Jump Activity Analysis for Affine Jump-diffusion Models: Evidences from the Commodity Market*

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Abstract

The objective of this paper is to perform a joint analysis of jump activity for commodities and their respective volatility indexes. Exploiting the property of the affine jump-diffusion models that a volatility index quoted on the market is an affine function of the instantaneous volatility state variable, thus turning this quantity into an observable, we perform a test of common jumps for multidimensional processes to assess whether an asset and its volatility jump together. Applying this test to the crude oil pair USO/OVX and the gold pair GLD/GVZ we find strong evidence that for these two markets the asset and its volatility have disjoint jumps. The results are further confirmed by analyzing jump size distributions using a copula methodology.

JEL Classification: G12, G13, C14
Keywords: Affine jump-diffusion models, Volatility indexes, Jump activity, Model specification

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1 Introduction

The affine stochastic volatility model proposed by Heston (1993) and extended by Bates (2000) and Duffie et al. (2000) provides an affine framework that is the most frequently used modeling framework for equity derivatives products available to date. From the initial continuous time model to the very general specification of Duffie et al. (2000) several improvements were proposed along the way to address some specific empirical properties of equity dynamics. Jump activity is certainly an important aspect as shown by a vast amount of research focusing on this feature. In Bates (2000), asset jumps are shown to be essential to capture crash risk while Eraker et al. (2003) and Eraker (2004) underpin the importance of volatility jumps. Many studies emphasize the importance of jumps, either in the asset or the volatility dynamics, or both, jumps in the asset and volatility dynamics for the model to capture equity empirical properties. Indeed, the affine framework offers a wide range of possibilities to incorporate jumps in the asset dynamics. However, the majority of the models incorporate jumps that can affect either the asset, or the volatility alone, as well as jumps that affect simultaneously the asset and the volatility. Although there exist several possible choices to incorporate jumps within the affine framework, no specific model stands out in the literature due to the different type of tests that can be used. Quite surprisingly, models where the asset and the volatility jumps are not contemporaneous, thus, disjoint, are considered less often.[1]

A large variety of model specifications allowed by the affine framework and the implementations obtained so far can be explained by the evolution of the equity derivatives markets. Starting from the equity (index) options traded in the early eighties, followed by the emergence of volatility indexes in the early nineties, and now by an increase of actively traded volatility derivatives products such as VIX futures and VIX options, the financial derivatives markets have grown considerably. This, in turn, lead to a rich data environment that allows, and maybe also requires, more sophisticated modeling frameworks. Many research works have been directed towards modeling volatility and jumps, and pricing in the VIX markets (see e.g. Zhang and Zhu (2006), Zhu and Zhang (2007), Sepp (2008), Lian and Zhu (2013)). Along with the increase of the number of derivatives products, the greater availability of high frequency data for financial assets, thus enhancing granularity of the data, has also triggered a large body of works devoted to model identification. In particular, jump activity analysis has attracted a lot of attention among academics, enabling a very accurate understanding of the impact of jumps on assets, let us quote without pretending to be exhaus-

[1]The noticeable exception are models based on the Hawkes process that naturally introduce disjoint jumps, see Aït-Sahalia et al. (2015a) or Kokholm (2016).

tive, Barndorff-Nielsen and Shephard (2004), Huang and Tauchen (2005), Barndorff-Nielsen and Shephard (2006), Jacod and Todorov (2009) and Corsi et al. (2010).

There is no doubt that the S&P500 option market along with the VIX, its volatility index, were the precursors of the evolutions observed in the equity derivatives markets. It is worth noting that commodity markets followed the same trend, though, with a certain delay. Among all the commodity products that are actively traded, crude oil and gold are certainly the most important ones. Crude oil is in fact the world's most actively traded commodity, while gold is famous as a currency, and is also know as the ultimate "safe haven" asset. As a result, it comes as no surprise that these two assets also possess volatility indexes, thus mimicking the development of the pair S&P500/VIX. For the crude oil an actively traded asset is the USO (United States Oil Fund), which is an exchange-traded fund (ETF), while its volatility index, computed using the VIX methodology, is the OVX (Oil Volatility Index). For the gold an actively traded asset is the GLD (Gold), which is also an ETF, and its volatility index is the GVZ (Gold Volatility Index). While the precedence of the S&P500/VIX pair over the other assets may explain why the number of studies focusing on it is larger compared to those dealing with the crude oil and its volatility index or the gold and its volatility index, there exists a fair amount of research available for the commodity markets, see e.g., Geman (2005) and Chevallier and Ielpo (2013) for a general overview. However, if we restrict our consideration to the more specific problem of the use of affine models for the commodity market and its corresponding volatility market then the literature is relatively sparse. For example, Trolle and Schwartz (2009) consider a stochastic volatility model without jumps to analyze commodity options (i.e. the authors do not consider the corresponding volatility index) or Chiarella et al. (2013) and Chiarella et al. (2016) dealing with a continuous stochastic volatility model for the commodity futures market. Regarding models for commodity derivatives, some effort has been directed towards modeling the convenience yield, see for example Gibson and Schwartz (1990), Cortazar and Schwartz (2003) and Liu and Tang (2010), as it is a very specific and important feature for that market. This consideration partially explains why stochastic volatility models are less often considered in the literature as it adds another complexity to the modeling. The availability of volatility indexes for the commodity market along with the associated derivatives products will, however, impose the requirement to model stochastic volatility. As a result, regarding the problem of finding a suitable affine model for both, the commodity market and its associated volatility market, the question remains largely unexplored, and taking into account numerical difficulties that arise when analyzing jointly these two markets, it is better to first approach the problem with a simple but robust test strategy.

Our paper contributes to the literature by performing a model specification analysis of jump activity for the commodity markets using the affine framework and the methodology proposed by Jacod and Todorov (2009). We apply the joint jumps test to the crude oil market, given by the USO, and its associated volatility market, given by the OVX, as well as to the gold market defined by the pair GLD/GVZ. We find that asset jumps and volatility jumps are disjoint. We further analyze the jump size distributions using a copula approach and confirm the hypothesis of disjoint asset jumps and volatility jumps. Overall, our results point towards a model specification that is rather different from those frequently used in the option pricing literature. It also suggests the presence of a volatility idiosyncratic factor that justifies the existence of sophisticated volatility derivatives.

The paper is organized as follows. We present the key results for affine models with jumps, econometric results for jump detection as well as joint and disjoint jumps test specifications and some well-know properties of copulas in Section 2. A description of the empirical data used in our analysis along with some basic descriptive statistics are provided in Section 3. Empirical analysis is performed in Section 4 and robustness checks are discussed in Section 5. Section 6 provides final remarks and Section 7 concludes the paper.

2 Modelling Framework

2.1 Model specification

We denote by $(F_t)_{t\geq 0}$ the forward price of the underlying and $(y_t)_{t\geq 0} = (\log(F_t))_{t\geq 0}$ its log return. We assume that under the risk neutral probability Q it follows the dynamics

$$dy_t = -\frac{v_t}{2}dt + \sqrt{v_t}(\rho dw_t^{1,Q} + \sqrt{1-\rho^2}dw_t^{1,Q}) + J_t^{a,Q}dN_t^{a,Q} - \nu_t^{a,Q}dt + \bar{J}_t^{a,Q}dN_t^{c,Q} - \bar{\nu}_t^{a,Q}dt, \tag{1}$$

$$dv_t = \kappa^Q(\theta^Q - v_t)dt + \sigma\sqrt{v_t}dw_t^{1,Q} + J_t^{v,Q}dN_t^{v,Q} + \bar{J}_t^{v,Q}dN_t^{c,Q} \tag{2}$$

with $(w_t^{1,Q}, w_t^{2,Q})_{t\geq 0}$ being standard Brownian motions, $(\kappa^Q, \theta^Q, \sigma) \in \mathbb{R}_+^3$, $(N_t^{a,Q}, N_t^{v,Q}, N_t^{c,Q})_{t\geq 0}$ being a three dimensional counting process with intensities $(\lambda_t^{a,Q}, \lambda_t^{v,Q}, \lambda_t^{c,Q})_{t\geq 0}$ while the random jump sizes affecting the asset and its volatility are given by $J_t^{a,Q}, J_t^{v,Q}$, $\bar{J}_t^{a,Q}$ and $\bar{J}_t^{v,Q}$. Thus, the notation $(N_t^{a,Q}, J_t^{a,Q})$ is used to represent the situation when jumps are affecting the asset (a); $(N_t^{v,Q}, J_t^{v,Q})$ is describing the scenario when jumps are affecting the volatility (v); while ($N_t^{c,Q}$,$\bar{J}_t^{a,Q}$,$\bar{J}_t^{v,Q}$) is used to represent the case of the common jumps, that is, when jumps in the asset and the volatility are contemporaneous (c). The parameter ρ captures the correlation between shocks to returns and volatility. Negative ρ signifies presence of the leverage effect meaning that decreasing prices are associated with increasing volatility, refer to Chiarella et al. (2016), Larsson and Nossman (2011) and Kristoufek (2014). Leverage should not be understood in a sense of a firm's capital structure whereby with a non-zero debt structure a decrease in the value of the company will lead to an increase of volatility. This can explain why a commodity asset (or a commodity) and its volatility index might not jump together: While for an equity the capital structure creates a strong relationship between the equity and its volatility, in case of a commodity asset such strong dependency does not exist. Hence, it is more likely that a commodity and its volatility have disjoint jumps than an equity and its corresponding volatility index.[2]

As a result, the compensators for the jumps affecting the asset are given by $\nu_t^{a,Q} = E_t^Q \left[\exp(J_t^{a,Q}) - 1\right] \lambda_t^{a,Q}$ and $\bar{\nu}_t^{a,Q} = \mathbb{E}_t^Q \left[\exp(\bar{J}^{a,Q}) - 1\right] \lambda_t^{c,Q}$ with $\mathbb{E}_t^Q$ [.] denoting the time-t conditional expectation under Q (the conditioning is performed with respect to the filtration generated by the Brownian motion and the jump process). The jump sizes $J_t^{a,Q}$ and $\bar{J}_t^{a,Q}$ are often considered as Gaussian under Q, so that we have $\mathbb{E}_t^Q \left[\exp(J_t^{a,Q}) - 1\right] = g^{a,Q} = e^{\mu^{a,Q}+(\sigma^a)^2/2} - 1$, $\mathbb{E}_t^Q \left[\exp(\bar{J}_t^{a,Q}) - 1\right] = \bar{g}^{a,Q} = e^{\bar{\mu}^{a,Q}+(\bar{\sigma}^a)^2/2} - 1$ while $J_t^{v,Q}$ and $\bar{J}_t^{v,Q}$ are often chosen as exponentially distributed under Q and we have $\mathbb{E}_t^Q \left[J_t^{v,Q}\right] = \mu^{v,Q}$ and $\mathbb{E}_t^Q \left[\bar{J}_t^{v,Q}\right] = \bar{\mu}^{v,Q}$ as the jump distributions are independent of the filtration generated by the Brownian motion and the jump processes. The square root nature of the volatility process imposes some strong constraints on the volatility jump distributions; they must be positive, which also justifies the choice of the exponential law.

The proposed formulation covers many research works on option pricing within the affine framework. The majority of the literature assumes only common jumps, that is, $N_t^{a,Q} = N_t^{v,Q} = 0$, which implies that both, the asset and the volatility jump together through the common jump factor $N_t^{c,Q}$, but with different jump size distributions. The most frequently used distributions are the normal distribution for the (log-)asset while for the volatility it is the exponential distribution in order to be compatible with the required positivity of this process. Notice that the common jumps hypothesis enables us to introduce a correlation between the jump size distribution affecting the asset and the jump size distribution affecting the volatility. Most often, this dependency is captured by choosing the jump size distribution affecting the asset to be a

[2] We thank an anonymous referee for bringing to us this point of view.

distribution that is conditional on the jump size affecting the volatility. This specification was first proposed in Duffie et al. (2000). However, Duffie et al. (2000) allows for a much more general specification regarding jump activity and, in particular, nothing prevents from having jumps affecting the asset without impacting the volatility (i.e. $N_t^{a,Q} \neq 0$) and vice-versa (i.e. $N_t^{v,Q} \neq 0$). In that simpler case, it makes no sense to correlate the jumps impacting the asset and its volatility. As the purpose of Duffie et al. (2000) was to develop a very general pricing framework involving different sources of noise (i.e. Brownian motion shocks on the asset, shocks on its volatility, jumps affecting the asset and/or the volatility), the mathematical challenge was to make these shocks correlated. In that perspective, the common jumps hypothesis was required, and all subsequent research works used this model specification. However, it does not mean that the hypothesis of common jumps holds in practice.

Without pretending to be exhaustive and in addition to the works mentioned in the introduction, let us cite the works of Bates (1996), Bakshi et al. (1997), Pan (2002), Broadie et al. (2007), Yun (2011) and Luo and Zhang (2012) where jumps can occur either in the asset dynamics, or in the asset and the variance dynamics (simultaneously), and the jump sizes can either be constant or have a distribution compatible with the state variable that they affect (asset or volatility). However, the literature always assumes that either the asset jumps (but not the volatility), or both, the asset and the volatility jump together. These rather strong assumptions might be justified by the data available in the markets. Indeed, if only equity options are available with only few strikes and maturities, then it might be wise to only consider very parsimonious models and jumps in the volatility should be disregarded. On the other hand, the growth of the volatility market can justify the usage of more elaborated models. The VIX in the equity markets is by far the most well known volatility index and VIX futures that are essential tools to trade volatility, entered the market in March 2004 and experienced a rapid increase in trading volumes by 2010. In addition to this richer financial market environment, the availability of high frequency data certainly opens the way to test more sophisticated models, and in particular, allows us to develop a better understanding of the correct model specification.

In line with the evolution of the equity index option market and its volatility market counterpart (i.e. the S&P500 option market and the VIX derivatives market) the commodity market has followed a similar development, though with some lag. Among the different commodity products actively traded on the market, crude oil and gold are certainly the most prominent ones. As a result, it is not surprising for these two assets to have volatility indexes computed using the methodology employed for the VIX. The volatility indexes (OVX and GVZ), along with the underlying commodities (USO and GLD) are available and quoted at a high frequency thanks to the high liquidity observed on the crude oil and gold option markets. This rich data environment allows us to improve our knowledge of these two key markets and extend previous existing studies on affine models applied to commodity markets, such as e.g. Larsson and Nossman (2011), Arismendi et al. (2016) or Baum and Zerilli (2016).

As mentioned above, the rapid growth of the option market led to the creation of several volatility indexes; we will use the generic symbol VOL to denote a volatility index, which is computed using a continuum of call and put options on a given asset. Thanks to the results presented, among others, in Carr and Wu (2006), Aït-Sahalia et al. (2015b) and Bardgett et al. (2016), we can write the following relationship:

$$\text{VOL}_t^2 = \frac{1}{\tau}\mathbb{E}_t^Q\left[\int_t^{t+\tau} v_u du + 2\left(e^{J_u^{a,Q}} - 1 - J_u^{a,Q}\right)dN_u^{a,Q} + 2\left(e^{\bar{J}_u^{a,Q}} - 1 - \bar{J}_u^{a,Q}\right)dN_u^{c,Q}\right], \tag{3}$$

where τ is in practice 30 days (expressed in years).

Under standard assumptions made for affine jump-diffusion models, Eq.(3) possesses a very simple form and it is possible to prove that

$$\text{VOL}_t^2 = \beta_0 v_t + \beta_1 \tag{4}$$

with β_0 and β_1 being constants that depend on the model parameters (see Aït-Sahalia et al. (2015b)). For the purpose of our analysis we do not need to compute these constants explicitly, thus, we refer to the literature for their expressions. Regarding Eq.(4) the key remark is that the volatility index, given by the left hand side of this equation, is observable and thanks to this relationship, it makes the right hand side (and more precisely, v_t, the volatility process) observable, too. Thus, the volatility process is no longer a latent factor and it opens a very interesting possibility to perform model specification tests for affine models (since the relationships in Eq.(3) and Eq.(4) heavily exploit the affine property of the model). We notice that Eq.(3) and Eq.(4) hold under the risk neutral measure but since the econometric tests involve the dynamics under the historical probability measure, it remains to investigate the market price of risks.

In order to specify the market price of risk, we follow Aït-Sahalia et al. (2015b) and references therein that provide the standard choices for these quantities, often motivated by parsimonious consideration to ease the estimation procedure. For the Brownian motion we will assume that the market price of risk is given by $\Lambda_t = (\gamma_1\sqrt{(1-\rho^2)v_t}, \gamma_2\sqrt{v_t})^\top$ (with $\top$ denoting the transpose). Regarding the counting processes, Aït-Sahalia et al. (2015b) propose to set the jump intensities under the risk neutral and the historical measures identical for each of the processes, and further specify an affine form of the volatility process v_t. As result, we assume that $\lambda_t^{a,Q} = \alpha_0^s + \alpha_1^a v_t$, $\lambda_t^{c,Q} = \alpha_0^c + \alpha_1^c v_t$ and $\lambda_t^{v,Q} = \alpha_0^v + \alpha_1^v v_t$ with $(\alpha_0^a, \alpha_1^a, \alpha_0^c, \alpha_1^c, \alpha_0^v, \alpha_1^v)$ being all positive. Also, regarding the jump sizes we also follow Aït-Sahalia et al. (2015b) and assume that the jump distributions under P possess the same law as those under Q but with different parameters. More precisely, when we set $\mathbb{E}_t^P[e^{J_t^{a,P}} - 1] = g^{a,P} = e^{\mu^{a,P}+(\sigma^a)^2/2} - 1$, $\mathbb{E}_t^P[e^{\bar{J}_t^{a,P}} - 1] = \bar{g}^{a,P} = e^{\bar{\mu}^{a,P}+(\sigma^a)^2/2} - 1$, the jumps have the same variance but different mean values, while $\mathbb{E}_t^P[J_t^{v,P}] = \mu^{v,P}$ and $\mathbb{E}_t^P[\bar{J}_t^{v,P}] = \bar{\mu}^{v,P}$. All these hypotheses lead to an asset jump risk premium given by $(g^{a,P} - g^{a,Q})(\alpha_0^a + \alpha_1^a v_t) + (\bar{g}^{a,P} - \bar{g}^{a,Q})(\alpha_0^c + \alpha_1^c v_t)$ while for the volatility jump risk premium we have $(\mu^{v,P} - \mu^{v,Q})(\alpha_0^v + \alpha_1^v v_t) + (\bar{\mu}^{v,P} - \bar{\mu}^{v,Q})(\alpha_0^c + \alpha_1^c v_t)$. Therefore, under the historical probability measure the dynamics for (y_t, v_t) take the same form as in Eq.(1)-(2), and by analyzing the joint jumps activity of the asset and its corresponding volatility index we can draw conclusions on model specification regarding the important aspect of jumps.

2.2 Daily jump detection

The importance of jumps was actively analyzed during the past decade or so thanks to the availability of high frequency data. For the commodity market, Chevallier and Ielpo (2012), Sévi (2014) and Brooks and Prokopczuk (2013) provide interesting overviews. The first task consists in identifying the days for which the asset jumps and the days for which its corresponding volatility index jumps. To this end we will follow methodology proposed in Huang and Tauchen (2005) and report the main equations of the methodology when applied to the asset $y_t = \log(F_t)$. Define

$$r_{t,i} = y_{t,i\Delta} - y_{t,(i-1)\Delta}, \tag{5}$$

where $r_{t,i}$ refers to the i^{th} intra-day return on day t, with Δ being the sampling frequency within each day such that $m = 1/\Delta$ observations occur every day and as $\Delta \to 0$ we have that $m \to \infty$.

Barndorff-Nielsen and Shephard (2004) propose two measures for quadratic variation process namely, the realized variance (RV) and the realized bipower variation (BV) that converge uniformly as $\Delta \to 0$ to different quantities of the jump diffusion process such as

$$RV_t = \sum_{i=1}^{m} r_{t,i}^2 \rightarrow \int_{t-1}^{t} v_u du + \int_{t-1}^{t} ({J_u}^{a,P})^2 dN_u^{s,P} + \int_{t-1}^{t} ({\bar{J}_u}^{a,P})^2 dN_u^{c,P}, \tag{6}$$

$$BV_t = \frac{\pi}{2}\frac{m}{m-1}\sum_{i=2}^{m} |r_{t,i}||r_{t,i-1}| \rightarrow \int_{t-1}^{t} v_u du. \tag{7}$$

As it is evident from Eq.(6) and Eq.(7), the difference between the realized variance and the realized bipower variation is zero when there is no jump and strictly positive when there is a jump. For detecting jumps, we adopt the ratio test, proposed in Huang and Tauchen (2005) and Andersen et al. (2007), where the test statistic

$$RJ_t \equiv \frac{RV_t - BV_t}{RV_t} \tag{8}$$

is an indicator for the contribution of jumps to the total within-day variance of the process. This test statistic converges in distribution to a standard normal distribution when using an appropriate scaling

$$ZJ_t = \frac{RJ_t}{\sqrt{\left\{\left(\frac{\pi}{2}\right)^2 + \pi - 5\right\}\Delta \max\left(1, \frac{TP_t}{BV_t^2}\right)}} \rightarrow N(0,1). \tag{9}$$

In Eq.(9) TP_t is the tripower quarticity that is robust to jumps; it is defined in Barndorff-Nielsen and Shephard (2004) as

$$TP_t \equiv m\mu_{4/3}^{-3}\frac{m}{m-2}\sum_{i=3}^{m} |r_{t,i-2}|^{4/3}|r_{t,i-1}|^{4/3}|r_{t,i}|^{4/3} \rightarrow \int_{t-1}^{t} v_u^2 du, \tag{10}$$

where

$$\mu_k \equiv 2^{k/2}\frac{\Gamma\left((k+1)/2\right)}{\Gamma\left(1/2\right)}, \quad k > 0.$$

Assuming that there is at most one jump per day and that jump size dominates the return when a jump occurs (Andersen et al. (2007)), daily realized jump sizes can be obtained as

$$\hat{J}_t = \text{sign}(r_t) \times \sqrt{(RV_t - BV_t) \times I_{(ZJ_t \geq \Phi_{\tilde{\alpha}}^{-1})}}, \tag{11}$$

where $\Phi(\cdot)$ is the cumulative standard normal distribution function with $\tilde{\alpha}$ being the level of significance and $I_{(ZJ_t \geq \Phi_{\tilde{\alpha}}^{-1})}$ is an indicator function which takes the value of one if there is a jump on a given day, and zero otherwise.

Once the realized jumps have been established, we can compute the jump mean $\hat{\mu}_J$, the variance $\hat{\sigma}_J$ and intensity $\hat{\lambda}_J$ as follows

$$\hat{\mu}_J = \text{Mean of } \hat{J}_t, \tag{12}$$

$$\hat{\sigma}_J = \text{Standard deviation of } \hat{J}_t, \tag{13}$$

$$\hat{\lambda}_J = \frac{\text{Number of jump days}}{\text{Number of trading days}}. \tag{14}$$

It has been shown in Huang and Tauchen (2005) that such an approach for estimation of realized jump parameters is robust with respect to the drift and the diffusion function specifications. It makes it easy to specify the jump arrival rate, avoiding sophisticated estimation methods, and yields reliable results under various settings, for instance, when the sample size is either finite, increasing or shrinking.

It is essential to notice that if we consider both, Eq.(2) and Eq.(4), the methodology can also be applied to detect jumps of the process $(v_t)_{t\geq 0}$ using the observations VOL_t. As result, despite the fact that v_t is a latent factor if we only observe the asset $(F_t)_{t\geq 0}$, the availability of a pure volatility product such as VOL_t allows us to turn the volatility $(v_t)_{t\geq 0}$ into an observable process. This, in turn, allows us to greatly simplify the test of whether the asset and its volatility jump together as we can use the methodology proposed by Jacod and Todorov (2009) that we now present.

2.3 Joint and disjoint jumps test specifications

Given the relationship in Eq.(4) and the dynamics in Eq.(1)-(2) that is also valid under the historical probability measure P (but with different parameters), it seems natural to apply the test for common arrival of jumps proposed by Jacod and Todorov (2009) to the following variables:[3]

$$x_t^1 = \log F_t,$$

$$x_t^2 = \text{VOL}_t^2,$$

where (F_t) is the forward price of the underlying. We denote the vector $x_t = (x_t^1, x_t^2)$. We fix the time horizon T and assume that the processes x_t^1 and x_t^2 are observed over a given time interval $[0, T]$ at discrete times $i = 0, 1, ..., [T/\Delta]$ where Δ is some small time lag, with $\Delta \to 0$. Since in our empirical analysis we will

[3]We notice that one could alternatively consider $x_t^1 = F_t$, i.e. not to apply the log-transformation to the underlying series.

compute test statistics for each day, we set $T = 1$. We can then consider the increments of those processes:

$$\Delta_i x^1 = \log F_{i\Delta} - \log F_{(i-1)\Delta}, \tag{15}$$

$$\Delta_i x^2 = \text{VOL}^2_{i\Delta} - \text{VOL}^2_{(i-1)\Delta}. \tag{16}$$

We follow the notation of Jacod and Todorov (2009) and define

$$V(f,\Delta)_T = \sum_{i=1}^{[T/\Delta]} f(\Delta_i x) \tag{17}$$

$$V(f,k\Delta)_T = \sum_{i=1}^{[T/k\Delta]} f(x_{ik\Delta} - x_{(i-1)k\Delta}), \tag{18}$$

with $f(x) = (x^1x^2)^2$ and the integer k in Eq.(18) is set $k = 2$. The rejection regions C^j for the null of the joint jumps (H_0 : Jumps in the time series occur at the same time (common jumps)) is specified as a function of the following test statistic:

$$\Phi_n^{(j)} = \frac{V(f,k\Delta)_T}{V(f,\Delta)_T}. \tag{19}$$

Here, the superscript (j) in the test statistic $\Phi_n^{(j)}$ stands for "joint". To specify the rejection region C^d for the null of the disjoint jumps (H_0 : Jumps in the time series do not occur at the same time (disjoint jumps)), we have the following test statistic:

$$\Phi_n^{(d)} = \frac{V(f,\Delta)_T}{\sqrt{V(g_1,\Delta)_T V(g_2,\Delta)_T}}, \tag{20}$$

where $g_1(x) = (x^1)^2$, $g_2(x) = (x^2)^2$ and the superscript (d) in the test statistic $\Phi_n^{(d)}$ stands for "disjoint".

In order to determine the critical values, we introduce the following notation:

$$\hat{A}'_T = \frac{1}{\Delta}\sum_{i=1}^{[T/\Delta]} f(\Delta_i x)\mathbf{1}_{\{|\Delta_i x^1| \le \alpha_1 \Delta^{\bar{\omega}}, |\Delta_i x^2| \le \alpha_2 \Delta^{\bar{\omega}}\}}, \tag{21}$$

where we choose $\alpha_1 > 0$, $\alpha_2 > 0$, and $\bar{\omega} \in (0, 1/2)$ arbitrary (here, we set $\bar{\omega} = 0.49$). For the parameters α_1, α_2, the choice is made following the Remark 5.4 on p. 1807 in Jacod and Todorov (2009). Thus, we set $\alpha_i = 3\sqrt{\overline{BV_i}}$, where $\overline{BV_i}$ denotes an average BV for series i with $i = 1$ representing the commodity index, and $i = 2$ representing the corresponding volatility index. We further define

$$\hat{F}_T = \frac{1}{2\tilde{k}\Delta}\sum_{i=1+\tilde{k}}^{[T/\Delta]-\tilde{k}-1}\sum_{j\in I(i)}\left(\left(\Delta_i x^1 \Delta_j x^2\right)^2 + \left(\Delta_i x^2 \Delta_j x^1\right)^2\right)\mathbf{1}_{\{M_1 \cap M_2\}} \tag{22}$$

with $\tilde{k}$ chosen in such a way that $\tilde{k}\Delta \to 0$ (here, we choose $\tilde{k} = [1/\sqrt{\Delta}]$); $M_1 = \{|\Delta_i x^1| \geq \alpha_1 \Delta^{\bar{\omega}}\} \cup \{|\Delta_i x^2| \geq \alpha_2 \Delta^{\bar{\omega}}\}$, $M_2 = \{|\Delta_j x^1| \leq \alpha_1 \Delta^{\bar{\omega}}\} \cap \{|\Delta_j x^2| \leq \alpha_2 \Delta^{\bar{\omega}}\}$ and $I(i) = \{i - \tilde{k}, i - \tilde{k} + 1, \ldots, i - 1\} \cup \{i + 2, i + 3, \ldots, i + \tilde{k} + 1\}$.[4] In addition, we denote

$$\hat{F}'_T = \frac{2}{\tilde{k}\Delta} \sum_{i=1+\tilde{k}}^{[T/\Delta]-\tilde{k}-1} \sum_{j \in I(i)} \Big(\left(\Delta_i x^1 \Delta_j x^2\right)^2 \times \left(\Delta_i x^1 \Delta_j x^2 + \Delta_i x^2 \Delta_j x^1\right)^2 \Big) \mathbf{1}_{\{M_1 \cap M_2\}}. \tag{23}$$

Finally, we set

$$\hat{V}^{(j)} = \frac{\sqrt{\Delta(k-1)\hat{F}'_T}}{V(f,\Delta)_T} \tag{24}$$

for the common jumps test, and

$$\hat{V}'^{(d)} = \frac{\Delta(\hat{F}_T + \hat{A}'_T)}{\sqrt{V(g_1,\Delta)_T V(g_2,\Delta)_T}} \tag{25}$$

for the disjoint jumps test. The corresponding critical values are given by

$$c^{(j)} = \hat{V}^{(j)}/\sqrt{\alpha} \tag{26}$$

for the common jump test and

$$c^{(d)} = \hat{V}'^{(d)}/\alpha \tag{27}$$

for the disjoint jumps test. In Eq.(26)-(27) α represents the significance level, which will be specified in Section 4 (Empirical Results).[5] Finally, the critical region where the null of common jumps is rejected is determined as

$$C^{(j)} = \{|\Phi^{(j)} - 1| \geq c^{(j)}\}. \tag{28}$$

Similarly, the critical region where the null of disjoint jumps is rejected is determined by

$$C^{(d)} = \{\Phi^{(d)} \geq c^{(d)}\}. \tag{29}$$

Remark 2.1 *Let us stress the fact that Jacod and Todorov (2009)'s test can be applied to any kind of stochastic differential equations (some constraints on the coefficients are required, though). In particular, they do not have to be of the affine type. Also, the increments (i.e. $\Delta_i x^1$ and $\Delta_i x^2$) do not have to be serially uncorrelated. However, if we wish to restrict our consideration to a model that is tractable for both,*

[4] Note, to guarantee a positive index, $I(i)$ is only defined for $i > \tilde{k}$.

[5] Note, the significance level α is not related to the choice of the parameters α_1, α_2 specified above.

the commodity (or equity index) option market and the volatility index option market, the affine framework is unavoidable. As an example, the model proposed on page 8 by Mencía and Sentana (2013), or Eq.(3.3) of Todorov and Tauchen (2011) that describes the dynamics of the logarithm of the asset's volatility using an Ornstein-Uhlenbeck process does not lead to a tractable pricing formula for the option written on that asset. In fact, this is the model proposed by Scott (1987) and it is well known, at least since the early nineties, for not admitting a closed form solution for option prices.

2.4 Copula dependency

This section focuses on the copula methodology, the main concepts of dependence modelling and techniques for copula estimation. It will be used to quantify the dependence between jump sizes affecting the underlying and jump sizes affecting the respective volatility index.

Copulas are multivariate distribution functions connecting d one-dimensional uniform-(0,1) marginals to a joint cumulative distribution. According to *Sklar's theorem*, if F_X is a d-dimensional distribution function for the vector $X = (X_1, \dots, X_d)^\top$ with marginals $F_1 \dots, F_d$, then under some general conditions there exists a copula C with

$$F_X(x_1, \dots, x_d) = C\{F_1(x_1), \dots, F_d(x_d)\} \tag{30}$$

for every $x_1, \dots, x_d \in \overline{\mathbb{R}}$.

Throughout the paper we will concentrate on two popular copula families: the *elliptical copulas family* and the *Archimedean copulas family*. Elliptical copulas have a dependence structure generated by elliptical distributions such as normal or Student-t. The Gaussian copula generates the dependence structure given by the multivariate normal distribution. In the case of the normal marginals, that is, if $X_j \sim N(0,1)$ and $X = (X_1, \dots, X_d)^\top \sim N_d(0, \Psi)$, where Ψ denotes a correlation matrix, the explicit expression for the Gaussian copula is given by

$$C_\Psi^{Ga}(u_1, \dots, u_d) = F_X\{\Phi^{-1}(u_1), \dots, \Phi^{-1}(u_d)\}, \tag{31}$$

where Φ^{-1} is the quantile function from the univariate standard normal distribution. Thus, combining normal marginals by using the Gaussian copula leads to the multivariate normal distribution. Note that the Gaussian copula can be used with any other marginal distribution (in which case the resulting multivariate distribution will not be normal).

The Student-t copula generates the dependence structure from the multivariate Student-t distribution. If $X = (X_1, \ldots, X_d)^\top \sim t_d(\nu, \mu, \Sigma)$, i.e. X has a multivariate Student-t distribution with ν degrees of freedom, mean vector μ and positive-definite covariance matrix Σ, the Student-t copula is given by

$$C^t_{\nu,\Psi}(u_1, \ldots, u_d) = t_{\nu,\Psi}\{t_\nu^{-1}(u_1), \ldots, t_\nu^{-1}(u_d)\}, \tag{32}$$

where $t_{\nu,\Psi}$ is the multivariate Student-t distribution with a correlation matrix Ψ and ν degrees of freedom, and t_ν^{-1} is the quantile function from the univariate t-distribution with ν degrees of freedom.[6] Modelling dependence by using elliptical distributions can be found e.g. in McNeil et al. (2005).

In our empirical analysis we will also use the Gumbel and Clayton copulas that belong to the family of Archimedean copulas. The Clayton copula with the dependence parameter $\theta \in (0, \infty)$ is defined by

$$C_\theta(u_1, \ldots, u_d) = \left\{ \left(\sum_{j=1}^{d} u_j^{-\theta} \right) - d + 1 \right\}^{-1/\theta}. \tag{33}$$

As the copula parameter θ goes to infinity, the dependence becomes maximal and as θ goes to zero, we have independence.

The Gumbel copula with the dependence parameter $\theta \in [1, \infty)$ is given by

$$C_\theta(u_1, \ldots, u_d) = \exp\left[- \left\{ \sum_{j=1}^{d} (-\log u_j)^\theta \right\}^{1/\theta} \right]. \tag{34}$$

For $\theta = 1$ it reduces to the product copula (i.e. independence): $C_\theta(u_1, \ldots, u_d) = \prod_{j=1}^d u_j$. Maximal dependence is achieved when θ goes to infinity.

Note that the literature suggests different approaches to the copula estimation. In our empirical analysis, we estimate copulas using the *inference for marginals* (IFM) method. The IFM suggests to estimate the marginal parameters in the first step, and then substitute them into a copula to obtain the pseudo log-likelihood function, which is then maximized with respect to the copula dependence parameter θ. The two-step procedure employed by this method makes it computationally efficient. For details on the IFM and the alternative estimation techniques refer to Joe (1997).

[6] Since copula functions remain invariant under strictly increasing transformations of X (e.g. standardisation of the marginal distributions), see Nelsen (1998), the copula of a $t_d(\nu, \mu, \Sigma)$ distribution is identical to that of a $t_d(\nu, 0, \Psi)$.

3 Data Description

In this study we consider commodity exchange-traded funds (ETFs) and the respective volatility indexes for the crude oil (Tickers: USO/OVX) for the period from 15/07/2008 to 18/11/2015 and gold (Tickers: GLD/GVZ) for the period from 04/08/2008 to 18/11/2015. The chosen time frames provide the longest series available from SIRCA.[7] Both underlying commodities (USO and GLD) are the ETFs quoted on New York Stock Exchange (NYSE) Arca; the options used to build the corresponding implied volatility indexes (OVX and GVZ) are traded on Chicago Board Options Exchange (CBOE). More precisely, the options written on the USO are used to compute OVX using methodology underlying the construction of the VIX. Thereby, all the out-of-the money put and call options with maturities under 30 days and maturities between 30 and 60 days (i.e. those lying within the next maturity bracket) are utilised. Similar construction method applies to gold. In our analysis we are restricted to use these commodities as the respective volatility indexes are the longest time series that are readily constructed and available from CBOE. Indeed, a long time frame is required to ensure that the sample contains sufficient number of days on which both series, the ETF and its volatility index jump, in order to guarantee the validity of statistical test results. If we were interested in analyzing other commodities, we would have to construct volatility indexes from options written on those commodities using the methodology developed for the VIX. In that case, most likely, the options would be quoted on NYMEX. In our case, the data are provided by the CBOE and are easily available, which avoids the time consuming and error prone construction of volatility indexes. We restrict our computations to 5-minute interval quotes from 10:00 am to 4:00 pm as it is well known that this sampling frequency avoids microstructure noise effects that can cause biases in the estimation of the realized volatility.

The USO is an ETF that tracks the WTI light sweet crude oil, the fund mainly invests in the front running WTI crude oil futures traded on the NYMEX. In terms of liquidity we report in Figure 1 the 1-month moving averages in dollars of the daily trading activity for the USO for the period considered in this work along with the same quantity for the front running WTI light sweet crude oil futures in order to ease the comparison. In both cases the values are stable over time, and in dollar terms the USO is approximately 50 times smaller (the average values for the USO and WTI crude oil futures are $418,581,702$ and $22,426,584,137$ USD, respectively). Although the USO displays much smaller values we need to keep in mind that the funding cost of a futures position is much smaller than the funding cost for an ETF position. For works that underpin

[7] See `http://www.sirca.org.au/`

the importance of the OVX for predicting WTI light sweet crude oil futures see Chevallier and Sévi (2013) and Haugom et al. (2014). The correlation between the USO and the WTI light sweet crude oil futures is high but not perfect. The discrepancies might occur between the returns when there are jumps. However, it is not really our concern as our objective is to analyze the joint behavior of the USO and its volatility index, the OVX index.

[Insert Figure 1 here]

A legitimate concern arising with high frequency data is the impact of sampling frequency on the key quantities that are the realized variance (RV) given by Eq.(6) and the realized bipower variation (BV) given by Eq.(7). It can be visualized by considering the signature plots, that is to say, the RV and BV computed as a function of the sampling frequency. It is known that sampling at a frequency higher than five minutes often has a tremendous impact on these two quantities. In Figure 2 we show signature plots for RV and BV along with a signature plot for the median RV for the pair USO/OVX. Consistently with known results, whether we consider the USO or the OVX, all the quantities stabilize when the sampling frequency is 5 minutes (300 seconds) or larger. If we restrict our consideration to the USO our results are consistent with those obtained in Chevallier and Sévi (2012) for the front running futures contract on the WTI light sweet crude oil.

[Insert Figure 2 here]

[Insert Table 1 here]

Table 1 shows summary statistics for the log-returns of the underlying (defined in Eq.(15)) and the corresponding differences of squares for the volatility (defined in Eq.(16)) for both pairs of commodities and volatility indexes. These include means, standard deviations, skewness, kurtosis, minimum and maximum, computed first for each day using 5-minute log-returns (for the indexes) and difference in squares (for the volatility indexes), and then averaging the resulted quantities across all days in the sample. We observe a negative relationship between the average return and the average standard deviation for the pairs of USO/OVX and GLD/GVZ, attributed to the leverage effect, i.e. with a drop in the underlying, the fear of market crashing kicks in, and volatility rises. Volatility index for the crude oil exhibits higher variability compared to gold, which justifies gold's reputation as the ultimate "safe haven" asset, which is especially important during the crisis periods, see Baur and McDermott (2010). We further notice that gold ETF (i.e. GLD) is positively skewed, which is different from equity indexes. It indicates that large declines in demand do not decrease prices as sharply as large increases in demand can increase them at the right tail of the price distribution. USO, on the contrary, has negative skewness, which points out more extreme negative returns observed in the crude oil market. Large skewness in volatility indexes, OVX and GVZ, indicates high volatility movements occurring in the crude oil and gold markets. Notice, however, that the mean level of the volatility, if restricted to the same period (16/03/2011 to 20/11/2015), corresponds to 31.90% and 19.03% per annum for OVX and GVZ, respectively.[8] Large positive kurtosis for both, ETFs and volatility indexes suggests non-normality of the data. Finally, high variability in the data is evident from the large range between the minimum and the maximum values. In particular, we confirm our previous observation that the volatility movements in the crude oil market are more pronounced compared to the volatility movements in the gold market.

[Insert Figure 3 here]

Figure 3 shows evolution of USO and GLD ETFs over the considered time period. We notice a sharp drop in the ETF value for the crude oil at the beginning of the sample period (year 2008), which is consistent with the start of the Global Financial Crisis (GFC). At the same, the value of the ETF for gold exhibits a minor increase towards the end of 2008, indicating that when the fear of financial or energy markets crashing kicks in, the demand for gold surges, which in turns generates an increase in the gold share price. GLD continues to increase until end of 2011 (end of the GFC), which reinforces again the identification of gold as a "safe haven" asset. During the post-GFC stage (from 2012 until end of the sample) gold share price experiences a slight decrease. Crude oil prices have been fairly stable up until mid-2014, but then started to decrease reaching its absolute minimum for the first time since 2008. The reason for this drop is manifold: weak demand for oil due to insipid economic growth, in combination with surging US production, and OPEC's[9] decision not to cut production as a way to prop up prices.

[Insert Figure 4 here]

[8] Note, these quantities are computed by annualizing daily volatilities.

[9] Organization of the Petroleum Exporting Countries, see `http://www.opec.org`.

Figure 4 shows annualised volatility indexes for the crude oil (OVX) and gold (GVZ). We observe that the volatilities range between 10% to 100% per annum. We confirm our previous results regarding higher variability in the crude oil market, compared to the gold market.

4 Empirical Results

In this part we develop the empirical analysis regarding joint jumps for the pairs USO/OVX and GLD/GVZ using the methodology outlined in the previous sections. As the graphs for the two pairs are similar we will report only those for the USO/OVX and refer to the online supplementary material for those related to the pair GLD/GVZ.

4.1 Daily jump activity

To perform a joint analysis of ETF and its volatility index jump activity we first need to identify the days for which both series jump. To this end we use Huang and Tauchen (2005)'s approach presented in Section 2.2. We compute daily realized variance (RV_t) and realized bipower variation (BV_t) using Eq.(6) and (7), respectively. From these two quantities we extract daily jumps using Eq.(11) (where we set $\tilde{\alpha} = 0.95$). Whenever J_t is non-zero, there is a jump in the underlying time series. While jumps in the index returns can be either positive or negative, we restrict jumps in volatility to be positive. This is consistent with the model specification in Eq.(1) - (2) that imposes a constraint on the sign of the volatility jumps.

[Insert Figure 5 here]

Figure 5 shows in the left panel the jumps sizes (J_t's extracted from Eq.(11)) in USO (top panel) and OVX (bottom panel) across the entire series (1852 observations). The right panel shows jump sizes for days where both series, USO and OVX, jump (49 observations).

[Insert Table 2 here]

In Table 2 we report in lines (i) the total number of days in the samples under consideration; (ii) the number of days when the ETF jumps regardless of whether the corresponding volatility index jumps or not; (iii) the number of days when volatility index jumps (positively) regardless of whether the corresponding ETF jumps or not. For the USO/OVX pair, there are 393 days over 1852 for which the ETF jumps regardless of the evolution of the volatility index while there are 183 days for which the volatility jumps with no constraints on the evolution for the ETF. For the GLD/GVZ pair the corresponding numbers are 427 and 220 out of 1838. Already from these numbers we can conclude that many jumps in the ETF and its volatility do not occur simultaneously. It is a first evidence that the usual common jump hypothesis is not supported by the commodity market data.

Further in that direction, if we consider the USO/OVX pair, the days for which the ETF jumps but not its volatility index corresponds to 276, given by line (iv) in Table 2, while there are 134 days with jumps in the volatility but not in the asset (line (v) in Table 2). For the pair GLD/GVZ, these numbers correspond to 300 and 158. As a result, there are quite a few days for which jumps occur only in one of the markets (i.e. either ETF or volatility index).

[Insert Table 3 here]

We report in Table 3 the descriptive statistics for the jumps in ETF returns and the squared volatility index returns. These include mean jump sizes, volatility of the jump size and jump intensity. Panel A reports the results computed separately for the days where jumps occur in the commodity ETF (the number of those days is reported in line (ii) of Table 2) and days when jumps occur in the volatility index (the number of those days is reported in line (iii) of Table 2). On average, we observe positive jumps for USO and GLD returns with jumps in USO returns being more than four times larger in magnitude (5.52×10^{-5}) compared to GLD returns (1.31×10^{-5}). The jump direction for commodity indexes is consistent with the sign of the mean returns reported in Table 1 (positive for USO and GLD). The intensity of jumps in the ETF returns are 0.2122 for USO and 0.2323 for GLD. For the volatility indexes we observe that the mean jump sizes and respective standard deviations correspond to (66.3433/212.9379) for OVX and (15.3733/39.5035) for GVZ. These results are consistent with high volatilities (standard deviations) documented for OVX, compared to GVZ, reported in Table 1 and Figure 4. Jump intensity for volatility indexes ranges from 0.0988 for OVX to 0.1197 for GVZ. Panel B of Table 3 reports the results computed for the days on which both series, the commodity ETF and the corresponding volatility index, jump (the number of the joint jumps days is reported in line (vi) of Table 2). We observe that the average jumps for USO (-0.0014) and GLD

(-7.62×10^{-4}) are negative and of low intensity (0.0265 for USO; 0.0337 for GLD).[10] The average jump sizes in the volatility index correspond to 40.787 for OVX to 11.9312 for GVZ. The lowest mean jump sizes and the lowest standard deviation are observed for GVZ reinforcing again the identification of gold as a ”safe haven” asset.

4.2 Intraday joint and disjoint jumps activity

Although for many days the ETF and its volatility do not jump together, there are a few days for which jumps occur in both markets. Table 2 line (vi) reports the number of those days, which corresponds to 49 for the USO/OVX pair and 62 for the GLD/GVZ pair. The next step is to determine whether jumps occur at the same time. To this end, we follow the methodology described in Section 2.3 for the joint and disjoint jumps test specification, and investigate whether each ETF and its corresponding volatility index exhibit jumps occurring simultaneously. For all jump tests we set the significant level $\alpha = 5\%$ in Eq.(26) and Eq.(27).

[Insert Figure 6 here]

[Insert Figure 7 here]

Figure 6 shows using a black solid line the test statistic $C_n^j = |\Phi^{(j)} - 1|$ computed from Eq.(19) (and used to determine the critical region in Eq.(28)) to test the null of joint jumps in the times series of USO and OVX; the red dotted line represents the corresponding critical value $c^{(j)}$ computed from Eq.(26). We notice that the value of the test statistic exceeds the critical value for the majority of cases. More precisely, the number of rejections of the null corresponds to 81.63%, which is reported in Table 4.

[Insert Table 4 here]

This rejection rate for the pair GLD/GVZ reaches 85.48%, indicating that the joint jumps in the ETF and respective volatility index occur only in 18.37% and 14.52% of all cases for USO/OVX and GLD/GVZ, respectively. Figure 7 shows the distribution of the test statistic (top panel) and the critical value (bottom

[10] This result has been already reported based on Table 2. We also notice that jump intensities for commodity ETFs and respective volatility indexes are identical, as we assume that jumps in the ETF and the volatility index occur on the same day.

panel) estimated in a non-parametric way. Given n sample points $(x_{t_1}, ..., x_{t_n})$, a kernel estimator for the probability density $f(x)$ is defined by

$$\hat{f}(x, h_n) = \frac{1}{nh_n} \sum_{i=1}^{n} K\left(\frac{x - x_{t_i}}{h_n}\right), \tag{35}$$

where $K(\cdot)$ is a kernel function and h_n is a bandwidth parameter controlling the degree of smoothness of the estimator. We use a Gaussian kernel given by $K(z) = (2\pi)^{-1/2} \exp\{-\frac{1}{2}z^2\}$ and the 'rule-of-thumb' bandwidth, which suggests to set $h_c = 1$ in bandwidth ($h_n = h_c \hat{\sigma} n^{-1/5}$), which also depends on the dispersion of the observations $\hat{\sigma}$ and the total number of data points n, see e.g. Epanechnikov (1969). We observe from Figure 7 that despite having a long right tail, the distribution of the test statistic indicates that the majority of values concentrate in range 0 to 5, which is much above the critical values ranging from 0 to 2.5, leading to a rejection rate of 81.63%.

[Insert Figure 8 here]

[Insert Figure 9 here]

Figure 8 shows the test statistic $\Phi_n^{(d)}$ from Eq.(20) (black solid line) and the critical value $c^{(d)}$ computed from Eq.(27) (red dotted line) to test the null of jumps arriving disjointly in USO and OVX. The bottom panel is a zoomed-in version of the top panel. We observe that the value of the test statistic is below the critical value for the majority of cases, which results in a relatively low rejection rate corresponding to 6.12% (reported in Table 4). For the pair GLD/GVZ this value corresponds to 3.12%. In other words, we conclude that joint jumps in the ETF and respective volatility index occur only in 6.12% and 3.12% of all cases for USO/OVX and GLD/GVZ, respectively. Figure 9 shows distribution of the test statistic (top panel) and the critical value (bottom panel) estimated in a non-parametric way. Again we confirm that the critical values typically fall in range between 0 to 19, thus, substantially exceeding the values of the test statistics taking values in the range between 0 to 1, leading to a low rejection rate of only 6.12%.

Overall, the test results prove our conjecture that for the days when both series jump, the jumps in the ETF and the respective volatility index tend to occur disjointly.

4.3 Joint jumps size analysis

The final stage of the empirical analysis investigates whether jump sizes in the ETF and the respective commodity volatility index follow some dependence structure. Again, we consider only those days where both series experience a jump, i.e. 49 days for the pair USO/OVX and 62 days for the pair GLD/GVZ.

[Insert Table 5 here]

We report in Table 5 a linear correlation coefficient computed for the jump sizes in ETF and its volatility, as well as copula dependence parameter for the four copula models discussed in Section 2.4. Thereby, we use the IFM procedure for the estimation of copula parameters, assuming an empirical distribution for the marginals (i.e. the ETF and the volatility index). We observe nearly zero correlation (0.0993) between the jump sizes for the commodity pair USO/OVX. Copula parameter, which also measures non-linear dependencies in the time series, also points towards independence (recall that for the Clayton copula independence is achieved when the copula parameter reaches zero, while for the Gumbel case independence is achieved when the dependence parameter reaches one). This result is consistent with the fact reported above that most of the jumps occurring in the time series are disjoint. For the pair GLD/GVZ we observe a weak negative dependence measured by the linear correlation coefficient (it corresponds to -0.14); as well as Gaussian and Student-t copulas, while Clayton and Gumbel copulas point towards independence. This is also in line with the results in Eraker et al. (2003) and Chernov et al. (2003) that estimate double-jump models restricting them to joint arrival of jumps in the underlying and its volatility but show, however, that the correlation between the jump sizes in the price and the volatility is not statistically different from zero.

[Insert Figure 10 here]

Finally, Figure 10 shows a scatter plot and a histogram for the jump sizes in USO and OVX for the days where both series jump. Again, we confirm that there is no clear dependence structure between the jump sizes in USO and OVX.

Overall, our results from the analysis of dependence between jump sizes, in conjunction with analyzing intraday joint and disjoint jumps activity, indicate that most of the jumps occurring (even though on the same days) in the time series of ETFs and respective volatility indexes, are disjoint in their nature.

5 Robustness Checks

The model specification test used in this work combines the jump days identification test proposed by Huang and Tauchen (2005), to detect days when the asset jumps or its volatility jumps, and Jacod and Todorov (2009)'s test to determine whether jumps in both time series occur at the same time. A natural question is the robustness of our conclusions with respect to a change of the parameters controlling these tests. Also, as there are many alternatives to Huang and Tauchen (2005)'s test we perform a robustness check with respect to a change of the jump day identification technique. Lastly, the sampling frequency may affect the results although we were cautious and took a five-minute sampling frequency, which is known to avoid micro-structure noise and that fact was already confirmed by the signature plots presented earlier. Nevertheless, we can still verify that our conclusions are robust to a change of the sampling frequency. As these robustness checks involve many tables, for conciseness we only report the main conclusions and delegate the numerical values to the online supplementary material.

The first robustness check that we perform is with respect to a changing value of $\tilde{\alpha}$ that appears in Eq.(11) in Huang and Tauchen (2005)'s test. It defines the threshold for the difference between the realized variance and the bipower realized variation above which a jump is detected. One would expected that an increase of the value $\tilde{\alpha}$ reduces the number of jump days, so Jacod and Todorov (2009)'s test will be performed on fewer days. We find that for both pairs the conclusions remain valid, that is, (i) the rejection rate for the joint jumps hypothesis is high; (ii) the rejection rate for the disjoint jumps hypothesis is low; (iii) the percentages of rejections are of same magnitude as those found initially. Rather naturally, we find that sampling at higher frequency will increase the number of jumps days, and applying Jacod and Todorov (2009)'s test leads to the same conclusions. A low sample frequency (i.e. 10 minutes) can be rather problematic as the number of days available to perform Jacod and Todorov (2009)'s test might become too low. The extreme case being a 10-minute sampling frequency combined with a large value for $\tilde{\alpha} = 0.99$ leads to 6 days for which both, the USO and the OVX jump and 11 days for which both, the GLD and the GVZ jump. In this case, Jacod and Todorov (2009)'s test is applied to only very few days and statistics might become unreliable. In any case, changes of the sampling frequency and changes of $\tilde{\alpha}$ do not alter the conclusion: the joint jumps hypothesis is most often rejected while the disjoint jumps hypothesis is most often not rejected for both pairs.

The second robustness check consists in replacing Huang and Tauchen (2005)'s test with either Jiang and

Oomen (2008) or Andersen et al. (2012) to find the days for which both, the asset and its volatility jump and then use Jacod and Todorov (2009)'s test to determine whether both time series jump at the same time. We choose these two alternatives among many others available, see Dumitru and Urga (2012) for an overview. As for the previous case, we can also change the sampling frequency and $\tilde{\alpha}$, which controls the magnitude of the difference between the realized variance and the bipower variation implying a jump. Notice also that Andersen et al. (2012) leads to two jump detection tests, one based on the minimum realized variance and minimum quarticity estimators, the other based on the median realized variance and median quarticity estimators, see online supplementary material for further details. Lastly, the sampling frequency can also be changed; we will consider beyond the usual 5-minute sampling frequency also 1-minute and 10-minute sampling frequencies to provide an exhaustive robustness check. As in the previous cases, changes in the sampling frequency and $\tilde{\alpha}$ have the expected impacts on the results. More importantly, all the tests lead to the same conclusions: the joint jumps hypothesis is rejected most of the time, with a rejection rate ranging from 80% to 95% approximately, while the disjoint jumps hypothesis is most often not rejected, with a non-rejection rate ranging from 88% to 100% approximately. Notice that a low sampling frequency combined with a high value of $\tilde{\alpha}$ leads to a very small number of days for which both, the asset and its volatility jump, thus turning Jacod and Todorov (2009)'s test unreliable (we disregarded the values for those cases). There are no noticeable differences between the USO/OVX and the GLD/GVZ pairs. Obviously, there are some discrepancies between the test results but not to the point that we can consider that they convey different conclusions.

6 Final Remarks

Overall, the results suggest that the commodities considered in this work and their corresponding volatility indexes do not jump together if we look at the data at high frequency, that is, the common jump hypothesis is too strong. This conclusion seems to contradict the results in Todorov and Tauchen (2011) where the common jump hypothesis seems to be satisfied for the S&P500 and a different sample period (their sample starts in September 2003 and ends in December 2008, so there is nearly no overlap with ours). Indeed, they found that VIX jumps and S&P500 returns jumps occur at the same time. Notice that Bakshi et al. (2000) found that index call prices do not always move in the same direction with the underlying index while Kapetanios et al. (2014) found that S&P500 E-mini futures options and the underlying futures do not jump at the same time (this latter work uses a different methodology than ours). In order to sort out an answer it might be wise to return to the definition of a volatility index. Indeed, the relation in Eq.(3) is more a mathematical or ideal relation, when it comes to operational computations the volatility index value is given by

$$\mathrm{vol}^2 = 100^2 \times \left(T_1\sigma_1^2 \frac{n_{T_1} - n_{30}}{n_{T_2} - n_{T_1}} + T_2\sigma_2^2 \frac{n_{30} - n_{T_1}}{n_{T_2} - n_{T_1}}, \right) \frac{n_{365}}{n_{30}} \tag{36}$$

where n_{30} is the number of minutes in 30 days, n_{T_1} is the number of minutes to expiration of the near term options (the near term, denoted T_1, is the largest maturity smaller than 30 days), n_{T_2} is the number of minutes to expiration of the next term options (the next term, denoted T_2, is the smallest maturity larger than 30 days), n_{365} is the number of minutes in 365 days and the quantities $\{\sigma_i^2; i = 1, 2\}$ are given by

$$\sigma_i^2 = \frac{1}{T_i} \sum_j \frac{\Delta k_j}{k_j} e^{rT_i} Q(k_i) - \frac{1}{T_i} \left(\frac{F_i}{k_0} - 1 \right)^2, \tag{37}$$

where r is the risk free rate, $Q(k_i)$ is the (mid) price of the option with strike k_i (only out-of-the money options are considered), $\Delta k_j = (k_{j+1} - k_{j-1})/2$, F_i is the forward price for the maturity T_i and k_0 is the strike below the forward price. The relation in Eq.(36) clarifies the problem. Indeed, for the asset and its volatility to jump together one requires that all the options, or at least many of the options involved in the formula, jump with the asset. This property was not found by Kapetanios et al. (2014) who document that the option price drivers are unrelated to jumps in the underlying asset price, which also complements the result of Bakshi et al. (2000) who find that index option prices call (or put) do not always move in the same (or opposite) direction with the underlying index; and option movements differ across strikes and maturities. The formula in Eq.(36) also suggests that jumps in volatility are equivalent to observing simultaneous jumps in many options, and are less likely to occur than asset jumps, a fact that is consistent with the results in Table 2 (and all the other tables reporting jump frequency statistics). So the assumption of simultaneous jumps in the asset and its volatility dynamics might just be too strong. In fact, there might be a delay between an asset jump and the adjustments affecting all the options involved in the volatility index. Nevertheless, even if we consider this weak relationship, jumps in the asset and jumps in the volatility do not occur on the same days most of the time. This result points towards a presence of volatility idiosyncratic factor, that is to say, not spanned by the asset's shocks, and it certainly justifies the appearance of non-linear volatility products such as volatility options. As certain shocks mainly impact, or impact first, the volatility market while others mainly impact, or impact first, the commodity market, it confirms the statement made earlier that the volatility is an asset class by itself with its own specific drivers. It would be of interest to characterize or identify the differences between the jumps affecting the volatility market and those affecting the commodity market. Also, as these two markets are related through an arbitrage argument their evolutions are related, it can create some potential problems to market participants involved in hedging of derivatives. Indeed, a shock affecting one market (i.e. commodity or volatility) will have an impact on the other market (i.e. volatility or commodity) but with some delay. This delay can be exploited by high frequency traders as they know how hedgers have to adjust their hedging strategies and, therefore, they can front run them. The impact of high frequency trading on the hedging of derivatives is a problem that remains largely unexplored and jumps play a crucial role on that problem as they induce significant changes in the hedging positions that can be anticipated (with the concomitant aspect of illiquidity).

Some very recent research works started to look at the drivers specific to each market. Kapetanios et al. (2014) who document disjoint jumps in the asset and its volatility (implied by the call and put options), propose two possible explanations that drive option price jumps but do not impact the asset price; these include (i) macroeconomic news releases and (ii) illiquidity of the market. The authors find that the release of news could potentially trigger jumps through two channels: heterogeneous beliefs or market sentiment. Kapetanios et al. (2014) document that jumps that are unrelated to the release of news are triggered by market illiquidity, whereby the shrinking market liquidity occurring due to an increase in option bid-ask spread leads to the jumps in the option prices. These are diverse research opportunities, some of them of practical importance (i.e. hedging), that our results suggest to look at.

7 Conclusion

This paper presents a model specification test analysis on jump activity for the commodity markets, developed within the affine jump diffusion framework. Thanks to the affine property of the model, the volatility index associated with a set of options written on a commodity is an affine function of the instantaneous volatility, thus, turning this variable observable. Combining high frequency data for both, the commodity index and its volatility index and applying joint and disjoint jumps tests proposed by Jacod and Todorov (2009), we carry out a test on the simultaneity of jumps for the asset and its volatility. Applying this methodology to the actively traded crude oil pair USO/OVX and gold pair GLD/GVZ, we find that models with disjoint jumps most likely provide the correct specification for these two markets. Our findings point towards a model specification that is rather different from the one used in the option pricing literature.

Our results provide useful guidance on the affine model specifications to ensure consistency with the dynamics observed in the commodity and volatility markets. It suggests several extensions. First, the fact that we found strong evidence for disjoint jumps does not imply that they are not related. Indeed, "correlating" two jump processes is difficult, with a good example given by the problem of correlating default times for credit derivatives products, but it is still possible to perform such a task. Indeed, a good example is a two-dimensional Hawkes process, which is a vector jump process that enables a dependency between the two jump process components through the self-exciting property, and that nevertheless excludes simultaneous jumps. The recent works of Aït-Sahalia et al. (2015a) and Kokholm (2016) suggest that it might be a convenient tool for the commodity market. Second, the existence of a volatility idiosyncratic factor could be further analyzed using non-linear volatility products that are actively traded on the market. One could also investigate the possible explanations that drive volatility jumps (through option price movements) but do not impact the asset price, e.g. the impact of macroeconomic news and market volatility. Lastly, a natural extension of our work is a joint calibration of derivatives written on the commodity index and derivatives written on the volatility index associated with that commodity. It is far from being an exotic objective as, indeed, for both, the crude oil and gold, such derivatives markets exist and are places of important trading activity.

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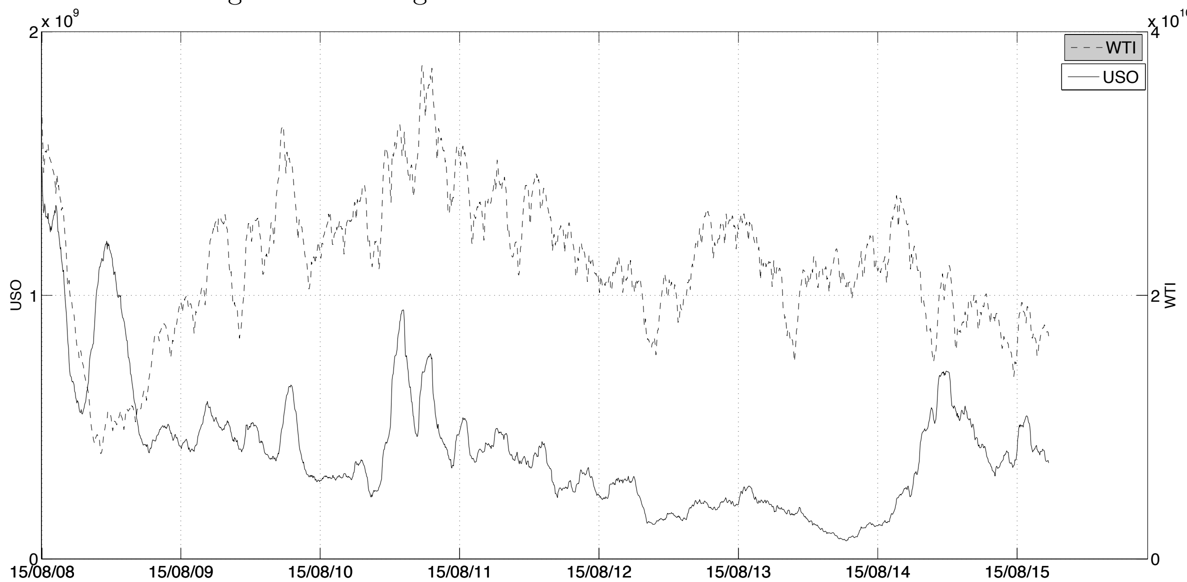
A Appendix

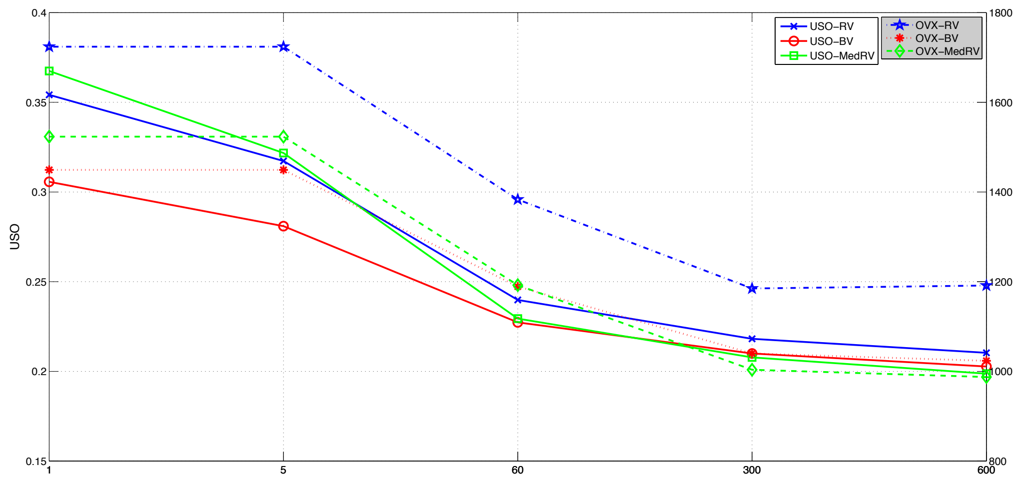
A.1 Figures

Figure 1: Trading volume in USD for USO and WTI crude oil



Note. One-month moving averages in dollars of the daily trading activity for the USO and the front running futures WTI light sweet crude oil. Period 15/07/2008 to 18/11/2015.

Figure 2: Signature plots for USO and OVX

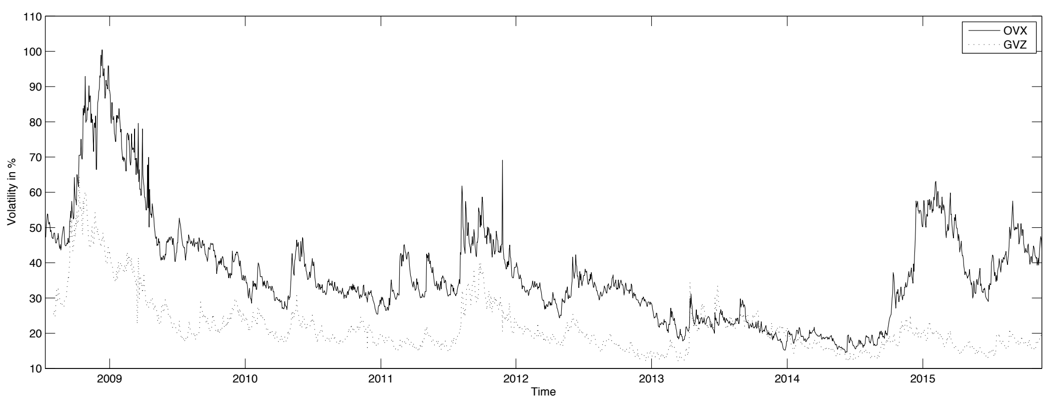


Note. Signature plots for USO (left vertical axes) and OVX (right vertical axes) as a function of time (depicted in seconds on the horizontal axis). RV and BV are computed using Eq.(6) and Eq.(7), respectively. MedRV is computed according to Andersen et al. (2012) as $\text{MedRV}_t = \frac{\pi}{6-4\sqrt{3}+\pi}\frac{m}{m-2}\sum_{i=3}^{m}\text{med}\left(|r_{t,i}|, |r_{t,i-1}|, |r_{t,i-2}|\right)^2$.

Figure 3: ETF plots

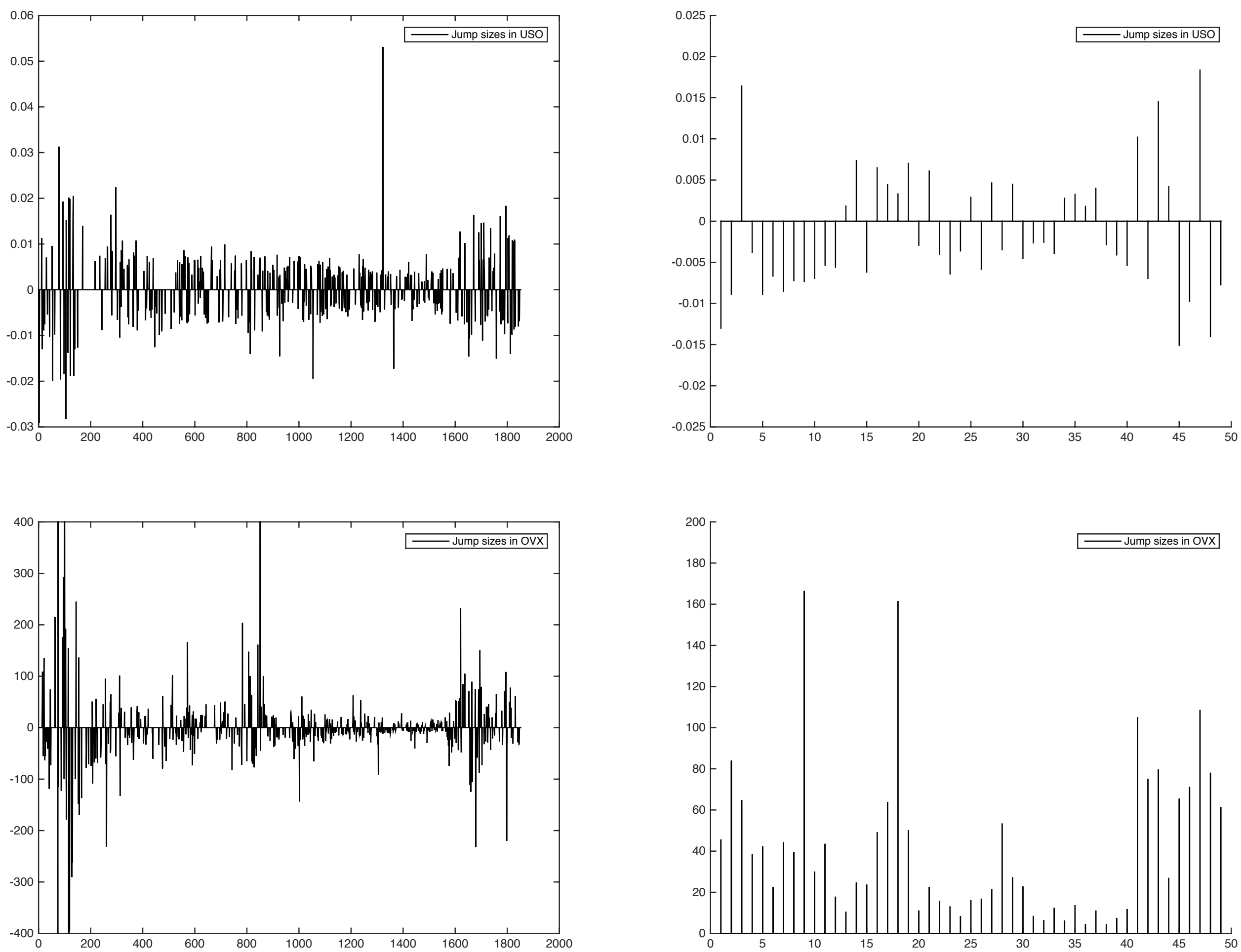
Note. ETF level plots for crude oil (USO) and gold (GLD).

Figure 4: Volatility plots



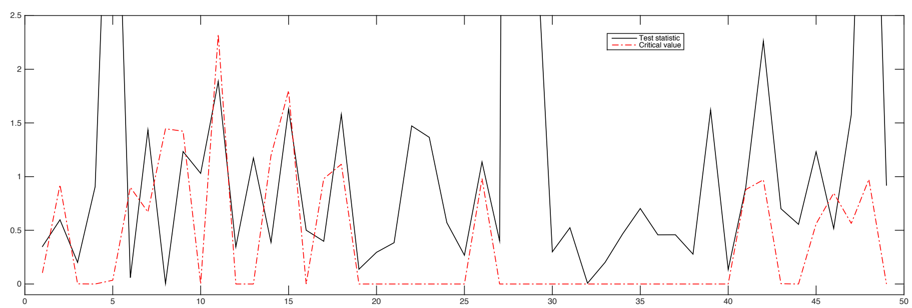
Note. Annualised volatility indexes for crude oil (OVX) and gold (GVZ).

Figure 5: Jump plots



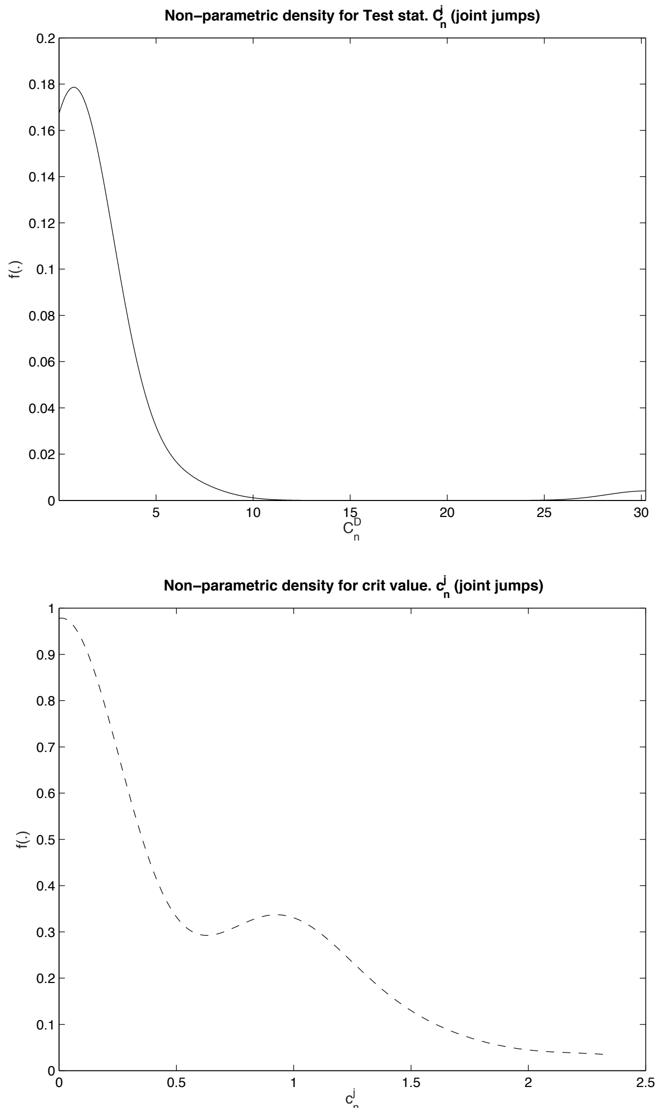
Note. Left panels: jumps sizes in USO (top panel) and OVX (bottom panel) across the entire series (1852 observations), extracted using Eq.(11). The jump is observed when J_t is non-zero. Right panel: jump sizes for days where both series, USO and OVX, jump (49 observations).

Figure 6: Test statistic and critical value for H_0 : Joint jumps



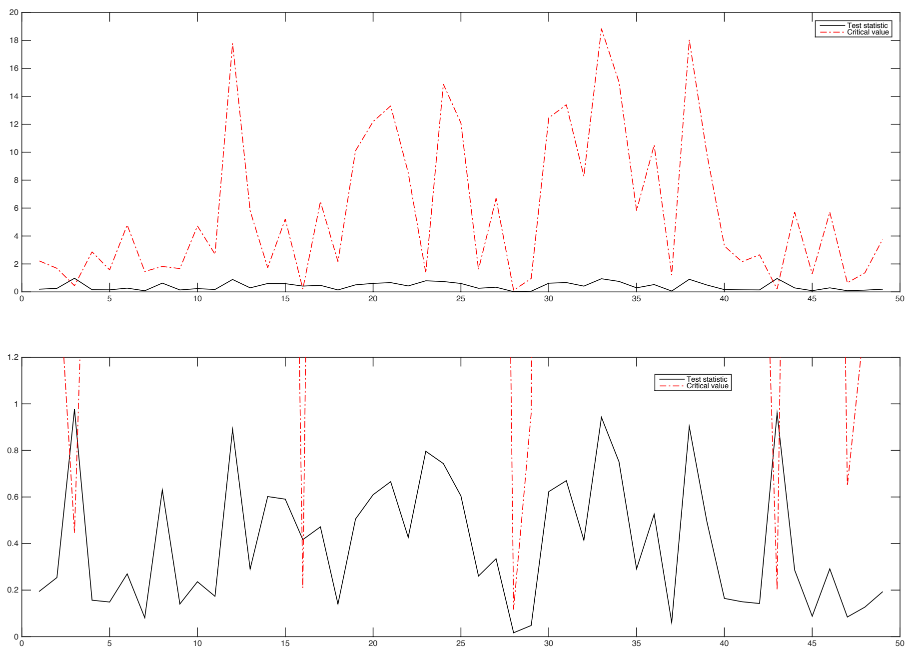
Note. For USO/OVX we plot the test statistics $C_n^j = |\Phi^{(j)} - 1|$ computed from Eq.(19) (black solid line) and the critical value $c^{(j)}$ computed from Eq.(26) (red dotted line) to test H_0 : Jumps arrive jointly in USO and OVX. We document the rejection of the null in 81.63% of all cases (reported in Table 4).

Figure 7: Distribution of the test statistic and the critical value for H_0 : Joint jumps



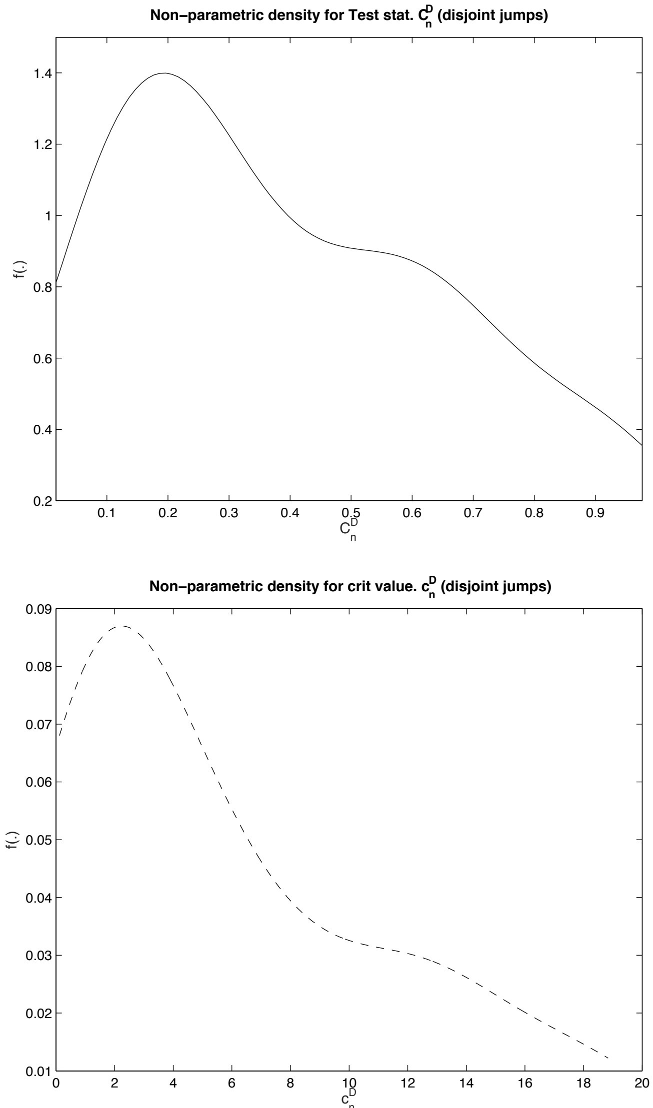
Note. For USO/OVX we plot in the top panel: the non-parametric density estimate for the test statistic $C_n^j = |\Phi^{(j)} - 1|$ computed from Eq.(19) for the H_0 : Joint jumps. Bottom panel: the corresponding non-parametric density estimate for the critical value $c^{(j)}$ computed from Eq.(26). We can observe that the magnitude of the values for the test statistic (upper panel) is typically much higher than the ones for the critical value (bottom panel), which leads to rejection of the H_0 : Joint jumps in 81.63% of all cases (reported in Table 4).

Figure 8: Test statistic and critical value for H_0 : Disjoint jumps



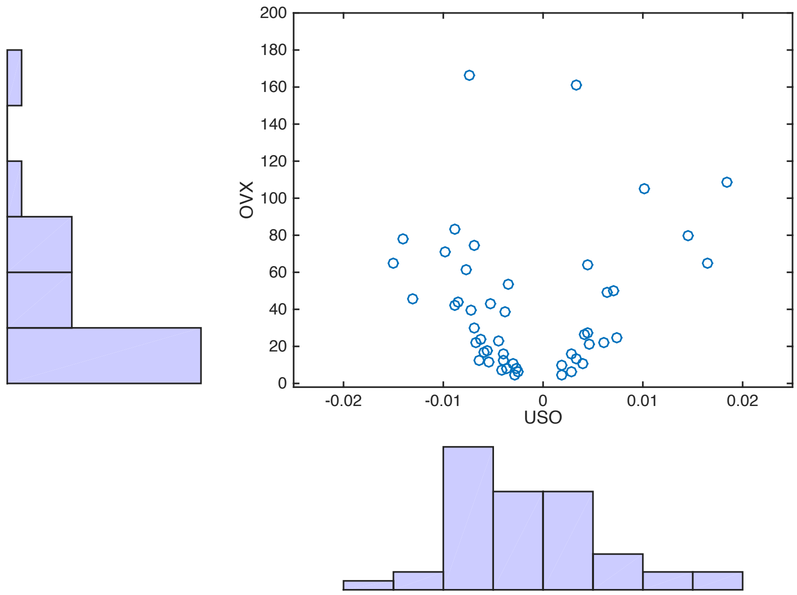
Note. For USO/OVX we plot the test statistics $\Phi_n^{(d)}$ computed from Eq.(20) (black solid line) and the critical value $c^{(d)}$ computed from Eq.(27) (red dotted line) to test H_0 : Jumps arrive disjointly in USO and OVX. The bottom panel is a zoomed-in version of the top panel. We document the rejection of the null in 6.12% of all cases (reported in Table 4).

Figure 9: Distribution of the test statistic and the critical value for H_0 : Disjoint jumps



Note. For USO/OVX we plot in the top panel: the non-parametric density estimate for the test statistic $\Phi_n^{(d)}$ computed from Eq.(20) for the H_0 : Disjoint jumps. Bottom panel: the corresponding non-parametric density estimate for the critical value $c^{(d)}$ computed from Eq.(27). We can observe that the magnitude of the values for the critical value (bottom panel) is typically much higher than the ones for the test statistic (top panel), which leads to rejection of the H_0 : Joint jumps in only 6.12% of all cases (reported in Table 4).

Figure 10: Scatter plot and histogram for jump sizes in USO and OVX



Note. Scatter plot and histogram for the jump sizes in USO and OVX for days where *both* series jump (but jumps in OVX are positive).

A.2 Tables

Table 1: Summary statistics

	Mean	Std. dev.	Skewness	Kurt.	Min	Max
USO	2.09×10^{-6}	0.0016	-0.0197	5.0314	-0.0048	0.0047
OVX	-0.1838	9.8192	0.1210	6.9646	-30.7985	32.9643
GLD	3.42×10^{-6}	8.01×10^{-4}	0.0020	5.8468	-0.0024	0.0024
GVZ	-0.0083	3.1599	0.1654	7.7313	-10.6269	11.3808

Note. Summary statistics are computed for the log-returns of the underlying ($\Delta_i x^1$ defined in Eq.(15)) and the corresponding squared differences of the volatility ($\Delta_i x^2$ defined in Eq.(16)) for each day and then averaged across days.

Table 2: Jump frequency statistics

Pairs	USO/OVX	GLD/GVZ
(i) Total days	1852	1838
(ii) Jump days (ETF jumps)	393	427
(iii) Jump days (vol. index jumps)	183	220
(iv) ETF jumps, vol. index does not jump	276	300
(v) Vol. index jumps; ETF does not jump	134	158
(vi) Jump days (both series jump)	49	62

Note. Jump frequency statistics include (i) the total number of days in the samples under consideration; (ii) the number of days when ETF jumps regardless of whether the corresponding volatility index jumps or not; (iii) the number of days when volatility index jumps (positively) regardless of whether the corresponding ETF jumps or not; (iv) the number of days when commodity ETF jumps and the corresponding volatility index does not jump; (v) the number of days when volatility index jumps (positively) and the corresponding ETF does not jump; (vi) the number of days when the commodity ETF jumps and the corresponding volatility index jumps (positively).

Table 3: Jump statistics

	USO	GLD	OVX	GVZ
Panel A:	Jumps in the commodity ETF		Jumps in volatility index	
Mean	5.52×10^{-5}	1.31×10^{-5}	66.3433	15.3733
Std. dev.	0.0083	0.0044	212.9379	39.5035
Intensity	0.2122	0.2323	0.0988	0.1197
Panel B:	Jumps in the commodity ETF and volatility index occurring on the same day			
Mean	-0.0014	-7.62×10^{-4}	40.787	11.9312
Std. dev.	0.0075	0.0035	37.427	15.0769
Intensity	0.0265	0.0337	0.0265	0.0337

Note. Descriptive statistics for the jumps in ETF returns and the squared volatility index returns. Panel A reports the results computed separately for the days where jumps occur in the commodity ETF (number of the days is reported in line (ii) of Table 2) and days when jumps occur in the volatility index (number of the days is reported in line (iii) of Table 2). Panel B reports the results computed for those days where both series, the commodity ETF and the corresponding volatility index jump (number of the days is reported in line (vi) of Table 2).

Table 4: Joint and disjoint test results

Pairs	USO/OVX	GLD/GVZ
H_0 : Joint	0.8163	0.8548
H_0 : Disjoint	0.0612	0.0312

Note. Test results: the number of rejections (in %) of the H_0 : Joint jumps (first line) and H_0 : Disjoint jumps (second line).

Table 5: Copula analysis

Model/Pair	USO/OVX	GLD/GVZ
Lin. correl.	0.0993	-0.1472
Gaussian	0.0096	-0.1417
Student-t	-0.0744	-0.1899
Clayton	0.0000	0.0000
Gumbel	1.0958	1.0398

Note. Copula dependence parameter measuring the dependence between jump sizes for ETF and respective volatility index on days when both series jump (but jumps in OVX are positive).

Jump Activity Analysis for Affine Jump-diffusion Models: Evidences from the Commodity Market - Online Supplementary Material

B Robustness Checks

In this section we report the results for the robustness checks that were discussed in Section 5. In the first part (B.1), we provide the results for the methodology presented in the paper based on a combination of Huang and Tauchen (2005)'s test and Jacod and Todorov (2009)'s test but we vary both, the sampling frequency by considering 1-minute and 10-minute time intervals, and the parameter $\tilde{\alpha}$ that appears in Eq.(11) and controls to which extent a difference between the realized variance and the realized bipower variation should be regarded as a jump.

In the second part (B.2.1), we replace the Huang and Tauchen (2005)'s test by the Jiang and Oomen (2008)'s test and report the results for the sampling frequencies 1-minute, 5-minute and 10-minute, and the values of $\tilde{\alpha}$ corresponding to $(0.95, 0.99, 0.90)$.

In the third part (B.2.2), we replace the Huang and Tauchen (2005)'s test by the Andersen et al. (2012)'s tests and present the results for the sampling frequencies 1-minute, 5-minute and 10-minute, and the values of $\tilde{\alpha}$ corresponding to $(0.95, 0.99, 0.90)$. We notice that Andersen et al. (2012) provide two tests to detect jumps, one based on the minimum realized variance and minimum realized quarticity estimators and another one based on the median realized variance and median realized quarticity estimators.

Lastly, all the figures presented in the paper are for the pair USO/OVX, in the last section (C) we report the corresponding figures for the pair GLD/GVZ. We notice however that the results obtained for GLD/GVZ are qualitatively similar to those obtained for USO/OVX.

B.1 Robustness with respect to the sampling frequency and the jump statistical threshold level $\tilde{\alpha}$ of Eq.(11)

Here we discuss the robustness of the results reported in the paper with respect to the sampling frequency and the jump statistical threshold $\tilde{\alpha}$, using methodology utilised in the paper. Tables 6 and 7 report the results for jump frequency statistics and joint/disjoint test results using data sampled at 1-minute and 10-minute frequencies and the same $\tilde{\alpha}$ of 0.95 used in the paper. These tables complement the results in Tables 2 and 4 of paper that are obtained using 5-minute data and $\tilde{\alpha} = 0.95$. Tables 8 and 9 correspond to $\tilde{\alpha} = 0.99$ and all frequencies (1-, 5- and 10-minute) while Tables 10 and 11 correspond to $\tilde{\alpha} = 0.90$ and all frequencies (1-, 5- and 10-minute). We observe that the results of the tests are qualitative similar to those obtained in the paper for 5-minute frequency and $\tilde{\alpha} = 0.95$, namely, we reject the hypothesis of joint jumps in the majority of the cases, while the hypothesis of disjoint jumps is most frequently not rejected.

Table 6: Jump frequency statistics ($\tilde{\alpha} = 0.95$)

	1-minute frequency		10-minute frequency	
Pairs	USO/OVX	GLD/GVZ	USO/OVX	GLD/GVZ
(i) Total days	1852	1838	1852	1838
(ii) Jump days (ETF jumps)	1011	703	279	338
(iii) Jump days (vol. index jumps)	442	551	165	181
(iv) ETF jumps, vol. index does not jump	415	178	212	245
(v) Vol. index jumps; ETF does not jump	192	323	141	138
(vi) Jump days (both series jump)	250	228	24	43

Note. Jump frequency statistics include (i) the total number of days in the samples under consideration; (ii) the number of days when ETF jumps regardless of whether the corresponding volatility index jumps or not; (iii) the number of days when volatility index jumps (positively) regardless of whether the corresponding ETF jumps or not; (iv) the number of days when commodity ETF jumps and the corresponding volatility index does not jump; (v) the number of days when volatility index jumps (positively) and the corresponding ETF does not jump; (vi) the number of days when the commodity ETF jumps and the corresponding volatility index jumps (positively).

Table 7: Joint and disjoint test results ($\tilde{\alpha} = 0.95$)

	1-minute frequency		10-minute frequency	
Pairs	USO/OVX	GLD/GVZ	USO/OVX	GLD/GVZ
H_0 : Joint	0.9280	0.9561	0.9167	0.7907
H_0 : Disjoint	0.0120	0.0000	0.0833	0.0233

Note. Test results: the number of rejections (in %) of the H_0 : Joint jumps (first line) and H_0 : Disjoint jumps (second line).

Table 8: Jump frequency statistics ($\tilde{\alpha} = 0.99$)

	1-minute frequency		5-minute frequency		10-minute frequency	
Pairs	USO/OVX	GLD/GVZ	USO/OVX	GLD/GVZ	USO/OVX	GLD/GVZ
(i) Total days	1852	1838	1852	1838	1852	1838
(ii) Jump days (ETF jumps)	685	386	193	232	111	149
(iii) Jump days (vol. index jumps)	330	446	101	133	88	94
(iv) ETF jumps, vol. index does not jump	395	148	153	188	97	125
(v) Vol. index jumps; ETF does not jump	204	345	86	110	82	83
(vi) Jump days (both series jump)	126	101	15	23	6	11

Note. Jump frequency statistics include (i) the total number of days in the samples under consideration; (ii) the number of days when ETF jumps regardless of whether the corresponding volatility index jumps or not; (iii) the number of days when volatility index jumps (positively) regardless of whether the corresponding ETF jumps or not; (iv) the number of days when commodity ETF jumps and the corresponding volatility index does not jump; (v) the number of days when volatility index jumps (positively) and the corresponding ETF does not jump; (vi) the number of days when the commodity ETF jumps and the corresponding volatility index jumps (positively).

Table 9: Joint and disjoint test results ($\tilde{\alpha} = 0.99$)

	1-minute frequency		5-minute frequency		10-minute frequency	
Pairs	USO/OVX	GLD/GVZ	USO/OVX	GLD/GVZ	USO/OVX	GLD/GVZ
H_0 : Joint	0.9127	0.9505	0.7333	0.9565	1.0000	0.8182
H_0 : Disjoint	0.0079	0.0000	0.0667	0.0000	0.0000	0.0909

Note. Test results: the number of rejections (in %) of the H_0 : Joint jumps (first line) and H_0 : Disjoint jumps (second line).

Table 10: Jump frequency statistics ($\tilde{\alpha} = 0.90$)

	1-minute frequency		5-minute frequency		10-minute frequency	
Pairs	USO/OVX	GLD/GVZ	USO/OVX	GLD/GVZ	USO/OVX	GLD/GVZ
(i) Total days	1852	1838	1852	1838	1852	1838
(ii) Jump days (ETF jumps)	1209	884	549	591	401	472
(iii) Jump days (vol. index jumps)	521	612	250	290	226	246
(iv) ETF jumps, vol. index does not jump	361	178	336	360	272	314
(v) Vol. index jumps; ETF does not jump	161	302	166	176	173	172
(vi) Jump days (both series jump)	360	310	84	114	53	74

Note. Jump frequency statistics include (i) the total number of days in the samples under consideration; (ii) the number of days when ETF jumps regardless of whether the corresponding volatility index jumps or not; (iii) the number of days when volatility index jumps (positively) regardless of whether the corresponding ETF jumps or not; (iv) the number of days when commodity ETF jumps and the corresponding volatility index does not jump; (v) the number of days when volatility index jumps (positively) and the corresponding ETF does not jump; (vi) the number of days when the commodity ETF jumps and the corresponding volatility index jumps (positively).

Table 11: Joint and disjoint test results ($\tilde{\alpha} = 0.90$)

	1-minute frequency		5-minute frequency		10-minute frequency	
Pairs	USO/OVX	GLD/GVZ	USO/OVX	GLD/GVZ	USO/OVX	GLD/GVZ
H_0 : Joint	0.9278	0.9548	0.8333	0.9737	0.8392	0.8649
H_0 : Disjoint	0.0056	0.0032	0.0476	0.0088	0.0755	0.0135

Note. Test results: the number of rejections (in %) of the H_0 : Joint jumps (first line) and H_0 : Disjoint jumps (second line).

B.2 Robustness to a change of the jump day detection methodology

Here we choose two alternatives to Huang and Tauchen (2005)'s test, the first one is the Jiang and Oomen (2008)'s test discussed in Section B.2.1 while the second one is the Andersen et al. (2012)'s test discussed in B.2.2 (see Dumitru and Urga (2012) for an overview of other alternatives). Each one is combined with the Jacod and Todorov (2009)'s test applied to different sampling frequencies (1-, 5- and 10-minute) and different values of $\tilde{\alpha}$ corresponding to 0.95, 0.99 and 0.9.

B.2.1 The Jiang and Oomen (2008) methodology

We report the main equations of Jiang and Oomen (2008) and refer to that paper for further details. We use the notations introduced in Section 2.2. We introduce the quantity SwV (i.e. swap variance) as

$$SwV_t = 2\sum_{i=1}^{m}(R_{t,i} - r_{t,i}),$$

where

$$R_{t,i} = \frac{y_{t,i\Delta}}{y_{t,(i-1)\Delta}} - 1$$

is a simple intra-day return and $r_{t,i}$ is given by Eq.(5). Using the realized variance and the realized bipower variation

$$BV_t = \frac{\pi}{2}\frac{m}{m-1}\sum_{i=2}^{m}|r_{t,i}||r_{t,i-1}|,$$

the test is based on the comparison:

$$ZJ_t^c = \frac{mBV_t}{\sqrt{\Omega_{SwV}}}\left(1 - \frac{RV_t}{SwV_t}\right) \xrightarrow{d} \mathcal{N}(0,1),$$

where

$$\Omega_{SwV} = \frac{\mu_6}{9}\frac{m^3\mu_{6/p}^{-p}}{m-3}\sum_{i=0}^{m-p}\prod_{k=1}^{p}|r_{i+k}|^{6/p}$$

with the moments $\mu_p = E(|\chi|^p)$ with $\chi \sim \mathcal{N}(0,1)$. We select $p = 4$ and notice that the moments are known in a closed form as we have

$$\mu_p = \frac{2^{p/2}}{\sqrt{\pi}}\Gamma\left(\frac{p+1}{2}\right), \quad p > 0,$$

see also Dumitru and Urga (2012). Thus, we obtain for Ω_{SwV}:

$$\Omega_{SwV} = 3.05\frac{m^3}{m-3}\sum_{i=0}^{m-4}\prod_{k=1}^{4}|r_{i+k}|^{6/4}.$$

Within this approach the jump size is given by

$$\hat{J}_t^c = \text{sign}(r_t) \times \sqrt{(RV_t - BV_t) \times I_{(ZJ_t^c \geq \Phi_{\tilde{\alpha}}^{-1})}}, \tag{38}$$

where $I_{(ZJ_t^c \geq \Phi_{\tilde{\alpha}}^{-1})}$ is the indicator function that depends on ZJ_t^c and $\tilde{\alpha}$, and determines whether there is a jump.

Robustness with respect to the sampling frequency and the jump statistical threshold $\tilde{\alpha}$ of Eq.(38)

Tables 12, 14 and 16 report the results for the jump frequency statistics for all frequencies and $\tilde{\alpha}$ corresponding to 0.95, 0.99 and 0.9, respectively. The results of the tests are reported in Tables 13, 15 and 17 across all frequencies for different levels of $\tilde{\alpha}$ corresponding to 0.95, 0.99 and 0.9, respectively. We observe that the results of these tests are qualitative similar to those obtained using Huang and Tauchen (2005)'s test in the paper as well as the results reported in Section B.1, namely, the majority of jumps arriving in the asset and its volatility are disjoint.

Table 12: Jump frequency statistics ($\tilde{\alpha} = 0.95$)

	1-minute frequency		5-minute frequency		10-minute frequency	
Pairs	USO/OVX	GLD/GVZ	USO/OVX	GLD/GVZ	USO/OVX	GLD/GVZ
(i) Total days	1852	1838	1852	1838	1852	1838
(ii) Jump days (ETF jumps)	173	184	158	188	119	149
(iii) Jump days (vol. index jumps)	697	701	485	539	483	499
(iv) ETF jumps, vol. index does not jump	108	102	137	133	93	105
(v) Vol. index jumps; ETF does not jump	632	622	464	484	457	456
(vi) Jump days (both series jump)	65	79	21	55	26	43

Note. Jump frequency statistics include (i) the total number of days in the samples under consideration; (ii) the number of days when ETF jumps regardless of whether the corresponding volatility index jumps or not; (iii) the number of days when volatility index jumps (positively) regardless of whether the corresponding ETF jumps or not; (iv) the number of days when commodity ETF jumps and the corresponding volatility index does not jump; (v) the number of days when volatility index jumps (positively) and the corresponding ETF does not jump; (vi) the number of days when the commodity ETF jumps and the corresponding volatility index jumps (positively).

Table 13: Joint and disjoint test results ($\tilde{\alpha} = 0.95$)

	1-minute frequency		5-minute frequency		10-minute frequency	
Pairs	USO/OVX	GLD/GVZ	USO/OVX	GLD/GVZ	USO/OVX	GLD/GVZ
H_0 : Joint	0.8923	0.9494	0.9524	0.7818	0.8462	0.8372
H_0 : Disjoint	0.0154	0.0000	0.1429	0.0000	0.0000	0.0465

Note. Test results: the number of rejections (in %) of the H_0 : Joint jumps (first line) and H_0 : Disjoint jumps (second line).

Table 14: Jump frequency statistics ($\tilde{\alpha} = 0.99$)

	1-minute frequency		5-minute frequency		10-minute frequency	
Pairs	USO/OVX	GLD/GVZ	USO/OVX	GLD/GVZ	USO/OVX	GLD/GVZ
(i) Total days	1852	1838	1852	1838	1852	1838
(ii) Jump days (ETF jumps)	108	125	93	115	77	98
(iii) Jump days (vol. index jumps)	689	694	480	534	473	486
(iv) ETF jumps, vol. index does not jump	65	70	80	77	60	68
(v) Vol. index jumps; ETF does not jump	646	640	467	496	456	457
(vi) Jump days (both series jump)	43	54	13	38	17	29

Note. Jump frequency statistics include (i) the total number of days in the samples under consideration; (ii) the number of days when ETF jumps regardless of whether the corresponding volatility index jumps or not; (iii) the number of days when volatility index jumps (positively) regardless of whether the corresponding ETF jumps or not; (iv) the number of days when commodity ETF jumps and the corresponding volatility index does not jump; (v) the number of days when volatility index jumps (positively) and the corresponding ETF does not jump; (vi) the number of days when the commodity ETF jumps and the corresponding volatility index jumps (positively).

Table 15: Joint and disjoint test results ($\tilde{\alpha} = 0.99$)

	1-minute frequency		5-minute frequency		10-minute frequency	
Pairs	USO/OVX	GLD/GVZ	USO/OVX	GLD/GVZ	USO/OVX	GLD/GVZ
H_0 : Joint	0.8605	0.9444	0.9231	0.7632	0.8235	0.8621
H_0 : Disjoint	0.0233	0.0000	0.1538	0.0000	0.0000	0.0690

Note. Test results: the number of rejections (in %) of the H_0 : Joint jumps (first line) and H_0 : Disjoint jumps (second line).

Table 16: Jump frequency statistics ($\tilde{\alpha} = 0.90$)

	1-minute frequency		5-minute frequency		10-minute frequency	
Pairs	USO/OVX	GLD/GVZ	USO/OVX	GLD/GVZ	USO/OVX	GLD/GVZ
(i) Total days	1852	1838	1852	1838	1852	1838
(ii) Jump days (ETF jumps)	225	235	196	229	156	183
(iii) Jump days (vol. index jumps)	702	706	490	543	485	504
(iv) ETF jumps, vol. index does not jump	139	130	170	162	123	130
(v) Vol. index jumps; ETF does not jump	616	605	464	477	452	453
(vi) Jump days (both series jump)	86	101	26	66	33	51

Note. Jump frequency statistics include (i) the total number of days in the samples under consideration; (ii) the number of days when ETF jumps regardless of whether the corresponding volatility index jumps or not; (iii) the number of days when volatility index jumps (positively) regardless of whether the corresponding ETF jumps or not; (iv) the number of days when commodity ETF jumps and the corresponding volatility index does not jump; (v) the number of days when volatility index jumps (positively) and the corresponding ETF does not jump; (vi) the number of days when the commodity ETF jumps and the corresponding volatility index jumps (positively).

Table 17: Joint and disjoint test results ($\tilde{\alpha} = 0.90$)

	1-minute frequency		5-minute frequency		10-minute frequency	
Pairs	USO/OVX	GLD/GVZ	USO/OVX	GLD/GVZ	USO/OVX	GLD/GVZ
H_0 : Joint	0.8721	0.94604	0.9615	0.7727	0.8485	0.8431
H_0 : Disjoint	0.0116	0.0000	0.1154	0.0000	0.0000	0.0392

Note. Test results: the number of rejections (in %) of the H_0 : Joint jumps (first line) and H_0 : Disjoint jumps (second line).

B.2.2 The Andersen, Dobrev, and Schaumburg (2012) methodology

We report the main equations of Andersen et al. (2012) and refer to that paper for further details. As for the previous case we follow the notations introduced in Section 2.2.

We define the minimum and the median realized variance as

$$\text{MinRV}_t = \frac{\pi}{\pi-2}\frac{m}{m-1}\sum_{i=2}^{m}\min\left(|r_{t,i}|,|r_{t,i-1}|\right)^2,$$
$$\text{MedRV}_t = \frac{\pi}{6-4\sqrt{3}+\pi}\frac{m}{m-2}\sum_{i=3}^{m}\text{med}\left(|r_{t,i}|,|r_{t,i-1}|,|r_{t,i-2}|\right)^2, \tag{39}$$

respectively. Similarly to Huang and Tauchen (2005), we have

$$ZJ_t^a = \frac{1-\text{MinRV}_t/\text{RV}_t}{\sqrt{1.81\Delta\max\left(1,\frac{\text{MinRQ}_t}{\text{MinRV}_t^2}\right)}} \xrightarrow{d} \mathcal{N}(0,1),$$
$$ZJ_t^b = \frac{1-\text{MedRV}_t/\text{RV}_t}{\sqrt{0.96\Delta\max\left(1,\frac{\text{MedRQ}_t}{\text{MedRV}_t^2}\right)}} \xrightarrow{d} \mathcal{N}(0,1)$$

where

$$\text{MinRQ}_t = \frac{\pi}{3\pi-8}\frac{m^2}{m-1}\sum_{i=2}^{m}\min\left(|r_{t,i}|,|r_{t,i-1}|\right)^4,$$
$$\text{MedRQ}_t = \frac{3\pi}{9\pi+72-52\sqrt{3}}\frac{m}{m-2}\sum_{i=3}^{m}\text{med}\left(|r_{t,i}|,|r_{t,i-1}|,|r_{t,i-2}|\right)^4.$$

Within this methodology, the jump sizes, depending on which estimator is used, are given by

$$\hat{J}_t^a = \text{sign}(r_t)\times\sqrt{(RV_t-BV_t)\times I_{(ZJ_t^a\geq\Phi_{\tilde{\alpha}}^{-1})}}, \tag{40}$$
$$\hat{J}_t^b = \text{sign}(r_t)\times\sqrt{(RV_t-BV_t)\times I_{(ZJ_t^b\geq\Phi_{\tilde{\alpha}}^{-1})}}. \tag{41}$$

The indicator $I_{(ZJ_t^a\geq\Phi_{\tilde{\alpha}}^{-1})}$ based on ZJ_t^a determines whether there is a jump, and depends on $\tilde{\alpha}$. A similar remark applies to $I_{(ZJ_t^b\geq\Phi_{\tilde{\alpha}}^{-1})}$.

Robustness with respect to the sampling frequency and the jump statistical threshold $\tilde{\alpha}$ of Eq.(40) and Eq.(41)

Tables 18, 20 and 22 report the results for the jump frequency statistics for both test specifications (based on MedRV and MinRV), all frequencies and $\tilde{\alpha}$ corresponding to 0.95, 0.99 and 0.9, respectively. The results of the tests are reported in Tables 19, 21 and 23 across all frequencies and different levels of $\tilde{\alpha}$ corresponding to 0.95, 0.99 and 0.9, respectively. The results for both test specifications are qualitative similar to those obtained using Huang and Tauchen (2005)'s methodology and the results reported in Sections B.1 and B.1.1. Overall, we confirm that the majority of jumps arriving in the asset and its volatility are disjoint.

Table 18: Jump frequency statistics ($\tilde{\alpha} = 0.95$)

	Tests based on MedRV					
	1-minute frequency		5-minute frequency		10-minute frequency	
Pairs	USO/OVX	GLD/GVZ	USO/OVX	GLD/GVZ	USO/OVX	GLD/GVZ
(i) Total days	1852	1838	1852	1838	1852	1838
(ii) Jump days (ETF jumps)	751	646	386	443	293	390
(iii) Jump days (vol. index jumps)	340	298	189	214	186	175
(iv) ETF jumps, vol. index does not jump	425	409	268	310	217	298
(v) Vol. index jumps; ETF does not jump	191	189	142	151	155	137
(vi) Jump days (both series jump)	149	109	47	63	31	38
	Tests based on MinRV					
	1-minute frequency		5-minute frequency		10-minute frequency	
Pairs	USO/OVX	GLD/GVZ	USO/OVX	GLD/GVZ	USO/OVX	GLD/GVZ
(i) Total days	1852	1838	1852	1838	1852	1838
(ii) Jump days (ETF jumps)	529	476	314	341	221	278
(iii) Jump days (vol. index jumps)	268	230	142	143	142	130
(iv) ETF jumps, vol. index does not jump	365	330	236	274	175	227
(v) Vol. index jumps; ETF does not jump	198	169	107	109	123	105
(vi) Jump days (both series jump)	70	61	35	34	19	25

Note. Jump frequency statistics include (i) the total number of days in the samples under consideration; (ii) the number of days when ETF jumps regardless of whether the corresponding volatility index jumps or not; (iii) the number of days when volatility index jumps (positively) regardless of whether the corresponding ETF jumps or not; (iv) the number of days when commodity ETF jumps and the corresponding volatility index does not jump; (v) the number of days when volatility index jumps (positively) and the corresponding ETF does not jump; (vi) the number of days when the commodity ETF jumps and the corresponding volatility index jumps (positively).

Table 19: Joint and disjoint test results ($\tilde{\alpha} = 0.95$)

	Tests based on MedRV					
	1-minute frequency		5-minute frequency		10-minute frequency	
Pairs	USO/OVX	GLD/GVZ	USO/OVX	GLD/GVZ	USO/OVX	GLD/GVZ
H_0 : Joint	0.9329	0.9725	0.8511	0.7937	0.8710	0.8684
H_0 : Disjoint	0.0000	0.0000	0.0638	0.0317	0.0645	0.0263
	Tests based on MinRV					
	1-minute frequency		5-minute frequency		10-minute frequency	
Pairs	USO/OVX	GLD/GVZ	USO/OVX	GLD/GVZ	USO/OVX	GLD/GVZ
H_0 : Joint	0.8857	0.9672	0.8286	0.7647	0.8421	0.8000
H_0 : Disjoint	0.0000	0.0000	0.0857	0.0000	0.1053	0.0400

Note. Test results: the number of rejections (in %) of the H_0 : Joint jumps (first line) and H_0 : Disjoint jumps (second line).

Table 20: Jump frequency statistics ($\tilde{\alpha} = 0.99$)

	Tests based on MedRV					
	1-minute frequency		5-minute frequency		10-minute frequency	
Pairs	USO/OVX	GLD/GVZ	USO/OVX	GLD/GVZ	USO/OVX	GLD/GVZ
(i) Total days	1852	1838	1852	1838	1852	1838
(ii) Jump days (ETF jumps)	435	383	181	239	120	173
(iii) Jump days (vol. index jumps)	235	205	106	125	107	93
(iv) ETF jumps, vol. index does not jump	318	288	147	200	102	141
(v) Vol. index jumps; ETF does not jump	181	160	90	101	97	77
(vi) Jump days (both series jump)	54	45	16	24	10	16
	Tests based on MinRV					
	1-minute frequency		5-minute frequency		10-minute frequency	
Pairs	USO/OVX	GLD/GVZ	USO/OVX	GLD/GVZ	USO/OVX	GLD/GVZ
(i) Total days	1852	1838	1852	1838	1852	1838
(ii) Jump days (ETF jumps)	262	229	110	145	52	72
(iii) Jump days (vol. index jumps)	162	139	61	69	50	48
(iv) ETF jumps, vol. index does not jump	210	186	96	131	48	66
(v) Vol. index jumps; ETF does not jump	138	121	55	59	49	45
(vi) Jump days (both series jump)	24	18	6	10	1	3

Note. Jump frequency statistics include (i) the total number of days in the samples under consideration; (ii) the number of days when ETF jumps regardless of whether the corresponding volatility index jumps or not; (iii) the number of days when volatility index jumps (positively) regardless of whether the corresponding ETF jumps or not; (iv) the number of days when commodity ETF jumps and the corresponding volatility index does not jump; (v) the number of days when volatility index jumps (positively) and the corresponding ETF does not jump; (vi) the number of days when the commodity ETF jumps and the corresponding volatility index jumps (positively).

Table 21: Joint and disjoint test results ($\tilde{\alpha} = 0.99$)

	Tests based on MedRV					
	1-minute frequency		5-minute frequency		10-minute frequency	
Pairs	USO/OVX	GLD/GVZ	USO/OVX	GLD/GVZ	USO/OVX	GLD/GVZ
H_0 : Joint	0.9259	0.9778	0.8125	0.7917	0.9000	0.8750
H_0 : Disjoint	0.0000	0.0000	0.1250	0.0833	0.1000	0.0625
	Tests based on MinRV					
	1-minute frequency		5-minute frequency		10-minute frequency	
Pairs	USO/OVX	GLD/GVZ	USO/OVX	GLD/GVZ	USO/OVX	GLD/GVZ
H_0 : Joint	0.8750	0.9444	1.0000	0.8000	1.0000	0.6667
H_0 : Disjoint	0.0000	0.0000	0.3333	0.0000	0.0000	0.0000

Note. Test results: the number of rejections (in %) of the H_0 : Joint jumps (first line) and H_0 : Disjoint jumps (second line).

Table 22: Jump frequency statistics ($\tilde{\alpha} = 0.90$)

	Tests based on MedRV					
	1-minute frequency		5-minute frequency		10-minute frequency	
Pairs	USO/OVX	GLD/GVZ	USO/OVX	GLD/GVZ	USO/OVX	GLD/GVZ
(i) Total days	1852	1838	1852	1838	1852	1838
(ii) Jump days (ETF jumps)	946	836	528	609	431	512
(iii) Jump days (vol. index jumps)	429	348	250	282	239	248
(iv) ETF jumps, vol. index does not jump	430	465	319	372	294	340
(v) Vol. index jumps; ETF does not jump	202	184	164	165	183	164
(vi) Jump days (both series jump)	227	164	86	117	56	84
	Tests based on MinRV					
	1-minute frequency		5-minute frequency		10-minute frequency	
Pairs	USO/OVX	GLD/GVZ	USO/OVX	GLD/GVZ	USO/OVX	GLD/GVZ
(i) Total days	1852	1838	1852	1838	1852	1838
(ii) Jump days (ETF jumps)	728	667	463	511	360	416
(iii) Jump days (vol. index jumps)	353	291	191	210	208	205
(iv) ETF jumps, vol. index does not jump	411	417	306	365	246	297
(v) Vol. index jumps; ETF does not jump	213	180	130	139	158	148
(vi) Jump days (both series jump)	140	111	61	71	50	57

Note. Jump frequency statistics include (i) the total number of days in the samples under consideration; (ii) the number of days when ETF jumps regardless of whether the corresponding volatility index jumps or not; (iii) the number of days when volatility index jumps (positively) regardless of whether the corresponding ETF jumps or not; (iv) the number of days when commodity ETF jumps and the corresponding volatility index does not jump; (v) the number of days when volatility index jumps (positively) and the corresponding ETF does not jump; (vi) the number of days when the commodity ETF jumps and the corresponding volatility index jumps (positively).

Table 23: Joint and disjoint test results ($\tilde{\alpha} = 0.90$)

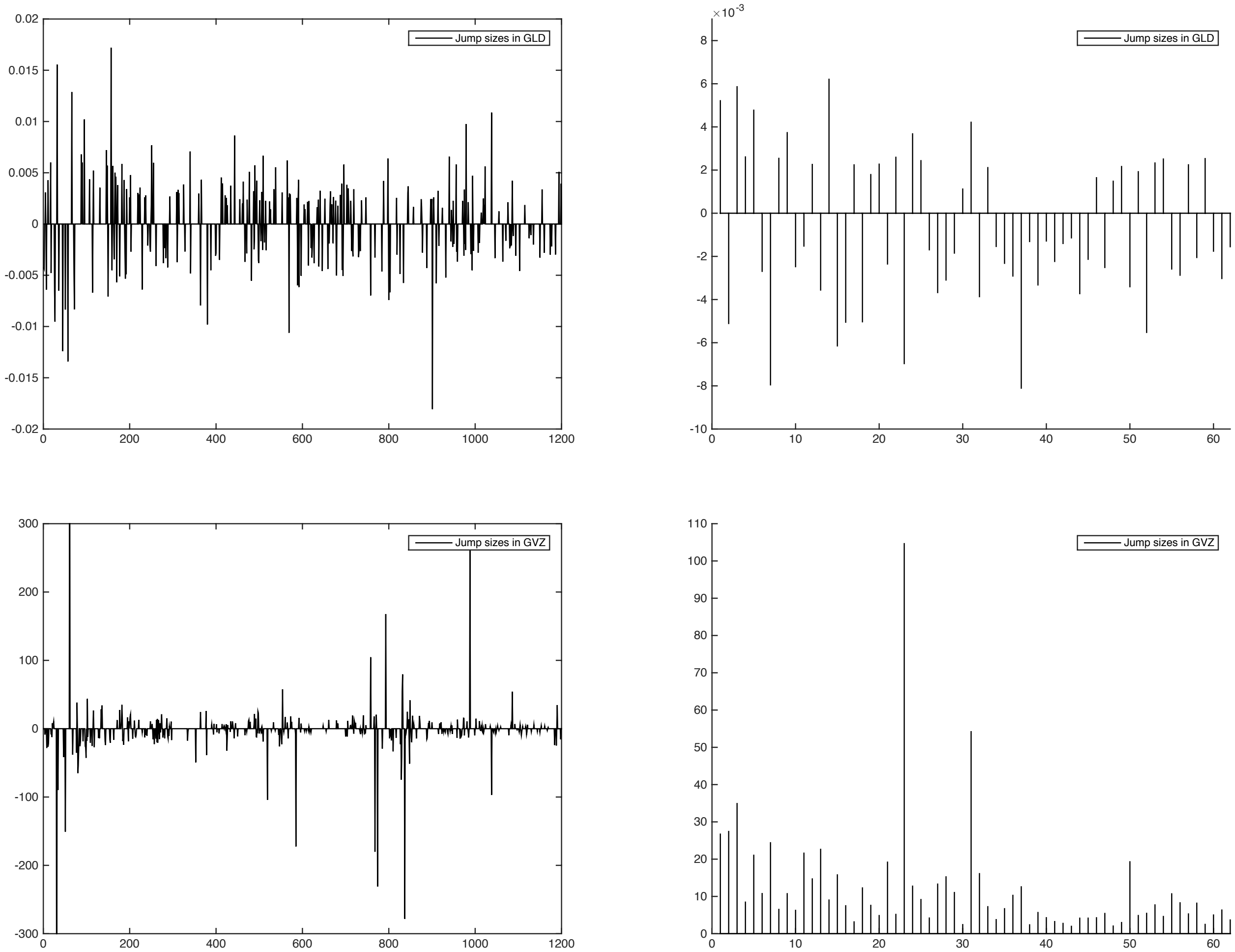
	Tests based on MedRV					
	1-minute frequency		5-minute frequency		10-minute frequency	
Pairs	USO/OVX	GLD/GVZ	USO/OVX	GLD/GVZ	USO/OVX	GLD/GVZ
H_0 : Joint	0.9339	0.9634	0.8256	0.7949	0.8750	0.8810
H_0 : Disjoint	0.0000	0.0000	0.0465	0.0171	0.0357	0.0238
	Tests based on MinRV					
	1-minute frequency		5-minute frequency		10-minute frequency	
Pairs	USO/OVX	GLD/GVZ	USO/OVX	GLD/GVZ	USO/OVX	GLD/GVZ
H_0 : Joint	0.9214	0.9640	0.8197	0.7887	0.9000	0.8246
H_0 : Disjoint	0.0000	0.0000	0.0656	0.0141	0.0600	0.0351

Note. Test results: the number of rejections (in %) of the H_0 : Joint jumps (first line) and H_0 : Disjoint jumps (second line).

C Figures for the Pair GLD/GVZ

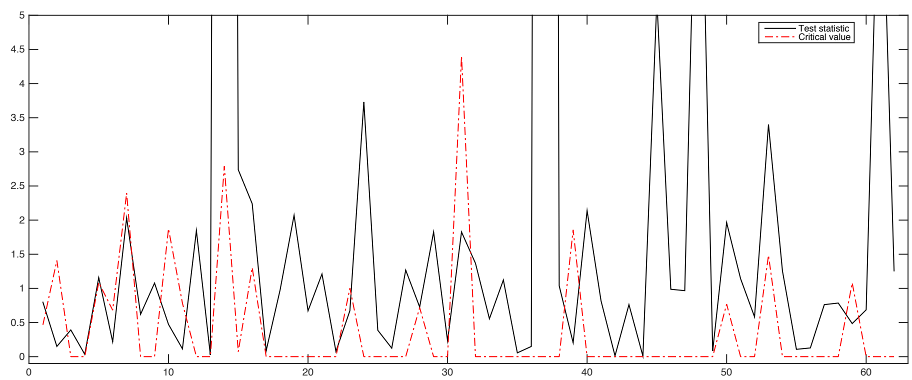
In the paper we reported figures for the pair USO/OVX, here we present similar figures for the pair GLD/GVZ. Figures 11-17 shown here for GLD/GVZ are the counterparts of Figures 5-10 and Figure 2 presented in the paper for the pair USO/OVX. We notice that all figures for the pair GLD/GVZ demonstrate that the results for this pair are qualitatively similar to those obtained for the pair USO/OVX.

Figure 11: Jump plots



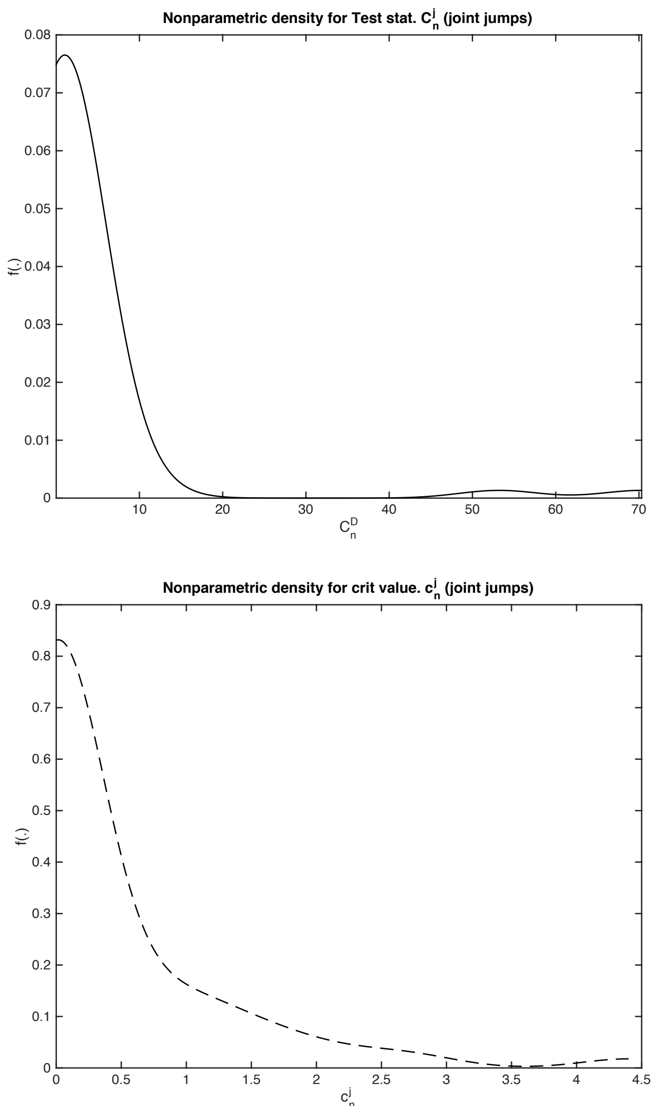
Note. Left panels: jumps sizes in GLD (top panel) and GVZ (bottom panel) across the entire series (1838 observations), extracted using Eq.(11). The jump is observed when J_t is non-zero. Right panel: jump sizes for days where both series, GLD and GVZ, jump (62 observations).

Figure 12: Test statistic and critical value for H_0 : Joint jumps



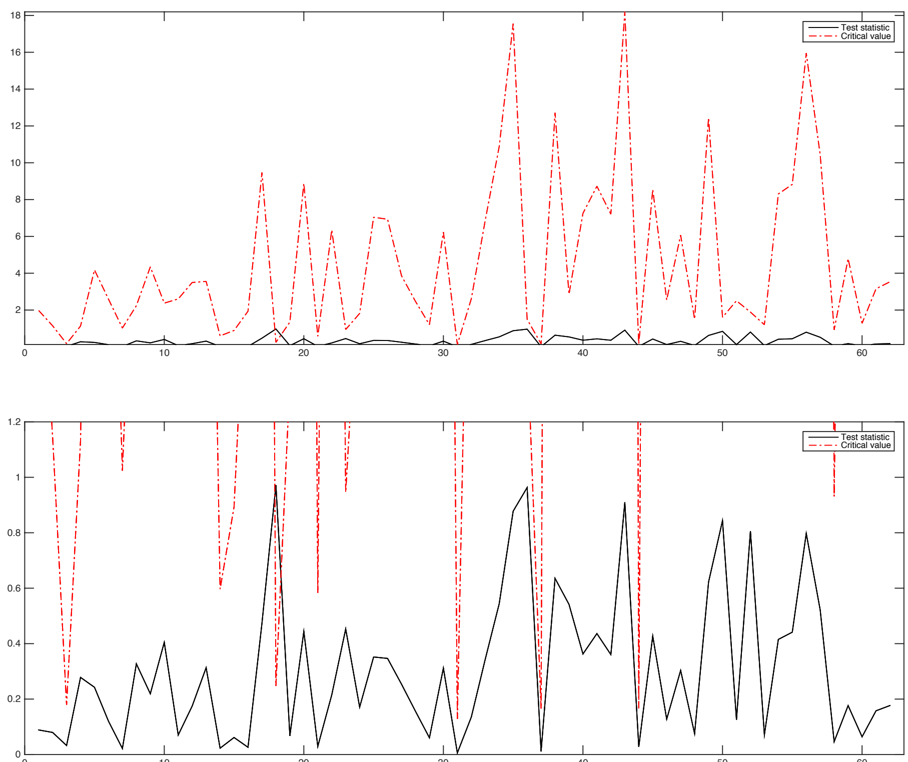
Note. For GLD/GVZ we plot the test statistics $C_n^j = |\Phi^{(j)} - 1|$ computed from Eq.(19) (black solid line) and the critical value $c^{(j)}$ computed from Eq.(26) (red dotted line) to test H_0 : Jumps arrive jointly in GLD and GVZ. We document the rejection of the null in 85.48% of all cases (reported in Table 4).

Figure 13: Distribution of the test statistic and the critical value for H_0 : Joint jumps



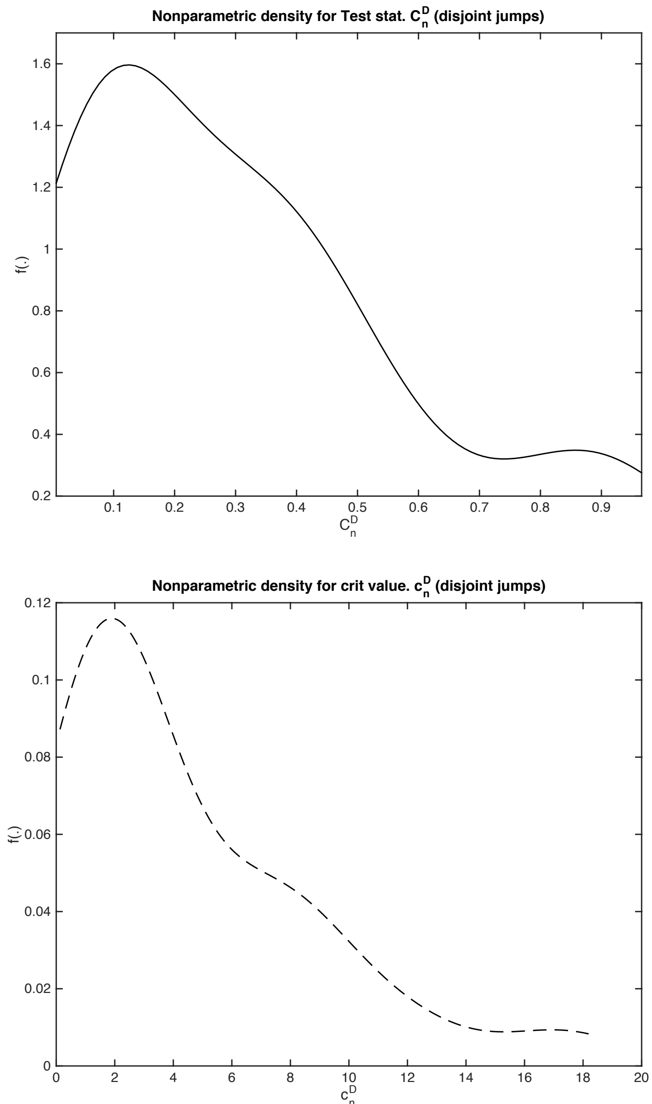
Note. For GLD/GVZ we plot in the top panel: the non-parametric density estimate for the test statistic $C_n^j = |\Phi^{(j)} - 1|$ computed from Eq.(19) for the H_0 : Joint jumps. Bottom panel: the corresponding non-parametric density estimate for the critical value $c^{(j)}$ computed from Eq.(26). We can observe that the magnitude of the values for the test statistic (upper panel) is typically much higher than the ones for the critical value (bottom panel), which leads to rejection of the H_0 : Joint jumps in 85.48% of all cases (reported in Table 4).

Figure 14: Test statistic and critical value for H_0 : Disjoint jumps



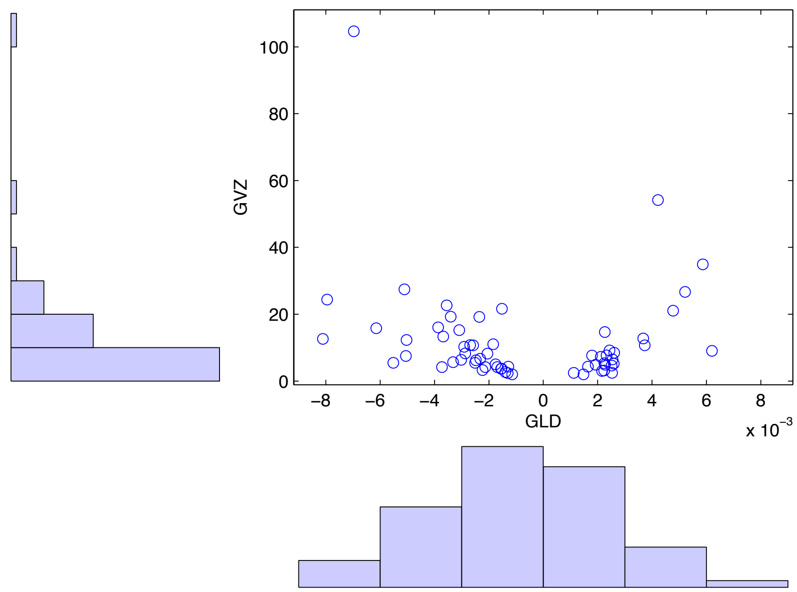
Note. For GLD/GVZ we plot the test statistics $\Phi_n^{(d)}$ computed from Eq.(20) (black solid line) and the critical value $c^{(d)}$ computed from Eq.(27) (red dotted line) to test H_0 : Jumps arrive disjointly in USO and OVX. The bottom panel is a zoomed-in version of the top panel. We document the rejection of the null in 3.12% of all cases (reported in Table 4).

Figure 15: Distribution of the test statistic and the critical value for H_0 : Disjoint jumps



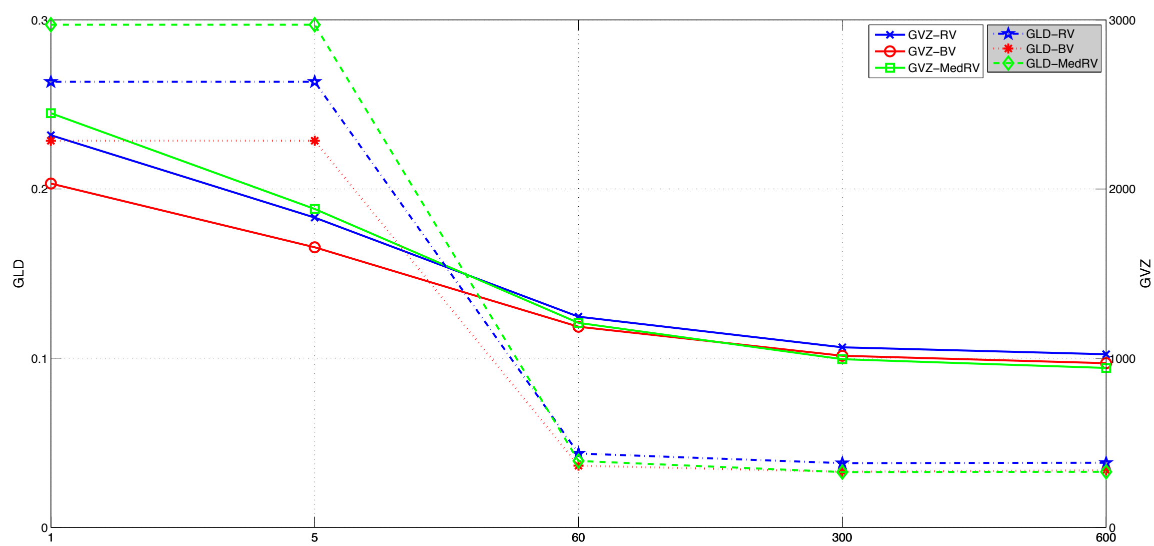
Note. For GLD/GVZ we plot in the top panel: the non-parametric density estimate for the test statistic $\Phi_n^{(d)}$ computed from Eq.(20) for the H_0 : Disjoint jumps. Bottom panel: the corresponding non-parametric density estimate for the critical value $c^{(d)}$ computed from Eq.(27). We can observe that the magnitude of the values for the critical value (bottom panel) is typically much higher than the ones for the test statistic (top panel), which leads to rejection of the H_0 : Joint jumps in only 3.12% of all cases (reported in Table 4).

Figure 16: Scatter plot and histogram for jump sizes in GLD and GVZ



Note. Scatter plot and histogram for the jump sizes in GLD and GVZ for days where *both* series jump (but jumps in GVZ are positive).

Figure 17: Signature plots for GLD and GVZ



Note. We plot signature plots for GLD (left vertical axes) and GVZ (right vertical axes) as a function of time (depicted in seconds on the horizontal axis). RV and BV are computed using Eq.(6) and Eq.(7), respectively. MedRV is computed using Eq.(39).