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Aggregate Fluctuations, Network Effects and Covid-19

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Keywords

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JEL Classification

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Aggregate Fluctuations, Network Effects, and Covid-19

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September 20, 2022

Abstract

We decompose the macroeconomic impact of Covid-19 in the US using three production network measures. First, we estimate the aggregate indirect effect of sectoral employment shocks, finding these "network spillovers" to account for $\approx 72\%$ of the decline in real GDP over the second quarter of 2020. Second, we show that downstream propagation explains most of the aggregate effect of the sector-specific disruptions. Specifically, 77% of the GDP decline constitutes the effect of shocks to supplier sectors on downstream customers. Finally, higher-order feedback is mostly inconsequential in explaining the depth of the contraction: only 5% of the aggregate impact is attributed to second-, third- and higher-round effects of the initial shocks.

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1 Introduction

At the onset of the Covid-19 crisis, some sectors of the US economy (such as retail trade) experienced historically high unemployment rates due to lockdown laws and social restrictions. Yet, other sectors (such as food/grocery stores) grew employment to meet the surge in demand for their goods and services. Overall, aggregate employment and real GDP decreased by 11.9% and 8.9%, respectively, over the second quarter of 2020, though these figures mask substantial heterogeneity in unemployment and output growth rates across sectors. Furthermore, sector-specific labor supply shocks induced output shortfalls in neighboring industries as the production of vital intermediate inputs ground to a halt. Disruptions along supply chains deepened the contraction, resulting in the sharpest quarterly decline in real GDP on record (The Wall Street Journal, 2020). Therefore, to understand the macroeconomic impact of Covid-19, it is important to understand the role of input-output linkages in amplifying the effects of the crisis.

We propose a framework that quantifies the contribution of sector-level supply chains to the decline in real GDP at the start of the pandemic in the US. We derive three nonparametric production network measures, which capture "network spillover", "downstream", and "feedback" effects of heterogeneous sectoral shocks. Network spillovers encompass the *indirect* impact of shocks on real GDP, that is, through output spillovers to other industries in the network. For example, a shock to industry i is said to have a significant network spillover if it primarily affects final consumption indirectly through other sectors' reliance on i's product. Next, downstream effects entail the extent to which sectoral shocks transmit down supply chains, from suppliers to customers. This measure quantifies the importance of a given sector's downstream customers in amplifying shocks. Finally, our measure of feedback effects isolates the role of second, third, and higher-round effects of an initial sectoral disturbance. Intuitively, feedback occurs if the shocked sector disrupts industries that it itself relies on for production, creating a propagation channel back to the shocked sector and reducing output multiple times over.

In line with Baqaee and Farhi (2020a), Guerrieri et al. (forthcoming) and Baqaee and Farhi (2022), we model Covid-19 disruptions as heterogeneous and sector-specific employment shocks. We focus on the beginning of the pandemic when government-mandated lockdowns were first implemented to stem the spread of the virus. Notably, our model-implied GDP decline (-8.70%) matches the observed GDP growth rate over the second quarter of 2020 (-8.99%), as measured by the Bureau of Economic Analysis (BEA), allowing for a meaningful decomposition. Our exercise reveals three key findings. First, network spillovers account for $\approx 72\%$ of the overall decline in GDP, translating to \$1.19 trillion of lost output. Second, employment shocks predominately affected final consumption by propagating down supply chains: assuming a sector's output is not used as an intermediate input by any other sector reduces the aggregate impact of its shock by 77%, on average. Finally, the fall in aggregate output was largely independent of shocked sectors' reliance on upstream suppliers. Indeed, assuming no reliance on intermediate inputs still explains, on average, 95% of a sector's overall effect on GDP, implying higher-order feedback is of little importance in explaining the contraction. Together, our findings highlight key pathways through which sectoral shocks propagate and magnify through production networks, consequentially affecting aggregate output.

Our paper is most closely related to the literature that studies the role of input-output linkages as a driver of aggregate fluctuations. Studies such as Long and Plosser (1983), Horvath (1998), Acemoglu et al. (2012), Acemoglu et al. (2017), and Baqaee and Farhi (2019) characterize the conditions under which microeconomic disturbances can propagate through the production network and impact aggregate variables such as GDP, employment and inflation. We contribute to this literature by showing how to isolate specific input-output linkages in the production network to quantify their systemic importance at the macroeconomic level. We derive our theoretical results in the context of a constant-elasticity-of-substitution (CES) production network model à la Atalay (2017), Baqaee and Farhi (2019) and Carvalho et al. (2021), highlighting the assumptions required to implement our framework. We first define a counterfactual production network that omits specific input-output links in order to isolate the network spillover, downstream, and feedback effects of shocks. We then characterize the aggregate impact of shocks in the actual (observed) economy and compare it with the outcome under the counterfactual network structure. The difference between these two effects underscores the importance of the omitted linkages. We take Hulten's (1978) theorem as the starting point of our analysis, which states that labor income as a share of nominal GDP sufficiently characterizes the first-order impact of labor-augmenting shocks on real GDP.[1] Notably, a sector's sales as a share of GDP (also known as its Domar weight) sufficiently summarizes all direct and indirect pathways through the network of a given sector to final demand (Acemoglu et al., 2012). We show that a sector's labor income share implicitly embodies its Domar weight, allowing us to isolate the three key network effects which are the central focus of this paper.

A running theme in the production networks literature is the quantitative importance of input-output linkages in generating aggregate fluctuations (see, for example, Foerster et al., 2011, di Giovanni et al., 2014, di Giovanni et al., 2018, Atalay, 2017, Grassi, 2017, Baqaee, 2018, Altinoglu, 2021, and Huo et al., 2022). In line with this literature, we find input-output linkages amplified the negative impact of sectoral Covid shocks. Under a counterfactual where shocks cannot propagate through the network, we find GDP would have only declined by one-quarter of the observed rate (or $\approx$ -2.25%). This result underscores the importance of accounting for the production network in multi-sector macroeconomic models.

Our paper also relates to the literature on the macroeconomic impact of Covid-19. Particularly relevant is Barrot et al. (2021), who study the effects of reductions in labor supply in response to nonessential business shutdowns and school closures. Using a production network model of the US economy, they find that labor supply shocks reduce GDP by approximately 30%. We contribute to this literature by providing an *ex-post* decomposition of aggregate output, highlighting the specific sectors that amplified Covid-19 disruptions through input-output linkages. In particular, we find that the most significant contributors to the contraction were food and beverage industries, employment services, accommodation, and transportation industries. Relatedly, Bonadio et al. (2021) study the impacts of Covid-19 using a

[1] Baqaee and Farhi (2020b) generalize Hulten's aggregation theorem to inefficient production network economies. The authors find that cost-based Domar weights are the correct statistics for aggregating productivity shocks in the presence of distortions.

global production network model and calibrated labor supply shocks. The authors find that lifting lockdowns in the largest economies would have increased the GDP growth of these countries' smaller trade partners by up to 2.5%. Our counterfactual analysis in Section 4 quantifies each US sector's contribution to real GDP in the absence of downstream (upstream) linkages. We find that shocks to industries such as apparel, leather, and allied product manufacturing would have decreased GDP by only 13% (88.9%) of the actual effect had downstream (upstream) sectors not been reliant upon these manufacturing industries. Crucially, our framework not only identifies the most significant sectors but also provides insight into why these industries were so influential in shaping aggregate output. Other papers that study the economics of Covid-19 include Eichenbaum et al. (2020), Guerrieri et al. (forthcoming), Kaplan et al. (2020), Fornaro and Wolf (2020), Carvalho et al. (2020), Bodenstein et al. (2020), Baqaee and Farhi (2020a), Baqaee et al. (2020), and Baqaee and Farhi (2022), among others. Our paper complements this literature by identifying the various network channels that amplified disruptions caused by the pandemic.

The outline of the paper is as follows. In Section 2, we present our three network measures and discuss their interpretation. In Section 3, we set up the model, characterize the equilibrium and introduce a variant of Hulten's theorem that serves as the basis of our empirical application. In Section 4, we derive the three network effects and highlight their usefulness in an application to Covid-19. Section 5 concludes. Proofs and supplementary results are relegated to the Appendix.

2 Production Network Measures

In this section, we provide intuition for the three measures that deconstruct the aggregate effects of sector-specific disruptions due to Covid-19. The measures capture i) the spillover effect of the shock, ii) the downstream propagation effect, and iii) the feedback effects that reverberate through the production network back to the disrupted sector.

Network Spillovers. Our measure of network spillovers captures the *total indirect effect* of a labor shock to sector i on final demand. For example, if a negative shock to i reduces real GDP because many other industries rely on i (directly or indirectly) for intermediate inputs, then sector i will have a consequential spillover effect. In other words, GDP is affected not because households directly consume i's product but because they consume products that depend upon the supply of good i.

Panels A and B of Figure 1 graphically illustrate our measure of network spillovers. Panel A shows a complete production network, where all input-output relationships between sectors are observed. Under this network structure, sectoral labor shocks can propagate to any other industry, directly and indirectly affecting household consumption. By contrast, Panel B shows a "self-sufficient" economy in which sectors do not rely on one another to produce. In this economy, shocks to sectors $1, 2, ..., N$ impact GDP *directly* by affecting the final consumption of goods produced by these sectors; propagation to other industries is not possible under this counterfactual structure. Network spillovers comprise only the propagation to other sectors, omitting the direct effect of shocks. In this sense, the aggregate impact of output disruptions in the complete production network, less the direct effect captured by the

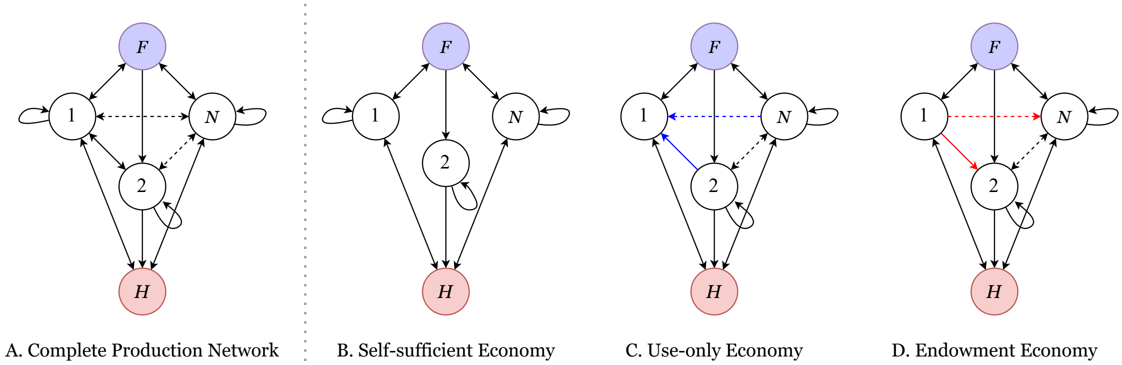

Figure 1: Visual Decomposition of a Production Network

Note: The lilac nodes, F, are a composite of factors (including labor), and the red nodes, H, represent the household. Blue (red) arrows indicate the flow of goods to (away from) producer 1, respectively.

self-sufficient economy, delivers the network spillover effect.

Downstream Effects. Our second construct, the *downstream effect*, measures the aggregate importance of a given sector's downstream customers in amplifying its shock. For example, suppose sector i is a consequential input supplier to many other sectors of the economy. In that case, a disturbance to i will propagate down the supply chain, decreasing the output of i's customers and ultimately contracting aggregate output. The more significant a sector's downstream effect, the greater the importance of the industry as an intermediate input supplier to other producers.

The downstream effect corresponds to the network shown in Panel C of Figure 1, where shocks cannot propagate to downstream customers. Under this production structure, a disruption to sector 1 only impacts final consumption, not the output of sector 1's customers. The difference between the impact on real GDP under the complete production network (Panel A) and the "use-only economy" (Panel C) delivers the importance of downstream linkages. This difference captures all propagation effects attributed to sector 1's role as an input supplier to other industries.[2]

Feedback Effects. Our third measure, the *feedback effect*, captures the role of higher-order feedback in generating aggregate output fluctuations. The intuition is as follows. When sector i uses intermediate inputs from other sectors, a negative shock to i will propagate throughout the economy and reverberate back to i via its upstream linkages. This process occurs *ad infinitum* whenever a shock indirectly impacts at least one of i's upstream suppliers. The greater the reliance of i on intermediates from other sectors, the more significant the feedback effect of the shock to i.

Feedback effects relate to the network shown in Panel D of Figure 1, whereby the upstream linkages of sector 1 are severed. In this economy, a negative shock to sector 1 cannot propagate back to sector 1 and reduce its output a second time. In this sense, there are no feedback effects under this structure.

[2]Note that in the economy shown in Panel C of Figure 1 sector 1 *uses* intermediates from other industries but does not supply intermediates.

The impact of a shock in the complete production network economy of Panel A, less the effect of the same disturbance in the "endowment economy" of Panel D, captures the importance of feedback effects in amplifying Covid-19 disruptions.[3] Put another way, feedback effects capture the role of upstream input-output linkages in the propagation of shocks.

In the next section, we provide a theoretical framework that allows us to extract these network effects by comparing the observed economy with a counterfactual economy that omits specific links. The difference between the aggregate effect of shocks in the actual and counterfactual economies delivers the extent of network spillovers, downstream, and feedback effects.

3 Model

We outline a general equilibrium model of production networks à la Long and Plosser (1983) and Acemoglu et al. (2012) to derive the three network measures discussed in the previous section. To this end, we first set up the environment and define the equilibrium. Then, building on Hulten's theorem, we show how to derive each measure by first defining a counterfactual network structure.

3.1 Environment and Equilibrium

Production. There are N sectors in the economy, each producing one distinct good using capital, labor, and intermediate goods. Following Wasmer (2006), we assume the existence of employees with either specific or general skills. In our model, each sector is endowed with a quantity of specific labor L_{iS} and general labor L_{iG}, where the endowment of each labor type is proportional to the sector's output. Unlike specific labor, the importance of general labor in each sector's production process is constant across all sectors and is captured by the parameter μ_G. By contrast, the importance of sector i's specific labor is governed by μ_i, which is a parameter unique to sector i. Output (y_i) is produced using labor (L_{iS}, L_{iG}), intermediate goods ($\{x_{ij}\}_{j=1}^N$) and capital (K_i). Producer i's technology is described by a constant-returns CES production function of the form

$$y_i = \left(\mu_i^{\frac{1}{\theta}} (z_i L_{iS})^{\frac{\theta-1}{\theta}} + \mu_G^{\frac{1}{\theta}} L_{iG}^{\frac{\theta-1}{\theta}} + \omega_{iK}^{\frac{1}{\theta}} K_i^{\frac{\theta-1}{\theta}} + \sum_{j=1}^{N} \omega_{ij}^{\frac{1}{\theta}} x_{ij}^{\frac{\theta-1}{\theta}} \right)^{\frac{\theta}{\theta-1}}$$

where ω_{ij} is a parameter measuring the importance of sector j's product in i's production process. The term $\omega_{iK}^{\frac{1}{\theta}} K_i^{\frac{\theta-1}{\theta}}$ relates to sector i's capital use. In particular, ω_{iK} captures the intensity with which capital is used to produce good i. Finally, z_i is a sector-specific labor shock, where $z_i < 1$ captures an exogenous reduction in sector i's specific labor endowment due to Covid-19 disruptions. A similar approach to modeling the pandemic is adopted by Baqaee and Farhi (2022) and Guerrieri et al. (forthcoming), although

[3]Our choice of terminology reflects the idea that under this production structure, producer i is treated as if it were a factor of production that transforms primary inputs into output but does not itself demand intermediates.

these studies do not distinguish between sector-specific and general labor.[4] Shocks to sector-specific labor capture the idea that some occupations experienced higher unemployment rates than others at the onset of the pandemic due to the feasibility of working from home (Dingel and Neiman, 2020). For simplicity, we assume sectors' endowment of general labor to be fixed exogenously, invariant to Covid-19 disruptions. Therefore, the only perturbation in the model is to sector-specific labor.
The profits earned by sector i are

$$\pi_i = p_i y_i - w_i L_{iS} - w_G L_{iG} - rK_i - \sum_{j=1}^{N} p_j x_{ij}$$

where w_i (w_G) is the wage of labor type i (general labor), r is the rental price of capital and p_j is the price of good j. The market-clearing conditions for goods $1 \leq i \leq N$, and capital are

$$y_i = c_i + \sum_{j=1}^{N} x_{ji} \quad \text{and} \quad \sum_{i=1}^{N} K_i = K$$

where c_i is the household's consumption of good i, and K is the aggregate supply of capital (which is inelastically supplied).

Households. The representative household has CES preferences over final goods and maximizes its utility $\mathcal{U}$ subject to its budget constraint. Formally, the household's problem is

$$\max_{\{c_i\}_{i=1}^N} \quad \mathcal{U} = \left(\sum_{i=1}^{N} a_i^{\frac{1}{\sigma}} c_i^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}} \quad \text{subject to} \quad \sum_{i=1}^{N} w_i L_{iS} + \sum_{i=1}^{N} w_G L_{iG} + \sum_{i=1}^{N} rK_i = \sum_{i=1}^{N} p_i c_i$$

where a_i is a consumption weight and σ is the elasticity of substitution between final goods.

Real GDP. We define *changes* in real GDP using the Divisia index

$$d \log Y = \sum_{i=1}^{N} b_i d \log c_i$$

where $b_i \equiv p_i c_i / \text{GDP}$ is the household's expenditure on good i as a fraction of nominal GDP. Notably, changes in real GDP and welfare coincide in our model. In what follows, we characterize the impact of labor shocks on real GDP, which is the central object of our analysis.

Equilibrium. The competitive equilibrium is defined such that producers maximize profits taking prices as given, the representative household maximizes utility subject to its budget constraint, and the markets for capital and goods $1 \leq i \leq N$ clear.

[4]Relatedly, Barrot et al. (2021) compute labor shocks as the number of workers forced into inactivity due to school closures plus the number of workers in nonessential sectors that could not work from home during the pandemic. However, since our Covid-19 application is ex-post, we remain agnostic regarding the exact cause of labor supply contractions across sectors.

3.2 Input-Output Definitions

Before discussing our theoretical results, we introduce some input-output notation. Specifically, we define the economy's input-output and Leontief inverse matrices, Domar weights, and sector-specific labor and final goods expenditure shares, all of which are measured at the initial (pre-shock) equilibrium.

Final expenditure shares. Let $\mathbf{b}$ be an $N \times 1$ vector of *final expenditure shares*, whose i^{th} element is defined as

$$b_i = \frac{p_i c_i}{\sum_{j=1}^{N} p_j c_j}.$$

The numerator is the household's expenditure on good i, and the denominator is nominal GDP. A given element b_i measures the importance of sector i's product in the household's consumption in equilibrium.

Labor expenditure shares. We also define an $N \times 1$ vector of *sector-specific labor expenditure shares* $\mathbf{\Lambda}$, with i^{th} element given by

$$\Lambda_i = \frac{w_i L_{iS}}{p_i y_i}.$$

The vector $\mathbf{\Lambda}$ captures each sector's expenditure on *specific* labor as a fraction of its total sales. Notably, the assumption that L_{iS} is endowed in proportion to i's size (y_i) allows us to isolate the role of the *intermediate goods* network in shaping real GDP in response to labor shocks.[5]

Domar weights. We define an $N \times 1$ vector of *Domar weights* (or sales shares) $\boldsymbol{\lambda} \equiv [\lambda_i]$, where

$$\lambda_i = \frac{p_i y_i}{\sum_{j=1}^{N} p_j c_j}$$

and $\sum_{i=1}^{N} \lambda_i > 1$ when there are intermediates. Domar weights summarize all direct and indirect paths from sector i to final demand. The network measures discussed in Section 2 can be interpreted as decompositions of the Domar weight vector $\boldsymbol{\lambda}$.

Input-output matrix. Let $\boldsymbol{\omega} \equiv [\omega_{ij}]$ be the economy's $N \times N$ input-output matrix, which captures all direct interdependencies between sectors of the economy. The input-output matrix $\boldsymbol{\omega}$ has a $N \times N$ general equilibrium counterpart, denoted by $\mathbf{\Omega}$, with a generic element given by

$$\Omega_{ij} = \frac{p_j x_{ij}}{p_i y_i}.$$

Notably, Ω_{ij} captures the expenditure by sector i on sector j's product as a fraction of sector i's total revenue and is a measure of the direct exposure of sector i to sector j in terms of revenues/costs (see

[5]The assumption that L_{iS}/y_i is constant across all sectors i can be relaxed without qualitatively bearing on our results in Section 4.

Carvalho and Tahbaz-Salehi, 2019, Baqaee and Farhi, 2019 and Baqaee and Farhi, 2020b for a more detailed discussion of the input-output matrix and its properties).

Leontief inverse. Associated with $\mathbf{\Omega}$ is an $N \times N$ *Leontief inverse matrix* $\mathbf{\Psi} \equiv [\Psi_{ij}]$, defined as

$$\mathbf{\Psi} \equiv (I - \mathbf{\Omega})^{-1} = I + \mathbf{\Omega} + \mathbf{\Omega}^2 + \ldots$$

The ij^{th} element of the Leontief inverse $\mathbf{\Psi}$ records all direct and indirect linkages connecting sector j to sector i in equilibrium. Specifically, $(\mathbf{\Omega}^n)_{ij}$ measures the weighted sum of all paths of length n linking sector j to sector i through the production network. The Leontief inverse is related to the notion of *influence* in Acemoglu et al. (2012), capturing the systemic importance of any given production unit.

From the goods market-clearing condition $y_i = c_i + \sum_{j=1}^{N} x_{ji}$, we derive the following identity linking Domar weights to the Leontief inverse[6]

$$\lambda_i = \sum_{j=1}^{N} b_j \Psi_{ji}. \tag{1}$$

Equation (1) highlights that sector i's Domar weight captures all the possible ways that final demand is linked to industry i through the production network. In the following section, we exploit the relationship between Domar weights and the Leontief inverse in deriving our network measures.

3.3 Theoretical Results

In this subsection, we introduce a variant of Hulten's (1978) theorem for a sector-specific labor shock $d\log z_i$, which forms the basis of our empirical application.

Theorem 1. *The first-order macroeconomic impact of a shock $d\log z_i$ is given by*

$$\frac{d\log Y}{d\log z_i} = \Lambda_i \lambda_i \tag{2}$$

where $\Lambda_i \lambda_i$ is sector i's expenditure on specific labor, as a fraction of nominal GDP.

Proof. See Appendix A.

Equation (2) states that i's specific-labor costs as a share of GDP sufficiently characterize how a labor shock to i impacts real GDP to a first-order of approximation. This result is a variant of Hulten's theorem, which summarizes the impact of a Hicks-neutral productivity shock to a producer on real GDP by that producer's nominal sales as a fraction of GDP.

[6] Multiplying both sides of $y_i = c_i + \sum_{j=1}^{N} x_{ji}$ by $p_i \cdot \text{GDP}^{-1}$, we can write $\lambda_i = b_i + \sum_{j=1}^{N} \lambda_j \Omega_{ji}$. Writing this new equation in matrix form and solving for the vector of Domar weights, we get $\boldsymbol{\lambda}' = \mathbf{b}'\mathbf{\Psi}$, where the i^{th} element is given by equation (1).

Equation (2) is the natural starting point for our analysis since it characterizes the elasticity of aggregate output with respect to a sector-specific shock $d\log z_i$ in terms of the economy's input-output network. To measure the spillover, downstream, and feedback effects of shocks, we first construct counterfactual production networks that omit specific input-output linkages. Then, we compute each sector's Domar weight under the new network structure, allowing us to measure the importance of the omitted links by comparing the counterfactual Domar weights with the economy's actual Domar weights. Counterfactual Domar weights are unobservable, and we characterize them in terms of observable statistics such as intermediate and final good expenditure, allowing their computation. However, before deriving our network measures, we provide two interim results that enable us to implement our approach. The first result, Proposition 1, shows that the economy's sector-specific labor shares $\{\Lambda_i\}_{i=1}^N$ are independent of the structure of the production network.

Proposition 1. *The economy's sector-specific labor shares $\{\Lambda_i\}_{i=1}^N$ are invariant to the structure of the intermediate goods network $\boldsymbol{\omega}$,*

$$\Lambda_i = \tilde{\Lambda}_i = \alpha^{\frac{\theta-1}{\theta}} \mu_i^{\frac{1}{\theta}} z_i^{\frac{\theta-1}{\theta}}$$

where $\tilde{\Lambda}_i$ is the labor share of sector i under the network structure $\tilde{\boldsymbol{\omega}} \neq \boldsymbol{\omega}$, and α is a constant given by

$$\alpha = \frac{L_{1S}}{y_1} = \frac{L_{2S}}{y_2} = \ldots = \frac{L_{NS}}{y_N} = \frac{\tilde{L}_{1S}}{\tilde{y}_1} = \frac{\tilde{L}_{2S}}{\tilde{y}_2} = \ldots = \frac{\tilde{L}_{NS}}{\tilde{y}_N}$$

Proof. See Appendix A.

In Proposition 1, tilde variables represent quantities in the counterfactual economy defined by the production network $\tilde{\boldsymbol{\omega}}$ that is different from $\boldsymbol{\omega}$. The result highlights that each sector i's labor expenditure share depends on the importance of labor in i's production process μ_i, i's initial productivity level z_i, the elasticity of substitution θ, and an economy-wide constant α that is invariant to the structure of the input-output network.[7] Crucially, since we assume μ_i, z_i, and θ to be the same in the economies defined by $\tilde{\boldsymbol{\omega}}$ and $\tilde{\boldsymbol{\omega}}$, it follows that Λ_i is not a function of $\boldsymbol{\omega}$. Put another way, changes in $\boldsymbol{\omega}$ will materialize as changes in the Domar weights $\{\lambda_i\}_{i=1}^N$ and not in the labor expenditure shares $\{\Lambda\}_{i=1}^N$.

The second key result, Proposition 2, allows us to measure each network effect using *observed* information on final and intermediate goods expenditure.

Proposition 2. *For some arbitrary linear transformation of the input-output matrix $T(\boldsymbol{\omega})$, the equilibrium input-output network and final expenditure shares are respectively given by*

$$T(\boldsymbol{\Omega}) \quad \text{and} \quad \mathbf{b}.$$

[7] For example, suppose $\tilde{\boldsymbol{\omega}}$ is a null matrix, meaning there are no intermediate goods in the economy. Then, $\tilde{y}_i = \tilde{c}_i < y_i$ for all i. Therefore, in the economy characterized by $\tilde{\boldsymbol{\omega}}$, each sector's specific labor endowment is less than or equal to that of an economy with intermediates, $\tilde{L}_{iS} \leq L_{iS}$ for all i. However, the ratio of specific labor to output is the same in both economies $\frac{L_{iS}}{y_i} = \frac{\tilde{L}_{iS}}{\tilde{y}_i}$ for all sectors i. Furthermore, the initial level of real GDP is the same in both economies.

Proof. See Appendix A.

Proposition 2 states that for any linear transformation of the economy's input-output matrix $\boldsymbol{\omega}$, we can apply the same transformation to the economy's equilibrium input-output matrix $\boldsymbol{\Omega}$. This result provides a direct mapping between $\boldsymbol{\omega}$ and $\boldsymbol{\Omega}$, meaning counterfactual Domar weights can always be expressed in terms of *observed* equilibrium input-output parameters. To see this, recall that sector i's Domar weight is given by $\lambda_i = \sum_{k=1}^{N} b_k \Psi_{ki} = \sum_{k=1}^{N} b_k \left(I - \boldsymbol{\Omega}\right)_{ki}^{-1}$. Proposition 2, therefore, implies that we can always compute i's counterfactual Domar weight as $\tilde{\lambda}_i = \sum_{k=1}^{N} b_k \left(I - T\left(\boldsymbol{\Omega}\right)\right)_{ki}^{-1}$ for some counterfactual network defined by $\tilde{\boldsymbol{\omega}} = T(\boldsymbol{\omega})$. Additionally, the linear transformation has no impact on the final expenditure shares $\{b_i\}_{i=1}^{N}$ since these depend solely on the weights in the consumption aggregator $\{a_i\}_{i=1}^{N}$.

4 Measuring Network Effects and Application to Covid-19

In this section, we derive our three network measures (discussed in Section 2) that respectively quantify the importance of network spillovers, downstream propagation, and feedback effects in shaping the response of real GDP to Covid-19 disruptions. We begin with a description of the data before deriving each measure and discussing the related empirical results.

4.1 Data

We use 2019 input-output data from the Bureau of Labor Statistics (BLS), which contains inter-industry data for 205 industries/commodities.[8] After dropping government and "special" sectors, we are left with 181 industries (excluding the final demand sector).[9] The final expenditure shares (b_i's) are calibrated using final consumption data in the 2019 use matrix.

Using data from the BLS, we calibrate our model to match each industry's end-of-period change in employment between Q1 and Q2 of 2020. Lockdown laws, government-mandated working-from-home orders, and behavioral responses to the fear of contagion resulted in substantial (and heterogeneous) changes in the quantity of labor supplied across sectors. For example, the motion picture, video, and sound recording industries (NAICS 512) experienced a 52% decline in the raw number of employees, whereas couriers and messengers' (NAICS 492) employment increased by over 6%.[10]

For most sectors, employment is measured at the four-digit NAICS level. Where possible, we match sectors for which we have employment data directly to those in the input-output table. However, in

[8] BLS input-output data are more disaggregated than the annual BEA input-output tables, which only contain data for 71 industries. Furthermore, we use 2019 input-output data since Covid disruptions likely altered the structure of the intermediate goods network in the latter half of 2020. Using the 2019 input-output table allows us to accurately capture the system of intersectoral relationships at the *onset* of the pandemic.

[9] These sectors are owner-occupied dwellings, noncomparable imports, scrap, used and secondhand goods, and rest of the world adjustment.

[10] We view labor supply contractions as the driving force behind the heterogeneity in sectoral output decline. Richer models (similar to that of Baqaee and Farhi, 2020a) can also capture how changes in the composition of households' demand for final goods affect the aggregate output function. The treatment of unstable final demand is outside the scope of this paper.

some cases, we do not have data at the same level of disaggregation as the input-output table. To correct this, we assign the average employment change at the lowest common level of disaggregation. For example, we do not have employment numbers for agencies, brokerages, and other insurance-related activities (NAICS 5242), so we assign the average change in employment for the finance and insurance industry (NAICS 52) as a whole. Finally, labor income shares are calibrated using 2019 BLS KLEMS data. Again, when there is no data at the four-digit NAICS level, we assign the average share at the lowest level of disaggregation. This process allows us to estimate labor income shares for all 181 sectors.

4.2 Network Measures and Empirical Results

Aggregation. To provide a benchmark for our decomposition, we use equation (2) to first get an expression that allows us to estimate the total impact of the shocks on real GDP: $\Delta \log Y = \sum_{i=1}^{N} \Lambda_i \lambda_i \cdot \Delta \log z_i$. Plugging sectoral employment growth rates from the BLS into the above expression suggests a quarter over quarter (Q/Q) real GDP contraction of -8.70%.[11] Comparatively, the Q/Q real GDP growth rate for 2020Q2, as measured by the BEA, was -8.99%.[12]

Network Measures. We now derive each network measure and present our empirical results. As discussed above, our approach relies on defining a production network that omits specific input-output linkages and characterizing the economy's Domar weights under this counterfactual structure. Comparing the aggregate impact of employment shocks under the counterfactual network with the actual effect (as measured by $\frac{d \log Y}{d \log z_i} = \Lambda_i \lambda_i$) gives a measure of the aggregate importance of the omitted links.

Measure 1: Network Spillovers. As discussed in Section 2, the network spillover effect captures the *total indirect effect* of a labor shock to sector i on final demand. To measure network spillovers resulting from labor shocks, we diagonalize the input-output matrix, defining the counterfactual network as $\boldsymbol{\omega}_{NS} \equiv \text{diag}(\boldsymbol{\omega})$ and characterize the economy's Domar weights under $\boldsymbol{\omega}_{NS}$. The extent to which the shock to i affects real GDP in the observed economy ($\Lambda_i \lambda_i$), less the impact of the shock under the counterfactual economy $\boldsymbol{\omega}_{NS}$, delivers the network spillover resulting from the shock. Formally, the network spillover effect of a labor shock to industry i is defined

$$\textit{Network Spillover}_i \equiv \Lambda_i \left(\lambda_i - \lambda_i^{NS}\right) \cdot \Delta \log z_i \tag{3}$$

where λ_i^{NS} is sector i's Domar weight under the input-output network structure $\boldsymbol{\omega}_{NS} = \text{diag}(\boldsymbol{\omega})$. While the Domar weights $\{\lambda_i^{NS}\}_{i=1}^{N}$ are unobserved, by applying Proposition 2, these statistics can be constructed using *observed* sector-level input-output and final expenditure data.

[11] In Table B.1 in Appendix B, we show that the model generates a real GDP decline of 11.75% when we compute the shocks as average (as opposed to end-of-period) employment changes. Specifically, we first calculate mean employment between January to March (Q1) and April to June (Q2) and then take the (log) difference between these average employment figures.

[12] For the same period, the annualized GDP growth rate according to the BEA is -32.9%. Our aggregation equation implies an annualized real GDP growth rate of -30.5%.

Remark 1. *The Domar weights* $\{\lambda_i^{NS}\}_{i=1}^{N}$ *are given by*

$$\lambda_i^{NS} = \sum_{j=1}^{N} b_j \left(I - \text{diag}\left(\mathbf{\Omega}\right)\right)_{ji}^{-1}.$$

The above result is a direct consequence of equation (1), which states that an industry's Domar weight can be expressed in terms of final expenditure shares and the Leontief inverse. Under $\boldsymbol{\omega}_{NS}$, the ji^{th} element of the Leontief inverse is given by $(I - \text{diag}(\mathbf{\Omega}))_{ji}^{-1}$, which can be constructed directly from the BLS input-output data, allowing for the computation of λ_i^{NS}. Notably, as *Network Spillover*$_i$ tends towards $\Lambda_i \lambda_i$, indirect spillovers increasingly account for the aggregate impact of the labor shock to sector i.

Network Spillovers: Empirical Results. Using equation (3), we first estimate aggregate network spillovers resulting from the labor shocks to be

$$\sum_{i=1}^{N} \Lambda_i (\lambda_i - \lambda_i^{NS}) \cdot \Delta \log z_i \approx 72\%.$$

Remarkably, nearly three-quarters of the aggregate output decline is attributed to spillover effects. It is important to note that there is no *a priori* reason to expect network spillovers to be substantial. The result reflects the fact that a large volume of intermediate goods is traded between producers in the US; the aggregate gross output to GDP ratio is approximately 1.8, suggesting intermediate sales are almost equal to final expenditure.[13]

Figure 2 shows the top twenty sectors by contribution to the output decline. Each bar measures a sector's total effect (as measured by Hulten's theorem, equation (2)), which is decomposed into the network spillover and direct effects.[14] The most important sectors are food services, accommodation, manufacturing, construction, and transportation, all of which (except for construction) experienced employment declines of more than 10%. A key revelation is the substantial heterogeneity of network spillover effects across sectors. For example, apparel, leather, and allied product manufacturing industries primarily affect GDP indirectly through the network, recording a network spillover of 96.4%. In contrast, only 13.3% of wholesale trade's total effect can be accounted for by its network spillover.

[13] Our estimate of aggregate network spillovers uses seasonally-adjusted employment series from the BLS. Table B.1 in Appendix B shows that the result holds for unadjusted sectoral employment data. Model-implied GDP growth is -7.29% and network spillovers account for $(-5.25\%/-7.29\%) = 72\%$ of the total effect when we use the unadjusted series. Panels B and C of Table B.1 also report the GDP growth rate and extent of network spillovers in the third and fourth quarters of 2020. Network spillovers account for 68% and 65% of the total effect in Q3 and Q4, respectively.

[14] Figure B.1 in Appendix B plots the employment shocks to the same twenty sectors.

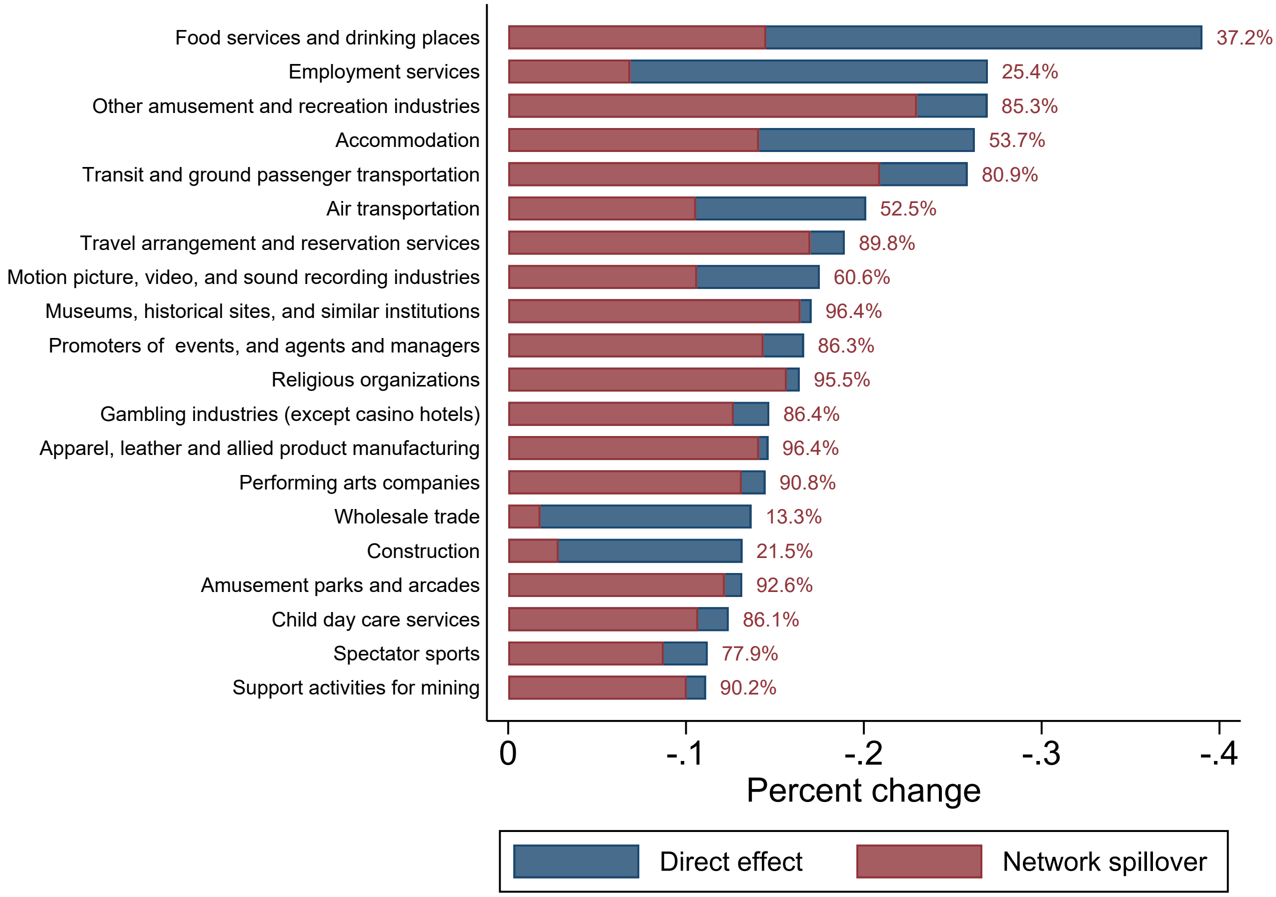

Figure 2: Network Spillover and Direct Effects of Heterogeneous Employment Shocks

Note: Each bar decomposes the total change in GDP growth (associated with a labor shock to the sector listed on the vertical axis) into its network spillover (red bar) and direct (blue bar) components. Percentages to the right of each bar state the network spillover effect as a percentage of the total impact, or $\frac{\Lambda_i(\lambda_i - \lambda_i^{NS})}{\Lambda_i \lambda_i} \times 100$. Only the top 20 sectors (by the total impact on GDP) are shown.

Measure 2: Downstream Effect. Our second construct, the *downstream effect*, measures the aggregate importance of a given sector's downstream customers in amplifying its shock. The difference between the impact on real GDP in the observed economy and a counterfactual economy where sector i does not supply intermediate goods to other industries measures the importance of downstream propagation. To measure the downstream effect of a shock to i, we define an input-output network $\boldsymbol{\omega}^D_{(i)} = \boldsymbol{\omega} - \boldsymbol{\omega}_{(i)}$ where $\boldsymbol{\omega}_{(i)}$ is an $N \times N$ matrix with the i^{th} column equal to that of $\boldsymbol{\omega}$ and zeros elsewhere. We construct N matrices of this kind, each corresponding to a different sector of the economy. The downstream effect of a labor shock to sector i is defined

$$Downstream\ Effect_i \equiv \Lambda_i \left(\lambda_i - \lambda_i^D\right) \cdot \Delta \log z_i$$

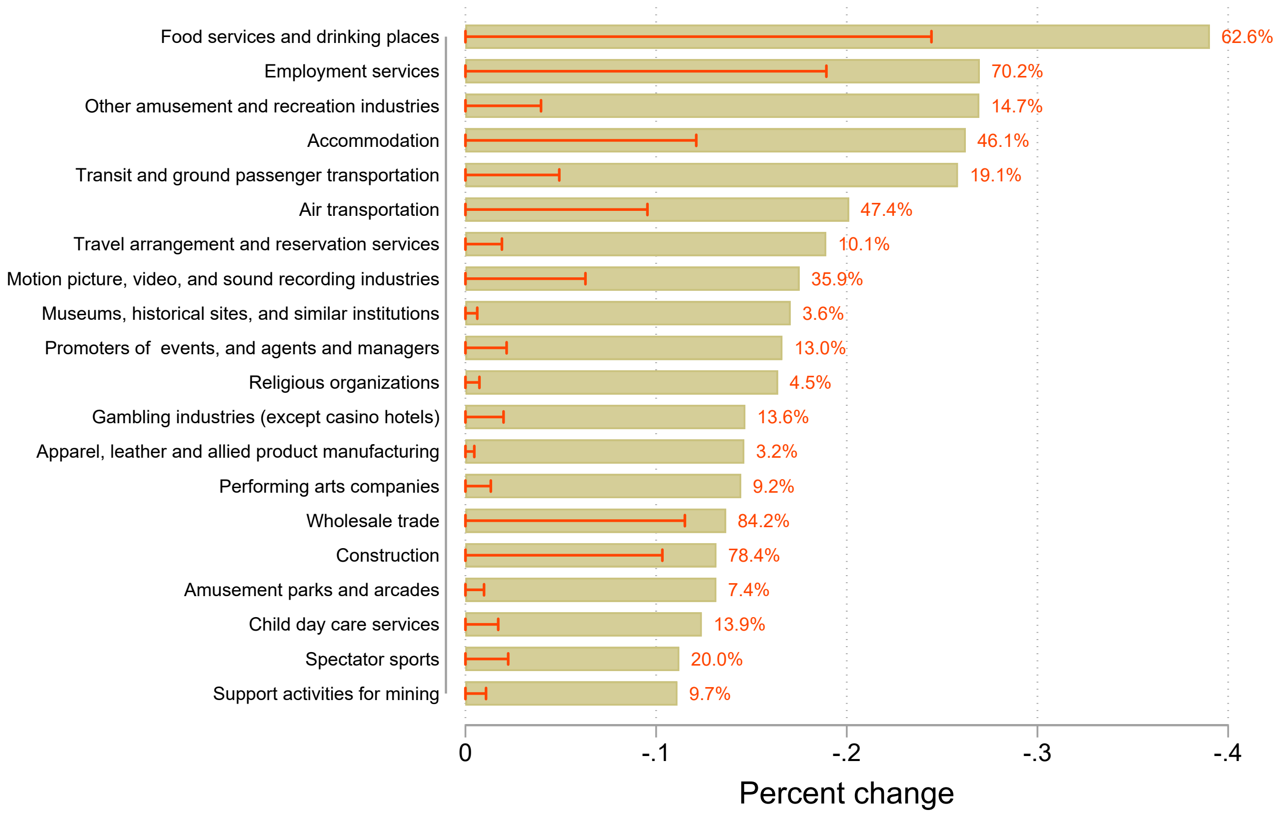

Figure 3: Downstream Effects of Employment Shocks

Note: Yellow bars measure the macroeconomic impact of each sector's shock. Red bars capture the shock's impact, assuming intermediates are not supplied to downstream industries, as measured by $\Lambda_i \lambda_i^D$. Percentages at the end of each bar show the GDP decline in the absence of downstream linkages as a fraction of the total impact under the "complete production network" of Figure 1, Panel A.

where λ_i^D is sector i's Domar weight under the input-output network structure $\boldsymbol{\omega}_{(i)}^D$. By exploiting the direct mapping between $\boldsymbol{\omega}_{(i)}^D$ and the corresponding equilibrium input-output matrix $\left(\boldsymbol{\Omega} - \boldsymbol{\Omega}_{(i)}\right)$, the Domar weights $\{\lambda_i^D\}_{i=1}^N$ are computed as in the following remark.

Remark 2. *The Domar weights $\{\lambda_i^D\}_{i=1}^N$ are given by*

$$\lambda_i^D = \sum_{j=1}^N b_j \left(I - \left(\boldsymbol{\Omega} - \boldsymbol{\Omega}_{(i)}\right)\right)_{ji}^{-1}.$$

The Leontief inverse matrix associated with $\boldsymbol{\omega}_{(i)}^D$ is $\left(I - \left(\boldsymbol{\Omega} - \boldsymbol{\Omega}_{(i)}\right)\right)^{-1}$, and (as with our previous measures) is built from the BLS input-output data. Large downstream effects (which are recorded when $Downstream\ Effect_i$ approaches $\Lambda_i \lambda_i$) imply propagation to downstream customer sectors is an increasingly important determinant of real GDP fluctuations.

Downstream Effect: Empirical Results. Figure 3 highlights the significance of downstream linkages in propagating sector-specific employment shocks. The yellow bars reflect the percentage change in real GDP due to sector-specific Covid disruptions over the second quarter of 2020. The red bars estimate the percentage change in GDP in the absence of downstream linkages. Percentages at the end of each bar measure the counterfactual GDP decline as a fraction of the total effect. The smaller these percentages are, the more substantial is the downstream propagation resulting from the sector's shock. As the figure shows, for many industries, downstream effects explain a large proportion of the total impact of the employment shocks. For example, air transportation records a downstream effect of (100% $-$ 47.4%)=52.5%. Intuitively, if other sectors did not rely on air transportation services, real GDP would have only declined by less than half of the actual decline. More than 50% of the output reduction is attributed to other industries' reliance on air transportation. This striking result underscores the importance of accounting for the flow of intermediate goods when making quantitative predictions about the welfare effects of sectoral disturbances. Other industries with substantial downstream effects include the apparel, leather, and allied product manufacturing sector (96.8%), support activities for mining (90.3%), and transit and ground passenger transportation (80.9%). On average, omitting the downstream propagation of shocks reduces a sector's aggregate impact by 76.7%, attesting to the significance of this channel.

Measure 3: Feedback Effect. Our final construct, the *feedback effect*, captures the role of higher-order feedback in generating aggregate output fluctuations. If sector i relies on inputs from other industries to produce, a labor shock to i propagates throughout the network and reverberates back to i through its upstream input-output linkages. We call this a feedback effect since the shock has a second-round impact on i's output. In the presence of upstream linkages, feedback occurs *ad infinitum*.

To measure the impact of feedback effects on real GDP, we define an input-output matrix in which sector i does not depend upon intermediate inputs from any other sector. Formally, we define an input-output network $\boldsymbol{\omega}_F^{(i)} = \boldsymbol{\omega} - \boldsymbol{\omega}^{(i)}$ where $\boldsymbol{\omega}^{(i)}$ is an $N \times N$ matrix with the i^{th} row equal to that of $\boldsymbol{\omega}$ and zeros elsewhere. There are N matrices of this kind, one for each sector. The feedback effect of a sector-specific labor shock to sector i is then defined as

$$Feedback\ Effect_i \equiv \Lambda_i \left(\lambda_i - \lambda_i^F\right) \cdot \Delta \log z_i \tag{4}$$

where λ_i^F is sector i's Domar weight under the input-output network structure $\boldsymbol{\omega}_F^{(i)}$. The difference $\lambda_i - \lambda_i^F$ captures the total impact of feedback of a shock to sector i. As with our measure of network spillovers and downstream effects, the Domar weights $\{\lambda_i^F\}_{i=1}^N$ are unobserved and must be constructed using observed input-output and final expenditure data.

Remark 3. *The Domar weights $\{\lambda_i^F\}_{i=1}^N$ are given by*

$$\lambda_i^F = \sum_{j=1}^{N} b_j \left(I - \left(\boldsymbol{\Omega} - \boldsymbol{\Omega}^{(i)}\right)\right)^{-1}_{ji}$$

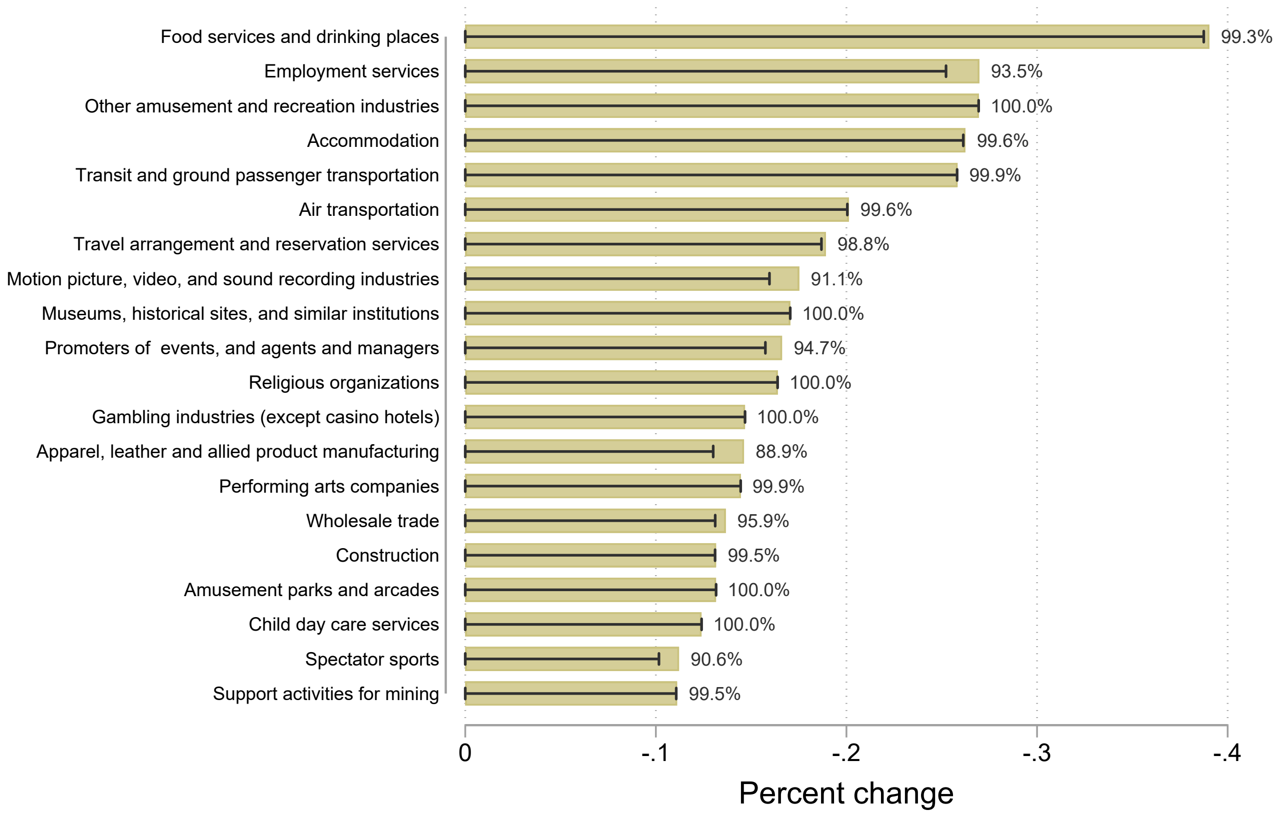

Figure 4: Feedback Effects of Employment Shocks

Note: Yellow bars capture the macroeconomic impact of each sector's shock. Black bars signify the effect of the shock, assuming no reliance on upstream sectors. Percentages at the end of each bar show the GDP decline in the absence of upstream linkages as a fraction of the total impact under the "complete production network" of Figure 1, Panel A.

By Proposition 2, the direct mapping between $\boldsymbol{\omega}_F^{(i)}$ and the corresponding equilibrium input-output matrix, $\left(\boldsymbol{\Omega} - \boldsymbol{\Omega}^{(i)}\right)$, implies the Leontief inverse $\left(I - \left(\boldsymbol{\Omega} - \boldsymbol{\Omega}^{(i)}\right)\right)^{-1}$ can again be computed from the BLS input-output data, thus permitting the measurement of λ_i^F. As $Feedback\ Effect_i$ approaches $\Lambda_i \lambda_i$, real GDP is increasingly influenced by higher-order feedback in response to the shock to i.

Feedback Effect: Empirical Results. As Figure 4 shows, feedback effects are relatively unimportant in explaining the aggregate output decline resulting from Covid-19 disruptions. In the figure, the yellow bars measure each sector's contribution to the total output decline (similar to Figure 2 and Figure 3), whereas the black bars capture the counterfactual change in real GDP in the absence of upstream linkages. Percentages at the end of each bar express the counterfactual GDP growth rate (without feedback effects) as a fraction of the total effect. The smaller this percentage, the greater the importance of upstream linkages in amplifying the sector's shock. Figure 4 shows some heterogeneity in feedback effects across sectors. For example, the apparel, leather, and applied manufacturing sector records a feedback effect of (100% − 88.9%) = 11.1%, suggesting the industry's consumption of intermediates from industries such as textile mills and textile product mills, and printing and related support activities (among others) to be crucial for the production of apparel. Other sectors with non-negligible feedback effects include spectator sports (9.4%), motion picture, video, and sound recording industries (8.9%), employment services (6.5%), and wholesale trade (4.1%). Overall, however, feedback effects do not appear to be consequential in explaining the observed output decline. Assuming no reliance on intermediate inputs still explains, on average, 95% of a sector's overall effect on GDP.

5 Conclusion

We present a theoretical framework to study the role of the US intermediate goods network in amplifying Covid-19 disruptions at the onset of the pandemic. We derive three nonparametric network measures using a CES production network model, which captures "network spillover", "downstream", and "feedback" effects of heterogeneous sectoral labor shocks. Our quantitative exercise reveals the aggregate indirect impact of Covid-19 disruptions (network spillovers) accounts for approximately three-quarters of the decline in real GDP over the second quarter of 2020. We also find employment shocks predominately impacted final demand by propagating from upstream suppliers to downstream customers. Nearly 80% of the aggregate impact of labor shocks is due to such downstream effects. Finally, higher-order feedback cannot explain the depth of the contraction in GDP: only 5% of the overall decline is attributed to second-, third-, and higher-round effects of the employment shocks.

Appendix

Appendix A. Proofs

Proof of Theorem 1. As in Baqaee and Farhi (2020b), throughout the proof we take nominal GDP to be the numeraire (GDP = 1), implying $d\log \mathrm{GDP} = 0$. Since $PY = \mathrm{GDP} = 1$, we have $d\log Y = -d\log P = -\sum_{j=1}^{N} b_j d\log p_j$.

From the producer's optimization problem, the first-order condition with respect to x_{ij}, L_{iS} and L_{iG} yield the following demand functions for intermediate inputs, labor, and capital, respectively: $x_{ij} = p_i^{\theta} y_i \omega_{ij} p_j^{-\theta}$, $L_{iS} = p_i^{\theta} y_i \mu_i z_i^{\theta-1} w_i^{-\theta}$, $L_{iG} = p_i^{\theta} y_i \mu_G w_G^{-\theta}$, and $K_i = p_i^{\theta} y_i \omega_{iK} r^{-\theta}$. Plugging these demand functions into i's production function, and solving for $p_i^{1-\theta}$, we get

$$p_i^{1-\theta} = z_i^{\theta-1}\mu_i w_i^{1-\theta} + \mu_G w_G^{1-\theta} + \omega_{iK} r^{1-\theta} + \sum_{j=1}^{N} \omega_{ij} p_j^{1-\theta}. \tag{5}$$

From the demand functions for x_{ij}, L_{iS}, L_{iG} and K_i we derive expressions for Ω_{ij}, Λ_i, Λ_{iG} and Ω_{iK}:

$$\Omega_{ij} = p_i^{\theta-1}\omega_{ij} p_j^{1-\theta}, \quad \Lambda_i = p_i^{\theta-1}\mu_i z_i^{\theta-1} w_i^{1-\theta}, \quad \Lambda_{iG} \equiv \frac{w_G L_{iG}}{p_i y_i} = p_i^{\theta-1}\mu_G w_G^{1-\theta}, \quad \text{and} \quad \Omega_{iK} \equiv \frac{rK_i}{p_i y_i} = p_i^{\theta-1}\omega_{iK} r^{1-\theta}.$$

Total (log) differentiation of equation (5) therefore yields

$$d\log p_i = \Lambda_i d\log w_i - \Lambda_i d\log z_i + \Lambda_{iG} d\log w_G + \Omega_{iK} d\log r + \sum_{j=1}^{N} \Omega_{ij} d\log p_j$$

Solving for $d\log p_i$, we get

$$d\log p_i = \sum_{m=1}^{N} \Psi_{im}\Lambda_m d\log w_m - \sum_{m=1}^{N} \Psi_{im}\Lambda_m d\log z_m + \sum_{m=1}^{N} \Psi_{im}\Lambda_{mG} d\log w_G + \sum_{m=1}^{N} \Psi_{im}\Omega_{mK} d\log r$$

Noting that $d\log Y = -\sum_{j=1}^{N} b_j d\log p_j$, we can write

$$\begin{aligned} d\log Y = & \sum_{j=1}^{N}\sum_{m=1}^{N} b_j \Psi_{jm}\Lambda_m d\log z_m - \sum_{j=1}^{N}\sum_{m=1}^{N} b_j \Psi_{jm}\Lambda_m d\log w_m \\ & - \sum_{j=1}^{N}\sum_{m=1}^{N} b_j \Psi_{jm}\Lambda_{mG} d\log w_G - \sum_{j=1}^{N}\sum_{m=1}^{N} b_j \Psi_{jm}\Omega_{mK} d\log r. \end{aligned}$$

Furthermore, since $\lambda_m = \sum_{j=1}^{N} b_j \Psi_{jm}$, we can rewrite the above expression as

$$d\log Y = \sum_{m=1}^{N} \lambda_m \Lambda_m d\log z_m - \left(\sum_{m=1}^{N} \lambda_m \Lambda_m d\log w_m + \sum_{m=1}^{N} \lambda_m \Lambda_{mG} d\log w_G + \sum_{m=1}^{N} \lambda_m \Omega_{mK} d\log r \right).$$

Since all factor supplies are fixed, and nominal GDP is taken to be the numeraire, we can write the above equation as

$$\begin{aligned} d\log Y = & \sum_{m=1}^{N} \lambda_m \Lambda_m d\log z_m \\ & - \left(\sum_{m=1}^{N} \lambda_m \Lambda_m d\log(\lambda_m \Lambda_m) + \sum_{m=1}^{N} \lambda_m \Lambda_{mG} d\log(\lambda_m \Lambda_{mG}) + \sum_{m=1}^{N} \lambda_m \Omega_{mK} d\log(\lambda_m \Omega_{mK}) \right). \end{aligned}$$

Finally, because $\sum_{m=1}^{N} \lambda_m \Lambda_m + \sum_{m=1}^{N} \lambda_m \Lambda_{mG} + \sum_{m=1}^{N} \lambda_m \Omega_{mK} = 1$ (because total factor income equals nominal GDP), the term in parentheses is equal to zero, hence

$$d\log Y = \sum_{m=1}^{N} \lambda_m \Lambda_m d\log z_m.$$

■

Proof of Proposition 1. We denote by $\boldsymbol{\omega}$ the *observed* input-output matrix, and $\tilde{\boldsymbol{\omega}}$ the *counterfactual* input-output network with omitted linkages. Throughout the proof, a tilde denotes a variable in the counterfactual economy. The assumption that i's specific labor is endowed in proportion to its size, implies that $\alpha = \frac{L_{1S}}{y_1} = \frac{L_{2S}}{y_2} = ... = \frac{L_{NS}}{y_N} = \frac{\tilde{L}_{1S}}{\tilde{y}_1} = \frac{\tilde{L}_{2S}}{\tilde{y}_2} = ... = \frac{\tilde{L}_{NS}}{\tilde{y}_N}$, where α is a constant. In other words, the ratio of each sector's specific-labor endowment to its output is an economy-wide constant invariant to the underlying input-output structure.

In the proof of Theorem 1, we derived $L_{iS} = p_i^{\theta} y_i \mu_i z_i^{\theta-1} w_i^{-\theta}$, which implies $\alpha = \frac{L_{iS}}{y_i} = p_i^{\theta} \mu_i z_i^{\theta-1} w_i^{-\theta}$. Next, noting that $\Lambda_i = p_i^{\theta-1} \mu_i z_i^{\theta-1} w_i^{1-\theta}$, we can express i's (specific) labor expenditure share as

$$\Lambda_i = \frac{w_i L_{iS}}{p_i y_i} = \frac{w_i}{p_i}\alpha = \frac{w_i}{p_i} w_i^{-\theta} p_i^{\theta} \mu_i z_i^{\theta-1}$$

Using the result $\alpha = p_i^{\theta} \mu_i z_i^{\theta-1} w_i^{-\theta}$, we can rearrange to solve for $\frac{w_i}{p_i}$ to get

$$\frac{w_i}{p_i} = \alpha^{-\frac{1}{\theta}} \mu_i^{\frac{1}{\theta}} z_i^{\frac{\theta-1}{\theta}}.$$

Finally, plugging the above equation into our expression for Λ_i, gives

$$\Lambda_i = \alpha^{\frac{\theta-1}{\theta}} \mu_i^{\frac{1}{\theta}} z_i^{\frac{\theta-1}{\theta}}.$$

Since α is a constant, we get $\Lambda_i = \tilde{\Lambda}_i$ for all i.

■

Proof of Proposition 2. We first show that the vector of final expenditure shares **b** is invariant to the underlying input-output structure of the economy.

The Lagrangean associated with the household's problem is

$$\mathcal{L} = \left(\sum_{i=1}^{N} a_i^{\frac{1}{\sigma}} c_i^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}} + \lambda \left(\sum_{i=1}^{N} w_i L_{iS} + \sum_{i=1}^{N} w_G L_{iG} + \sum_{i=1}^{N} r K_i - \sum_{i=1}^{N} p_i c_i \right)$$

where λ is a Lagrange multiplier. The first-order condition with respect to c_i implies

$$c_i = \text{GDP} \cdot a_i p_i^{-\sigma} P^{\sigma-1}$$

where P is the CPI price index, defined $P \equiv \left(\sum_{i=1}^{N} a_i p_i^{1-\sigma} \right)^{\frac{1}{1-\sigma}}$. Therefore,

$$b_i = \frac{p_i c_i}{\text{GDP}} = a_i p_i^{1-\sigma} P^{\sigma-1}.$$

In the proof of Theorem 1, we derived $L_{iG} = p_i^{\theta} y_i \mu_G w_G^{-\theta}$. Since general labor is endowed in proportion to each sector's size, we get $\beta = \frac{L_{iG}}{y_i} = p_i^{\theta} \mu_G w_G^{-\theta}$, where β is an economy-wide constant. This implies that output prices at the initial equilibrium are given by $p_i = \beta^{\frac{1}{\theta}} \mu_G^{-\frac{1}{\theta}} w_G$ for all i. We can then write the CPI price index as

$$P = \left(\beta^{\frac{1-\sigma}{\theta}} \mu_G^{\frac{\sigma-1}{\theta}} w_G^{1-\sigma} \left(\sum_{i=1}^{N} a_i \right) \right)^{\frac{1}{1-\sigma}}.$$

Since $\sum_{i=1}^{N} a_i = 1$, we get

$$P = p_i = \beta^{\frac{1}{\theta}} \mu_G^{-\frac{1}{\theta}} w_G.$$

Plugging the above expression into $b_i = a_i p_i^{1-\sigma} P^{\sigma-1}$ implies that $b_i = a_i$ for all i. In words, final expenditure shares equal the consumption weights in the household's utility function, which are assumed to be exogenous. This result completes the first part of the proof.

We now prove that $T(\boldsymbol{\omega}) = T(\boldsymbol{\Omega})$ where $T(\cdot)$ denotes some arbitrary linear transformation. In the proof of Theorem 1, we derived an expression for Ω_{ij} in terms of prices and the input-output matrix $\boldsymbol{\omega}$. Specifically,

$$\Omega_{ij} = p_i^{\theta-1} \omega_{ij} p_j^{1-\theta}.$$

Furthermore, since $p_i = p_j = \beta^{\frac{1}{\theta}} \mu_G^{-\frac{1}{\theta}} w_G$, we get

$$\Omega_{ij} = \omega_{ij}$$

for all i, j. The above result therefore implies that $\boldsymbol{\Omega} = \boldsymbol{\omega}$, which in turn delivers the result that $T(\boldsymbol{\omega}) = T(\boldsymbol{\Omega})$.

■

Appendix B. Supplementary Results

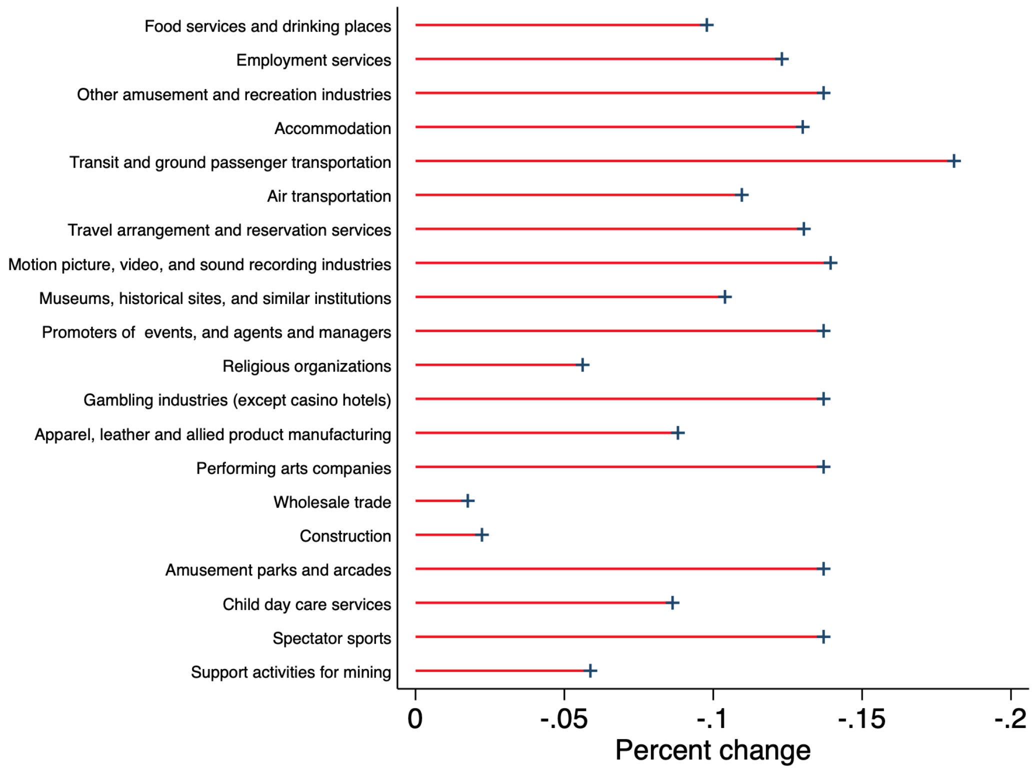

Figure B.1: Employment Growth Rate by Sector, 2020 Q2

Note: The figure shows, for the top 20 sectors, the labor shocks used to calibrate the model. Shocks are computed as the end-of-period employment change between the second and third quarters of 2020 in the United States, as measured by the BLS.

Table B.1: Model-Implied Real GDP Growth and Total Network Spillover Effect

	Seasonally Adjusted		Not Seasonally Adjusted	
	Real GDP	Network Spillover	Real GDP	Network Spillover
Panel A: Q2 2020				
End-of-period	-8.70%	-6.25%	-7.29%	-5.25%
Mean employment change	-11.75%	-8.44%	-10.71%	-7.72%
Panel B: Q3 2020				
End-of-period	2.60%	1.77%	1.85%	1.24%
Mean employment change	5.14%	3.61%	5.48%	3.84%
Panel C: Q4 2020				
End-of-period	1.01%	0.66%	1.30%	0.80%
Mean employment change	1.68%	1.13%	1.54%	0.97%

Notes: The table shows the model-implied real GDP growth rate and aggregate network spillover effect using seasonally adjusted and unadjusted employment series. It also compares the calibration of shocks to end-of-period versus mean employment changes. The observed real GDP growth rates, according to the BEA, for Q2, Q3 and Q4 of 2020 are, respectively, −8.99%, 7.48% and 0.99%.

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