Fertility and economic growth: the role of workforce skill composition and child care prices

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Abstract
This paper presents an overlapping generations model that incorporates choice of occupation (education), fertility, how to rear children, and a market for child care. The dynamic interplay between economic growth and fertility is examined as an economy moves through two phases distinguished by the skill composition of the workforce. In the initial phase, the economy comprises skilled and unskilled workers. In the second phase, all workers are skilled. Skilled workers are shown to have fewer children than less educated workers. Aggregate fertility initially declines as the fraction of skilled workers rises with economic growth, and then may recover as the fertility of a skilled workforce rises with skilled wages, for given child care prices. However, in equilibrium, child care prices rise proportionally to skilled wages when child care is produced with constant returns to skilled labour. Results indicate that whether or not the rise in fertility witnessed in high-income countries will continue depends on each country's structure of child care.

JEL classifications: J11, J13, J24, O41

1. Introduction
Following an emerging empirical debate over whether fertility decline reverses at advanced stages of development, the main goal of this paper is to theoretically address how fertility changes over time as an economy develops. This paper proposes a consistent and cohesive explanation of fertility decline, its recent reversal in several high-income economies, and discusses the possible relation between the recent rise in fertility and the structure of child care. In the model presented in this paper, the relationship between aggregate fertility and per capita income reflects within-country differences in workforce skill composition and household choice of occupation, as well as fertility and child care—the price of which is endogenously determined by market forces.
Since 2000, some advanced economies have experienced an upturn in fertility (see Fig. 1). There is also evidence of a change in the relationship between fertility and the level of development in advanced economies. Based on cross-sectional and longitudinal data covering more than 100 countries, Myrskyla et al. (2009) find that the once negative relationship between fertility and development has become reverse-J shaped. In 2005, the correlation between fertility and the Human Development Index (HDI) was positive for advanced economies with high HDI levels.

However, recent evidence suggests that a reversal in the relationship between fertility and development is not robust. Luci and Thevenon (2011) and Luci-Greulich and Thevenon (2014) focus on the association between fertility and the HDI standard of living sub-index for OECD economies. After controlling for birth postponement and country-specific effects, Luci and Thevenon (2011) confirm a convex impact of per capita income on fertility, but observe that advanced economies have divided into two distinct groups with either near replacement or low total fertility rates. Family-friendly policies, such as child care subsidies, play an important role in determining to which of these two groups an advanced economy belongs (McDonald and Moyle, 2010).

1 Harttgen and Vollmer (2014) show that reversal in the total fertility rate-HDI association at a HDI of around 0.9 is not robust to a decomposition of the HDI into standard of living, education, and health sub-indices. For most OECD economies, HDI levels exceed 0.9, which corresponds to a 75-year life expectancy, per capita GDP of US$25,000 in year 2000 PPP, and a 0.95 human capital index (based on literacy and enrolment rates).
Moreover, Luci-Greulich and Thevenon (2014) find that increases in fertility with per capita income are likely to be negligible if economic growth is not accompanied by changes to institutions that facilitate the combination of child rearing and work. This is a theme developed in this paper by exploring the role that the structure of child care plays in the relationship between fertility and economic development at high levels of per capita income.

The paper hypothesizes that an explanation for fertility initially declining and then rising with economic development lies in the combined effects of differential fertility rates, the opportunity cost of time spent rearing children, and the relative price of child care. In the model, economic development is driven by the accumulation of capital, which complements skilled labour. Skilled workers have lower fertility than unskilled workers, but fertility is increasing with skilled wages when parents can substitute purchased goods and services for their own time.

As an economy becomes more capital intensive, increasing demand for skilled workers is initially absorbed by an increasing share of skilled workers, with the skilled wage linked to the constant unskilled wage by an arbitrage condition. Ultimately, as all workers become skilled, increasing demand for skilled workers is absorbed by increasing skilled wages; skilled workers find children more affordable, but more costly to rear using unpaid time. 2 Substitution of fixed price goods and services for parental time mitigates the opportunity cost of rearing children and the fertility of skilled workers rises with their wages. 3 In the first phase, overall fertility declines. In the second phase, it rises. However, the latter no longer occurs if the rise in skilled wages leads to a proportionate rise in child care prices. 4

The observation that parents with a lower (higher) level of skill have more (fewer) children is consistent with international evidence from both long-run historical data (for a review, see Sato et al., 2008) and micro data (Rosenzweig, 1990; Beenstock, 2007). The observation that a skilled workforce experiences a rise in fertility is consistent with evidence that parental tertiary education and fertility rates are now positively associated across OECD countries (d'Addio and d’Ercole, 2005).

This is the first model where endogenous increases in the price of child care and workforce skill composition play important roles in the interrelationship between fertility and economic growth. Two particularly interesting results of the model presented in this paper are, firstly, that fertility decline may reverse at advanced stages of economic development, as the fertility of skilled workers rises with wages, for given child care prices. Secondly, economic growth can only sustain the reversal of fertility decline if child care prices rise proportionately less than skilled wages, be it because child care is subsidized, because child care is produced with increasing returns to skilled labour, or because child care labour is relatively less skilled than the average worker. These results suggest that an emerging positive association between fertility and economic growth will then depend on the country’s structure of child care, and have implications for the role of child care policy.

2 Cette et al. (2007) find evidence corroborating the simultaneous and opposing forces of these two effects on fertility.

3 Child rearing is a relatively goods-intensive activity in the United States (Gronau and Hamermesh, 2006) suggesting that substitution can indeed mitigate the rising opportunity cost in high-income economies.

4 Recent empirical evidence suggests that rising child care prices would in turn reduce demand for children and household labour supply (Guest and Parr, 2013). By endogenizing the price of child care, this paper analyses both directions of causality.
In existing models, growth in per capita income induces fertility decline by raising either the opportunity cost of time (Barro and Becker, 1988; Galor and Weil, 1996), the relative return to investing in education per child (Becker et al., 1990) or the fraction of households having fewer children (Kimura and Yasui, 2007; Chen, 2010). Although Kimura and Yasui (2007) and Chen (2010) predict that fertility and economic growth are unambiguously negatively related, their model that includes an individual’s choice to become skilled or remain unskilled is a useful starting point for the analysis in this paper. Parental time spent rearing children is fixed in these models. This assumption implies that wages do not affect household fertility choice.

In existing models with endogenous child care, the real price of child care is constant by assumption (Apps and Rees, 2004; Hirazawa and Yakita, 2009) or by assuming that child care uses unskilled labour (Martinez and Iza, 2004). This is a shortcoming when assessing the role of child care in reversing fertility decline since increased demand may raise the price of child care. The model in this paper allows for endogenous increases in the price of child care.

To incorporate the opposing effects of higher wages on the fertility of skilled workers, a household optimization problem is embedded within a growing economy where fertility, skill composition, and wages evolve over time. Households choose the cost-minimizing mix of time and purchased goods and services to rear children. To endogenize the price of child care, the model allows for the child care sector to draw on the educated workforce of a high-income economy.

Section 2 of this paper provides the theoretical model of production and household choice of occupation, fertility and child care, which underpins a dynamic interplay between workforce skill composition, aggregate fertility, and economic growth. Section 3 examines the robustness of fertility decline reversal in relation to endogenous child care prices and the structure of child care. Section 4 concludes.

2. Basic structure of the model

Consider an overlapping generations model in which people live for three periods. In childhood, an agent consumes time, as well as goods and services, from their parents. In adulthood, an agent supplies labour, raises children, and may receive further education. The agent decides whether or not to undertake training to become a skilled worker, how many children to have, and how to rear them. In old age, an agent is retired from the labour force and consumes the proceeds of their savings from the previous period.

The closed economy identity of savings and investment provides the link with growth in the capital stock, skill composition of the workforce, and wages, which in turn influence fertility. Rising skill intensity is a consequence of economic growth. Skilled labour is complementary to physical capital. Combined with the neoclassical capital intensity effect, this final feature generates a feedback loop between growth in output per worker and fertility.

The economy is modelled as it moves through two phases, distinguished by the skill composition of the labour force. The analysis sets out to explore the behaviour of fertility, and its interrelationship with economic growth, during an initial phase when the labour force comprises skilled and unskilled workers, before entering a second phase when every agent chooses to become skilled. Whilst a threshold level of capital per household distinguishes the

Like Chen (2010), although the model is based on Kimura and Yasui (2007), the focus of this paper differs and the modifications alter the dynamic analysis.
two phases, capital per household is endogenously determined by savings and fertility, which
are in turn determined by the accumulation of capital per household, which raises the share
of skilled workers in the initial phase and skilled wages in the second phase.

Economic development is both a cause and a consequence of the behaviour of fertility. The driving force of economic development is the complementarity of skilled labour and
physical capital, which engenders a positive feedback loop between capital accumulation
and rising skill intensity, increasing savings, and declining fertility in the initial phase, and
rising wages, increasing savings, and rising fertility in the second phase.

2.1 Production of final output
Physical capital ($K$), unskilled labour ($L_u$) and skilled labour ($L_s$) are factors of production,
all with non-increasing marginal products. The greater the capital-labour ratio in the econ-
omy, the more highly-rewarded is skilled labour relative to unskilled labour. This is consist-
ent with the relative rise in rewards to skilled labour characterizing economic growth.

The production function is given by

$$Y_t = A[K_t^a(L_s^t)^{(1-a)} + bL_u^t]; \quad A > 0; \quad b > 0; \quad a \in (0, 1),$$  

where the separability captures the assumption that, whereas capital complements skilled
labour, unskilled labour is a perfect substitute for other factors of production.

Each adult household is endowed with a unit of time, which can be allocated to child
rearing, labour force participation, and education (training). Each agent chooses whether
to become a skilled worker or an unskilled worker. To supply skilled labour, an agent must
spend a fraction of their time endowment, $\tau \in (0, 1)$, to acquire higher education.

Each agent chooses the number of children, where $n^t_s$ and $n^t_u$ denote the pairs of children
of skilled and unskilled agents, respectively. To raise children, an agent purchases goods
and services and employs a fraction of their time endowment, denoted $\hat{x}$ and $\hat{z}$, respectively.
Purchased goods and services per pair of children and time per pair of children are $\hat{x}$ and $\hat{z}$,
where $\hat{x}, \hat{z} \in (0, 1)$.  

Let $N_t$ be the number of working-age agents and $\varphi$ be the ratio of skilled workers
to all workers. The aggregate supply of skilled labour and unskilled labour at time $t$ is

$L_s^t = (1 - \hat{z} n^t_s)/(1 - \varphi) N_t$ and $L_u^t = (1 - \hat{z} n^t_u)/(1 - \varphi) N_t$, respectively.

Perfectly competitive factor markets imply

$$w_s^t = \partial Y_t/\partial L_s^t = A(1 - z)[k_t/b_t \varphi]^z$$ \hspace{1cm} (2)

$$w_u^t = A b$$ \hspace{1cm} (3)

$$R_{t+1} \equiv (1 + r_{t+1}) = \partial Y_t/\partial K_t = A x[k_t/b_t \varphi]^{x-1},$$ \hspace{1cm} (4)

where $k_t \equiv K_t / N_t$ and $b_t \equiv L_s^t/L_u^t \varphi N_t$ denote physical capital per working age agent and
skilled labour per skilled working age agent, respectively. 

\textit{Ceteris paribus}, an increase in capital intensity will therefore raise the wage\(^7\) for skilled labour, $(w_s^t)$ while the wage for
unskilled labour $(w_u^t)$ is constant.

\hspace{1cm} 6 For the moment, $\hat{x}$ is measured in units of market goods and services (Apps and Rees, 2004;
Hirazawa and Yakita, 2009). Section 3 focuses on the child care component, where households
may buy out all of their time in child-rearing: $\hat{x} \in [0, 1]$.

\hspace{1cm} 7 This is the real wage, with the price of the aggregate good normalized to 1.
### 2.2 Household optimization

Each agent (household) derives utility directly from the number of children. Skilled and unskilled agents are assumed to have the same preferences. The household utility function is

\[ u_t = \gamma \ln(n_t) + \lambda \ln(c_t) + \delta \ln(c_{t+1}), \]

where \( c_t, c_{t+1} \) and \( n_t \) denote working-age (or current period) consumption, old-age consumption, and pairs of children, respectively, chosen by the household at time \( t \); \( \gamma \in (0, 1) \) and \( \lambda \in (0, 1) \) capture the relative weights given to children and working-age consumption, and \( \delta = (1 - \gamma - \lambda) \in (0, 1) \) is the discount factor.

Assume that \( n_t \) pairs of children are produced at home by combining parental time and goods and services purchased from the market, both of which are essential for production. Let the “production function” for child rearing be specified as

\[ n_t = (zt)^a(x_t)^{1-a}; a \in (0, 1), \]

where \( x \) and \( z \) denote total child rearing goods and services and total time input, respectively. This production function determines how to rear a given number of children, meaning the optimal input mix. The optimal number of children is determined by maximizing (5) subject to a household budget constraint.

Because the child-rearing production function is homogeneous of degree one, the household optimization problem can be solved in two stages. The household first chooses, for a given \( n_t \), the cost-minimizing input mix and then chooses \( n_t \), given the efficient input mix, so as to maximize utility subject to a budget constraint.

The total cost of rearing children for agent \( i \) is

\[ C_i = w_i'z_i' + p_x(1 - \beta)x_i' ; \beta \in (0, 1), \]

where \( i = s, u \) denotes skilled and unskilled, respectively, and \( p_x \) and \( \beta \) denote the price and government subsidy per unit of purchased goods and services, respectively. \(^8\)

The household first chooses \( x_i' \) and \( z_i' \) so as to minimize (7) subject to (6), yielding input demands for child-rearing time and goods, respectively,

\[ z_i' \equiv z_i'^a n_i'^{1-a} = \frac{p_x(1 - \beta)a}{w_i'(1 - a)} n_i'^a, \]

\[ x_i' \equiv x_i'^a n_i'^{1-a} = \frac{w_i'(1 - a)}{p_x(1 - \beta)a} n_i'^a. \]

Substituting the optimal input mix, (8a) and (8b), into (7), we obtain an implicit cost per \( n_i' \) as

\[ p(w_i', p_x, \beta) = [w_i']^a [p_x(1 - \beta)]^{1-a} B, \]

where \( B = a^{-a}(1 - a)^{-(1-a)} \), \( z_i' = \partial p(\cdot)/\partial w_i' \) and \( x_i' = \partial p(\cdot)/\partial p_x(1 - \beta) \).

Each agent faces the second-period budget constraint

\[ c_{t+1} = s_t(1 + r_{t+1})(1 - T_{t+1}), \]

\(^8\) As with \( w_i' \), \( p_x \) is the real price of child-rearing goods and services. Subsidies are financed by a tax on old age consumption (see second period budget constraint below).
where \( r_{t+1} \) denotes the rate of return on savings, \( s_t \), and \( T_{t+1} \) denotes the rate of taxation on old age consumption.\(^9\)

A skilled worker faces the first-period budget constraint

\[
p(u^s_t, p_x, \beta)n_t + c_t + s_t \leq u^s_t(1 - \tau). \tag{11}
\]

Maximizing (5) with respect to (10) and (11), it follows that the number of children, consumption, and savings of a skilled worker are

\[
n^s_t = \frac{\gamma(1 - \tau)}{B} \left[ \frac{u^s_t}{p_x(1 - \beta)} \right]^{1-a} \tag{12a}
\]

\[
c^s_t = \lambda(1 - \tau)u^s_t \tag{12b}
\]

\[
s^s_t = (1 - \gamma - \lambda)(1 - \tau)u^s_t. \tag{12c}
\]

An unskilled worker faces the first-period budget constraint

\[
p(u^u_t, p_x, \beta)n_t + c_t + s_t \leq u^u_t. \tag{13}
\]

Maximizing (5) with respect to (10) and (13), it follows that the number of children, consumption, and savings of an unskilled worker are

\[
n^u_t = \frac{\gamma}{B} \left[ \frac{u^u_t}{p_x(1 - \beta)} \right]^{1-a} \tag{14a}
\]

\[
c^u_t = \lambda u^u_t \tag{14b}
\]

\[
s^u_t = (1 - \gamma - \lambda)u^u_t. \tag{14c}
\]

Since agents choose to become skilled or to remain unskilled, agents are indifferent between becoming skilled and remaining unskilled in an equilibrium where both types of workers exist. Equating the indirect utility of both types of workers yields the arbitrage condition

\[
\frac{u^u_t}{u^s_t} = (1 - \tau)^{1/(1-a)}, \tag{15}
\]

where the equilibrium wage rate of a skilled worker is higher than that of an unskilled worker, since \( \tau \in (0, 1); a \in (0, 1) \), and \( \gamma \in (0, 1) \).

Intuitively, each agent derives utility from children and consumption when young and old. A skilled worker has lower fertility than an unskilled worker. Therefore, in order to be indifferent between becoming skilled or remaining unskilled, a skilled worker must earn a higher wage to fund higher young and old-age consumption than their unskilled counterpart.

This discussion is summarized with the following:

**Remark 1** The equilibrium wage rate of a skilled worker is higher than that of an unskilled worker.

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\(^{9}\) The rate of subsidy and taxation are set so as to satisfy the balanced-budget constraint: \( \beta \delta n_t[n_{t-1}L_{t-1}] = s_{t-1}(1 + r_t)T_tL_{t-1} \). Although endogenous at the aggregate level, the rate of subsidy and taxation is treated as exogenous by each individual household.
Equations (12a) and (14a) imply that a skilled worker has fewer children despite a skilled wage premium:

\[ w_t^s < \left(1 - \tau\right)^{-1/(1-a)} w^u_t \Rightarrow n_t^s < n_t^u, \]

where \( w_t^s \) is given by the arbitrage condition provided by condition (15). Intuitively, on the one hand, to become a skilled worker, an agent must spend a fraction of their time training, and therefore has less time to rear children. On the other hand, higher wages make children more affordable for a skilled worker. The former effect dominates, consistent with evidence that the fertility of skilled workers is less than the fertility of unskilled workers (Rosenzweig, 1990; Beenstock, 2007).

This discussion is summarized with the following:

**Remark 2** A skilled worker has fewer children than an unskilled worker.

Like Kimura and Yasui (2007) and Chen (2010), modelling the education sector is outside the scope of this paper. Section 3 models the child care sector because whether or not fertility rises depends on the structure of child care. However, we would expect that the key results regarding fertility are robust to an education sector that soaks up labour. Allowing skilled agents to work in education or final production may introduce an additional arbitrage condition, but if the fertility of skilled educators is the same as other skilled workers, then fertility would initially decline as the share of skilled workers rises, and then rise as the skilled wage rises.

2.3 Dynamic system

In equilibrium, all markets clear in every period. The labour market clears, with the fraction of skilled workers determined by the indifference condition between being skilled and unskilled, in (15). The final goods market clears, \( Y_t = C_t + I_t \) or equivalently, the capital market clears, \( K_{t+1} = s_t N_t \).

The dynamics of the economy can be summarized through the law of motion for the capital stock per working age household. The clearing condition of the capital market implies that the capital stock at period \( t+1 \) is determined by the savings of working-age households at the end of period \( t \),

\[ K_{t+1} = \left[ \varphi_s s^u_t + \left(1 - \varphi_s\right) s^u_t \right] N_t. \]  

(16)

The number of working age households at time \( t + 1 \) is

\[ N_{t+1} = f_t N_t, \]

where the total fertility rate at time \( t \), is

\[ f_t = \varphi_s n_t^s + \left(1 - \varphi_s\right) n_t^u \]  

(17)

Rewriting (16) in terms of capital stock per working-age household,

\[ k_{t+1} = \frac{K_{t+1}}{N_{t+1}} = \frac{s_t}{f_t}. \]  

(18)

From (18), an equation of motion \( k_{t+1} = \phi(k_t) \) is obtained, since overall savings and fertility is determined by \( \varphi_s \), which we will see is a function of \( k_t \) during an initial phase, and savings and fertility of a skilled worker are determined by \( w_t^s \), which in turn is a function of \( k_t \) during a second phase.
It follows from (2), substituting for \( \hat{z} n_t^* \), (3), and (15) that the fraction of skilled workers, \( \varphi_t \), is a function of \( k_t \), as implied by the following expression:

\[
\varphi_t(k_t) = \left( \frac{1 - \tau}{1 - \alpha} \right)^{1/\alpha} \left( \frac{1 - \alpha}{b} \right) \frac{1}{k_t} \equiv \theta k_t,
\]

where \( \varphi_t(k_t) \) has an upper bound of 1. Once \( k_t \) reaches a sufficiently high level, \( \bar{k} \), all agents choose to become skilled.

Thus, the fraction of skilled workers is

\[
\varphi_t(k_t) = \begin{cases} 1 & \text{if } k_t \geq \bar{k} \\ \theta k_t & \text{if } k_t < \bar{k} \end{cases},
\]

where \( \bar{k} = \theta^{-1} \), yielding the following:

**Remark 3** The fraction of skilled workers is non-decreasing in the capital stock per household, and strictly increasing in the capital stock per household for \( k_t < \bar{k} \).

Substituting into (18) from (16) and (17), the equation of motion for the system is derived from

\[
k_{t+1} = B \left( 1 - \gamma - \lambda \right) \frac{1}{\gamma} \left\{ \begin{array}{l}
0 k_t \varphi_t^*(1 - \tau) + (1 - \theta k_t) \varphi_t^* \\
0 k_t \left( \frac{\varphi_t^*}{p_x(1 - \beta)} \right) (1 - \tau) + (1 - \theta k_t) \left( \frac{\varphi_t^*}{p_x(1 - \beta)} \right) \end{array} \right\} \text{ if } k_t < \bar{k},
\]

where

\[
\begin{align*}
\varphi_t^* &= A (1 - \alpha) \left\{ \begin{array}{l}
[k_t/(1 - \alpha\gamma)(1 - \tau)]^2 & \text{if } k_t \geq \bar{k} \\
[(1 - \alpha\gamma)(1 - \tau)\bar{k}]^{-2} & \text{if } k_t < \bar{k} \end{array} \right. \\
\end{align*}
\]

Referring to the Appendix, capital per household evolves over time from a historically given initial level according to

\[
k_{t+1} = \Phi(k_t) = \left\{ \begin{array}{l}
\left( \frac{1 - \alpha}{1 - \alpha\gamma} \right)^{1/\alpha} \frac{\gamma}{(1 - \alpha\gamma)} \frac{1}{k_t} \text{ if } k_t \geq \bar{k} \\
1 + \theta \left\{ (1 - \tau)(1 - \alpha\gamma) - 1 \right\} k_t \text{ if } k_t < \bar{k} \end{array} \right. \]

where

\[
\bar{k} k_t = a^{-\alpha} (1 - a)^{-(1 - \alpha)} \left( p_x(1 - \beta) \right)^{1 - \alpha}.
\]
The following properties of the equation of motion (23) ensure the existence of a steady state equilibrium:

\[ \phi(0) = (Ab)^\gamma B \left( 1 - \frac{\gamma}{\lambda} \right) > 0 \quad \text{and} \quad \lim_{k_t \to \infty} \phi'(k_t) = 0. \quad (24) \]

Furthermore, the equation of motion has curvature properties that are summarized with the following:

**Proposition 1** \( \phi(k_t) \) is increasing and convex in \( k_t \) over the interval \((0, \bar{k})\), and increasing and concave over the interval \((\bar{k}, \infty)\).

**Proof** Refer to the Appendix.

Intuitively, the curvature of the equation of motion follows from the dynamic behaviour of savings and fertility. First, consider Phase 1, where the economy comprises skilled and unskilled workers. If \( k_t < \bar{k} \), the equilibrium skilled wage rate is constant, by (22). Intuitively, an agent chooses to spend a fraction of their time in higher education in order to become skilled. In an equilibrium where both skilled and unskilled workers exist, agents are indifferent between becoming skilled or remaining unskilled. Equating the indirect utility of workers yields a skilled wage premium that, referring to the appendix, is constant over time. Referring to (2), capital complements skilled labour, raising its marginal product. The supply of skilled workers increases, lowering its marginal product. The offsetting effect of a rising fraction of skilled workers maintains a constant skilled wage premium.

Thus, the dynamics of overall savings and fertility is solely determined by a rising fraction of skilled workers. By the simple linear relationship in (20), the higher \( k_t \), the larger the fraction of skilled workers. Since a skilled worker saves more and has less children than an unskilled worker, overall savings will rise and overall fertility will fall, as the number of skilled workers increases. From (18), increasing savings and decreasing fertility implies that capital per household accumulates at an increasing rate. That is, \( \phi(k_t) \) is strictly convex.

Now, consider Phase 2, where all workers are skilled. If \( k_t \geq \bar{k} \), the skilled wage rate is increasing in \( k_t \), by (22). Intuitively, capital complements skilled labour, raising its marginal product. Fertility and savings are, in turn, increasing in the skilled wage rate. Over time, both savings and fertility will rise. Proportionate to the increase in the efficient bought in child care input, fertility rises less than does savings, ensuring that capital per household accumulates: \( \phi'(k_t) > 0 \). From (18), increasing savings and increasing fertility implies capital per household accumulates at a decreasing rate. That is, \( \phi(k_t) \) is strictly concave, enabling convergence to a steady state equilibrium.

Thus, \( \phi(k_t) \) is increasing in \( A \), the initial state of technology. For the purposes of this paper, assume \( A \) is sufficiently high to ensure a unique steady state equilibrium, \( k^* \), describing a developed economy comprising skilled workers and a stationary fertility rate, \( f^* \). The behaviour of fertility in convergence to this equilibrium is now explored.

**Proposition 2** In transition to the high income steady state, \( k^* \), overall fertility decreases with growth in per capita income over the interval \((0, \bar{k})\), and increases with per capita income over the interval \((\bar{k}, k^*)\).

Further, \( \phi(k_t) \) will follow the path of the per unit child-rearing cost function, \( p(w_t^c, p_x, \beta) \) which, referring to (9), is increasing and concave in \( k_t \).
Proof. Refer to the Appendix.

Referring to Fig. 2, up until \( t \), the fraction of skilled workers rises with the accumulation of capital per household. Skilled workers have less children than unskilled workers. Up until \( t \), increasing weight given to the relatively low fertility of skilled workers reduces the overall fertility rate.

At time \( t \), a sufficiently high level of capital per household has been accumulated such that the workforce is skilled. Overall fertility is given by the fertility of skilled workers. From \( t \), the skilled wage rate rises with the accumulation of capital per household. As skilled wages rise, the opportunity cost of raising children rises, but so too does income. Substitution of bought-in child care for parental time mitigates some of the rising opportunity cost for skilled workers, so that the income effect dominates the substitution effect and a skilled worker chooses to have more children as the skilled wage rate rises. After \( t \), the overall fertility rate increases until it converges to the stationary rate, \( f^* \). This discussion is summarized with the following:

Remark 4. Ultimately, the fertility of skilled workers rises with increasing per capita income, for a constant real price of child care.

The model predicts not only a negative relationship between fertility and per capita income, observed as growth in per capita income takes off and the fraction of skilled workers rises, but also a positive relationship between fertility and income per capita in high income...
economies with skilled workforces. The prediction that fertility initially declines as the share of skilled workers rise is corroborated by the experience of developing China and Thailand, where average years of schooling have doubled to 8.5 years between 1980 and 2009 (Morrison and Murtin, 2009), and the total fertility rate has fallen to around 1.6 births per woman. The prediction that fertility may rise when all workers are skilled is consistent with the rise in completed fertility amongst tertiary educated parents in Denmark (Esping-Andersen et al., 2007) and Norway (Kravdal and Rindfuss, 2008), where family-friendly policies allow higher educated workers to choose to have more children.

Regarding the eventual rise in fertility, we have thus far held the price of child-rearing goods and services fixed. Referring to (12a), the fertility of skilled workers rises with real skilled wages, but decreases with the subsidized real price of child-rearing goods and services. Child-rearing goods and services encompass child care and other household commodities to rear children. *A priori*, if the rise in skilled wages leads to a higher real price of child care, the positive link between skilled wages and fertility of skilled workers is then broken. In the following section, the price of child care is endogenized in Phase 2 of the economy to explore a possible relationship between the rise in fertility and the structure of child care.\(^{11}\)

3. Child care sector and market

Section 2 analyses the decision making of a representative skilled household for a given price of child care in Phase 2. To endogenize the price of child care, this section introduces a child care sector and allows for intertemporal changes in \(p_x^t\) relative to the opportunity cost of parental time. Consider herein that \(p_x^t\) is given by equilibrium in the child care market, and how demand and supply affect the price of child care.

From eq. (7), the total cost of rearing children for a skilled household is

\[
C_t^s = w_t^s \hat{z} n_t + p_x^t (1 - \beta) \hat{x} n_t^s, \tag{25}
\]

where \(\hat{x}\) denotes child rearing time per \(n_t\), bought on the child care market for a price per unit of time of \(p_x^t\), subsidized at the rate \(\beta\). Since \(\hat{x}\) and \(\hat{z}\) are measured in the same unit of time, a household may reduce their child rearing time per child, \(\hat{z}\), by paying a subsidized fee of \((1 - \beta) p_x^t \hat{x}\).

Final goods are produced by \(Y_t = AK^a_t L_t^x (1 - z)^{-1}\), since all workers are skilled in Phase 2. Aggregate child care services, \(X_t\), are produced by \(X_t = \rho L_t^x\), where \(L_t^x\) is the total amount of labour employed in child care and \(\rho\) is a parameter that affects total productivity in the child care sector. The child care sector profit is \(\pi_t = p_x^t \rho L_t^x - w_t^s L_t^{\rho x}\). Under perfect competition, we obtain \(w_t^s = \rho p_x^t\).

The aggregate supply of child care services is

\[
X_t^s = \rho L_t^x = \rho [\omega N_t (1 - \hat{z} n_t^s)], \tag{26}
\]

where \(\omega\) denotes the fraction of skilled agents working in child care. Skilled wages in the child care and final goods sectors are linked by the arbitrage condition \(w_t^s = w_t^x = w_t^x (k_t)\).

\(^{11}\) Endogenizing the price of child care in Phase 1 would be a redundant exercise since equilibrium skilled wages are constant.
All skilled households demand parental time and child care to rear children. The aggregate demand for child care services is

$$X_t^D = \hat{x}n_t^sN_t,$$

where $N_t$ is the number of skilled agents.

The real price of child care services is determined so that demand coincides with supply. Equating (26) and (27), substituting for $\hat{x}n_t^s$ and $\hat{x}n_t^c$ from (8) and (12a) yields the subsidized market price of child care

$$p_t^s(1 - \beta) = \left[\frac{1 - a}{1 - a_\tau}\right]^{\gamma} w_t^s(k_t),$$

where $w_t^s(k_t) = A(1 - \alpha)[k_t/(1 - \tau)(1 - a\gamma)(1 - \omega)]^\gamma$. Here, $p_t^s$ changes over time in response to market demand and supply forces to satisfy (28). On the one hand, $p_t^s$ is increasing in the demand for child care, which is increasing in skilled wages, $w_t^s$, the rate of child care subsidy, $\beta$, the share of child care in child rearing, $(1 - a)$, and the preference weight for children, $\gamma$. On the other hand, $p_t^s$ is decreasing in the supply of child care, which is increasing in the fraction of workers in child care, $\omega$, and productivity in the sector, $\rho$.

In equilibrium, all markets, including child care and skilled labour, clear in every period. Substituting from eq. (28) for the market price of child care, the arbitrage condition $w_t^s(k_t) = \rho p_t^s$ is satisfied when $\omega = [(1 - \alpha)\gamma]/[(1 - a\gamma)(1 - \beta)]$, which implies that the equilibrium share of skilled labour in child care is increasing in the rate of child care subsidy, $\beta$. Once the sectoral allocation of labour stabilizes, in equilibrium, $p_t^s$ increases proportional to growth in $w_t^s(k_t)$. The fertility of skilled workers now remains constant.

The market for capital clears, $K_{t+1} = s_t^cN_t$. Referring to the appendix, in Phase 2 capital per household now evolves according to

$$k_{t+1} = \phi(k_t) = A\bar{\beta}\frac{(1 - \gamma - \lambda)}{\gamma}(1 - a)\frac{(1 - \alpha)}{(1 - a\gamma)(1 - \tau)^2} k_t^2 \quad \text{if} \quad k_t \geq \bar{k},$$

where $\bar{\beta} = a^{-\alpha}(1 - a)^{-(1 - \alpha)}\left(\frac{1 - a}{1 - a\gamma}\right)^{-\alpha}(1 - \omega)^{-\gamma}$ and $\phi(k_t)$ is increasing and concave in $k_t$. The behaviour of fertility over the interval $(\bar{k}, k^*)$ is explored when the price of child care is endogenously determined.

**Proposition 3** For an endogenous price of child care produced with constant returns to skilled labour, overall fertility does not increase with per capita income over the interval $(\bar{k}, k^*)$.

**Proof** Refer to the Appendix.

Referring to Fig. 3, the overall fertility rate no longer rises with per capita income after 7. In the high-income economy, individual demand for child care, given by $\hat{x}n_t^s$, is decreasing in the subsidized real price of child care, $p_t^s(1 - \beta)$, consistent with Guest and Parr (2013), and increasing in $w_t^s$, consistent with Myrskyla *et al.* (2009) and Luci and Thevenon (2011). As $w_t^s$ increases over time, demand for children increases for a given $p_t^s(1 - \beta)$. The market demand for child care shifts out, pushing up $p_t^s$. In equilibrium, the market price of child care increases proportional to skilled wages, so that $n_t^s$ remains constant.

The possibility that the overall fertility rate no longer rises after 7 under endogenous child care prices arises when the market price of child care increases proportional to skilled
wages. This in turn depends on two assumptions. First, the production of child care services features constant returns to labour. Second, the skill level of labour employed in child care reflects the average skill level in the economy. For the child care production function, \( X_t = \rho L^*_t \), a constant \( \rho \) reflects a constant average and marginal product of labour with an average skill level. Consider the possibilities arising from other modelling assumptions.

Referring to eq. (28), \( d \ln p^*_t \mid_{d\tilde{a}=d\tilde{w}=0} = d \ln \tilde{w}^*_t - d \ln \rho \), where \( d \ln p^*_t \geq d \ln \tilde{w}^*_t \) if \( d \ln \rho \leq 0 \). A fall (rise) in \( \rho \) captures decreasing (increasing) returns in child care production, since \( \partial X_t / \partial L^*_t = X_t / L^*_t = \rho \). Under increasing returns, the market price of child care rises proportionately less than skilled wages in equilibrium, so that individual demand for child care now rises as \( \tilde{w}^*_t \) rises after \( t \).

The parameter \( \rho \) also measures the productivity of labour employed in the child care sector, \( L^*_t \), that has the same skill level as labour employed in final goods, \( L^*_t \). All agents are skilled in Phase 2. However, we could intuit the implications of introducing a skill premium to labour in final goods relative to child care by adjusting \( \rho \). If child care required labour that was less skilled than that used in final production, then \( \rho \), the productivity of labour in child care that is as skilled as that used in final production, rises. A higher marginal product of same-skilled labour in child care would imply a lower marginal cost, thereby reducing the proportional rise in the market price of child care, so that individual demand for child care now rises after \( \bar{t} \).
This discussion is summarized with the following:

**Remark 5** A positive relationship between fertility and per capita income in a high-income economy is possible if the real price of child care increases less than proportionately to the average skilled wage, due to government subsidies or the structure of child care.

When the production of child care is linear in skilled labour, an increase in the rate of government subsidy, $\beta$, implies that the cost of child care rises proportionately less than skilled wages. Extending the child care production function to allow for increasing returns to skilled labour or the input of less skilled labour further explains that an increase in productivity of skilled labour, $\rho$, has the same effect. Some testable empirical predictions arise. The recent rise in fertility may not continue in high income countries where child care subsidies are low, child care features non-increasing returns to skilled labour, or where less skilled labour is in limited supply.

While the model in this paper does not provide a rationale for child care subsidies, it shows how such subsidies would distort fertility and the accumulation of capital that complements skilled labour. Consider the effects of subsidies under exogenous and endogenous child care prices. Referring to eq. (12a), subsidies raise fertility for exogenous child care prices by reducing the cost of rearing children. Referring to eq. (23), subsidies reduce $k_{t+1}$ accumulated at any given $k_t$. Intuitively, higher fertility has a capital dilution effect. Referring to eq. (19), the fraction of skilled workers falls with the steady-state level of capital per worker.

Under endogenous child care prices, subsidies raise fertility by increasing the share of labour in child care, $\omega$. Intuitively, subsidies increase the demand for child care, which offsets the effect of subsidies at the initial $\omega$. In equilibrium, $\omega$ increases, which increases the supply of child care and reduces the subsidized market price relative to skilled wages, thereby raising fertility. Referring to eq. (29), the increase in $\omega$ has an ambiguous effect on the equation of motion. Intuitively, $k_{t+1}$ is determined by the ratio of savings to fertility in period $t$. On the one hand, fertility increases with the share of labour in child care. On the other hand, savings increases with skilled wages as the share of labour in final goods, $(1 - \omega)$, falls.

**4. Conclusion**

The model of individual skill, fertility, child-rearing and child care market presented in this paper yields a rich dynamic interplay between aggregate fertility and economic growth as a high-income economy moves through two phases distinguished by the skill composition of the workforce. Within an economy, skilled workers are shown to have fewer children than low skilled workers. The analysis predicts that:

1. Depending on the availability of a child-rearing alternative to parental time, fertility of skilled workers increases with skilled wages.
2. As an economy grows, overall fertility follows a non-monotonic path. Fertility initially declines with rising skill intensity of the workforce; it then may recover with rising wages of a skilled workforce, for a constant real price of child care.

12 This paper does not engage in the normative debate over whether the government should subsidize child care. The existing literature explains that child care subsidies help alleviate the costs of population ageing by encouraging fertility (Guest and Parr, 2013) and help avoid non-distortionary commodity taxation (Bastani et al., 2015).
3. However, a positive relationship between fertility and per capita income in a high-income economy depends on the structure of child care. If the production of child care is linear in skilled labour, the real price of child care increases proportionally with skilled wages, so that fertility remains constant as per capita income rises over time. However, if child care is produced with increasing returns to skilled labour, or using less skilled labour, then the real price of child care increases proportionally less than the average skilled wage, and fertility rises with per capita income over time.

These predictions are consistent with recent evidence that suggests increases in per capita income need not reverse fertility decline (Luci-Greulich and Thevenon, 2014). Some interesting policy implications arise for a high-income economy with a skilled workforce. The possibility of a fertility upturn, after a prolonged period of fertility decline, depends on real wages growing relative to the real cost of child care, suggesting a role for government policy to contain increases in the subsidized real price of child care where the production of child care is linear in skilled labour.

The model can be extended in several ways to explain more aspects of fertility decline and upturn. For instance, the rise in fertility of wealthier households is partly attributed to the increased fertility of older households who delayed childbearing (Guest and Parr, 2013). The analysis here complements this assessment. The decision to train to become a skilled worker affects the timing of births. Furthermore, the cost of education could be endogenized by modelling an education sector that employs skilled workers. Extending the choice set to include timing of fertility and specifying the details behind the production of education is an interesting and feasible direction for further research.

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References
Appendix

A skilled worker chooses fertility

\[ n^* = \frac{\gamma(1 - \tau)(w^*)^{1-a}}{p_s^{1-a}(1 - \beta)^{1-a} B} \]

and total time spent rearing children

\[ \tilde{z}n^* = \gamma(1 - \tau) \left( \frac{a/(1 - a)}{a/(1 - a)} \right)^{1-a} + \left( (1 - a)/a \right)^a \]

\[ \equiv \gamma(1 - \tau)a. \]

Substituting for \( p(u^*, \beta) = |u^{\prime}|^2 |p_s(1 - \beta)|^{1-a} B \) in (15), the skilled wage differential is given by

\[ \frac{w^\prime}{w^a} = (1 - \tau)^{1/(1-a)}. \]
Equation of Motion (23)
Substituting the second line of (22) in (21) and noting that \( u_t' = Ab \) yields

\[
k_{t+1} = \frac{1 - \gamma}{\gamma} A^a B[p_x(1 - \beta)]^{1-a} \left[ b + \left\{ \frac{a^{1-a}(1-\gamma)\gamma}{(1-a)^2} \right\} \right] k_t.
\]

Further substituting for \( \theta^{-\gamma} \), using (19), and manipulating yields (23).

Proof of Proposition 1
Differentiating (23) and using (19) gives

\[
\phi'(k_t) = A^a B \frac{(1 - \gamma - \lambda)}{\gamma} \left\{ \begin{array}{l}
ax \left[ \frac{(1 - \alpha)(1 - \beta)}{(1 - a \gamma)^2(1 - \tau)^2} \right] k_{t}^{a-1} > 0 \quad \text{if } k_t \geq \bar{k} \\
b^a \theta \left[ \frac{-a \gamma}{(1 - \tau)(1 - a \gamma)} - \frac{a(1 - \gamma)}{(1 - \tau)(1 - a \gamma)} \right] \left[ \frac{a(1 - \gamma)}{1 - \theta \left(1 - (1 - \tau)(1 - a \gamma)\right) k_t} \right] > 0 \quad \text{if } k_t < \bar{k}.
\end{array} \right.
\]

Further differentiating, we obtain

\[
\phi''(k_t) = A^a B \frac{(1 - \gamma - \lambda)}{\gamma} \left\{ \begin{array}{l}
-(1 - ax)ax \left[ \frac{(1 - \alpha)(1 - \beta)}{(1 - a \gamma)^2(1 - \tau)^2} \right] k_{t}^{a-2} < 0 \quad \text{if } k_t \geq \bar{k} \\
b^a 2b^2 \left[ 1 - (1 - \tau)(1 - a \gamma) \right] \left[ \frac{a(1 - \gamma)}{1 - \theta \left(1 - (1 - \tau)(1 - a \gamma)\right) k_t} \right] > 0 \quad \text{if } k_t < \bar{k}.
\end{array} \right.
\]

Proof of Proposition 2
From (12a), (14a) and (17)

\[
f_t = \left\{ \begin{array}{ll}
\gamma \left[ \frac{(1 - \tau)(u_t')^{1-a}/[p_x(1 - \beta)]^{1-a} B}{[p_x(1 - \beta)]^{1-a} B} \right] \left[ \phi_t(1 - \tau)(u_t')^{1-a} + (1 - \phi_t)(1 - \tau)(u_t'^a) \right] & \text{if } k_t \geq \bar{k} \\
\gamma \left[ \frac{(1 - \tau)(u_t')^{1-a}/[p_x(1 - \beta)]^{1-a} B}{[p_x(1 - \beta)]^{1-a} B} \right] \left[ \phi_t(1 - \tau)(u_t')^{1-a} + (1 - \phi_t)(1 - \tau)(u_t'^a) \right] & \text{if } k_t < \bar{k}.
\end{array} \right.
\]
where, substituting for \( w_t \) from (22), the average fertility rate, \( f_n \), is expressed as a function of \( k_t \),

\[
f(k_t) = \begin{cases} 
\gamma \left[ \frac{Ab}{p_x(1-\beta)} \right]^{1-a} \left[ 1 - \theta \left( 1 - (1-\tau) \left( 1 - a' \right) \right) \right] k_t & \text{if } k_t < \bar{k} \\
\frac{\gamma(1-\tau)^{1-x(1-a)}}{B} \left[ \frac{A(1-\beta)}{p_x(1-\beta)(1-a')} \right]^{1-a} k_t^{1-a} & \text{if } k_t \geq \bar{k}
\end{cases}
\]

where

\[
f'(k_t) = -\gamma \left[ \frac{Ab}{p_x(1-\beta)} \right]^{1-a} \theta \left( 1 - (1-\tau) \left( 1 - a' \right) \right) < 0; \quad f''(k_t) = 0 \text{ if } k_t < \bar{k}
\]

\[
f'(k_t) = \alpha(1 - a) \frac{f(k_t)}{k_t} > 0; \quad f''(k_t) = [\alpha(1 - a) - 1] \frac{f'(k_t)}{k_t} < 0 \text{ if } k_t \geq \bar{k}
\]

**Equation of Motion (29)**

Rewriting \( K_{t+1} = s_t N_t \) in terms of capital stock per skilled working-age household,

\[
k_{t+1} = \frac{K_{t+1}}{N_{t+1}} = \frac{s_t}{n_t} \text{ if } k_t \geq \bar{k}.
\]

Substituting in the first line of (21) for \( w_t = A(1-\alpha)[k_t/(1-\tau)(1-a')(1-\omega)]^\gamma \) and \( p_x(1-\beta) \) from eq. (28) gives the equation of motion

\[
k_{t+1} = AB \left[ \frac{1 - \gamma \lambda}{\gamma} \right] \left[ \frac{(1 - a)(1 - \gamma)}{(1 - a')(1 - \omega)} \right]^{1-a} \left[ \frac{(1 - a)(1 - \gamma)}{(1 - a')(1 - \omega)^\gamma} \right] k_t^\gamma \text{ if } k_t \geq \bar{k},
\]

which simplifies to

\[
k_{t+1} = \phi(k_t) = AB \left[ \frac{1 - \gamma \lambda}{\gamma} \right] \left[ \frac{(1 - a)(1 - \gamma)}{(1 - a')(1 - \omega)^\gamma} \right] k_t^\gamma \text{ if } k_t \geq \bar{k},
\]

where \( \bar{B} = a^{-a}(1-a)^{-(1-a)} \left[ \frac{(1-a)(1-\gamma)}{(1-a')(1-\omega)^\gamma} \right] \left[ \frac{(1-\gamma)}{(1-\omega)^\gamma} \right] \left[ \frac{(1-\gamma)}{(1-\omega)^\gamma} \right] \). Substituting for \( \omega = \frac{(1-a)^\gamma}{(1-a')(1-\beta)} \),

\[
B = a^{-a}(1-a)^{-(1-a)} \left[ \frac{(1-a)(1-\gamma)}{(1-a')(1-\omega)^\gamma} \right] \left[ \frac{(1-\gamma)}{(1-\omega)^\gamma} \right] \left[ \frac{(1-\gamma)}{(1-\omega)^\gamma} \right].
\]

Differentiating (29) gives

\[
\phi'(k_t) = \alpha A B \left[ \frac{1 - \gamma \lambda}{\gamma} \right] \left[ \frac{(1 - a)(1 - \gamma)}{(1 - a')(1 - \omega)^\gamma} \right] k_t^{\gamma-1} > 0
\]

\[
\phi''(k_t) = -(1 - a) \alpha A B \left[ \frac{1 - \gamma \lambda}{\gamma} \right] \left[ \frac{(1 - a)(1 - \gamma)}{(1 - a')(1 - \omega)^\gamma} \right] k_t^{\gamma-2} < 0.
\]

**Proof of Proposition 3**

Recognizing that \( f_t = n_t \) if \( k_t \geq \bar{k} \), and substituting for \( p_x \) from eq. (28),

\[
f_t = \frac{\gamma(1 - \tau)}{B} \left[ \frac{(1 - a')(1 - \omega)}{(1 - a')^\gamma} \right]^{1-a}.
\]
Further substituting for $\omega = \frac{(1-a)_c}{(1-a_c)(1-\bar{b})}$ implies

$$f_t = \frac{\gamma(1 - \tau)}{B} \left( \frac{\rho}{(1 - \bar{b})} \right)^{1-a},$$

which is independent of $k_t$. 