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Abstract

We study the joint determination of fertility subsidies and Social Security taxes in an overlapping generations model where agents are heterogeneous in endowments. In equilibria where Social Security is valued, old and poor young agents form a coalition that sustains Social Security. When voting for fertility subsidies, the young take into account both the deadweight loss of such subsidies and the gains from a higher future tax base. They also take into account a third effect of increasing population growth: that of a decrease in future Social Security benefits as a consequence of a change in the identity of the future decisive voter.
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1 Introduction

In this paper we develop a political economy model of the joint evolution of unfunded Social Security (SS) systems and fertility policies. We use it to investigate the viability of unfunded Social Security and the role of fertility in securing it. We employ an overlapping-generations framework with endogenous fertility, in which agents vote on the size of fertility subsidies, on whether or not to continue with the SS system, and on the magnitude of pension taxes and payments. It is well known that a number of different policies - and in particular policies related to public education and immigration - can make SS viable in situations where otherwise it would be abandoned. However, our framework adds significantly to the literature by exploring the way in which current fertility subsidies can affect the identity of the future decisive (median) voter, and thereby affect future pension/SS entitlements. This has not been investigated before. And yet as we discuss below there are clear cases in which actual fertility policies have been designed to alter the identity of future decisive voters.

While this strategic effect of fertility subsidies has received little attention in the context of the sustainability of SS, there is at least one example where such subsidies have been implemented to affect the identity of the future decisive voter. Winckler [2003] describes pro-natalist policies in Israel. From 1970 to 1996, family allowances were largely conditional on family members serving in the army, effectively discriminating against the Arab-Israeli population. Such policies were clearly motivated to maintain a ‘demographic balance’ between Jews and Arabs, as discussed in Portuguese [1998], Friedlander [1973], and expressed in the political arena most eloquently by David Ben Gurion (see Friedlander and Goldscheider [1979]).

A form of strategic manipulation of tomorrow’s constituency by means of current public policies is also present in Hassler et al. [2003]. In that paper, the redistributive policies reduce the incentives to invest in human capital, creating future constituencies that are more inclined to vote for the continuation of the welfare state. Like that paper, we are interested in the conflict within and between generations over redistribution, and the conditions under which the welfare state will survive. However, unlike Hassler et al. [2003], we focus on the role that fertility policies may have in ensuring the survival of the Social Security system, which introduces an extra public policy decision to be made. Also, while in Hassler et al. [2003] young and old agents are homogeneous, we allow for a continuum of ability levels, and study the role of income heterogeneity in the sustainability of the Social Security system.

A number of recent contributions on the political economy of SS (see Galasso and Profeta [2002] for a survey) are also relevant for our purposes. First, Casamatta et al. [2000] and Tabellini [2000] study SS as a device that not only redistributes income from young to old, but also from wealthy to poor households, and therefore SS arrangements can be sustained as a political equilibrium without resorting to intergenerational (e.g. dynamic efficiency) considerations. Cooley and Soares [1999] and Galasso [1999] study SS as an institution with inherited rules that are costly to change, but their approaches are quite different from the one taken here. The paper that is closer to ours is Boldrin and Rustichini [2000]. In that model, agents are confronted with an existing promise of paying a SS tax to old agents, and may either abandon the SS system altogether, or pay the SS tax and vote for a level of SS benefits in the next period. The analysis in Boldrin and Rustichini [2000] gives an explicit dynamic dimension to the problem of sustaining a SS system, and provides a clear interpretation of it as one of unfunded SS liabilities, in line with most of the public policy debate.

A small number of studies examine the joint determination of SS along with another dimension of the welfare system, as we do. In particular, Conde-Ruiz and Galasso [2003] separate the redistributive aspect of SS into within and between cohort components, and consider the circumstances under which both aspects arise as an equilibrium. Boldrin and Montes [2002] show that unfunded SS creates incentives for the optimal provision of public education, a result generalized by Rangel [2003]. Finally, Kemnitz [2000] and Poutvaara [2003] study the joint determination of unfunded SS and education subsidies. We are aware of no paper that studies the joint determination of fertility policies and SS systems.

A number of contributions have linked however fertility decisions to the SS system. Sinn [1999] examines the properties of SS as an insurance device against being infertile or having ungrateful children. A related literature suggests that the SS sustainability problem could -at least partially- be solved by linking pension benefits to fertility. In particular, Sinn [1997] examines a reform where individuals with fewer than a set number of children would have to contribute an extra amount to a parallel funded system. Our paper can partly be seen as a study of the political sustainability of this type of reforms.

Our paper extends the literature on the political economy of SS systems to incorporate the endogenous determination of fertility policies. We find that strategic setting of fertility subsidies to limit the political influence of the newborn generation in the future decreases the levels of both fertility subsidies and SS taxes. We also obtain the result that the existence of fertility subsidies is a necessary condition for the existence of a unique, positive and globally stable steady state level of SS taxes.

This paper has three other sections. In the next section we present the model, and define the equilibrium in which we are interested. In section 3 we derive and present the results with endogenous policy variables, and in section 4 we conclude.

2 The voting equilibrium

We study an overlapping generations economy where households live for two periods. Households have preferences defined over their consumption when young c^y, the number of children to be born at the end of the first period n, and their consumption when old c^o.

An endowment of α_i is received when young and, as there is no technology to allow saving, agents must rely on SS transfers to consume when old. Households are heterogeneous in their endowments (α), with the distribution of α over young households summarized by the uniform CDF $G(\alpha)$, with mean θ and support $\Theta \equiv [\underline{\alpha}, \overline{\alpha}]$.[1] In this model income heterogeneity serves two purposes. First, it allows us to investigate both within generation and between generation redistribution, which turns out to have important implications for our results. Second, it ensures that Social Security is valued even in the absence of dynamic efficiency, which we believe introduces a stronger motivation for the existence of the Social Security system. In this framework, fertility subsidies are valued only because they serve to obtain higher Social Security payments when old.

There are two policy instruments, SS taxes τ^{ss} and fertility subsidies τ^f. The SS system is unfunded, or pay-as-you-go, so young households pay a proportional tax on their endowment, which finances the unique level of SS benefits received by old households at every period. Fertility subsidies (taxes)

[1]This implies:

$$G(\alpha_i) = \begin{cases} 0 & \text{if } \alpha_i < \underline{\alpha} \\ \frac{\alpha_i - \underline{\alpha}}{\overline{\alpha} - \underline{\alpha}} & \text{if } \alpha_i \in [\underline{\alpha}, \overline{\alpha}] \\ 1 & \text{if } \alpha_i > \overline{\alpha} \end{cases} \tag{1}$$

are designed to reduce (increase) the cost of having children bn, where b is the cost per child, and are financed by a lump sum tax (rebate) T.[2] Young household i at time t maximizes the direct utility function in (2) in which taxes and subsidies are exogenous to the household, subject to the per-period budget constraints.

$$\max_{\{c^y,c^o,n\}} \quad \hat{V}^y(c_t^y, c_{t+1}^o, n_t) = c_t^y + \beta c_{t+1}^o \quad + \gamma \ln n_t \tag{2}$$
$$s.t. \qquad (1-\tau_t^{ss})\alpha_i = c_t^y + b(1-\tau_t^f)n_t + T_t$$
$$c_{t+1}^o = ss_{t+1}.$$

Here, ss_{t+1} represents SS benefits at time $t+1$ and β is a time discount factor. Old households solve

$$\max_{c^o} \quad \hat{V}^o(c_t^o) = c_t^o \tag{3}$$
$$s.t. \qquad c_t^o = ss_t.$$

The use of quasilinear preferences, together with a lower bound on individual endowments α_i will ensure that the choice of n is independent of wealth, which simplifies aggregation greatly.

It is important to note that the private decisions made by each household are taken after the public decisions. Moreover, since there is a continuum of agents, their individual decisions do not affect the government's budget balance. This is why, in problem (2), taxes and subsidies are exogenously given to households.

In this arrangement, all households choose the same number of children n_t. Fertility and consumption when young and old follow [3]

$$n_t = \frac{\gamma}{b(1-\tau_t^f)}. \tag{4}$$
$$c_t^y = (1-\tau_t^{ss})\alpha_i - T_t - \gamma \tag{5}$$
$$c_{t+1}^o = ss_{t+1} \tag{6}$$

[2]Note that n is normalized so that the unit of measurement of the population is the young (two person) household, so $n=1$ implies 2 children per couple.

[3]We restrict the parameters to ensure an interior solution where consumption when young is strictly positive. The condition on the parameters is that disposable income when young is larger than γ:$\gamma < (1-\tau_t^{ss})\underline{\alpha} - T_t$

The government raises taxes and provides subsidies and SS benefits under a restrictive rule of budget balance: SS benefits are financed with SS contributions, and fertility subsidies are financed through a lump sum tax paid only by the young. These conditions are formalized below.

$$N_{t+1}\tau_{t+1}^{ss}\theta = N_t ss_{t+1} \quad (7)$$
$$T_t = bn_t\tau_t^f \quad (8)$$

Because endowments are exogenous and T is lump sum, both the fertility and SS programs are financed in a non distortionary fashion. However, as ss_{t+1} is the same for every old household, but SS taxes are paid according to wealth, the SS system is redistributive. Fertility subsidies on the contrary are distortionary in that they affect the number of children, but do not imply a redistribution of income. The redistributive aspect of SS and the distortionary aspect of fertility subsidies can therefore be isolated in this setting.[4]

The model is closed by specifying the law of motion for population. The working population (N_t) evolves according to

$$N_{t+1} = n_t N_t. \quad (9)$$

Once the individual decision rules (4), (5) and (6) are substituted into the objective functions in (2) and (3), we obtain the indirect utility functions. Making use of the budget constraints (7) and (8), we finally obtain the reduced indirect utility functions:

$$V_i^y(\tau_t^f, \tau_{t+1}^{ss}, \tau_t^{ss}) = (1-\tau_t^{ss})\alpha_i + \frac{\theta\beta\gamma\tau_{t+1}^{ss}}{(1-\tau_t^f)b} - \frac{\gamma}{1-\tau_t^f} + \gamma\ln\frac{\gamma}{(1-\tau_t^f)b}. \quad (10)$$

$$V^o(\tau_t^{ss}) = \frac{\theta w\tau_t^{ss}\gamma}{(1-\tau_{t-1}^f)b}. \quad (11)$$

[4]The non distortionary nature of the financing of both programs is a natural consequence of having an endowment economy, where the only meaningful private decision is that of consuming versus having children. At the same time, the separation of the SS and fertility budgets in this framework is a way to ensure the self-financing of the fertilitity program within each household. as will become clear in section 3, that the fertility program is not redistributive is crucial in ensuring that a well defined decisive voter exists. This rules out, for instance, financing fertility subsidies through a proportional tax on wealth.

Equations (10) and (11) represent the preferences of young and old households over the policy choices. Note that we use hats to differentiate direct utility (in (2) and (3)) and indirect utility functions (in (10) and (11)). Voting occurs each period over three dimensions $\{\lambda_t, \tau_{t+1}^{ss}, \tau_t^f\}$, where λ_t represents the continuation vs. abandoning of Social Security, and the remaining variables represent next period's SS taxes and current period's fertility subsidies respectively. In this paper we follow the literature in assuming sincere voting: Households vote for their preferred choices even if this will not change the equilibrium outcomes.[5]

Expression (10) shows that young household i's utility is increasing next period's SS tax rate τ_{t+1}^{ss}, and there is a trade-off in the choice of fertility subsidies, as these increase next period's Social Security payments but are otherwise distortionary and thus costly in terms of current utility.

Old households (expression (11)) are indifferent among different levels of the tax rates chosen in the current period $\{\tau_{t+1}^{ss}, \tau_t^f\}$, as these choices will only have consequences in the next period. Since they are indifferent, the old abstain from voting on tax rates.

As to the choice over the continuation vs. abandoning of Social Security, note that the old will always prefer to keep SS, as they stand to lose their benefits otherwise. The young in turn will be divided: as SS contributions are proportional to income, but benefits are not, the poorest among the young gain, while the wealthier lose, from the institution of Social Security.

We view these choices as occurring sequentially. First, households vote on λ_t, expressing their preference for honoring the promise τ_t^{ss} or defaulting, in which case τ_t^{ss} is set to zero. Households then proceed to vote for their preferred levels of future SS tax τ_{t+1}^{ss} and current level of fertility subsidy τ_t^f. It is also natural to think of public policy choices being made before individual decisions, as we do here. The timing of decisions is illustrated in figure 1.

Because individuals have incentives to abandon SS when young and reinstate it when old, an equilibrium arrangement must attach a penalty for

[5]From a life cycle perspective, a voter born at t makes four public choices, $\{\lambda_t, \tau_{t+1}^{ss}, \tau_t^f\}$ when young and $\{\lambda_{t+1}\}$ when old. To achieve a more concise notation we omit the lambdas from (10) and (11), and use the current SS tax as an argument instead, making the functional dependence of the lambdas implicit: if $\lambda_t = 0$ so that SS is abandoned at t, we have $\{\tau_t^{ss}, \tau_{t+1}^{ss}, \tau_t^f\} = \{0, 0, 0\}$ as neither are SS contributions paid nor benefits received, and there is no reason to vote for fertility subsidies different from zero, as will be seen in what follows. By the same token, if $\lambda_{t+1} = 0$ we have $\tau_{t+1}^{ss} = 0$.

defaulting on current SS payments. As in Boldrin and Rustichini [2000], we consider a trigger strategy where SS cannot be reinstalled after it has been abandoned. Given the threat of not receiving SS payments when old, the young decide on the issue of whether to vote for or against SS based on the predetermined level of current SS taxes τ_t^{ss}, which is the only state variable in this dynamic problem. This trigger strategy is imposed on the environment to study whether it can sustain SS. It is motivated by the observation that defaulting on SS unfunded liabilities is costly to the government. In our setup, such costs are given by the impossibility of reinstalling SS after defaulting.

We focus on a Markov Perfect Equilibrium where the policy functions for the voting choices depend only on the current state of the economy, summarized by the current SS tax. In addition, we impose the condition that individual decisions are made optimally given the tax rates.

Voting Equilibrium: A voting Equilibrium is a collection of public policy functions $\{\tau_{t+1}^{ss}, \tau_t^f, \lambda_t\}$, and optimal policies for consumption and fertility $\{n_t, c_t^y, c_t^o\}$, that satisfy:

1. $\{\tau_{t+1}^{ss}, \tau_t^f, \lambda_t\} = argmax\, V_m(\tau_t^f, \tau_{t+1}^{ss}, \tau_t^{ss})$ subject to $V_m(\tau_{t+1}^f, \tau_{t+2}^{ss}, \tau_{t+1}^{ss}) \geq V_m(0,0,0)$, where V_m is the (reduced) indirect utility function of the decisive voter.

2. Consumption when young and old and fertility follow 5, 6 and 4 respectively.

The first equilibrium condition requires that $\{\tau_{t+1}^{ss}, \tau_t^f, \lambda_t\}$ maximize the objective function of the decisive (median) voter, taking into account that the decisive voter in the next period must be at least as well off with SS than without it. The second equilibrium condition simply states that private decisions over fertility and consumption are chosen optimally.

3 Properties of the equilibrium

In this section we show that a decisive voter exists and examine the properties of the equilibrium. Existence of a decisive voter, is established in subsection 3.1. In general, the median voter theorem fails to hold when voting takes place over more than one dimension. In our model however, the only dimension over which derived preferences conflict is that of continuing versus abandoning Social Security. When choosing fertility subsidies and Social Security taxes, all young voters prefer and vote for the same tax levels.

Once existence of a decisive voter is established, we illustrate the two main tradeoffs present in the voting model. The first tradeoff involves the gains from increased fertility in the form of a larger tax base in the future, weighted against the deadweight loss from subsidizing fertility. The second tradeoff involves weighting the net gains from increased fertility via a larger tax base, as discussed above, against a lower level of SS taxes that tomorrow's decisive voter will be willing to pay.

3.1 The decisive voter

We begin by characterizing voting by young and old households [6]. An important first result is that there is no conflict of preferences over the levels of tax rates:

Lemma 1 *All (young) voters choose the same tax rates.*

Proof: note that because V_i^y is separable in α_i, the problem of choosing tax rates once SS is continued can be stated without α_i being an argument of the objective function or the constraints (see appendix A.1).

The reason why all young agents vote for the same fertility subsidy is that they all have the same fertility behavior, and the costs of subsidizing fertility are equally similar across young agents. At the same time, because the Social Security tax chosen is that which will be paid by the young in the next period, the level of this tax that maximizes utility for all young agents in the current period is the highest tax rate that ensures the continuation of Social Security in the next period. This Social Security tax level is independent of the identity of the young agent.

Preferences do conflict over the issue of continuing vs. abandoning Social Security, as formalized in the following lemma.

Lemma 2 *Voting over the existence of Social Security.*

[6] One possible equilibrium of this game has the young at t voting for fertility subsidies/taxes that imply $n_t = 1$, ensuring that they will constitute a majority when old in period $t+1$, and at the same time setting $\tau_{t+1}^{ss} = 1$, so that all the income is extracted from the young at $t+1$. We do not think that this is an interesting equilibrium, and in this section we will restrict the parameters of the model to eliminate it from the analysis. The propositions that follow refer then to an equilibrium where $n_t > 1$ for all t.

1. *The old vote* $\lambda_t = 1$

2. *There exists an endowment level* α^m *such that the young vote* $\lambda_t^i = 1$ *if* $\alpha^i \leq \alpha^m$ *and* $\lambda_t^i = 0$ *otherwise.*

Proof: for the fist point, note that the old will only receive SS benefits if SS is continued, so they vote $\lambda^m = 1$.

To prove the second point, note that the value of continuing SS for young household i is

$$V_i^y(\tau_t^f, \tau_{t+1}^{ss}, \tau_t^{ss}) - V_i^y(0,0,0) = -\tau_t^{ss}\alpha_i + \frac{\theta\beta\gamma\tau_{t+1}^{ss}}{(1-\tau_t^f)b} - \frac{\gamma\tau_t^f}{1-\tau_t^f} - \gamma\ln(1-\tau_t^f) \tag{12}$$

Which is decreasing in the endowment α_i *and is negative for all endowments larger than some critical value* α^m.

The above lemma implies that the SS system is always sustained by a coalition of the old (who are always beneficiaries) and the poor young. The old favor SS because, for them, all the costs associated with it are sunk. The poor young, in turn, favor SS because they gain from the redistributive aspect of the system.

The preceding two lemmas establish that the continuation of the SS system is the only decision on which agents alive in each period disagree, and therefore the decisive voter is clearly defined.

Proposition 1 *A decisive voter exists*

Proof: By lemmas 1 and 2, preferences can be represented along the single dimension of continuing vs. abandoning the Social Security system.

The decisive voter is then the same that decides, at the margin, on the issue of the continuation of SS. To characterize this voter, we begin by noting that, due to the dynamic nature of the model, she obtains no surplus from the SS system. This is formalized in the following lemma.

Lemma 3 *The decisive voter, with endowment* α^m, *is indifferent between the allocation with and without SS: Half of the voters choose* $\lambda = 1$ *and the other half* $\lambda = 0$.

Proof: the proof proceeds by contradiction. If the decisive voter at $t+1$ *receives a net surplus from continuing SS, the decisive voter at* t *could have increased next period's SS taxes* (τ_{t+1}^{ss}) *by a small amount, and SS would still* *be continued at $t+1$. If on the contrary, the decisive voter at $t+1$ receives a net loss from continuing SS, he will vote $\lambda^m = 0$, so the decision to continue SS by the decisive voter at t could not have been derived from an equilibrium policy.*

We make use of lemma 3 to solve for the Markov Perfect Equilibrium. The fact that the decisiver voter is indifferent between continuing or abandoning SS allows to obtain the equilibrium tax rates $\{\tau_{t+1}^{ss}, \tau_t^f\}$ by solving a program where households maximize their indirect utility subject to the constraint that the decisive voter next period will be indifferent in his choice over λ.

This effectively implies that the voters at t must know the realization of τ_{t+2}^{ss} when choosing τ_{t+1}^{ss}. In this paper, we will use the following interpretation of this paradox: the institution of SS makes a promise of tax rates into the future, and voters take this promise as their expected value of future SS tax levels. This interpretation will allow for a discussion of the role played by expected future benefits in the sustainability of SS. In equilibrium such expected values are self fulfilling. An expected SS tax rate will be denoted with a hat (e.g. $\widehat{\tau}_{t+2j}^{ss}$).

Using lemmas 2 and 3 we can obtain an expression for the endowment level of the median voter. Note that old voters will unanimously choose to honor the SS promise ($\lambda_{t+1} = 1$), since they are the beneficiaries. If we normalize the number of old households to 1, then the mass of voting households is $1 + n_t$. With $n_t > 1$, the decisive voter household is such that a proportion $\frac{n_t - 1}{2n_t}$ of young households will vote $\lambda_{t+1} = 1$ [7]. Together with the fact that the poorest young households are the ones to vote for continuing SS, which follows from lemma 2, this implies that the decisive voter household at time $t+1$ will have an endowment level

$$\alpha_{m,t+1} = \underline{\alpha} + (\overline{\alpha} - \underline{\alpha})\frac{n_t - 1}{2n_t}. \tag{13}$$

Regarding the interactions between the SS system and fertility subsidies, note that the only motivation to vote for fertility subsidies comes from the existence of unfunded SS. At the same time, it is the heterogeneity in endowments that makes Social Security valued in equilibria that are dynamically efficient. The following lemma formalizes these results.

[7] If $n_t > 1$, the decisive voter household for the two generations is such that $(1 + n_t)/2$ vote for the same choice. Thus the proportion of the young voting the same way as the old is given by $[\frac{1+n_t}{2} - 1]/n_t = \frac{n_t - 1}{2n_t}$.

Lemma 4 *Interactions between SS and fertility subsidies.*

1. *Social Security is necessary for fertility subsidies to be valued*
2. *If the economy is dynamically efficient, heterogeneity in endowments is necessary for Social Security to be valued.*

Proof: For point 1, the net value of voting for fertility subsidies in the absence of Social Security (from expression (12)) is

$$-\frac{\gamma\tau_t^f}{1-\tau_t^f} - \gamma\ln(1-\tau_t^f) \tag{14}$$

with a maximum at $\tau_t^f = 0$.

For point 2, note that if the economy is dynamically efficient the equilibrium without taxes is Pareto optimal, so SS can only be sustained through redistribution.

We present the Markov Perfect Equilibrium in three steps. First, we derive the evolution of SS in an economy without fertility subsidies. Then, we introduce fertility subsidies but examine the outcomes when agents do not internalize the effects of their choices on the identity of the future decisive voter. Finally, we study the model with fully rational agents.

3.2 No fertility subsidies

It is instructive to consider initially the model with no recourse to fertility subsidies. We proceed backwards by first deriving the tax rates chosen if SS is not abandoned, and then considering the choice of keeping the SS system. Young households choose $\{\tau_{t+1}^{ss}\}$ by maximizing their (reduced) indirect utility function subject to the incentive compatibility constraint that next period's decisive voter will be (weakly) beter off keeping the Social Security system.

$$\max_{\tau_{t+1}^{ss}} \quad V^y(0,\tau_{t+1}^{ss},\tau_t^{ss}) = (1-\tau_t^{ss})\alpha_i + \tfrac{\theta\beta\gamma\tau_{t+1}^{ss}}{b} - \gamma \quad +\gamma\ln\frac{\gamma}{b} \tag{15}$$

$$s.t. \qquad -\tau_{t+1}^{ss}\alpha_m + \tfrac{\theta\beta\gamma\widehat{\tau}_{t+2}^{ss}}{b} \geq 0 \qquad \text{if } \gamma/b > 1 \tag{16}$$

$$\tau_{t+1}^{ss} \leq 1 \qquad \text{otherwise.} \tag{17}$$

With the decisive voter having an endowment $\alpha_m = \underline{\alpha} + (\overline{\alpha} - \underline{\alpha})\frac{\gamma/b-1}{2\gamma/b}$. Constraint (16) applies in case of an interior solution, when next period's decisive voter is also a young agent. As mentioned above, this constraint states that, to next period's decisive voter, the value of keeping SS (or $V_i^y(0, \tau_{t+2}^{ss}, \tau_{t+1}^{ss})$ as defined in expression (10)) must be at last as high as the value of abandoning SS (or $V_i^y(0,0,0)$). Note that in this program τ_{t+1}^f is set to zero, as we are considering the case without fertility subsidies. Constraint (17) in turn represents a corner solution: if $n_t = \gamma/b \leq 1$, the current young will be in a majority in the next period, and therefore can set a expropriatory tax rate of $\tau_{t+1}^{ss} = 1$. In what follows we disregard this possibility and consider only interior solutions, where constraint (16) applies. The optimal choice for the SS tax rate is obtained from standard first order conditions:

$$\tau_{t+1}^{ss} = \frac{\theta}{\alpha_m}\frac{\gamma}{b}\beta\widehat{\tau}_{t+2}^{ss} \tag{18}$$

We can decompose the coefficient multiplying $\widehat{\tau}_{t+2}^{ss}$ into three parts: $\frac{\theta}{\alpha_m}$, $\frac{\gamma}{b}$, and β. Note that the ratio of average income to the income of the richest household voting for SS, denoted by $\theta/\alpha_m > 1$, is a measure of how redistributive the SS system is. Note also that the ratio γ/b is both the fertility rate and the dependency ratio in the next period. The intuition behind (18) is as follows: The larger θ/α_m is, the more do low endowment young households at $t+1$ have to gain from SS for any given tax rate $\widehat{\tau}_{t+2}^{ss}$. Consequently, they in turn can be taxed more and still want to preserve SS. A similar argument holds for the fertility rate, as a higher rate implies larger SS benefits for any given tax rate, since there are more young households to be taxed. Finally, since costs τ_{t+1}^{ss} are paid one period before SS benefits for the decisive voter at $t+1$, these benefits need to be discounted by β.

The policy function for SS taxes can be obtained by inverting (18), using the fact that, in equilibrium, $\{\widehat{\tau}_{t+2}^{ss}\}_{t=0}^{\infty} = \{\tau_{t+2}^{ss}\}_{t=0}^{\infty}$, and lagging the resulting expression one period.

$$\tau_{t+1}^{ss} = \tau_t^{ss}\frac{\alpha_m b}{\theta\beta\gamma} \tag{19}$$

Which defines the MPE for this model. Since the law of motion for SS taxes is linear, in general an interior steady state will not exist. This is formalized in the next lemma

Lemma 5 *In an economy without fertility subsidies, a stable interior steady state for Social Security taxes exists only in the case* $\frac{\alpha_m b}{\theta\beta\gamma} = 1$.

Proof: Without fertility taxes, the dynamics of the system depend on $x \equiv \frac{\alpha_m b}{\theta\beta\gamma}$. *If* $x > 1$, *no positive levels of SS benefits can be sustained as part of an equilibrium, as the economy reaches a point where tax rates higher than one are necessary to sustain the system. If* $x = 1$, *the economy is stable at any positive level of* τ^{ss} , *and if* $x < 1$, *SS tax rates converge monotonically to zero, the only steady state level of SS taxes.*

This result is similar to those in Boldrin and Rustichini [2000], who find that zero is the only stable steady state, and that therefore any SS system will gradually shrink as time progresses.

In this version of our model, a high fertility rate, high inequality, and high degree of patience (β close to one), all contribute to making SS implementable ($x < 1$) since they imply that any level of future SS taxes is associated with higher levels of benefits. Successive decisive voters may therefore prefer to continue the SS system even if the sequence of SS tax rates is decreasing.

3.3 Fertility subsidies

We now allow for voting over fertility subsidies. We begin by noting that fertility subsidies affect the identity of the decisive voter in the next period, as described in the following lemma.

Lemma 6 *Higher (lower) fertility subsidies at* t *imply a higher (lower) endowment decisive voter at* $t + 1$.

Proof: Expression (13) shows the endowment level for the decisive voter household at time $t + 1$:

$$\alpha_{m,t+1} = \underline{\alpha} + (\overline{\alpha} - \underline{\alpha})\frac{n_t - 1}{2n_t}. \tag{20}$$

By (4), we have $\frac{\partial n}{\partial \tau^f} > 0$, *so* $\frac{\alpha_{m,t+1}}{\tau_t^f} > 0$

This result is illustrated in figure 3, where the endowment level is measured on the vertical axis. The horizontal axis represents the number of voters, where the number of old households is normalized to one and young households are ranked from poorest to wealthiest. As fertility increases from n to n', the endowment level of the decisive voter household increases from α_m to α'_m.

To derive the MPE we begin by considering the choice of $\{\tau_t^f, \tau_{t+1}^{ss}\}$, conditional on the SS system being continued. Using the expression for α_m in

(13), and with n_t given by (4), the problem of choosing $\{\tau_t^f, \tau_{t+1}^{ss}\}$ for young households becomes:

$$\max_{\{\tau_t^f, \tau_{t+1}^{ss}\}} \quad V^y(\tau_t^f, \tau_{t+1}^{ss}, \tau_t^{ss})$$

$$= (1-\tau_t^{ss})\alpha_i + \tfrac{\theta\beta\gamma\tau_{t+1}^{ss}}{(1-\tau_t^f)b} - \tfrac{\gamma}{1-\tau_t^f} \qquad +\gamma \ln \frac{\gamma}{(1-\tau_t^f)b} \tag{21}$$

$$s.t. \quad -\tau_{t+1}^{ss}\{\overbrace{\frac{(1-(1-\tau_t^f)b/\gamma)(\overline{\alpha}-\underline{\alpha})}{2} + \underline{\alpha}}^{\alpha_{m,t+1}}\} \quad + \frac{\theta\beta\gamma\widehat{\tau}_{t+2}^{ss}}{(1-\tau_{t+1}^f)b}$$

$$-\tfrac{\gamma}{1-\tau_{t+1}^f} + \gamma + \gamma \ln \tfrac{1}{1-\tau_{t+1}^f} = 0 \qquad \text{if } \tau_t^f > 1-\gamma/b \tag{22}$$

$$\tau_{t+1}^{ss} \leq 1 \qquad \text{otherwise} \tag{23}$$

This problem is similar to (15)-(17) except that households can now vote over the level of τ_t^f, and therefore can affect the identity of the decisive voter in the next period. Expression (22) is then $V_i^y(\tau_t^f, \tau_{t+1}^{ss}, \tau_t^{ss}) \geq V_i^y(0,0,0)$ for a decisive voter with endowment $\alpha_{m,t+1}$, as defined in expression (13).

Figure 2 illustrates the problem of choosing $\{\tau_{t+1}^{ss}, \tau_t^f\}$, where the first part of the constraint (equation (22)) has been rearranged to highlight that τ_{t+1}^{ss} is a function of $\widehat{\tau}_{t+2}^{ss}$. The problem has an interior solution only for $\widehat{\tau}_{t+2}^{ss}$ above a critical value. In the figure, $\widehat{\tau'}_{t+2}^{ss}$ is above this critical value, but $\widehat{\tau}_{t+2}^{ss}$ is not.

We discuss the MPE of this model in two steps, always assuming an interior solution. First, we derive the equilibrium in a model where agents do not anticipate the effects of their actions on the identity of the future decisive voter. This myopic version of the equilibrium will allow for a discussion of the first tradeoff in the choice of fertility subsidies: that of a higher tax base and therefore larger future SS benefits, against the deadweight loss of the subsidy. We then discuss the equilibrium in the model with fully rational voters, which will allow for a discussion of the second effect: that of lower future SS benefits due to a change in the identity of the decisive voter.

3.3.1 Myopic voters

In this subsection we study a MPE with myopic voters. In this equilibrium, derived formally in appendix A.2, voters solve the problem in (21) to (22) taking $\alpha_{m,t+1}$ in equation (22) as parametric. That is, they do not internalize the effect of their chosen fertility policy on the identity of next period's decisive voter. In an interior solution, with μ denoting the multiplier assigned to constraint (22), tax rates are obtained from the following first order conditions:

$$
\begin{aligned}
&(\tau^{ss}_{t+1}) \quad \tfrac{\theta\beta\gamma}{(1-\tau^f_t)b} - \mu\{\underline{\alpha} + \tfrac{(\overline{\alpha}-\underline{\alpha})}{2}(1-(1-\tau^f_t)b/\gamma)\} = 0 && (24)\\
&(\tau^f_t) \quad -\tfrac{\gamma}{(1-\tau^f_t)^2} + \tfrac{\theta\beta\gamma\tau^{ss}_{t+1}}{b(1-\tau^f_t)^2} + \tfrac{\gamma}{(1-\tau^f_t)} = 0 && (25)\\
&(\mu) \quad -\tau^{ss}_{t+1}\{\underline{\alpha} + \tfrac{(\overline{\alpha}-\underline{\alpha})}{2}(1-(1-\tau^f_t)\tfrac{b}{\gamma})\} && (26)\\
&\quad + \tfrac{\theta\beta\gamma\hat{\tau}^{ss}_{t+2}}{(1-\tau^f_{t+1})b} - \tfrac{\gamma}{1-\tau^f_{t+1}} + \gamma + \gamma\ln\tfrac{1}{1-\tau^f_{t+1}} = 0
\end{aligned}
$$

The first condition is actually redundant in determining the equilibrium, which can be obtained from equations (25) and (26).

Equation (25) governs the choice of τ^f_t: the first term represents the cost of paying for higher subsidy levels for each birth, and the third represents the utility gains from having more children. Together these terms are the (net) deadweight loss from subsidizing fertility, and the myopic voter weights this loss against the utility gain from higher future SS benefit levels, represented by the second term.

The last condition, equation (26), is the incentive compatibility condition, and says that tomorrow's voter must be at least indifferent between keeping and abandoning SS, given the chosen tax rates.

The MPE for this model is given by

$$\tau^{ss}_{t+1} = \frac{b}{\theta\beta}\{1-\exp\{-\frac{\tau^{ss}_t\alpha_{\tilde{m},t}}{\gamma}\}\} \tag{27}$$

$$\tau^f_t = 1-\exp\{-\frac{\tau^{ss}_t\alpha_{\tilde{m},t}}{\gamma}\} \tag{28}$$

Where $\alpha_{\tilde{m},t}$ is an equilibrium object that maps the current SS tax to the endowment of the current decisive voter, and is obtained from the definition of $\alpha_{m,t}$ (expression (13)) and the FOC (25):

$$\alpha_{\tilde{m},t} \equiv \underline{\alpha} + \frac{(\overline{\alpha}-\underline{\alpha})}{2}(1-(1-\frac{\theta\beta\tau^{SS}_t}{b})b/\gamma) \tag{29}$$

Equations (27) and (28) describe the equilibrium sequence of SS taxes and fertility subsidies respectively. Because $\frac{b}{\theta\beta}$ may be larger than one, the sequence in (27) may reach tax rates higher than one in finite time, in which case no equilibrium exists.

3.3.2 Rational voters

We now consider the model with voters who anticipate the effect of their choice of τ^f on the identity of the future decisive voter. The equilibrium is the solution to the same first order conditions for τ^{ss}_{t+1} and μ in the previous version (equations (24) and (26)), plus the following first order condition that governs the choice of τ^f_t:

$$-\frac{\gamma}{(1-\tau^f_t)^2} + \frac{\theta\beta\gamma\tau^{ss}_{t+1}}{b(1-\tau^f_t)^2} + \frac{\gamma}{(1-\tau^f_t)} - \mu\tau^{ss}_{t+1}\frac{b}{\gamma}\frac{(\overline{\alpha}-\underline{\alpha})}{2} = 0 \qquad (30)$$

Note that the first three terms are the same as in the previous example (equation (25)). The fourth term adds a further important effect from increasing τ^f_t. As fertility in one period increases, the endowment level of tomorrow's decisive voter household will also increase. Because the young at $t+1$ now form a larger constituency, a larger proportion of them will be needed to form a majority pro-SS together with the old. By Lemma 6, the decisive voter in the next period will now be a household with a higher endowment level than before. Because SS is redistributive, a higher endowment decisive-voter household at $t+1$ obtains lower net gains from participating in SS, so she will be indifferent between maintaining or abandoning the SS system at a *lower* SS tax τ^{ss}_{t+1} for each promised tax rate $\widehat{\tau}^{ss}_{t+2}$.

Starting from a common initial condition, anticipating this effect implies that both equilibrium fertility subsidies and SS taxes are lower in the economy with rational voters than in the economy with myopic voters. For fertility subsidies, the rational voter anticipates the negative effect on welfare of higher population growth, so the result is intuitive. For Social Security taxes, we can provide the following intuition: by construction, a decisive voter who is rational obtains higher utility benefits from choosing fertility, so she will need a lower future SS tax in order to become indifferent between keeping or abandoning the SS system.

Because a closed form solution for the MPE cannot be obtained in the case of rational voters, we compare the equilibria for the three models using numerical simulations. We choose two sets of plausible parameter values to illustrate what we believe are the interesting equilibria. [8] The laws of motion

[8] We choose the following parameter values for Example 1: $\beta = .15$, which implies an annual interest rate of 6.5% for 30 years. $b = .12$ and $\gamma = .56$, which implies a dependency ratio $\frac{\gamma}{b}$ of 4.7 in the absence of fertility subsidies, and $\underline{\alpha} = .4$, $\overline{\alpha} = 2.6$ for the distribution of endowments. Example 2 increases γ to .8

of SS taxes and fertility subsidies are plotted in figure 4. Figures 4A and B represent an equilibrium where SS is not viable without fertility subsidies. Figure 4A shows the law of motion for SS taxes against a forty-five degree line. The horizontal axis represents SS taxes at time t, and the vertical axis represents SS taxes at $t+1$. The crossed line shows the law of motion for the model without fertility subsidies. For these parameters, SS taxes reach values higher than one in finite time, so no SS system can be implemented. The dotted and continuous lines show the laws of motion for the models with myopic and rational voters respectively. Note that in both cases the SS tax converges to an interior steady state.

Figure 4B shows the dynamics of fertility subsidies for the economy with myopic voters (dotted line) and rational voters (full line) against a forty-five degree line. The horizontal axis again represents SS taxes at time t, the state variable, and the vertical axis represents the rate of fertility subsidies. Note that the strategic effect on the identity of tomorrow's decisive voter implies that chosen fertility subsidies are lower for rational voters, as expected.

Figures 4C and D show the same dynamics for an example where SS is valued in the absence of fertility subsidies. In this case, the only steady state for all three economies is zero, and both SS and fertility subsidy systems shrink with time. [9]

Note that, since the path of SS taxes is lower with rational voters than in the myopic case, an equilibrium with valued SS is more likely to exist when voters fully anticipate the effect of subsidies on the identity of the future decisive voter. In this sense SS can be said to become more sustainable' when the effect introduced in this section is internalized.

In our model, strategic setting of fertility subsidies provide a further tool which current young voters may use to improve their future SS benefits. That with higher subsidies the future decisive voter will be less prone to accept a high level of SS contributions is a purely political economy effect which operates besides the well understood effect of a higher tax base.[10]

[9] For alternative parameterizations, we found that the ranking of taxes across models remains unchanged, but the concavity of the MPE with fertility subsidies is not guaranteed.

[10] The limit case of this effect being dominant does not imply in any way the demise of the SS system. Rather, it implies that current voters -anticipating this effect- may choose to vote for net taxes to fertility, to shape a future constituency from which they can extract higher contributions.

4 Conclusion

In this paper, we developed a political economy model of Social Security and fertility subsidies, where young generations confront promises made previously by older generations and in turn promise themselves future levels of SS benefits. A trigger strategy, with the threat of abandoning SS, sustains the voting equilibria where SS is valued.

We highlighted that, when choosing to subsidize fertility, young generations not only increase the tax base in the future but at the same time also limit their own political influence. Strategic setting of subsidies to fertility to change the identity of the future decisive voter imply that both fertility subsidies and SS taxes are lower than otherwise. In other respects, we find that fertility subsidies can sustain Social Security in cases where it would otherwise have to be abandoned, mimicking the effects of other, better understood policy tools such as public education and migration.

Our paper complements an existing literature which suggests, as in Sinn [1997], that the SS sustainability problem could be addressed by linking pension benefits to fertility. We find that such reforms are likely to be politically sustainable. Moreover, even though the political economy effect we introduce dampens the positive effect of fertility on SS, the possibility of strategically manipulating this effect makes the fertility subsidy program an even more powerful tool to help sustain an unfunded SS program.

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A Appendix

A.1 The choice of taxes is independent of individual endowments

Once the SS system has been continued, the tax rates are chosen by solving:

$$
\begin{aligned}
\max_{\{\tau_t^f,\tau_{t+1}^{ss}\}} \quad & V^y(\tau_t^f,\tau_{t+1}^{ss},\tau_t^{ss}) \\
& = (1-\tau_t^{ss})\alpha_i + \tfrac{\theta\beta\gamma\tau_{t+1}^{ss}}{(1-\tau_t^f)b} - \tfrac{\gamma}{1-\tau_t^f} \quad +\gamma\ln\frac{\gamma}{(1-\tau_t^f)b} && (31)\\
s.t. \quad & -\tau_{t+1}^{ss}\alpha_{t+1}^m + \tfrac{\theta\beta\gamma\hat{\tau}_{t+2}^{ss}}{(1-\tau_{t+1}^f)b} \\
& -\tfrac{\gamma}{1-\tau_{t+1}^f} + \gamma + \gamma\ln\tfrac{1}{1-\tau_{t+1}^f} = 0 \quad \text{if } \tau_t^f > 1-\gamma/b && (32)\\
& \tau_{t+1}^{ss} \le 1 \quad \text{otherwise} && (33)
\end{aligned}
$$

Where α_{t+1}^m is a function of n_t (equation (13). This problem is equivalent to the modified problem where $(1-\tau_t^{ss})w\alpha_i$ is eliminated from the objective function:

$$
\begin{aligned}
\max_{\{\tau_t^f,\tau_{t+1}^{ss}\}} \quad & V^y(\tau_t^f,\tau_{t+1}^{ss},\tau_t^{ss}) - (1-\tau_t^{ss})w\alpha_i \\
& = \tfrac{\theta\beta\gamma\tau_{t+1}^{ss}}{(1-\tau_t^f)b} - \tfrac{\gamma}{1-\tau_t^f} \quad +\gamma\ln\frac{\gamma}{(1-\tau_t^f)b} && (34)\\
s.t. \quad & -\tau_{t+1}^{ss}\alpha_{t+1}^m + \tfrac{\theta\beta\gamma\hat{\tau}_{t+2}^{ss}}{(1-\tau_{t+1}^f)b} \\
& -\tfrac{\gamma}{1-\tau_{t+1}^f} + \gamma + \gamma\ln\tfrac{1}{1-\tau_{t+1}^f} = 0 \quad \text{if } \tau_t^f > 1-\gamma/b && (35)\\
& \tau_{t+1}^{ss} \le 1 \quad \text{otherwise} && (36)
\end{aligned}
$$

Because the endowment (α_i) does not play a role in this problem, the solution cannot be a function of it.

A.2 Derivation of the MPE for the model with myopic voters

In an interior solution, the MPE is obtained by maximizing the objective function in (21) subject to the constraint in (22):

$$\max_{\{\tau_t^f, \tau_{t+1}^{ss}\}} \quad V^y(\tau_t^f, \tau_{t+1}^{ss}, \tau_t^{ss})$$

$$= (1-\tau_t^{ss})\alpha_i + \tfrac{\theta\beta\gamma\tau_{t+1}^{ss}}{(1-\tau_t^f)b} - \tfrac{\gamma}{1-\tau_t^f} + \gamma \ln \tfrac{\gamma}{(1-\tau_t^f)b} \tag{37}$$

$$s.t. \quad -\tau_{t+1}^{ss}\{\overbrace{\frac{(1-(1-\tau_t^f)b/\gamma)(\overline{\alpha}-\underline{\alpha})}{2}+\underline{\alpha}}^{\alpha_{m,t+1}}\} + \tfrac{\theta\beta\gamma\hat{\tau}_{t+2}^{ss}}{(1-\tau_{t+1}^f)b}$$
$$-\tfrac{\gamma}{1-\tau_{t+1}^f} + \gamma + \gamma\ln\tfrac{1}{1-\tau_{t+1}^f} = 0 \tag{38}$$

To derive constraint (38), note that it represents the condition

$$V_m^y(\tau_{t+1}^f, \tau_{t+2}^{ss}, \tau_{t+1}^{ss}) \geq V_m^y(0,0,0) \tag{39}$$

Where the indirect utilities are those of the decisive voter at time $t+1$ and the arguments in the function on the left hand side are chosen optimally. From (10), this expression is equivalent to

$$(1-\tau_{t+1}^{ss})\alpha_{t+1}^m + \tfrac{\theta\beta\gamma\tau_{t+2}^{ss}}{(1-\tau_{t+1}^f)b} - \tfrac{\gamma}{1-\tau_{t+1}^f} + \gamma\ln\tfrac{\gamma}{(1-\tau_{t+1}^f)b} \tag{40}$$
$$\geq \quad \alpha_{t+1}^m - \gamma + \gamma\ln\tfrac{\gamma}{b} \tag{41}$$

Which, after some algebra, leads to condition (38).

To obtain the first order conditions (24) to (26), we take the expression for $\alpha_{m,t+1}$ in the constraint (38) as parametric, which affects the derivation of the FOC for τ_t^f, expression (25).

From (25) we obtain:

$$\tau_t^f = \tau_{t+1}^{ss}\frac{\theta\beta}{b} \tag{42}$$

Note that equation (26) can be written as

$$-\tau_{t+1}^{ss}\{\underline{\alpha} + \tfrac{(\overline{\alpha}-\underline{\alpha})}{2}(1-(1-\tau_t^f)\tfrac{b}{\gamma})\} \tag{43}$$
$$+\tfrac{\gamma}{1-\tau_{t+1}^f}(\tfrac{\theta\beta}{b}\tau_{t+2}^{ss} - \tau_{t+1}^f) + \gamma\ln\tfrac{1}{1-\tau_{t+1}^f} = 0$$

The second term of this expression is zero, which comes from leading expression (42) one period. Using the identity in (29), equation (43) becomes

$$\tau_{t+1}^{ss}\tilde{\alpha_{m,t+1}} = \gamma\ln(\frac{1}{1-\frac{\theta\beta}{b}\tau_{t+2}^{ss}}) \tag{44}$$

This leads to

$$\tau^{ss}_{t+2} = \frac{b}{\theta\beta}\{1 - \exp\{-\frac{\tau^{ss}_{t+1}\alpha_{\tilde{m},t+1}}{\gamma}\}\} \tag{45}$$

The MPE in (27) is this same expression lagged one period. The function characterizing the evolution of fertility subsidies can be obtained from (45) and (42).

Figure 1: Timing of decisions

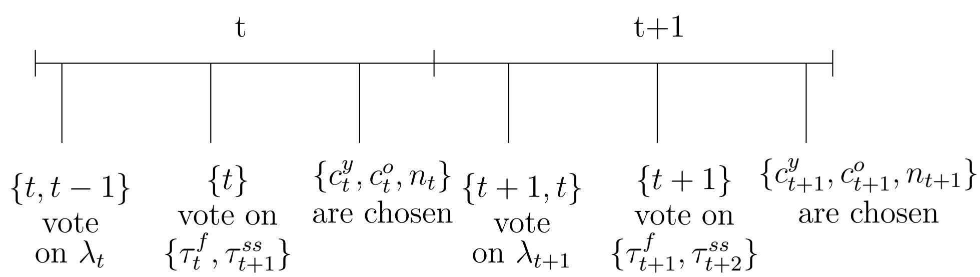


Figure 2: Choice of tax rates

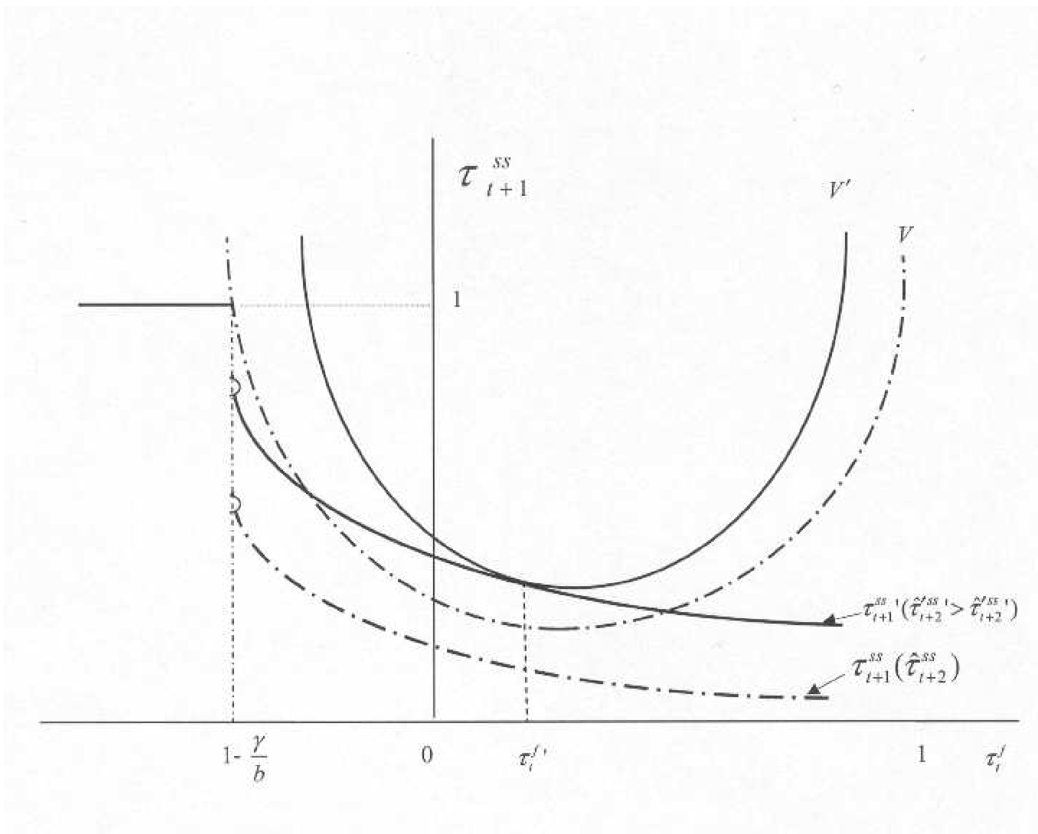


Figure 3: Who is the decisive voter

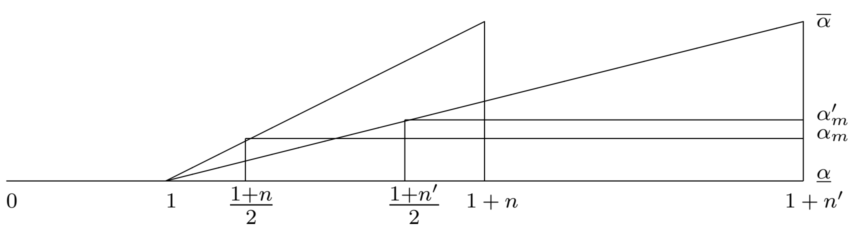
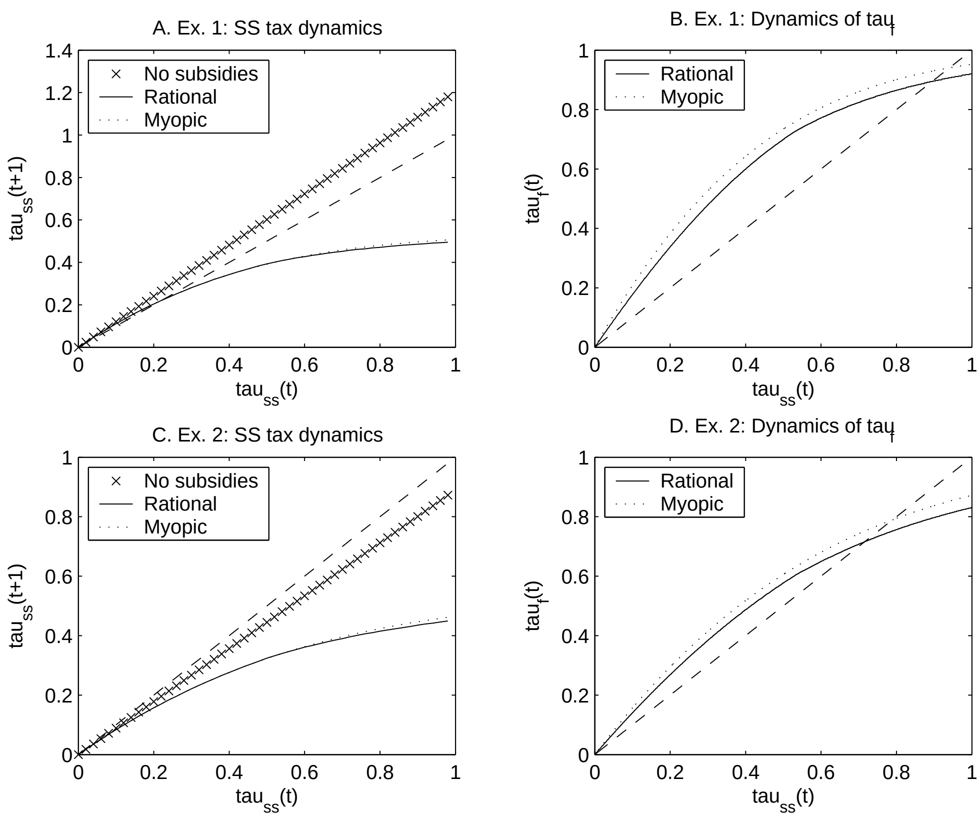


Figure 4: Tax rate dynamics



$Parameters\ Example\ 1: \quad \{\beta = .15, b = .12, \gamma = .56, \overline{\alpha} = 2.6, \underline{\alpha} = .4\}$
$Parameters\ Example\ 2: \quad \{\beta = .15, b = .12, \gamma = .8, \overline{\alpha} = 2.6, \underline{\alpha} = .4\}$