A Reliable Output Gap from the Beveridge-Nelson Decomposition Imposing a Low Signal-to-Noise Ratio

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Introduction

- The output gap is often conceived of as transitory movements in log real GDP at business cycle frequencies
- Methods like the Hodrick-Prescott (HP) filter that impose a low signal-to-noise ratio are convenient and produce intuitive results (similar to policymakers' estimates)
- But these methods are subject to a number of problems
  - "end point" problem for two-sided filters
  - "spurious cycles" in the presence of a stochastic trend

What we do

- Beveridge-Nelson (1981) trend-cycle decomposition, but with one major modification
- We impose a low signal-to-noise ratio for output growth (corresponds to a smoothing prior on trend)
- Our approach is easy to implement and produces intuitive estimates of the output gap
- Relatively small revisions over time and better out-of-sample forecasts of output growth and inflation than other methods

Greenbook and CBO

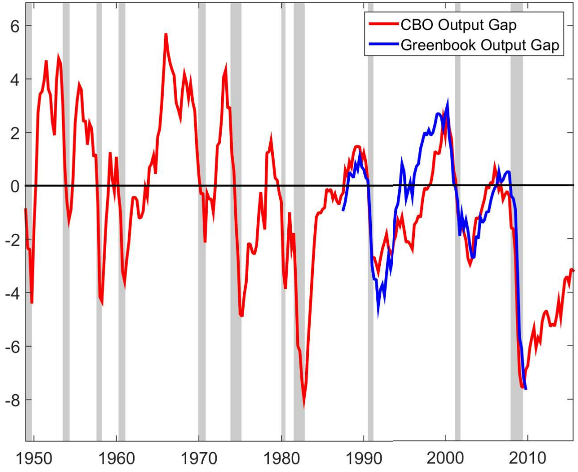
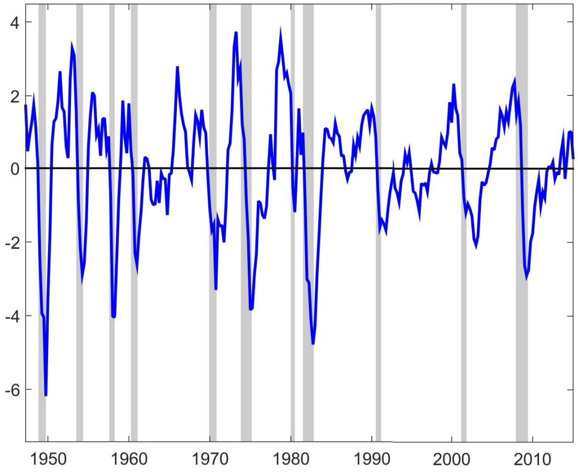


Figure: Cycle from HP (1600) US Real GDP



Beveridge-Nelson (BN) Decomposition

- *"Why not define trend as simply the long-horizon forecast? Rather than being fixed and pre-determined, this trend will shift as each new data point reveals new information about the future."* - Nelson (2008, JoE)
- In the long run, any current transitory dynamics are expected to die off and all that will be left is the trend
- Trend is the long-horizon conditional forecast (minus any deterministic drift) of a time series $\{y_t\}$:

$$\tau_t = \lim_{j\to\infty} \mathbb{E}_t \left[y_{t+j} - j \cdot \mathbb{E}\left[\Delta y\right]\right]$$

- Avoids spurious cycles given unbiased forecasts
- One-sided filter not subject to large revisions as long as the forecasting model is stable

Beveridge-Nelson (BN) Decomposition based on an estimated AR(1) Model

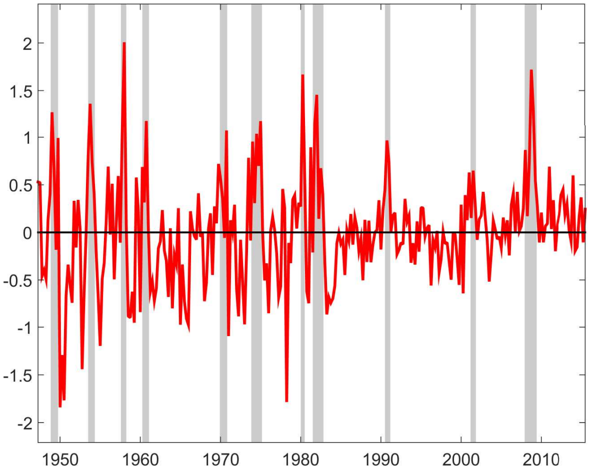


Figure: Real Time Gap, BN decomposition based on an estimated AR(1) model

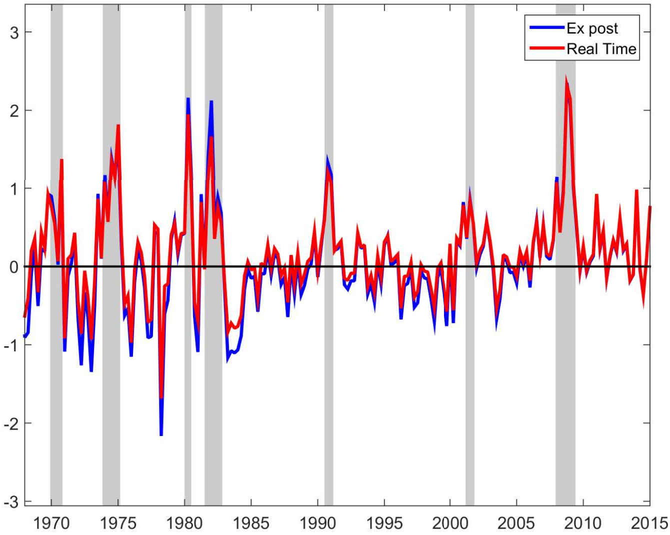
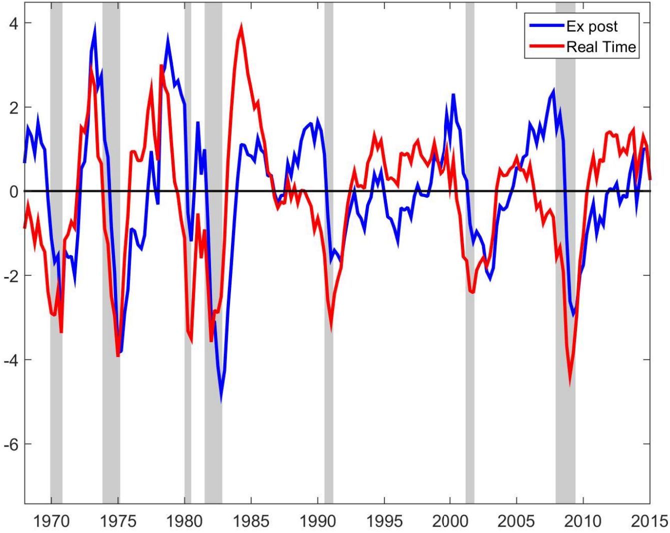


Figure: Real Time Gap, HP filter ($\lambda = 1600$)



Conceptually appealing and reliable, but...

BN decomposition based on an estimated AR(1) model produces an output gap that does not fit with most economists' beliefs

1. Amplitude is too small (i.e., implied trend is too volatile)
2. Does not capture any of the NBER recessions

Outline

1. Our proposed approach
2. Real-time evaluation (i.e., are the output gap estimates reliable?)
3. Ex post evaluation
4. Conclusions

The BN Decomposition

Suppose $\triangle y_t$ follows an AR(1) process, then

$$\begin{aligned}
(\triangle y_t - \mu) &= \phi(\triangle y_{t-1} - \mu) + \epsilon_t \\
\mathbb{E}_t[(\triangle y_{t+1} - \mu)] &= \phi(\triangle y_t - \mu) \\
\mathbb{E}_t[(\triangle y_{t+2} - \mu)] &= \phi^2(\triangle y_t - \mu) \\
\vdots &\quad \vdots \\
\mathbb{E}_t[(\triangle y_{t+j} - \mu)] &= \phi^j(\triangle y_t - \mu)
\end{aligned}$$

To work out the long run forecast for y_t, note that

$$\begin{aligned}
\sum_{i=1}^{\infty} \mathbb{E}_t[(\triangle y_{t+i} - \mu)] &= (\phi^1 + \phi^2 + \ldots)(\triangle y_t - \mu) \\
&= \phi(I - \phi)^{-1}(\triangle y_t - \mu)
\end{aligned}$$

The BN Decomposition

$$
\begin{aligned}
BN_t^{trend} &= y_t + \phi(I-\phi)^{-1}(\triangle y_t - \mu) \\
BN_t^{cycle} &= -\phi(I-\phi)^{-1}(\triangle y_t - \mu)
\end{aligned}
$$

Why does the BN decomposition based on an AR(p) model produce a counterintuitive output gap?

- According to MLE estimates, output growth is positively serially correlated with little persistence implying,
  1. cycle is small as $|\phi| \to 0$
  2. trend is volatile as $\phi > 0$

ϕ is linked to the signal-to-noise ratio ($\delta = \sigma^2_{\Delta\tau}/\sigma^2_{\epsilon}$)

The BN Decomposition

Large and persistent cycles with smooth trends are clearly at odds with the data from the perspective of a BN decomposition based on an estimated AR(1) model.

1. If we don't believe the BN cycle, then we have a prior about ϕ (possibly a very tight one)
2. We should be able to quantify this prior and compute cycle accordingly
3. Prior is linked to δ, the signal-to-noise ratio

Imposing a low signal-to-noise ratio

- We consider the BN decomposition, but place an implicit smoothing prior on trend when estimating forecasting model
- Bayesian estimation of an AR(p) model of output growth restricting the sum of AR coefficients
  - Corresponding signal-to-noise ratio $\delta = \sigma^2_{\Delta trend} / \sigma^2_{error}$ set to much less than one
  - Shrinkage prior (à la Minnesota) also placed on lags to avoid overfitting

Implementation

- Consider an AR(p) forecasting model:

$$\Delta y_t = c + \sum_{j=1}^{p} \phi_j \Delta y_{t-j} + e_t, e_t \sim N(0, \sigma_e^2)$$

- Let $\rho \equiv \phi_1 + \phi_2 + \ldots + \phi_p$ and $\phi_j^* \equiv -(\phi_{j+1} + \ldots + \phi_p)$
- Transform AR(p) model into its Dickey-Fuller representation:

$$\Delta y_t = c + \rho \Delta y_{t-1} + \sum_{j=1}^{p-1} \phi_j^* \Delta^2 y_{t-j} + e_t$$

- Define signal-to-noise ratio $\delta \equiv \sigma_{\Delta\tau}^2 / \sigma_e^2 = (1-\rho)^{-2}$
- For quarterly data, we set $\delta = 0.1$, fixing ρ accordingly

Figure: BN-Bayes estimate of US output gap for 1951-2014 ($p = 12$, $\delta = 0.1$)

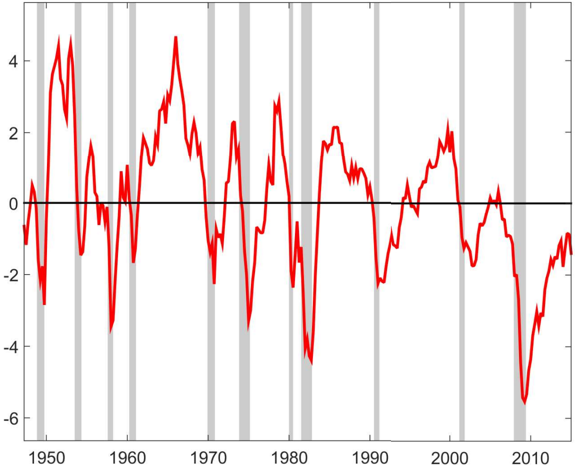


Figure: BN-MLE

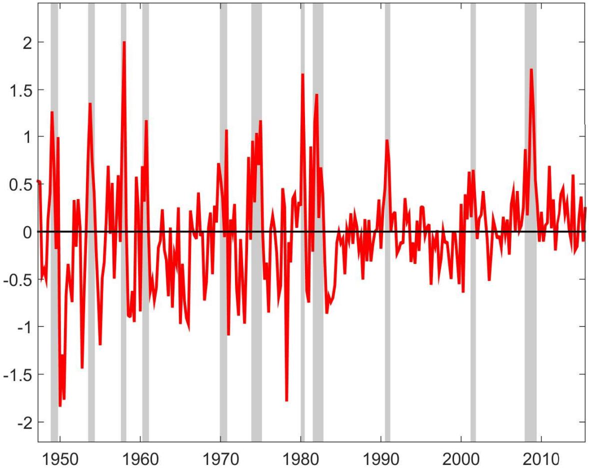
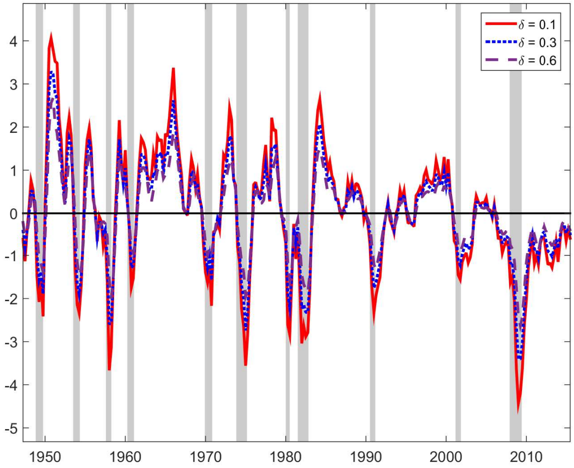


Figure: Different signal-to-noise ratios



Is it reasonable to restrict the signal-to-noise ratio?

- Signal-to-noise ratio decreases with more multivariate information (Evans and Reichlin, 1994)
- Economic slack is unobservable, which is why we are estimating the output gap

Figure: BN decomposition based on a VAR model ($\triangle Y, U$)

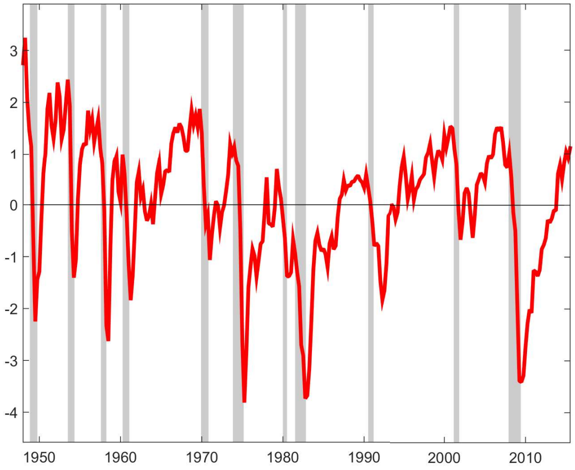


Figure: BN decomposition based on a VAR model ($\triangle$Y,CAPU)

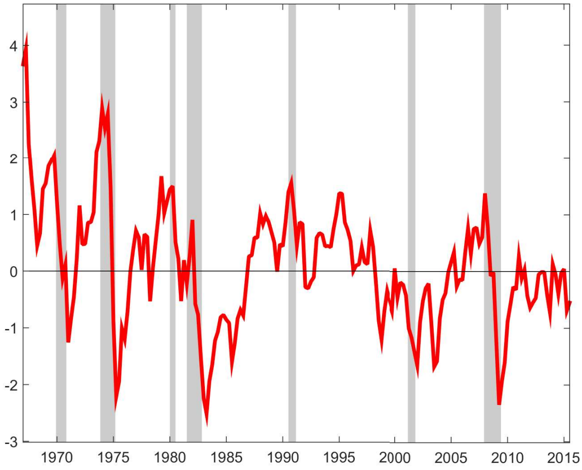
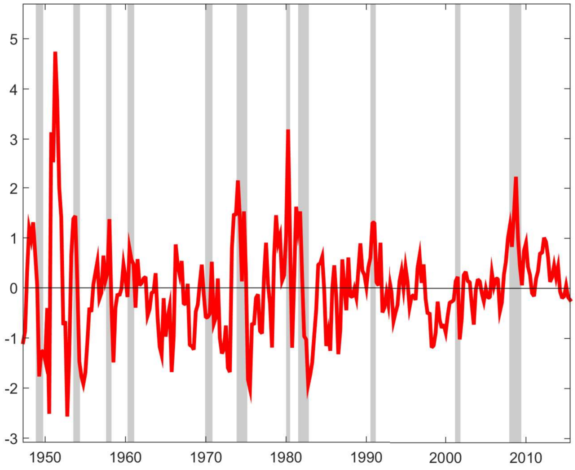


Figure: BN decomposition based on a VAR model (△Y,△C)



Shortcut

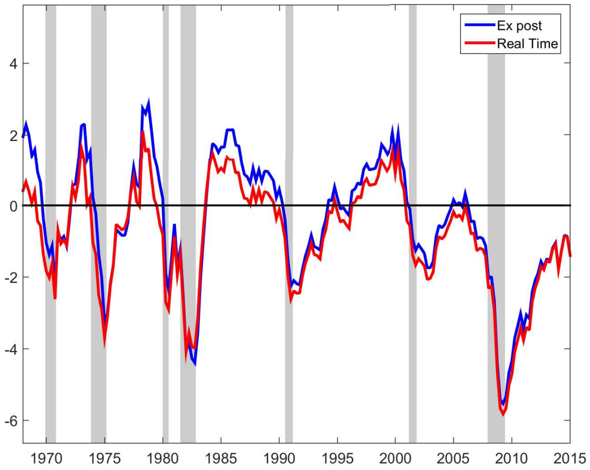
- Imposing a signal-to-noise ratio provides a shortcut to capturing beliefs about economic slack without needing the right multivariate information
- Easy to implement and works better than relying on observable multivariate information or MA terms

Real Time Evaluation of Reliability

- Why not use other methods that impose a low signal-to-noise ratio?
- Revisions and reliability (Orphanides and van Norden, 2002)
- Out-of-sample forecasts (expanding window estimation, with evaluation sample beginning in 1970Q1)
  - The output gap should forecast future output growth (Nelson, 2008)
    - $y_{t+h} - y_t = \alpha + \beta gap_t + \epsilon_t$, where $\beta < 0$
  - ARDL inflation forecast (Stock and Watson, 1999, 2008; Clark and McCracken, 2006)
    - $\pi_{t+h} - \pi_t = \gamma + \sum_{i=0}^{p} \theta_i \triangle \pi_{t-i} + \sum_{i=0}^{q} \kappa_i gap_{t-i} + \upsilon_t$

Revisions

Figure: Ex post versus real-time US output gap 1968-2015, BN-Bayes estimate ($p = 12$, $\delta = 0.1$)



Revisions

Figure: Ex post versus real-time US output gap 1968-2015, BN-Bayes estimate ($p = 12$, $\delta = 0.1$)

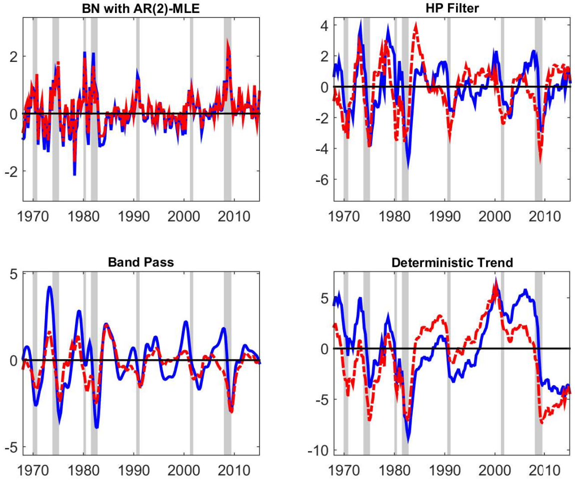


Table: Revision Statistics, 1970Q1 - 2012Q4

	Correlation	Size of Revisions		Same
		Std	RMS	Sign
BN Bayes	0.99	0.14	0.23	0.88
BN, AR(1) MLE	1.00	0.12	0.16	0.95
BN, AR(12) MLE	0.74	0.96	1.11	0.75
BN, ARMA(2,2) MLE	0.81	0.59	0.60	0.84
Harvey-Clark	0.75	0.66	1.00	0.66
Deterministic Trend	0.70	0.78	0.81	0.70
HP Filter	0.55	0.97	0.97	0.59
BP Filter	0.77	0.65	0.67	0.74

Notes: Correlation refers to the correlation of the psuedo-real time estimate to the final estimate of the output gap. Std and RMSE refers respectively to the standard deviation and root mean square of revisions to the psuedo-real time estimate of the output gap, with both subsequently normalised by the standard deviation of the final estimate of the output gap. Same sign refers to the proportion of psuedo-real time estimates which shares the same sign as the final estimate of the output gap.

Figure: Out of Sample Relative RMSE to BN-Bayes: $y_{t+h} - y_t = \alpha + \beta gap_t + \epsilon_t$

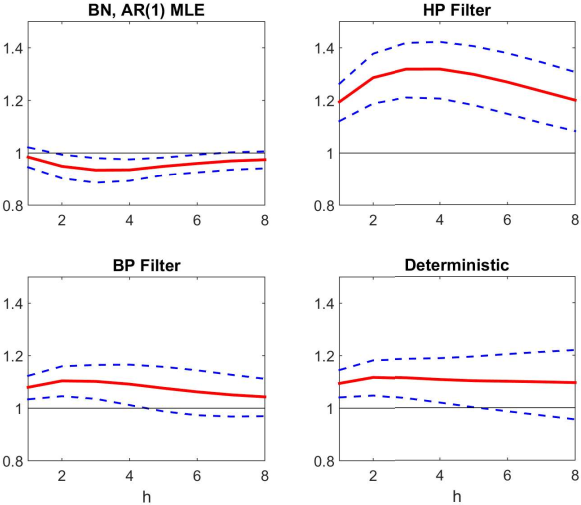


Figure: Out of Sample Relative RMSE to BN-Bayes:
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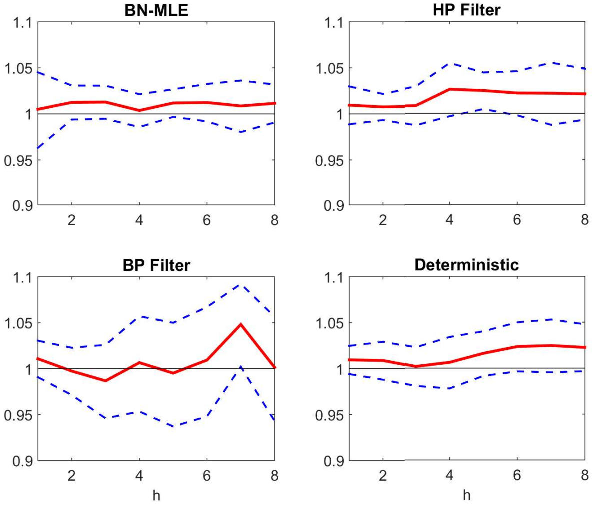
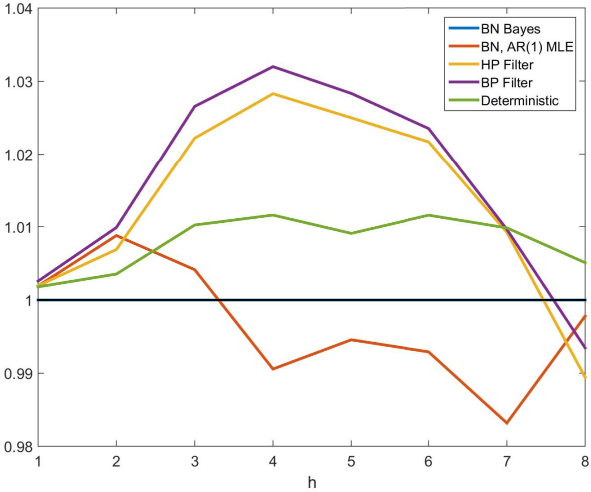


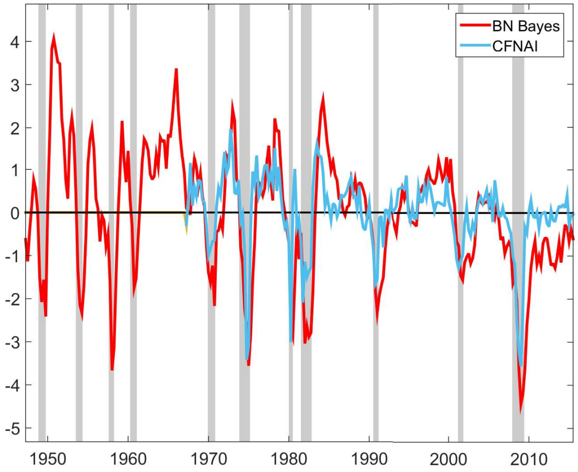
Ex Post Evaluation

- Are revisions useful even if they reduce reliability?

1. In sample inflation fit
2. Correlation with an independent measure of slack (Chicago Fed National Activity Index)

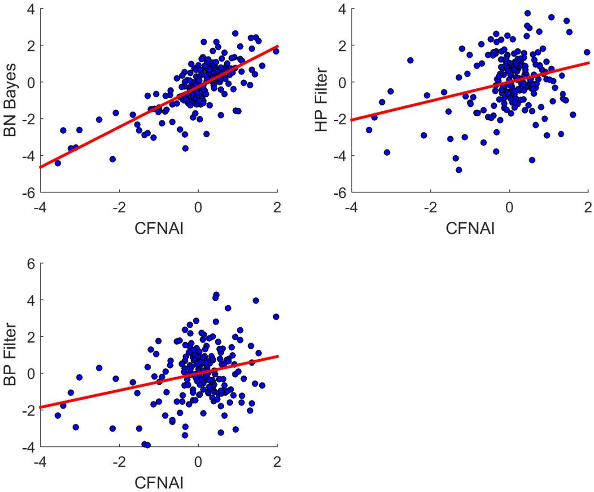
Figure: In Sample Inflation Fit Relative RMSE to BN-Bayes: $\pi_{t+h} - \pi_t = \gamma + \kappa gap_t + \upsilon_t$





BN Bayes
CFNAI
4
3
2
1
0
-1
-2
-3
-4
-5
1950
1960
1970
1980
1990
2000
2010

Figure: Scatter Plot against CFNAI



Conclusions

- When imposing a low signal-to-noise ratio, the BN decomposition can produce an intuitive and reliable estimate of output gap
  - Estimated output gap consistent with most economists' beliefs about amplitude, persistence, and direction of transitory movements in log real GDP
  - Estimated output gap are reliable in the sense of not being subject to large revisions