Abstract

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Keywords
Climate change, directed technical change, innovation policy

JEL Classification
O33, O44, Q30, Q54, Q56, Q58

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A comment on innovation in The Environment and Directed Technical Change

By Anthony Wiskich*

The framework used to endogenise technology growth by Acemoglu, Aghion, Bursztyn, and Hemous (2012), hereafter AABH, allows the existence of unstable equilibria and does not provide a rationale for specifying which equilibrium should apply when more than one exists. This paper: (i) suggests a rationale for choosing one corner solution used in AABH that constitutes a lower bound for the subsidy or tax required to direct clean research; (ii) argues against use of the other corner solution; and (iii) provides an alternative equilibrium that constitutes an upper bound to the policy required. The alternative methods can produce substantially different results when the elasticity of substitution between clean and dirty inputs is high. (JEL O33, O44, Q30, Q54, Q56, Q58)

Keywords: Climate change, directed technical change, innovation policy.

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The paper by Acemoglu et al. (2012), hereafter AABH, is prominent in the literature and many subsequent papers have built on the described integrated
assessment framework. The model considers just two sectors (clean and dirty) and optimal policy relies on both a distortionary carbon tax and costless research subsidies. The strength of this framework is the analytical tractability of the monopolistically competitive framework which allows profits to accrue, and therefore subsidies to be determined. The current paper highlights some limitations of the innovation framework used by AABH and proposes a resolution.

The AABH innovation framework

AABH derive optimal taxes and subsidies that redirect innovation towards clean inputs in a growth model with environmental constraints. For two sectors clean (c) and dirty (d), profits in sector j at time t \( \Pi_{jt} \), probability of innovation success \( \eta_j \) where innovation increases the quality of a machine by a factor \( 1 + \gamma \), a share of scientists researching in sector j \( s_{jt} \), average productivity \( A_{jt} \), and \( \varphi := (1 - \alpha)(1 - \varepsilon) \) where \( \varepsilon \) is the elasticity of substitution between the two sectors and \( \alpha \) is the share of income spent on machines, AABH describe conditions for equilibria in the following lemma.

**LEMMA 1:** Under laissez-faire, it is an equilibrium for innovation at time t to occur in the clean sector only when

\[
\eta_c A_{ct-1}^{\varphi} > \eta_d (1 + \gamma \eta_c) A_{dt-1}^{\varphi}
\]

in the dirty sector only when

\[
\eta_c (1 + \gamma \eta_d) A_{ct-1}^{\varphi} < \eta_d A_{dt-1}^{\varphi}
\]

and in both sectors when

\[
\eta_c (1 + \gamma \eta_d s_{dt}) A_{ct-1}^{\varphi} = \eta_d (1 + \gamma \eta_c s_{ct}) A_{dt-1}^{\varphi}\]

(with \( s_{ct} + s_{dt} = 1 \)).

To prove lemma 1, AABH define

\[
f(s) = \frac{\Pi_c(s)}{\Pi_d(s)} = \frac{\eta_c}{\eta_d} \left( \frac{1 + \gamma \eta_c s}{1 + \gamma \eta_d (1 - s)} \right)^{-\varphi} \left( \frac{A_{ct-1}}{A_{dt-1}} \right)^{-\varphi}
\]

for $s \in [0,1]$. They highlight that if $f(1) > 1$ then $s^* = 1$ is an equilibrium, if $f(0) < 1$ then $s^* = 0$ is an equilibrium, and if $f(s^*) = 1$ for some $s^* \in (0,1)$ then $s^*$ is an interior equilibrium. If $1 + \varphi > 0$ then $f(s)$ is strictly decreasing in $s$ and the equilibria are unique. If $1 + \varphi < 0$ then $f(s)$ is strictly increasing in $s$ and there are three potential equilibria if $f(0) < 1 < f(1)$: an interior one $s^* \in (0,1)$ and the two corner solutions $s^* = 0$ and $s^* = 1$.

*Limitations of the AABH innovation framework*

Two problems exist with this characterisation of the allocation of scientists when $1 + \varphi < 0$, described in the following remarks.

REMARK 1: An interior equilibrium is unstable in both directions and hence should not be an outcome in such a growth model.

Figure 1, fully explained later, shows the increasing clean(dirty) profits as clean research increases(decreases). An interior equilibrium is not Nash for any finite number of scientists, as any scientist changing research sectors from this equilibrium increases their expected profits. Thus, it seems implausible that this outcome could eventuate in a laissez-faire scenario or be induced by a tax or subsidy.

REMARK 2: Which corner solution applies is not clear from the framework.

AABH do not specify which equilibrium should apply when multiple potential equilibria exist and indeed make different assumptions for different scenarios. For first-best policy simulations, AABH use the critical ratio $\frac{\Pi_c(s)}{\Pi_d(s)} = 1$ when $s > 0$. This assumption for $s = 1$ is depicted in the first panel of Figure 1, where the
subsidy $q_1$ is required to direct clean research and clean profits $\Pi'_c$ are boosted by the subsidy to $(1 + q_1)\Pi'_c$. However, for tax-only scenarios, AABH use the other corner equilibrium where $\frac{\Pi_c(0)}{\Pi_d(0)} = 1$, shown in the second panel, involving a much larger required subsidy $q_0$.\(^2\)

**Addressing the limitations**

As remark 1 describes why the interior solution should not apply, there are potentially two applicable corner equilibria when $1 + \varphi < 0$, corresponding to the first two panels of Figure 1 which I label as ‘Lower’ and ‘Extreme Upper’ for reasons that will become clear. For the Lower approach, a slightly higher subsidy than $q_1$ implies that, when $s = 1$, the marginal researcher is not incentivized to switch to dirty research. However, researchers have an incentive to shift to dirty research en masse. Thus, this equilibrium seems reasonable in providing a lower bound to the subsidy required to direct clean research.

![Figure 1. Three methods of determining the subsidy required to direct clean research](image-url)

\(^2\) In tax-only simulations, the tax directs technical change rather than a subsidy.
The second ‘Extreme Upper’ panel shows the other corner solution where the critical ratio $\frac{\Pi_c(0)}{\Pi_d(0)} = 1$ applies to direct clean research. A larger subsidy is required and, even if the subsidy is reduced marginally, clean research is still more profitable than dirty when $s > 0$: the subsidy $q_0$ is the minimum required to induce the marginal researcher to switch to clean research when $s = 0$. While such a high subsidy may be required to induce a switch to clean research from a prior state of dirty research, I don't propose incorporating such previous state-dependence in the framework to keep things simple. It seems that a government wanting to induce energy transformation to clean technology would not require such a high subsidy to do so. In addition, if subsidies are costly or have even a small administrative burden, the government would have an incentive to keep subsidies as low as possible. Thus, this equilibrium does not seem appropriate to use as it exaggerates the required subsidy too much.

The third panel assumes the Pareto optimal solution where scientists (or their employers) talk to each other, recognise the externality from the research conducted by other scientists on their expected profits and thus allocate themselves to achieve maximum expected profits. This calculation involves the ratio $\frac{\Pi_c(1)}{\Pi_d(0)}$ and the required subsidy is $q_{1,0}$. Any reduction in subsidy from this level would mean that researchers are all better off undertaking dirty research than clean research. The critical ratio is

$$\frac{\Pi_{ct}(1)}{\Pi_{dt}(0)} = \frac{\eta_c}{\eta_d} \left( \frac{1 + y \eta_d}{1 + (1 + y \eta_d) \phi \tilde{A}_{t-1}^\phi} \right)^{\frac{1}{\phi}} \tilde{A}_{t-1}^{-\phi} \text{ where } \tilde{A}_t = \frac{A_{ct}}{A_{dt}}$$

I label this approach as ‘Upper’ as I consider it an upper bound to the required subsidy. The downside of this approach is greater complexity in the profit
calculation because the critical ratio now involves different values of \( s \) in the numerator and denominator. Including taxes \( \tau_t \) and research profit subsidies \( q_t \) leads to

\[
(3) \quad \frac{\Pi_{ct}(1)}{\Pi_{dt}(0)} = \frac{(1+q_t)\eta_c(1+\tau_t)^\sigma}{\eta_d} \left( \frac{(1+y\eta_d)^{\phi(1+\tau_d)^\sigma}+\lambda_{t-1}^\phi}{(1+\tau_d)^\sigma(1+y\eta_c)^\phi\lambda_{t-1}^{\phi}} \right) \left( \frac{(1+y\eta_d)^{\phi(1+\tau_d)^\sigma}+\lambda_{t-1}^\phi}{(1+\tau_d)^\sigma(1+y\eta_c)^\phi\lambda_{t-1}^{\phi}} \right) \frac{1}{\bar{A}_{t-1}}. 
\]

The question then is: are the implications of this alternative worth the complexity?

*Implications for the AABH numerical results*

The most important determinant of the difference in results using the alternative approaches is the elasticity: the higher the elasticity, the greater the slope of the profit lines shown in Figure 1 and hence the greater separation between resulting subsidies. For the high elasticity case with a high discount rate, the initial subsidy required increases by around 25 per cent using the upper bound method rather than the lower bound method. The difference when the elasticity is 3 is not material. In addition, there are no longer periods where allocation is split between clean and dirty (interior solutions) in the low elasticity case with a high discount rate, but this effect does not play an important role in AABH results.

As AABH assume that subsidies do not have any economic costs, the different methods of determining the required subsidy do not affect other variables in their first-best simulations and where policy is delayed. However, AABH also report results where a carbon price is the only policy available. The welfare costs for this second-best sensitivity differ between the methods as the (distortionary) tax is used to direct technical change as well as shift production. Unlike their approach under first-best, AABH assume the *Extreme Upper* approach under this policy which increases the tax (and the associated welfare cost) required to direct technical
change. The different computed equilibrium highlights the ambiguity in the specification of equilibrium in the AABH framework, and a different choice can lead to a substantial change in results as demonstrated below.

Table 1 shows the welfare costs for each method for different elasticities of substitution and discount rates as used by AABH. Compared to the results that AABH report, the welfare loss is reduced under the Upper method and even more so under the Lower method. The extent of the fall is greater with a high elasticity.

<table>
<thead>
<tr>
<th>Elasticity of substitution</th>
<th>10</th>
<th>0.015</th>
<th>0.001</th>
<th>0.015</th>
</tr>
</thead>
<tbody>
<tr>
<td>Discount rate</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>AABH (Extreme Upper)</td>
<td>1.02</td>
<td>1.66</td>
<td>1.92</td>
<td>1.78*</td>
</tr>
<tr>
<td>Upper</td>
<td>0.53</td>
<td>0.89</td>
<td>1.72</td>
<td>1.54</td>
</tr>
<tr>
<td>Lower</td>
<td>0.22</td>
<td>0.37</td>
<td>1.65</td>
<td>1.30</td>
</tr>
</tbody>
</table>

Notes: Percentage reductions in utility for second-best relative to first-best policy. *AABH report a value of 3.15 due to an apparent programming error.

No matter which method is used, the welfare loss is smaller when the elasticity is high as a smaller tax is required to direct technical change. The effect of the discount rate depends on the timing of clean research. For the high elasticity case where clean research occurs immediately, a high discount rate increases welfare costs under second best as greater weight is placed on earlier periods where a higher tax is imposed. For the low elasticity case, clean research is delayed when the discount rate is high and the associated loss at this time is therefore reduced, leading to a lower welfare loss.³

In summary, this comment shows that the AABH paper allows the existence of unstable equilibria and does not specify which equilibrium should apply when more than one exists. This note: provides a rationale for one corner equilibrium used by

³ A programming error mean that AABH miss this finding and they conclude that a high discount rate increases the welfare loss under both elasticities.
AABH as a lower bound to the required policy for directing technical change; argues that the other corner equilibrium should not be applied; and presents an alternative method that provides an upper bound estimate to required policy. The alternative methods can produce substantially different results when a high elasticity of substitution between clean and dirty inputs is assumed.

REFERENCES


