the first considers an exogenous level of returns to education, where η remains fixed, following de La Croix and Doepke (2003). The second part involves a comprehensive calibration of our unified model across the entire period, incorporating an endogenous level of returns to education, where η is a function of technological progress, following Galor and Weil (2000). The following table contains the values of the parameters taken from de la Croix and Doepke (2003), Bhattacharya and Chakroborty (2007) and Cordoba and Ripoll (2019).

Parameters	Value	Source
α	0.33	de la Croix and Doepke (2003)
β	0.37	de la Croix and Doepke (2003), Bhattacharya and Chakroborty (2007)
θ	0.0118	de la Croix and Doepke (2003)
γ	0.271	de la Croix and Doepke (2003)
ϕ	0.075	de la Croix and Doepke (2003), Cordoba and Ripoll (2019)
ψ	0.001	Bhattacharya and Chakroborty (2007)
δ	0.015	Bhattacharya and Chakroborty (2007)
η	0.635	de la Croix and Doepke (2003)

The parameter ϕ represents the time cost of raising a child, computed by de la Croix and Doepke (2003) and Cordoba and Ripoll (2019) from age 0 to 17, varying across income levels. On average, this time cost is approximately 7.5%. Focusing on growth, ϑ is set to $1 - \tau$, where τ indicates the direct effect of parental human capital on the child's human capital. Following de la Croix and Doepke (2003), τ is assumed to be 0.2.

5.1 Initial inequality, fertility and growth

In our first computational experiment, we examine how initial inequality affects human capital growth under the assumption of a fixed level of returns to education, i.e, starting with initial level of $\eta = 0.635$. By excluding the lifetime subsistence consumption constraint, Figure (6) demonstrates that as inequality increases, human capital growth declines. The blue line corresponds with the findings of de la Croix and Doepke (2003) (orange line), as their analysis also assumed a fixed level of returns to education and excluded the Malthusian regime. Consequently, the blue line becomes obscured due to its overlap with the orange line. After incorporating the Malthusian regime, Figure (7) illustrates that human capital growth initially rises with increasing inequality but ultimately declines. This pattern suggests that during the Malthusian regime, inequality fosters human capital growth; however, in the **MG** regime, further inequality has a suppressive effect on it. Figure (8) illustrate the trajectories of human capital, fertility, education, capital, GDP, inequality, and differential fertility over 240 years ($t = 1$ to $t = 8$), considering the Malthusian regime and fixed level of η. With a decrease in inequality, total population declines and eventually turns negative, while capital, human capital growth, and GDP growth rises, contradicting the **EP** outcome. With fixed level of η, population decline indirectly boosts human capital growth via education and directly raises GDP growth due to fewer individuals, without any adverse effects on these variables. However, if η is assumed to be a function of technological progress, which itself depends on total population and education, a negative population growth directly reduces human capital growth through declining technological progress and returns to education (η). This would align the economy with the **EP** outcome under negative population growth, as shown by Jones (2022).

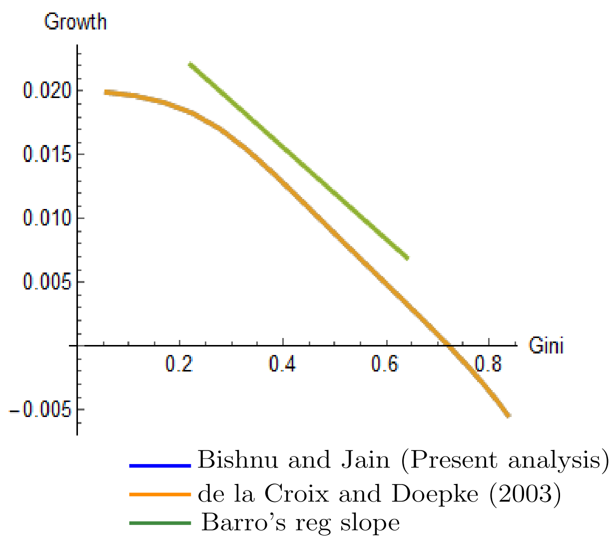


Figure 6: Gini vs Growth with no Malthusian regime and exogenous $\eta = 0.635$.

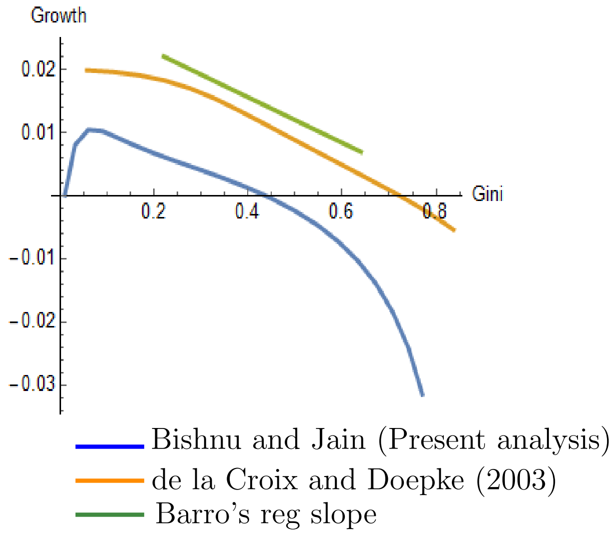


Figure 7: Gini vs Growth with Malthusian regime and exogenous $\eta = 0.635$.

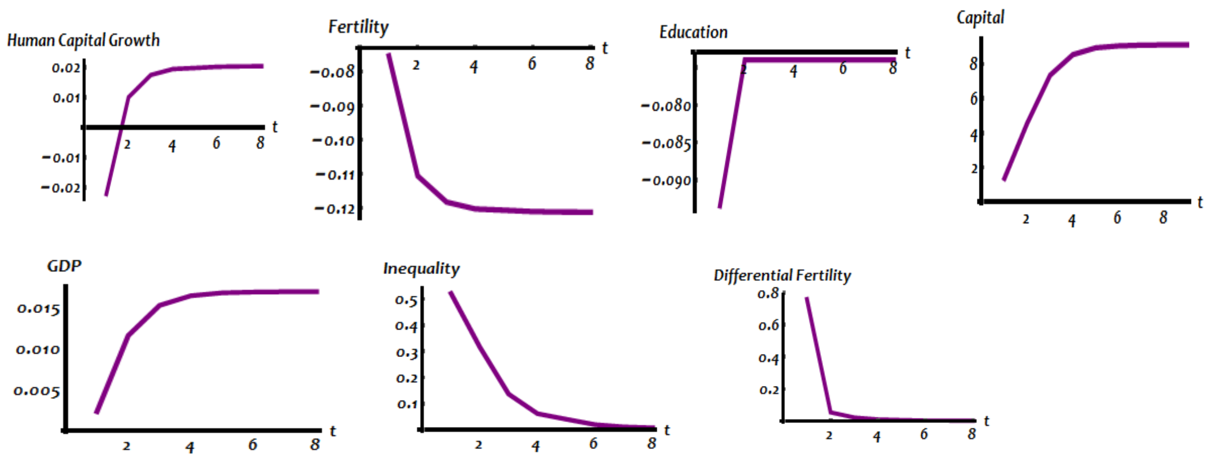


Figure 8: Human capital, Fertility, Education, Capital, GDP, Inequality, Differential fertility.

5.2 With technological progress

We now turn to dynamic implication of our model. We have assumed returns to education, η, as strictly concave and increasing function of technological progress which itself is strictly concave and increasing function of education and total population. We adhere to the following specific form of return to education:

$$\eta = a + (g_{t+1}(e_t, P_t))^{\varepsilon} \equiv a + \left((P_t)^{\zeta}(e_t)^{1-\zeta}\right)^{\varepsilon}.$$

We have chosen the parameter values, $\gamma = 0.01$, $\zeta = 0.65$ and $\varepsilon = 0.33$ such that the elasticity of human capital with respect to education lies with in the acceptable range of values mentioned in the literature, that is $\eta \in [0.635, 0.7]$. Figure (9) confirms our theoretical claim that fertility initially rises, then declines, and eventually starts rising with income. The *income-fertility* curve for 21 OECD countries, presented in **Appendix**

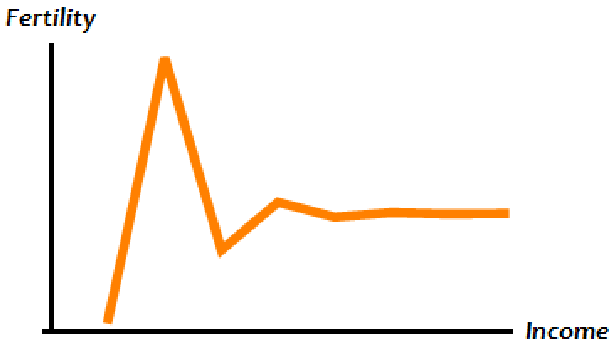
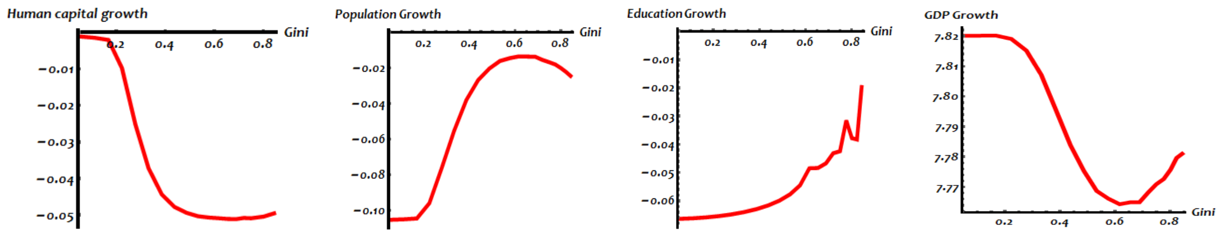
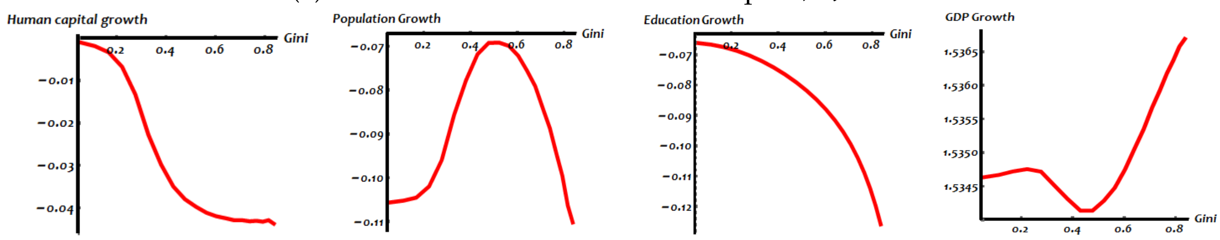


Figure 9: Income-fertility path.



(a) Initial level of relative human capital, $a_t = 0.21$



(b) Initial level of relative human capital, $a_t = 1$

Figure 10: Gini vs Growth with endogenous η.

B.3, also illustrates this trend of initial increase, subsequent decline, and slight rise. Figure (10) illustrates the impact of inequality on total population, education, and economic growth. Figure (10a) shows that if an economy starts with Malthusian regime that is initial level of relative human capital, $a_t = 0.21$, then rising inequality leads to an increase in population and education growth. However, as economy transitions into the **MG** regime, that is initial level of relative human capital, $a_1 = 1$ as in Figure (10b) population and education growth begins to decline. GDP growth, on the other hand, initially falls but eventually starts rising. This supports our claim that the influence of inequality on fertility and education is contingent on the regime in which an economy lies. Figure (11) shows how inequality and technological progress together can explain historical fertility path (green curve). We can see that from period $t = 1$ to $t = 3$,

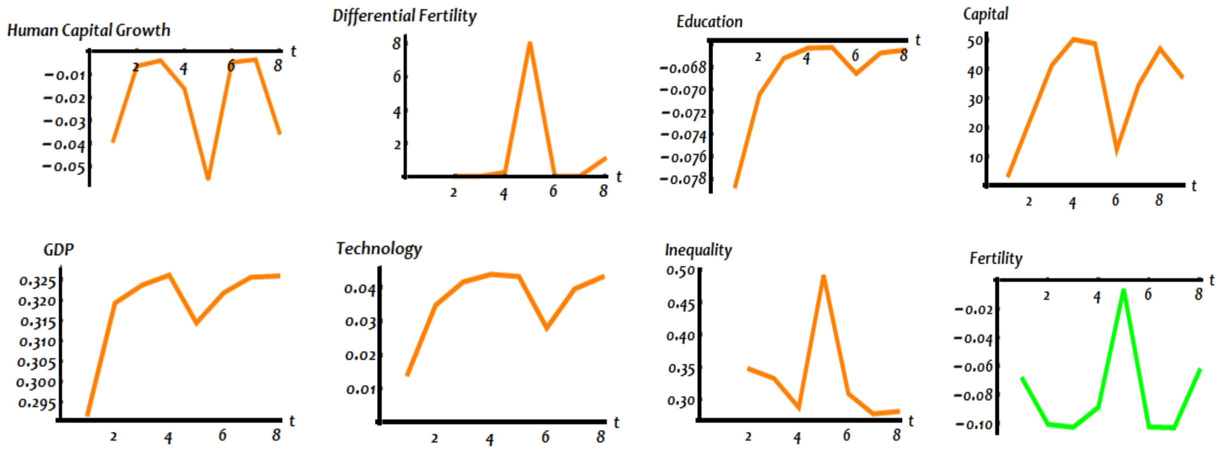


Figure 11: Human capital, Fertility, Education, Capital, GDP, Inequality, Differential fertility.

with decrease in inequality and increase in technological progress, total population falls. As during Malthusian regime decrease in inequality leads to decrease in fertility and with increase in technological progress fertility rises as income effect dominates substitution effect; overall fertility falls if inequality effect dominates the impact of technological progress. As during the **MG** regime with rise in technological progress and inequality, fertility rises and education falls during period $t = 4$ to $t = 5$. During this regime, rising inequality leads to a drop in education levels. If the impact of inequality outweighs technological advancements, education may decline despite technological growth. Together, inequality and technological progress shape the trajectory of fertility and economic growth. It is evident that between periods $t = 7$ and $t = 8$, increasing population growth combined with constant education growth leads to an increase in technological growth but inequality shows a declining trend. Consequently, GDP growth begin to stagnate, leading the economy into an **ML** outcome as the effect of rising population outpaces the effect of rising education. By altering parameter values and initial conditions, a variety of fertility trajectories can be generated, reflecting the significant variations observed across countries due to their unique starting points. Through computational exercises involving changes to these parameter values, we identified the possibility of an economy moving to an **EP** outcome.

6 Conclusion

This paper provides a unified theory of the income–fertility relationship across all stages of economic development. Technological progress operates on fertility through two distinct and opposing channels: an education channel that reduces desired family size by raising the opportunity cost of children, and an inequality channel that widens the distribution of relative human capital across households, which depending on the development stage, can raise aggregate fertility. Their interaction produces a non-monotonic historical fertility path — ascending in the Malthusian regime, declining during the demographic transition, and mildly ascending again at high income — within a single internally consistent framework.

Three propositions summarise the core results. First (Proposition 1), the sign of dn_t/dy_t switches twice along the development path, at thresholds determined by the relative strength of the opportunity-cost and income effects. Second (Proposition 2), inequality affects aggregate growth not only through individual education choices but through the composition of births across the human-capital distribution: in regimes where high-income households have more children, a mean-preserving spread raises fertility-weighted average education and accelerates growth; in the MG regime the opposite holds. This composition effect complements de la Croix and Doepke (2003), whose finding that inequality reduces growth is an unconditional result within the MG regime; our framework shows that this conclusion is conditional on the development stage, and outside the MG regime inequality can accelerate growth through the composition channel. Third (Proposition 3), the long-run outcome — sustained growth, Empty Planet, or Malthusian relapse — depends on whether inequality rises sufficiently to shift the dynamical system's loci above the Conditional Malthusian Frontier; technological progress alone is insufficient.

The quantitative analysis confirms these mechanisms. With an exogenous return to education, the Gini–growth relationship initially rises (Malthusian regime) and then falls (MG regime), matching the Barro regression benchmark and improving on de la Croix and Doepke (2003) by adding the Malthusian segment. With endogenous returns linked to population and education, the model generates the observed income–fertility path for 21 OECD countries and replicates the historical fertility trajectories over 240 years. Empirical evidence using US state-level data (1880–1940) is consistent with the proposed mechanism.

Several directions merit further work. The technological-progress function $g(E_t, P_t)$ is taken as given; a microfounded innovation model that endogenises the returns to population size and education would sharpen the structural conditions for EP versus ML outcomes. The empirical strategy could be extended to cross-country panel data spanning the full demographic transition, where the regime-dependent sign reversal of the inequality effect could be tested more directly. Finally, the composition-of-fertility channel identified here has policy implications: interventions that shift the distribution of educational investment across households — such as targeted schooling subsidies — affect not only average human capital but the trajectory of aggregate fertility, with effects that depend critically on the development stage of the economy.

References

Aiyar, S., Dalgaard, C-J., Moav, O. 2008. “Technological Progress and Regress in Pre-Industrial Times.” Journal of Economic Growth, 13(2): 125–144.

Amol, A., Bishnu, M., Ray, T. 2023., Pension, possible phaseout, and endogenous fertility in general equilibrium. J. Public Econ. Theory 25 (2), 376–406.

Aksan, A. M., and Chakraborty, S., (2014). “Mortality versus morbidity in the demographic transition". European Economic Review, 70, 470–492.

Barro, R. J., and Jong-Wha Lee, 2013, "A New Data Set of Educational Attainment in the World, 1950-2010". Journal of Development Economics, 104: 184-198.

Becker, G. S. and R. J. Barro (1988). “A reformulation of the economic theory of fertility". Quarterly Journal of Economics 103(1), 1–25.

Bishnu, M. (2013). “Linking consumption externalities with optimal accumula- tion of human and physical capital and intergenerational transfers". In: Journal of Economic Theory 148.2, pp. 720-742.

Bishnu, M., Garg, G., Garg, T. and Ray, T. (2021). “Optimal Intergenerational Transfers: Public Education and Pensions". In: Journal of Public Economics 198, 104411.

Bishnu, M., Garg, G., Garg, T. and Ray, T. (2023). “Optimal Intergenerational Transfers: Public Education and Pensions". In: Journal of Economic Dynamics and Control 153, 104697.

Bishnu, M., and Wang, M. (2017). “The Political Intergenerational Welfare State". In: Journal of Economic Dynamics and Control 77, pp. 93-110.

Boldrin, M., Montes, A. (2005). “The Intergenerational State Education and Pensions". In: Review of Economic Studies 72.3, pp. 651-664.

Boucekkine, R., de la Croix, D. and Licandro, O. (2003). “Early mortality declines at the dawn of modern growth". Scandinavian Journal of Economics, vol. 105, pp. 401–18.

Bhattacharya, J., Chakraborty, S. (2017). “Contraception and the demographic transition". Economic Journal, 127. 606, pp. 2263 -301.

Cordoba, J.C. and Ripoll, M. (2014). “Intergenerational Transfers and the Fertility-Income Relationships". Economic Journal, 126 (593), pp. 949-977.

Cordoba, J.C., Liu, X., Ripoll, M. (2016). “Fertility, social mobility and long run inequality". Journal of Monetary Economics, 77, pp. 103–124.

Cordoba, J.C., Ripoll, M. (2019). “The elasticity of intergenerational substitution, parental altruism and fertility choice". Review of Economic Studies 86, pp. 1935–1972.

de la Croix, D., Doepke, M. (2003). “Inequality and Growth: Why Differential Fertility Matters". American Economic Review, 93. 4, pp. 1091 - 1113.

de la Croix, David, and Licandro, O. (2013). “The Child is the Father of Man: Implications for the Demographic Transition”. Economic Journal, 123 (567): 236–261.

Doepke, M. (2004). “Accounting for Fertility Decline During the Transition to Growth”. Journal of Economic Growth, 9, 347-383.

Doepke, M., Hannusch, A., Kindermann, F., Tertilt, M. (2023). “The Economics of Fertility: A New Era". Handbook of the Economics of the Family, Vol. 1, Ch 4.

Fernandez-Villaverde, J. (2001). “Was Malthus Right? Economic Growth and Population Dynamics". Mimeo, University of Pennsylvania.

Feenstra RC., Inklaar R., Timmer MP. (2015). “The Next Generation of the Penn World Table.” American Economic Review, 105(10), pp. 3150–3182.

Galor, O. 2011. Unified Growth Theory. Princeton: Princeton University Press.

Galor O. and Moav, O. (2002). “Natural Selection and the Origin of Economic Growth". Quarterly Journal of Economics 117 (4): 1133-1191.

Galor O. and Weil, D. N. (2000). “Population, Technology, and Growth: From Malthusian Stagnation to the Demographic Transition and Beyond". American Economic Review 90. 4, pp. 806-828.

Galor, O., Moav, O., and Vollrath, D. (2009). "Inequality in Landownership, the Emergence of Human-Capital Promoting Institutions, and the Great Divergence". Review of Economic Studies, 76(1), pp. 143-179.

Glomm, G., and B. Ravikumar. (1992). “Public vs. private investment in human capital: Endogenous growth and income inequality". Journal of Political Economy, 100, no.4 (August):818-834.

Hansen, G. D., and Prescott, E. C. (2002). “Malthus to Solow". American Economic Review 92(4):1205–1217.

Jones, C. (2001). “Was an Industrial Revolution Inevitable? Economic Growth over the Very Long Run". Advances in Macroeconomics 1(2): Article 1.

Jones, C. (2022). “The End of Economic Growth? Unintended Consequences of a Declining Population". American Economic Review, 112(11):3489–3527.

Kogel, T. and Prskawetz, A. (2001). “Agricultural Productivity Growth and the Escape from the Malthusian Trap". Journal of Economic Growth 6:337–57.

Krueger, D. and Ludwig, A. (2007). “On the consequences of demographic change for rates of return to capital, and the distribution of wealth and welfare". Journal of Monetary Economics, 54 (1) (2007), pp. 49-87.

Madsen, J. and Strulik, H. (2023). "Testing unified growth theory: Technological progress and the child quantity-quality tradeoff". Quantitative Economics. Vol. 14, Issue 1, pp: 235-275.

Moav O. (2005). "Cheap children and the persistence of poverty". Economic Journal. Vol. 14, pp: 235-275.

Reher, D. S. (2004). “The demographic transition revisited as a global process". Population, Space and Place, 10(1), Wiley, 19-41.

Tamura, R. (2002). "Human Capital and the Switch from Agriculture to Industry". Journal of Economic Dynamics and Control 27:207–42.

Turner, C., Tamura, R., Mulholland, S. (2013). "How Important Are Human Capital, Physical and Total Factor Productivity for Economic Growth: The States of the United States, 1840 2000". Journal of Economic Growth 18: 319-371.

Online Appendices

A Appendix A

A.1 Proof of Proposition 1 (Non-monotonic Income–Fertility Path)

All the derivations are given in **Appendix B**; the optimal choice of fertility n_t is:

$$n_t = \begin{cases} \frac{\left(a_t w_t \bar{h}_t - \bar{c}\right)}{\left(a_t \phi w_t \bar{h}_t + \psi\right)} & \text{if } \frac{\bar{c}}{w_t \bar{h}_t} \leq a_t < \frac{\theta}{(\phi\eta(g_{t+1}) - \theta\delta)} - \frac{\eta\psi}{(\phi\eta(g_{t+1}) - \theta\delta) w_t \bar{h}_t}, \\ \frac{\left(a_t w_t \bar{h}_t - \bar{c}\right)(1-\eta(g_{t+1}))}{\left(a_t \phi w_t \bar{h}_t + \psi - w_t \bar{h}_t \theta(1+\delta a_t)\right)} & \text{if } \frac{\theta}{(\phi\eta(g_{t+1}) - \theta\delta)} - \frac{\eta\psi}{(\phi\eta(g_{t+1}) - \theta\delta) w_t \bar{h}_t} \leq a_t < \frac{\bar{c}(1+\beta+\gamma)}{(1+\beta) w_t \bar{h}_t}, \\ \frac{\gamma(1-\eta(g_{t+1})) a_t w_t \bar{h}_t}{(1+\beta+\gamma)\left[a_t \phi w_t \bar{h}_t + \psi - w_t \bar{h}_t \theta(1+\delta a_t)\right]} & \text{if } \frac{\bar{c}(1+\beta+\gamma)}{(1+\beta) w_t \bar{h}_t} \leq a_t. \end{cases} \tag{A.24}$$

Since $y_t = w_t \bar{h}_t \cdot a_t$, the effect of potential income y_t on fertility n_t is obtained via the chain rule:

$$\frac{dn_t}{dy_t} = \frac{1}{w_t \bar{h}_t} \frac{\partial n_t}{\partial a_t} + \frac{1}{a_t} \frac{\partial n_t}{\partial (w_t \bar{h}_t)}, \tag{A.25}$$

where $\frac{da_t}{dy_t} = \frac{1}{w_t \bar{h}_t}$ and $\frac{d(w_t \bar{h}_t)}{dy_t} = \frac{1}{a_t}$ follow directly from $y_t = w_t \bar{h}_t \cdot a_t$. Both scale factors carry units of inverse goods, ensuring dimensional consistency.

Computing the partial derivatives from (A.24), we obtain $\frac{dn_t}{dy_t}$ for each regime:

Case 1 $\left(\frac{\bar{c}}{w_t \bar{h}_t} \leq a_t < \frac{\theta}{(\phi\eta - \theta\delta)} - \frac{\eta\psi}{(\phi\eta - \theta\delta) w_t \bar{h}_t}\right)$:

$$\frac{dn_t}{dy_t} = \frac{2(\psi + \phi\bar{c})}{\left(a_t \phi w_t \bar{h}_t + \psi\right)^2} > 0, \tag{A.26}$$

since $\psi, \phi, \bar{c} > 0$. Fertility unambiguously *increases* with potential income in this regime.

Case 2 $\left(\frac{\theta}{(\phi\eta - \theta\delta)} - \frac{\eta\psi}{(\phi\eta - \theta\delta) w_t \bar{h}_t} \leq a_t < \frac{\bar{c}(1+\beta+\gamma)}{(1+\beta) w_t \bar{h}_t}\right)$:

$$\frac{dn_t}{dy_t} = \frac{(1-\eta)\left[2\psi - w_t \bar{h}_t \theta + 2\bar{c}(\phi - \theta\delta) - \frac{\bar{c}\theta}{a_t}\right]}{\left(a_t w_t \bar{h}_t (\phi - \theta\delta) + \psi - w_t \bar{h}_t \theta\right)^2}, \tag{A.27}$$

whose sign depends on $2a_t\psi - a_t w_t \bar{h}_t \theta + 2a_t \bar{c}(\phi - \theta\delta) \gtrless \bar{c}\theta$. In particular:

$$\frac{dn_t}{dy_t} \begin{cases} > 0 & \text{if } a_t > \frac{\bar{c}\theta}{2\psi - w_t \bar{h}_t \theta + 2\bar{c}(\phi - \theta\delta)}, \\ < 0 & \text{if } a_t < \frac{\bar{c}\theta}{2\psi - w_t \bar{h}_t \theta + 2\bar{c}(\phi - \theta\delta)}. \end{cases}$$

Case 3 $\left(a_t \geq \frac{\bar{c}(1+\beta+\gamma)}{(1+\beta)w_t\bar{h}_t}\right)$:

$$\frac{dn_t}{dy_t} = \frac{\gamma(1-\eta)\left(2\psi - w_t\bar{h}_t\theta\right)}{(1+\beta+\gamma)\left(a_t w_t\bar{h}_t(\phi-\theta\delta)+\psi-w_t\bar{h}_t\theta\right)^2}, \tag{A.28}$$

whose sign depends on $2\psi \gtrless w_t\bar{h}_t\theta$:

$$\frac{dn_t}{dy_t}\begin{cases} > 0 & \text{if } \psi > \dfrac{w_t\bar{h}_t\theta}{2}, \\ < 0 & \text{if } \psi < \dfrac{w_t\bar{h}_t\theta}{2}. \end{cases}$$

Therefore, along a development path where both a_t and $w_t\bar{h}_t$ increase with y_t, fertility first increases (Case 1), then may decline (Case 2 and/or Case 3 when $\psi < \frac{w_t\bar{h}_t\theta}{2}$), and then rises again (Case 3 when $\psi > \frac{w_t\bar{h}_t\theta}{2}$), establishing the non-monotonic income–fertility relationship claimed in Proposition 1.

A.2 Proof of Proposition 3 (EP and ML Outcomes)

This appendix derives the explicit threshold condition for Part (i) of Proposition 3 — the capital level below which population growth turns negative in the MG regime, generating the EP outcome. Parts (ii)–(iv) are established in the in-text proof using the phase-diagram geometry of Section 4.1.

During the **MG** regime, when fertility is declining with potential income, that is the region with $\frac{\bar{c}(1+\beta)}{(1+\beta+\gamma)w_t\bar{h}_t} < a_t < (w_t\bar{h}_t)\left(\frac{w_t\bar{h}_t\theta-\psi}{\psi}\right)$,

$$n_t = \frac{\gamma(1-\eta)a_t w_t\bar{h}_t}{(1+\beta+\gamma)(a_t\phi w_t\bar{h}_t+\psi-w_t\bar{h}_t\theta(1+\delta a_t))}.$$

Assuming no inequality, i.e., each household is endowed with same level of relative human capital, $a_t = a$ then population growth turns negative when $n_t - 1 < 0$, that is,

$$w_t(\kappa_t)h_t < \frac{(1+\beta+\gamma)\psi}{\gamma(1-\eta)a+(1+\beta+\gamma)\theta(1+\delta a)-(1+\beta+\gamma)a\phi}.$$

Hence, with low level of capital per effective labor, population growth turns negative. The optimal level of investment in education and growth rate of education is given by,

$$e_{t+1} = \frac{\eta(aw_t h_t\phi+\psi)-w_t h_t\theta(1+\delta a)}{(1-\eta)w_t h_t(1+\delta a)},$$

$$\dot{e}_{t+1} = \frac{\frac{1}{1-\eta}\left(\frac{d\eta}{dt}\left[\left(a\phi w_t \bar{h}_t + \psi - w_t \bar{h}_t \theta(1+\delta a)\right)\right]\right) - \frac{d(w_t \bar{h}_t)}{dt}\left(\frac{\eta\psi}{w_t \bar{h}_t}\right) + \frac{da}{dt}\left(\eta\phi w_t \bar{h}_t - \eta\delta\psi\right)}{(1-\eta)(1+\delta a)(w_t \bar{h}_t)} > 0.$$

The growth rate of education increases with a rise in relative human capital and returns to education, but it decreases with high fixed income. During the **MG** regime, the impact of the first two factors outweighs the latter. Consequently, the economy transitions to the **EP** outcome when the growth rate of education is positive, but population growth is negative. Technological progress begins to decline as the negative population growth rate's effect surpasses the positive impact of the rising education growth rate.

B Appendix B

B.1 Derivation of equations (9)-(10)

The optimization problem is given by,

$$argmax\{ln(c_t) + \beta ln(s_t R_{t+1}) + \gamma ln(w_{t+1}h_{t+1}n_t)\},$$

s.t.

$$c_t + \frac{d_{t+1}}{R_{t+1}} = w_t\bar{h}_t(a_t(1-(\phi+\delta e_{t+1})n_t) - e_{t+1}n_t) - \psi n_t \geq \bar{c},$$

and,

$$\{n_t, e_{t+1}\} \geq 0.$$

For households with $c_t + s_t = \bar{c}$, the optimal choice of fertility is given by,

$$n_t = \frac{a_t w_t \bar{h}_t - \bar{c}}{a_t(\phi+\delta e_{t+1})w_t\bar{h}_t + e_{t+1}w_t\bar{h}_t + \psi}. \tag{B.29}$$

The optimal choice of saving is given by,

$$\frac{dU_t}{ds_t} = -\frac{1}{c_t} + \frac{\beta}{s_t} = 0,$$

hence

$$s_t = \frac{\beta\bar{c}}{1+\beta},$$

precisely

$$s_t = \begin{cases} 0, & \text{if } a_t \leq \frac{\bar{c}}{w_t\bar{h}_t} \\ \frac{\beta\bar{c}}{(1+\beta)}, & \text{if } \frac{\bar{c}}{w_t\bar{h}_t} \leq a_t < \frac{\bar{c}(1+\beta+\gamma)}{(1+\beta)w_t\bar{h}_t}. \end{cases}$$

The optimal choice of investment in child's education is given by,

$$\frac{dU_t}{de_{t+1}} = \frac{\gamma}{n_t}\frac{dn_t}{de_{t+1}} + \frac{\gamma}{h_{t+1}}\frac{dh_{t+1}}{de_{t+1}} = 0,$$

$$\frac{dU_t}{de_{t+1}} = -\frac{\gamma w_t\bar{h}_t(1+\delta a_t)}{a_t(\phi+\delta e_{t+1})w_t\bar{h}_t + e_{t+1}w_t\bar{h}_t + \psi} + \frac{\gamma\eta}{\theta + e_{t+1}} = 0,$$

$$e_{t+1} = \frac{\eta(a_t\phi w_t\bar{h}_t + \psi) - w_t\bar{h}_t\theta(1+\delta a_t)}{(1-\eta)(1+\delta a_t)w_t\bar{h}_t}. \tag{B.30}$$

Substituting (B.30) in (B.29) we get,

$$n_t = \frac{(1-\eta)(a_t w_t \bar{h}_t - \bar{c})}{(a_t \phi w_t \bar{h}_t + \psi - w_t \bar{h}_t \theta(1+\delta a_t))}. \tag{B.31}$$

The threshold level of relative human capital for households with $c_t + s_t = \bar{c}$ is,

$$\frac{dU_t}{dn_t} = \frac{1}{c_t}\frac{dc_t}{dn_t} + \frac{\gamma}{n_t} > 0,$$

$$\frac{dU_t}{dn_t} = -\frac{1}{\bar{c} - s_t} + \frac{\gamma}{(a_t w_t \bar{h}_t - \bar{c})} > 0,$$

$$a_t < \frac{\bar{c}(1+\beta+\gamma)}{(1+\beta) w_t \bar{h}_t}. \tag{B.32}$$

The choice of e_{t+1} is positive if,

$$a_t > \frac{\theta}{\phi\eta - \theta\delta} - \frac{\eta\psi}{(\phi\eta - \theta\delta) w_t \bar{h}_t}. \tag{B.33}$$

Now, we have three main cases to consider,

i) Case 1: $\frac{\theta}{\phi\eta-\theta\delta} - \frac{\eta\psi}{(\phi\eta-\theta\delta)w_t\bar{h}_t} < \frac{\bar{c}(1+\beta+\gamma)}{(1+\beta)w_t\bar{h}_t}$, i.e., $w_t\bar{h}_t < \frac{\phi\eta-\theta\delta}{\theta}\left[\frac{\bar{c}(1+\beta+\gamma)}{(1+\beta)} + \frac{\eta\psi}{\phi\eta-\theta\delta}\right]$,

ii) knife edge case: $\frac{\theta}{\phi\eta-\theta\delta} - \frac{\eta\psi}{(\phi\eta-\theta\delta)w_t\bar{h}_t} = \frac{\bar{c}(1+\beta+\gamma)}{(1+\beta)w_t\bar{h}_t}$, i.e., $w_t\bar{h}_t = \frac{\phi\eta-\theta\delta}{\theta}\left[\frac{\bar{c}(1+\beta+\gamma)}{(1+\beta)} + \frac{\eta\psi}{\phi\eta-\theta\delta}\right]$,

iii) Case 2: $\frac{\theta}{\phi\eta-\theta\delta} - \frac{\eta\psi}{(\phi\eta-\theta\delta)w_t\bar{h}_t} > \frac{\bar{c}(1+\beta+\gamma)}{(1+\beta)w_t\bar{h}_t}$, i.e., $w_t\bar{h}_t > \frac{\phi\eta-\theta\delta}{\theta}\left[\frac{\bar{c}(1+\beta+\gamma)}{(1+\beta)} + \frac{\eta\psi}{\phi\eta-\theta\delta}\right]$.

Using (B.29), (B.30), (B.31), (B.32) and (B.33), the optimal choice of fertility and investment in child's education is given by,

Case 1: $w_t\bar{h}_t < \frac{\phi\eta-\theta\delta}{\theta}\left[\frac{\bar{c}(1+\beta+\gamma)}{(1+\beta)} + \frac{\eta\psi}{\phi\eta-\theta\delta}\right]$,

$$n_t = \begin{cases} 0, & \text{if } a_t \le \frac{\bar{c}}{w_t\bar{h}_t} \\ \frac{(a_t w_t\bar{h}_t - \bar{c})}{(a_t\phi w_t\bar{h}_t + \psi)}, & \text{if } \frac{\bar{c}}{w_t\bar{h}_t} \le a_t < \frac{\theta}{\phi\eta-\theta\delta} - \frac{\eta\psi}{(\phi\eta-\theta\delta)w_t\bar{h}_t} \\ \frac{(1-\eta)(a_t w_t\bar{h}_t - \bar{c})}{(a_t\phi w_t\bar{h}_t + \psi - w_t\bar{h}_t\theta(1+\delta a_t))}, & \text{if } \frac{\theta}{\phi\eta-\theta\delta} - \frac{\eta\psi}{(\phi\eta-\theta\delta)w_t\bar{h}_t} \le a_t < \frac{\bar{c}(1+\beta+\gamma)}{(1+\beta)w_t\bar{h}_t}, \end{cases}$$

$$e_{t+1} = \begin{cases} 0, & \text{if } a_t \le \frac{\bar{c}}{w_t\bar{h}_t} \\ 0, & \text{if } \frac{\bar{c}}{w_t\bar{h}_t} \le a_t < \frac{\theta}{\phi\eta-\theta\delta} - \frac{\eta\psi}{(\phi\eta-\theta\delta)w_t\bar{h}_t} \\ \frac{\eta(a_t\phi w_t\bar{h}_t + \psi) - w_t\bar{h}_t\theta(1+\delta a_t)}{(1-\eta)(1+\delta a_t)w_t\bar{h}_t}, & \text{if } \frac{\theta}{\phi\eta-\theta\delta} - \frac{\eta\psi}{(\phi\eta-\theta\delta)w_t\bar{h}_t} \le a_t < \frac{\bar{c}(1+\beta+\gamma)}{(1+\beta)w_t\bar{h}_t}. \end{cases}$$

We can see that, $\frac{dn_t}{da_t} \geq 0$, $\frac{de_{t+1}}{da_t} \geq 0$, $\frac{d^2 n_t}{da_t^2} \leq 0$ and $\frac{d^2 e_{t+1}}{da_t^2} \leq 0$.

Knife edge case: $w_t \bar{h}_t = \frac{\phi\eta - \theta\delta}{\theta}\left[\frac{\bar{c}(1+\beta+\gamma)}{(1+\beta)} + \frac{\eta\psi}{\phi\eta-\theta\delta}\right]$,

$$n_t = \begin{cases} 0, & \text{if } a_t \leq \frac{\bar{c}}{w_t \bar{h}_t} \\ \frac{(a_t w_t \bar{h}_t - \bar{c})}{(a_t \phi w_t \bar{h}_t + \psi)}, & \text{if } \frac{\bar{c}}{w_t \bar{h}_t} \leq a_t < \frac{\bar{c}(1+\beta+\gamma)}{(1+\beta) w_t \bar{h}_t}, \end{cases}$$

$$e_{t+1} = \begin{cases} 0, & \text{if } a_t \leq \frac{\bar{c}}{w_t \bar{h}_t} \\ 0, & \text{if } \frac{\bar{c}}{w_t \bar{h}_t} \leq a_t < \frac{\bar{c}(1+\beta+\gamma)}{(1+\beta) w_t \bar{h}_t}. \end{cases}$$

So we can see that, $\frac{dn_t}{da_t} \geq 0$, $\frac{de_{t+1}}{da_t} = 0$, $\frac{d^2 n_t}{da_t^2} \leq 0$ and $\frac{d^2 e_{t+1}}{da_t^2} = 0$.

Case 2: $w_t \bar{h}_t > \frac{\phi\eta - \theta\delta}{\theta}\left[\frac{\bar{c}(1+\beta+\gamma)}{(1+\beta)} + \frac{\eta\psi}{\phi\eta-\theta\delta}\right]$,

$$n_t = \begin{cases} 0, & \text{if } a_t \leq \frac{\bar{c}}{w_t \bar{h}_t} \\ \frac{(a_t w_t \bar{h}_t - \bar{c})}{(a_t \phi w_t \bar{h}_t + \psi)}, & \text{if } \frac{\bar{c}}{w_t \bar{h}_t} \leq a_t < \frac{\bar{c}(1+\beta+\gamma)}{(1+\beta) w_t \bar{h}_t}, \end{cases}$$

$$e_{t+1} = \begin{cases} 0, & \text{if } a_t \leq \frac{\bar{c}}{w_t \bar{h}_t} \\ 0, & \text{if } \frac{\bar{c}}{w_t \bar{h}_t} \leq a_t < \frac{\bar{c}(1+\beta+\gamma)}{(1+\beta) w_t \bar{h}_t}. \end{cases}$$

So we can see that, $\frac{dn_t}{da_t} \geq 0$, $\frac{de_{t+1}}{da_t} = 0$, $\frac{d^2 n_t}{da_t^2} \leq 0$ and $\frac{d^2 e_{t+1}}{da_t^2} = 0$.

For households with $c_t + s_t > \bar{c}$, the optimization problem is,

$$argmax\{ln(c_t) + \beta ln(d_{t+1}) + \gamma(w_{t+1} h_{t+1} n_t)\},$$

s.t.

$$c_t + \frac{d_{t+1}}{R_{t+1}} = w_t \bar{h}_t (a_t(1 - (\phi + \delta e_{t+1}) n_t) - e_{t+1} n_t) - \psi n_t > \bar{c},$$

and,

$$\{n_t, e_{t+1}\} \geq 0.$$

The optimal choice of saving is given by,

$$\frac{dU_t}{ds_t} = -\frac{1}{c_t} + \frac{\beta}{s_t} = 0,$$

$$s_t = \beta c_t. \tag{B.34}$$

The optimal choice of fertility is given by,

$$\frac{dU_t}{dn_t} = \frac{1}{c_t}\frac{dc_t}{dn_t} + \frac{\gamma}{n_t} = 0,$$

$$\frac{\gamma}{n_t} = \frac{(a_t(\phi + \delta e_{t+1})w_t\bar{h}_t + \psi + w_t\bar{h}_t e_{t+1})}{c_t},$$

$$n_t = \frac{\gamma c_t}{(a_t(\phi + \delta e_{t+1})w_t\bar{h}_t + \psi + w_t\bar{h}_t e_{t+1})}. \tag{B.35}$$

The optimal choice of investment in child's education is given by,

$$\frac{dU_t}{de_{t+1}} = \frac{1}{c_t}\frac{dc_t}{de_{t+1}} + \frac{\gamma}{h_{t+1}}\frac{dh_{t+1}}{de_{t+1}} = 0,$$

$$\frac{\gamma\eta}{(\theta + e_{t+1})} = \frac{(w_t\bar{h}_t n_t + w_t\bar{h}_t\delta a_t n_t)}{c_t},$$

$$e_{t+1} = \frac{\eta(a_t\phi w_t\bar{h}_t + \psi) - w_t\bar{h}_t\theta(1 + \delta a_t)}{(1-\eta)(1+\delta a_t)w_t\bar{h}_t}. \tag{B.36}$$

Substituting (B.34), (B.35) in budget constraint,

$$c_t = \frac{a_t w_t\bar{h}_t}{(1+\beta+\gamma)}. \tag{B.37}$$

Substituting (B.37) in (B.34),

$$s_t = \frac{\beta w_t h_t}{(1+\beta+\gamma)}.$$

Using (B.37), (B.36) and (B.35),

$$n_t = \frac{\gamma(1-\eta)a_t w_t\bar{h}_t}{(1+\beta+\gamma)(a_t\phi w_t\bar{h}_t + \psi - w_t\bar{h}_t\theta(1+\delta a_t))}.$$

We consider three important cases for which the optimal choice of fertility and investment in child's education is given by,

Case 1: $w_t\bar{h}_t < \frac{\phi\eta-\theta\delta}{\theta}\left[\frac{\bar{c}(1+\beta+\gamma)}{(1+\beta)} + \frac{\eta\psi}{\phi\eta-\theta\delta}\right]$,

$$n_t = \begin{cases}\frac{\gamma(1-\eta)a_t w_t\bar{h}_t}{(1+\beta+\gamma)(a_t\phi w_t\bar{h}_t+\psi-w_t\bar{h}_t\theta(1+\delta a_t))}, & \text{if } \frac{\bar{c}(1+\beta+\gamma)}{(1+\beta)w_t\bar{h}_t} \le a_t,\end{cases}$$

$$e_{t+1} = \begin{cases}\frac{\eta(a_t\phi w_t\bar{h}_t+\psi)-w_t\bar{h}_t\theta(1+\delta a_t)}{(1-\eta)(1+\delta a_t)w_t\bar{h}_t}, & \text{if } \frac{\bar{c}(1+\beta+\gamma)}{(1+\beta)w_t\bar{h}_t} \le a_t.\end{cases}$$

Knife edge case: $w_t\bar{h}_t = \frac{\phi\eta-\theta\delta}{\theta}\left[\frac{\bar{c}(1+\beta+\gamma)}{(1+\beta)} + \frac{\eta\psi}{\phi\eta-\theta\delta}\right]$,

$$n_t = \begin{cases}\frac{\gamma(1-\eta)a_t w_t\bar{h}_t}{(1+\beta+\gamma)(a_t\phi w_t\bar{h}_t+\psi-w_t\bar{h}_t\theta(1+\delta a_t))}, & \text{if } \frac{\bar{c}(1+\beta+\gamma)}{(1+\beta)w_t\bar{h}_t} \le a_t,\end{cases}$$

$$e_{t+1} = \begin{cases}\frac{\eta(a_t\phi w_t\bar{h}_t+\psi)-w_t\bar{h}_t\theta(1+\delta a_t)}{(1-\eta)(1+\delta a_t)w_t\bar{h}_t}, & \text{if } \frac{\bar{c}(1+\beta+\gamma)}{(1+\beta)w_t\bar{h}_t} \le a_t.\end{cases}$$

Case 2: $w_t\bar{h}_t > \frac{\phi\eta-\theta\delta}{\theta}\left[\frac{\bar{c}(1+\beta+\gamma)}{(1+\beta)} + \frac{\eta\psi}{\phi\eta-\theta\delta}\right]$,

$$n_t = \begin{cases}\frac{\gamma a_t w_t\bar{h}_t}{(1+\beta+\gamma)\left[a_t\phi w_t\bar{h}_t+\psi\right]}, & \text{if } \frac{\bar{c}(1+\beta+\gamma)}{(1+\beta)w_t\bar{h}_t} \le a_t < \frac{\theta}{\phi\eta-\theta\delta} - \frac{\eta\psi}{(\phi\eta-\theta\delta)w_t\bar{h}_t} \\ \frac{\gamma(1-\eta)a_t w_t\bar{h}_t}{(1+\beta+\gamma)(a_t\phi w_t\bar{h}_t+\psi-w_t\bar{h}_t\theta(1+\delta a_t))} & \text{if } \frac{\theta}{\phi\eta-\theta\delta} - \frac{\eta\psi}{(\phi\eta-\theta\delta)w_t\bar{h}_t} \le a_t,\end{cases}$$

$$e_{t+1} = \begin{cases}0, & \text{if } \frac{\bar{c}(1+\beta)}{(1+\beta-\gamma)w_t\bar{h}_t} \le a_t < \frac{\theta}{\phi\eta-\theta\delta} - \frac{\eta\psi}{(\phi\eta-\theta\delta)w_t\bar{h}_t} \\ \frac{\eta(a_t\phi w_t\bar{h}_t+\psi)-w_t\bar{h}_t\theta(1+\delta a_t)}{(1-\eta)(1+\delta a_t)w_t\bar{h}_t}, & \text{if } \frac{\theta}{\phi\eta-\theta\delta} - \frac{\eta\psi}{(\phi\eta-\theta\delta)w_t\bar{h}_t} \le a_t.\end{cases}$$

B.2 Derivation of equations (18)-(23)

For households lying within Malthusian regime, the value of κ_{t+1} assuming uniform distribution is given by,

$$K_{t+1} = P_t\int_{\frac{\bar{c}}{w_t\bar{h}_t}}^{\frac{\bar{c}(1+\beta+\gamma)}{(1+\beta)w_t\bar{h}_t}} \frac{\beta\bar{c}}{1+\beta}\, g(a_t)da_t,$$

$$K_{t+1} = \frac{P_t(1+\beta)\beta w_t\bar{h}_t\bar{c}}{(1+\beta)(\gamma)(\bar{c})}\int da_t.$$

Substituting the limits we get,

$$K_{t+1} = \frac{P_t\beta\bar{c}}{(1+\beta)} \equiv P_t * A,$$

$$K_t = P_{t-1} * A;$$

$$\frac{K_{t+1}}{K_t} = \frac{P_t}{P_{t-1}}.$$

For households lying within **MG** regime,

$$K_{t+1} = P_t\int_{\frac{\bar{c}(1+\beta+\gamma)}{(1+\beta)w_t\bar{h}_t}}^{\frac{(w_t\bar{h}_t\theta-\psi)w_t\bar{h}_t}{\psi}} \frac{\beta w_t\bar{h}_t a_t}{1+\beta+\gamma}\, g(a_t)da_t,$$

$$K_{t+1} = P_t\left[\frac{\bar{c}\beta}{(1+\beta)} + \frac{\beta(w_t\bar{h}_t)^2(w_t\bar{h}_t\theta-\psi)}{(1+\beta+\gamma)\psi}\right] \equiv P_t * B(w_t\bar{h}_t).$$

For households lying within **MMG** regime,

$$K_{t+1} = P_t \int_{\frac{(w_t \bar{h}_t \theta - \psi) w_t \bar{h}_t}{\psi}}^{\frac{4(w_t \bar{h}_t \theta - \psi) w_t \bar{h}_t}{\psi}} \frac{\beta w_t \bar{h}_t a_t}{(1+\beta+\gamma)} g(a_t) da_t,$$

$$K_{t+1} = \frac{5\beta (w_t \bar{h}_t)^2 (w_t \bar{h}_t \theta - \psi) P_t}{(1+\beta+\gamma)\psi} \equiv P_t * C(w_t \bar{h}_t).$$

For households lying with in Malthusian regime, we know,

$$L_t = \frac{P_t \bar{h}_t z_t (1+\beta)}{\gamma \bar{c}} \left[\int_{\frac{\bar{c}}{z_t}}^{\frac{\theta}{\phi\eta - \theta\delta} - \frac{\eta\psi}{(\phi\eta - \theta\delta) z_t}} a_t (1 - \phi n_t) + \int_{\frac{\theta}{\phi\eta - \theta\delta} - \frac{\eta\psi}{(\phi\eta - \theta\delta) z_t}}^{\frac{\bar{c}(1+\beta+\gamma)}{(1+\beta) z_t}} a_t (1 - (\phi + \delta e_{t+1}) n_t) - e_{t+1} n_t \right] da_t,$$

$$\begin{aligned} L_t = {} & \frac{P_t(1+\beta)}{\gamma \bar{c} w_t} \left[\frac{\psi + \phi \bar{c}}{\phi^2} \left[\frac{\theta \phi z_t}{\phi\eta - \theta\delta} - \frac{\eta\psi\phi}{\phi\eta - \theta\delta} - \bar{c}\phi - \psi log \left[\frac{\theta\phi z_t - \psi\theta\delta}{(\phi\eta - \theta\delta)(\bar{c}\phi + \psi)} \right] \right] \right] + \frac{P_t(1+\beta)}{\gamma \bar{c} w_t} \\ & \left[\bar{c} \left[\frac{\bar{c}(1+\beta+\gamma)}{(1+\beta)} - \frac{\theta z_t}{\phi\eta - \theta\delta} + \frac{\eta\psi}{(\phi\eta - \theta\delta)} \right] \right. \\ & \left. + \frac{\psi(1-\eta)}{(\phi - \theta\delta)^2} \left[\frac{\bar{c}(1+\beta+\gamma)(\phi - \theta\delta)}{(1+\beta)} - \left[\frac{\theta z_t (\phi - \theta\delta)}{\phi\eta - \theta\delta} - \frac{(\phi - \theta\delta)\eta\psi}{\phi\eta - \theta\delta} \right] \right] \right] \\ & - \frac{P_t(1+\beta)}{\gamma \bar{c} w_t} \left[\frac{\psi(1-\eta)}{(\phi - \theta\delta)^2} \left[\bar{c}(\phi - \theta\delta) + \psi - z_t \theta \right] \right. \\ & \left. log \left[\frac{(\phi\eta - \theta\delta) \left(\bar{c}(1+\beta+\gamma)(\phi - \theta\delta) + (1+\beta)(\psi - z_t\theta) \right)}{(1+\beta)(1-\eta)\theta(\phi z_t - \delta\psi)} \right] \right]. \end{aligned}$$

Hence, we can write that

$$L_t = \frac{P_t(1+\beta)}{(\bar{c}\gamma) w_t} f \left(w_t \bar{h}_t = z_t(\kappa_t, e_t, g_t), \eta(g_{t+1}) \right),$$

where,

$$\frac{dL_t}{dz_t} < 0,$$

and,

$$\frac{dL_t}{d\eta} > 0.$$

Similarly we can solve for L_t when economy lies within **MG** and **MMG** regime and calculate κ_{t+1} in all the three regimes and arrive at Equation (20). We define $\kappa_{t+1} = \frac{K_{t+1} L_t}{L_{t+1} K_t (1+g_{t+1})} \kappa_t$. Now, calculating optimal value of e_{t+1} when economy lies within

Malthusian regime,

$$e_{t+1} = \int_{\frac{\theta}{\phi\eta-\theta\delta}-\frac{\eta\psi}{(\phi\eta-\theta\delta)z_t}}^{\frac{\bar{c}(1+\beta+\gamma)}{(1+\beta)z_t}} \frac{\eta(a_t\phi w_t\bar{h}_t+\psi)-w_t\bar{h}_t\theta(1+\delta a_t)}{(1-\eta)w_t\bar{h}_t(1+\delta a_t)} da_t,$$

$$\begin{aligned}
e_{t+1} = &\left[\frac{1}{(1-\eta)z_t}\left[\left[\frac{\eta}{\delta^2}\right]\left[\frac{\bar{c}(1+\beta+\gamma)}{(1+\beta)} - \frac{\theta z_t}{\phi\eta-\theta\delta} + \frac{\eta\psi}{\phi\eta-\theta\delta}\right]\right.\right.\\
&\left[\frac{(1+\beta)z_t(\phi\eta-\theta\delta)+\delta\bar{c}(1+\beta+\gamma)(\phi\eta-\theta\delta)-\theta(1+\beta)z_t+\eta\psi(1+\beta)}{(1+\beta)z_t(\phi\eta-\theta\delta)}\right.\\
&\left.\left. -\frac{1}{z_t}log\left[\frac{(1+\beta)z_t(\phi\eta-\theta\delta)+\delta\bar{c}(1+\beta+\gamma)(\phi\eta-\theta\delta)-\theta(1+\beta)z_t+\eta\psi(1+\beta)}{(1+\beta)z_t(\phi\eta-\theta\delta)}\right]\right]\right]\\
&\left. -\frac{\theta}{2}\left[\frac{\bar{c}(1+\beta+\gamma)}{(1+\beta)} + \frac{\theta z_t}{\phi\eta-\theta\delta} - \frac{\eta\psi}{(\phi\eta-\theta\delta)}\right]\right]\\
&\equiv \Phi^a(e_t, g_t, \kappa_t; P).
\end{aligned}$$

Similarly we can solve for an economy lying within **MG** and **MMG** regime and calculate e_{t+1} in all three regimes and arrive at Equation (21).

B.3 Income-fertility path of 21 OECD countries

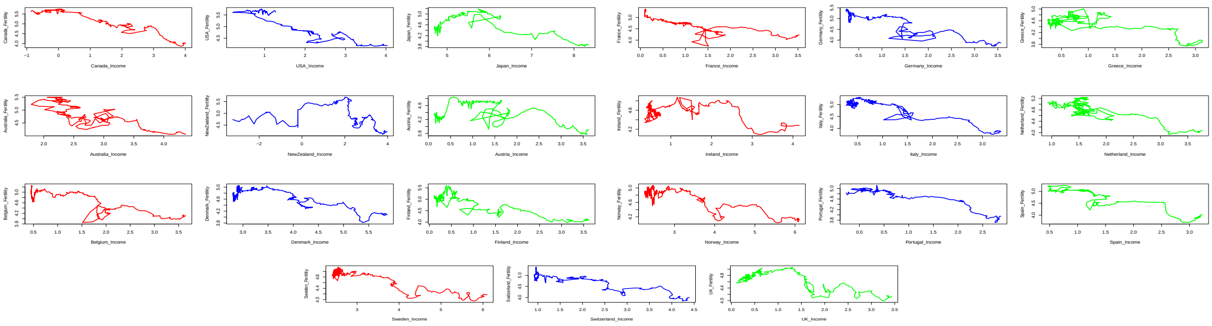


Figure 12: Income-fertility path of 21 OECD countries

B.4 Historical fertility path (over time) of 21 OECD countries

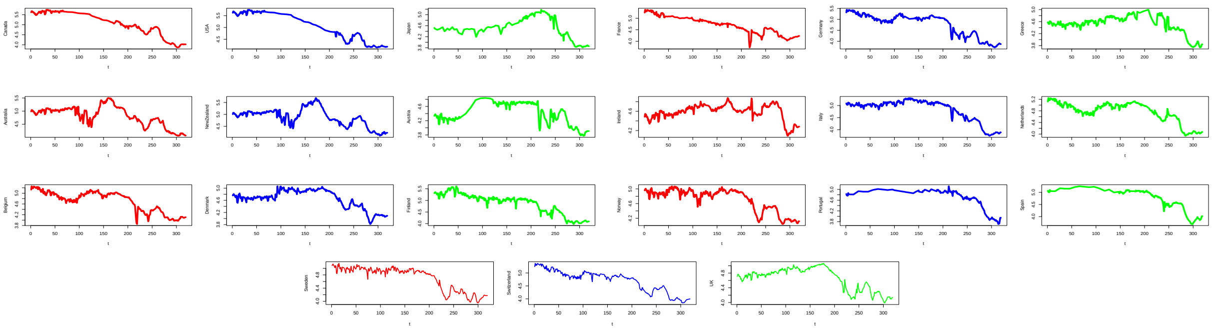


Figure 13: Historical fertility path (over time) of 21 OECD countries.

B.5 Rising return to education with technological progress

We describe how returns to education varies with technological progress. To calculate Mincer returns to education, we use the Penn World Table (PWT) 10.1 and the Barro and Lee (2018) dataset. The formula used to compute the Mincer returns to education is as follows:

$$\text{human capital} = e^{\text{Mincer returns to education * year of schooling}},$$

$$\text{Mincer returns to education} = \frac{log(\text{human capital})}{\text{year of schooling}}.$$

The data on years of schooling is taken from the Barro and Lee (2018) dataset, while human capital is obtained from the Penn World Table (PWT) 10.1 to calculate the Mincer returns to education. Information on total factor productivity (TFP) is also taken from the PWT. Specifically, the PWT provides the data on variable *ctfp*, which measures TFP at current purchasing power parity (PPP) prices. Furthermore, we compute Solow residual TFP growth using the PWT variables *rgdpna*, *rnna*, *emp*, *avh*, and *labsh*, which represent real GDP at constant 2017 national prices, capital stock at constant prices, number of persons employed, average annual hours worked per worker, and the labor share of income, respectively. Importantly, we use total hours worked (derived from $emp \times avh$) rather than employment alone to account for variations in hours worked. The Solow residual TFP growth can then be calculated from PWT using the following standard growth accounting formula:

$$TFP\ growth = \Delta ln(Y) - \alpha \Delta ln(K) - (1-\alpha)\Delta ln(L),$$

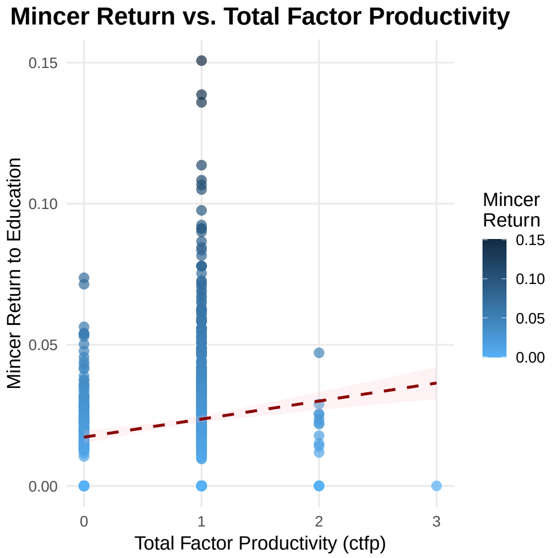
where Y is real GDP at constant prices (*rgdpna* in PWT), K is capital stock (*rnna* in PWT), L is labor input (Total hours worked, *avh* * *emp* in PWT) and α is capital share of income (from PWT: *labsh* is the labor share, so $\alpha = 1 - labsh$). In addition, we calculate labor-augmented TFP growth using PWT variables. The formula for labor-augmented TFP (also referred to as TFP per effective worker) is given by:

$$TFP\ growth = \frac{Y}{K^{\alpha}(h.L)^{1-\alpha}},$$

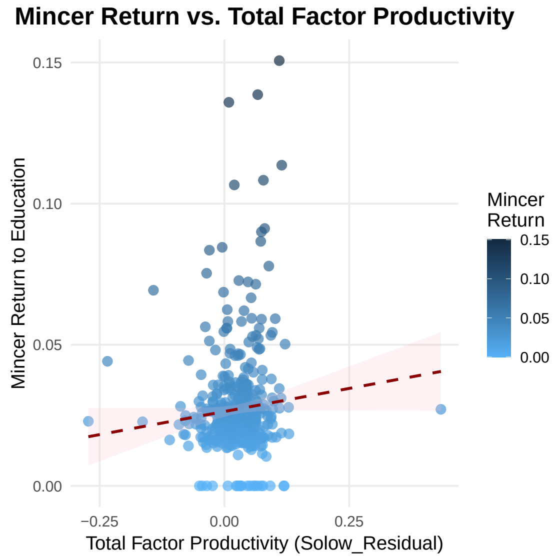
where Y is output-side real GDP at constant 2017 national prices (*rgdpo*), K is capital stock (*rnna* in PWT), h is human capital index (*hc* in PWT), L is employment (*emp* in PWT) and α is capital share. Figure 14a, 14b, and 14c show a positive relationship between total factor productivity and the Mincer returns to education where TFP is measured using *ctfp*, Solow residual TFP growth and labor-augmented TFP growth using PWT 10.1 and Barro and lee (2018) dataset which is a cross country analysis. In contrast, Figure 14d indicates a positive relationship between the Mincer returns to education and TFP for USA using Turner et al. (2013) dataset. The dataset cover 28 states of USA for a period of 1840-2000. They have estimates for years of schooling and also constructed original per worker estimates of human capital, physical capital and output. They then used growth accounting technique to estimate TFP to output growth. The formula used to calculate Mincer returns to education follows the same approach as in PWT 10.1. Total Factor Productivity (TFP) growth is calculated using the following expression:

$$TFP\ growth = y - \alpha k - (1 - \alpha)h$$

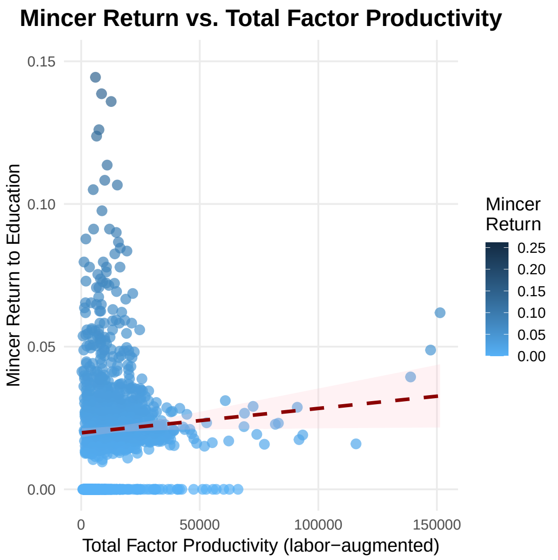
where y is output per worker, k is capital per worker and h is human capital per worker.



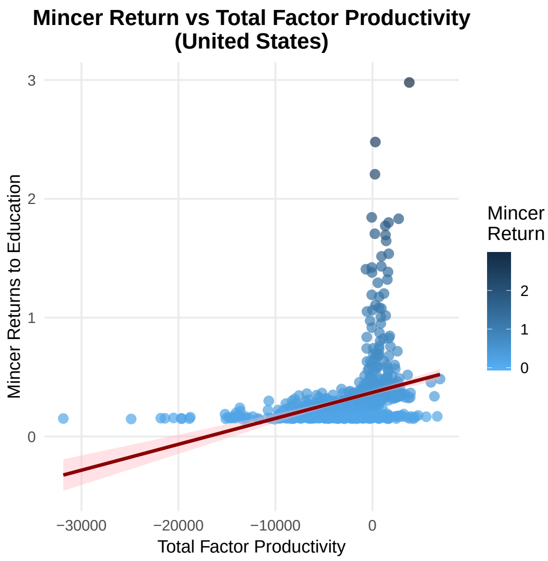
(a) TFP level at current PPPs.



(b) Solow residual TFP growth.



(c) Labor-augmented TFP growth.



(d) TFP (Turner et al. (2013)).

Figure 14: Mincer Return vs. Total factor Productivity.

C Appendix C

This appendix provides additional details on the empirical specification, data construction, identification strategy, and estimation results supporting the mechanism developed in the main text.

C.1 Empirical Specification

We estimate the relationship between inequality, fertility, and educational expenditure using a two-stage least squares (2SLS) framework. Technological progress, measured by total factor productivity (TFP), is used as an instrument for inequality.

The first-stage specification is

$$\log(\text{Inequality}_{i,t}) = \lambda_1 + \lambda_2 \text{TFP}_{i,t} + \lambda_3 \log(X_{i,t}) + \epsilon_{i,t}, \tag{C.38}$$

where $X_{i,t}$ includes demographic controls, specifically the share of Black population and the share of urban population. The index i denotes US states and t denotes census years.

The second-stage fertility regression is

$$\log(\text{Fertility}_{i,t}) = \beta_1 + \beta_2 \log(\widehat{\text{Inequality}}_{i,t}) + \beta_3 \log(X_{i,t}) + u_{i,t}, \tag{C.39}$$

while the education regression is

$$\log(\text{Education}_{i,t}) = \delta_1 + \delta_2 \log(\widehat{\text{Inequality}}_{i,t}) + \delta_3 \log(X_{i,t}) + v_{i,t}. \tag{C.40}$$

To examine whether TFP retains explanatory power for fertility conditional on inequality and education, we additionally estimate

$$\log(\text{Fertility}_{i,t}) = \gamma_1 + \gamma_2 \log(\text{Inequality}_{i,t}) + \gamma_3 \text{TFP}_{i,t} + \gamma_4 \log(\text{Education}_{i,t}) + \gamma_5 \log(X_{i,t}) + z_{i,t}. \tag{C.41}$$

The coefficient γ_3 captures any residual direct effect of TFP on fertility after conditioning on inequality and education expenditure.

C.2 Identification

The empirical strategy relies on the exclusion restriction that TFP affects fertility primarily through inequality and educational investment rather than through additional direct channels.

This assumption is motivated by the theoretical structure of the model, in which technological progress affects fertility decisions through two mechanisms: (i) changes in the return to education and (ii) changes in the distribution of relative human capital across households. These channels are captured empirically through the inequality and education variables included in the second-stage regressions.

Table 3 provides additional evidence consistent with this restriction. Once inequality and education expenditure are included simultaneously, the coefficient on TFP becomes statistically insignificant across all specifications. This suggests that TFP does not retain substantial residual explanatory power for fertility conditional on the proposed mediating channels.

Because education expenditure is itself potentially endogenous to TFP and inequality, the specification in Table 3 is interpreted as a descriptive mediation exercise rather than a formal causal test of the exclusion restriction.

C.3 Data

The data combine state-level observations from Galor et al. (2009) and Turner et al. (2013) for 28 US states over the census years 1880, 1900, 1920, and 1940.

TFP is constructed following Turner et al. (2013) using output per worker, physical capital per worker, and human capital per worker.

Inequality is measured using land concentration, defined as the share of land held by the largest farms in 1880. Formally,

$$S_{i,t} = 1 - \left(1 - \frac{\text{Topfarms}_{i,1880}}{\text{Farms}_{i,t}}\right)^{\varrho_{i,t}}, \tag{C.42}$$

where $\varrho_{i,t}$ denotes the Lorenz-curve coefficient for state i in year t.

Education expenditure data are obtained from the Historical Statistics of the United States and the US Bureau of Education. Fertility is measured as live births per 1,000 women aged 15–44 using US Census data.

C.4 Regime Interpretation

The 1880–1940 US state panel primarily corresponds to the transition from the Post-Malthusian regime to the Modern Growth (MG) regime. During this period, income per worker increased while fertility rates declined and investment in education expanded.

The model predicts that, in the MG regime, higher inequality reduces educational investment among lower-income households and increases fertility through reduced human-capital accumulation. The estimated coefficients reported in Tables 1 and 2 are consistent with these predictions.

Land concentration serves as a proxy for inequality in relative human capital. In the agrarian US economy of this period, large landowners possessed substantially greater resources for educational investment than smallholders or tenant farmers. Consequently, greater land concentration is associated with wider dispersion in educational opportunities and human-capital accumulation across households.

Table 1: IV Regressions (2SLS). Dependent variable: log(Fertility).

	1	2	3	4
		Second Stage (Fertility)		
blackpopulation		1.165***		0.535*
		[0.000]		[0.026]
urbanpopulation			-1.17***	-0.98***
			[0.000]	[0.000]
Inequality	3.91**	3.85**	3.14**	3.15**
	[0.006]	[0.003]	[0.004]	[0.004]
		First Stage (Inequality)		
TFP	0.000009***	0.000009***	0.00001***	0.00001***
	[0.000]	[0.000]	[0.000]	[0.000]
		Diagnostic Tests		
Weak Instruments	26.57***	28.93***	32.12***	32.21***
	[0.000]	[0.000]	[0.000]	[0.000]
Wu-Hausman	58.54***	56.55***	42.31***	42.05***
	[0.000]	[0.000]	[0.000]	[0.000]
Observations	175	172	167	167

p-values in parentheses.
*** $p<0.001$, ** $p<0.01$, * $p<0.05$.

C.5 Additional Discussion

The first-stage estimates indicate that higher TFP is associated with greater land concentration inequality. Across all specifications, the first-stage F-statistics exceed conventional weak-instrument thresholds.

Table 2: IV Regressions (2SLS). Dependent variable: log(Education).

	1	2	3	4
		Second Stage (Education)		
blackpopulation		-6.17***		-3.81***
		[0.000]		[0.000]
urbanpopulation			5.08***	3.74***
			[0.000]	[0.000]
Inequality	-19.56***	-18.84***	-17.49***	-17.56***
	[0.000]	[0.000]	[0.000]	[0.000]
		First Stage (Inequality)		
TFP	0.000009***	0.000009***	0.00001***	0.00001***
	[0.000]	[0.000]	[0.000]	[0.000]
		Diagnostic Tests		
Weak Instruments	25.13***	27.28***	32.23***	32.32***
	[0.000]	[0.000]	[0.000]	[0.000]
Wu-Hausman	85.35***	140.86***	98.06***	167.92***
	[0.000]	[0.000]	[0.000]	[0.000]
Observations	169	164	168	168

p-values in parentheses.
*** $p<0.001$, ** $p<0.01$, * $p<0.05$.

Table 3: Fertility Regression.

	1	2	3	4
blackpopulation		-0.56***		-0.60***
		[0.000]		[0.000]
urbanpopulation			0.08	0.145
			[0.494]	[0.22]
Inequality	-1.716***	-1.789***	-1.763***	-1.887***
	[0.000]	[0.000]	[0.000]	[0.000]
TFP	0.00001	-0.0000007	0.00001	-0.000004
	[0.119]	[0.9]	[0.208]	[0.60]
Education	-0.2***	-0.28***	-0.21***	-0.30***
	[0.000]	[0.000]	[0.000]	[0.000]
R Squared	0.57	0.60	0.57	0.60
Observations	169	164	168	168

p-values in parentheses.
*** $p<0.001$, ** $p<0.01$, * $p<0.05$.

The fertility regressions in Table 1 show a positive and statistically significant relationship between inequality and fertility. The education regressions in Table 2 show a negative and statistically significant relationship between inequality and educational expenditure.

Table 3 additionally shows that TFP becomes statistically insignificant once inequality and education are included simultaneously. This pattern is consistent with the mechanism proposed in the model, according to which technological progress affects fertility primarily through inequality and educational investment.

The sign difference between the 2SLS fertility regressions and the conditional OLS regressions reflects the role of education as a mediating channel. Conditioning on education partially removes the mechanism through which inequality affects fertility, leaving only the residual direct relationship.