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Critically Assessing Estimated DSGE Models: A Case Study of a Multi-Sector Model*

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Abstract

We describe methods for assessing estimated Dynamic Stochastic General Equilibrium (DSGE) models. One involves the computation of alternative impulse responses from models constrained to have an identical likelihood and the same contemporaneous signs as responses in the DSGE model. Others ask how well the model matches the data generating process; whether there is weak identification; the consequences of including measurement error with growth rates of non-stationary variables; and whether the model can reproduce features of the data that involve combinations of moments. The methods are applied to a large-scale small-open economy DSGE model, typical of those used at policy institutions.

1 Introduction

Estimated Dynamic Stochastic General Equilibrium (DSGE) models today are commonplace both in academia and in policy institutions, such as central banks. There are standard ways in which these models are used, such as the production of impulse response functions and a decomposition of the observed variables used in estimation into the contributions from each of the structural shocks. Arguably, there is often much less attention paid to assessing the output of these DSGE models post-estimation. This paper presents a collection of methods that can be used to do so.

Throughout the paper we focus our analysis on a multi-sector model of the Australian economy (MSM in this paper) which has been developed by Rees et al (2016) at the Reserve Bank of Australia. This DSGE model features seventeen shocks and three production sectors: (i) non-traded commodities and services, (ii) traded non-resource commodities and services, and, (iii) traded resources. It has a unit root process for the log of technology so that some

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of the variables in the model follow integrated processes that are also co-integrated. What makes the model innovative is the presence of three sectors, and in many ways it can be seen as an extension of the dependent economy model whose origins are strongly Australian - see Metaxas and Weber (2016). A key element in it is a real exchange rate, but it also allows for nominal rigidities, so it is potentially a very useful model for policy analysis.[1]

Data on seventeen variables were used to estimate the parameters of MSM. In addition to those commonly used for small-open economy DSGE models, such as GDP growth, inflation, the real exchange rate and the policy rate, a variety of others included reflecting sector-specific variables. The foreign sector was captured through three core variables - GDP growth, inflation and a policy rate.[2] An extra variable in MSM that is external to the Australian economy is the price of resources. The foreign variables (and resource prices) are strictly exogenous to the Australian economy and so the model is of a small dependent economy. Estimation was performed with Bayesian methods, requiring some prior distributions for the DSGE model parameters to be stated. After estimation some experiments were done with the model in order to assess features such as the impact of monetary and risk premium shocks.

There are many issues raised when assessing output from any DSGE model such as MSM.[3] Section 2 looks at one of these - whether there are other models which have the same generating process for the variables as that given by the estimated DSGE model, but which have different impulse responses to shocks. We find such alternative models, after imposing two restrictions. First the alternative model must fit the data equally well as the DSGE model. Secondly, the impulse responses to the named shocks from any alternative model must have the same contemporaneous signs as those given by the estimated DSGE model, in this case the MSM. To perform this task we utilize the fact that a structural model like a DSGE model solves for a Vector Autoregression in the variables, and this can be written in such a way as to highlight the structural shocks from the DSGE model. We refer to the resulting representation as a *semi-structural VAR (SSVAR)* model. Because many DSGE models, including MSM, imply that there is co-integration between certain variables, Section 3 considers different representations of the basic SSVAR from MSM, moving towards a semi-structural Vector Error Correction. Such a representation is useful for a number of analyses of MSM output.

Another reason it might be possible to find multiple models with the same generating process is that there could be many values for the estimated parameters which produce the same likelihood. If so, then there would be many different values for impulse responses. This would be an example of a failure of structural identification of the DSGE model parameters,

[1]Another reason for studying MSM is that its documentation is outstanding.

[2]Throughout this paper we try to use the same notation as in the MSM, with $*$ denoting the foreign sector; for example p and p^* would be the logs of the domestic and foreign price levels respectively. The complete list of variables whose data are used in estimation includes the growth rates in GDP (Δy^{va}), consumption (Δc), investment (Δi), public demand (Δg), resource exports (Δz^x), non-resource exports (Δy_m^x), non-tradeable value added Δy_n^{va}, non-resource tradeable value added (Δy_m^{va}), resources valued added (Δy_z^{va}), domestic inflation π_t, non-tradeable inflation (π_n), the Australian cash rate (r) and the change in the nominal exchange rate (Δs). There are also data on growth in foreign GDP (Δy^*), inflation (π^*), a short term interest rate (r^*) and resource price inflation (Δp_z^*).

[3]The approaches adopted here are not meant to be exhaustive - see, for example, Schorfheide (2013) for an earlier discussion. The methods we advance can be thought of as complements to existing techniques.

and so it is important is to check for such problems. Basically one wants to check the shape of the likelihood (or whatever function is being optimized to produce parameter estimates). Some identification measures are now in Dynare and these were set out in Ratto and Iskrev (2010). In Section 4 we discuss these measures and apply them to the MSM model. Another approach, described in Koop et al (2013), uses a convergence rate indicator to flag parameters that are possibly only weakly identified. This involves simulation of the estimated DSGE model and, in Section 4, we suggest another way that the simulated data can be fruitfully used to judge identification issues. It appears that some of the crucial parameters in the MSM, namely the slopes of Phillips curves, may be weakly identified.

Finally, we have the fundamental question of how the generating process of the selected DSGE model (the *model generating process MGP*) matches the *data generating process (DGP).* That there can be a gap between these comes from the fact that some of the DSGE model variables may be unobserved i.e. do not have an exact analogue in the data. In that situation, although the MGP for all variables may be a VAR, this may not be true of the DGP for the observed variables. Section 5 looks at this in a number of stages. Given a distinction between the data and the model generated variables, it seems natural to ask how one might bridge these? In many DSGE models, including MSM, reconciling the model and data is often partially done by allowing for "measurement error" in the data. Section 5.1 examines how productive this approach is, pointing out the difficulties with it when data is measured as growth rates in I(1) variables.

Section 5.2 moves on to ask whether dropping the unobservable variables means that the form of the MGP for the complete MSM differs from the format of the generating process for a reduced number of variables. Using impulse responses to measure the correspondence between these two generating mechanisms we find that the SSVAR(2) implied by the MSM is reasonably well approximated by an SSVAR(2) in just the observable variables. In Section 5.3 we ask how well certain features of the MGP match the DGP of the observable variables. This is a quantitative test. Simple statistics such as moments can be informative about this question, and more complex ones, such as business cycle outcomes, can be important for conceptualizing what any failure to match the data means. Finally, at various times we utilize the different representations of Section 3 in order to shed light on a failure of data and the MSM model moments to match.

Many of the issues we will address are common to a large number of DSGE models. What made the MSM an attractive vehicle for analysis is that it is a fairly large model, and so poses questions that don't arise to the same degree for many of the smaller New Keynesian models one sees in use in academia, largely because they feature only one sector. But there is a great need to develop multi-sector models for policy analysis, as seen by the use of large scale computable general equilibrium models in analyzing many issues such as the shifting of taxes, carbon pricing etc.; Murphy (2017) has a good discussion of these uses. Hence there seems to be a need to have methods that can analyze the answers provided by such models to policy-type questions. In DSGE models these typically revolve around impulse response functions, and so our paper essentially examines how robust these are to the assumptions employed in constructing the models they are derived from as well as the quality of the match with the data.

2 Examining the Model Generating Process of MSM

2.1 Generating a Range of Models Compatible with a Given Model Generating Process

Let the DSGE model have parameters θ. Then its variables, z_t, will be generated using the model with these parameters set to some estimated values θ^*. We will call this the *Model Generating Process (MGP)*. Suppose there are other models that have the same generating process as that found with θ^*, i.e. the MGP, but with different impulse responses to shocks. In that case, the location of the responses of the DSGE model in this range is a useful indicator of the uncertainty surrounding such responses when used for policy analysis. This is *different to the statistical uncertainty* coming from the fact that the parameters are estimated. Rather it is *model uncertainty*, reflecting the fact that there are other models compatible with the MGP but which produce different reactions to shocks. The focus of this sub-section is to demonstrate how it is possible to quantify the extent of model uncertainty.[4] In particular, we show how to produce such a range of models and define what we mean by compatibility.

Variables in DSGE models can be taken to be $I(0)$, perhaps after some transformation. The most common transformation needed is to convert $I(1)$ variables into $I(0)$ variables by de-trending them with the level of technology. We will focus on this later but, for now, assume that the z_t in a DSGE model are all $I(0)$ variables. In most instances the DSGE model has the structural equations[5]

$$A_0 z_t = CE_t(z_{t+1}) + A_1 z_{t-1} + Hu_t, \tag{1}$$

where u_t are shocks possibly following a VAR(1), $u_t = \Phi u_{t-1} + \varepsilon_t$, and ε_t is a vector of white noise structural shocks with covariance matrix Ω that is diagonal. The latter are generally referred to as innovations to the structural shocks and we will use that terminology here. A_0, C, A_1 and H are matrices which are functions of θ. This system can then be solved for z_t by using (for example) the method of undetermined coefficients. This produces a solution

$$z_t = Bz_{t-1} + Gu_t.$$

Binder and Pesaran (1995) present the two relevant conditions for this solution to exist, namely a rank condition and the Blanchard-Kahn stability conditions. Hence

$$\begin{aligned}
z_t &= Bz_{t-1} + G(\Phi u_{t-1} + \varepsilon_t) \\
&= Bz_{t-1} + G\Phi(G^+(z_{t-1} - Bz_{t-2})) + G\varepsilon_t \\
&= (B + G\Phi G^+)z_{t-1} - G\Phi G^+ Bz_{t-2} + G\varepsilon_t \\
&= B_1 z_{t-1} + B_2 z_{t-2} + e_t,
\end{aligned}$$

where $e_t \equiv G\varepsilon_t$ are the VAR error terms, G^+ is the (possibly generalised) inverse of G, $B_1 \equiv (B + G\Phi G^+)$ and $B_2 \equiv -G\Phi G^+ B$. This is a VAR(2) in which the VAR errors have been written as functions of the structural shocks ε_t. For convenience we will refer to this

[4] The presentation of the solution of the DSGE model draws on Pagan and Robinson (2016).

[5] DSGE models with long lags can be accommodated by expanding z_t to include lagged variables. The subsequent analysis is similar; the current values of variables would have to be selected from z_t.

as a *semi-structural VAR (SSVAR)*. As we will see later the MSM can be expressed in this form.

Now let us write

$$\begin{aligned} z_t &= B_1 z_{t-1} + B_2 z_{t-2} + G\Sigma\Sigma^{-1}\varepsilon_t \\ &= B_1 z_{t-1} + B_2 z_{t-2} + G\Sigma\eta_t, \\ &= B_1 z_{t-1} + B_2 z_{t-2} + F\eta_t, \end{aligned}$$

where Σ is a diagonal matrix containing the standard deviations of ε_t and $F \equiv G\Sigma$. The resulting η_t will have unit variances but the impulse responses to η_t *are the same as those for a one standard deviation perturbation to whatever the shocks* ε_t are named.

That there are other models with different structural impulse responses can be seen by writing

$$\begin{aligned} z_t &= B_1 z_{t-1} + B_2 z_{t-2} + FQ'Q\eta_t \\ &= B_1 z_{t-1} + B_2 z_{t-2} + D\tilde{\eta}_t, \end{aligned}$$

where Q is a matrix with the property that $QQ' = Q'Q = I, D = FQ'$ and $\tilde{\eta}_t = Q\eta_t$.[6] Then the new shocks $\tilde{\eta}_t$ will be uncorrelated and the impulse responses to them will also be to to a one standard deviation perturbation in whatever they are named. So the contemporaneous impulse responses have been changed from F to D, i.e. we have a new model.

In what sense is the new model compatible with the MGP of the original model with shocks η_t? The answer is that, since the $cov(e_t) = cov(G\Sigma\eta_t) = cov(G\Sigma Q'\tilde{\eta}_t)$, the model with $\tilde{\eta}_t$ shocks produces the same covariance matrix for the VAR errors. Because B_1 and B_2 have not changed, the density function for z_t must be the same for both models i.e. the likelihood has not changed. The new and existing model fit the data equally as well.[7]

Alternative Q matrices will therefore be the approach used to study the range of impulse responses that are compatible with the MSM model, which is a way to quantify the extent of model uncertainty present. The nature of this analysis with the semi-structural VAR has strong parallels with sign-restricted VARs, but B_1 and B_2 here are anchored by the DSGE model.

2.2 The Nature of Shocks in the Estimated MSM Model

Before we proceed to further analysis it is necessary to make clear what the constraints are when generating any new set of impulse responses. Specifically, we *will not be generating* impulse responses that produce a *better match* to the data. Instead, we will be generating responses that constrain the shocks to be uncorrelated and which replicate certain hypothetical results from the MSM model. In these hypothetical results the dynamic parameters of

[6] Fry and Pagan (2011) discuss Q matrices that have this property. The best known of them is the Givens matrix used in the sign restriction literature by Canova and de Nicolo (2002) while Rubio-Ramirez et al (2010) give a general way of finding a Q matrix with the requisite properties using simulation methods. We use an adaption of that method in what follows.

[7] In this respect the SSVAR is distinctly different to the DSGE-VAR literature, such as Del Negro and Schorfheide (2004).

the model B_1 and B_2 are fixed at values estimated from data, while the structural shocks are *assumed* to be uncorrelated. We raise this issue since it is not the case that the MSM shocks estimated from the data are uncorrelated. Table 1 shows some of the larger correlations.[8]

Table 1: Correlations of Selected Shocks from the Estimated MSM Model

Shock Pair	Correlation
corr($\varepsilon_r, \varepsilon_{y^*}$)	.67
corr($\varepsilon_{r^*}, \varepsilon_\psi$)	-.36
corr($\varepsilon_{\pi^*}, \varepsilon_\psi$)	.44
corr($\varepsilon_{p^*}, \varepsilon_r$)	.34
corr($\varepsilon_g, \varepsilon_\Upsilon$)	-.56
corr(ε_{r^*}, μ)	-.53
corr($\varepsilon_f, \varepsilon_n$)	.78
corr($\varepsilon_{m^*}, \varepsilon_{a_m}$)	.62
corr($\varepsilon_m, \varepsilon_n$)	.32

To understand why Table 1 can record such non-zero correlations, even though they are assumed zero in estimation, take the two regression equations

$$\begin{aligned} y_{1t} &= x_{1t}\beta_1 + \varepsilon_{1t} \\ y_{2t} &= x_{2t}\beta_2 + \varepsilon_{2t}, \end{aligned}$$

and *assume* that ε_{1t} and ε_{2t} are uncorrelated when estimating β_1 and β_2. Then, based on that assumption, the maximum likelihood estimates of β_1 and β_2 would be the OLS estimates. However, nothing guarantees that the residuals $\hat{\varepsilon}_{jt}$ formed from these are orthogonal. For this to be asymptotically true it would be necessary that the assumption of zero correlation between the shocks is correct. If this assumption is not true in the data, then $\hat{\varepsilon}_{jt}$ will not be orthogonal. Of course the estimators of β_j in the example above are consistent, even if there is correlation.

The reason for the lack of orthogonality is that we have two moment conditions defining the parameter estimates - $E(x_{jt}\varepsilon_{jt}) = 0, (j = 1, 2)$ and two other conditions defining the shock variances - $E(\varepsilon_{jt}^2) = \sigma_j^2$ - meaning that the moment condition $E(\varepsilon_{1t}\varepsilon_{2t}) = 0$ has not been needed for estimation of the parameters. So the moment conditions deliver more than is needed to get estimators of the parameters i.e. we have an over-identified model.

Suppose instead we had written

$$\begin{aligned} y_{1t} &= x_{1t}\beta_1 + \varepsilon_{1t} \\ y_{2t} &= x_{2t}\beta_2 + \gamma y_{1t} + \varepsilon_{2t}. \end{aligned}$$

Then three moment conditions are needed to estimate all the unknown parameters, the system is *exactly identified*, and the residuals are orthogonal.

The last scenario is a characteristic of just-identified SVAR models and it is this that ensures shocks in such models are orthogonal. In contrast, DSGE models typically are heavily

[8]The symbols are as in the MSM.

over-identified. To see this, note that even in large DSGE models often only a small number of structural parameters are estimated. For example, in Smets and Wouters (2007), 36 parameters are estimated with 7 observed variables; a just-identified SVAR(2), in contrast, would include 126 estimated parameters. In MSM it is even more stark; 45 parameters are estimated, but there are 600 moment conditions. The theory built into these models results in them often being very tightly parameterised.

In an overidentified system, not all of the moment conditions are used in estimation or can be satisfied at once. Consequently, even though it is *assumed that the shocks are uncorrelated*, this restriction may not be used in estimation and the shocks obtained from an estimated DSGE model may well be correlated.[9] This provides one explanation of the results in Table 1.

A second, supplementary, reason involves the use of Bayesian estimation. The Bayesian estimates of parameters can be thought of as a weighted average of the Maximum Likelihood Estimates (MLE) and the prior. In the exactly identified case the MLE would satisfy all moment conditions, including those that define uncorrelated shocks. However, because the Bayesian estimates weight the MLE and the prior mean they will clearly not necessarily satisfy the conditions which make the shocks uncorrelated. So another possible explanation is that Table 1 reflects Bayesian estimation.[10] Because the MSM is highly over-identified it seems more likely that the results are due to that feature.

It should be said that this is not the only problem that can arise for model shocks from over-identifying information. The restriction that the *innovations* to the structural shocks have no serial correlation may not be enforced. Table 2 shows that this is clearly evident for the MSM; for example the supposed innovation to technology growth actually has an autocorrelation coefficient of 0.64. The phenomenon is not isolated to the innovations for the structural shocks, but can also be true for the measurement errors. As Table 2 shows the autocorrelation for those shocks for resource prices is particularly high (0.76). So the shocks and the measurement errors are often very different from what has been assumed about them. When it comes to data, assuming that they are white noise does not make them so, as regression users learnt many years ago.

What are the consequences of the shocks being correlated? From the perspective of using the model, it makes variance decompositions problematic, and one also really needs shocks to be innovations in order to utilize impulse responses, for these work under a *ceteris paribus* assumption that would not hold when shocks are correlated. Thus, if two shocks, say money and technology, are correlated, then we don't know what the common component to them represents - is it money or technology? Decompositions of variables according to shocks are then impossible to interpret.

[9] Andrle, M. (2014) has made this point as well and says - "Yet, the actually estimated 'structural' shocks are strongly correlated as a rule rather than exception. Correlated structural shocks are a sign of misspecification". As our simple example shows this need not be the case, although it is certainly inconsistent with the statistical assumptions being made for estimation.

[10] We have looked at the shocks in the Smets and Wouters (2007) model and find that there the shocks are correlated but the correlations are smaller than for the MSM, probably because the degree of over-identification is very much smaller. Of the 21 shock correlations 4 exceed .2 in absolute value.

Table 2: Autocorrelation of Selected Shock Innovations from the Estimated MSM Model

Innovation	Mnemomic	Autocorrelation
Technology Growth	ε_{μ}	.64
Investment Efficiency	ε_{Υ}	-.33
Resource Price	$\varepsilon_{p_z^*}$	.40
Foreign Output	ε_{y^*}	.64
Monetary Policy	ε_r	.59
Measurement Error		
Nominal Exchange Rate		.39
Foreign GDP		.52
Resource Price		.76

Of course we can take the estimated parameters B_1, B_2 as given and then ask what the impulse responses are in this hypothetical context, that is *assuming* that the shocks are uncorrelated even though this assumption may not be compatible with the data. The variance decompositions presented in Rees et al (2016) are a hypothetical experiment and relate to consistency of the model itself, as the experiment does not represent the situation with the estimated shocks and the data variables. Essentially they are constructing a scenario i.e. asking what would the results be if the estimated parameter values were used for B_1 and B_2, and then uncorrelated shocks are applied rather than the actual shocks found from the data. In what follows we will adopt this same scenario, constructing models and impulse responses which utilize the estimated B_1, B_2 and which reproduce the $cov(e_t)$ from the experiment they performed.

2.3 Constructing the Alternative Models in Practice

One difficulty in constructing alternative models that have the same fit as the hypothetical MSM model, but which embody different impulse responses, is to make decisions about which shocks should be recombined. To understand the issue consider the three variable SSVAR given by Equations 2 to 4 that we wish to capture with an alternative model:

$$y_{1t} = b_{11}^1 y_{1t-1} + b_{12}^1 y_{2t-1} + b_{13}^1 y_{3t-1} + f_{11}\eta_{1t} \quad (2)$$
$$y_{2t} = b_{21}^1 y_{1t-1} + b_{22}^1 y_{2t-1} + b_{23}^1 y_{3t-1} + f_{21}\eta_{1t} + f_{22}\eta_{2t} + f_{23}\eta_{3t} \quad (3)$$
$$y_{3t} = b_{31}^1 y_{1t-1} + b_{32}^1 y_{2t-1} + b_{33}^1 y_{3t-1} + f_{31}\eta_{1t} + f_{32}\eta_{2t} + f_{33}\eta_{3t}. \quad (4)$$

In this model only one shock affects y_{1t}. Now consider constructing new shocks $\tilde{\eta}_{jt} = q_{j1}\eta_{1t} + q_{j2}\eta_{2t} + q_{j3}\eta_{3t}$ using a Q matrix. Then we see that all of the shocks $\tilde{\eta}_{jt}$ will have an impact upon y_{1t} because they are formed with η_{1t}. However, we may not want all of the $\tilde{\eta}_{jt}$ shocks to impact upon y_{1t}, as this would not preserve the fact that only one of the $\tilde{\eta}_{jt}$ should have a non-zero impact on it. Consequently a Q matrix must be designed that preserves the zero impact of some shocks. The simplest way to do this is to set $q_{11} = 1, q_{12} = 0, q_{13} = 0, q_{21} = 0$ and $q_{31} = 0$. Consequently, $\tilde{\eta}_{1t} = \eta_{1t}$ and $\tilde{\eta}_{jt} = q_{j2}\eta_{2t} + q_{j3}\eta_{3t}$ $(j = 2, 3)$. This means that we use a $(3{\times}3)$ Q matrix of the form

$$Q = \begin{bmatrix} 1 & 0 \\ 0 & Q_2 \end{bmatrix},$$

where the (2×2) matrix Q_2 has the properties $Q_2'Q_2 = I_2 = Q_2Q_2'$. Clearly Q has the requisite properties of $Q'Q = I_3 = QQ'$.[11]

As an illustration of applying this methodology in the context of the MSM we note that in it the government expenditure variable is strongly exogenous, i.e. it has the form of y_{1t} in (2) above but with $b_{12}^1 = 0, b_{13}^1 = 0$. So we would not want to form new shocks by combining the government expenditure shock with others.

A similar issue arises with the external sector. The MSM has a simple New-Keynesian representation for the foreign sector of the form

$$\begin{aligned} \tilde{y}_t^* &= E_t(\tilde{y}_{t+1}^*) - (r_t^* - E_t(\pi_{t+1}^*)) + u_{yt}^* && (5) \\ \pi_t^* &= \beta E_t(\pi_{t+1}^*) + \kappa \tilde{y}_t^* + u_{\pi t}^* && (6) \\ r_t^* &= \rho_r r_{t-1}^* + (1-\rho_r)(\gamma_y \tilde{y}_t^* + \gamma_\pi \pi_t^*) + \delta \Delta \tilde{y}_t^* + \varepsilon_{rt}^*. && (7) \end{aligned}$$

Here the variables in the model are foreign ones and so are distinguished with an asterisk. u_{yt}^* and $u_{\pi t}^*$ are AR(1) shocks driven by innovations ε_{yt}^* and $\varepsilon_{\pi t}^*$, y_t^* is the log level of foreign output, $\tilde{y}_t^* = y_t^* - a_t$, where a_t is the log of the level of technology, and π_t^*, r_t^* are the foreign inflation and interest rate. There is a separate equation, namely

$$\Delta y_t^* = \Delta \tilde{y}_t^* + \Delta a_t = \Delta \tilde{y}_t^* + \mu_t, \quad (8)$$

where μ_t is the innovation into technology described in the MSM model. We will not re-combine this technology shock μ_t with the other external shocks in forming new shocks. The reason is that μ_t is a permanent shock and the other three are transitory. As Fry and Pagan (2011) observed one cannot combine permanent with transitory shocks if you want some of the final shocks $\tilde{\eta}_t$ to be transitory. Given that all shocks (apart from technology) are transitory in MSM, it is not sensible to include the technology shock in the set to be re-combined.[12]

Finally, we have foreign and domestic shocks. In the MSM Australia is assumed to be a small open economy, that is, the foreign sector is strictly exogenous. However, if some of the newly created shocks were obtained by combining the external and domestic shocks then the small open economy assumption would be violated. Consequently, by constructing new uncorrelated shocks from the hypothetical uncorrelated shocks of the MSM those shocks that are not re-combined are left at the MSM estimates, and so they will be uncorrelated with any combination of the other shocks.

The MSM external sector has a SSVAR(1) format

$$z_t = B_1 z_{t-1} + \varepsilon_t^{MSM},$$

[11] Arias et. al. (2014) have another way of generating Q that preserves zero restrictions. Consequently, our choice may not exhaust the range of possible Q matrices and alternative models that might be generated, but that would only mean that our range is possibly smaller. Because it turns out that our range is large this does not seem a crucial issue, but further investigation seems warranted.

[12] If we combined μ_t with the transitory shocks to create new ones $\tilde{\eta}_t$, these would have permanent effects and that would mean there is more than one supply-side shock.

where $z_t' = \left[\begin{array}{ccc} \tilde{y}_t^* & \pi_t^* & r_t^* \end{array} \right]$. Hence, using the parameter values given by Rees et al the solution to the external system is an SSVAR(1) of the form[13]

$$\begin{aligned} \tilde{y}_t^* = & \quad .865\tilde{y}_{t-1}^* - .248\pi_{t-1}^* + .083r_{t-1}^* + .003\varepsilon_{yt}^{*MSM} - .005\varepsilon_{\pi t}^{*MSM} - .049\varepsilon_{rt}^{*MSM} && (9) \\ \pi_t^* = & \quad .108\tilde{y}_{t-1}^* + .269\pi_{t-1}^* + .123r_{t-1}^* + .0006\varepsilon_{yt}^{*MSM} + .013\varepsilon_{\pi t}^{*MSM} - .009\varepsilon_{rt}^{*MSM} && (10) \\ r_t^* = & \quad .954r_{t-1}^* + .005\tilde{y}_{t-1}^* - .0095\pi_{t-1}^* + .0005\varepsilon_{yt}^{*MSM} + .0005\varepsilon_{\pi t}^{*MSM} + .001\varepsilon_{rt}^{*MSM}. && (11) \end{aligned}$$

The equations (9)-(11) are identities. The shocks, such as ε_{yt}^{*MSM}, can be converted to the corresponding unit variance shocks, η_{yt}^*, simply by re-scaling the coefficients attached to ε_{yt}^{*MSM} by the standard deviations of ε_{yt}^{*MSM} etc.. As an example, the interest rate equation becomes

$$r_t^* = .954r_{t-1}^* - .0095\pi_{t-1}^* + .005\tilde{y}_{t-1}^* + .000098\eta_t^{r^*} + .0001\eta_t^{\pi^*} + .0065\eta_t^{y^*}. \qquad (12)$$

Then a new set of shocks $\tilde{\eta}_t^*$ needs to be constructed that are linear combinations of the original normalised shocks, η_{yt}^* etc.. This must be done in such a way as to ensure that they are uncorrelated with unit variance. To be clear, the new SSVAR will still have the same dynamics i.e. B_1 is fixed at the MSM estimated values, and the covariance matrix for the hypothetical reduced-form errors will be replicated by the new set of innovations $\tilde{\eta}_t^*$. Consequently the alternative model fits the data equally well as the MSM external sector specification. However, these new shocks $\tilde{\eta}_t^*$ will have different impulse responses, and it is useful to look at the range of responses that can be generated. Of course a wider range of impulse responses might be found that produce a superior fit by allowing changes in the dynamics as well. Our focus here, however, is to gauge the extent of model uncertainty for a given fit with the data.

Some of the alternative models that we generate with uncorrelated shocks might be ruled out. This could be because they produce responses of unrealistic magnitude. A weaker constraint is to eliminate models that do not produce the same signs for contemporaneous impulse responses as the MSM does. Table 3 gives the latter for (positive) shocks to the structural equations (5)-(7).

The contemporaneous impulse response functions for the new shocks will be compared to the signs from those in the MSM given in Table 3. If they agree the impulse responses corresponding to the new shocks are accepted. If they don't then we draw a new Q, and once again combine together the three MSM shocks. It is important to emphasize that we have retained the MSM dynamics in this operation, i.e. B_1 is fixed.

Generating 1000 models with uncorrelated shocks by re-combining the MSM ones we find that 118 satisfy the sign restrictions in Table 3. Now, we will be finding impulse responses in the alternative models to a one standard deviation perturbation. But in each model there will be a different standard deviation for shocks such as the monetary one. Consequently we need to re-scale these impulse responses so that they are comparable to the MSM results (we will refer to these as the *standardized shocks*).[14]

Focussing on the results for a monetary policy shock, these are shown in Table 4. Considering first the contemporaneous impact on output, it is apparent that the MSM responses

[13]This was obtained using the simulation method in Pagan and Robinson (2016).

[14]This requires us to find the implied standard deviation of the monetary shock in the alternative models. To do this we use the method given in Ouliaris and Pagan (2016).

Table 3: Signs for Contemporaneous Responses To Three Shocks in the MSM External Sector

Variable	Shocks		
	Demand	Cost	Interest Rate
$\tilde{y}^*$	>0	<0	<0
π^*	>0	>0	<0
r^*	>0	>0	>0

are very much at the high end of the scale for monetary effects. In fact there are only three impulse responses of the 118 that are larger (in absolute terms) than that given by the estimated MSM. Therefore, it is possible to find models that are observationally equivalent to MSM (in the sense of replicating second moments) but which deliver a much lower impact for monetary policy. This is the type of result that can be produced from the semi-structural VAR approach that we believe will be of interest to policymakers when assessing the output produced from estimated DSGE models.

Table 4: Contemporaneous Responses to a Standardized Monetary Policy Shock in the MSM Foreign Sector

Magnitude	Variables	
	Output	Inflation
Maximum	-.437	-.276
Minimum	-.0004	-.0001
MSM	-.346	-.066

Turning to inflation, it appears that the result predicted by MSM is less extreme. As has been argued by Ouliaris and Pagan (2016), *inter alia,* it is useful to think of the average of the maximum and minimum values as a representative value. With that choice the MSM results are found to be about 1/2 of the representative value.[15] These results suggest that one either might need to look more closely at the specification of the New-Keynesian model which is at the heart of the foreign sector of MSM or at least perform experiments with the equivalent models producing the minimum and maximum responses when conducting policy analysis. Of course based on the magnitudes of the responses one might be able to rule out some of the range of estimates.[16]

[15]Baumeister and Hamilton (2015) pointed out that statistics such as the median of the range of outcomes depended upon the simulation method employed and so were not especially informative. This is illustrated in a simple way in Ouliaris and Pagan (2016).

[16]Regarding the foreign interest rate identity, Equation (12) shows that the foreign demand shock is the dominant force in the evolution of the foreign interest rate. Indeed, in the variance decomposition for the external system the foreign demand shock η_{y^*} explains 99.12% of the variance of the foreign interest rate, while the monetary shock explains virtually nothing. This seems a little odd. Furthermore, 41% of the foreign output gap variance is due to the monetary policy shock, an impact of magnitude rarely seen in small New-Keynesian models.

Now it is important to emphasize that we have only considered alternative models that have the impulse response signs of Table 3 for a limited range of variables. But the MSM also produces impulse responses of domestic variables to these external shocks and they have a set of signs implied by the MSM model. Hence we might reject an alternative model if it fails to reproduce the signs of *all variables*, both domestic and foreign. When we do this some of the models above will be rejected. Indeed, of the 118 models found above, only 3 are now retained as agreeing with all the signs for impulse responses. Therefore, this requires the generation of many more models than the 1000 used before in order to study the range of impulse responses that are possible. Accordingly, we generated 100000 models, 77 of which were retained as providing a complete match with the signs of the impulse responses from the MSM (ignoring the level of government expenditure variable where there are zero effects of all shocks). Now the biggest and smallest effects are at -.0353 and -.001, so the range has narrowed, with the MSM output response being virtually the largest in the complete set of alternative models. The same outcome is true of the inflation response, where the minimum is now -.0359 and the maximum is -.1072.

Moving on to alternative domestic shocks, for the reasons given above we will only combine together the domestic MSM shocks, excluding the foreign ones. A further issue is how many shocks to combine together when forming the new shocks? We will use three of these from the MSM - the monetary shock, the marginal efficiency of capital shock and the risk premium shock - keeping all the others at their MSM values. One reason for using a smaller number than the total is that, once one starts combining a large number of shocks with very different impulse responses, it may be very hard to find models that satisfy a set of signs for (say) monetary policy effects, and it might be necessary to simulate many millions of models. Indeed, in the following work, 10 million models had to be simulated in order to get a reasonable number of acceptances.

Reflecting the small-open economy assumption, the domestic shocks must have a zero impact on foreign variables, and consequently we look at the signs of the responses of the non-foreign variables. Table 5 shows the MSM responses to the three domestic shocks. It is worth observing that, since the MSM shocks which are being combined all have a zero impact upon foreign variables, then so will any combination of them. It is also true that the shocks that are not combined must have zero correlation with any combination of those that are, simply because they are uncorrelated with the individual shocks making up the combination (this being the nature of the hypothetical experiment).

In assessing the response of output and inflation to a monetary shock we examine the results when either we impose the signs on just 3 variables – GDP growth, inflation and the interest rate – or on all 18 domestic variables (Table 6). The MSM value for the output response is around the middle of the range obtained, in contrast to the extreme results obtained from the foreign sector. This, however, is not so true for the inflation response, which appears to be quite large. Little difference was found in the range of impulse responses obtained when either 3 or 18 signs need to be matched; a benefit of the former is that it is likely to require less draws to obtain a suitable number of acceptances. Overall, our analysis using the SSVAR suggests that the responses obtained from the MSM for domestic monetary policy shocks are reasonable, whereas their foreign counterparts seem extreme.

Table 5: Signs of Impact of Three Domestic MSM Shocks

Variable		Shocks		
Description	**Mnemonic**	**Monetary**	**Investment Efficiency**	**Risk Premium**
		ε_r	ε_Υ	ε_ψ
GDP	$\tilde{y}_{va}$	<0	>0	<0
Inflation	π	<0	<0	<0
Interest rate	r	>0	>0	<0
Real exchange rate	q	<0	<0	>0
Exports:				
- non-resource	$\tilde{y}_m^x$	>0	<0	<0
- resource exports	$\tilde{z}_x$	>0	<0	>0
Consumption	$\tilde{c}$	<0	<0	>0
Value-added:				
- non-traded	$\tilde{y}_n^{va}$	<0	>0	>0
- resource	$\tilde{y}_z^{va}$	>0	>0	<0
Inflation:				
- non-traded	π_n	<0	<0	>0
- import price	π_f	<0	<0	>0
Investment	$\tilde{\imath}$	<0	>0	>0
Capital:				
- non-traded	$\tilde{k}_m$	<0	>0	<0
- resources	$\tilde{k}_z$	<0	>0	>0
Tobin's Q:				
- resources	λ_z	>0	<0	<0
- non-traded	λ_n	<0	<0	<0
- non-resource exports	λ_m	>0	<0	<0
Net foreign assets to GDP ratio	b^*	>0	<0	>0

Note: ~ denotes normalised by the level of non-stationary technology.

Table 6: Contemporaneous Responses to a Standardized Monetary Policy Shock: Domestic Sector MSM

Magnitude	**Variables**			
	Output		**Inflation**	
	3 Variables	All	3 Variables	All
Maximum	-.303	-.298	-.029	-.029
Minimum	-.05	-.05	-.006	-.006
MSM	-.1195		-.0196	

3 The Nature of Variables and Their Representation

3.1 The Nature of Variables

In DSGE models, particularly as they become large, many of their variables have no counterpart in the data used to estimate their parameters. A useful distinction that can be made between the variables is to separate them into the following categories: (i) observable, i.e. data is available on them; (ii) partially observable, i.e. some data is available which contains information about them; (iii) redundant, namely they can be substituted out as a function of other variables, and (iv) strictly unobservable, for which there is no equivalent data.

Turning to MSM, an example of an observed variable is the interest rate r_t, while stationized GDP, $\tilde{y}_t^{va}$, is partially observed. In total, the MSM model contains more than 80 variables, of which 55 are redundant. Consequently in the SSVAR representation of the MSM discussed above there were 24 variables, of which 17 are observable or partially observable and seven are unobserved. Examples of the latter are the net foreign assets to GDP ratio and the sectoral capital stocks.[17] MSM also includes 17 shocks (excluding measurement errors).

3.2 A VAR/VECM Representation of MSM Via Error-Correction terms

Many DSGE models now include permanent shocks, such as non-stationary technology, and therefore they feature cointegration between many of the variables. This is true of MSM. Our objective here is to look at what the Vector Error Correction representation of it might be. This was done theoretically for DSGE models in Christensen at al (2011), but it is much simpler to use the simulation approach in Pagan and Robinson (2016) that we have used earlier to construct the underlying SSVAR. Indeed what we will develop could be called a semi-structural VECM, since the system will include changes in some variables as well as error-correction terms.

As MSM is large, in order to demonstrate the semi-structural VECM representation we first consider a simple example. Suppose we had a DSGE model that had three variables that were $I(1)$ - the logs of domestic output y_t, consumption c_t and foreign output y_t^*. Then these are integrated processes because of the log level of technology a_t being $I(1)$. Just as for the MSM, in this DSGE model we would have variables $\tilde{y}_t = y_t - a_t, \tilde{c}_t = c_t - a_t$ and $\tilde{y}_t^* = y_t^* - a_t$. Imposing exogeneity of the foreign sector the SSVAR would have a form such as

$$
\begin{aligned}
\tilde{y}_t^* &= b_{11}\tilde{y}_{t-1}^* + \varepsilon_{y_t^*} \\
\tilde{y}_t &= b_{21}\tilde{y}_{t-1}^* + b_{22}\tilde{y}_{t-1} + b_{23}\tilde{c}_{t-1} + g_{21}\varepsilon_{y_t^*} + g_{22}\varepsilon_{yt} + g_{23}\varepsilon_{ct} \\
\tilde{c}_t &= b_{31}\tilde{y}_{t-1}^* + b_{32}\tilde{y}_{t-1} + b_{33}\tilde{c}_{t-1} + g_{31}\varepsilon_{y_t^*} + g_{32}\varepsilon_{yt} + g_{33}\varepsilon_{ct}.
\end{aligned}
$$

Now there are four $I(1)$ variables here - y_t, c_t, y_t^* and a_t - and there are three error-correction terms - $\tilde{y}_t, \tilde{c}_t$ and $\tilde{y}_t^*$. Rather than use this SSVAR form we want to rewrite the equations above in terms of observable error-correction terms, since $\tilde{y}_t$ and $\tilde{c}_t$ are only partially observable due to the technology shock. It should be noted, however, that it is not possible to write the

[17] The complete list of unobserved variables includes $\pi_{f,t}, b_t^*, k_{m,t}, k_{z,t}, \lambda_{z,t}, \lambda_{n,t}$ and $\lambda_{m,t}$.

model in terms of observable error-correction (EC) terms alone. We will use two observable ones, namely $\xi_{1t} = y_t - y_t^*$ and $\xi_{2t} = c_t - y_t$, and one partially observable, $\xi_{3t} = \tilde{y}_t^*$. The equation for $\Delta\tilde{y}_t$ can then be expressed as

$$\begin{aligned}
\Delta\tilde{y}_t &= b_{21}\tilde{y}_{t-1}^* + (b_{22}-1)\tilde{y}_{t-1} + b_{23}\tilde{c}_{t-1} + g_{21}\varepsilon_{y_t^*} + g_{22}\varepsilon_{yt} + g_{23}\varepsilon_{ct} \\
&= b_{21}\tilde{y}_{t-1}^* + (b_{22}+b_{23}-1)\tilde{y}_{t-1} + b_{23}\xi_{2t-1} + g_{21}\varepsilon_{y_t^*} + g_{22}\varepsilon_{yt} + g_{23}\varepsilon_{ct} \\
&= (b_{21}+b_{22}+b_{23}-1)\tilde{y}_{t-1}^* + (b_{22}+b_{23}-1)\xi_{1t-1} + b_{23}\xi_{2t-1} + g_{21}\varepsilon_{y_t^*} + g_{22}\varepsilon_{yt} + g_{23}\varepsilon_{ct} \\
&= \phi_{21}\tilde{y}_{t-1}^* + \phi_{22}\xi_{1t-1} + \phi_{23}\xi_{2t-1} + g_{21}\varepsilon_{y_t^*} + g_{22}\varepsilon_{yt} + g_{23}\varepsilon_{ct}.
\end{aligned}$$

Consequently, in terms of the observable Δy_t,

$$\begin{aligned}
\Delta y_t &= \Delta\tilde{y}_t + \varepsilon_t^a \\
&= \phi_{21}\tilde{y}_{t-1}^* + \phi_{22}\xi_{1t-1} + \phi_{23}\xi_{2t-1} + g_{21}\varepsilon_{y_t^*} + g_{22}\varepsilon_{yt} + g_{23}\varepsilon_{ct} + \varepsilon_{at}. \qquad (13)
\end{aligned}$$

This is an identity. There will be a similar equation for $\tilde{c}_t$. The advantage is the separation of the EC terms into those that are observable - ξ_{1t}, ξ_{2t} - and only one that is partially unobservable - $\tilde{y}_t^*$.

This approach can also be applied to the MSM. In it there is $\tilde{y}_t^{va} = y_t^{va} - a_t$; we would form EC terms $\xi_t^{va} = y_t^{va} - y_t^*$, $\xi_t^c = \tilde{c}_t - y_t^{va} = c_t - y_t^{va}$, $\xi_t^i = i_t - y_t^{va}$ etc. so that the domestic variables are related to GDP while aggregate GDP depends on foreign GDP. There will then be only one partially unobservable EC term $(y_t^* - a_t)$. Indeed, such a term will always exist; it is not possible to express all the error-correction terms using observed variables alone as there is always one latent EC term left.[18] Fully unobserved variables such as a sectoral capital stock, $\tilde{k}_{m,u}$, are left in this form, as there is no point in expressing them relative to an observed variable. The representation for domestic output growth in MSM equivalent to Equation 18 is presented in Appendix A.

What can we learn from this representation? First, we can gauge, among the many factors influencing GDP growth, the importance of the strictly unobservable variables.[19] If these are omitted from the regression, the R^2 goes from unity (recall this representation is an identity) to .998, so these contribute little to the explanation of GDP growth. Second, we can also look at the importance of the innovations to the shocks. Indeed, when all innovations ε_t are deleted the R^2 drops to .22. Hence *current shocks* are the most important factors affecting GDP growth. It is this fact that explains why recessions are so hard to predict, as future shocks must be known in order to predict whether future growth rates are negative. Essentially, this is an informal way to judge the likely performance of a model at forecasting.

It is also possible to examine which of these shocks is the most important. One might expect growth in non-stationary aggregate productivity, μ_t, to be important, but deleting only that shock reduces the R^2 from unity to .99. Deleting the domestic industry productivity shocks $\varepsilon_{a_{nt}}$ etc. has a much greater impact, with the R^2 going from unity to .82. But by far the most important single shock is the marginal efficiency of investment ε_{Υ_t}, since removing

[18] To see this, suppose for simplicity that we manipulate the error-correction terms so that they are relative to domestic GDP, rather than technology. This can be done for all of the variables except domestic GDP itself, so it would remain relative to technology and therefore partially observed.

[19] The unobserved variables are π_{t-1}^f, $\tilde{k}_{m,t-1}$, b_{t-1}^*, b_{t-2}^*, $\tilde{k}_{m,t-2}$, $\tilde{k}_{z,t-1}$, $\tilde{k}_{z,t-2}$, $\lambda_{z,t-1}$, $\lambda_{n,t-1}$ and $\lambda_{m,t-1}$.

it reduces the R^2 to .71.[20] The R^2 available from the SSVAR, in this case reformulated using error-correction terms due to the presence of a permanent shock, is a useful metric for looking at the importance of unobserved variables or the innovations.[21]

The analysis above can be repeated for inflation. Removing the innovations results in a substantial drop in the R^2 to .23. Again this suggests that it will be difficult to predict inflation. In contrast to the results for output growth, deleting the unobservable variables from the regression results in an R^2 of .55. So this points to a problem of matching data and model variables. Indeed the first order serial correlation of inflation from the model is .34 and the data is .48.

There are other comparisons we might make, such as comparing the statistics on the observable EC terms in the model to the data (Table 7). Doing so, it is apparent that the model generally produces much greater volatility than that evident in the data. With exactly identified models the estimated model variance would match that of the data. We return to this issue in section 5. It is also evident for many of the error-correction terms, but particularly for investment and government expenditure.

Table 7: **Volatility of Selected Error-Correction Terms from the Estimated MSM Model**

Error-Correction	**Mnemonic**	**Standard Deviation**	
		Model	Data
Non-Traded GDP	ξ_t^n	1.17	0.79
GDP	ξ_t^{va}	1.99	1.72
Investment	ξ_t^i	7.66	4.97
Government Expenditure	ξ_t^g	5.12	2.60
Consumption	ξ_t^c	2.75	2.08

4 Identification Issues in the MSM

Multiple models with an equivalent fit may alternatively occur if some of the DSGE parameters, θ, are not well identified. In DSGE models identification issues can be a reflection of the model solution being largely invariant to different values of the elements of θ, or the likelihood being insensitive to the solution (see Iskrev 2010). In this section we discuss two methods that have been applied in the literature to ascertain if weak identification exists, practical issues that arose in their application to MSM, and how these can be handled.[22]

Consider the log likelihood $L(\theta)$ as a function of parameters θ. Then a second-order approximation around the maximum likelihood estimate, $\hat{\theta}$ yields

$$L(\theta) = L(\hat{\theta}) + \frac{1}{2}(\theta - \hat{\theta})' H_{\theta\theta}(\hat{\theta})(\theta - \hat{\theta}) + terms, \tag{14}$$

[20] One can see this effect as well from Rees et al.'s figure 10.

[21] Of course when one turns to data rather than the hypothetical scenario the shocks are actually correlated, so that it is not possible to uniquely attribute any part of actual GDP movements to particular shocks.

[22] Canova and Sala (2009) demonstrate that simply comparing the prior and posterior may not detect identification issues.

where $H_{\theta\theta}$ is the Hessian $\frac{\partial^2 L}{\partial\theta\partial\theta'}$. The omitted "terms" should be smaller than the other elements. Hence

$$\frac{1}{T}\{L(\theta) - L(\hat{\theta})\} \approx \frac{1}{2}(\theta - \hat{\theta})'(T^{-1}H_{\theta\theta}(\hat{\theta}))(\theta - \hat{\theta}). \tag{15}$$

Now the $\lim_{T\to\infty} -\frac{1}{T}E[H_{\theta\theta}(\theta)] = I_{\theta\theta}$, the asymptotic information matrix. Hence we might replace the above with

$$\frac{1}{T}\{L(\theta) - L(\hat{\theta})\} = -\frac{1}{2}(\theta - \hat{\theta})'I_{\theta\theta}(\hat{\theta})(\theta - \hat{\theta}) + o_p(1).$$

So then the magnitude of the right hand side indicates how much a change in $\hat{\theta}$ to another value θ would change the log likelihood (scaled by T). If we put $\theta = 0$ then $\sqrt{\hat{\theta}_i^2 I_{ii}(\hat{\theta})/2}$ might be used as an index of this for the $i'th$ parameter θ. If this is low then the likelihood does not change much when θ_i values depart from zero, a characteristic of weak identification.

The quantity $\ln(\sqrt{\hat{\theta}_i^2 I_{ii}(\hat{\theta})})$ is the "sensitivity" index that Dynare produces to assess the identification of the parameters - see Ratto and Iskrev (2010) - except that $\hat{\theta}_i$ in their case is not the MLE but rather the prior mean. A difficulty with the latter choice is that, while $\frac{\partial L}{\partial\theta}(\hat{\theta}) = 0$, this will not be zero at the prior mean, and so (14) has another term in it. If one is analyzing a DSGE model that has been estimated with Bayesian methods then the Bayes posterior mode would be a more appropriate choice.[23] Fundamentally, this is a scaling issue. Another problem is that we probably should evaluate the information matrix at $\hat{\theta}$ and not at a prior mean. Again, using the Bayes mode makes sense in a Bayesian context.

Although there is no threshold value of $\sqrt{\hat{\theta}_i^2 I_{ii}(\hat{\theta})/2}$ that might signal weak identification, the relative magnitudes provide a guide to which of the parameters are likely to be weakly identified. Using a value of $\sqrt{\hat{\theta}_i^2 I_{ii}(\hat{\theta})/2}$ of less than 1.7 we find that a number of parameters are potentially weakly identified, in particular the response of all inflation rates to marginal cost pressures (the "slopes" of the Phillips curves).[24] This suggests that one should examine more closely the estimated slopes of the Phillips curves.

Reverting back to Equation (15), we could alternatively have used $T^{-1}H_{\theta\theta}(\hat{\theta})$ as our criterion. Koop et al (2013) suggested a "learning rate indicator" for identification which involves simulating the DSGE model with the estimated parameters, and then studying the rate at which the precision changes. They argue that, when there is more than a single parameter, this is a better check of identification than simply looking at the closeness of the posterior and prior for any single parameter and observe that, in an identified model, $H_{\theta\theta}(\hat{\theta})$ should rise at rate T, meaning that $T^{-1}H_{\theta\theta}(\hat{\theta})$ will tend to a constant. In contrast, if the parameter is weakly identified, it will rise at a slower rate, and so $T^{-1}H_{\theta\theta}(\hat{\theta})$ will decline. Hence, by simulating the model and then estimating $H_{\theta\theta}$ for a range of T we can determine

[23] This is because the mode is the θ that maximizes $C(\theta) = L(\theta) + \log p(\theta)$, where L is the log likelihood and $p(\theta)$ is a prior density. Consequently the mode sets $\frac{\partial C}{\partial\theta} = 0$ and we could apply the same expansion as above to $C(\theta)$ rather than $L(\theta)$. The negative of the expected value of the second derivatives of $C(\theta)$ will be the information matrix since the prior is dominated as the sample size grows.

[24] In contrast, the parameter ρ_{r*} which is the inertia effect in the foreign interest rate rule, has the sensitivity index at 76.

whether there might be a weak identification problem.[25] In practice, a weakly identified model will become identified as T becomes very large, so that the index stops declining with very large T, which can be seen in Koop et al's tables.

The implementation of the test is not entirely straight-forward for MSM since the simulated $H_{\theta\theta}$ was sometimes not negative definite.[26] Consequently, we use a variant, focussing on the identification issue with each parameter individually. Data is simulated from the MSM using the parameters estimated by Rees et al. i.e. these are treated as the correct ones. Then a particular parameter is selected, say θ_i, and it is estimated by maximum likelihood and Bayesian methods, with all other parameters not being estimated but set to the true values. Designating the computed standard deviation of this estimated coefficient by $\hat{\sigma}_i$ we know that $\hat{\sigma}_i = -(H_{ii}\ )^{-1/2}$, so that studying the rate of convergence of $\hat{\sigma}_i$ should tell us about identification. Koop et al referred to this in their work on the New Keynesian Phillips Curve and they noted that an identified parameter had $\hat{\sigma}_i$ tending to zero much faster than for the unidentified ones. This approach circumvents the problem above and, as only one parameter is being estimated, it is quick to implement, even with large data samples.

Estimation was performed on a simulated sample of $M = 1500$ data points using subsamples rising from 100 to 1500 observations by 100 at a time.[27] It is useful to assess the speed of convergence by regressing the resulting estimated standard deviations for the single parameter being estimated against the sample size. Specifically, we regress $\ln \hat{\sigma}_i$ against a constant and $\ln(M)$. The coefficient on $\ln M, \hat{\gamma}$, should be -.5 for an identified parameter. In fact, the estimates for $\hat{\gamma}$ for the Phillips curve slopes are values between -.63 (non-resource exports) and -.91 (foreign economy), so this does suggest some weak identification issues. By comparison a parameter which has a very high sensitivity index, namely ρ_{r^*}, gives an estimate of -.56, with a standard deviation of .02.

A related criterion which uses the same simulated data is to study how the simulated data recursive estimate $\hat{\theta}_i$ changes as the sample size grows. Since all parameters except for θ_i are set at the true values, i.e. those used for simulating the data, we would expect that $\hat{\theta}_i$ should converge to its true value for strongly identified parameters, but show slow convergence when there is weak identification. Moreover, it is often the case that with weak identification one sees "jumps" in the estimated $\hat{\theta}_i$ as the sample size gets larger.

Figure 1 shows a plot of the estimated Phillips curve slope for the non-traded sector, κ_{π_n}, as the sample is expanded from 100 to 1500. The true value is .2902. Two estimators are given - unconstrained maximum likelihood and the Bayesian mode using the MSM priors. As expected, we see that there is very little difference between all three estimators as the sample size grows, since the prior gets dominated. Even in small samples the differences are

[25] The logic of this last remark comes from noting that $H_{\theta\theta}(\theta) = \sum_{t=1}^{T} \frac{\partial^2 L_t}{\partial\theta\partial\theta'}$ and so $E[H_{\theta\theta}(\theta)] = \sum E(\frac{\partial^2 L_t}{\partial\theta\partial\theta'}) = TE(\frac{\partial^2 L_t}{\partial\theta\partial\theta'})$ under stationarity i.e. $T^{-1}H_{\theta\theta}(\theta)$ should converge to a constant as $T \to \infty$. In analyzing weak identification the local to zero approach puts $E(\frac{\partial^2 L_t}{\partial\theta\partial\theta'}) = \frac{D}{\sqrt{T}}$. Consequently, $E[H_{\theta\theta}(\theta)] = \frac{TD}{\sqrt{T}} = \sqrt{T}D$, and so it grows at a slower rate than T. Under such conditions, when there is weak identification, $\frac{1}{T}H_{\theta\theta}(\theta)$ will decline to zero as T increases.

[26] The problem was also encountered by Caglar et al (2011) in their application of the Koop et al approach to the Smets and Wouters (2007) model. Their strategy to address this was to simply vary the sample until they got a $H_{\theta\theta}$ that was negative definite.

[27] In other words, we are essentially performing a recursive estimation using samples 1-100,1-200, 1-300..... We actually simulated 10000 observations and dropped the first 8500 to eliminate any initial condition effects.

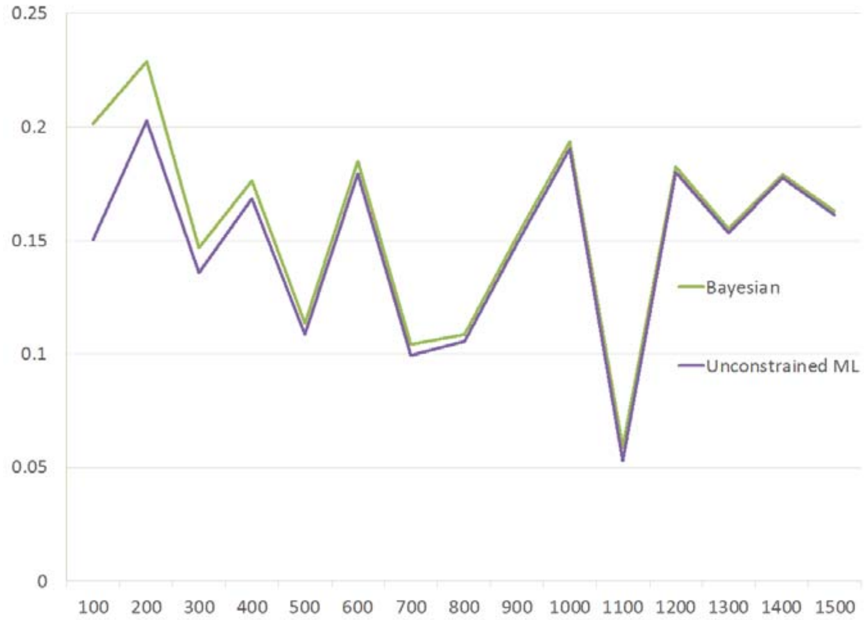

Figure 1: Recursive MLE and Bayesian Estimates of the Slope of the Phillips Curve for the Non-traded sector

not great. It should be noted that there is very little evidence of convergence to the true value of .29. This contrasts with what one sees for ρ_{r*} where the true value is .928 and the estimated quantities start at .918 (for 100 observations) and finish at .93 (for 1500). We note that there is a sample where the estimated parameter for κ_{π_n} dropped to a very small value and, as mentioned above, that type of behaviour is consistent with weak identification. This pattern is repeated for the slope coefficients of all the Phillips curves. A difficulty in identifying the slope of the Phillips curves is not isolated to MSM; for a discussion see Schorfheide (2008).

In summary, if the data had been generated with the parameter values estimated by Rees et al there would be a bias in some of the Phillips curves slopes.[28] Moreover it is clear that we could get values that are close to zero. This is despite the fact that the prior on κ_{π_n} that Rees et al used had a mean of 50. As the slope of the Phillips curve is a crucial parameter for policy assessments and forecasting it would seem that one would need to look at a wide range of parameter values for the slopes when conducting policy assessments, since the weak identification analysis suggests that they are very hard to estimate.

[28]In the light of recent discussion about the decline in values for slopes of Phillips curves it is interesting to note that the bias would be downward.

5 Comparing the Model and Data Generating Processes

It emerged from the VECM representation of MSM constructed above that the variance of GDP growth in the model considerably exceeds that in the data used in estimation.[29] What are the implications of this?

Suppose that we had a model with two shocks and one of the variables in the model was generated by

$$y_{1t}^M = d_1\varepsilon_{1t} + d_2\varepsilon_{2t},$$

where the ε_{jt} are uncorrelated. Assuming that the same structure is used for estimation we would end up with (in large samples)

$$y_{1t}^D = d_1 u_{1t} + d_2 u_{2t},$$

where the u_{jt} are the shocks found after estimation. In large samples the variances of u_{jt} will equal that of ε_{jt}. Then, if $var(y_{1t}^D) < var(y_{1t}^M)$, it must be that there is a negative correlation between u_{1t} and u_{2t}. If it was positive then $var(y_{1t}^D) > var(y_{1t}^M)$. So this explains why the negative correlations between shocks found earlier can occur, and it reflects the fact that one of the assumptions used in estimation is incorrect. One possible response to this is to argue that the data has measurement error in it, and that is why the model variances do not match the data variances. The MSM model does incorporate such a feature, so we look at the issues of bridging data and model via measurement error in Section 5.1. Then in Section 5.2 we ask whether the presence of unobservable variables in the MSM would mean that we could not easily capture the impulse responses of the model by just using observable data. Finally, Section 5.3 asks whether the business cycles that would be produced by the MSM would resemble those of the Australian economy.

5.1 Bridging Data and Model via Measurement Error

One development in estimating DSGE models has been to build a bridge to the data via measurement errors. That is, if the model variable is y_t^M and the data is y_t^D, the equation $y_t^D = y_t^M + \zeta_t$ is added to the system. The implications of including measurement error ζ_t were analysed in Pagan (2017); here we discuss the results for the MSM.

A ζ_t will exist that reconciles the data and model variables. Watson (1993) considered this and, as he noted, some assumption has to be made about the relationship of y_t^M and ζ_t, i.e. how do the reconciliation shocks ("measurement errors") and the model shocks interact? One specification is that they are uncorrelated and that is the primary assumption used in the MSM.

There are also other decisions that need to be made. Two stand out. First, how does ζ_t evolve, i.e. what is its nature? Second, do we fix or estimate the parameters of the generating process for the ζ_t? The answer to the first of these questions given in the MSM was to assume that ζ_t are white noise processes that are uncorrelated with one another. This means that

[29] This is also evident from Table 5 in Rees et al.

the only parameters involved in the ζ_t processes are the variances of the shocks, and they were set in the MSM to values that were connected to the magnitude of y_t^D. The motivation for this approach seems to be that the model shocks would explain a certain percentage of the data while the "measurement error" accounted for the rest.

5.1.1 Parameter Choices for the Measurement Error Process

To examine the consequences of fixing the variance of the shocks ζ_t consider the following calculation. From Equation A27 of Rees et al (2016)

$$\tilde{y}_t^{va} = \omega_1 \tilde{y}_{nt}^{va} + \omega_2 \tilde{y}_{mt}^{va} + \omega_3 \tilde{y}_{zt}^{va}.$$

The left-hand side is model-based GDP and the others are the sectoral value-added measures.[30] The tildes denote that the unit root technology process is subtracted from each and, as $\omega_1 + \omega_2 + \omega_3 = 1$, this implies that

$$\begin{aligned} y_t^{va} &= \omega_1 y_{nt}^{va} + \omega_2 y_{mt}^{va} + \omega_3 y_{zt}^{va} \\ \Longrightarrow &\ \Delta y_t^{va} = \omega_1 \Delta y_{nt}^{va} + \omega_2 \Delta y_{mt}^{va} + \omega_3 \Delta y_{zt}^{va}. \end{aligned}$$

In MSM the data is said to differ from the model variables by measurement errors (here "*obs*" indicates the observed data)

$$\begin{aligned} \Delta y_t^{va,obs} &= \Delta y_t^{va} + \zeta_t^{va} \\ \Delta y_{nt}^{va,obs} &= \Delta y_{nt}^{va} + \zeta_{nt}^{va} \\ \Delta y_{mt}^{va,obs} &= \Delta y_{mt}^{va} + \zeta_{mt}^{va} \\ \Delta y_{zt}^{va,obs} &= \Delta y_{zt}^{va} + \zeta_{zt}^{va}. \end{aligned}$$

Hence

$$\begin{aligned} \Delta y_t^{va,obs} - \zeta_t^{va} &= \omega_n(\Delta y_{nt}^{va,obs} - \zeta_{nt}^{va}) + \omega_m(\Delta y_{mt}^{va,obs} - \zeta_{mt}^{va}) \\ &\quad + \omega_z(\Delta y_{zt}^{va,obs} - \zeta_{zt}^{va}), \end{aligned}$$

so that

$$\psi_t \equiv \Delta y_t^{va,obs} - \Delta \bar{y}_t^{va,obs} = \zeta_t^{va} - \omega_n \zeta_{nt}^{va} - \omega_m \zeta_{mt}^{va} - \omega_z \zeta_{zt}^{va}, \quad (16)$$

where $\Delta \bar{y}_t^{va,obs} \equiv (\omega_n \Delta y_{nt}^{va,obs} + \omega_m \Delta y_{mt}^{va,obs} + \omega_z \Delta y_{zt}^{va,obs})$. Consequently ψ_t can be constructed from the observed data and it has a standard deviation of .49. Its standard deviation implied by the measurement error shocks can be computed from the right-hand side of Equation (16) as Rees et al set $\omega_n = .64$, $\omega_m = .23$, $w_z = .13$, $\sigma_n = .18, \sigma_m = .36, \sigma_z = .74$, and the $\text{std}(\eta_t^{va}) = \sigma_y^{va}$ to .18. This means that the implied standard deviation of ψ_t is

$$\sqrt{(\sigma_y^{va})^2 + \omega_n^2 \sigma_n^2 + \omega_m^2 \sigma_m^2 + \omega_z^2 \sigma_z^2} = .25.$$

Since the standard deviation of ψ_t from the data is .49, it is clear that the measurement error shocks do not provide a reconciliation of the model with the data.

[30] The sectors are non traded (n), non-resource exportables (m) and resource exportables (z).

How this can be so? One possible reason is that the standard deviations of the measurement errors are being set rather than being estimated. Estimating these, however, does not substantially change the result.[31] Other possible reasons relate to the other assumptions being made. One assumption is that the measurement errors are uncorrelated with the model variables. In fact, this is not the case; e.g. the correlation between the technology shock and measurement error in resource exports is .4, that between the risk premium shock and the nominal exchange rate measurement error is .83. Another assumption is that the measurement error shocks are uncorrelated with each other. Again, there are actually many substantial correlations; for example those between η_t^{va} and the three industry GDP measurement error shocks η_{nt}^{va} $(-.79)$, η_{mt}^{va} $(-.82)$ and η_{zt}^{va} $(-.74)$, while the correlation between measurement errors in investment and consumption is .7.[32]

5.1.2 The Interrelationship of Measurement Error Specification and Data

The nature of the ζ_t needed to reconcile the data and model variables may also be an issue. The only variables exempt from measurement error in the MSM are the nominal interest rates. To see the possible unintended consequences of including measurement error shocks we look at the change in the real exchange rate (q_t), which is defined as (from their Equation A31)

$$\Delta q_t = \Delta s_t + \pi_t - \pi_t^*,$$

where s_t is the log of the nominal exchange rate. Then the data variables are $\pi_t^D = \pi_t^M + \zeta_t^{\pi}, \pi_t^{*D} = \pi_t^{*M} + \zeta_t^{\pi^*}$, where the ζ_t are measurement errors. Hence we have for the data

$$\begin{aligned}\Delta q_t^D &= \Delta s_t^D + \pi_t^D - \pi_t^{*D} \\ &= \Delta s_t^M + \zeta_t^{\Delta s} + \pi_t^M - \pi_t^{*M} + \zeta_t^{\pi} - \zeta_t^{\pi^*}.\end{aligned}$$

Cumulating these produces

$$q_t^D = s_t^M + P_t^M - P_t^{*M} + \sum_{j=1}^{t} (\zeta_j^{\Delta s} + \zeta_j^{\pi} - \zeta_j^{\pi^*}).$$

In the model $q_t^M = s_t + P_t^M - P_t^{*M}$ is an I(0) process since there is co-integration between the nominal exchange rate and the relative prices. Because $\sum_{j=1}^{t}(\zeta_j^{\Delta s} + \zeta_j^{\pi} - \zeta_j^{\pi^*})$ is an I(1) process it follows that, unless the variance of $(\zeta_j^{\Delta s} + \zeta_j^{\pi} - \zeta_j^{\pi^*})$ is zero, the real exchange rate *in the data* is predicted to be $I(1)$. Because in the MSM measurement errors are taken to be

[31] One further aspect to note is that unless these standard deviations are estimated observed GDP is redundant. To see this, recall that the observed data for MSM consists of seventeen variables. These include the aggregate GDP growth Δy_t^{va} and the sectoral ones Δy_{jt}^{va} $(j = m, n, z)$. Given Δy_{jt}^{va} and the other 13 variables on can set up a likelihood to find estimates of the MSM parameters θ. Defining $\Delta y_t^S = \sum_j \omega_j \Delta y_{jt}^{va}$, where ω_j are weights used in the MSM then we have $\Delta y_t^{va} = \Delta y_t^S + \zeta_t$, where ζ_t is a reconciliation or measurement error. So Δy_t^{va} is not used in estimating θ but would only be used to find the variance of ζ_t. Hence it means that there are only 16 observable variables being used to estimate the 17 MSM shocks and so one cannot uniquely recover the latter. This also meant that in the weak identification analysis above, which was conducted without measurement error, observed GDP growth was omitted to avoid a singularity.

[32] Because the DSGE model is over-identified we can estimate these correlations.

independent and white noise processes the variance of $(\zeta_j^{\Delta s} + \zeta_j^{\pi} - \zeta_j^{\pi^*})$ will not be zero. In other words the introduction of measurement errors predicts a lack of co-integration between the nominal exchange rate and relative prices in the data.

Similar implications extend to many other variables in the model. A noteworthy example is that y_t^{va} and y_t^* in the model co-integrate but they will be predicted to not have that property in the data. Clearly measurement error could be a useful device to reconcile data and model variables when there is co-integration implied by the model but it is not present in the data. Viewed in this way ζ_t is best thought of as a reconciliation shock, since it aims to reconcile the model and data, rather than measurement error.

In order for the reconciliation shocks ζ_t to actually achieve a reconciliation between the model and data the way they are modelled is important. For example, if none of the error-correction terms defined by the model are $I(0)$ in the data, then one can choose ζ_t to be white noise. However, if a model-defined error-correction term is $I(0)$ in the data, an assumption of white noise measurement errors is incorrect, since the data and model are already reconciled on that dimension.

5.1.3 The Nature of the MSM-Implied Error-correction Terms in the Data

There are other aspects to reconciling the model and the data, such as the nature of deterministic trends and their implications for the error-correction terms. In the MSM model the deterministic growth rates in all of the real variables are the same. However, this may not be true in the data. To handle this problem Rees et al adopted a commonly used approach, namely they mean corrected the data growth rates and then modelled the resulting series. Essentially this removes a different linear deterministic trend from each of the log levels data and thereby avoids a lack of co-trending between the variables. It means that the error-correction terms should not have any trend. However, if there are breaking trends in the data, rather than a constant one, this can show up as a trend in the error-correction terms. Such a breaking trend would need to be allowed for in estimation, otherwise there is a mis-specification and the likelihood is incorrect. These issues are evident in the MSM.

Let us look at data on some of the error-correction terms implied by the model. First, Figure 2 shows the error-correction term involving the logs of investment and aggregate GDP. This has an ADF test of -3.597 with a 1% critical value of -3.507, so the evidence is against there being any need for a reconciliation shock to replicate a lack of co-integration. For the error-correction term involving GDP in the non-traded sector relative to aggregate GDP, the ADF test is -3.463 and the 1 and 5% critical values are -3.51 and -2.90, so it is a mixed result. Plotting the data though shows a clear trend in it. An ADF test allowing for a trend is -4.434 with the 1% critical value of -4.07. So it would seem better to either provide an explanation of this or to have a breaking trend.

Previously we demonstrated that the MSM's use of white noise measurement errors implies that the real exchange rate in the data would be $I(1)$. Applying an ADF test to the latter series (based on four lags) gives -1.58, which does seem to point to an I(1) process, and implying that this could be handled by allowing for measurement errors in the domestic and foreign inflation rates. Looking at the plot of the real exchange rate in Figure 3 (this is after mean correcting the change in the nominal exchange rate and the domestic and foreign inflation rates) it is apparent that there is a trend in the data i.e. a shift in the mean of the

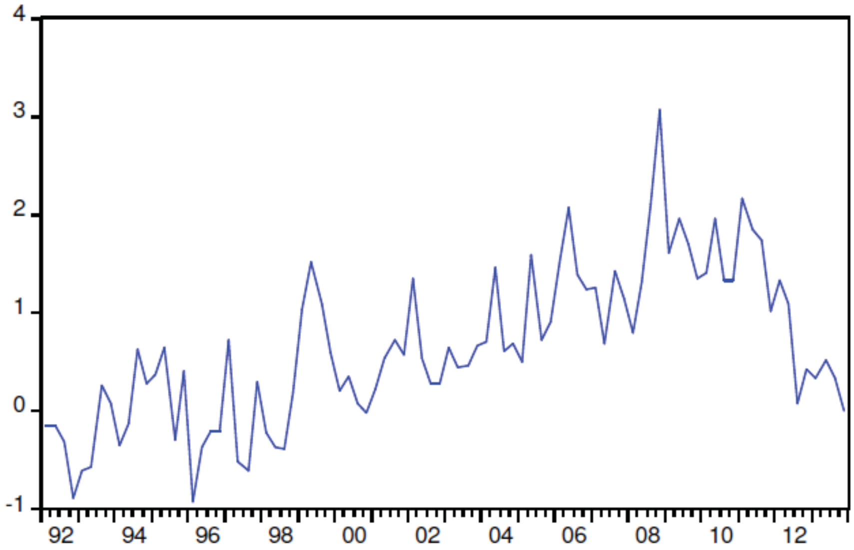

Figure 2: The MSM Implied Error Correction Term Connecting GDP in the Non-Traded Sector to Aggregate GDP

growth rates after 2003. Kulish and Rees (2017) noted this and proposed that one allow for such trend shifts in an MSM type model. So the incorporation of white noise measurement errors does not provide a satisfactory solution. One needs to effect any reconciliation with a different model. Relatedly, if a model is to be used for policy analysis one needs to make some assumptions about the nature of the reconciliation shock into the future, as the policy maker is ultimately interested in the implications for the actual data, rather than the model variables.

There is an argument for re-structuring the MSM model like we did for the SSVAR and expressing it in terms of EC terms plus $y_t^* - a_t$. Then we could work with the growth rates, error-correction terms and Δy_t^* as the data, and have the reconciliation shocks placed on the error correction terms. That enables more flexibility than placing them on the growth rate data.

5.2 Approximating the MSM with an Observable Variables SSVAR

The MSM model has an SSVAR involving both observable and unobservable variables, but what happens if one only fits an SSVAR with observable variables? This is of interest because if it is found that the MSM can be approximated well by a SSVAR(2), then it suggests one could obtain similar results from other models as long as they also have such a representation. There could be advantages to working with these other models, in particular institutional features of the Australian economy may be captured more easily with them.

Pagan and Robinson (2016) looked at this for a range of DSGE models in the literature and found that in many cases an SSVAR(2) fitted quite well, in the sense of being able to

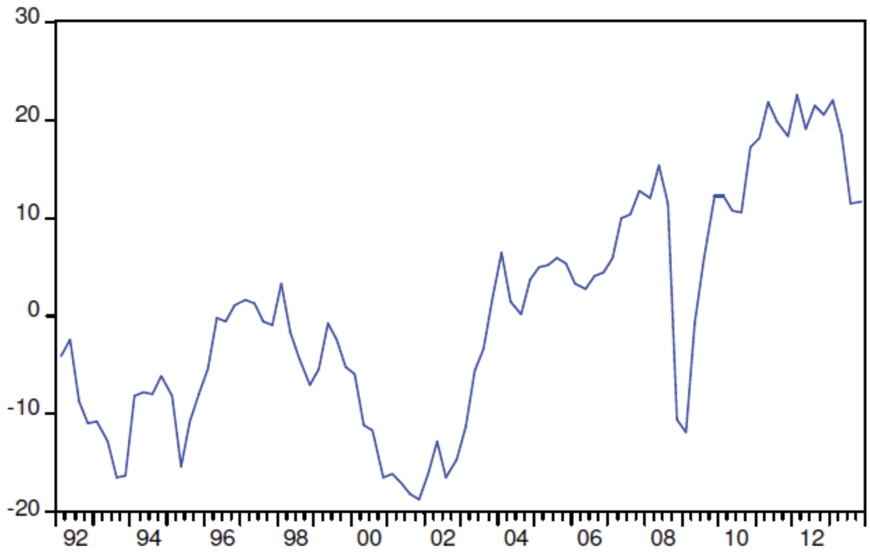

Figure 3: Log of the Real Exchange Rate

generate impulse responses that were those of the underlying DSGE model. To see why this might be the case take the $\tilde{y}_t^*$ of the external sector of MSM and set up its connection with observed foreign GDP growth Δy_t^*

$$\Delta y_t^* = \Delta \tilde{y}_t^* + \Delta a_t = \Delta \tilde{y}_t^* + \mu_t. \tag{17}$$

Equation (9) provides an expression for $\tilde{y}_t^*$ above from the solution to the MSM and, combining this with (17), we get

$$\Delta y_t^* = -.248\pi_{t-1}^* + .083r_{t-1}^* + \{-.135\tilde{y}_{t-1}^* + .003\varepsilon_{yt}^* - .005\varepsilon_{\pi t}^* - .049\varepsilon_{rt}^* + \mu_t\}.$$

If the term in brackets is close to white noise then a VAR(1) in $\Delta y_t^*, \pi_t^*$ and r_t^* would fit this equation quite well. Essentially the question of how important the approximation error is boils down to comparing the relative variances of the omitted term $-.135\tilde{y}_{t-1}^*$ and the disturbance $.003\varepsilon_{yt}^* - .005\varepsilon_{\pi t}^* - .049\varepsilon_{rt}^* + \mu_t$. Since $var(\tilde{y}_t^*) = .0001$ and $var(\mu_t) = .0001$, it follows that the series in brackets will look like white noise. Indeed if one fits to it a MA(1) process it yields a coefficient of .07. The situation differs for the π_t^* equation, where the MA(1) is .24. So we might expect some difficulties in capturing the inflation responses. Figures 4 and 5 below show that this seems to be true; in these figures we look at the ability of an SSVAR(2) in observables $\Delta y_t^*, \pi_t^*$ and r_t^* to capture the monetary impulse responses for the foreign sector of the MSM. Note that in order to study the pure approximation error we begin the impulse responses with the values of the contemporaneous responses - the argument for this is given in Pagan and Robinson (2016).

We can also study impulse responses for the domestic sector. Figures 6-10 give a collection of impulse responses - inflation to monetary policy, GDP growth to technology, real exchange rate to risk, GDP growth to the marginal efficiency of investment and the real exchange rate to the monetary shock. Generally, the correspondence is quite good.

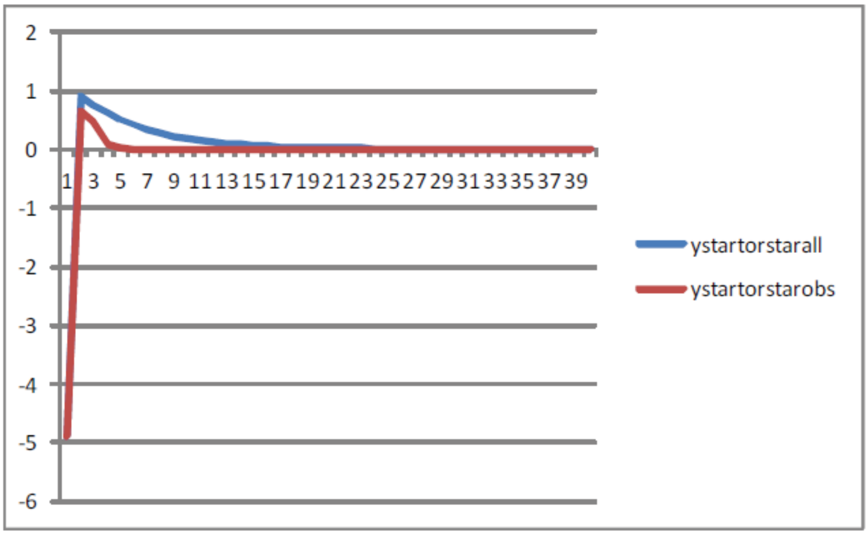

Figure 4: Impulse Response of Foreign GDP Growth to a Foreign Monetary Policy Shock, All and Just Observed Variables

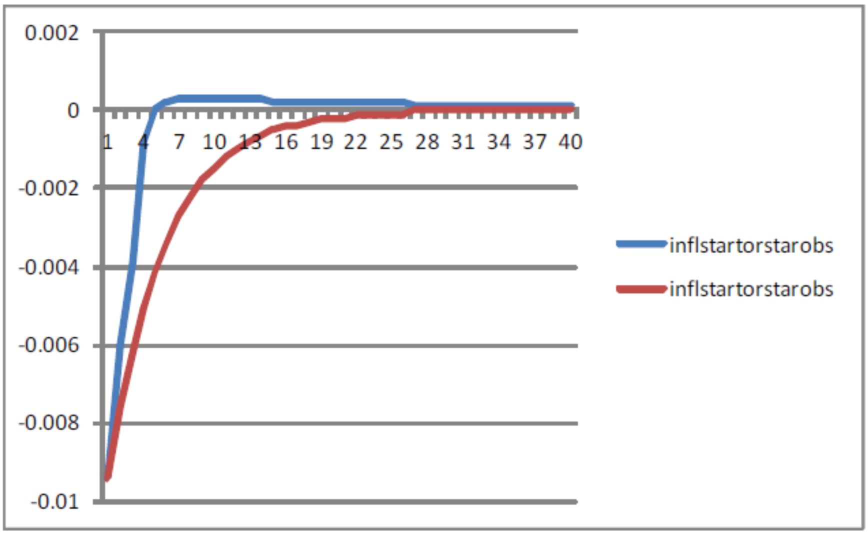

Figure 5: Impulse Response of Foreign Inflation to a Foreign Monetary Policy Shock, All and Just Observed Variables

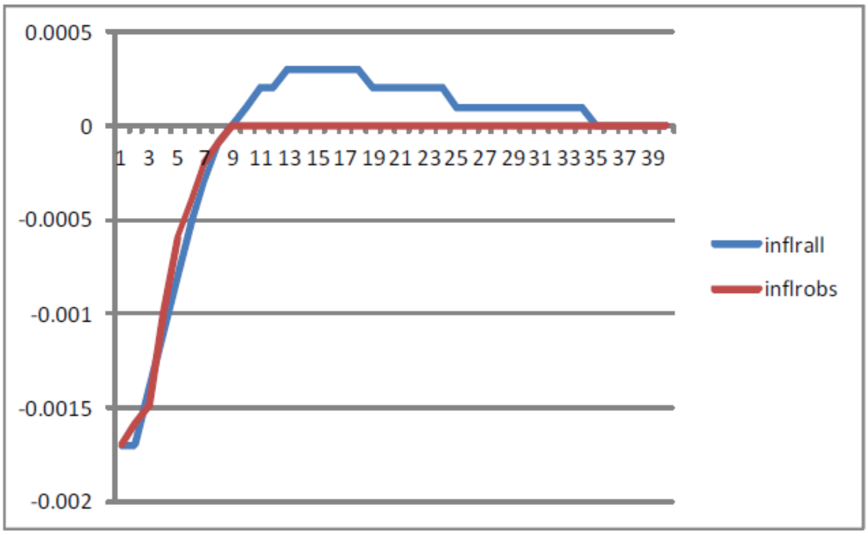

Figure 6: Impulse Response of Domestic Inflation to a Domestic Monetary Policy Shock, All and Just Observed Variables

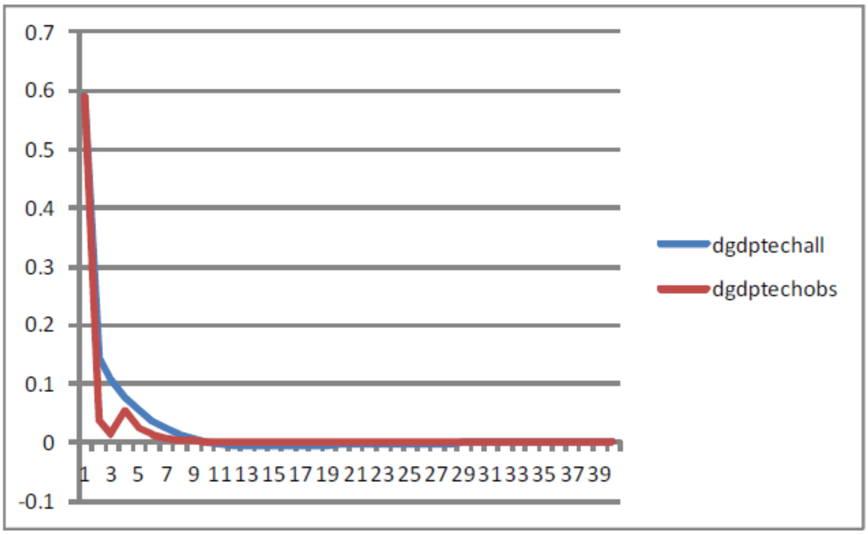

Figure 7: Impulse Response of Domestic GDP Growth to a Technology Shock, All and Just Observed Variables

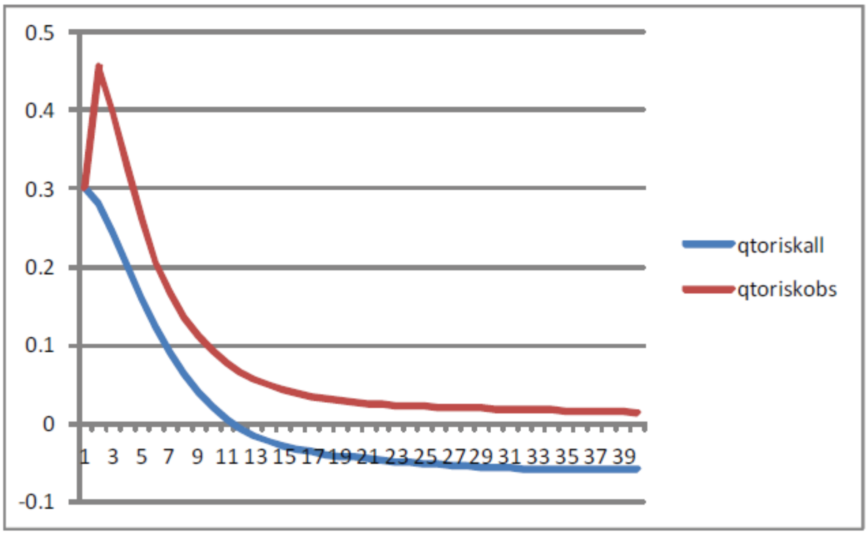

Figure 8: Impulse Response of the Real Exchange Rate to a Risk Premium Shock, All and Just Observed Variables

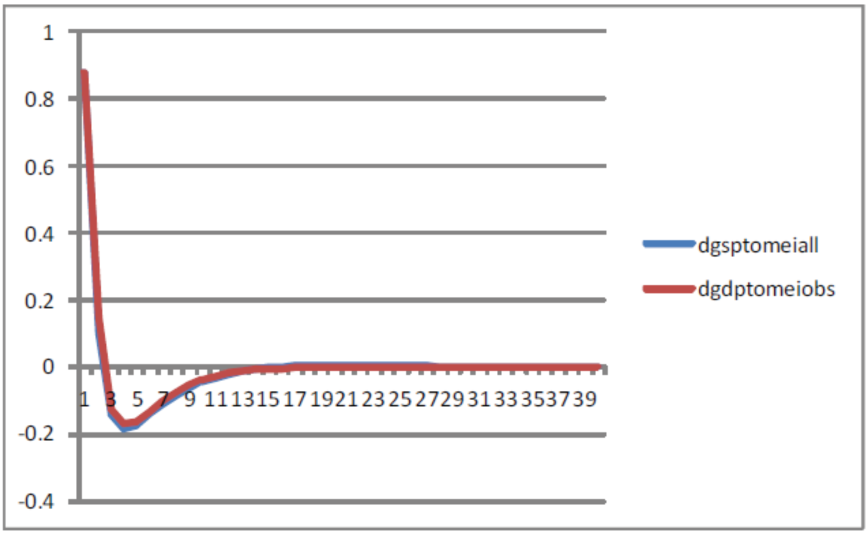

Figure 9: Impulse Response of Domestic GDP Growth to a Marginal Efficiency of Capital Shock, All and Just Observed Variables

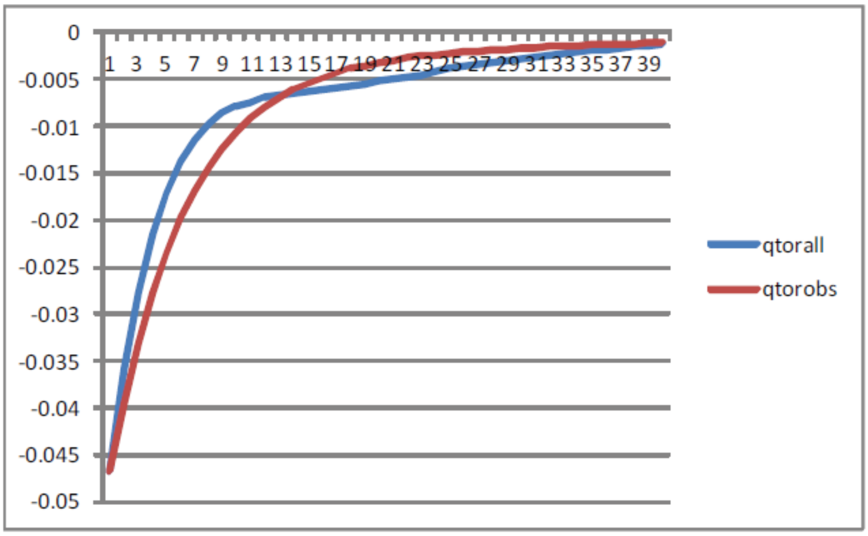

Figure 10: Impulse Response of the Real Exchange Rate to a Domestic Monetary Shock, All and Just Observed Variables

5.3 Examining Some Features of the MSM Generating Process in Relation to Data

The excessive volatility of GDP growth in MSM has consequences for its business cycle properties. One of these is the number of negative growth rates we would get from the model versus that in the data. To examine this we add back the mean growth rate to the simulated model series and then ask how many negative growth rates would be produced, finding that there are nearly four times as many as in the data. This suggests that the model will produce quite a lot of recessions. Of course any recession starts with a negative growth rate, but they are often defined as having two or more. Alternatively, one can identify recessions using the turning points in the level of the series, found with rules like those used by the NBER to date business cycles. This literature is surveyed in Harding and Pagan (2016). Pagan and Robinson (2014) used this to assess financial models, finding that they were unable to produce realistic characteristics of business and financial cycles. Specifically, we simulate GDP data from the MSM model (with uncorrelated shocks) and apply the MBBQ algorithm to identify turning points to look at the business cycle characteristics. These are presented below.

We see from this that the model produces a complete cycle on average every 7 years (Table 8). This is considerably different to the recent Australian experience, as the current expansion has lasted more than twenty years. Applying the dating algorithm to the actual data used in the MSM estimation we also find that there were no recessions. Consequently, it is clear that the model has an in-built feature that would predict recessions too often. This is almost certainly the consequence of the high volatility of GDP growth given by the model versus what is in the data.

Business cycle dating can also be used to ascertain the importance of particular shocks

Table 8: The MSM Business Cycle Characteristics

	All Shocks (MSM)	Omitting: MEI	Foreign
Durations (qtrs)			
Contractions	2.51	2.33	2.45
Expansions	27.26	54.50	27.49
Amplitudes (%)			
Contractions	-1.19	-0.80	-1.17
Expansions	25.04	46.29	25.12

Notes: Foreign shocks include: ε_{y^*}, ε_{r^*}, ε_{π^*}, ε_{p^*} and ε_{μ}. Excludes measurement errors.

in determining the business cycle characteristics. This is simple to implement, in that it involves simulating data from the model with that shock turned off, applying the dating algorithm, and then comparing the cycle characteristics to those obtained with all of the shocks. Such exercises are shown in Table 8. The relationship between this and variance decompositions is discussed in Pagan and Robinson (2014). Table 8 shows that the characteristics are dramatically altered when the marginal efficiency of investment shock is omitted - in particular, the duration and amplitude of expansions approximately double. Using different methods Justiniano et al. (2011) found this shock to be important in U.S. models, and they argue that it is proxying for financial shocks. In contrast to the MEI shocks, Table 8 shows that omitting external shocks has relatively little impact.[33]

6 Conclusion

The construction of the MSM is an impressive achievement. It provides a feasible way of implementing the tradeable/non-tradeable model of a small open econom while at the same time handling a number of sectors that are an important institutional feature of the Australian economy.

The analysis in Rees et al suggested that the MSM did not match some characteristics of the data. This is not surprising; ultimately all empirical macroeconomic modelling involves making compromises along some dimensions. Our work, however, suggests that a failure to match data on some levels can have broad implications for the nature of the shocks that are in the data, and these may not be adequately recognized in the estimation and use of the model. The MSM model (and many other DSGE models) are being used as if the shocks have exactly the same properties as were assumed about them in estimation. When DSGE models are exactly identified this is correct. However, DSGE models are typically over-identified and, consequently, experiments being performed under the estimation assumptions about the shocks are hypothetical, and may be contrary to the data. Such experiments may be

[33]See Justiniano and Preston (2010), who show that small-open economy DSGE models often attribute a surprisingly small role to foreign disturbances.

suitable for understanding the ways in which shocks potentially work through the Australian economy, but whether the data-compatible shocks enter in the same way is unknown.

The essential point is that once any DSGE model is established and estimated, one needs to spend time investigating its fit to the data. This involves more than just looking at a few moments or (in a Bayesian context) a marginal likelihood. Simulating the model so as to study other ways of judging adequacy, such as business cycle outcomes, often provides insight into weaknesses that are not apparent from studying a few moments. If there is a failure along these dimensions then the question that needs to be raised is whether something important has been missed. In addition to investigating the fit to the data, whether parameters are well identified or not should also be examined. In the case of MSM our analysis suggests that the slopes of the Phillips curves in all of the sectors may only be weakly identified. This means that when the model is used one should assume a range of scenarios with different parameter values being used in the model.

The practice of adding measurement error into DSGE models (which has become quite common) often leads to unintended implications that are assessable from the data. This problem is particularly acute when such errors are associated with growth rates in variables (as in the MSM), since then there are testable co-integration implications. This was set out in Pagan (2017) and here it was applied in the context of the MSM. Our feeling is that it is generally not a good idea to claim that there is measurement error. In the event that one does, it needs to be done with great care. In particular, one should check that the implications of doing so are valid.

Many DSGE models today, including MSM, include a unit root technology process. This implies that co-integration exists between many of the real variables. We re-formulated MSM as a SVAR with growth rates and error-correction terms. This had the advantage that one could focus on error-correction terms between observable variables rather than the unobserved technology level. Using this reformulation it was possible to easily compare the properties of error-correction terms in the model to those in the actual data.

MSM is a policy model. It was suggested that a way of reporting results from such a model which is potentially useful to policy makers is to present the range of impulse responses from models that provide the same fit to the data as MSM. This is a way to communicate the degree of model uncertainty that exists. It demonstrated, for example, that the contemporaneous response of GDP in the external sector to a foreign monetary policy shock is very large compared to that from many equivalent models.

Finally, we looked at whether it would be possible to approximate the MSM impulse responses with a SVAR. Here the complication is that there are variables in the MSM that have no counterpart in the data used in its estimation, such as foreign debt and capital stocks in each of the sectors. We found that an SVAR(2) in just the observable variables could do quite well in capturing the responses. This suggests that one might reproduce many of the MSM results with alternative types of models, provided they have a SVAR(2) representation. This might be appealing as it could be easier to incorporate institutional features of the Australian economy into these alternative models.

In summary, in this paper we have examined ways in which a DSGE model can be assessed after estimation, focussing particularly on the nature of shocks and what can be learnt from alternative representations of the DSGE model.

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8 Appendix

8.1 VECM Representation of Domestic GDP Growth in MSM

The expression for the growth in domestic GDP from the MSM model, omitting variables whose coefficient is zero, is

$$\begin{aligned}\Delta y_t^{va} = {} & .23\Delta y_{t-1}^{va} - .015\Delta y_{t-1}^{*} - .007\xi_{t-1}^{zxy} - .01\xi_{t-1}^{yxm} - .015\xi_{t-2}^{yxm} - \\ & .03\pi_{t-1} + .08\xi_{t-1}^{c} - .171\xi_{t-2}^{c} - .25\xi_{t-1}^{va} + .02q_{t-1} + .04p_{t-1}^{*z} - .088\xi_{t-1}^{van} \\ & +.11r_{t-1}^{*} + .033\xi_{t-1}^{vaz} - .02\pi_{t-1}^{n} - .41r_{t-1} - .008\pi_{t-1}^{*} + .006\xi_{t-1}^{i} \\ & -043\xi_{t-1}^{i} - .019\xi_{t-1}^{g}\{-.22\tilde{y}_{t-1}^{*} + .02\pi_{t-1}^{f} - .169b_{t-1}^{*} + .168b_{t-2}^{*} + \\ & 086\tilde{k}_{m,t-1} - .063\tilde{k}_{m,t-2} - .082\tilde{k}_{z,t-1} + .062\tilde{k}_{z,t-2} - .094\lambda_{z,t-1} + \\ & .041\lambda_{n,t-1} + .074\lambda_{m,t-1}\} - .01\varepsilon_{r,t} - .0033\varepsilon_{r_t^*} + .0005\varepsilon_{ft} - .0004\varepsilon_{mt} \\ & -.003\varepsilon_{nt} - .0015\varepsilon_{m_t^*} + .0004\varepsilon_{y_t^*} + .0004\varepsilon_{\pi_t^*} + .0009\varepsilon_{\Upsilon_t} + .0005\varepsilon_{a_{mt}} \\ & +.0007\varepsilon_{a_{nt}} - .59\mu_t + .001\varepsilon_{a_{zt}} + .0015\varepsilon_{gt} - .0007\varepsilon_{\psi t} + .0002\varepsilon_{p_t^*} + .0008\varepsilon_{\xi_{c,t}}\end{aligned}$$

The terms in the braces are the unobserved variables.